Aug 22th. 2024

Quark deconfinement in neutron stars by Color-Molecular-Dynamics

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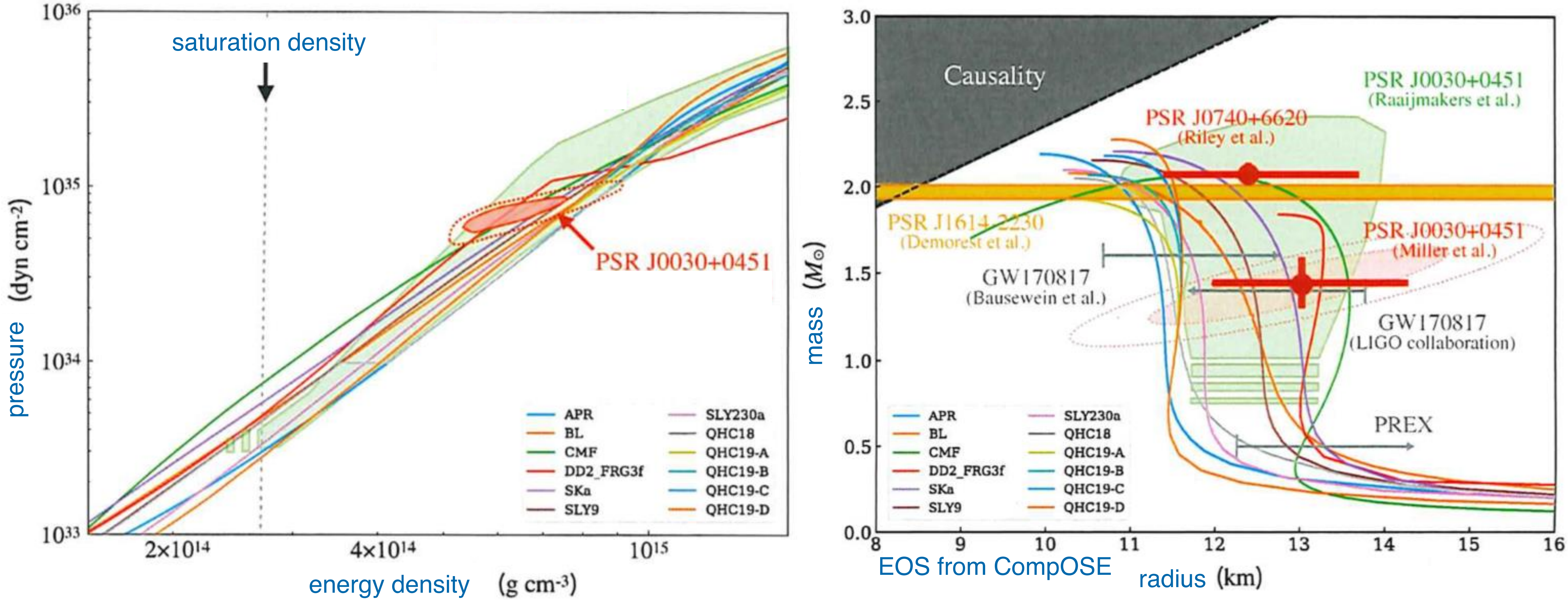
PRD 109. 043056 (2024)

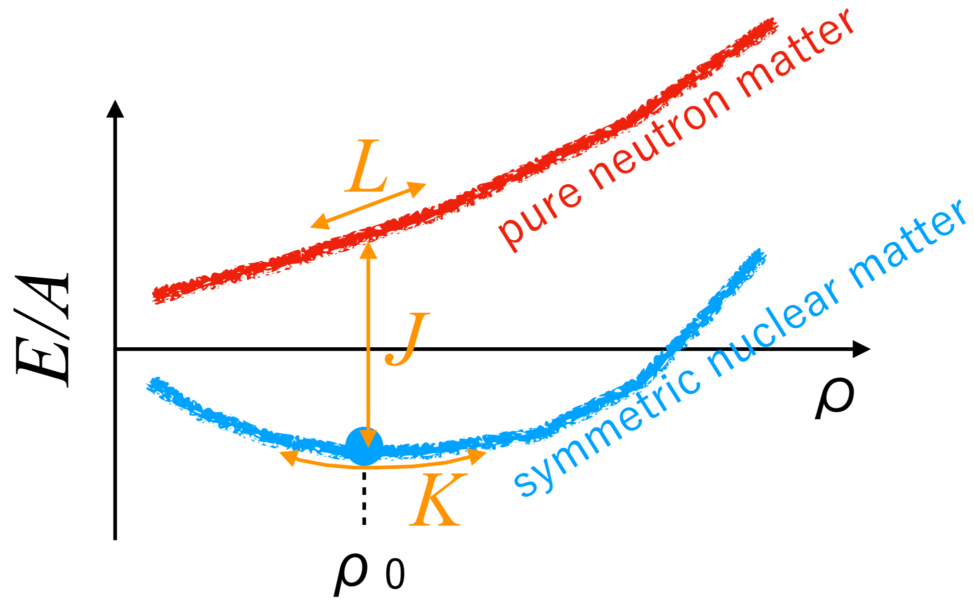
EOS and M-R relation

Enoto & **NY** (2021) Oct. Journal of Japanese Physical Society



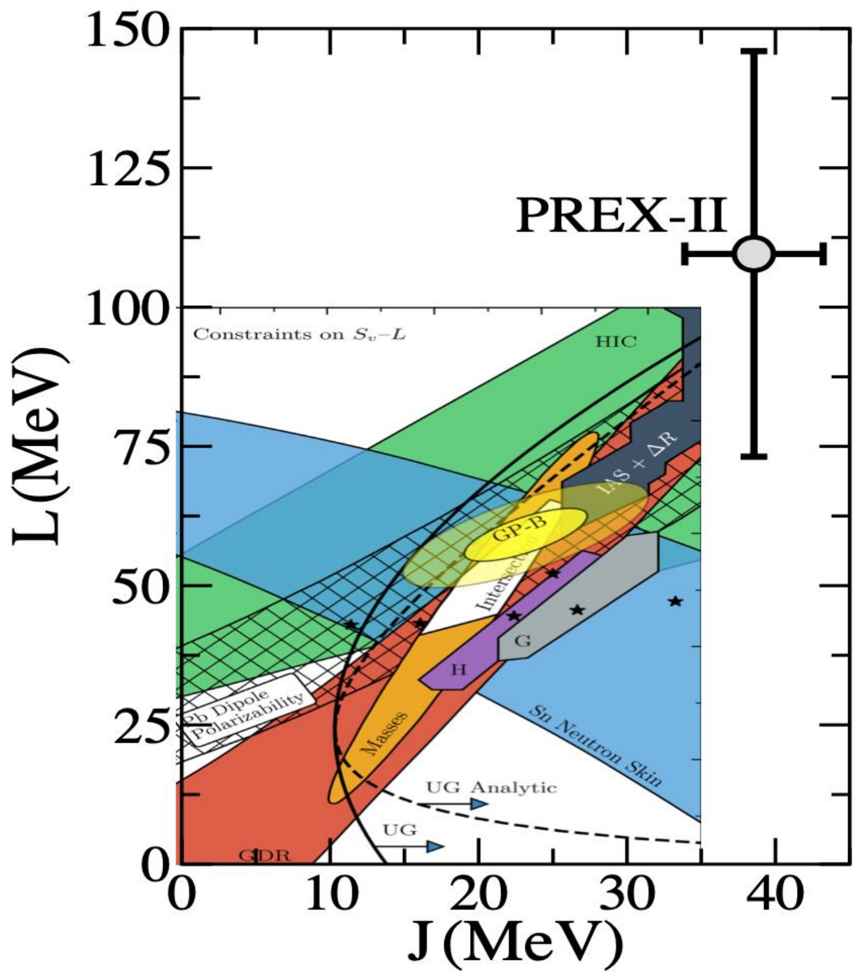
EOS Constraints
around saturation density

$$E/A = S + \frac{L(\rho-\rho_0)}{3\rho_0} + \frac{K_{sym}(\rho-\rho_0)^2}{18\rho_0} \ldots$$



$$E/A = S_0 + \frac{K(\rho-\rho_0)^2}{18\rho_0^2} \ldots$$

Reed et al. PRL 126,172503 (2021)



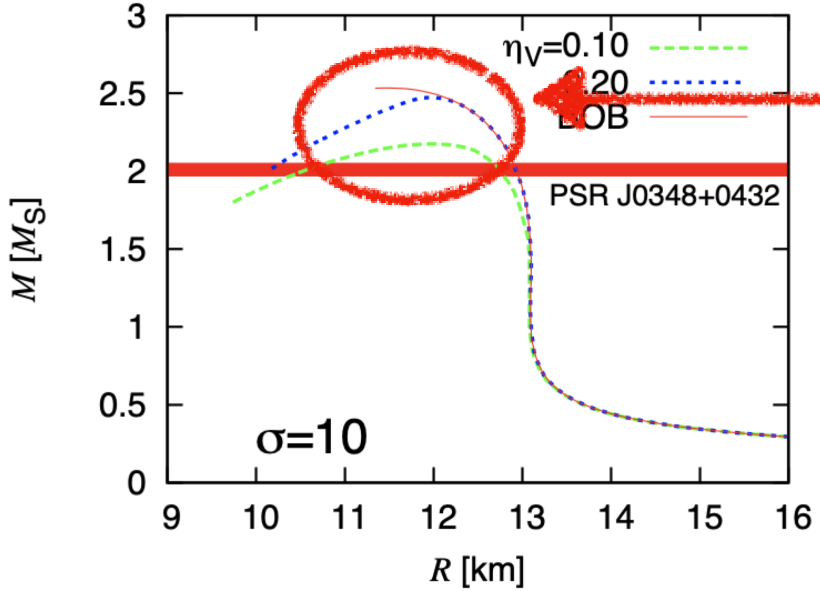
- The result of PREX-II depends on the analysis methods. (Reinhard et al. PRL 127.232501(2021))
- **CREX** results suggest ordinary L values, not large as PREX-II. (Adhikari et al. PRL 129.042501(2022))

EOS from quarks to neutron stars

?

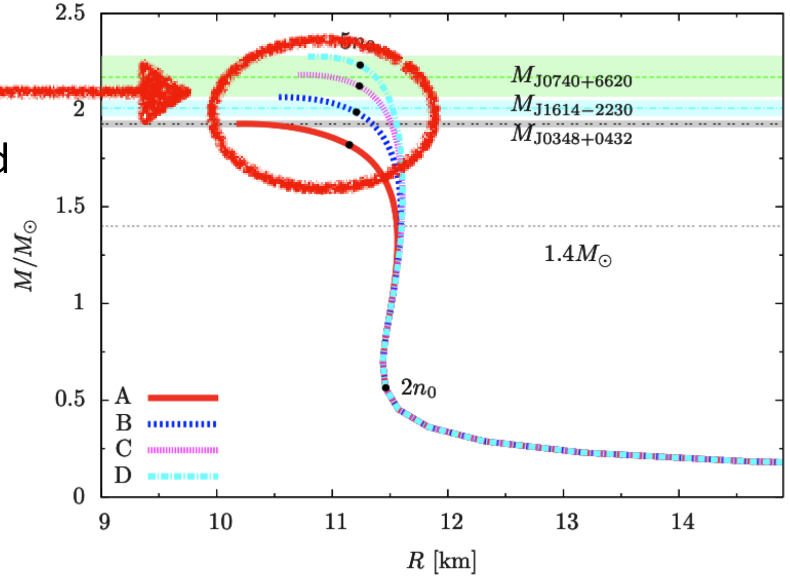
1st oder PT

cross over



N

overlapped



NY, et al. PRC 90, 067302 (2014)

G. Baym, et al. ApJ 885, 42 (2019)

nucleon(BHF model) + quark(eNJL model)

nucleon(Variational method) + quark(NJL model)

See also Xia, Maruyama, **NY**, Tatsumi, Shen, Togashi (2020), Phys. Rev. D 102, 023031

Problems

1. How much can we believe hadron models at high density region?
2. 1st oder or cross over?

→ It is dependent on the assumption (not results).

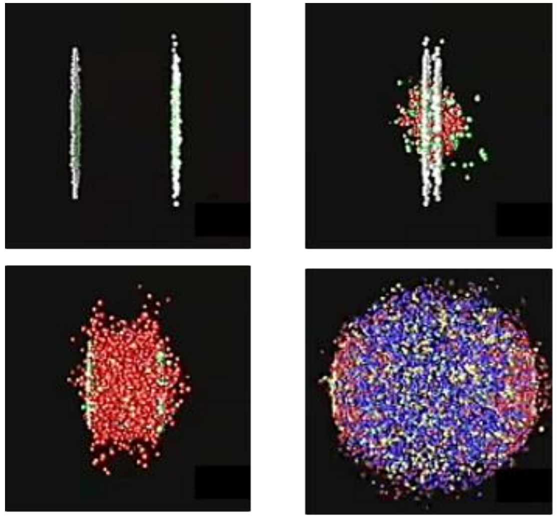
→ We can not get unified understanding.

cf.) We can not obtain the other physical properties for crossover.

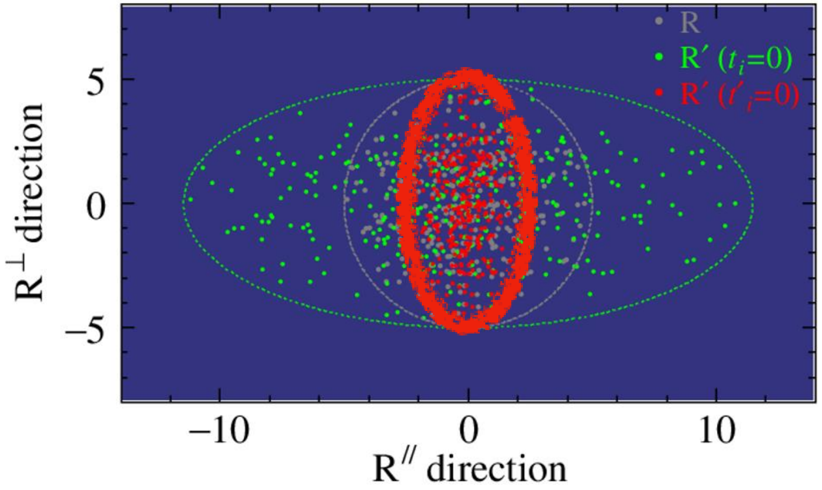
Molecular dynamics

Why molecular dynamics ?

① We focus on Quark-Molecular-Dynamics.
→We can describe EOS and NS physics only with quark system.
② MD enables us to know also dynamical behaviors, fluctuation, clustering.
→We can apply MD directly to Heavy Ion Collisions.



UrQMD for RHIC
https://www.bnl.gov/rhic/physics.php



Relativistic color molecular dynamics
(on going project)

Formulation

NY, Maruyama (2024) PRD
Maruyama, Hatsuda (2000) PRD

number of variables

$$\{\boxed{x, y, z}, \boxed{Px, Py, Pz}, \alpha, \beta, \theta, \varphi\}_i$$

10 variables on each particle

time evolution

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} \qquad \mathcal{L} = \left\langle \Psi \middle| i\hbar\frac{d}{dt} - \hat{H} \middle| \Psi \right\rangle$$

cooling

$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$
$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

wave functions

$$\Psi = \prod_{i=1}^{3A} \phi_i(\mathbf{r})\chi_i \qquad \chi_i = f_i s_i c_i$$

f_i ... flavors (fixed)

s_i ... spins (fixed. But unfixed later section)↑or↓

$$c_i \equiv \begin{pmatrix} \cos\alpha_i e^{-i\beta_i}\cos\theta_i \\ \sin\alpha_i e^{+i\beta_i}\cos\theta_i \\ \sin\theta_i e^{i\varphi_i} \end{pmatrix}$$

R
G
B

$$\phi_i(\mathbf{r}) \equiv (\pi L^2)^{-\frac{3}{4}} \exp[-(\mathbf{r}-\mathbf{R}_i)^2/2L^2 - i\mathbf{P}_i\mathbf{r}]$$

confinement conditions

$$\begin{cases} |\mathbf{R}_i - \mathbf{R}_j| < d_{cluster} \quad (i, j = 1, 2, 3) \\ \sum_{a=1}^{8}\left[\sum_{i=1}^{3} \langle \chi_i \,|\, \lambda^a \,|\, \chi_i \rangle\right]^2 < \varepsilon \end{cases}$$

λ^a being the Gell-Mann matrices

Interactions

The system follows the Hamiltonian,

$$H = H_0 + V_{\text{Pauli}} - T_{\text{spur}},$$

where H_0 is the conventional Hamiltonian expressed as

$$H_0 = \left\langle \Psi \left| \sum_{i=1}^{N} \sqrt{m + \hat{\mathbf{p}}_i^2} + \frac{1}{2} \sum_{i,j\neq i}^{N} \hat{V}_{ij} \right| \Psi \right\rangle .$$

$$\hat{V}_{i,j} = -\sum_{a=1}^{8} \tau_i^a \tau_j^a \hat{V}_{\text{C}} + \hat{V}_{\text{M}},$$

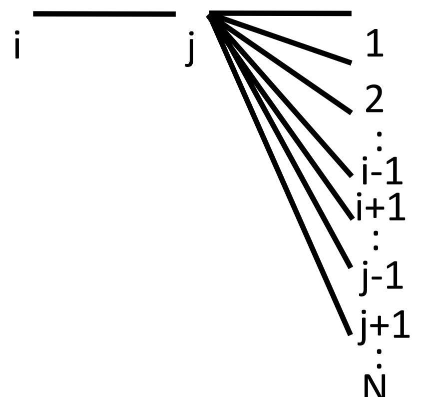
$\tau_i^a = \lambda_i^c/2$ with λ_i^c being the Gell-Mann matrices.

$$\hat{V}_{\text{C}} = \boxed{\kappa \hat{r}_{ij}} - \frac{\alpha_s}{\hat{r}_{ij}},$$

quark many body effects

$$\hat{V}_{\text{M}} = \frac{g_\omega^2 C_\omega}{4\pi} \left(\sum_{j\neq i}^{n} \frac{e^{-\mu_\omega \hat{r}_{ij}}}{\hat{r}_{ij}} \right)^{\boxed{1+\epsilon_\omega}} - \frac{g_\sigma^2 C_\sigma}{4\pi} \left(\sum_{j\neq i}^{n} \frac{e^{-\mu_\sigma \hat{r}_{ij}}}{\hat{r}_{ij}} \right)^{\boxed{1+\epsilon_\sigma}}$$

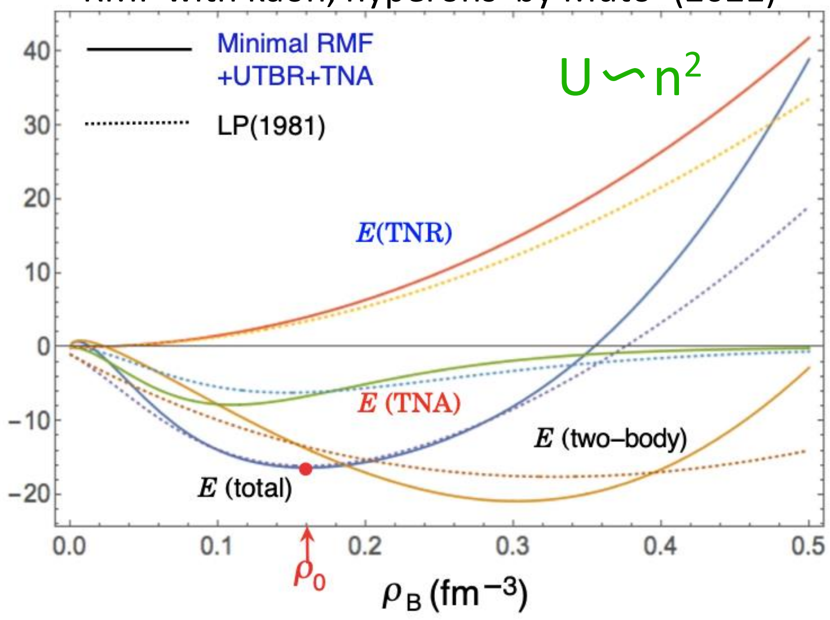
$$+ \frac{\sigma_i^3 \sigma_j^3}{4} \frac{g_\rho^2}{4\pi} \frac{e^{-\mu_\rho \hat{r}_{ij}}}{\hat{r}_{ij}}$$



Many-body effects on EOSs

$n^2 \backsim (1/r)^6$

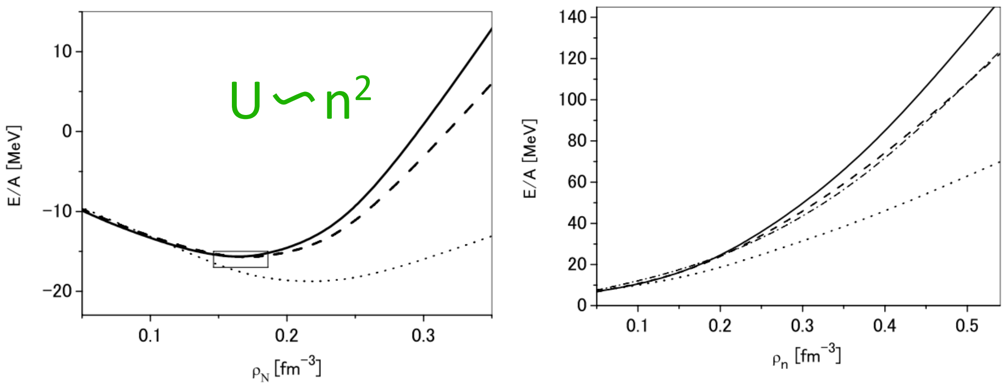
RMF with kaon, hyperons by Muto+(2021)



$$U_{\mathrm{SJM2}}(r;\rho_{\mathrm{B}}) = V_r\rho_{\mathrm{B}}(1 + c_r\rho_{\mathrm{B}}/\rho_0)\exp[-(r/\lambda_r)^2)]$$

introduced to be consistent with MR relations.

BHF with hyperons by Yamamoto, **NY,** Rijiken (2022)



$$V_{eff}^{(N)}(\mathbf{x}_1,\mathbf{x}_2) = \rho_{NM}^{N-2}\int d^3x_3 ... \int d^3x_N\ V(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_N)$$

$$= g_P^{(N)} g_P^N \frac{\rho_{NM}^{N-2}}{\mathcal{M}^{3N-4}} \frac{1}{\pi\sqrt{\pi}} \left(\frac{m_P}{\sqrt{2}}\right)^3 \exp\left(-\frac{1}{2}m_P^2 r_{12}^2\right), (3)$$

N=3,4

introduced to be consistent with MR relations

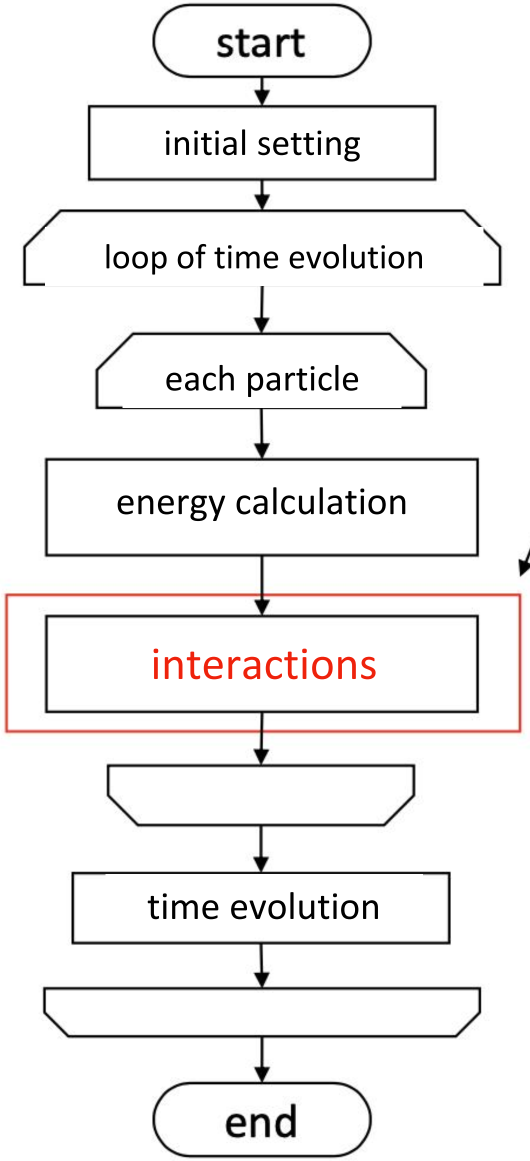
and/or cross sections of nuclei.

These examples are adopted to hadron interactions.
Why not to quark-quark interactions?

$Uqqq \backsim (1/r)^{1+\varepsilon}$

Parallel computings in MD

No money(GPUs), No study….



- The main part of calculations (〜90%).
  → The main target to be improved.

- Super computer in JAEA
- parallel computings

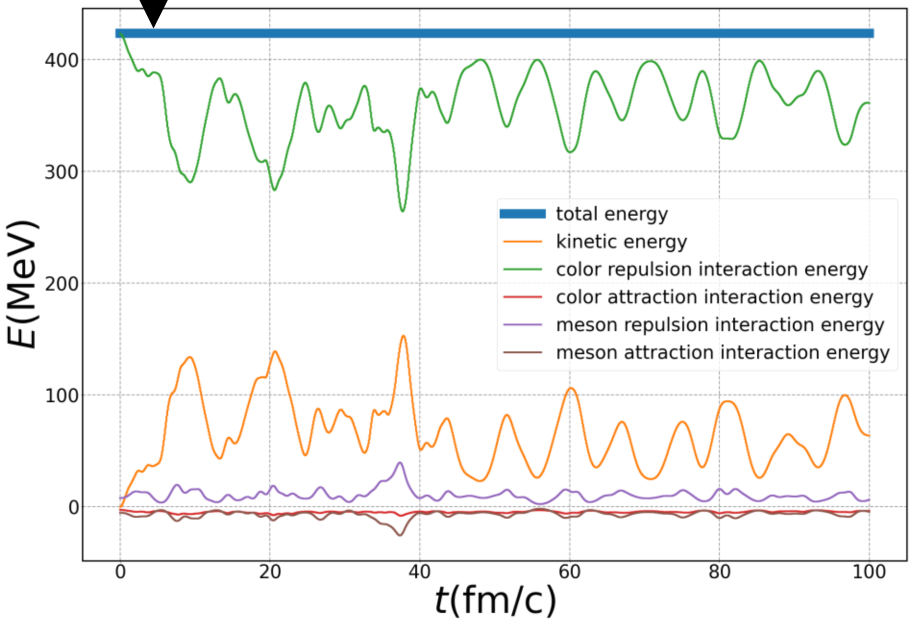
Device : HPE SGI 8600 in JAEA
CPU : Intel Xeon Gold 6428R
(3.0GHz, 35.75MB cache)X 2CPU
Core number per node : 48
GPU : NVIDIA Tesla V100 SX2 32GB memory
(FP64/GPU : 2560)x4GPU
Memory per node : 384 GB

Energy conservation

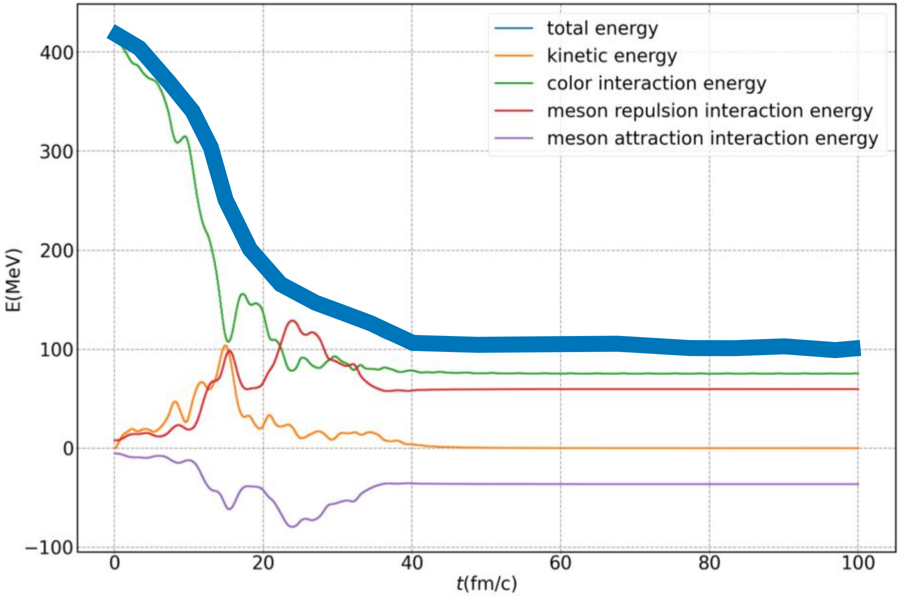
as an accuracy check

total energy

without dumping (cooling) terms



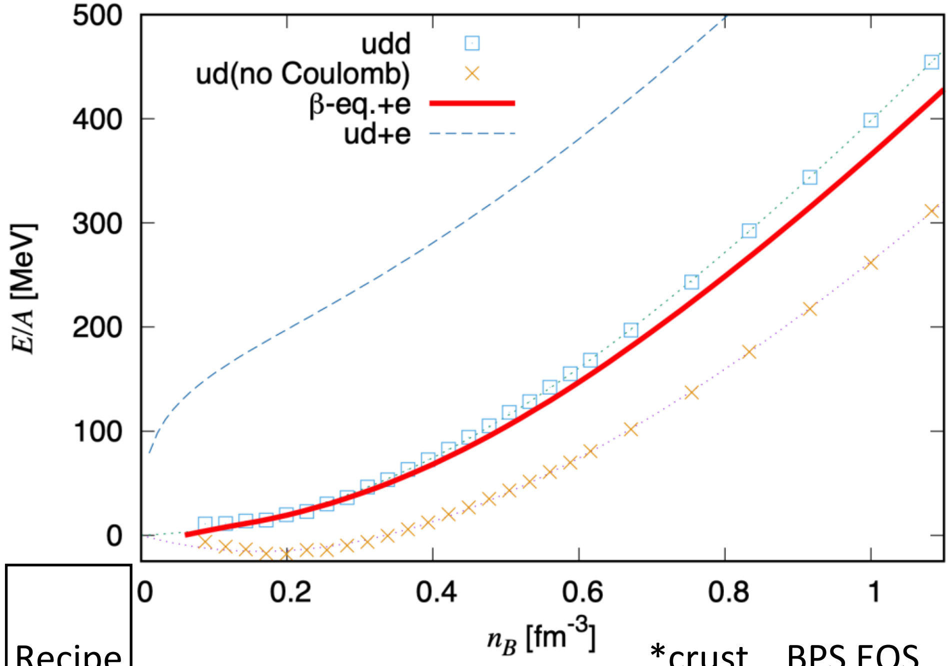
including dumping (cooling) terms

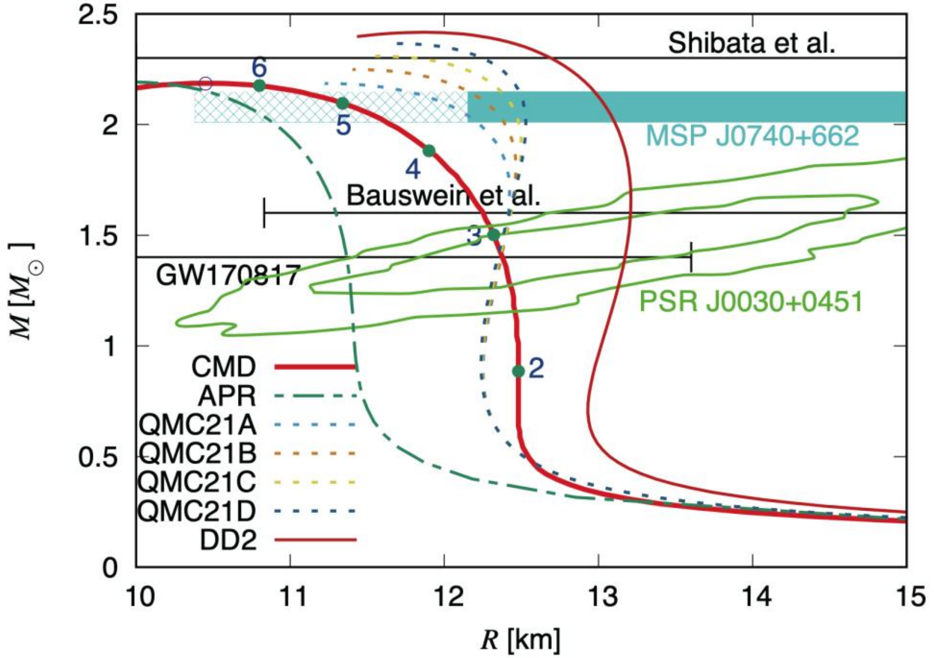


$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$

$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

EOS by Color-Molecular-dynamics





Recipe

*crust …BPS EOS

try next parameter

① We derive fitting formulae from numerical data points. (~ 1day for 1 parameter set)

② We obtain EOS using the formulae under beta equilibrium and charge neutrality conditions.

beta eq.

$$\mu_u + \mu_e = \mu_d \,,$$

$$\mu_p + \mu_e = \mu_n = \mu_u + 2\mu_d \,.$$

charge neutrality

$$e n_{\rm ch}(\boldsymbol{r}) = \sum_{q=u,d,e} Q_q n_q(\boldsymbol{r})$$

$$e n_{\rm ch}(\boldsymbol{r}) = \sum_{i=n,p,e} Q_i n_i(\boldsymbol{r})$$

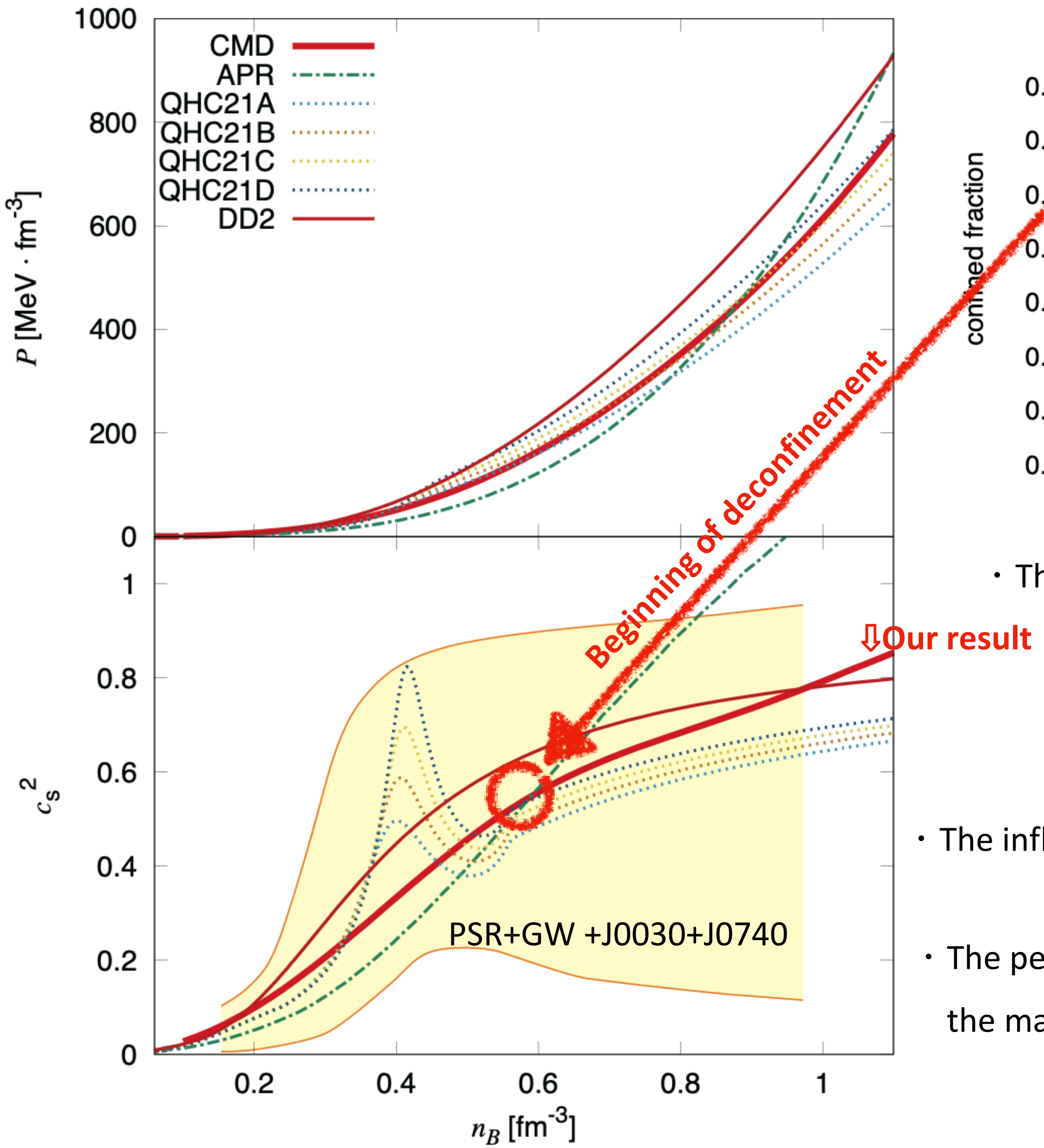
③ We check whether the EOSs are consistent with astrophysical constraints, and the nuclear experiments result.

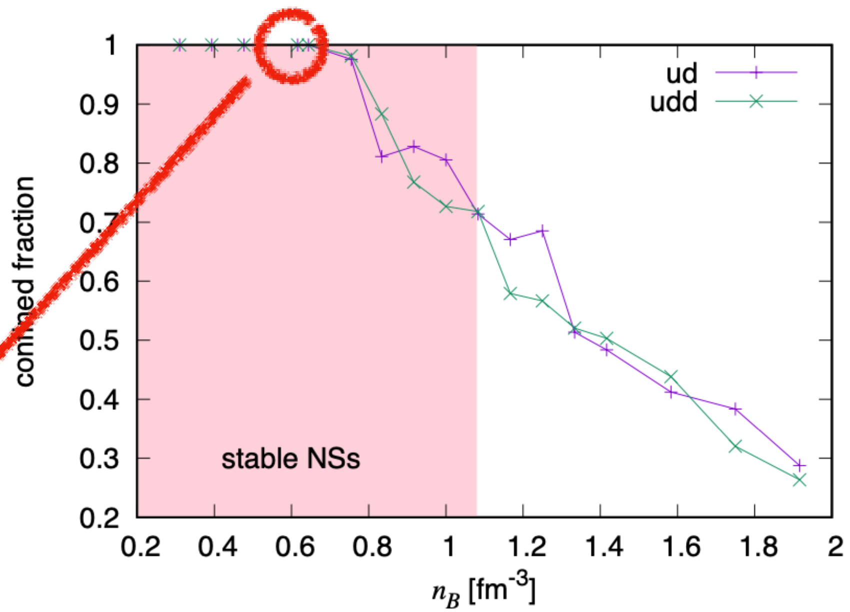
cf.) the above result

n_0=0.167fm^{-3}, S_0=-15.8 MeV, J=31.0MeV, L=74.2MeV, K =260 MeV, M_{max}=2.19 Ms, $\Lambda_{1.4}$=458, nc=1.08 fm^{-3}

ref) Danielewicz et al., Science 298 (02) 1592: K = 167-300 MeV

Pressure & Sound velocity

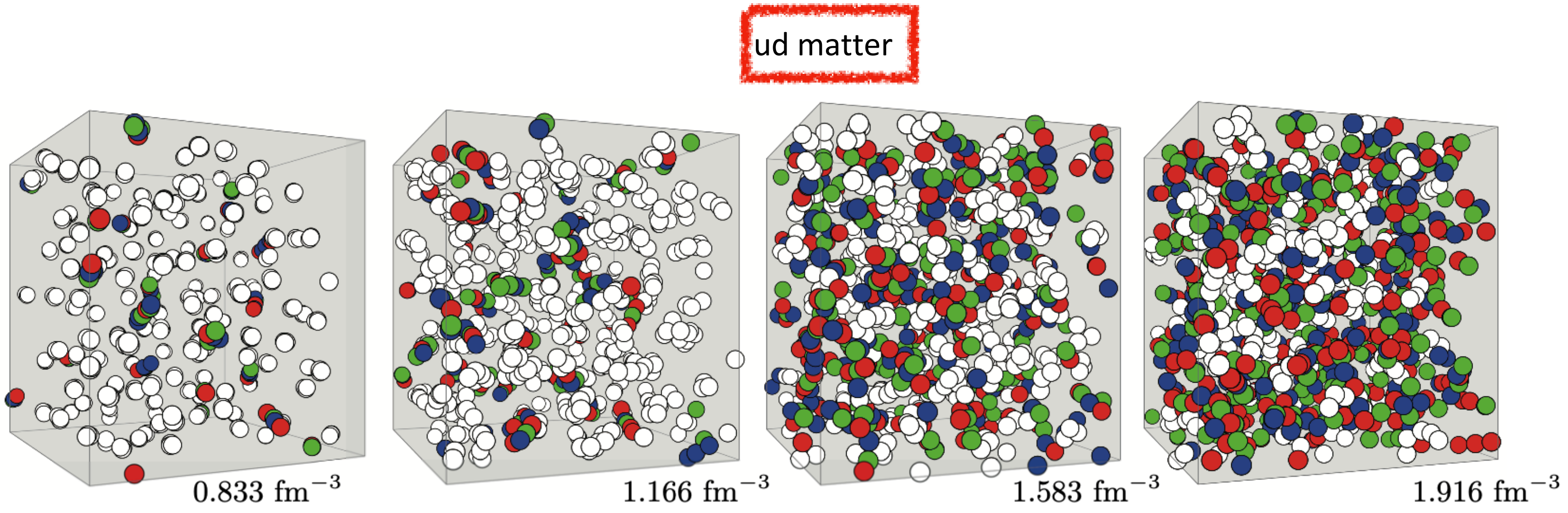




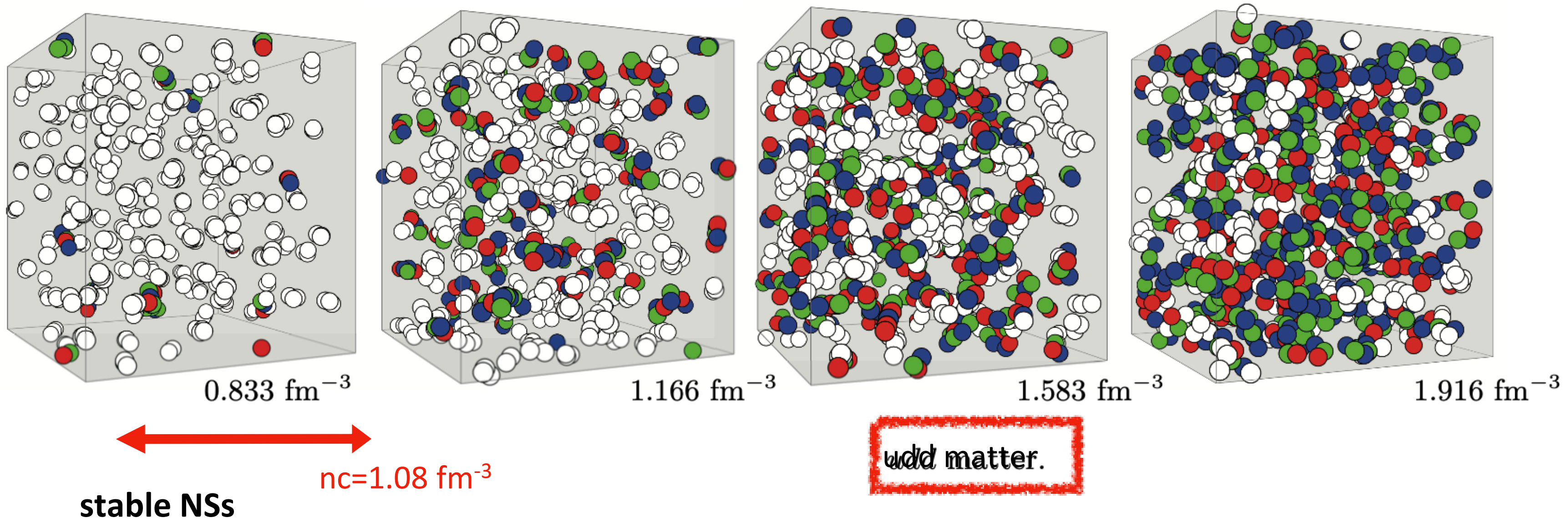
・The main component of NSs are hadron matter.

・The inflection point is the quark deconfinement origin.

・The peak of sound speed appears higher density than the maximum density of NSs in our models.

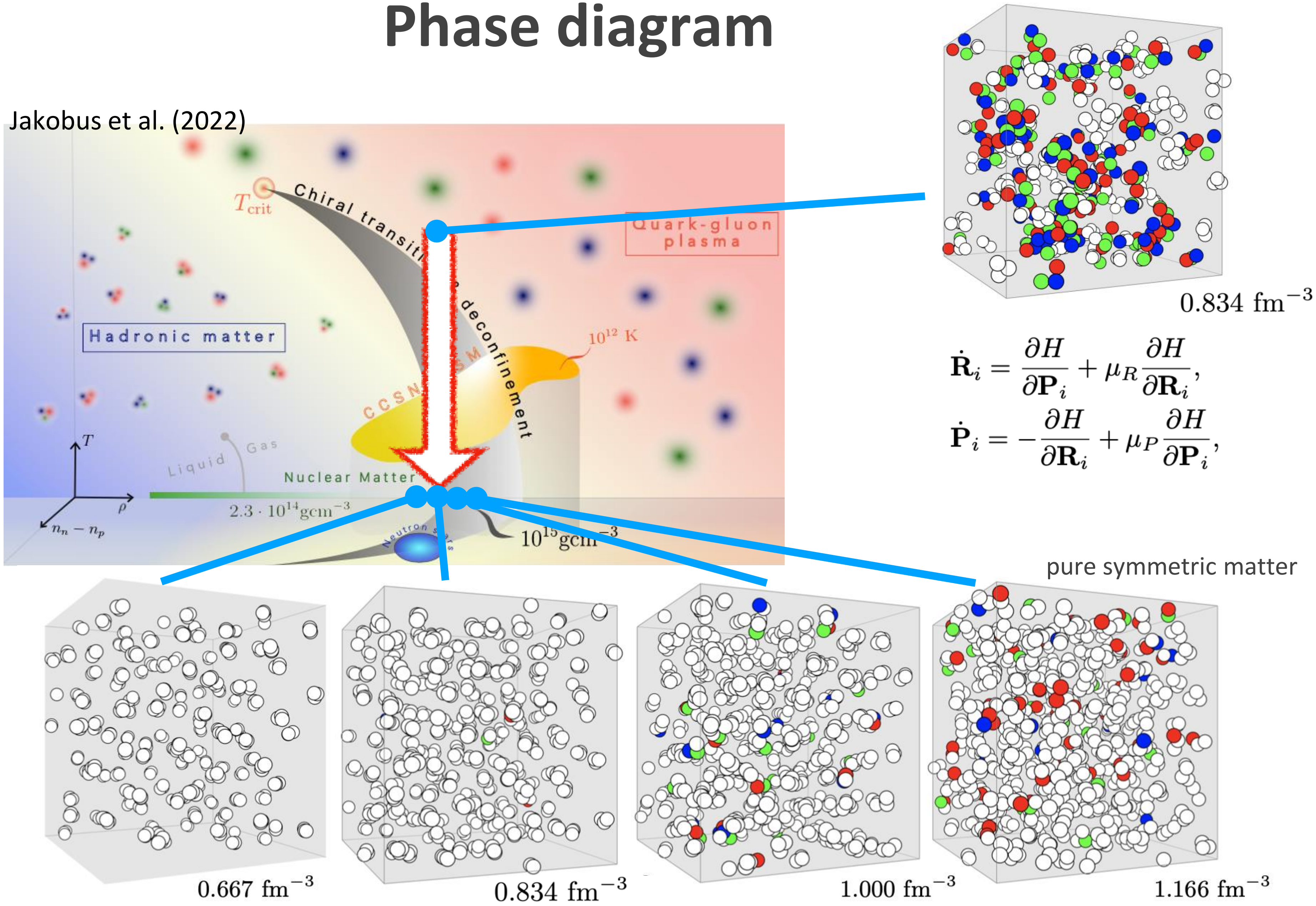


The perspectives for *ud* matter depended on density. Each color corresponds to the color's internal degree of freedom for each quark: the white color balls represent the quarks in the baryon state, while red, blue, and green ones do in the deconfined quark matter.



Phase diagram

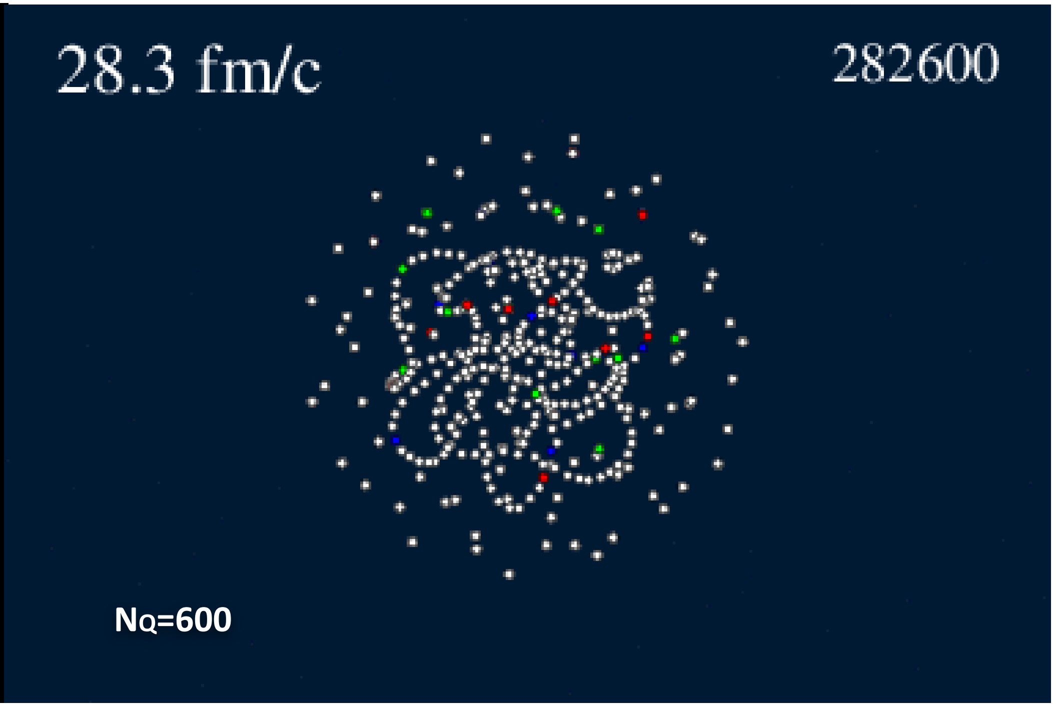
Jakobus et al. (2022)



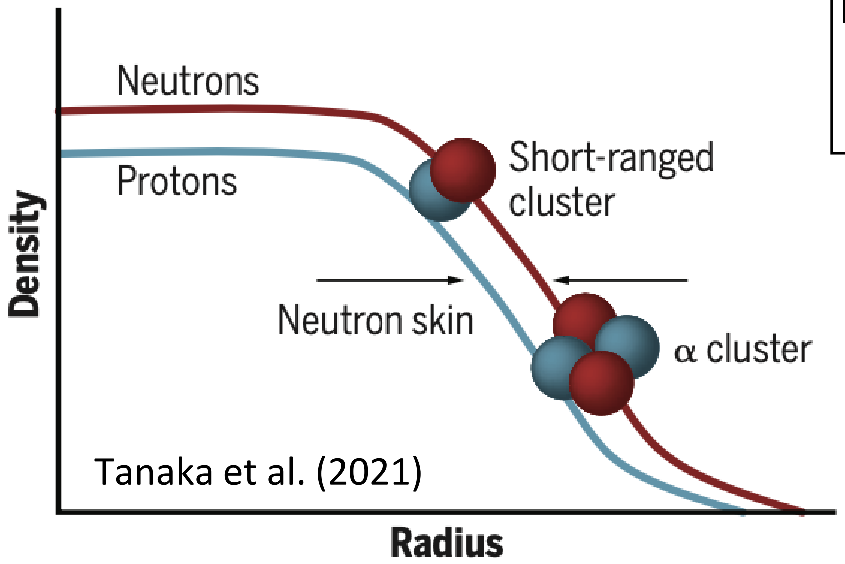
$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$

$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

Finite system consistent with finite system



Nucleon density in neutron-rich nuclei



RMF + alpha cluster(local correlation) neutron skin change 10-40%

"DD2" Typel, Ropke, Klahn, Blaschke, Wolter, PRC. 81, 015803 (2010).

Typel, PRC.89,064321(2014).

Theoretical cross sections (nb)	Effective number of α clusters	Theoretical Δr_{np} without α (fm)	Theoretical Δr_{np} with α (fm)	Relative Δr_{np} change (%)
0.160	0.3876	0.0495	0.0277	−44
0.127	0.3304	0.0843	0.0580	−31
0.095	0.2668	0.1179	0.0912	−23
0.065	0.1958	0.1505	0.1275	−15

Color magnetic interaction and baryon mass

interactions with colors

$$\hat{V}_{\text{color}} = \frac{1}{2}\sum_{i=1,j\neq i}^{n}\left(-\sum_{a=1}^{8}\frac{\lambda_i^a\lambda_j^a}{4}\left(K\hat{r}_{ij} - \frac{\alpha_s}{\hat{r}_{ij}}\right)\right)$$

↑or↓

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2}\frac{1}{r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\lambda_i^c\lambda_j^c\,\sigma_i\cdot\sigma_j$$

$$r_{0ij} = (\alpha + \beta\mu_{ij})^{-1}, \quad \mu_{ij} = m_i m_j/(m_i + m_j)$$

Aaron et al. EPJA 56,93(2020)

∽1/(2m)

αs	κ'	K	α	β	$m_{u,d}$	m_s	L
1.25	0.5	750 MeVfm^{-1}	2.1 fm^{-1}	0.552	0.362 GeV	0.538 GeV	0.346 fm

Akimura et al. EPJA 25,405(2005) etc.

Optimized in this study.

2-body spin correlations

$$<\frac{\sigma_i}{2}\cdot\frac{\sigma_j}{2}>=\frac{1}{4}(\uparrow\uparrow), -\frac{3}{4}(\uparrow\downarrow)$$

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N,P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.906	1.245	1.075	1.075	1.415	1.217	1.584
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

2-body spin correlations consistent with 3-body spin correlations

$$<\frac{\sigma_i}{2}\cdot\frac{\sigma_j}{2}>=\frac{1}{4}(\uparrow\uparrow), -\frac{1}{2}(\uparrow\downarrow)$$

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.938	1.233	1.115	1.177	1.379	1.328	1.531
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

others

quark-meson coupling

$$V_{\text{meson}}(r) \equiv -\frac{g_{\sigma q}^2}{4\pi}\frac{e^{-\mu_\sigma r_{ij}}}{r_{ij}} + \frac{g_{\omega q}^2}{4\pi}\frac{e^{-\mu_\omega r_{ij}}}{r_{ij}} + \frac{\sigma_i^3\sigma_j^3}{4}\frac{g_{\rho q}^2}{4\pi}\frac{e^{-\mu_\rho \hat{r}_{ij}}}{\hat{r}_{ij}}$$

pauli interaction

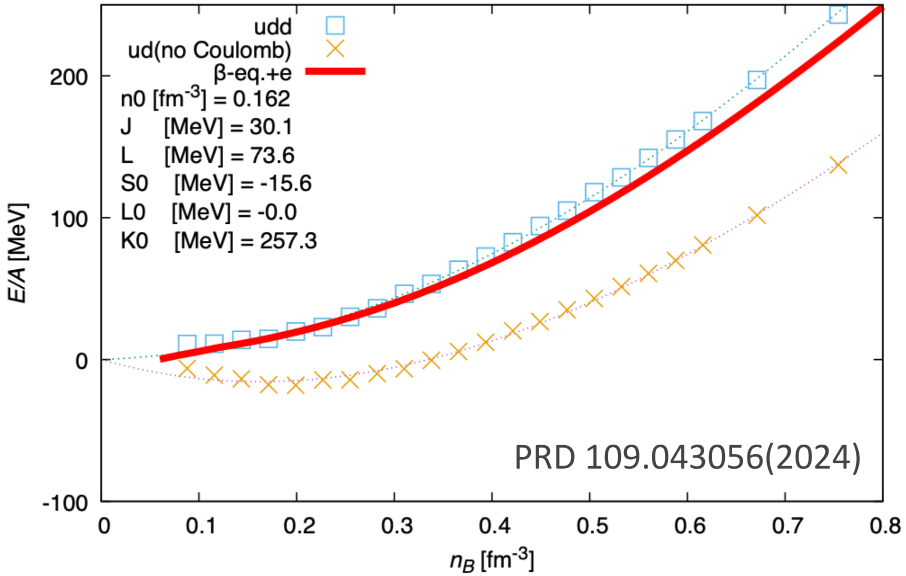
$$\langle V_{\text{pauli}}(r)\rangle \equiv \frac{C_p}{(q_0p_0)^3}\exp\left[-\frac{(\mathbf{R}_i-\mathbf{R}_j)^2}{2q_0^2} - \frac{(\mathbf{P}_i-\mathbf{P}_j)^2}{2p_0^2}\right]\delta_{\chi_i\chi_j}$$

Preliminary results

Color magnetic interaction and EOS

without color-magnetic interaction

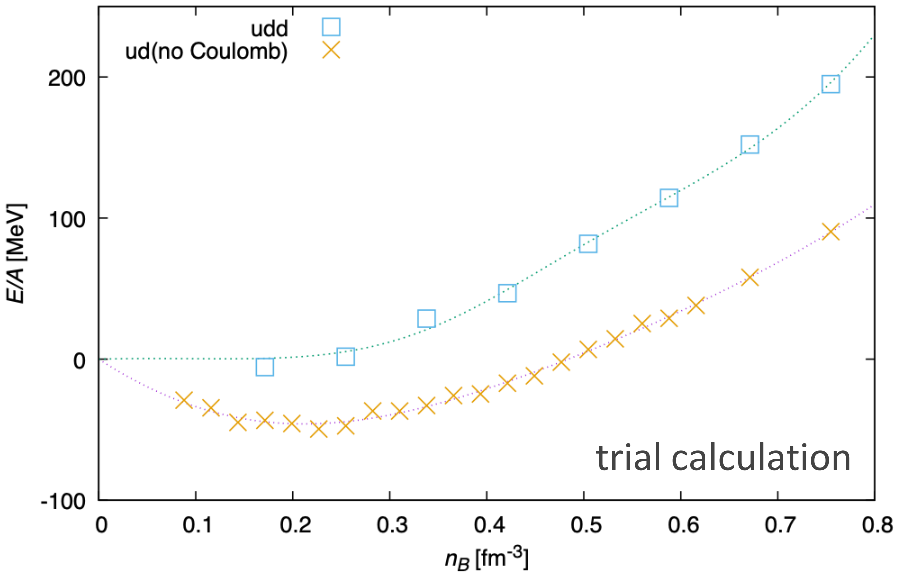
NY, Maruyama



PRD 109.043056(2024)

with color-magnetic interaction

NY, Maruyama, Park, Lee



trial calculation

$$s_i = \begin{pmatrix}1\\0\end{pmatrix} \text{ or } \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow s_i = \begin{pmatrix} e^{-i\beta_i^{(s)}} \cos\alpha_i^{(s)} \\ e^{i\beta_i^{(s)}} \sin\alpha_i^{(s)} \end{pmatrix}$$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \underbrace{\lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j}_{\text{time dependent}},$$

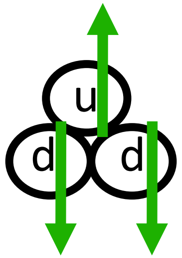
Preliminary results

Color magnetic interaction and spins

NY, Maruyama, Park, Lee

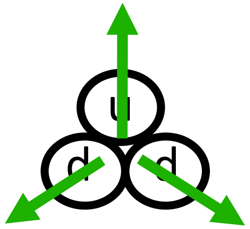
3-quark system

$$< \frac{\sigma_i}{2} \cdot \frac{\sigma_j}{2} >= \frac{1}{4}(\uparrow\uparrow), -\frac{3}{4}(\uparrow\downarrow)$$

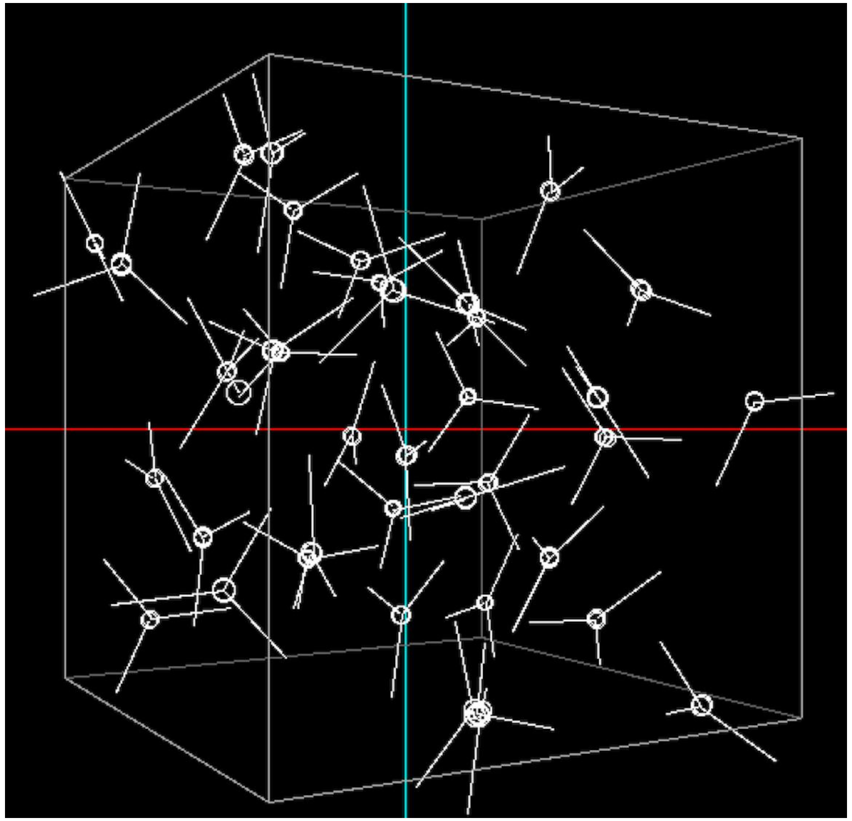


Large N-quark system

$$< \frac{\sigma_i}{2} \cdot \frac{\sigma_j}{2} >= \frac{1}{4}(\uparrow\uparrow), -\frac{1}{4}(\uparrow\downarrow)$$



Mercedes-Benz



Around saturation density
with periodic boundary (infinite system)

Physical properties by MD

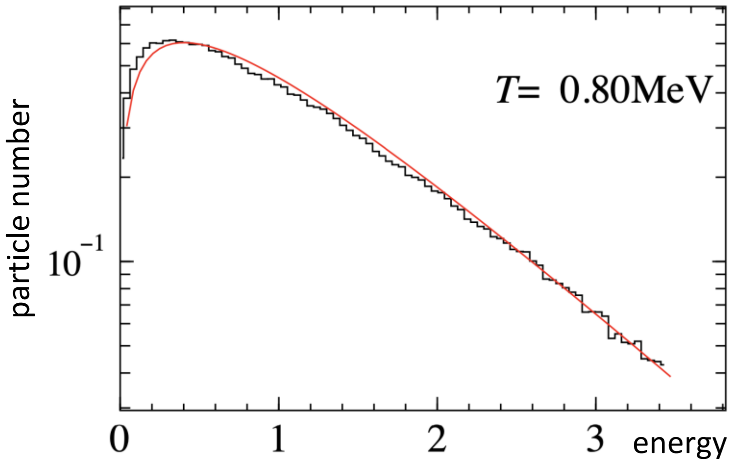
We can obtain physical quantities
by Green-Kubo formula.
(1st. oder of fluctuation)

$$\alpha = \int_0^\infty \left\langle \dot{U}(t)\dot{U}(0) \right\rangle dt.$$

e.g. thermal conductivity λ

$$\begin{aligned} \alpha &= VT^2\lambda, \\ U_x(t) &= \sum_{i=1}^{N} x_i(t)E_i(t), \\ \dot{U}_x(t) &= \frac{d}{dt}\sum_{i=1}^{N} x_i(t)E_i(t). \end{aligned}$$

Statistical temperature from CMD is
consistent with theoretical distribution.



→(Proto)Neutron Star coolings / evolutions

Evolution of (Proto)Neutron Stars

Neutron Star evolution:

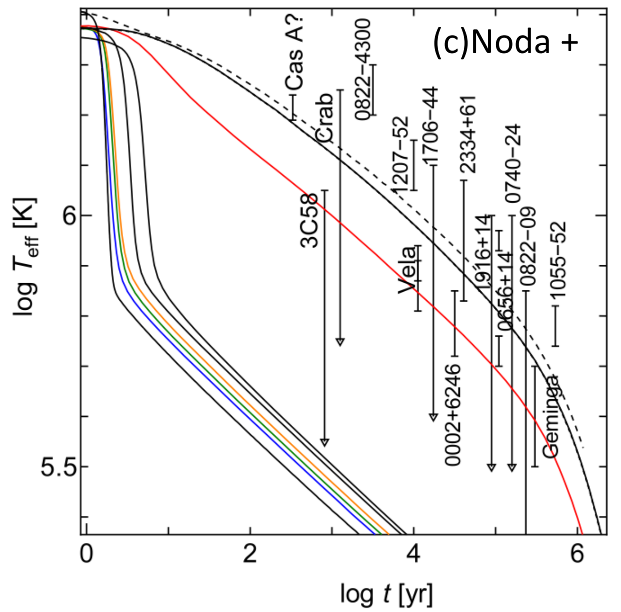
$$c_v e^{\Phi}\frac{\partial T}{\partial t} + \nabla\cdot(e^{2\Phi}F) = e^{2\Phi}Q$$

thermal diffusion eq.

heat capacity

flux (thermal conductivity)

cooling rate (neutrino) + heating rate (magnetic field)



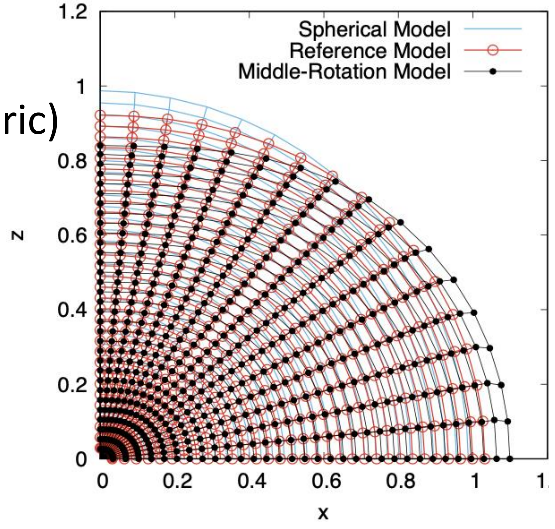
Toward Proto Neutron Star evolution:

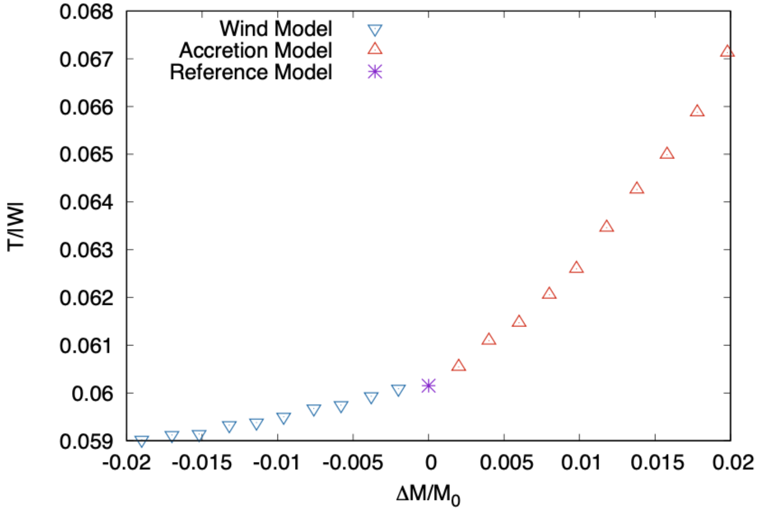
- hydrostatic equilibria in GR
- Lagrange scheme(axisymmetric)

+

on going …

- energy eq.
- neutrino transports





Okawa, Fujisawa, **NY**, Ogata, Yamamoto,Yamada (2023) MNRAS

Summary and Discussion

- We present CMD results, which is from QCD to neutron stars (hadron+quarks)
- EOS (MR relation) constraints

  Low density ←nuclear experiments

  High density ← astrophysical observations
- As the results, our CMD simulations provide a neutron star (NS) EoS.
- We find cross over deconfinement.

  GW, Supernovae… ← Finite temperature behavior
- We need more realistic set up: relativistic effects, strangeness effects, vacuum effects.
- Now, we focus on CMD with Color-magnetic interactions.
- Our CMD will provide thermal properties for NS evolution in the future.