Aug 22th. 2024

Quark deconfinement in neutron stars by Color-Molecular-Dynamics

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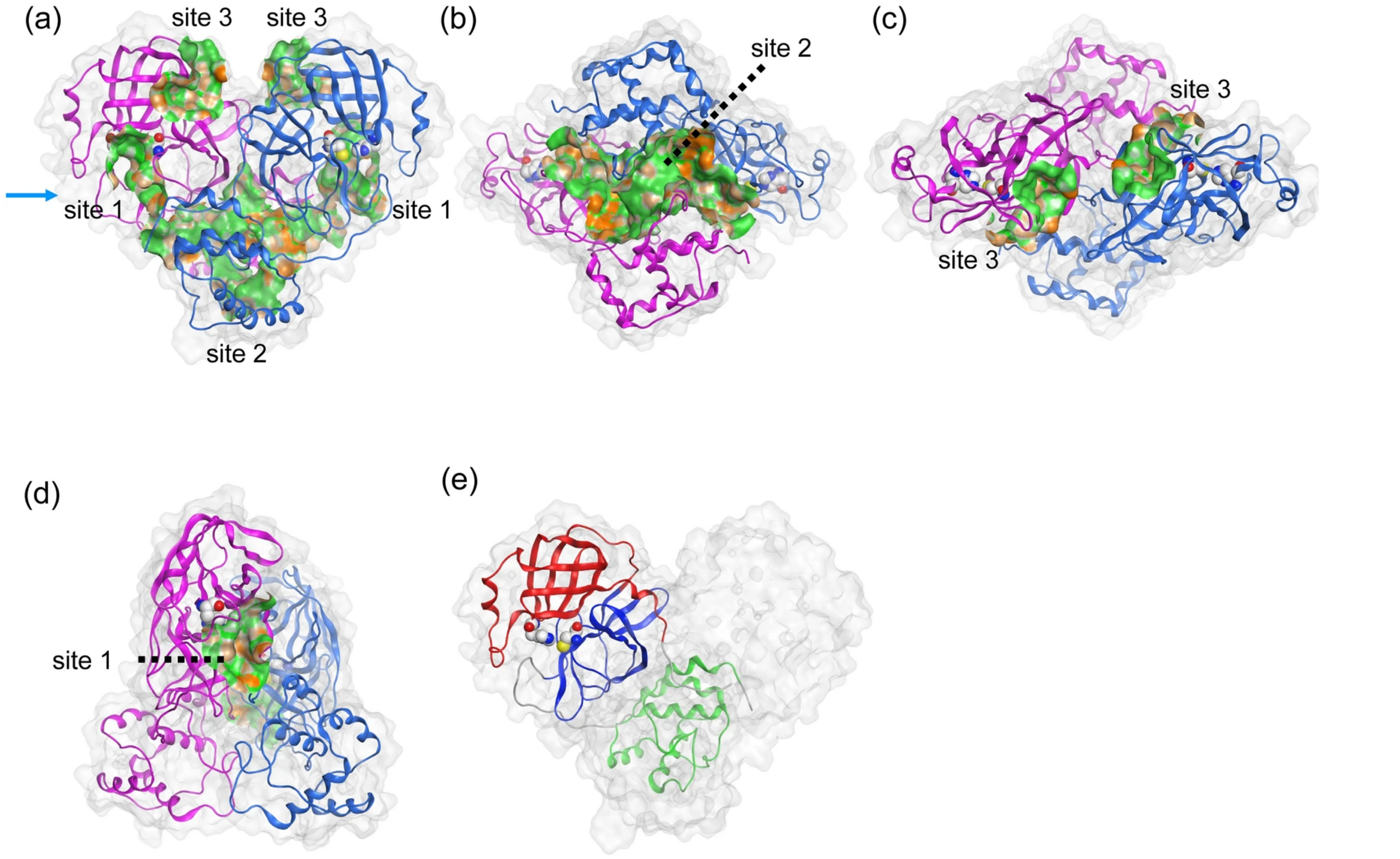
T.Maruyama(JAEA)

PRD 109. 043056 (2024)

Application of Molecular dynamics

DNA, medicine, virus….

→ Without the fundamental theory in physical meaning, they give useful information.



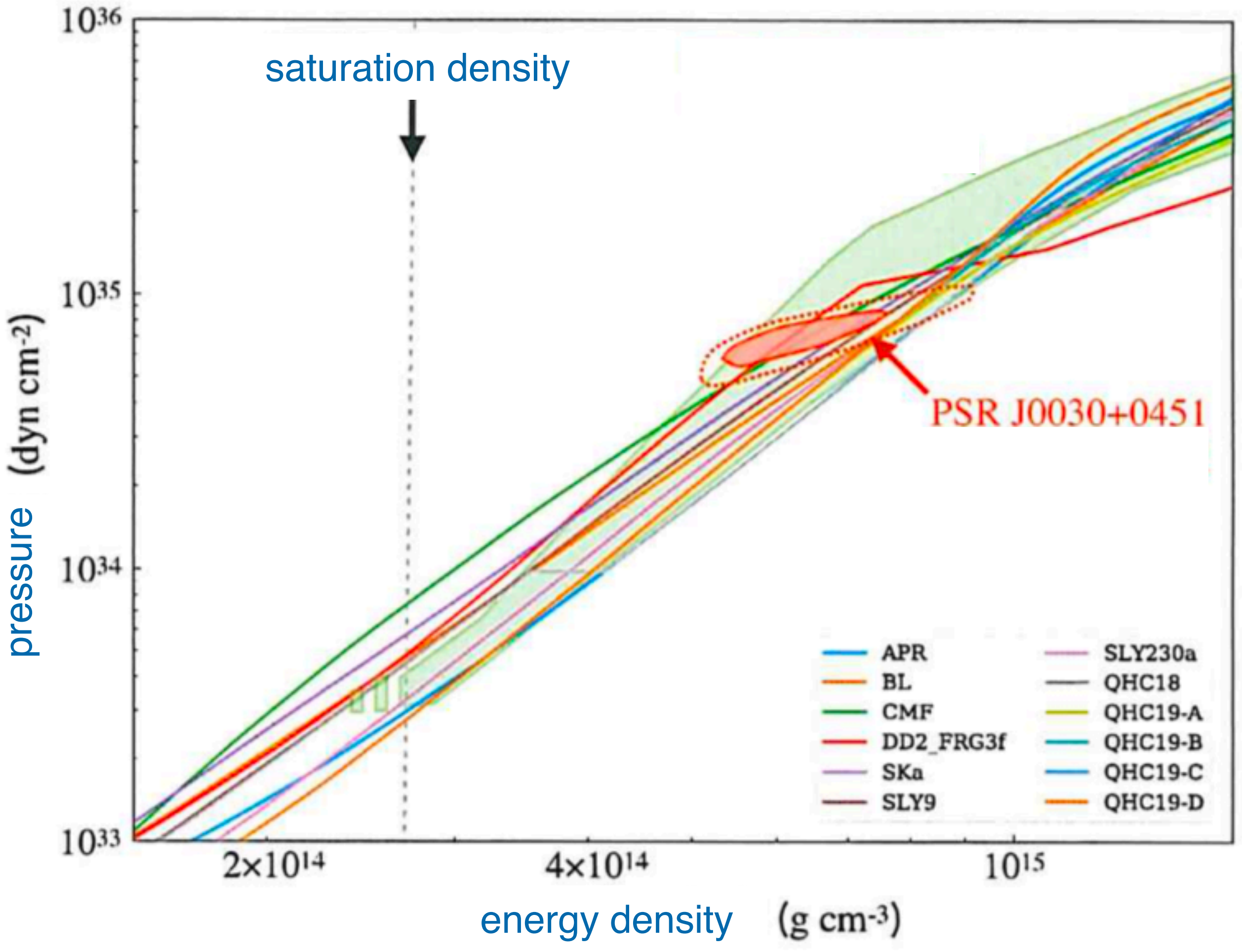
Drug binding dynamics of the dimeric SARS-CoV-2 main protease, determined by molecular dynamics simulation
Scientific Reports 10, 16986 (2020)

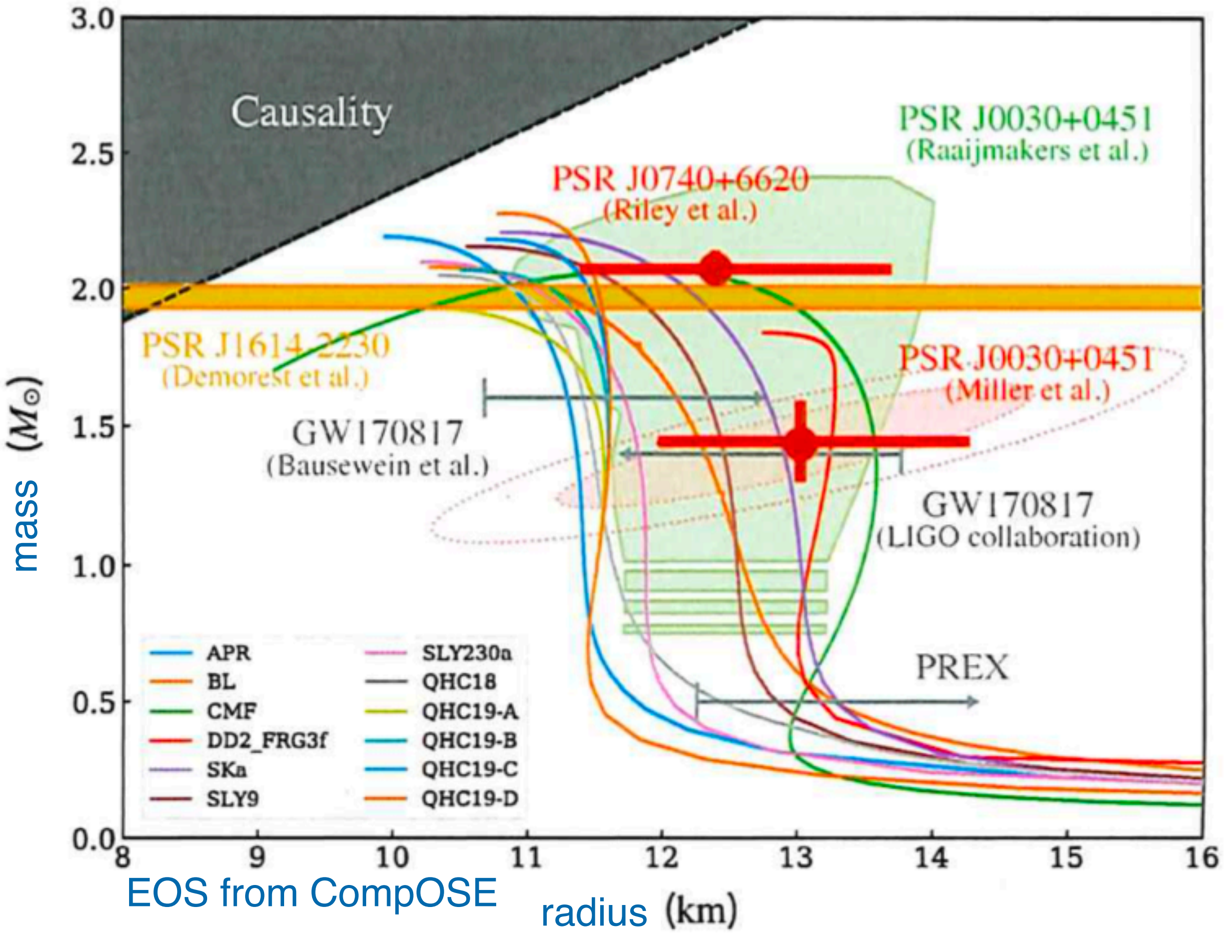
I. Background

EOS under some constraints

EOS and M-R relation

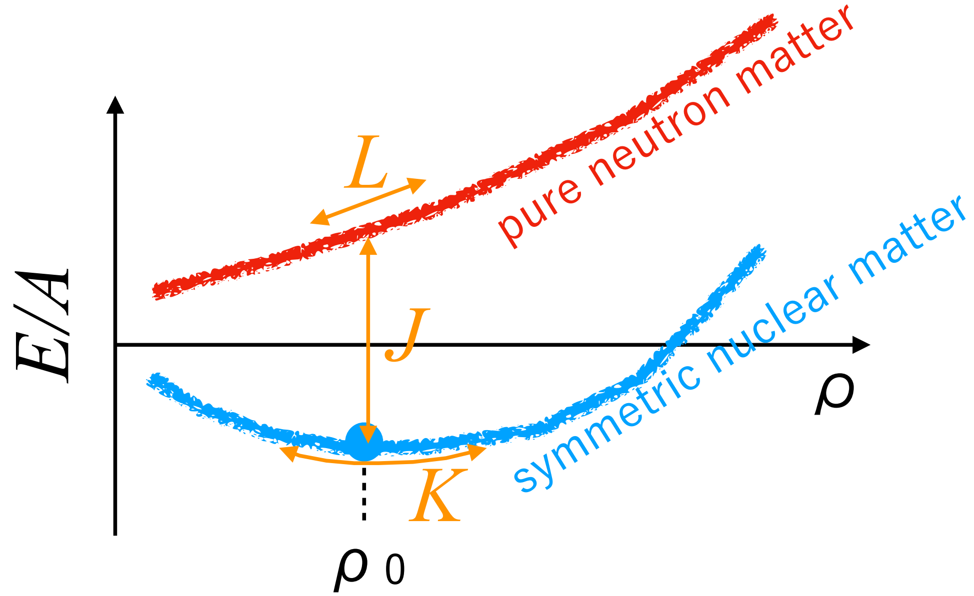
Enoto & **NY** (2021) Oct. Journal of Japanese Physical Society





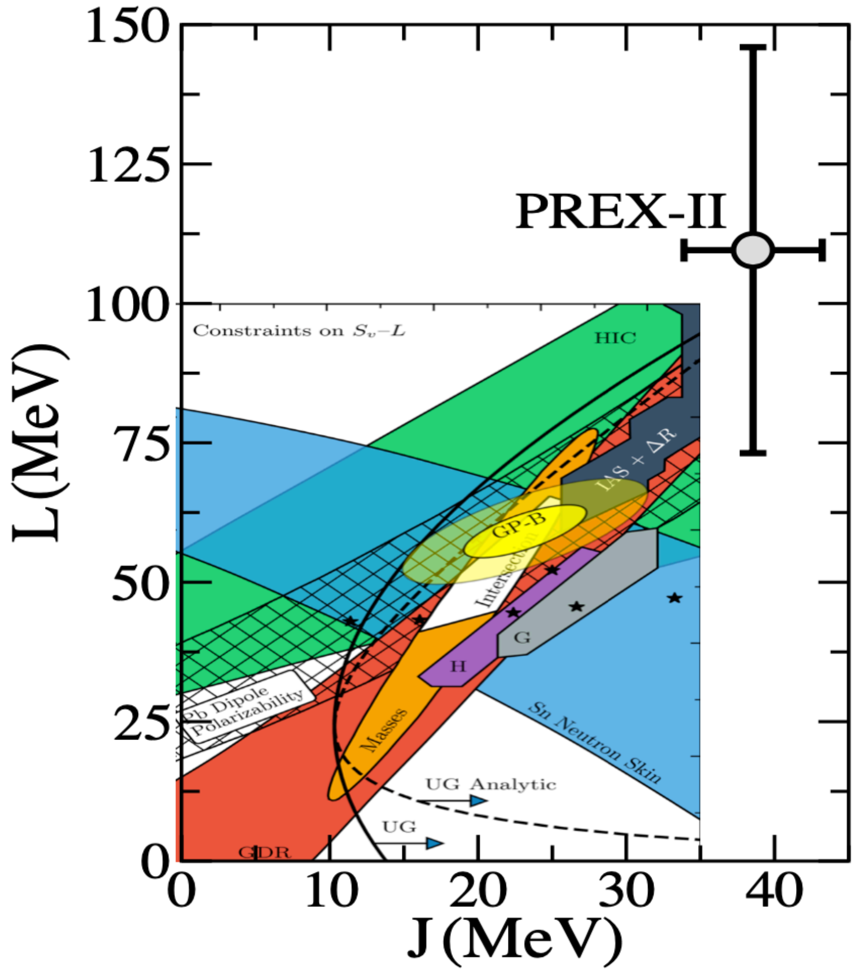
EOS Constraints
around saturation density

$$E/A = S + \frac{L(\rho - \rho_0)}{3\rho_0} + \frac{K_{sym}(\rho - \rho_0)^2}{18\rho_0} \ldots$$



$$E/A = S_0 + \frac{K(\rho - \rho_0)^2}{18\rho_0^2} \ldots$$

Reed et al. PRL 126,172503 (2021)

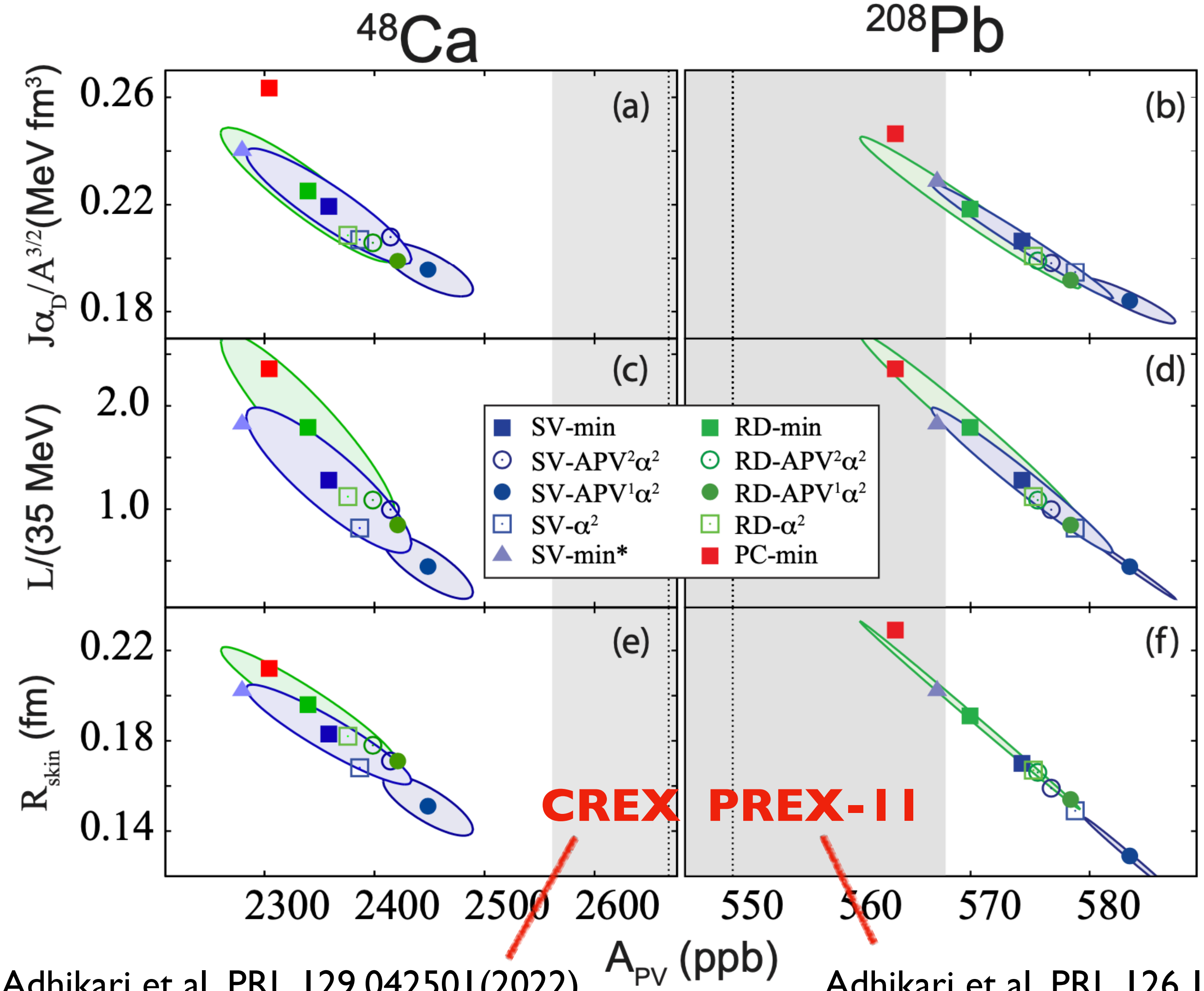


・The result of PREX-II depends on the analysis methods. (Reinhard et al. PRL 127.232501(2021))

・**CREX** results suggest ordinary L values, not large as PREX-II. (Adhikari et al. PRL 129.042501(2022))

$$A_{\rm PV}(Q^2) = \frac{d\sigma_R/d\Omega - d\sigma_L/d\Omega}{d\sigma_R/d\Omega + d\sigma_L/d\Omega} \xrightarrow{\mathcal{M}} F_W(q) \xrightarrow{\mathcal{M}} R_{\rm skin}, J, L$$

Reinhard et al. PRL 129,232501(2022)



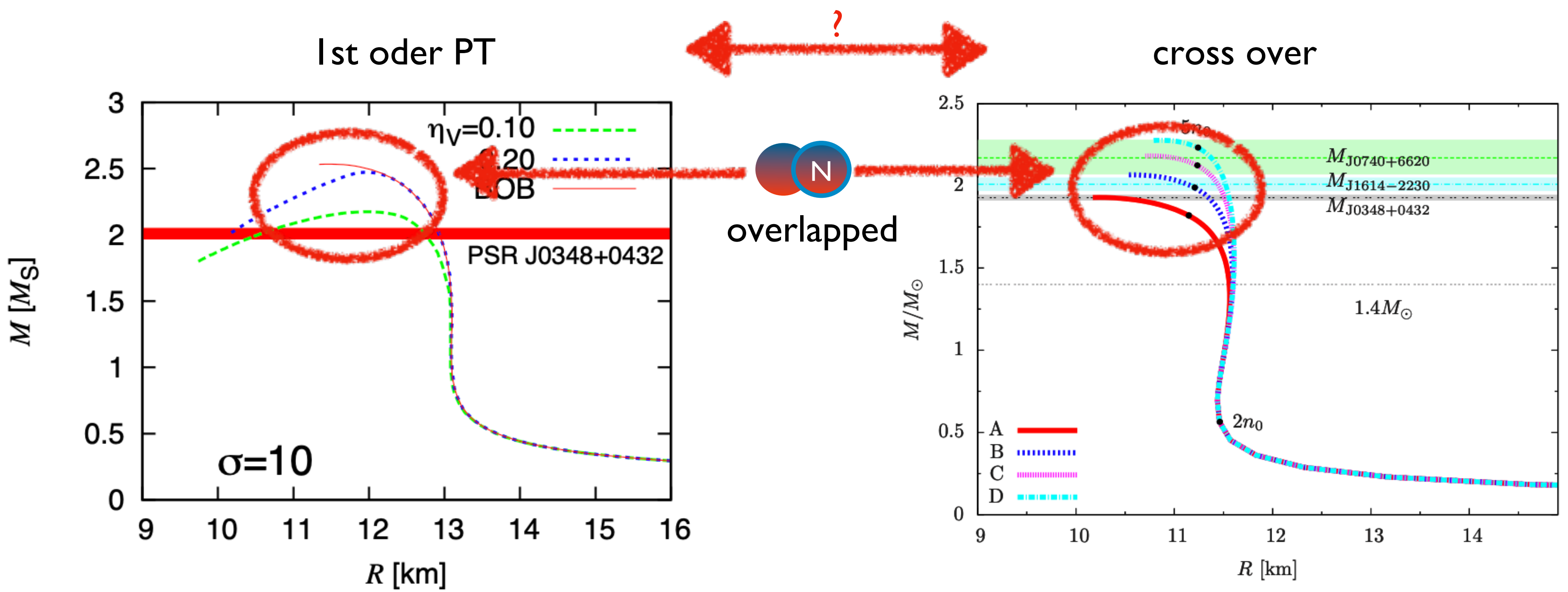
Adhikari et al. PRL 129.042501(2022)

Adhikari et al. PRL 126.172501(2021)

We can not conclude anything because of uncertainty for Apv→Rskin, L, J.

We needs some good models consistent for Ca and Pb.

EOS from quarks to neutron stars



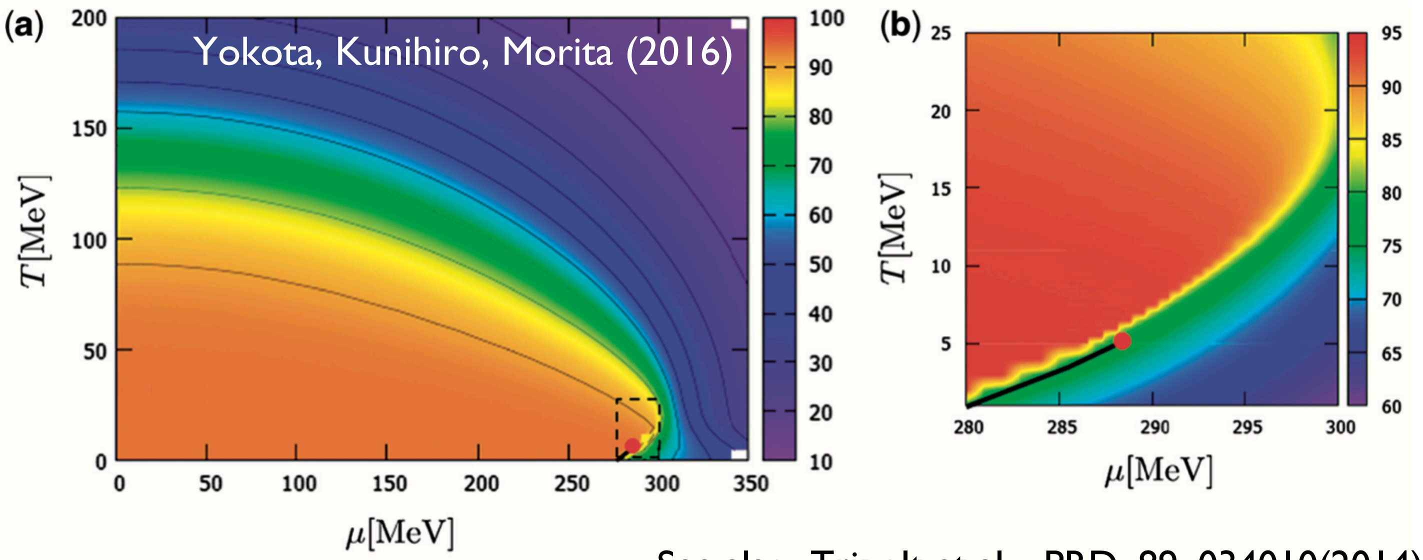
NY, et al. PRC 90, 067302 (2014) G. Baym, et al. ApJ 885, 42 (2019)

nucleon(BHF model) + quark(eNJL model) nucleon(Variational method) + quark(NJL model)

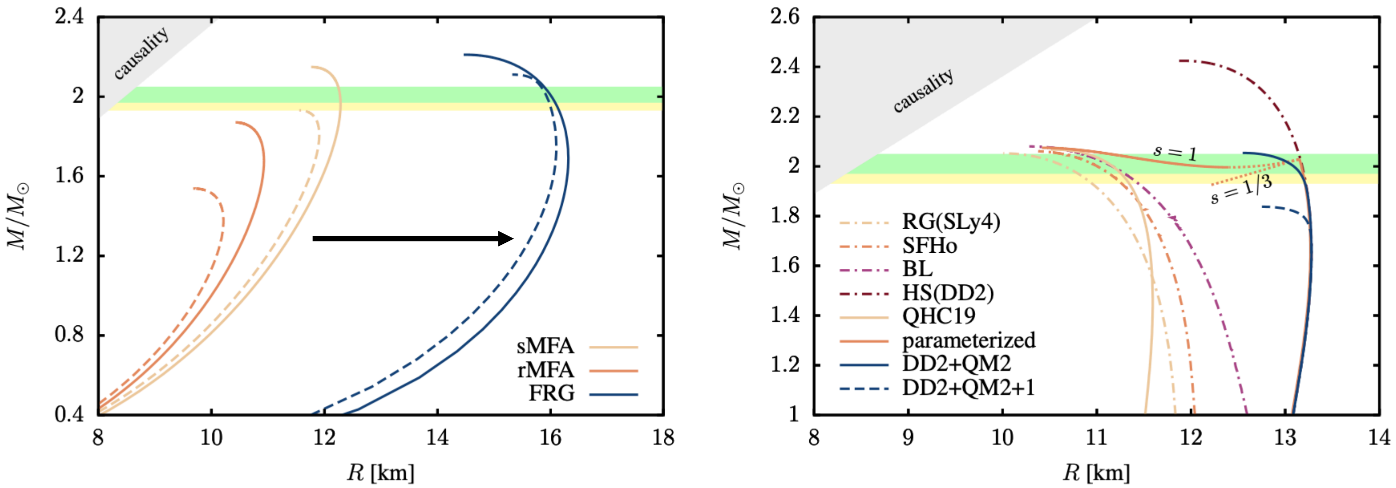
See also Xia, Maruyama, **NY**, Tatsumi, Shen, Togashi (2020), Phys. Rev. D 102, 023031

Problems
① How much can we believe hadron models at high density region?
② 1st oder or cross over?
→ It is dependent on the assumption (not results).
→ We can not get unified understanding.
cf.) We can not obtain the other physical properties for crossover.

Effects of Fluctuation



See also Tripolt et al. , PRD, 89, 034010(2014)



(a) Pure quark stars

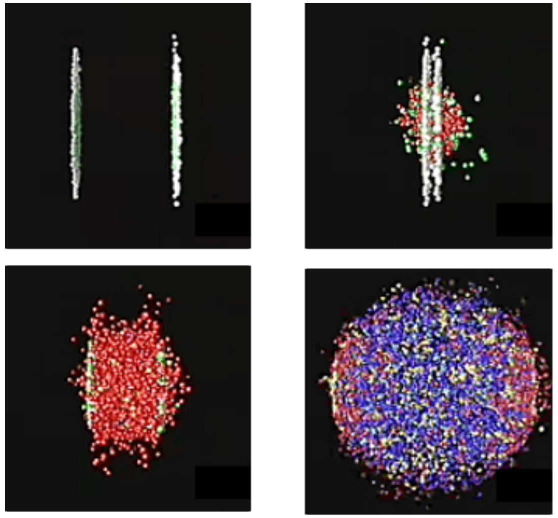
(b) Hadronic (dash-dotted) and hybrid (solid, dashed) stars

Otto, Oertel, Schaefer, PRD, 101, 103021 (2020)

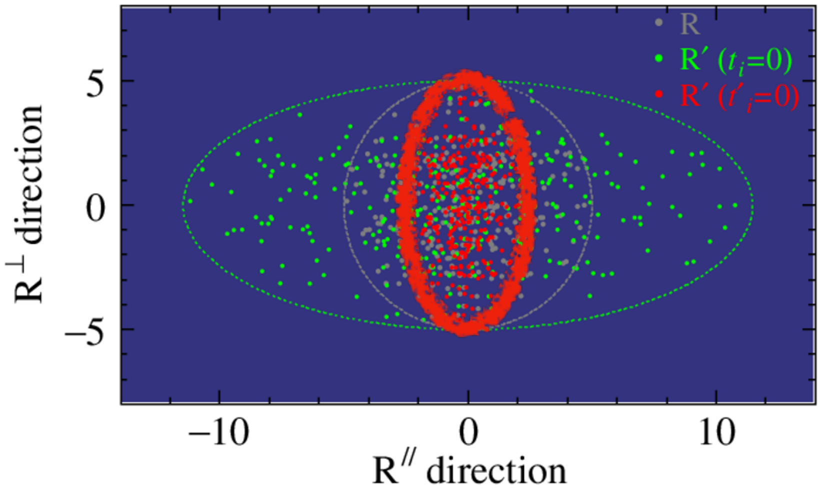
Molecular dynamics

Why molecular dynamics ?

① We focus on Quark-Molecular-Dynamics.
→We can describe EOS and NS physics only with quark system.
② MD enables us to know also dynamical behaviors, fluctuation, clustering.
→We can apply MD directly to Heavy Ion Collisions.



UrQMD for RHIC
https://www.bnl.gov/rhic/physics.php



Relativistic color molecular dynamics
(on going project)

II. Color molecular dynamics

Current status

Formulation

NY, Maruyama (2024) PRD
Maruyama, Hatsuda (2000) PRD

number of variables

$$\{x, y, z, Px, Py, Pz, \alpha, \beta, \theta, \varphi\}_i$$

10 variables on each particle

wave functions

$$\Psi = \prod_{i=1}^{3A} \phi_i(\mathbf{r})\chi_i \qquad \chi_i = f_i s_i c_i$$

f_i ... flavors (fixed)

s_i ... spins (fixed. But unfixed later section) ↑or↓

$$c_i \equiv \begin{pmatrix} \cos\alpha_i e^{-i\beta_i} \cos\theta_i \\ \sin\alpha_i e^{+i\beta_i} \cos\theta_i \\ \sin\theta_i e^{i\varphi_i} \end{pmatrix} \begin{matrix} \text{R} \\ \text{G} \\ \text{B} \end{matrix}$$

$$\phi_i(\mathbf{r}) \equiv (\pi L^2)^{-\frac{3}{4}} \exp[-(\mathbf{r} - \mathbf{R}_i)^2/2L^2 - i\mathbf{P}_i\mathbf{r}]$$

time evolution

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} \qquad \mathcal{L} = \left\langle \Psi \middle| i\hbar\frac{d}{dt} - \hat{H} \middle| \Psi \right\rangle$$

cooling

$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$

$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

confinement conditions

$$\begin{cases} |\mathbf{R}_i - \mathbf{R}_j| < d_{cluster} \quad (i, j = 1, 2, 3) \\ \sum_{a=1}^{8} \left[\sum_{i=1}^{3} \langle \chi_i | \lambda^a | \chi_i \rangle\right]^2 < \varepsilon \end{cases}$$

λ^a being the Gell-Mann matrices

Interactions

The system follows the Hamiltonian,

$$H = H_0 + V_{\text{Pauli}} - T_{\text{spur}},$$

where H_0 is the conventional Hamiltonian expressed as

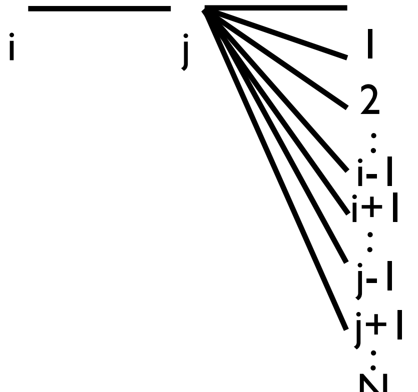
$$H_0 = \left\langle \Psi \left| \sum_{i=1}^{N} \sqrt{m + \hat{\mathbf{p}}_i^2} + \frac{1}{2} \sum_{i,j \neq i}^{N} \hat{V}_{ij} \right| \Psi \right\rangle .$$

$$\hat{V}_{i,j} = -\sum_{a=1}^{8} \tau_i^a \tau_j^a \hat{V}_{\text{C}} + \hat{V}_{\text{M}},$$

$\tau_i^a = \lambda_i^c/2$ with λ_i^c being the Gell-Mann matrices.

$$\hat{V}_{\text{C}} = \kappa \hat{r}_{ij} - \frac{\alpha_s}{\hat{r}_{ij}},$$

quark many body effects



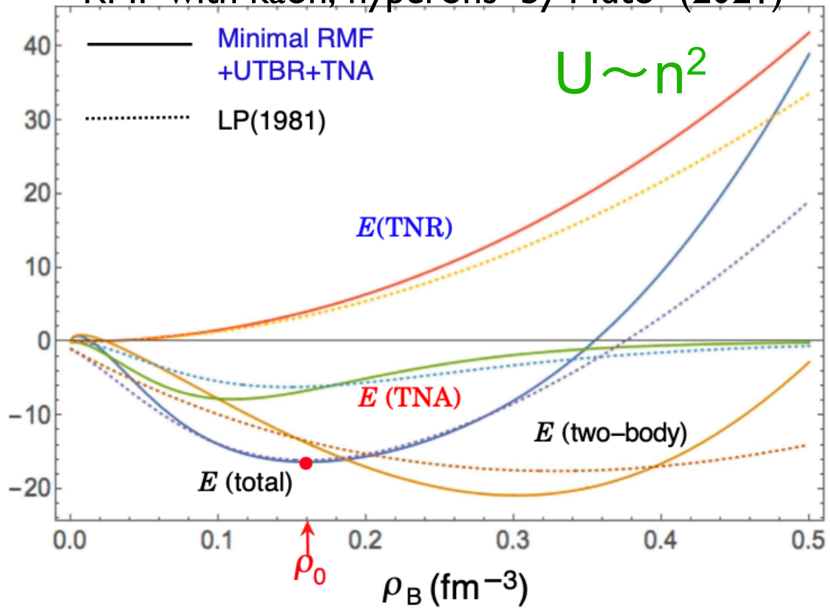
$$\hat{V}_{\text{M}} = \frac{g_\omega^2 C_\omega}{4\pi} \left(\sum_{j \neq i}^{n} \frac{e^{-\mu_\omega \hat{r}_{ij}}}{\hat{r}_{ij}} \right)^{1+\epsilon_\omega} - \frac{g_\sigma^2 C_\sigma}{4\pi} \left(\sum_{j \neq i}^{n} \frac{e^{-\mu_\sigma \hat{r}_{ij}}}{\hat{r}_{ij}} \right)^{1+\epsilon_\sigma}$$

$$+ \frac{\sigma_i^3 \sigma_j^3}{4} \frac{g_\rho^2}{4\pi} \frac{e^{-\mu_\rho \hat{r}_{ij}}}{\hat{r}_{ij}}$$

Many-body effects on EOSs

$n^2 \sim (1/r)^6$

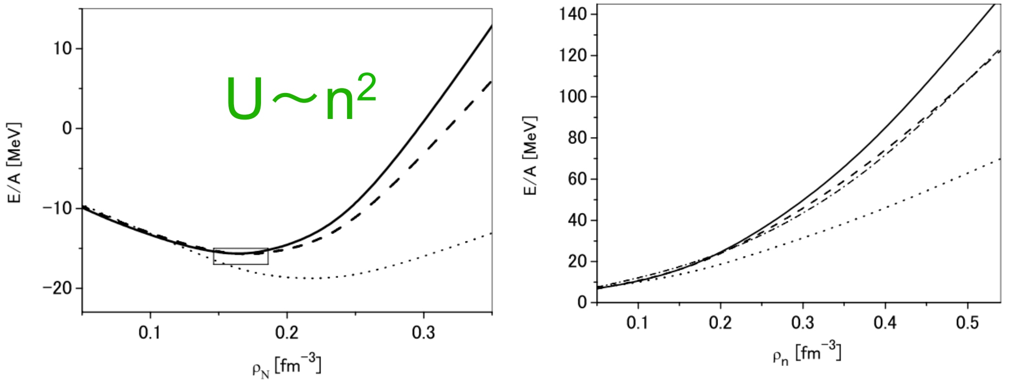
RMF with kaon, hyperons by Muto+(2021)



$$U_{SJM2}(r;\rho_B) = V_r\rho_B(1 + c_r\rho_B/\rho_0)\exp[-(r/\lambda_r)^2)]$$

introduced to be consistent with MR relations.

BHF with hyperons by Yamamoto, **NY,** Rijiken (2022)



$$V_{eff}^{(N)}(\mathbf{x}_1,\mathbf{x}_2) = \rho_{NM}^{N-2}\int d^3x_3 \ldots \int d^3x_N \; V(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N)$$

$$= g_P^{(N)} g_P^N \frac{\rho_{NM}^{N-2}}{\mathcal{M}^{3N-4}} \frac{1}{\pi\sqrt{\pi}} \left(\frac{m_P}{\sqrt{2}}\right)^3 \exp\left(-\frac{1}{2}m_P^2 r_{12}^2\right), (3)$$

N=3,4

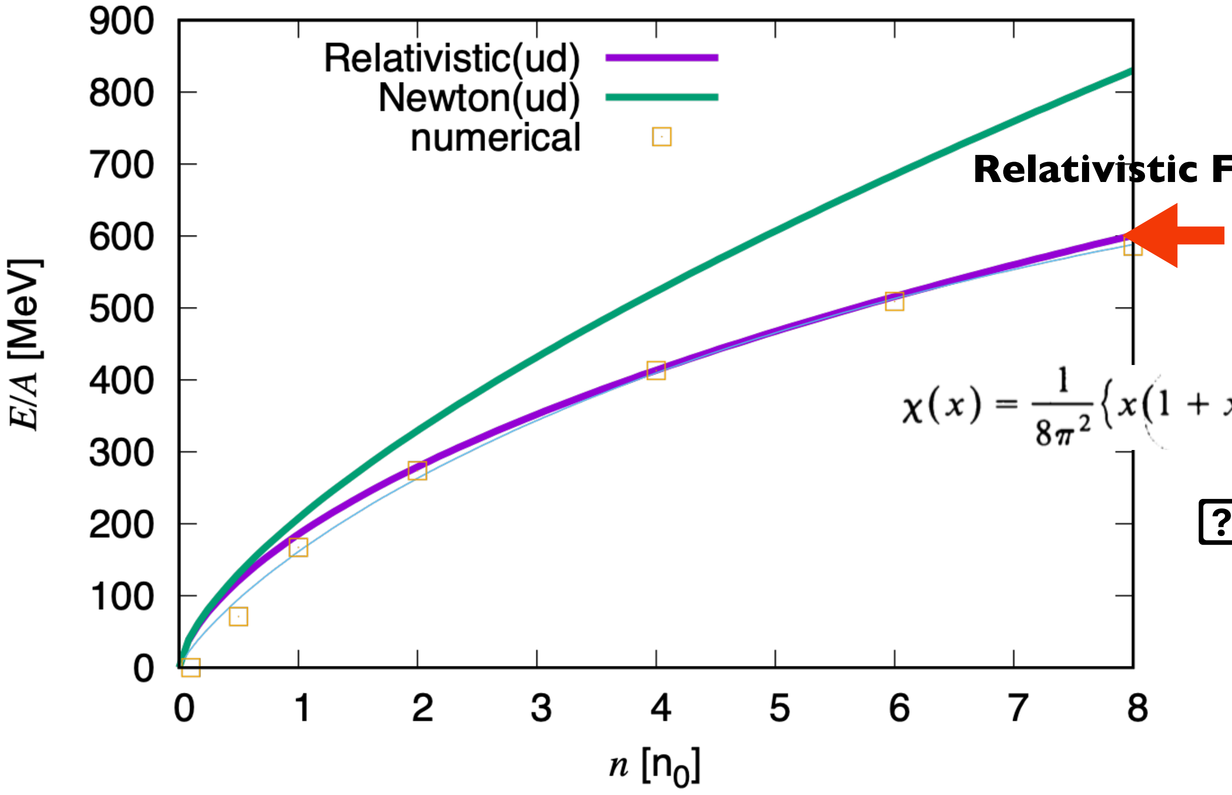
introduced to be consistent with MR relations

and/or cross sections of nuclei.

These examples are adopted to hadron interactions.
Why not to quark-quark interactions?

$U_{qqq} \sim (1/r)^{1+\varepsilon}$

Free kinetic energy and Pauli interaction



Relativistic Free kinetic energy for fermions

?=???↑2 /?↑3 ?(?)

$$\chi(x) = \frac{1}{8\pi^2}\{x(1+x^2)^{1/2}(1+2x^2) - \ln[x+(1+x^2)^{1/2}]\}$$

?=4?↑2 ?/3?↑3 ?=??????↑3??

?=spin×color×flavor

Effective Pauli interaction

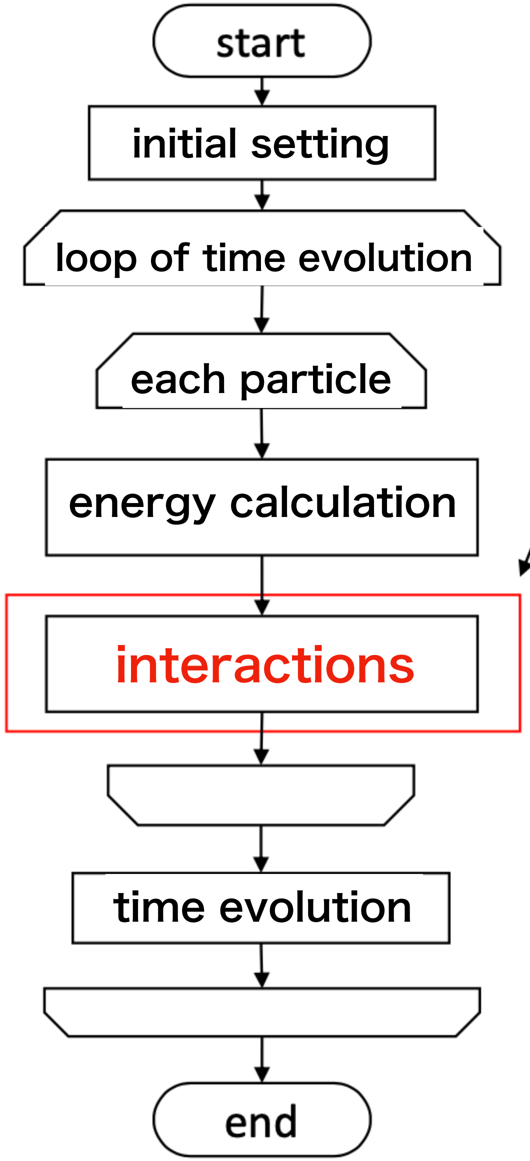
$$V_{\text{Pauli}} = \frac{C_p}{(q_0 p_0)^3} \exp\left[-\frac{(\mathbf{R}_i - \mathbf{R}_j)^2}{2q_0^2} - \frac{(\mathbf{P}_i - \mathbf{P}_j)^2}{2p_0^2}\right] \delta_{\chi_i \chi_j}$$

Introduced to show the antisymmetric effects.

These parameters, Cp, q0, p0 are optimized to reproduce the kinetic energy for fermions

Parallel computings in MD

No money(GPUs), No study….



- The main part of calculations (~90%).

→ The main target to be improved.

- Super computer in JAEA
- parallel computings

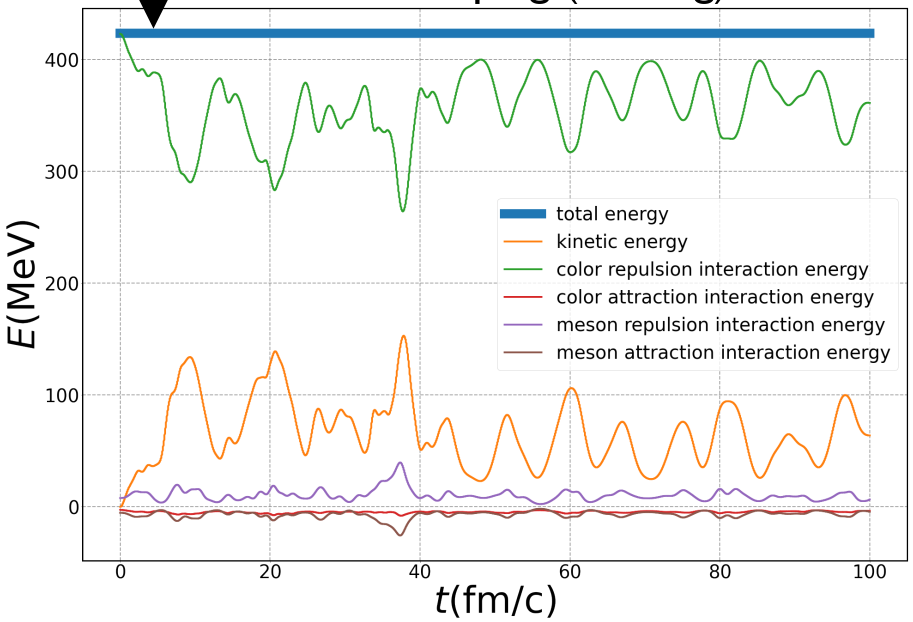
Device : HPE SGI 8600 in JAEA
CPU : Intel Xeon Gold 6428R
(3.0GHz, 35.75MB cache)X 2CPU
Core number per node : 48
GPU : NVIDIA Tesla V100 SX2 32GB memory
(FP64/GPU : 2560)x4GPU
Memory per node : 384 GB

Energy conservation

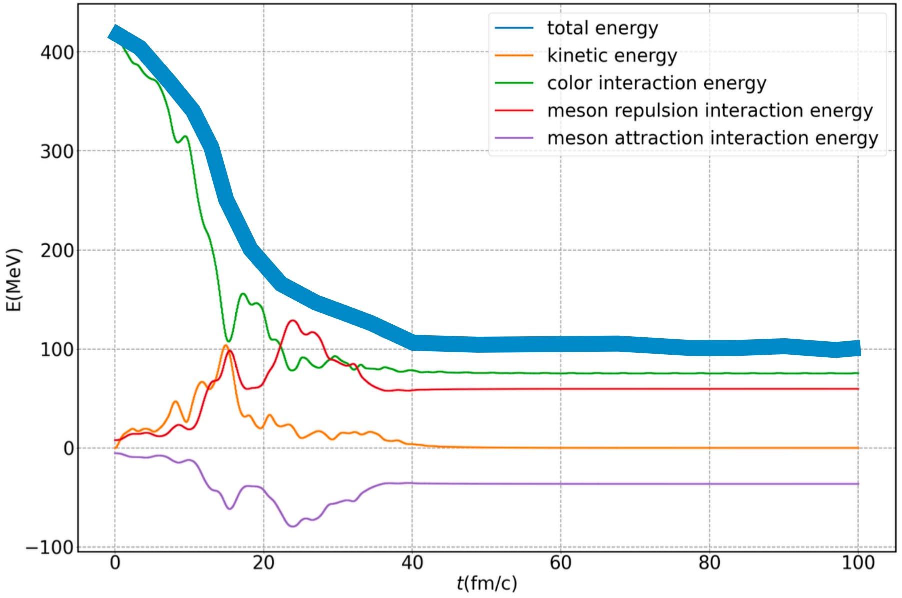
as an accuracy check

total energy

without dumping (cooling) terms



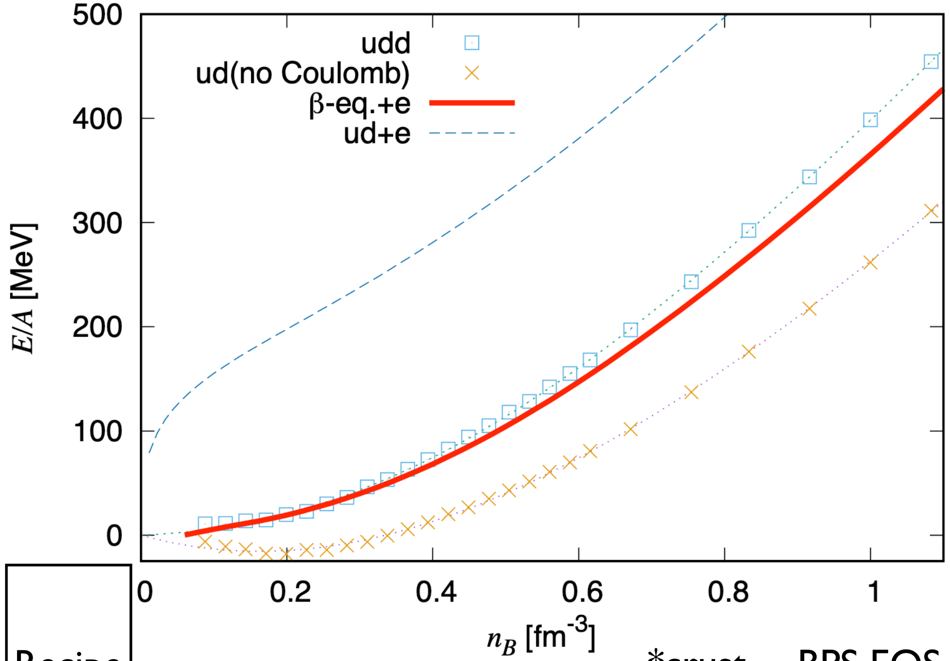
including dumping (cooling) terms

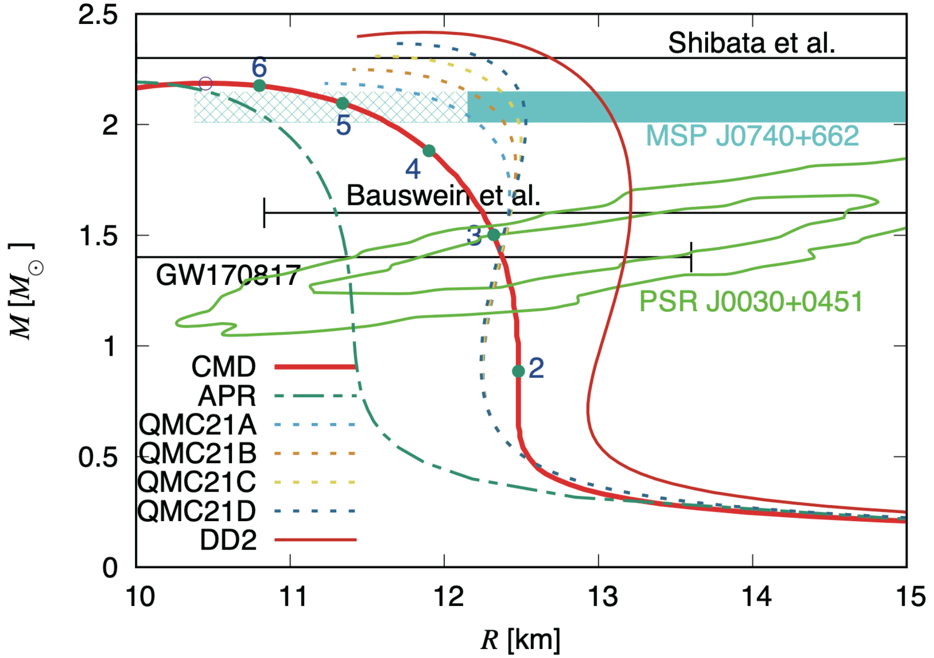


$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$

$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

EOS by Color-Molecular-dynamics





Recipe

*crust …BPS EOS

try next parameter

① We derive fitting formulae from numerical data points. (~ 1day for 1 parameter set)

② We obtain EOS using the formulae under beta equilibrium and charge neutrality conditions.

beta eq.

$$\mu_u + \mu_e = \mu_d \, ,$$

$$\left(\mu_p + \mu_e = \mu_n = \mu_u + 2\mu_d \, . \right)$$

charge neutrality

$$en_{\rm ch}(\boldsymbol{r}) = \sum_{q=u,d,e} Q_q n_q(\boldsymbol{r})$$

$$\left(en_{\rm ch}(\boldsymbol{r}) = \sum_{i=n,p,e} Q_i n_i(\boldsymbol{r}) \right)$$

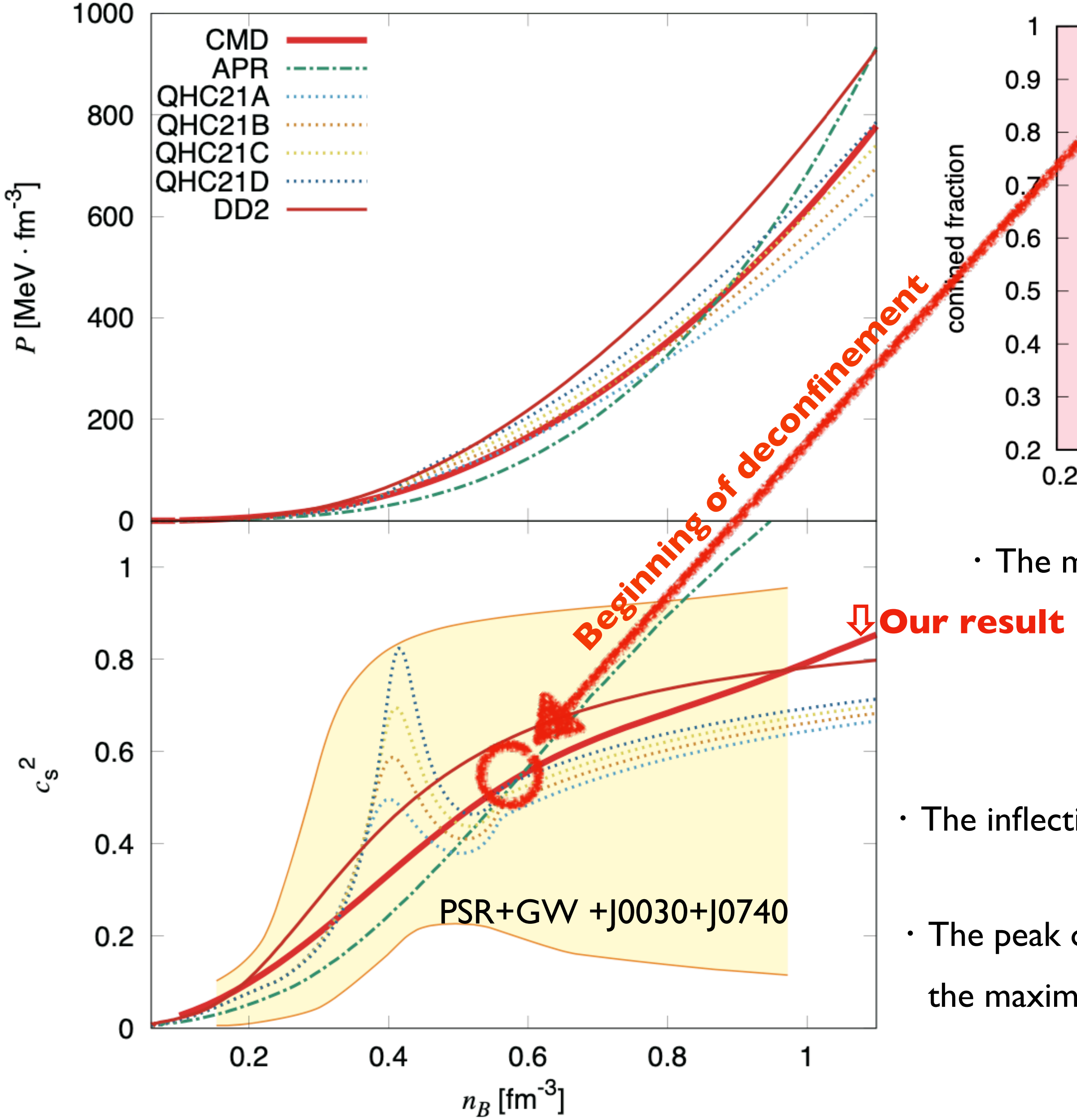
③ We check whether the EOSs are consistent with astrophysical constraints, and the nuclear experiments result.

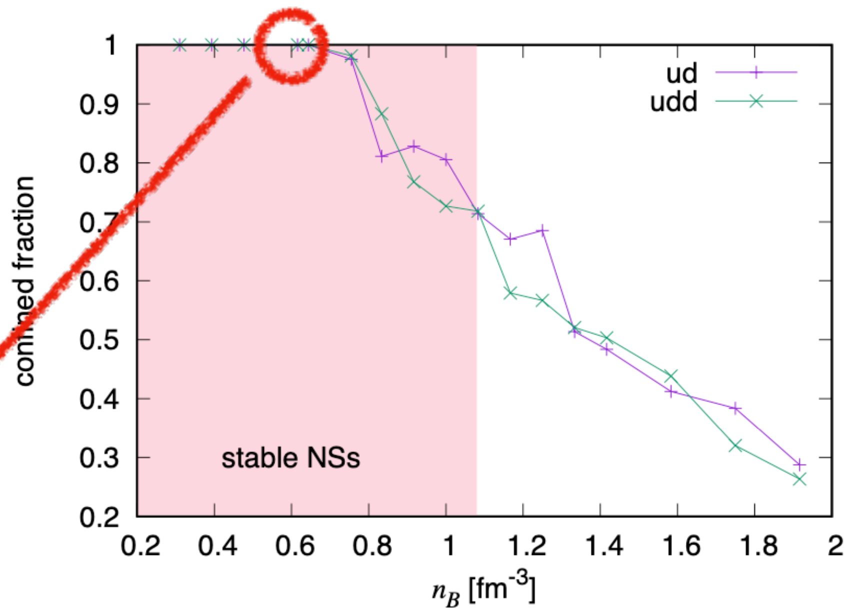
cf.) the above result

n_0=0.167fm^{-3}, S_0=-15.8 MeV, J=31.0MeV, L=74.2MeV, K =260 MeV, M_{max}=2.19 Ms, $\Lambda_{1.4}$=458, nc=1.08 fm^{-3}

ref) Danielewicz et al., Science 298 (02) 1592: K = 167-300 MeV

Pressure & Sound velocity

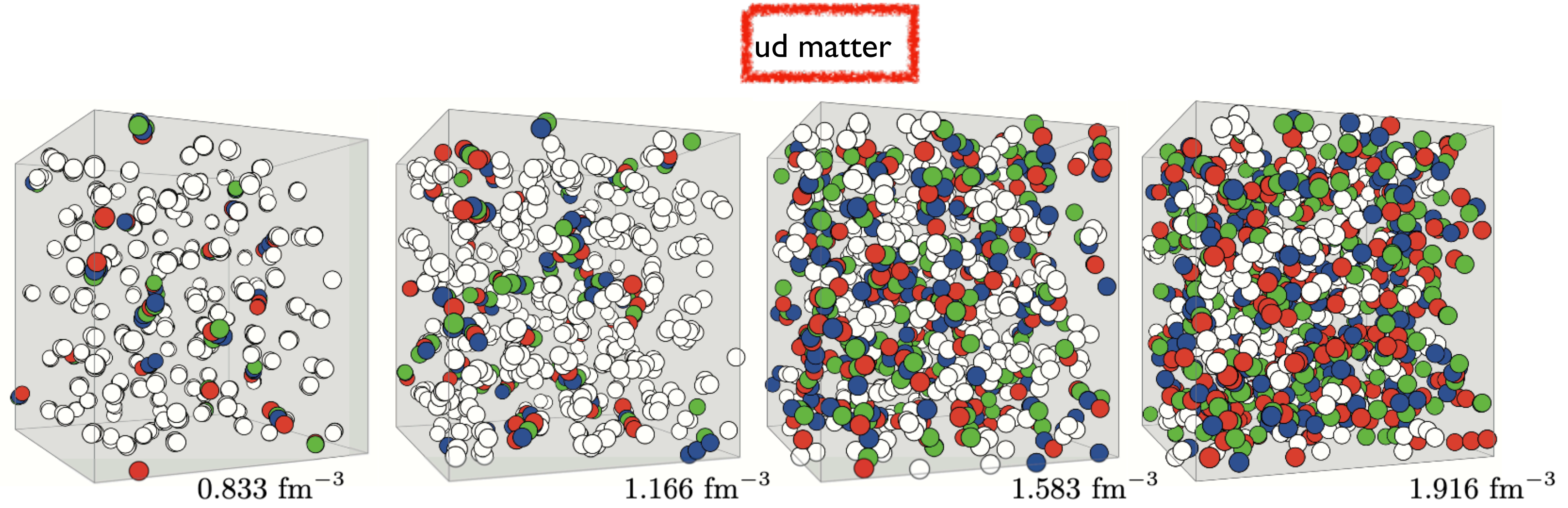




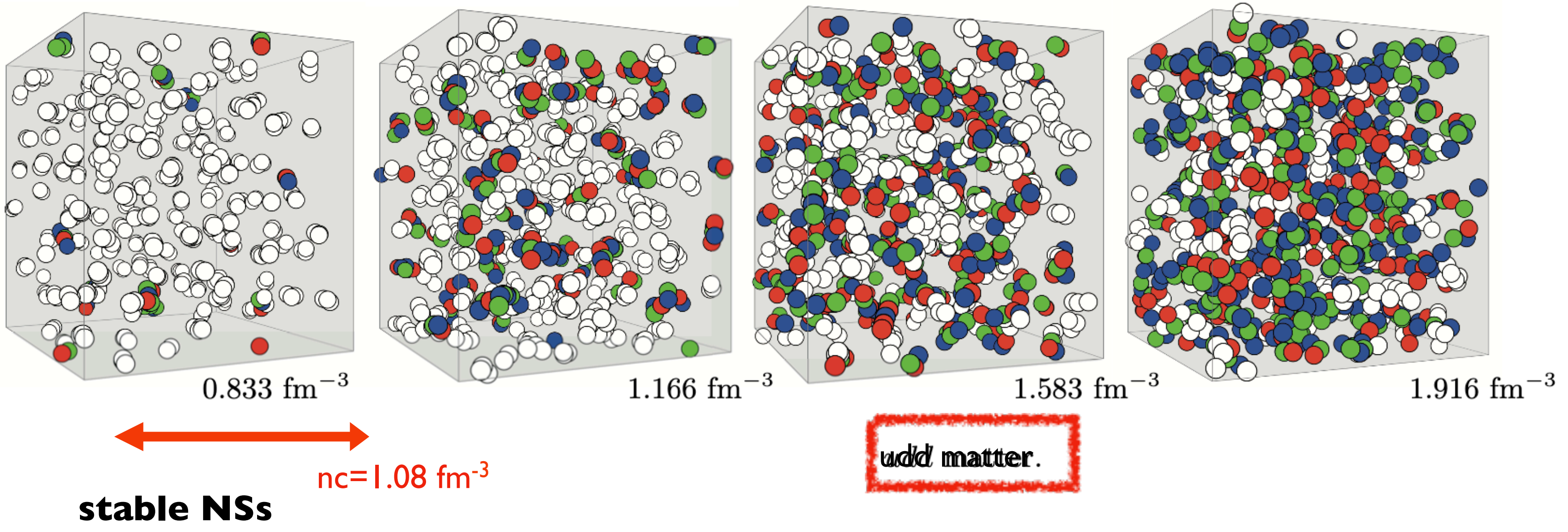
・The main component of NSs are hadron matter.

・The inflection point is the quark deconfinement origin.

・The peak of sound speed appears higher density than the maximum density of NSs in our models.

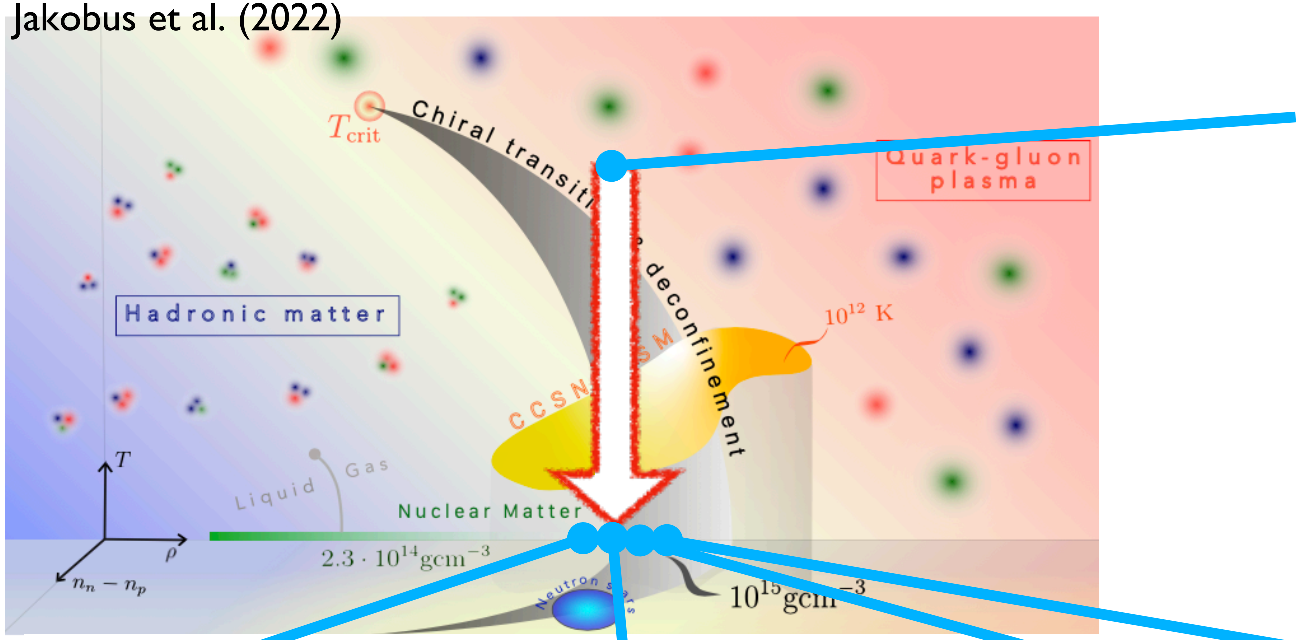


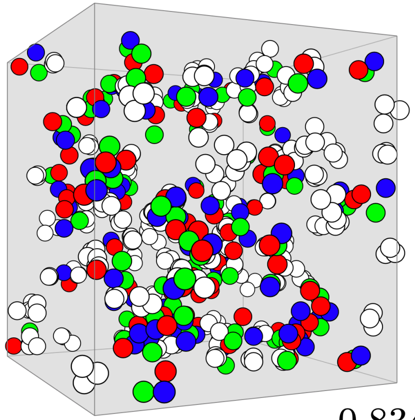
The perspectives for *ud* matter depended on density. Each color corresponds to the color's internal degree of freedom for each quark: the white color balls represent the quarks in the baryon state, while red, blue, and green ones do in the deconfined quark matter.



Phase diagram

Jakobus et al. (2022)



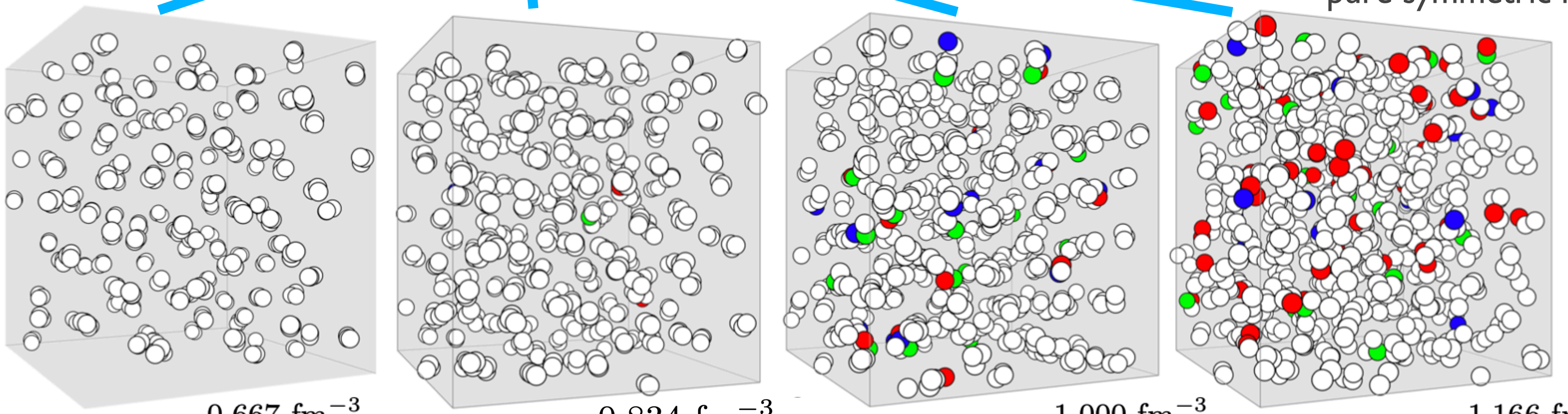


0.834 fm^{-3}

$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i} + \mu_R \frac{\partial H}{\partial \mathbf{R}_i},$$

$$\dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i} + \mu_P \frac{\partial H}{\partial \mathbf{P}_i},$$

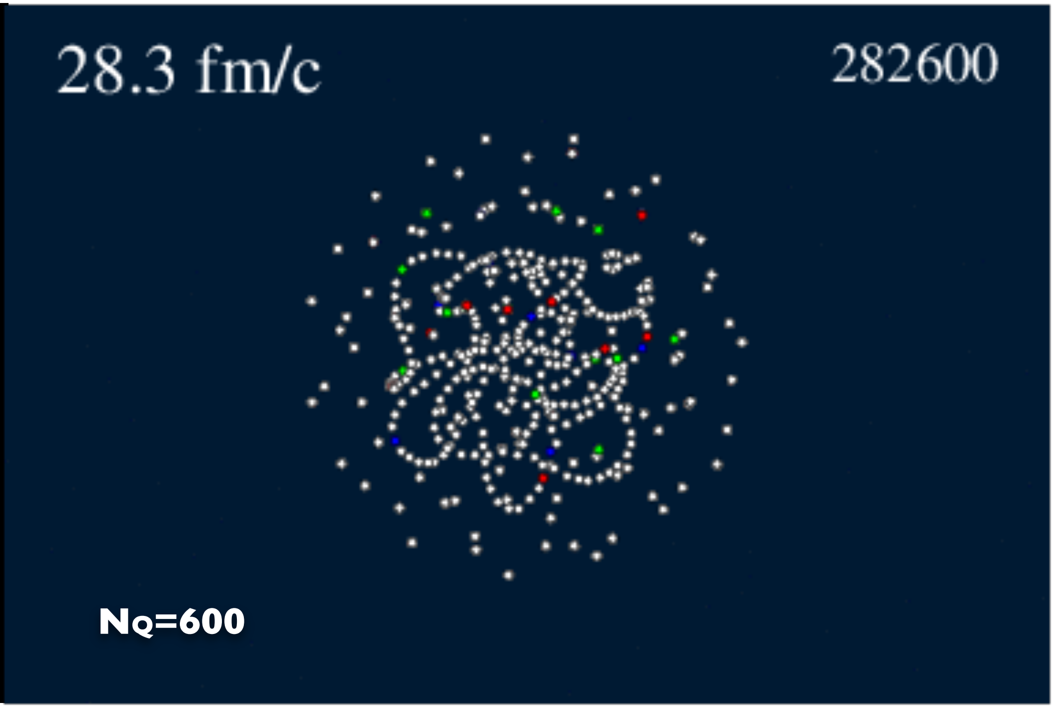
pure symmetric matter

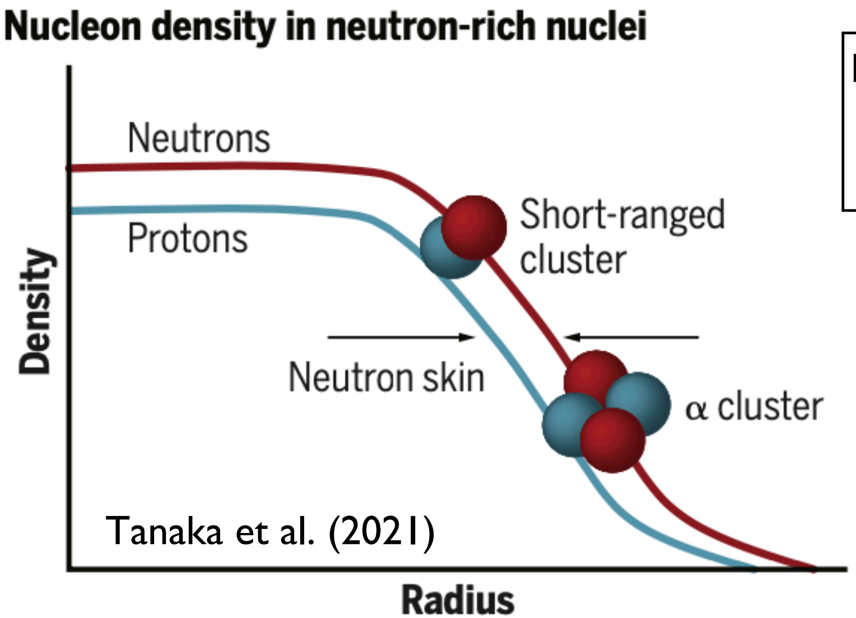


0.667 fm^{-3} 0.834 fm^{-3} 1.000 fm^{-3} 1.166 fm^{-3}

III. On going project

Finite system consistent with finite system





RMF + alpha cluster(local correlation) neutron skin change 10-40%

"DD2" Typel, Ropke, Klahn, Blaschke, Wolter, PRC. 81, 015803 (2010).

Typel, PRC.89,064321(2014).

Theoretical cross sections (nb)	Effective number of α clusters	Theoretical Δr_{np} without α (fm)	Theoretical Δr_{np} with α (fm)	Relative Δr_{np} change (%)
0.160	0.3876	0.0495	0.0277	−44
0.127	0.3304	0.0843	0.0580	−31
0.095	0.2668	0.1179	0.0912	−23
0.065	0.1958	0.1505	0.1275	−15

Color magnetic interaction and baryon mass

interactions with colors

$$\hat{V}_{\rm color} = \frac{1}{2}\sum_{i=1, j\neq i}^{n}\left(-\sum_{a=1}^{8}\frac{\lambda_i^a\lambda_j^a}{4}\left(K\hat{r}_{ij} - \frac{\alpha_s}{\hat{r}_{ij}}\right)\right)$$

↑or↓

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2}\frac{1}{r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\lambda_i^c\lambda_j^c\,\sigma_i\cdot\sigma_j$$

$r_{0ij} = (\alpha + \beta\mu_{ij})^{-1}$. $\mu_{ij} = m_i m_j/(m_i+m_j)$

Aaron et al. EPJA 56,93(2020)

～1/(2m)

αs	κ'	K	α	β	$m_{u,d}$	m_s	L
1.25	0.5	750 MeVfm^{-1}	2.1 fm^{-1}	0.552	0.362 GeV	0.538 GeV	0.346 fm

Akimura et al. EPJA 25,405(2005) etc. **Optimized in this study.**

2-body spin correlations

<?↓??/2 · ?↓??/2 >=1/4 (↑↑),−3/4 (↑↓)

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N,P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.906	1.245	1.075	1.075	1.415	1.217	1.584
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

2-body spin correlations consistent with 3-body spin correlations

<?↓??/2 · ?↓??/2 >=1/4 (↑↑),−1/2 (↑↓)

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.938	1.233	1.115	1.177	1.379	1.328	1.531
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

others

quark-meson coupling

$$V_{\rm meson}(r) \equiv -\frac{g_{\sigma q}^2}{4\pi}\frac{e^{-\mu_\sigma r_{ij}}}{r_{ij}} + \frac{g_{\omega q}^2}{4\pi}\frac{e^{-\mu_\omega r_{ij}}}{r_{ij}} + \frac{\sigma_i^3\sigma_j^3}{4}\frac{g_{\rho q}^2}{4\pi}\frac{e^{-\mu_\rho \hat{r}_{ij}}}{\hat{r}_{ij}}$$

pauli interaction

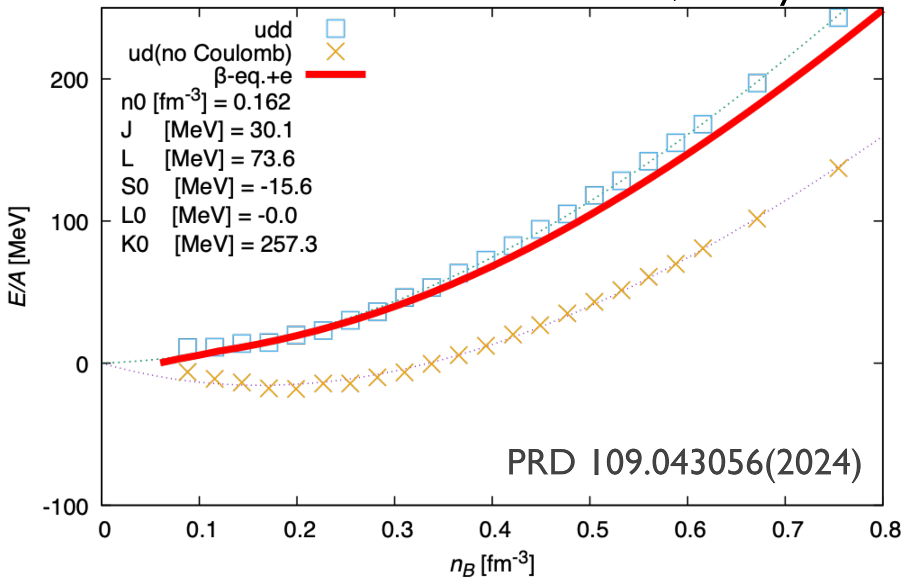
$$\langle V_{\rm pauli}(r)\rangle \equiv \frac{C_p}{(q_0p_0)^3}\exp\left[-\frac{(\mathbf{R}_i-\mathbf{R}_j)^2}{2q_0^2} - \frac{(\mathbf{P}_i-\mathbf{P}_j)^2}{2p_0^2}\right]\delta_{\chi_i\chi_j}$$

Preliminary results

Color magnetic interaction and EOS

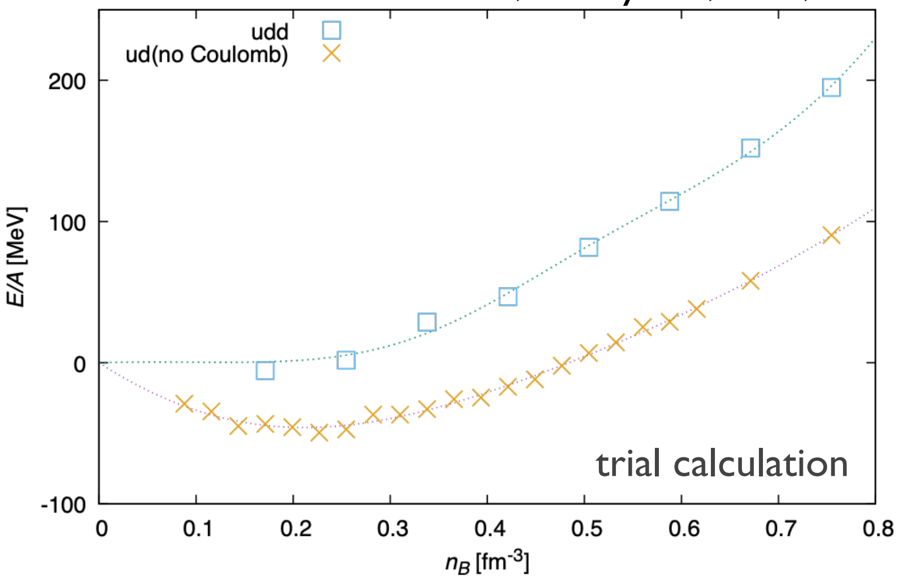
without color-magnetic interaction

NY, Maruyama



with color-magnetic interaction

NY, Maruyama, Park, Lee



? ⇂ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ → ? ⇂ ? ? $\begin{pmatrix} e^{-i\beta_i^{(s)}} \cos\alpha_i^{(s)} \\ e^{i\beta_i^{(s)}} \sin\alpha_i^{(s)} \end{pmatrix}$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \underline{\lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j,}$$

time dependent

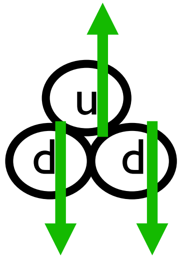
Preliminary results

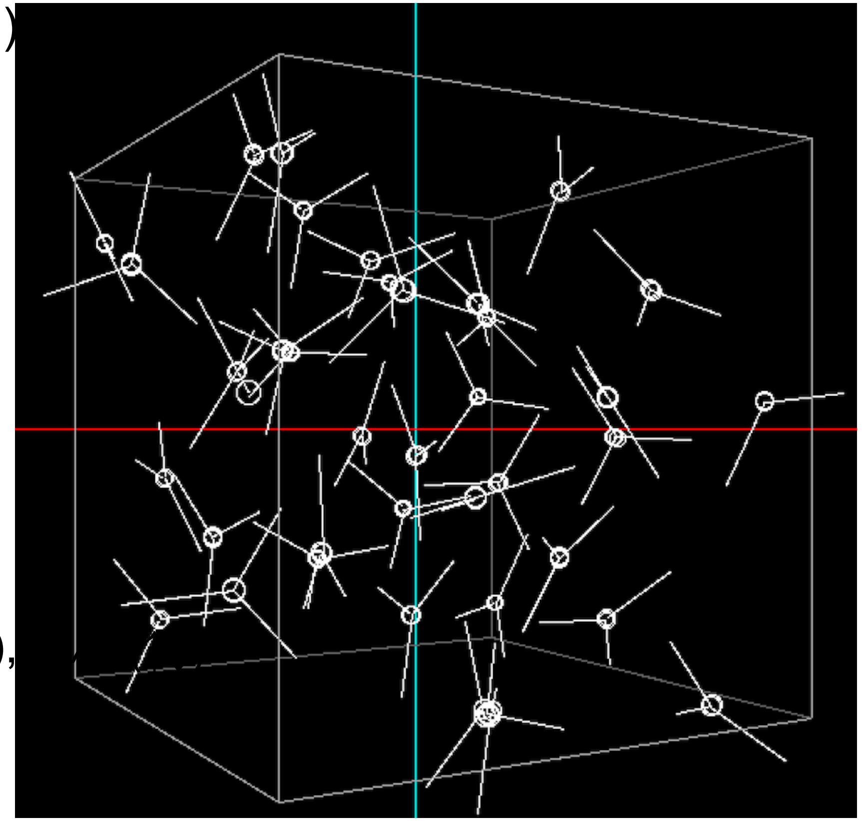
Color magnetic interaction and spins

NY, Maruyama, Park, Lee

3-quark system

<?↓??/2 · ?↓??/2 >=1/4 (↑↑)

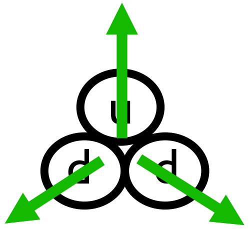




Around saturation density
with periodic boundary (infinite system)

Large N-quark system

<?↓??/2 · ?↓??/2 >=1/4 (↑↑),



Mercedes-Benz

Color magnetic interaction for N quarks

Jaffe, PRD 15, 281 (1978),   Oka & Yazaki, PTP 66, 556 15, 281 (1981)

$$-\sum_{i\neq j}^{N}\{\underset{\sim}{\lambda}\vec{\sigma}\}_i\cdot\{\underset{\sim}{\lambda}\vec{\sigma}\}_j \;=\; 8N-\tfrac{1}{2}C_6^N+\tfrac{4}{3}S_N(S_N+1)+C_3^N$$

where we use quadratic Casimir operators

$$C_6^N=\sum_{r=1}^{35}\Big(\sum_{i=1}^{N}\mu_i^r\Big)^2\;,\quad C_2^N=4S_N(S_N+1)=\sum_{k=1}^{3}\Big(\sum_{i=1}^{N}\sigma_i^k\Big)^2,\quad C_3^N=\sum_{a=1}^{8}\Big(\sum_{i=1}^{N}\lambda_i^a\Big)^2.$$

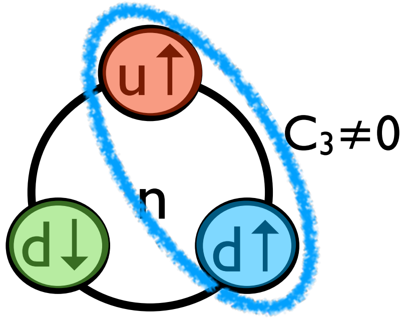
e.g.

$n\cdots$ N=3, C_6=33*2, S=1/2, C_3=0 ⇨ -8

$\Delta\cdots$ N=3, C_6=21*2, S=3/2, C_3=0 ⇨ +8

But what about N>>3 ?

In our molecular dynamics,
we need 2-body effective interaction
corresponding N-body systems.



$n\cdots$ Σ sij=-1/2-1/2+1/4=-3/4=-S(S+1)

QUARK MODEL

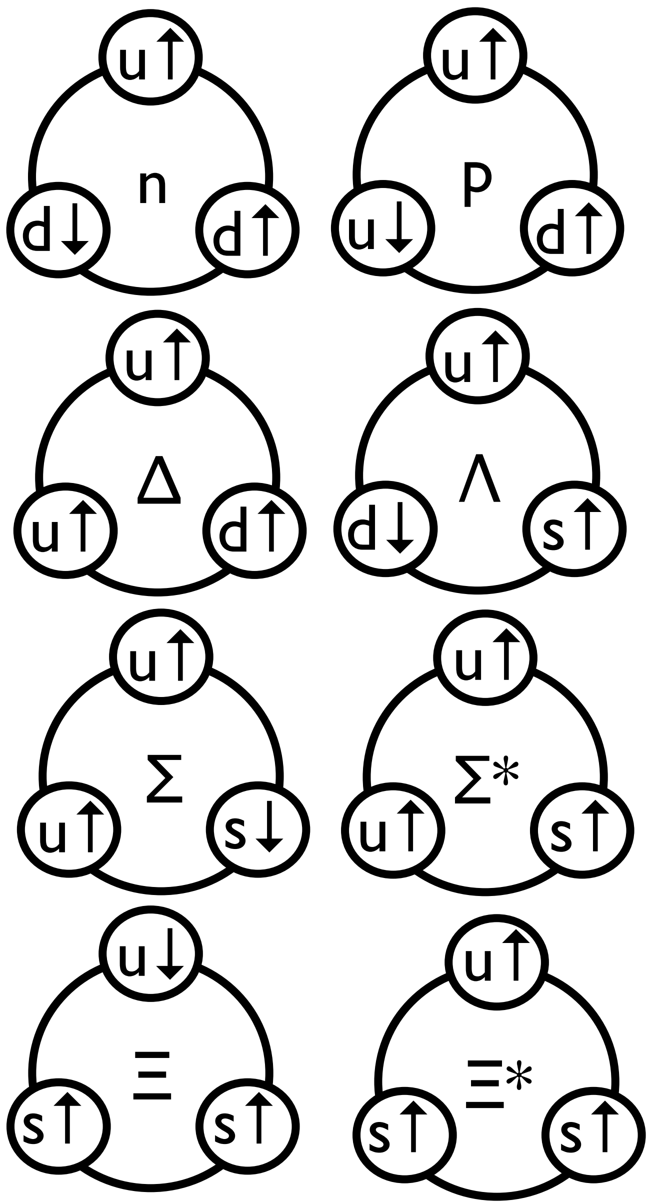
$$M = m_1 + m_2 + m_3 + b \sum_{i<j} \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j} \qquad i,j = 1,2,3$$

QQQ	mass	expectation	experiment
n	$3m - \frac{3b}{4m^2}$	939	938
Δ	$3m + \frac{3b}{4m^2}$	1232	1232
Λ	$2m + m_s - \frac{3b}{4m^2}$	1116	1115
Σ	$2m + m_s + \frac{b}{4}\left(\frac{1}{m^2} - \frac{4}{mm_s}\right)$	1180	1189
Σ^*	$2m + m_s + \frac{b}{4}\left(\frac{1}{m^2} + \frac{2}{mm_s}\right)$	1377	1382
Ξ	$m + 2m_s + \frac{b}{4}\left(\frac{1}{m_s^2} - \frac{4}{mm_s}\right)$	1331	1315
Ξ^*	$m + 2m_s + \frac{b}{4}\left(\frac{1}{m_s^2} + \frac{2}{mm_s}\right)$	1528	1532

* b → strength of color interactions (calculated by CMD).
* coefficients → from sum rule of spins
* Λ → obtained from following (3rd eq.)

$$M_\Delta - M_N \sim \delta(\mathrm{color-spin\ correlations})$$
$$M_\Delta + M_N \sim 6m$$
$$M_\Lambda - M_N \sim m_s - m$$

configurations set (fixed)



Physical properties by MD

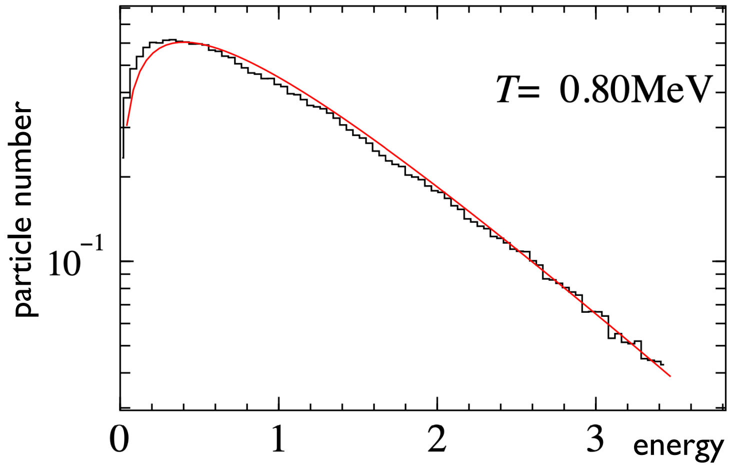
We can obtain physical quantities
by Green-Kubo formula.
(1st. oder of fluctuation)

$$\alpha = \int_0^\infty \left\langle \dot{U}(t)\dot{U}(0) \right\rangle dt.$$

e.g. thermal conductivity λ

$$\begin{aligned} \alpha &= VT^2\lambda, \\ U_x(t) &= \sum_{i=1}^{N} x_i(t)E_i(t), \\ \dot{U}_x(t) &= \frac{d}{dt}\sum_{i=1}^{N} x_i(t)E_i(t). \end{aligned}$$

Statistical temperature from CMD is consistent with theoretical distribution.



→(Proto)Neutron Star coolings / evolutions

Thermal conduction in strong magnetic field

$$c_v e^{\Phi}\frac{\partial T}{\partial t} + \nabla\cdot(e^{2\Phi}\boldsymbol{F}) = e^{2\Phi}Q$$

$$\boldsymbol{F}_e = -e^{\Phi}\kappa_e^{\perp}\left[\nabla\tilde{T} + (\omega_B\tau)^2\left(\boldsymbol{b}\cdot\nabla\tilde{T}\right)\cdot\boldsymbol{b} + \omega_B\tau\left(\boldsymbol{b}\times\nabla\tilde{T}\right)\right]$$

Geppert et al.2004

the thermal conductivity

$$\kappa = \begin{pmatrix} \kappa_\perp & \kappa_\wedge & 0 \\ -\kappa_\wedge & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

here

$$\kappa_0 = \frac{1}{3}c_v\bar{v}^2\tau = \frac{\pi^2 k_B^2 T n_e}{3m_e^*}\tau$$

$$\kappa_\parallel = \kappa_0$$

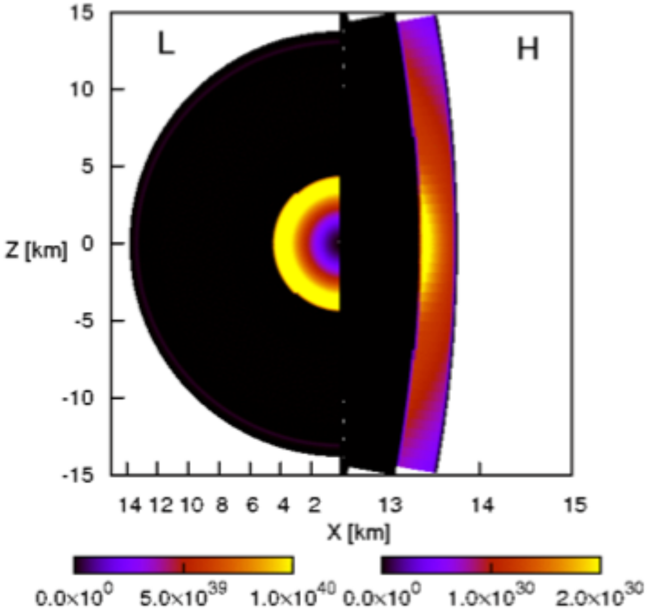
$$\kappa_\perp = \frac{\kappa_0}{1+(\omega_B\tau)^2}$$

$$\kappa_\wedge = \frac{\kappa_0\,\omega_B\tau}{1+(\omega_B\tau)^2}$$

- implicit scheme
- operator splitting
- neglect induction equation

cooling rate(L) & heating rate(H)

hyperon matter



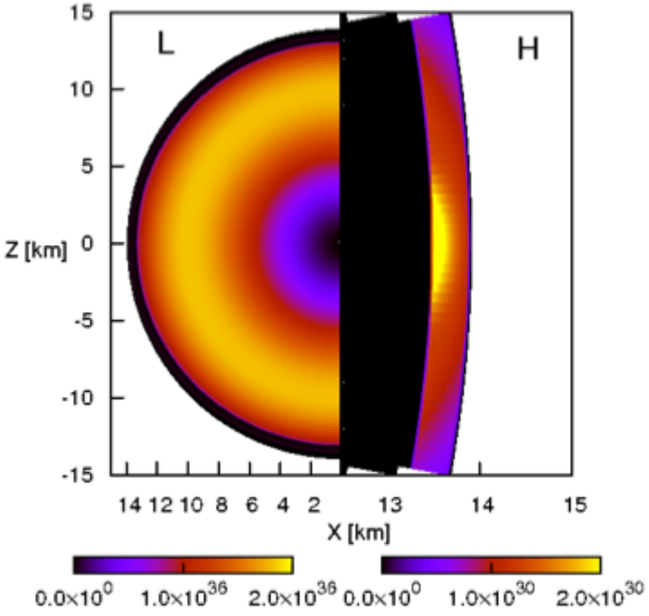


TABLE III: Cooling ratio in the cores and crusts we adopt
The details are shown in the references.

process	ratio	referenc
Core		
"Modified URCA processes (n-branch)"		
$nn \rightarrow nn\nu\bar{\nu}$		
$pne \rightarrow nn\bar{\nu}_e$	$8\times10^{21}(n_p)^{1/3}T_9^8$	[31]
"Modified URCA processes (p-branch)"		
$nn \rightarrow nn\nu\bar{\nu}$	$7\times10^{19}(n_n)^{1/3}T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1\times10^{20}(n_p)^{1/3}T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7\times10^{19}(n_p)^{1/3}T_9^8$	[31]
"$N-N$ Bremsstrahlung"		
$nn \rightarrow nn\nu\bar{\nu}$	$7\times10^{19}Zn_e^{1/3}T_9^8$	[31]
$np \rightarrow np\nu\bar{\nu}$	$1\times10^{20}Zn_e^{1/3}T_9^8$	[31]
$pp \rightarrow pp\nu\bar{\nu}$	$7\times10^{19}Zn_e^{1/3}T_9^8$	[31]
Crust		
"$e-A$ Bremsstrahlung"		
$e(A,Z) \rightarrow e(A,Z)\nu\bar{\nu}$	$3\times10^{12}Zn_eT_9^8$	[32]
"$N-N$ Bremsstrahlung"		
$nn \rightarrow nn\nu\bar{\nu}$	$7\times10^{19}Zn_e^{1/3}T_9^8$	[32]

Evolution of (Proto)Neutron Stars

Neutron Star evolution:

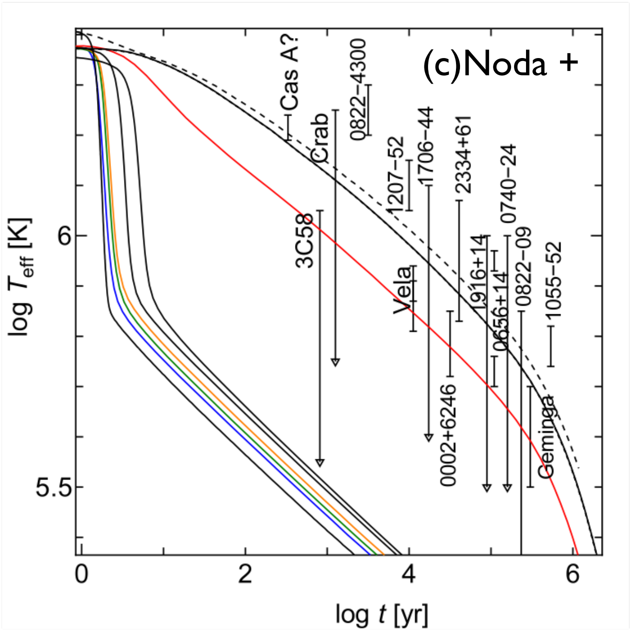
$$c_v e^{\Phi} \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} \boldsymbol{F}) = e^{2\Phi} Q$$

thermal diffusion eq.

heat capacity

flux (thermal conductivity)

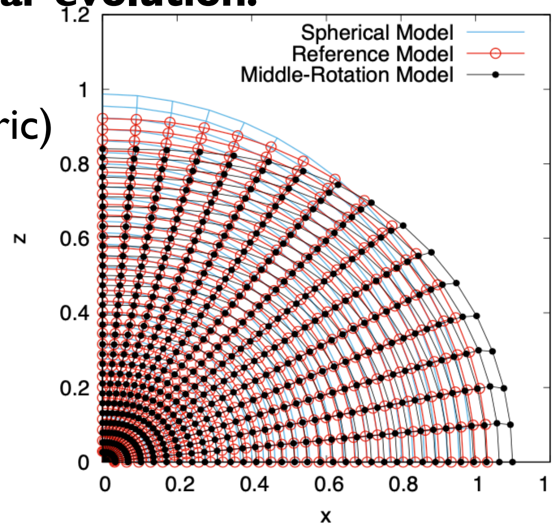
cooling rate (neutrino) + heating rate (magnetic field)

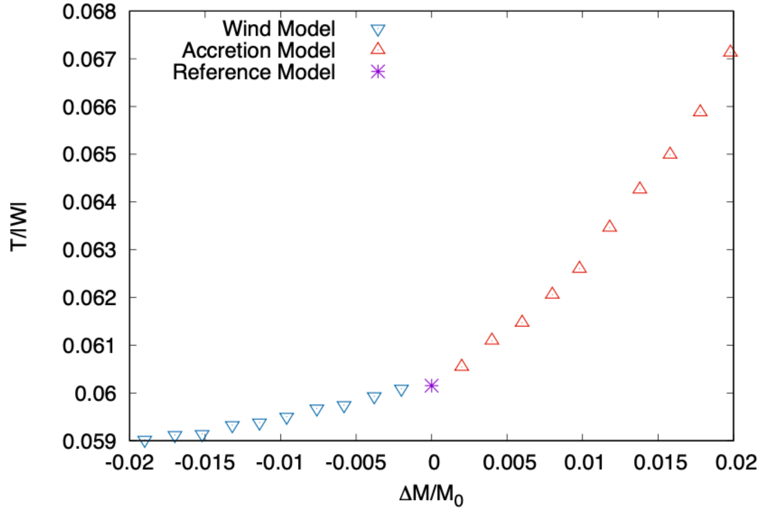


Toward Proto Neutron Star evolution:

・hydrostatic equilibria in GR
・Lagrange scheme(axisymmetric)

+
on going …
・energy eq.
・neutrino transports





Okawa, Fujisawa, **NY**, Ogata, Yamamoto,Yamada (2023) MNRAS

Summary and Discussion

- We present CMD results, which is from QCD to neutron stars (hadron+quarks)
- EOS (MR relation) constraints

  Low density ←nuclear experiments

  High density ← astrophysical observations
- As the results, our CMD simulations provide a neutron star (NS) EoS.
- We find cross over deconfinement.

  GW, Supernovae… ← Finite temperature behavior
- We need more realistic set up: relativistic effects, strangeness effects, vacuum effects.
- Now, we focus on CMD with Color-magnetic interactions.
- Our CMD will provide thermal properties for NS evolution in the future.