Hydrodynamics for symmetry broken phases

Masaru Hongo (Niigata University/RIKEN iTHEMS)

2024/8/20, The XVIth Quark Confinement and the Hadron Spectrum Conference

WHAT and WHY hydro?

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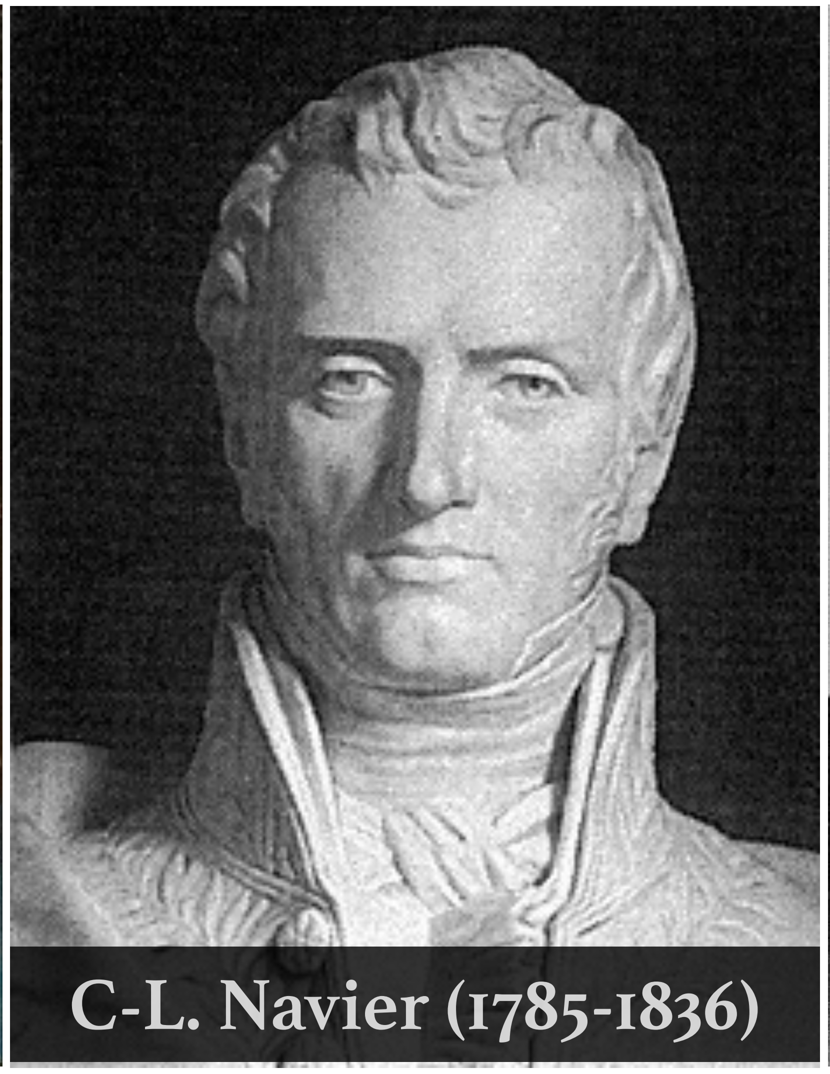

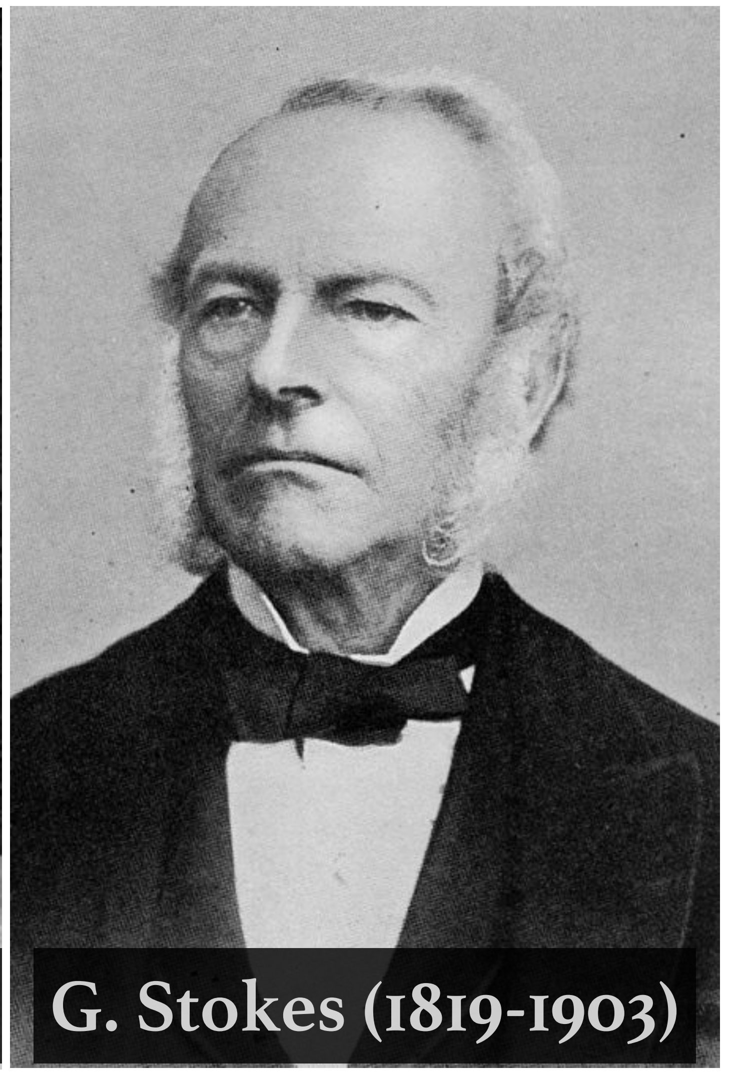

Pascal's law

Hydro*dynamics*

Euler equations
(Perfect fluid)

Navier-Stokes equations
(Viscous fluid)

1600 1700 1800 1900

WHAT and WHY hydro?

Hydrodynamics = Low-energy EFT for real-time dynamics

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- Effective theory for macroscopic dynamics
- Universal description, not depending on details
- Only conserved quantity ~ symmetry of system

[redacted]

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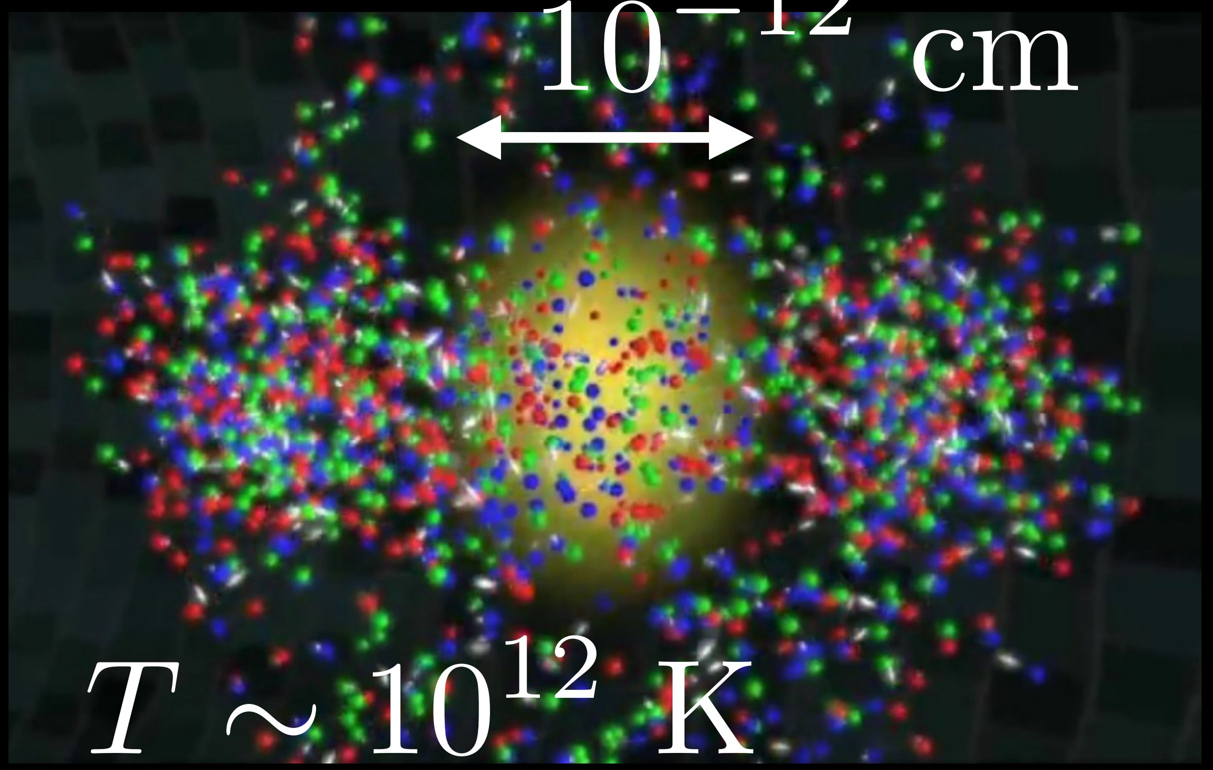
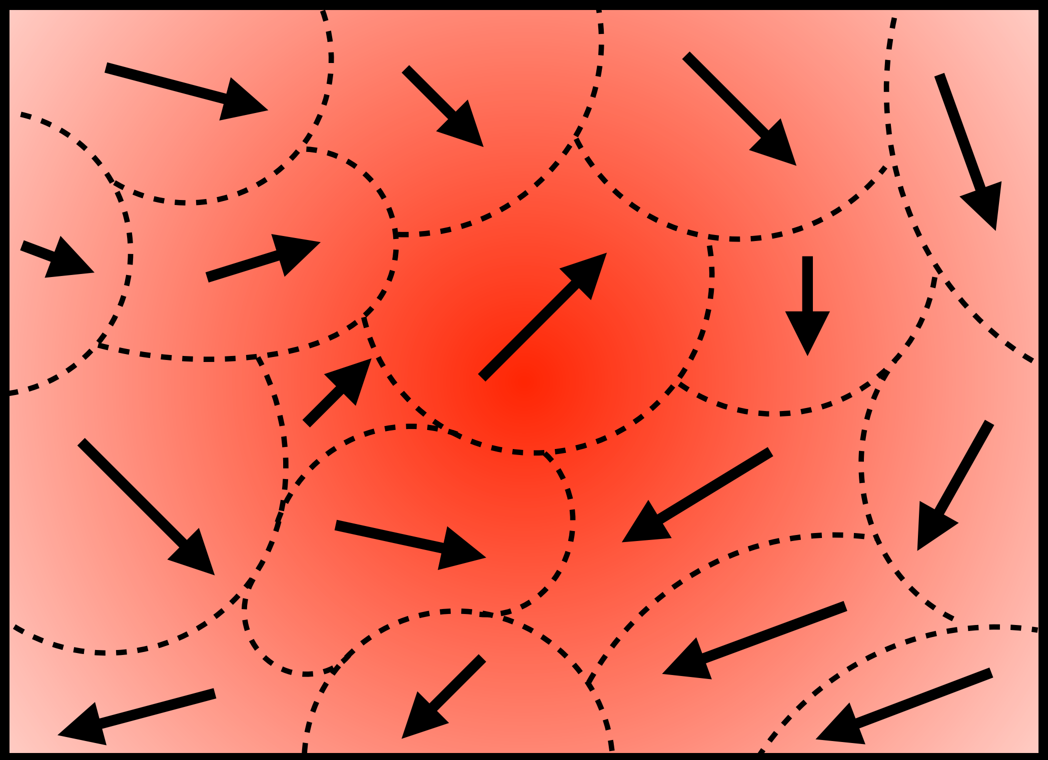
Quark-Gluon Plasma



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

$\simeq$

Hydro: $\{\beta(x),\ \vec{v}(x)\}$



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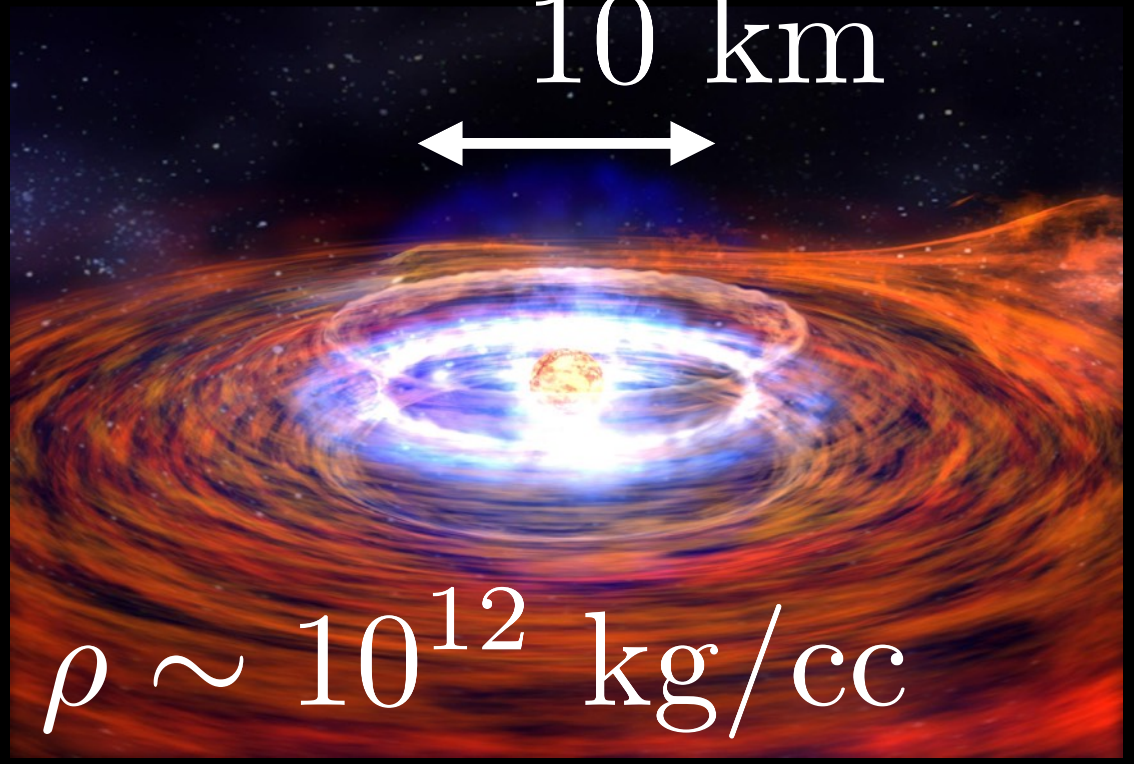
Neutron Star



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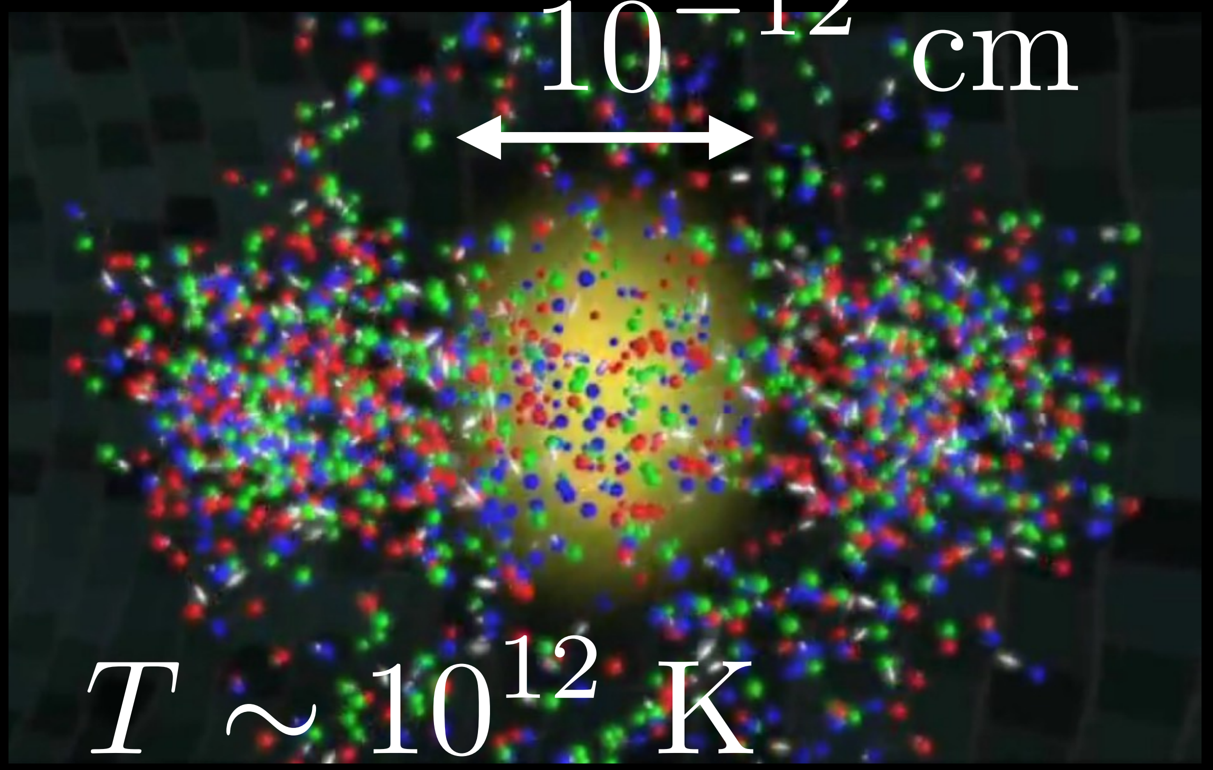

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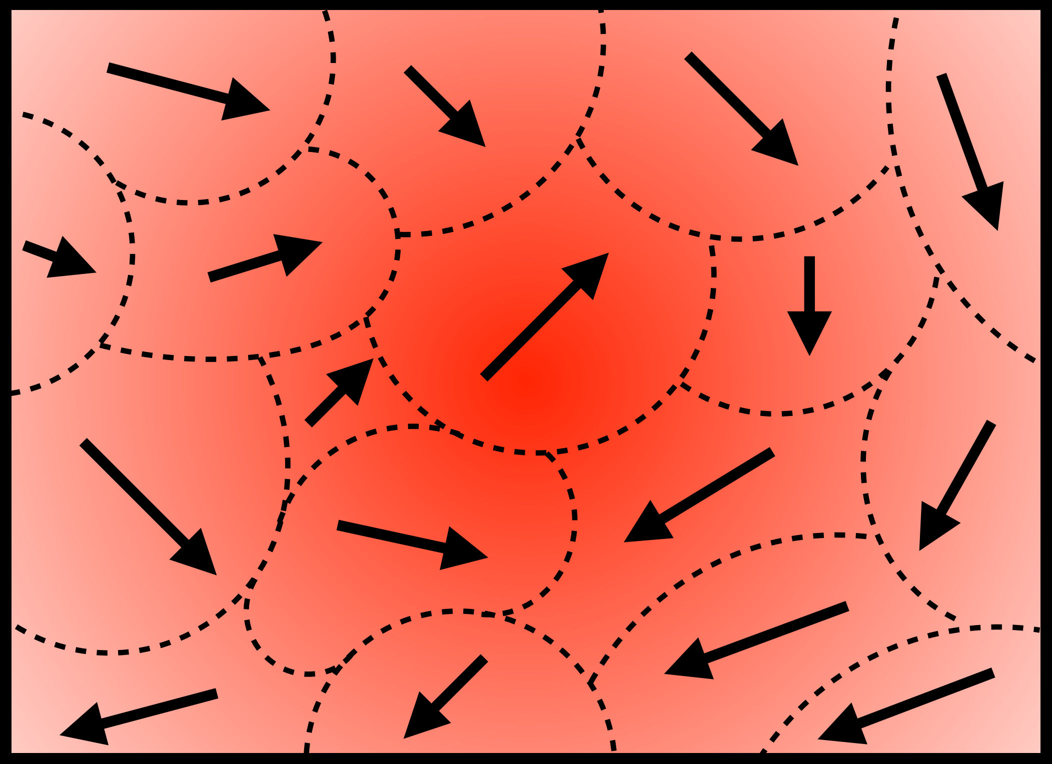

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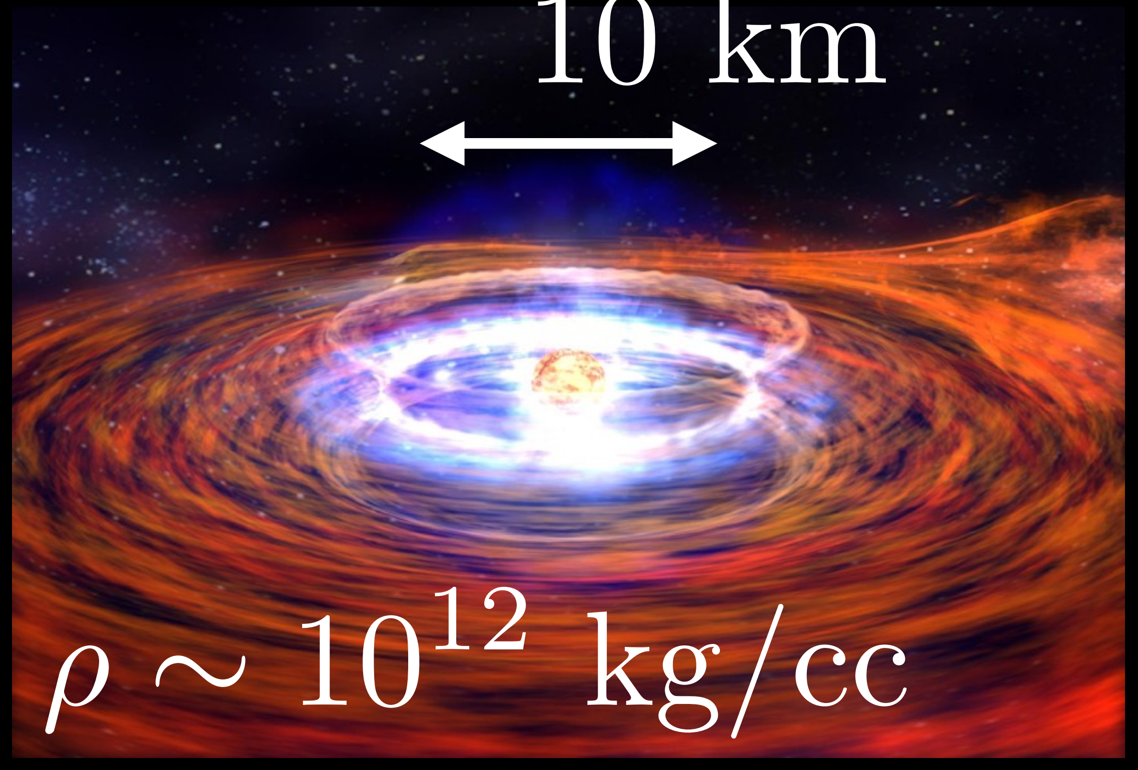

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When continuous global symmetry is spontaneously broken,
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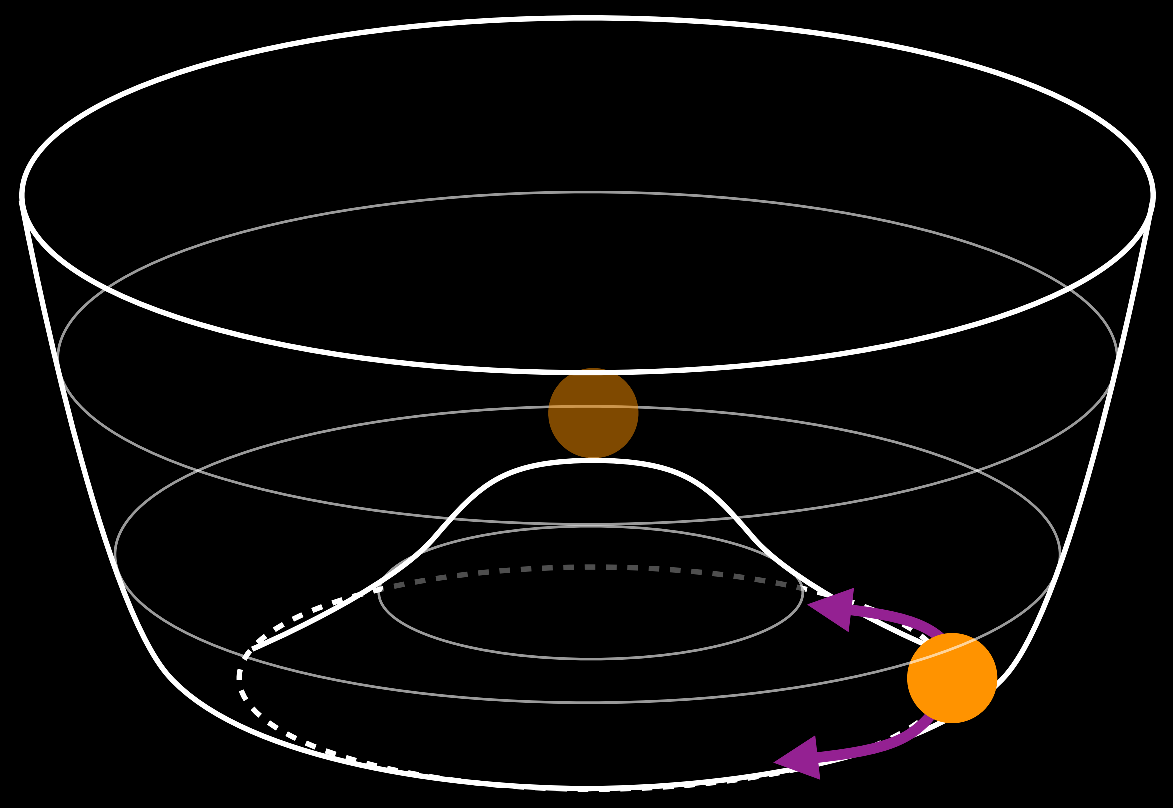

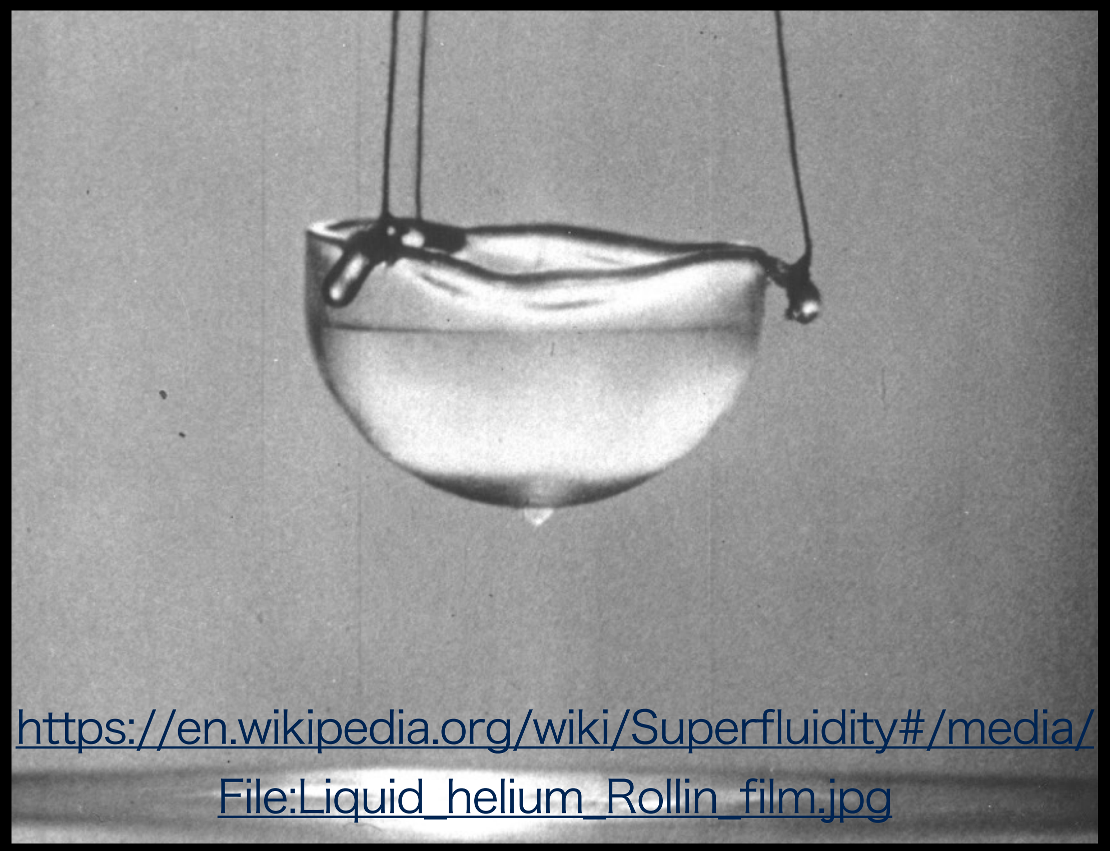

https://en.wikipedia.org/wiki/Superfluidity#/media/File:Liquid_helium_Rollin_film.jpg

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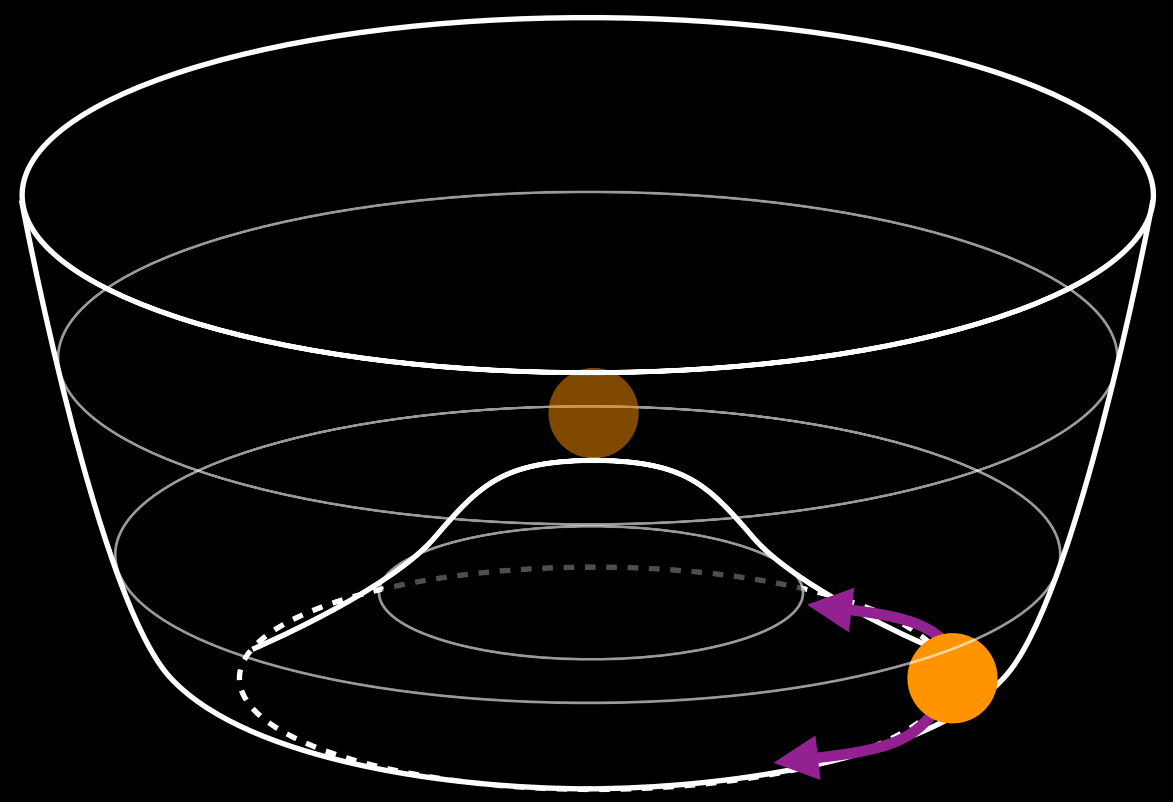
Superfluid Hydrodynamics (Two-fluid model) by Tisza, Landau

General consequence resulting from the Nambu-Goldstone theorem



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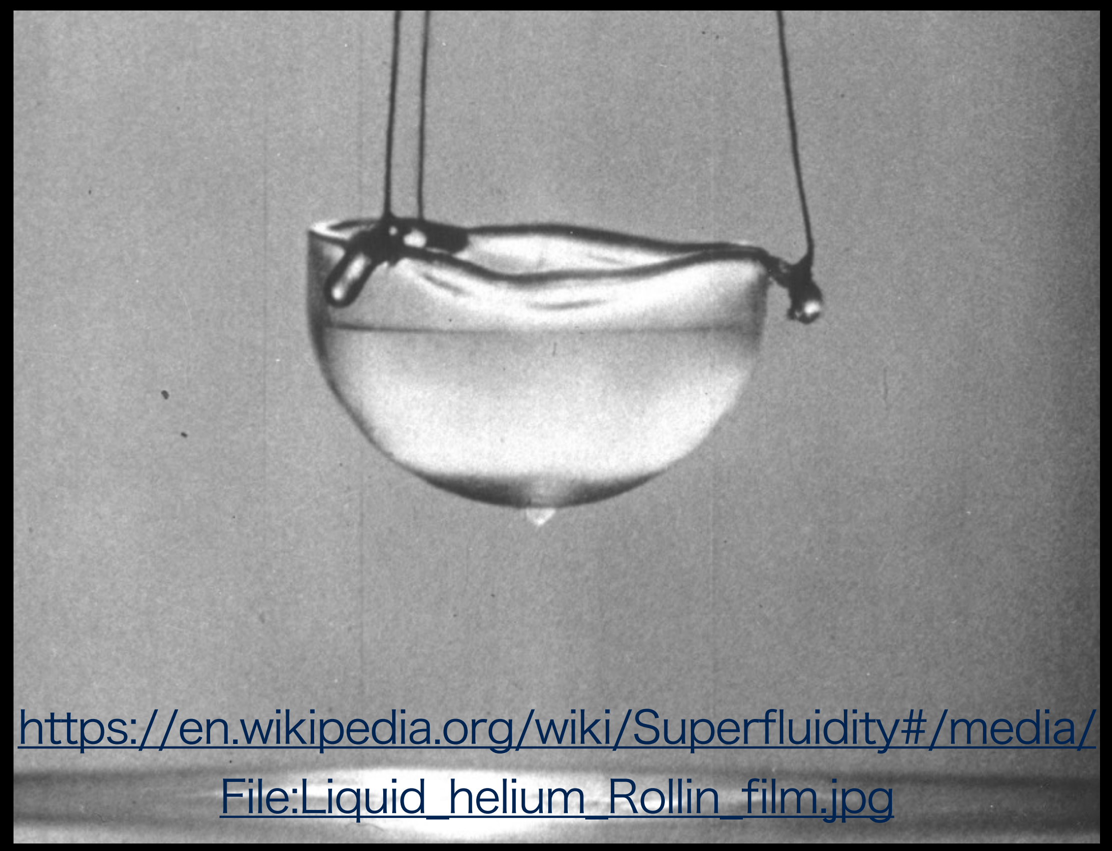

SSB patterns of QCD matter

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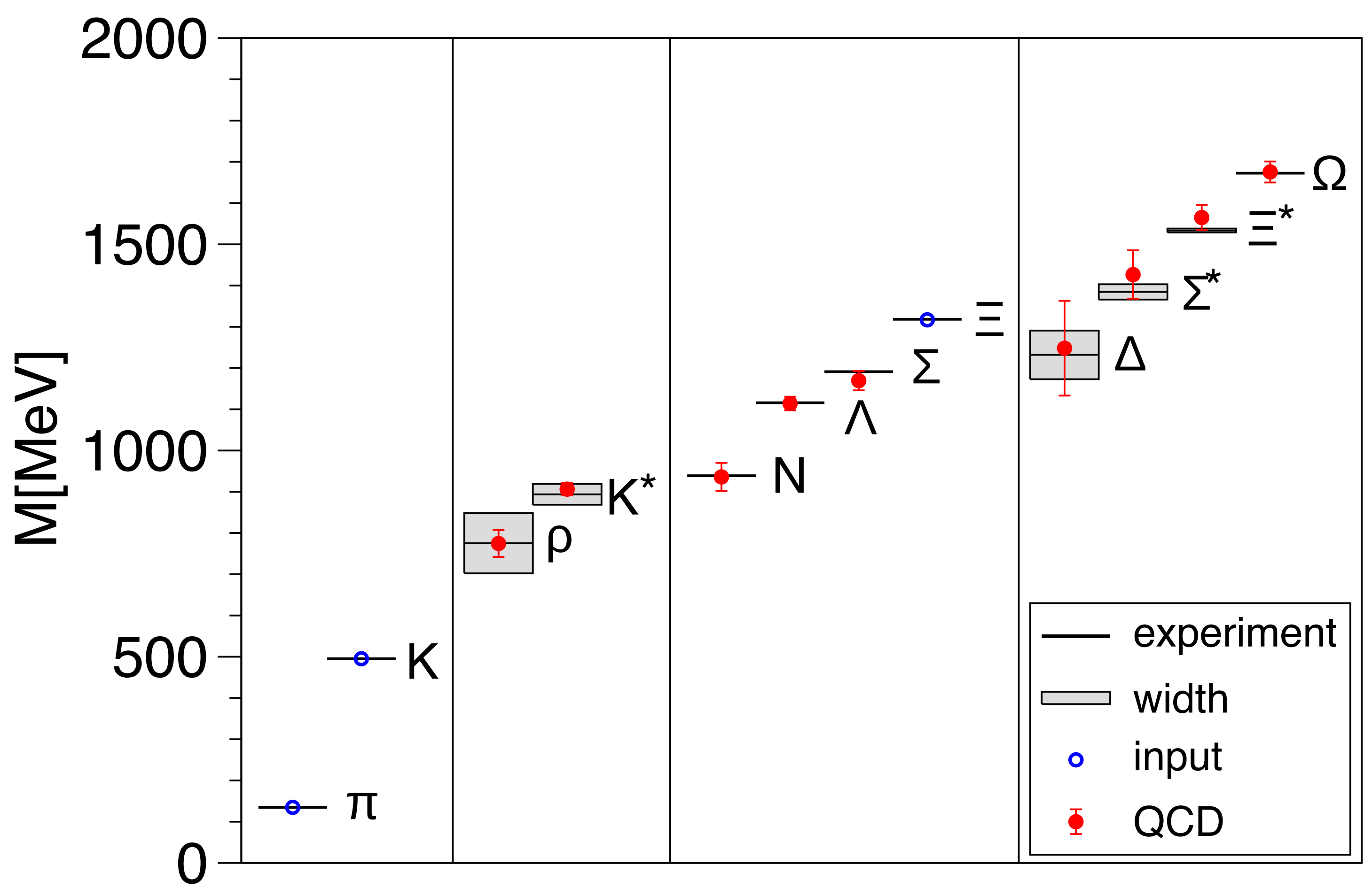

From Budapest-Marseille-Wuppertal Collaboration (2008)

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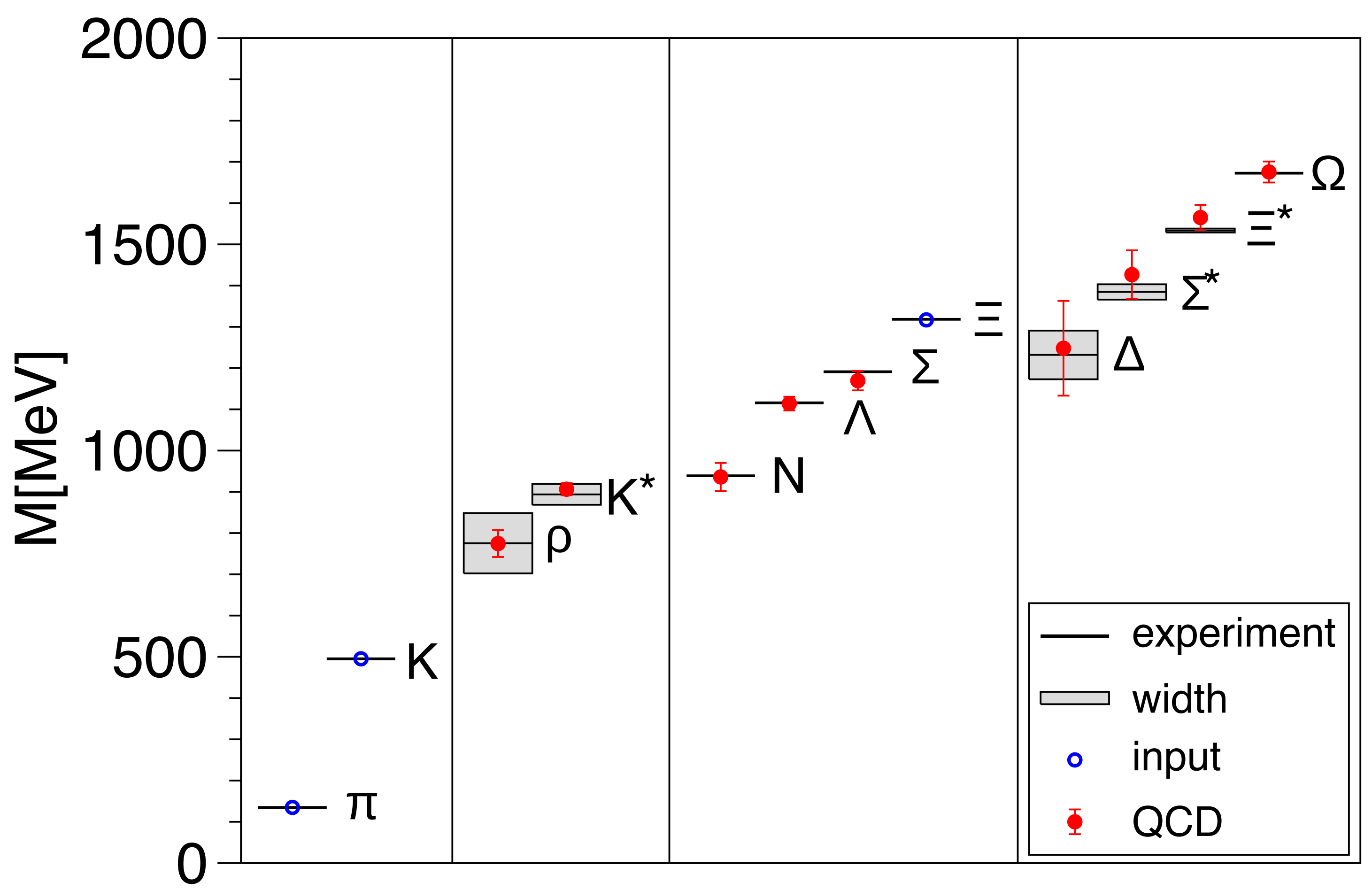

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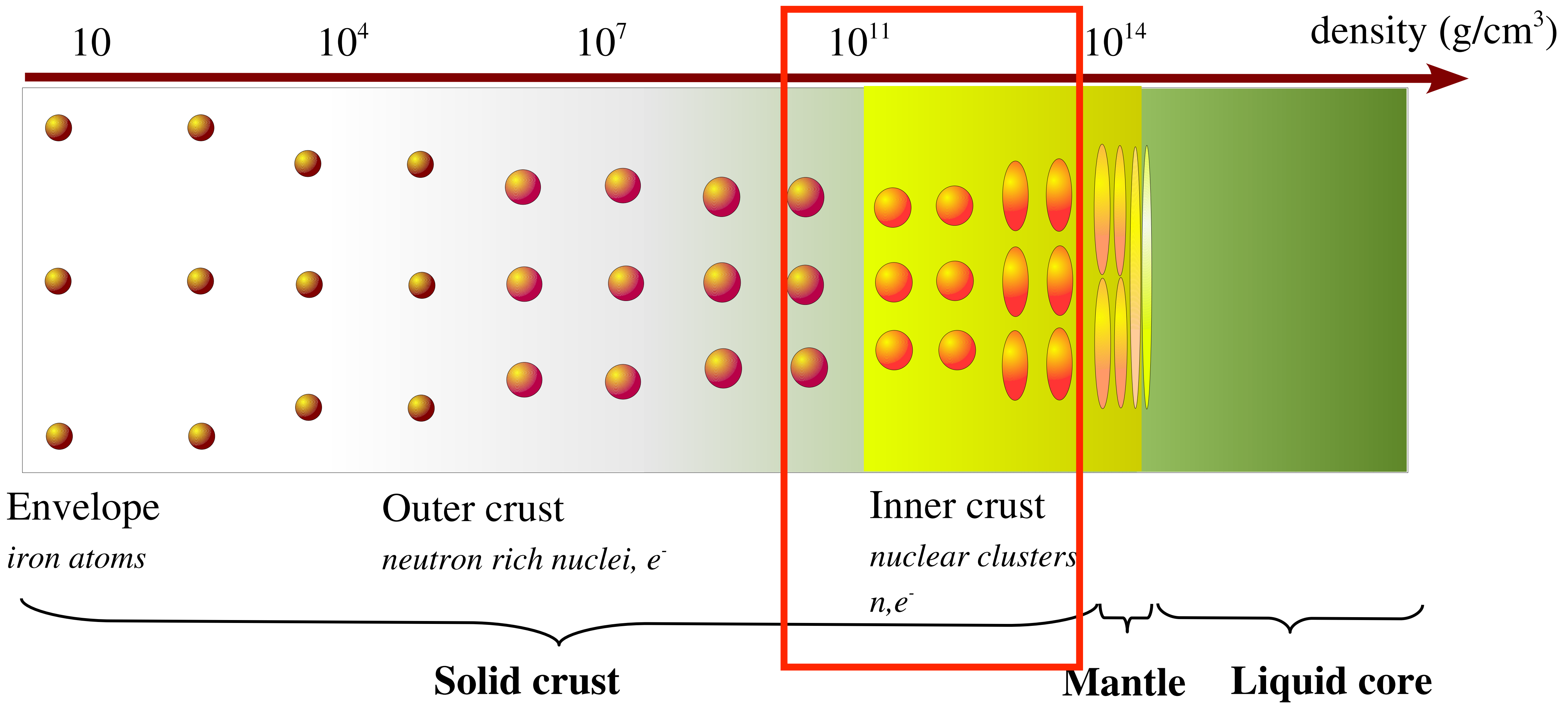

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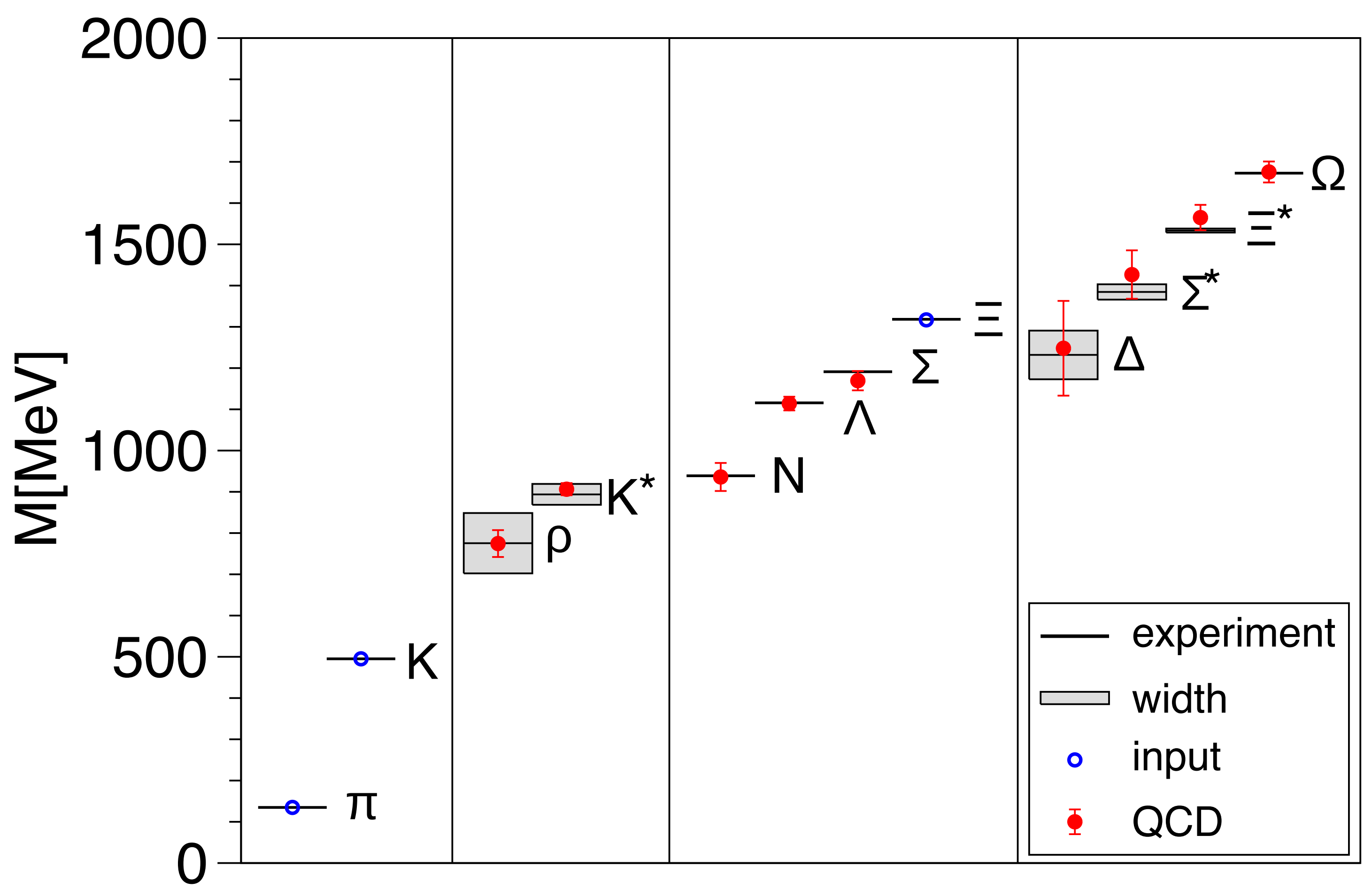

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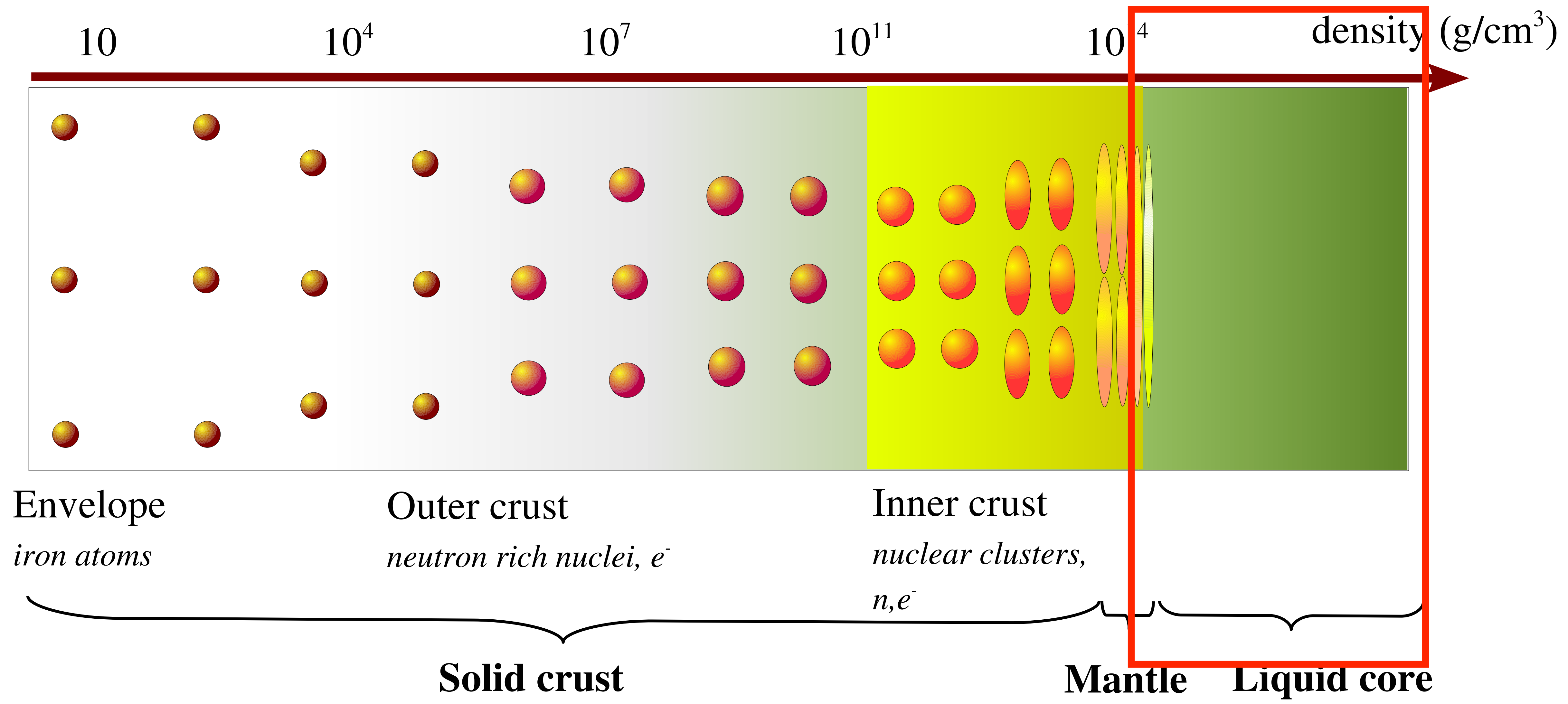

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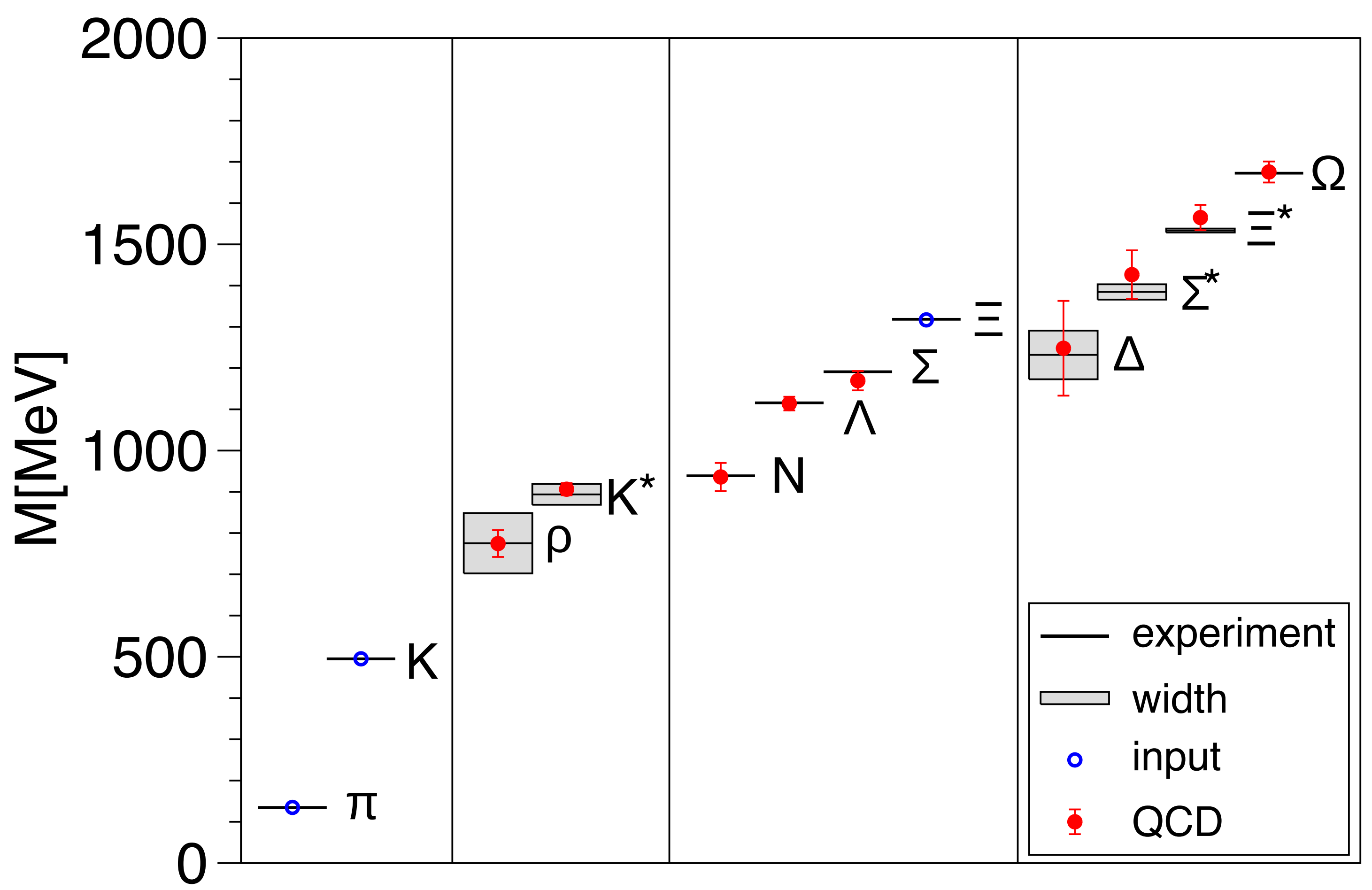
Q. What are hydrodynamic equations in symmetry-broken phases?



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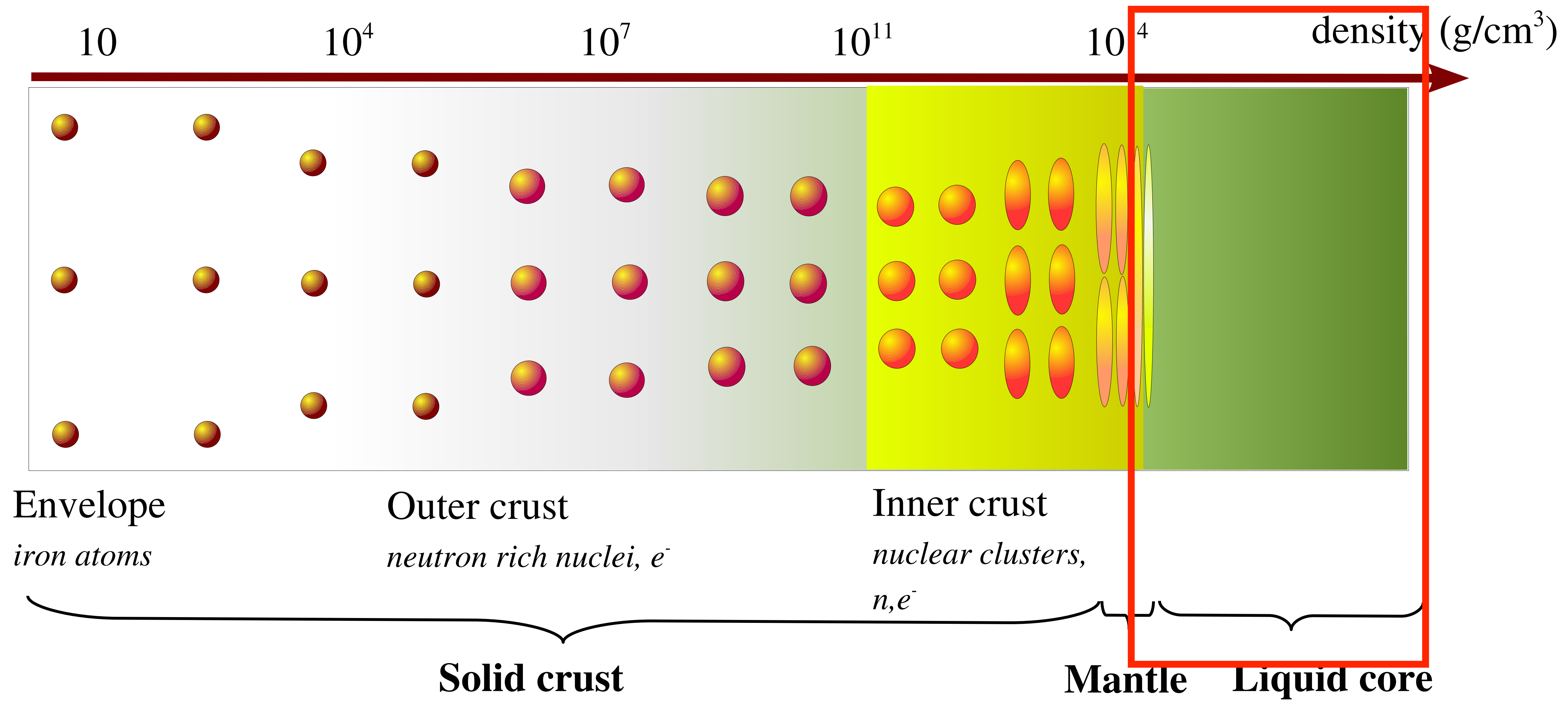

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Outline

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Hydrodynamics for
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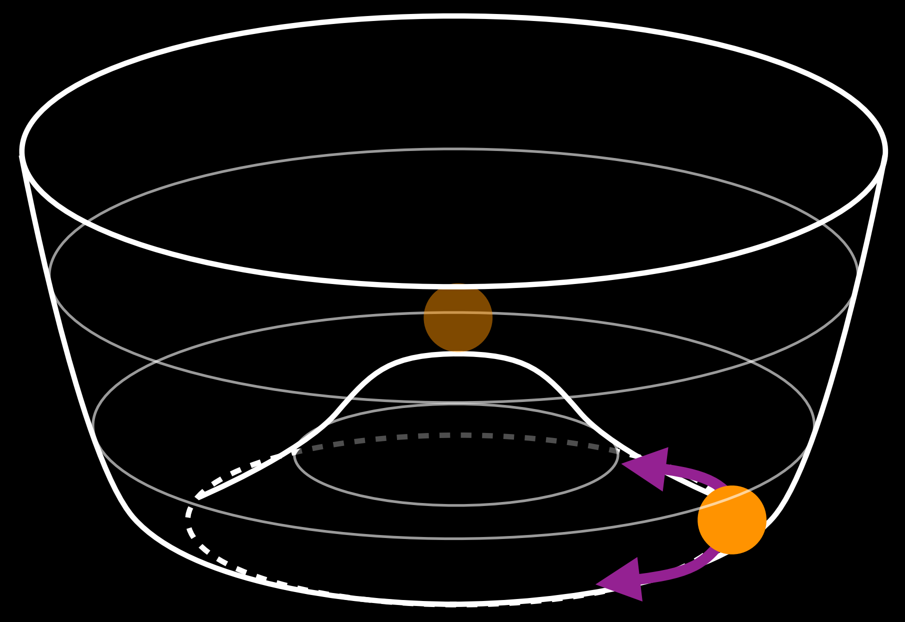

Approach:

Semi-phenomenology based on local thermodynamics

Result & Outlook:

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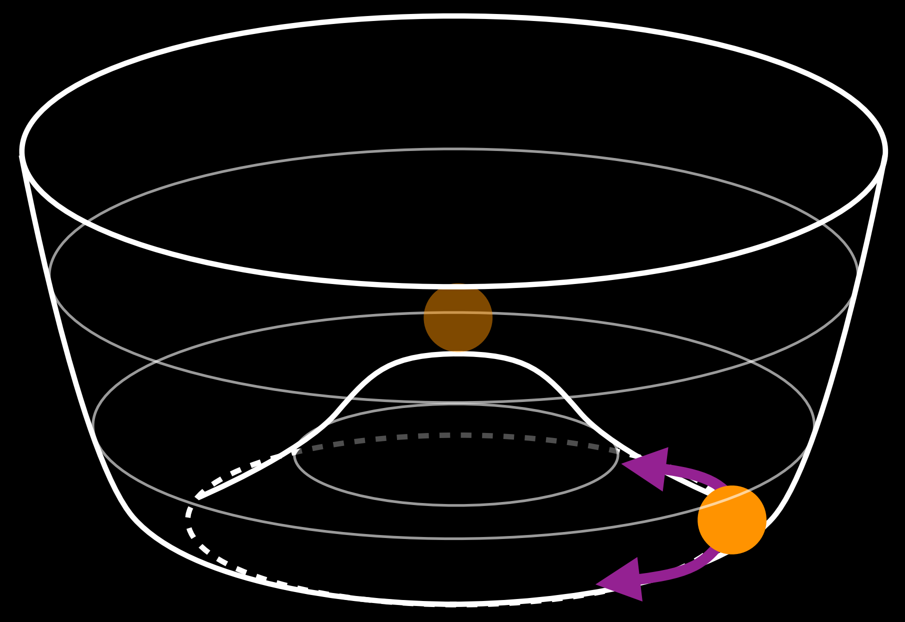

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- **Kinetic-theory derivation based on the Boltzman equation**

  [Tsumura et al, PLB (2007), Denicol et al, PRD (2012), ⋯]

- **Nonequilibrium statistical operator approach**

  [Becattini et al, EPJC (2008), Hayata et al, PRD (2015), ⋯]

- **Holographic-derivation based on fluid/gravity correspondence**

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- **Projection operator/Poisson bracket approach**

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- **Phenomenological derivation based on local thermodynamics**

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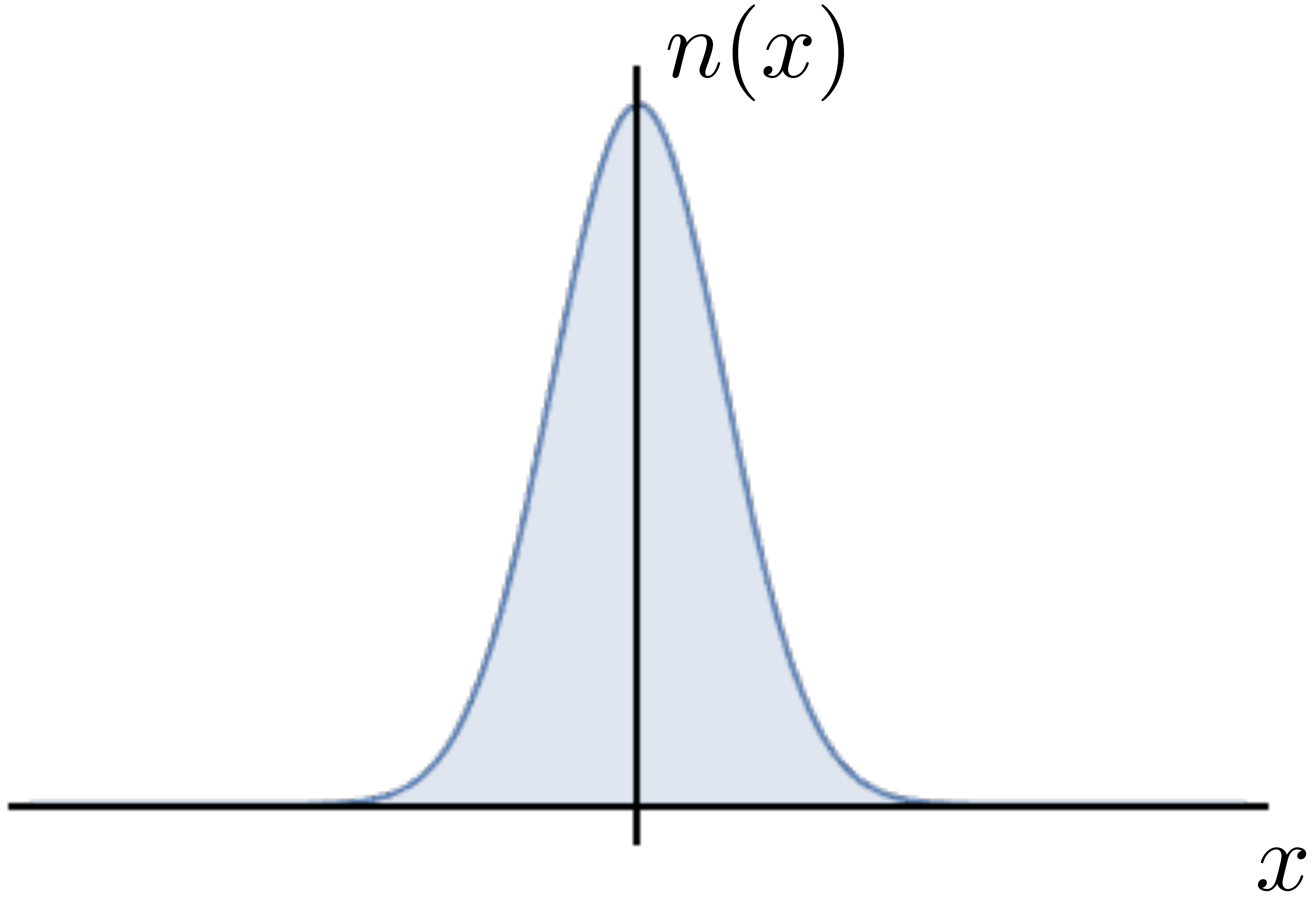

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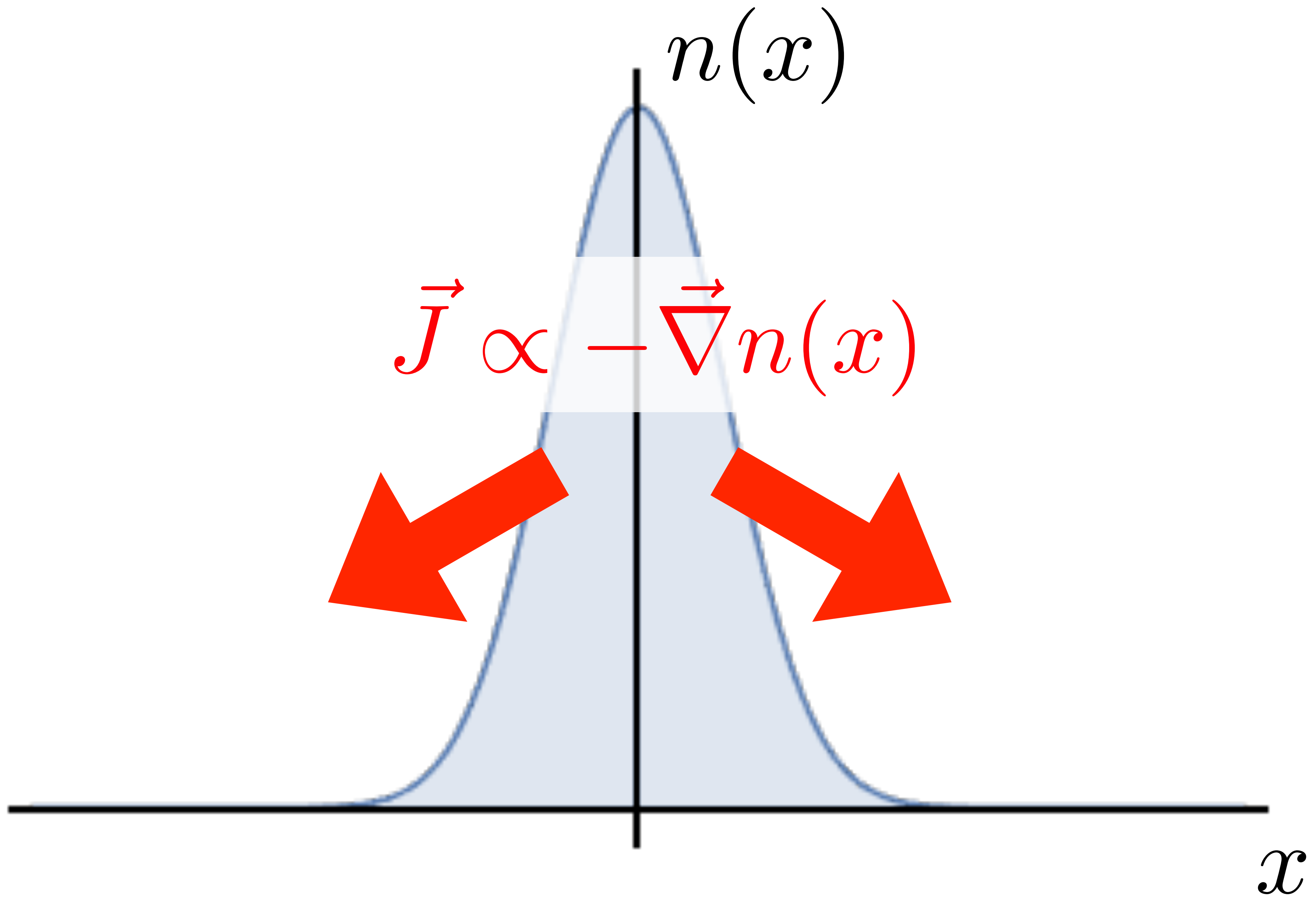

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◆ **Bulding blocks of hydrodynamic equation**

(1) **Conservation law:** $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

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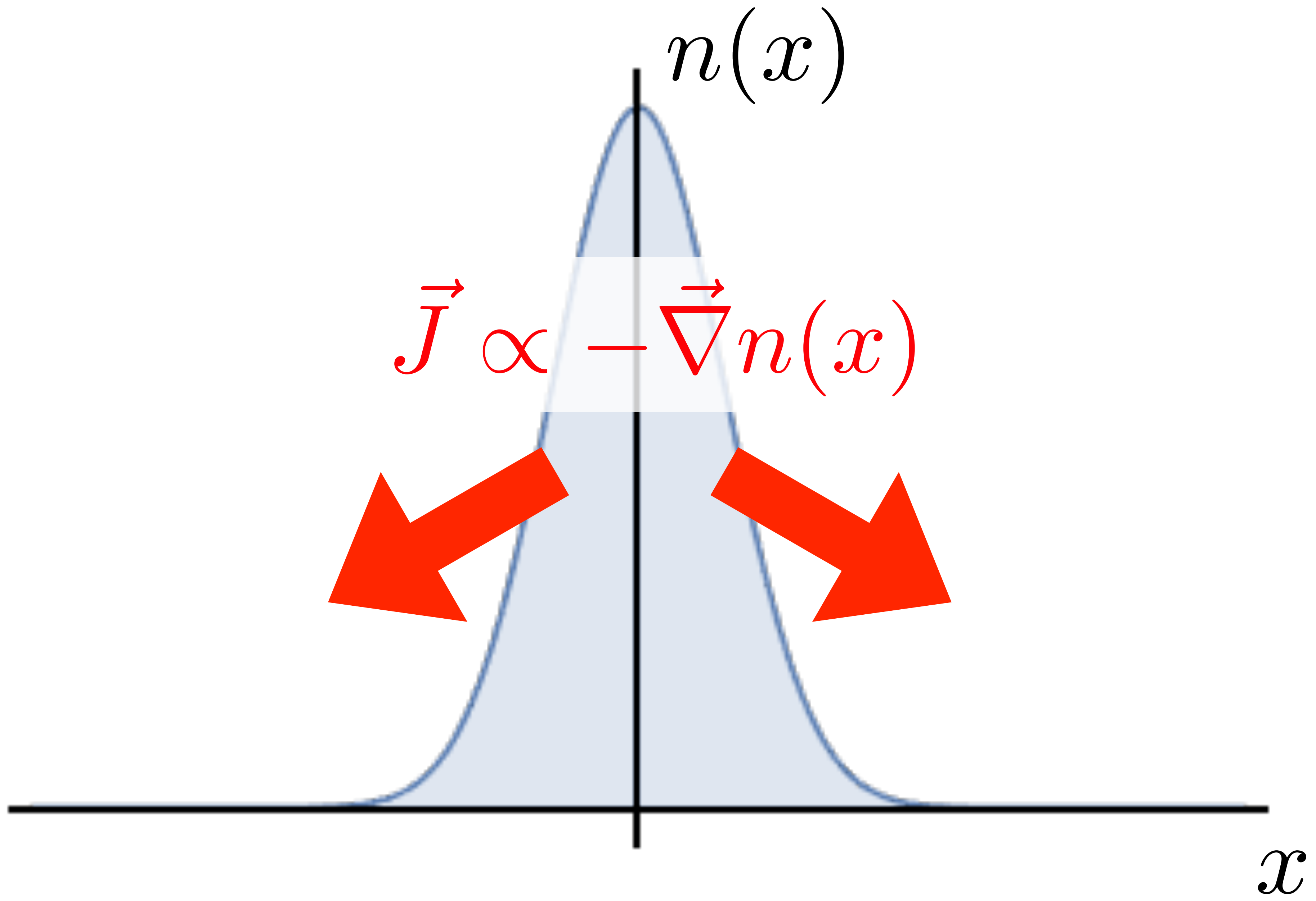

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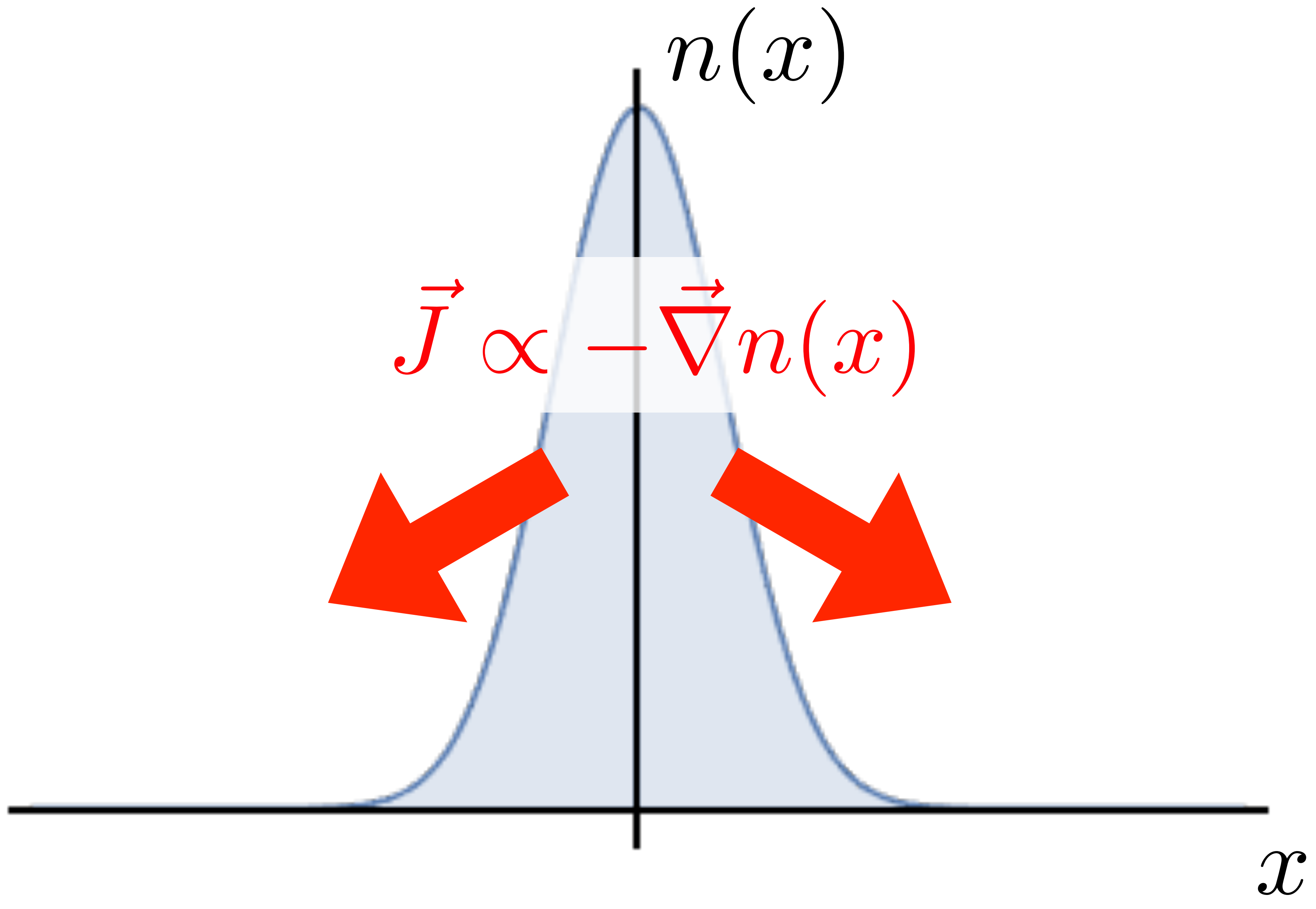

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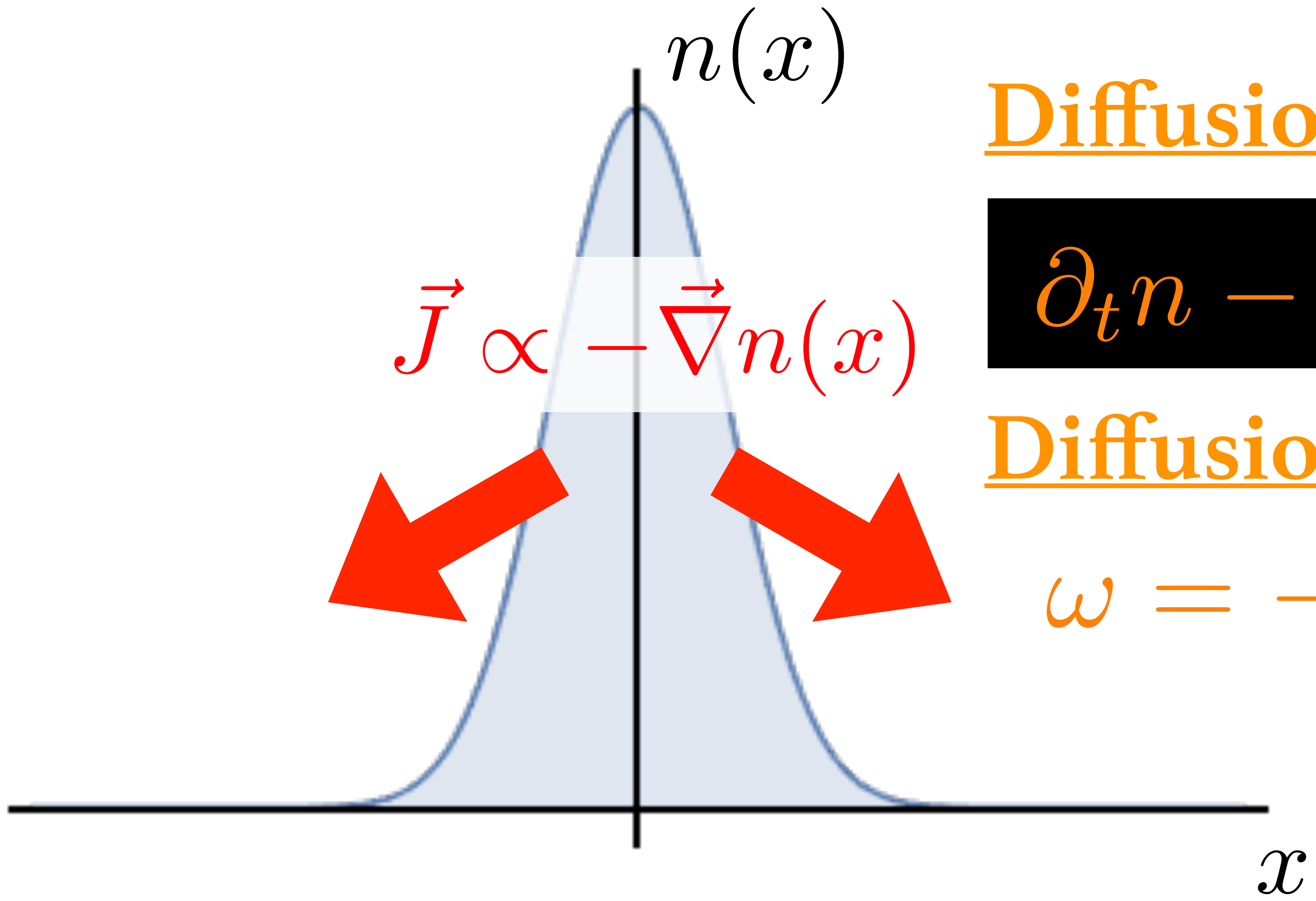
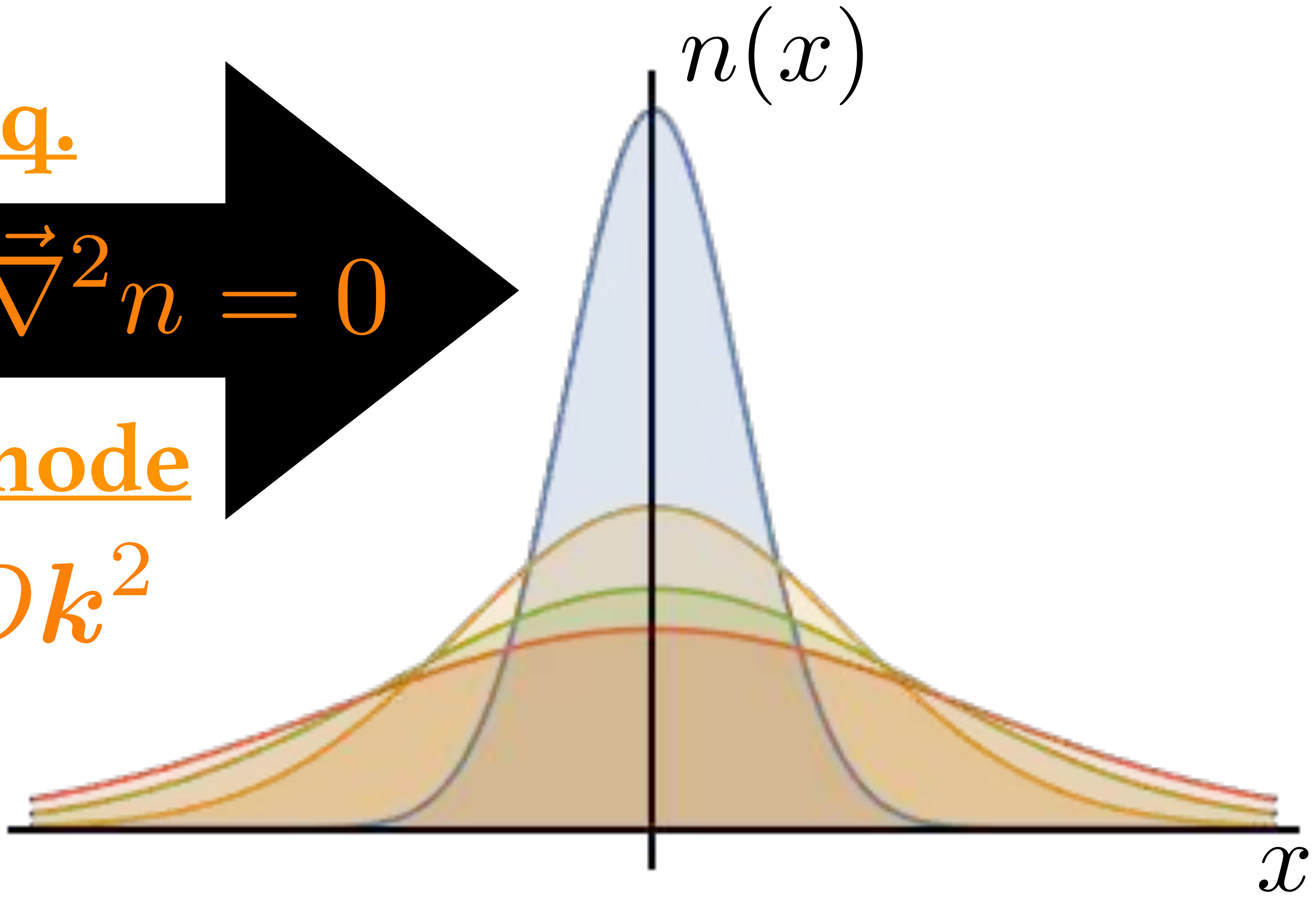


Diffusion eq.

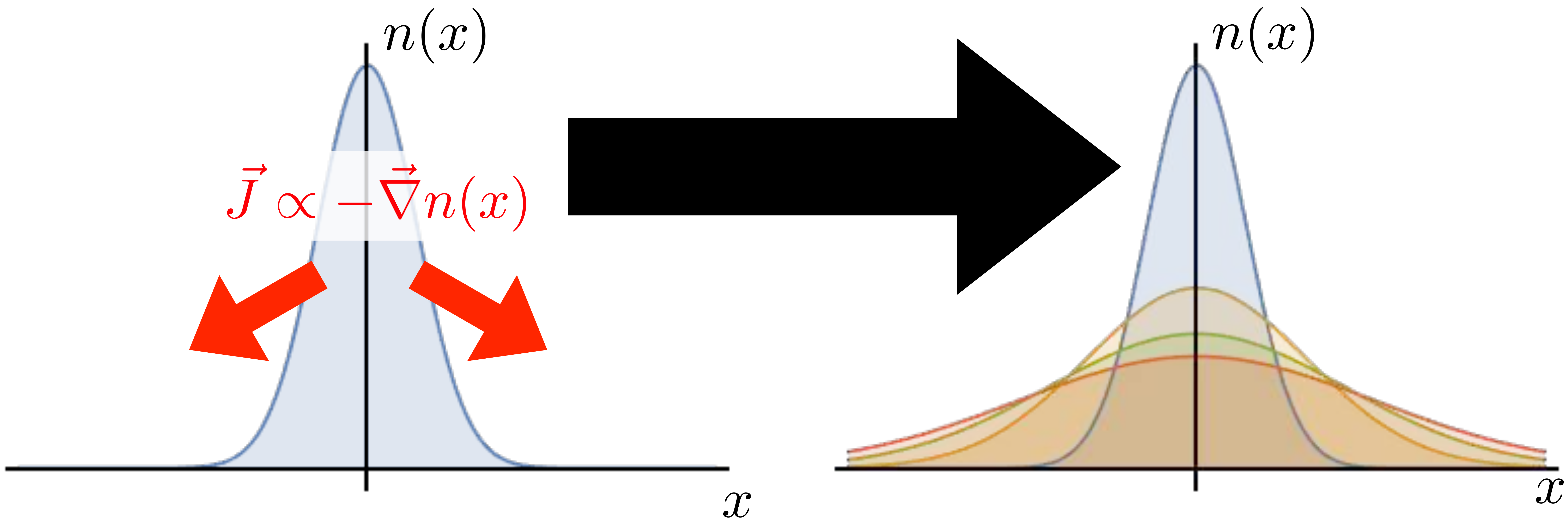
$$\partial_t n - D\vec{\nabla}^2 n = 0$$

Diffusion mode

$$\omega = -\mathrm{i}D\boldsymbol{k}^2$$



Irreversibility of diffusion



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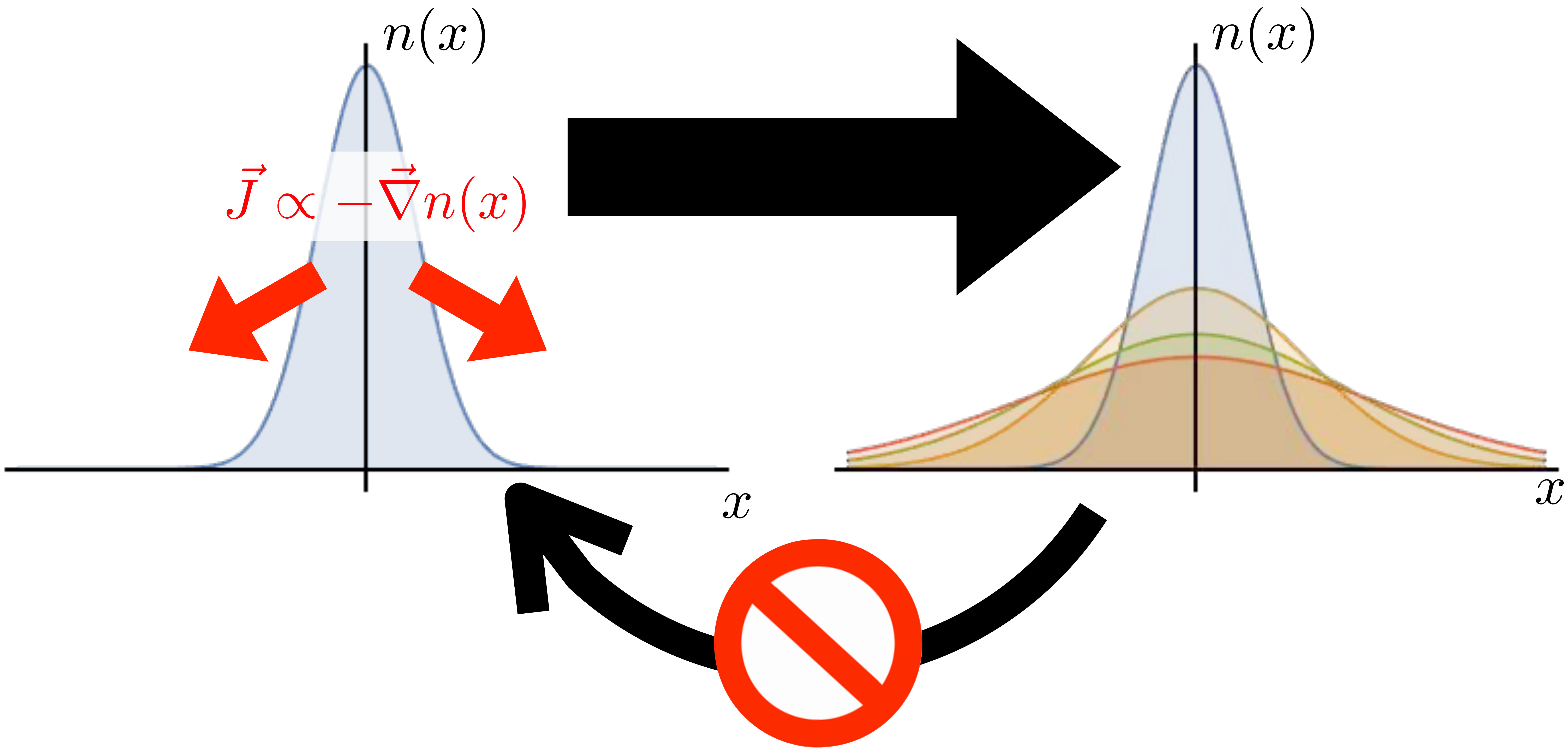

No-go for time-reversal process!

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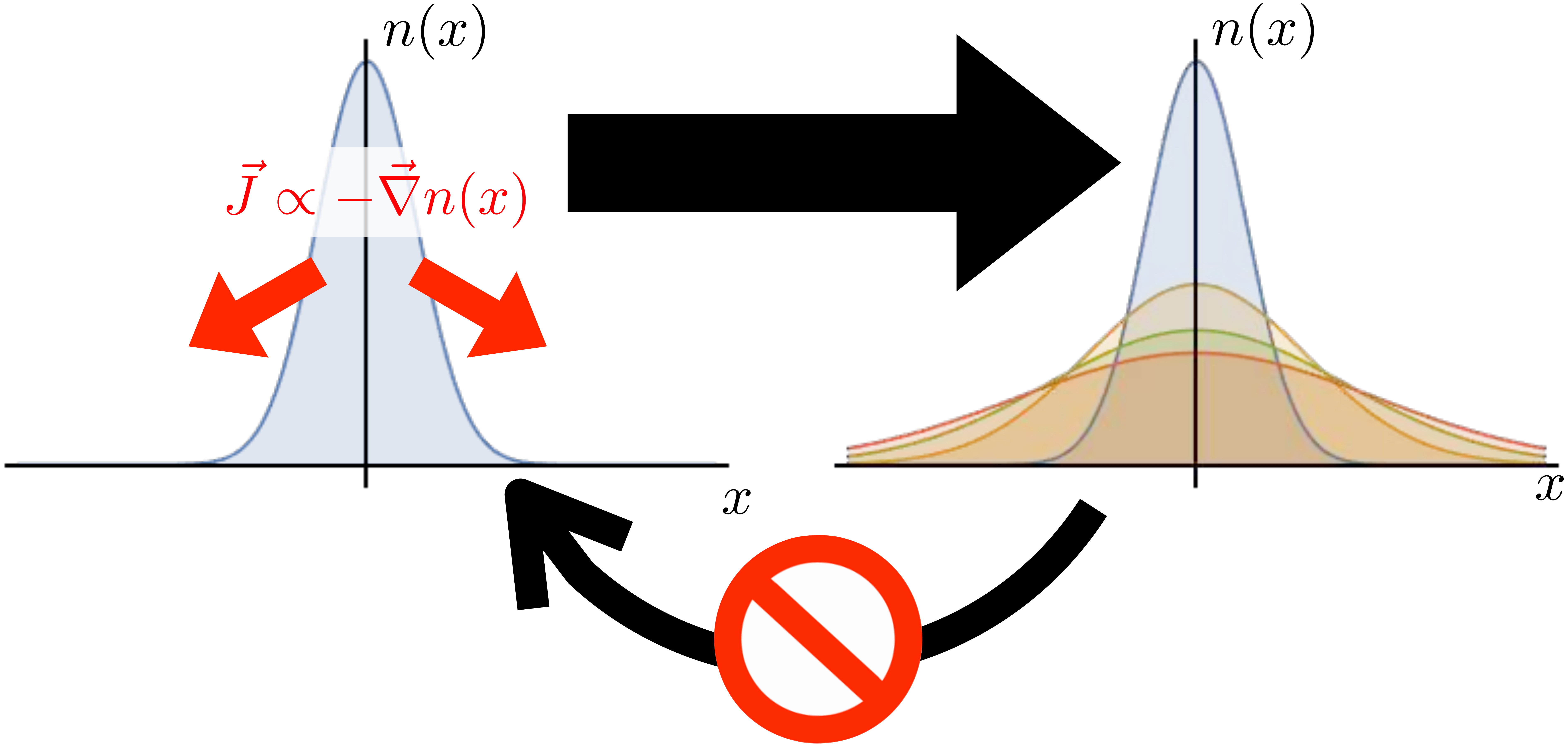

No-go for time-reversal process!

Thermodynamic concepts, especially, The 2_{nd} law, should be there!

Phenomenological derivation

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Step 1. Determine dynamical d.o.m (& its equation of motion)

Charge density: $n(x)$ EoM: $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

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Entropy density: $s(n)$ $\xrightarrow{T\mathrm{d}s = -\mu \mathrm{d}n}$ Chemical pot.: $\beta\mu \equiv -\frac{\partial s}{\partial n}$

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Step 4. Identify how parameters (e.g., κ_n) can be matched

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≃ Ward-Takahashi identity

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≃ Effective Lagrangian (Hamiltonian)

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≃ Matching condition for low-energy coeff.

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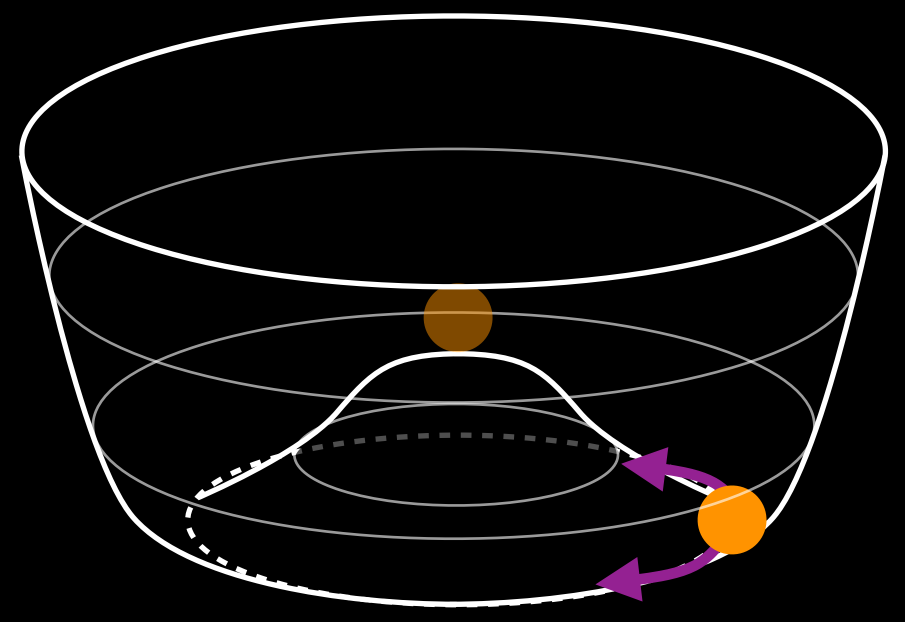

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Semi-phenomenology based on local thermodynamics

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Derivation of hydrodynamics for symmetry-broken phases

Matching condition (Kubo formula) for all Onsager coeff.

Application to NS physics (e.g., neutrino reaction, ...)

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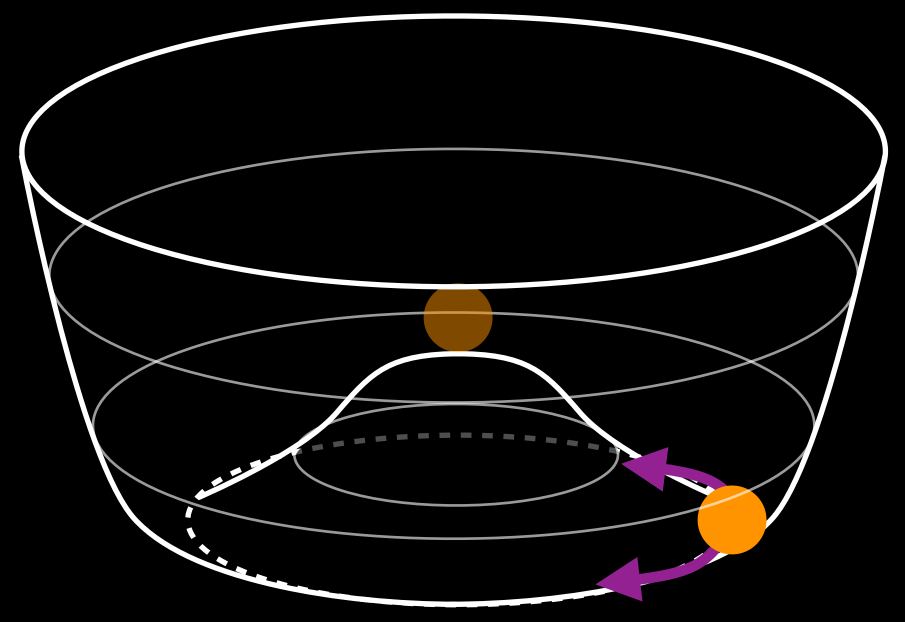

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Application to U(1)-symmetry breaking

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In addition to the conserved charge density

superfluid phonon φ appears!

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Entropy density: $s = s(n, v)$ with $v = \frac{1}{2}(\partial_i \varphi)^2, \quad \beta\mu = -\frac{\partial s}{\partial n}, \quad \beta f^2 = -\frac{\partial s}{\partial v}$

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Step 3. Find $\{J^i_a, \Pi, f^i\}$ up to finite derivatives compatible with 2nd law

$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ $\Rightarrow$ $J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \; \Pi = -\mu + \zeta_s \partial_i u (f^2 \partial^i \varphi)$

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Charge density: n & Superfluid phonons: φ EoM: $\partial_t n + \partial_i J^i = 0$ & $\partial_t \varphi = \Pi$

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$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ $\Rightarrow$ $J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \ \Pi = -\mu + \zeta_s \partial_i u(f^2 \partial^i \varphi)$

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Choosing $s^i := -\beta\mu J^i + \beta f^2 \Pi \partial^i \varphi, \ J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \ \Pi = -\mu + \zeta_s \partial_i u (f^2 \partial^i \varphi)$ works!

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Supercurrent/Diffusion Josephson eq./Damping effect

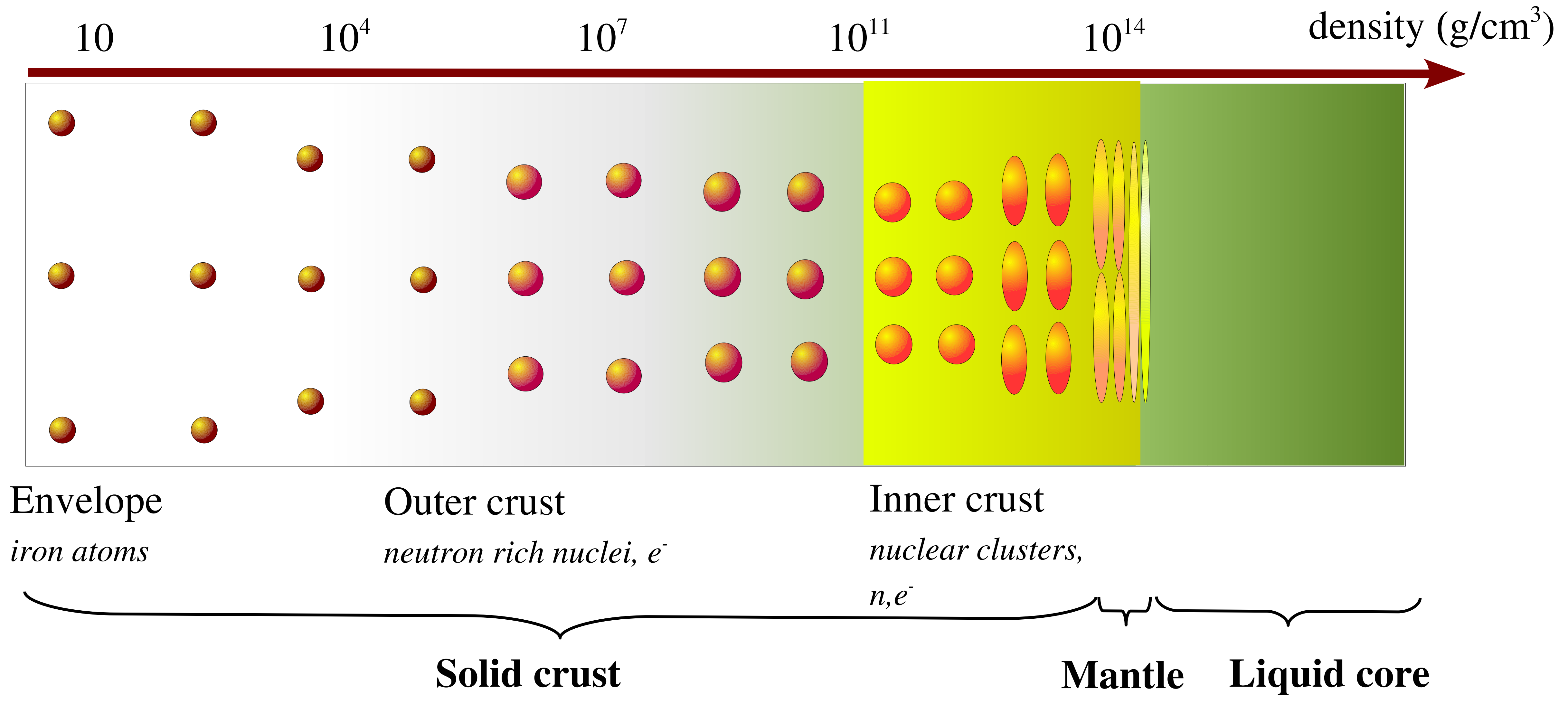
◆ **Onsager coefficient**

Charge conductivity: κ_n, Damping coefficient: ζ_s

➡ **Gapless mode:** $\omega \simeq \pm c_s |\boldsymbol{k}| - \frac{\mathrm{i}}{2}(D + f^2 \zeta_s)\boldsymbol{k}^2$ **appears!** $\left[c_s := \frac{f}{\sqrt{\chi}},\ D := \frac{\sigma}{\chi} \right]$

Application to Hydrodynamics in the neutron star inner crust

SSB pattern in the NS inner crust



From Chamel-Haensel (2008)

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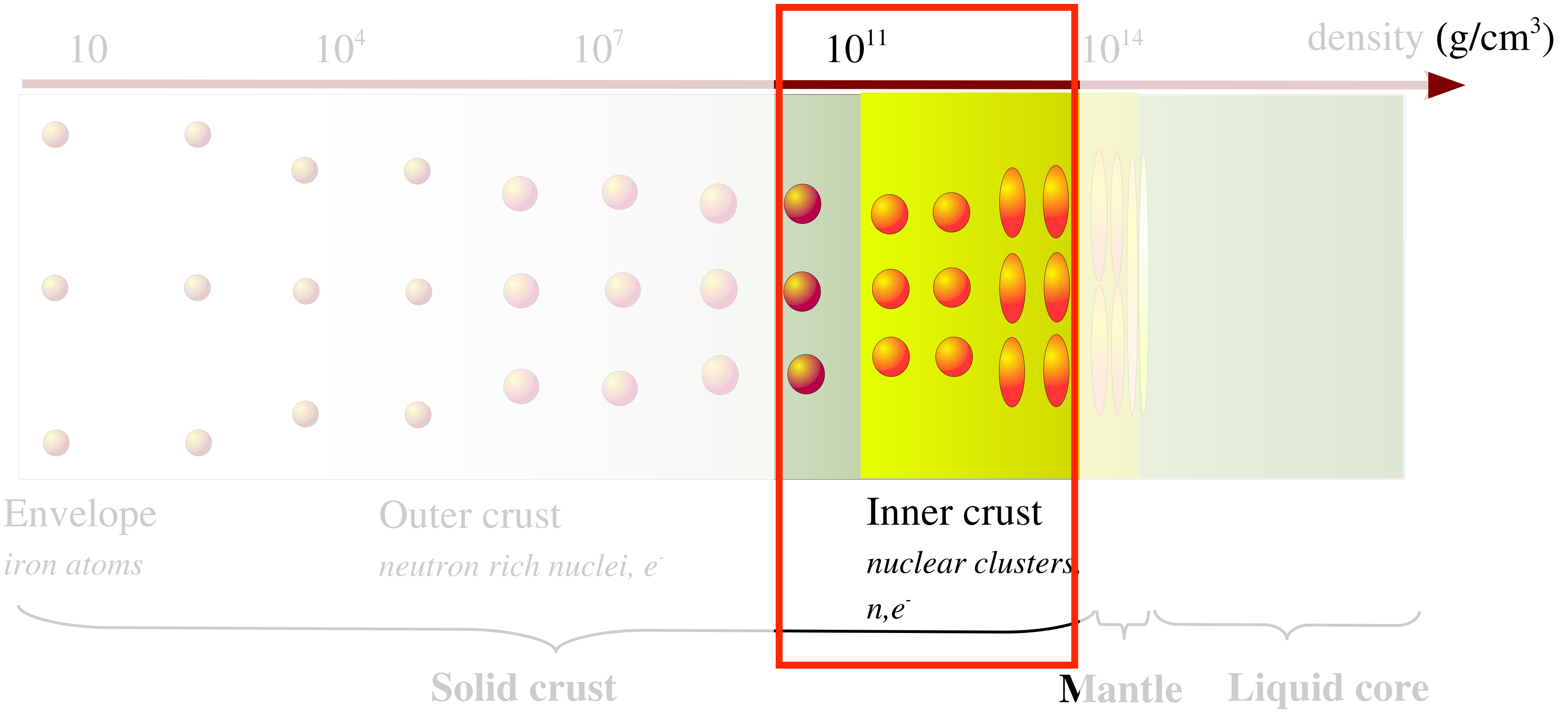

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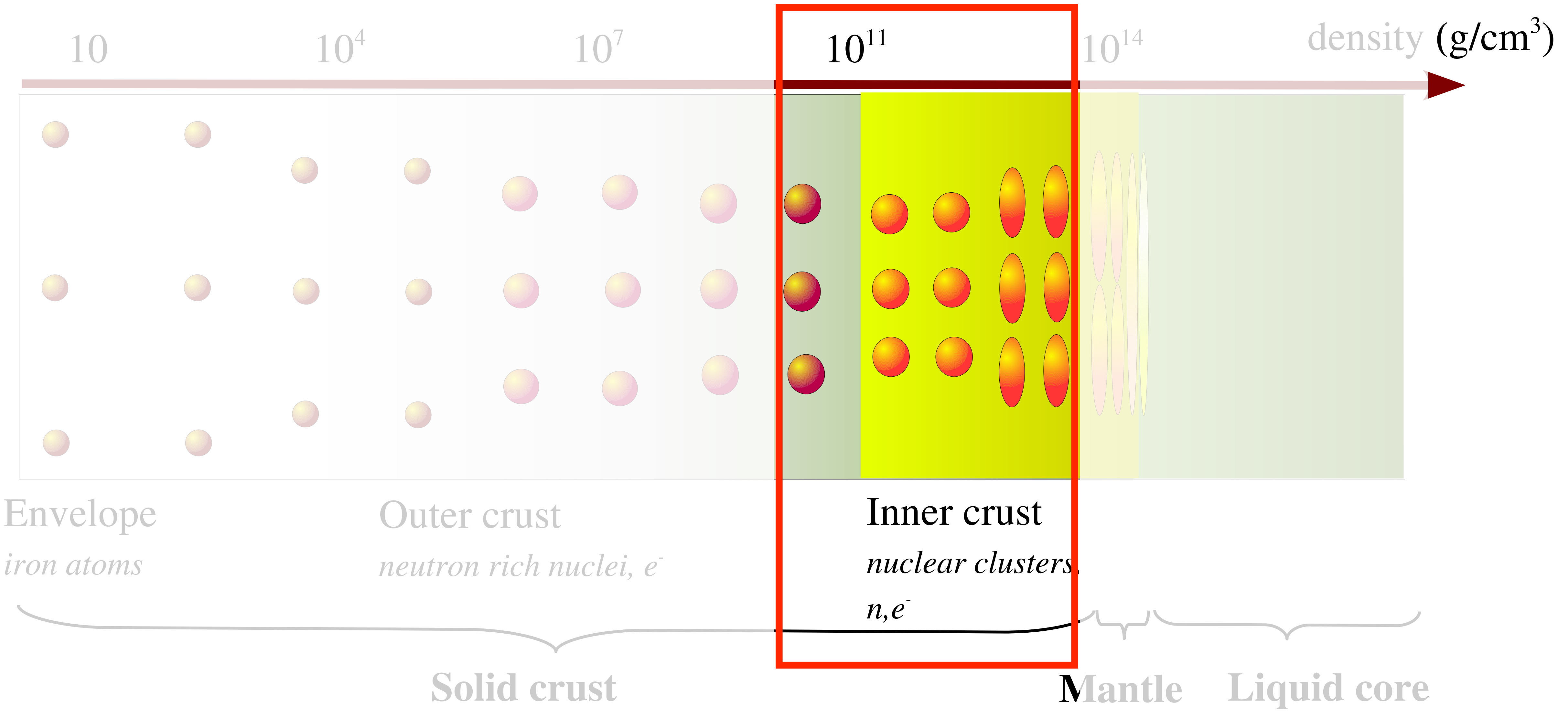
Key properties: [See Cirigliano et al. et al, PRC (2011) for low-energy EFT]

- **Nuclei form a Coulomb lattice:**
  Translational symmetry is spontaneously broken → Lattice phonons ξ^i appears!
- **Cooper pairs of dripped neutrons realize an s-wave condensate:**
  $U(1)_n$ symmetry is spontaneously broken → Superfluid phonon φ appears!



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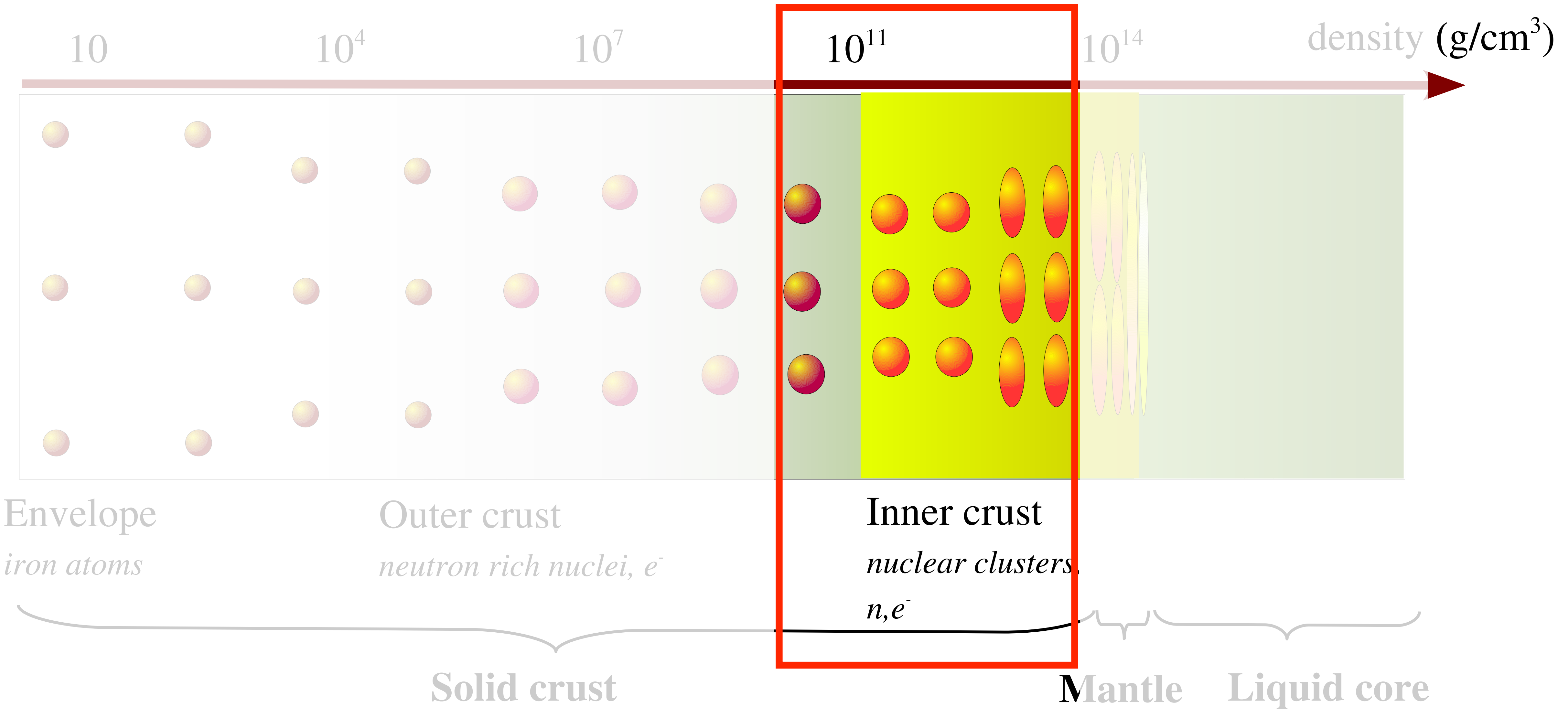
➡ What is the corresponding hydrodynamics at $T \neq 0$ for the inner crust?



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Entropy density $s \simeq s_0(e, \rho_n - g\partial_i \xi^i) - \dfrac{\beta f^2}{2}(\partial_i \varphi)^2 - \dfrac{\beta}{2}\mu^{ijkl}\partial_i \xi_j \partial_k \xi_l$ with $\beta = \dfrac{\partial s}{\partial e},\ \beta\mu_n = -\dfrac{\partial s}{\partial \rho_n}$

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The procedure looks complicated in this case, but we can do it!

Hydrodynamics for inner crust (preliminary)

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$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J_n^\mu = 0, \quad u^\mu \partial_\mu \varphi = \Pi, \quad u^\mu \partial_\mu \xi^i = h^i$$

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$$-T\eta^{\mu\nu\rho\sigma}\partial_\rho(\beta u_\sigma) - T\zeta_\times h^{\mu\nu}\beta\partial_\mu(f^2\partial^\mu\varphi)$$

$$J^\mu = nu^\mu + f^2\partial^\mu\varphi - T\kappa_n \partial_{\perp\mu}\nu$$

$$\left[v_{\mu\nu} = \frac{1}{2}(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)\right]$$

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$$\left[v_{\mu\nu} = \frac{1}{2}(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)\right]$$

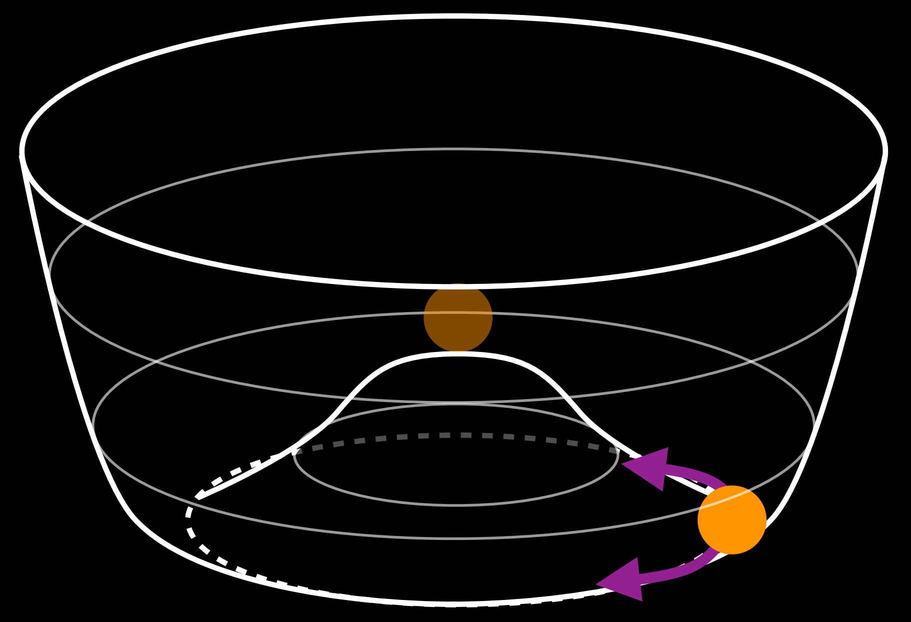
◆ Onsager coefficient

$$\eta,\ \zeta,\ \kappa_n,\ \zeta_s,\ \zeta_\times,\ \gamma_{ij}$$

Summary

Motivation:

Hydrodynamics for
symmetry-broken phases?



Approach:

Semi-phenomenology based on local thermodynamics

Result & Outlook:

Derivation of hydrodynamics for symmetry-broken phases

Matching condition (Kubo formula) for all Onsager coeff.

Application to NS physics (e.g., neutrino reaction, ...)