GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN IN PUBLICA COMMODA SEIT 1737

SFB 1073

Universal features of non-Fermi liquids & application to black holes

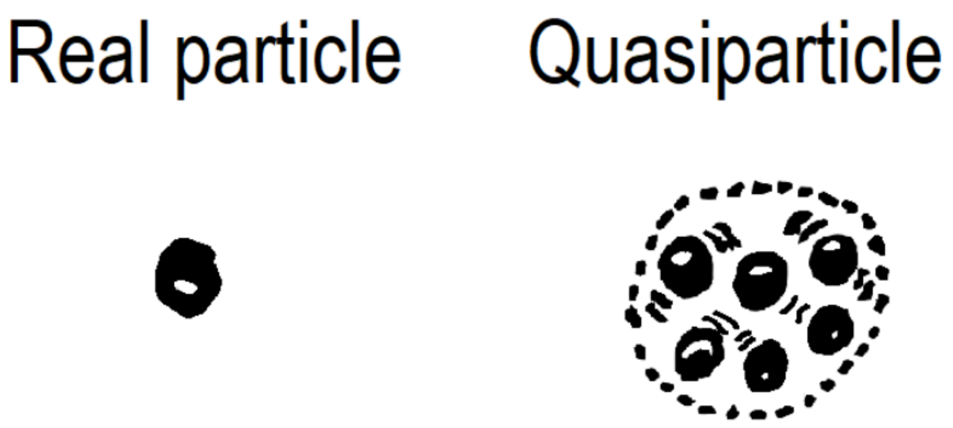
XVIth Quark Confinement and the Hadron Spectrum
Cairns Convention Center, Cairns, Queensland, Australia

Wednesday, August 21, 2024

Rishabh Jha
Institute for Theoretical Physics,
Georg-August-Universität Göttingen, Germany

Ordinary Metals

- Fermi Liquid Theory:



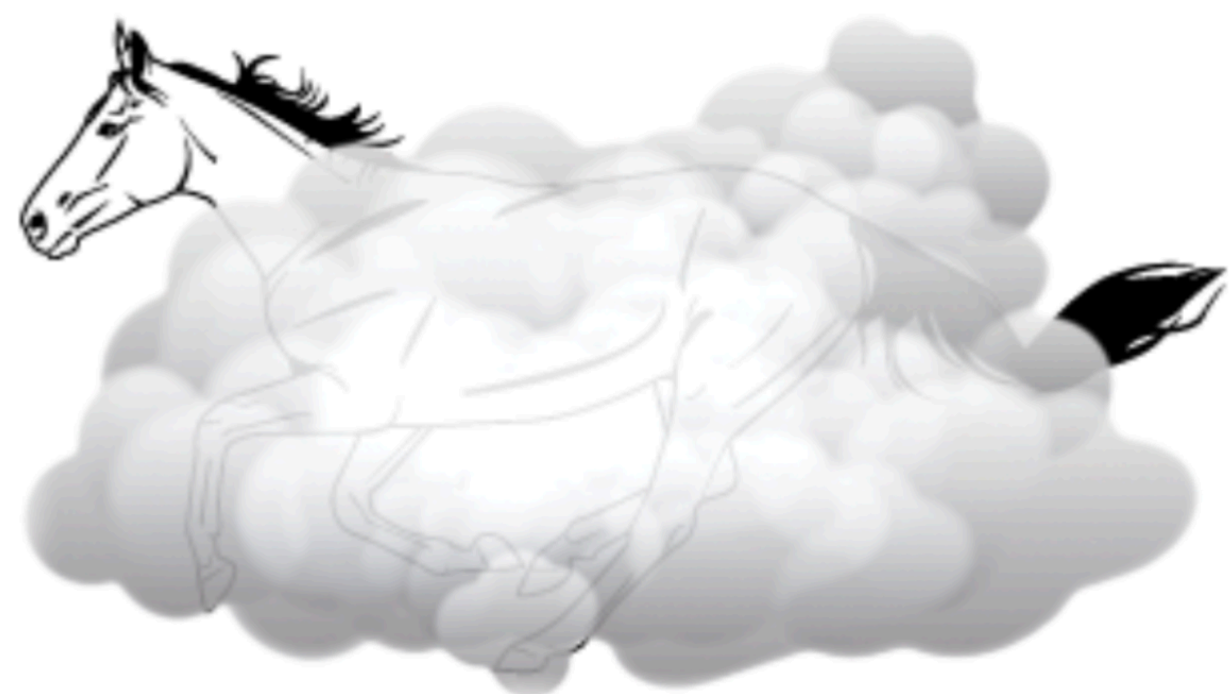
Ordinary Metals

Real Particle

Quasiparticle

Real Horse

Quasihorse



D. Mattuck. A Guide to Feynman Diagrams in the Many-Body Problem. *Dover Publications, 1992*

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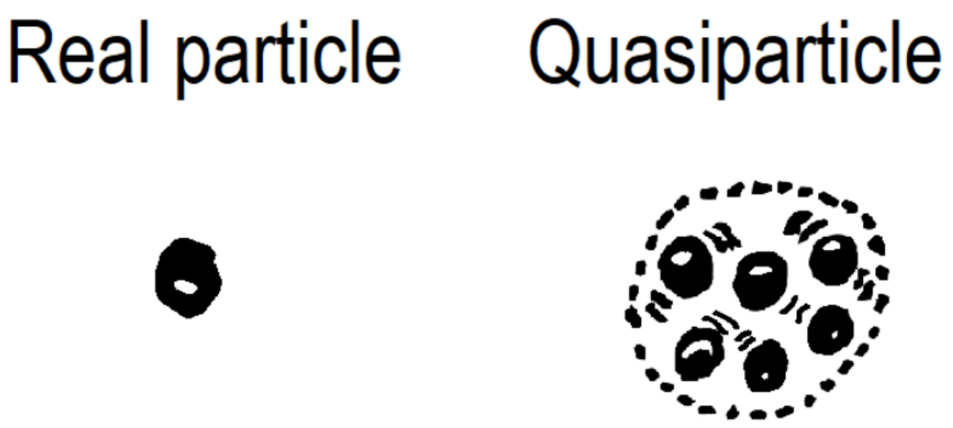

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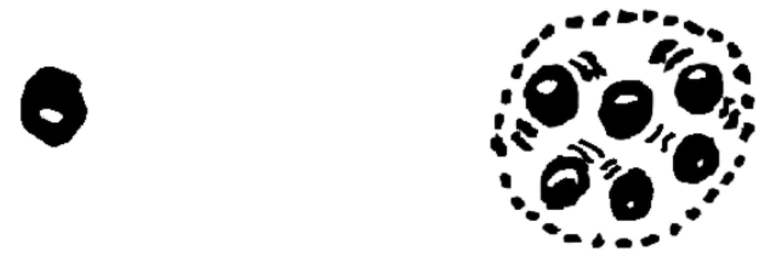
Real particle Quasiparticle



$$E_{\text{low energy}}[\vec{n}] = \sum_{k=1}^{N} \underbrace{\epsilon_k}_{\text{quasiparticle energies}} n_k + \sum_{k,p} \underbrace{f_{kp} n_k n_p + \cdots}_{\text{weak scattering}}$$

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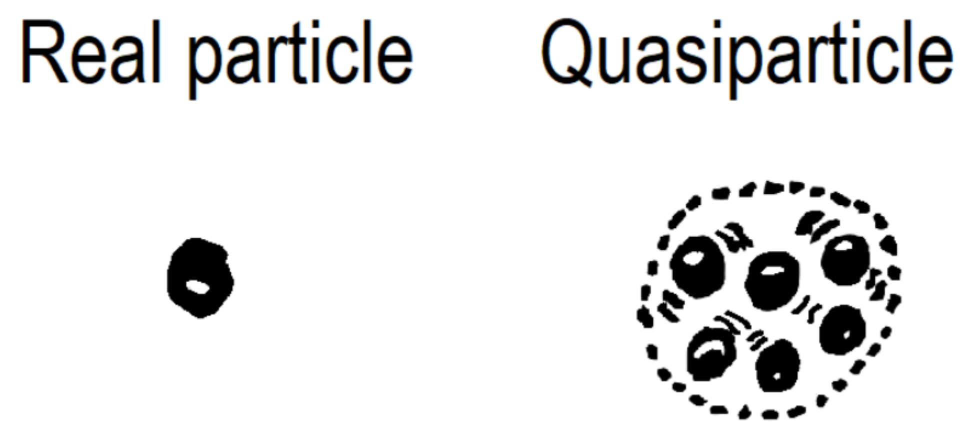

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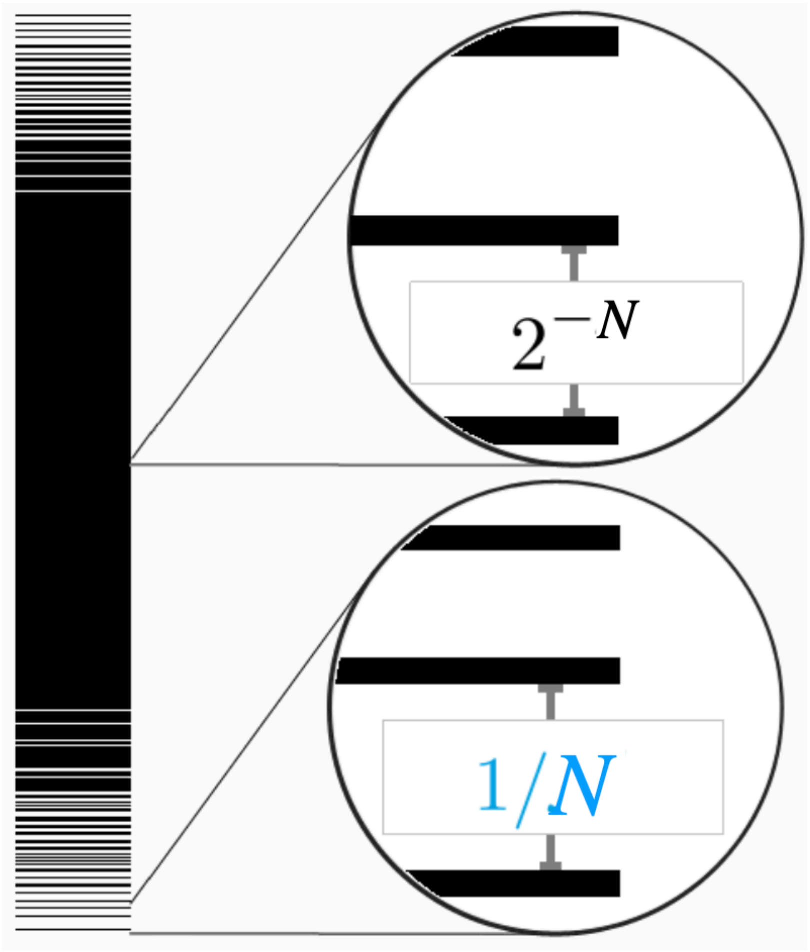

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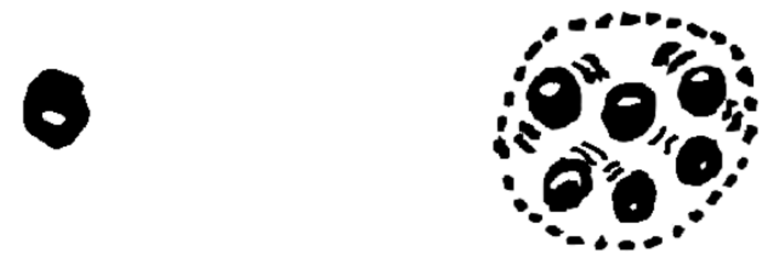

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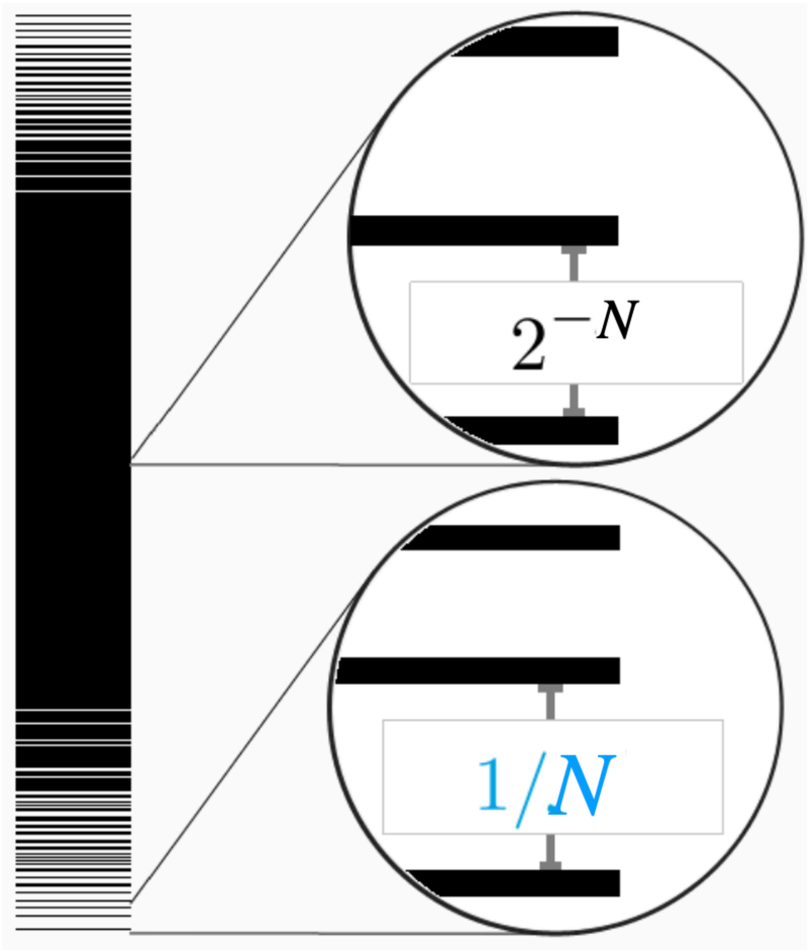

$\Longrightarrow$ **Quasiparticles**

Ordinary Metals

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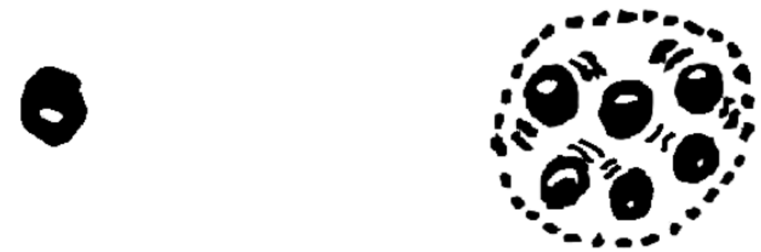

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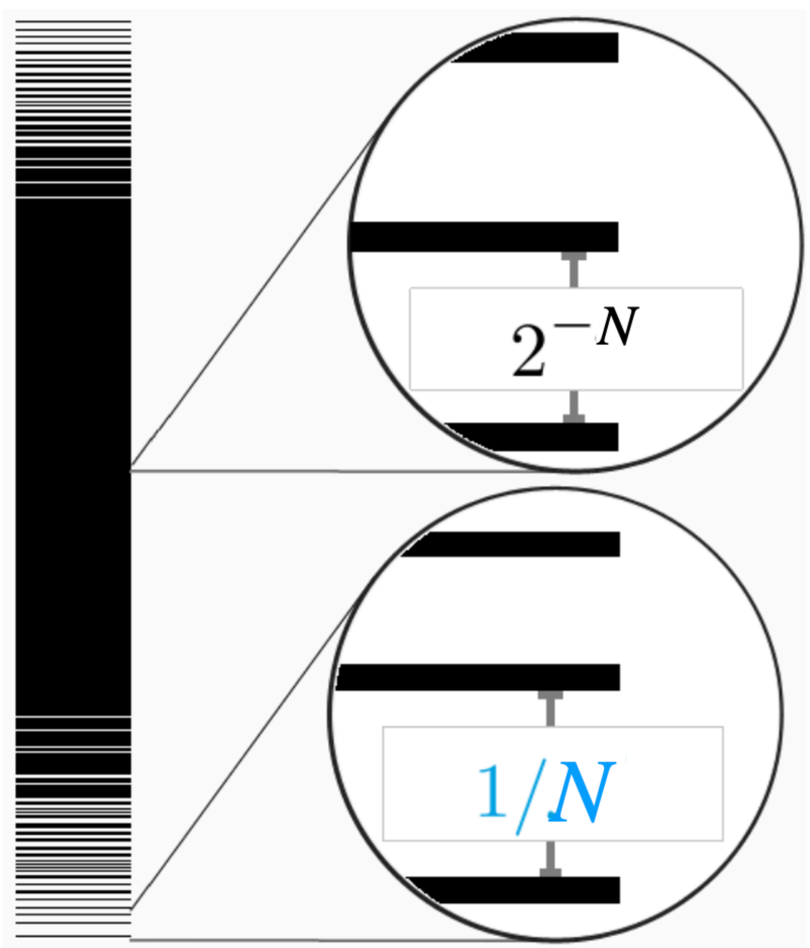
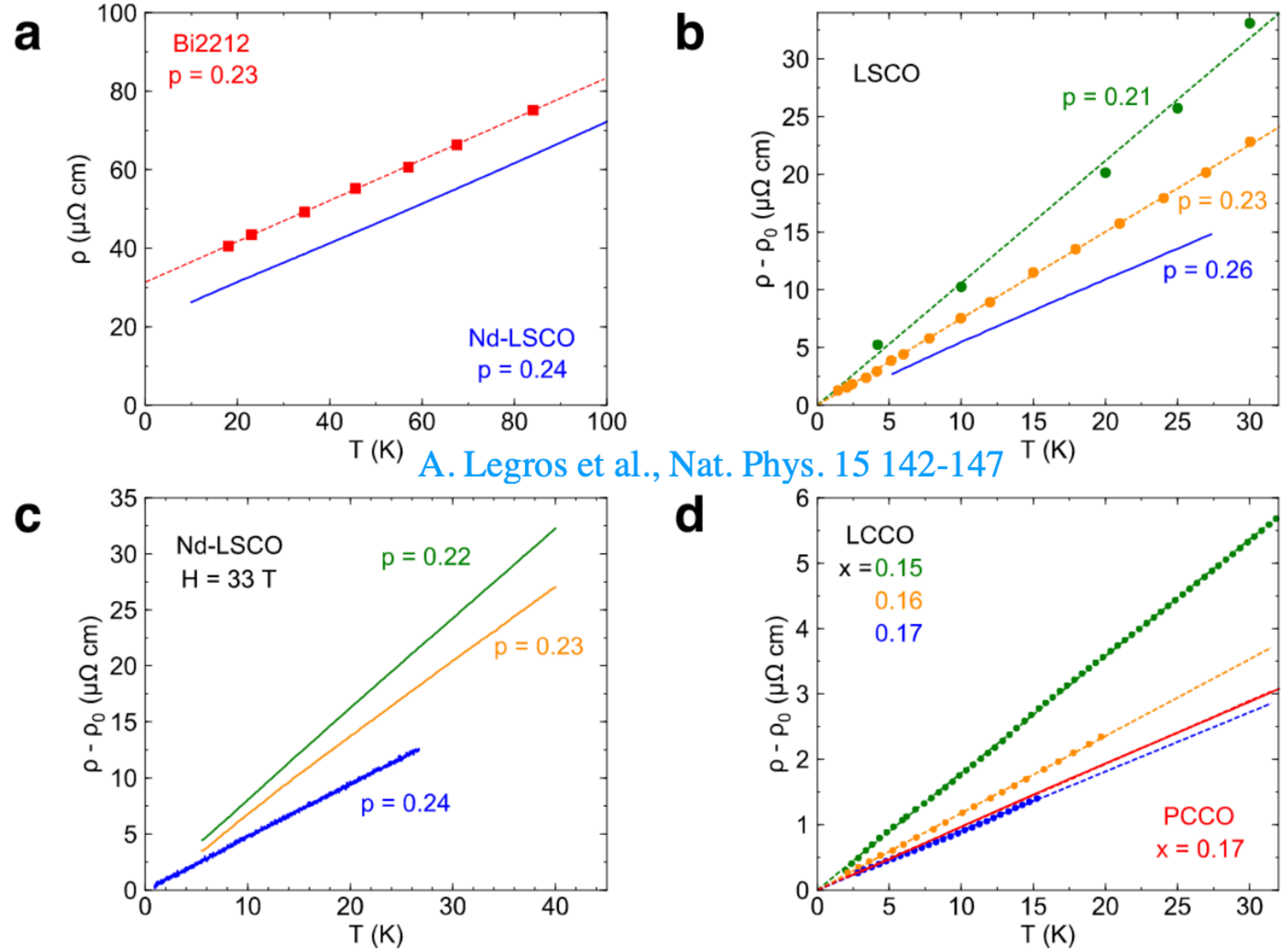


$\Longrightarrow$ **Quasiparticles**

1. Level spacing $1/N \Rightarrow$ Quasiparticles
2. Electrical resistivity $\rho \propto T^2$
3. Equilibration rate $\tau_{\text{eq}}^{-1} \propto T^2$

Strange Metals

- Strange resistivity $\rho \propto T$ in high-T_c cuprates:



A. Legros et al., Nat. Phys. 15 142-147

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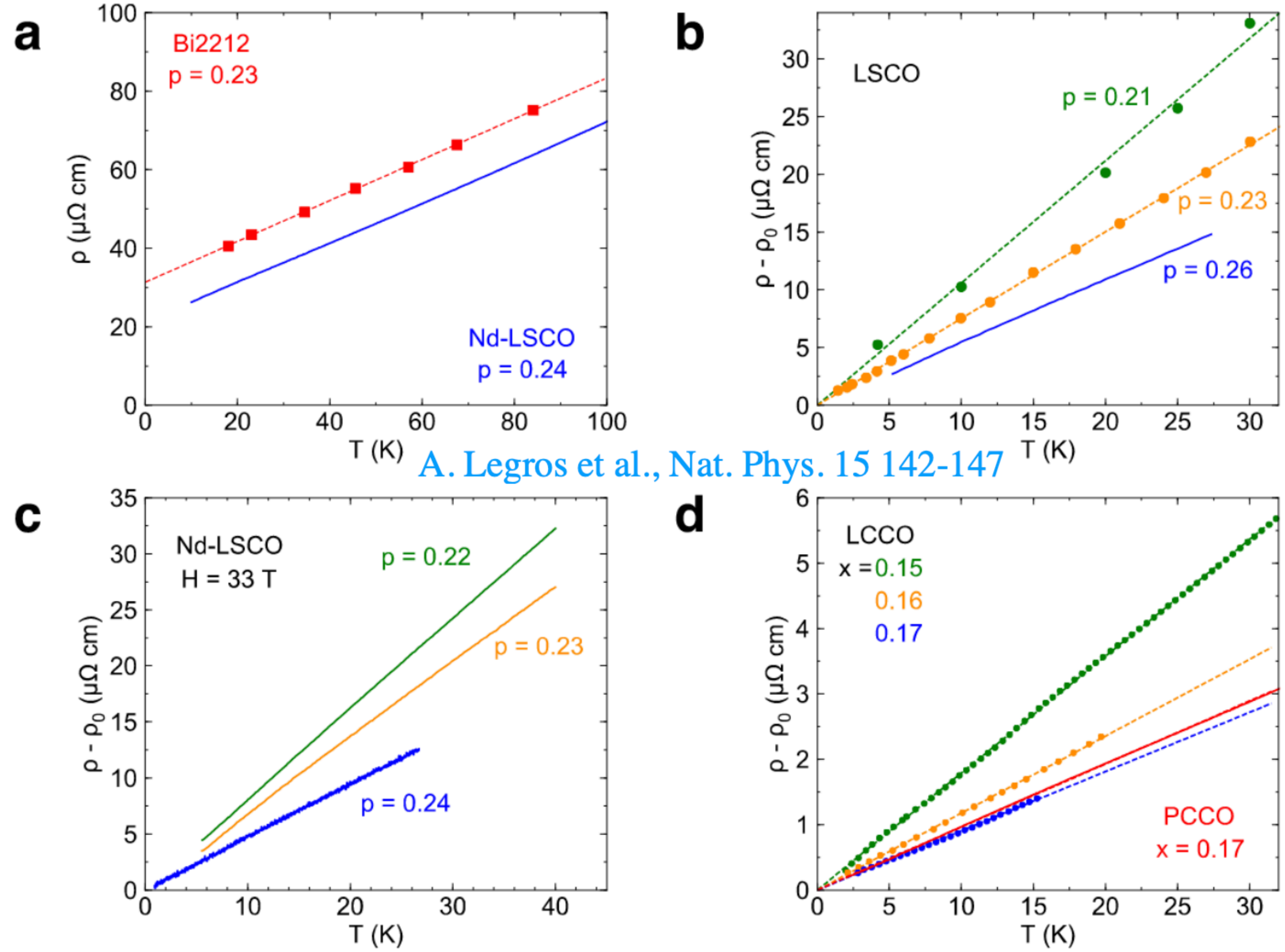

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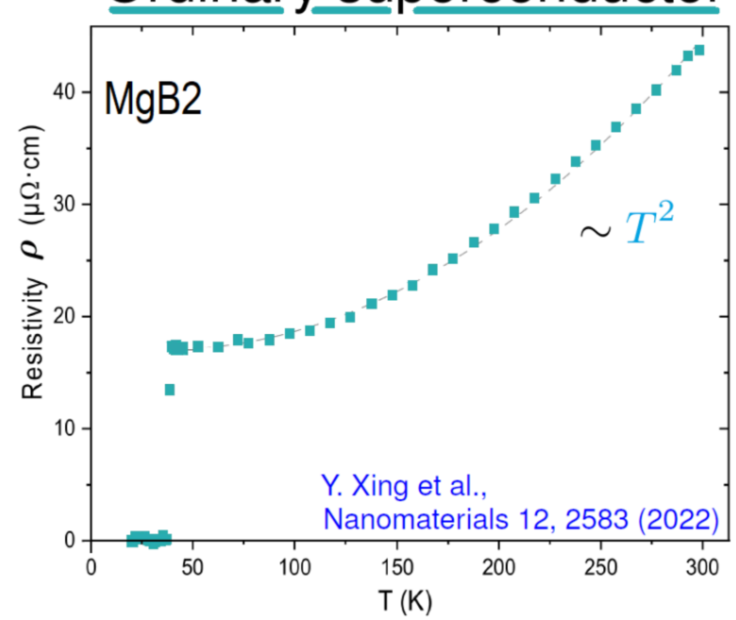
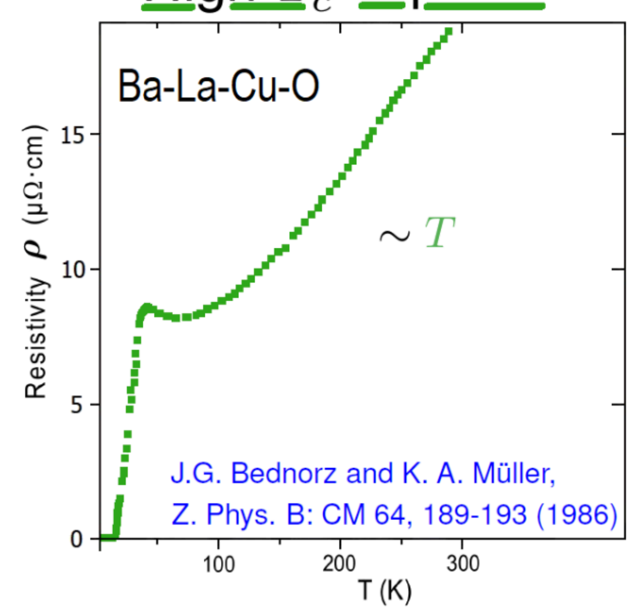
- Ordinary vs. High-T_c superconductors:

Ordinary superconductor



High-T_c cuprate



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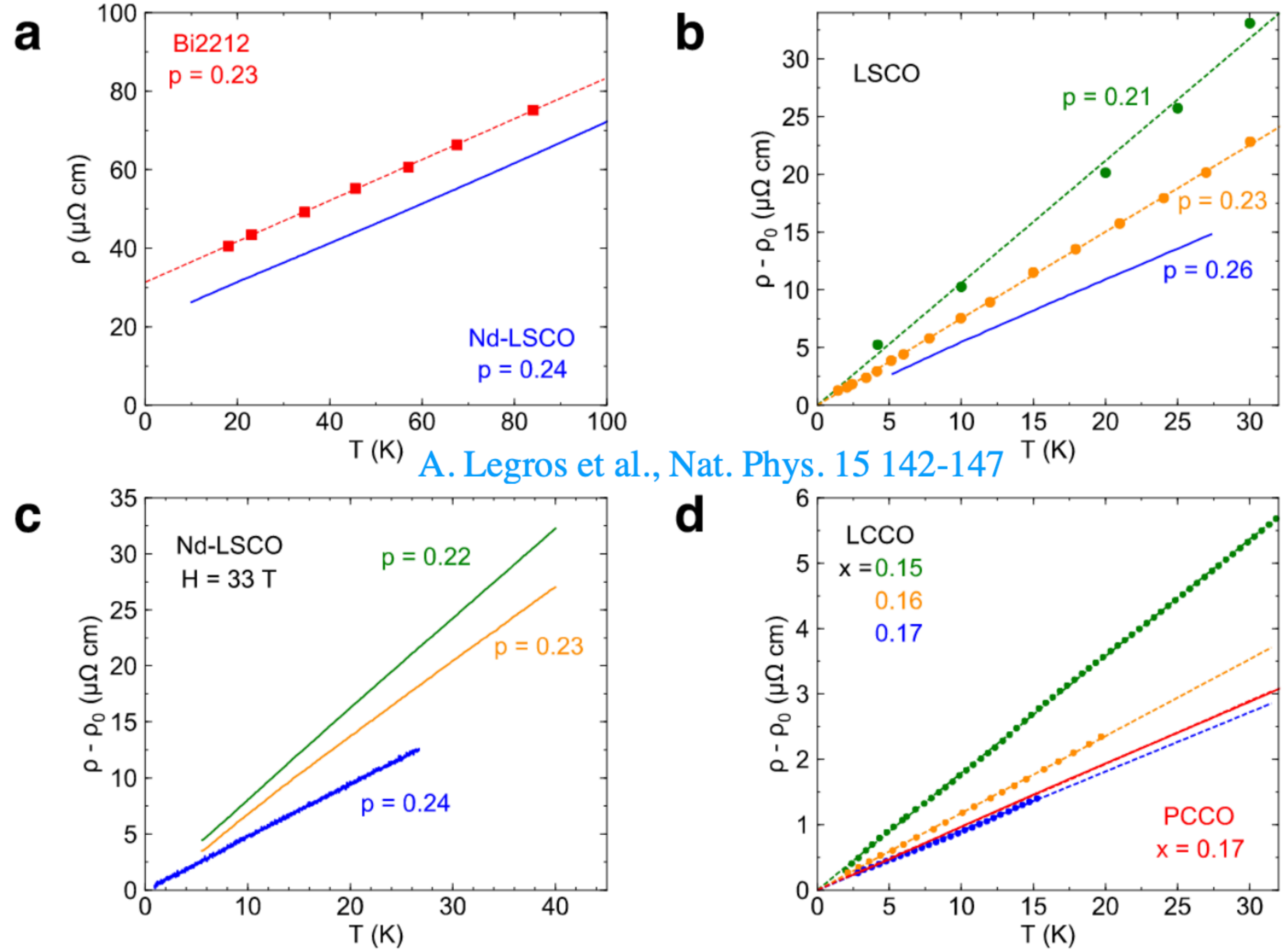

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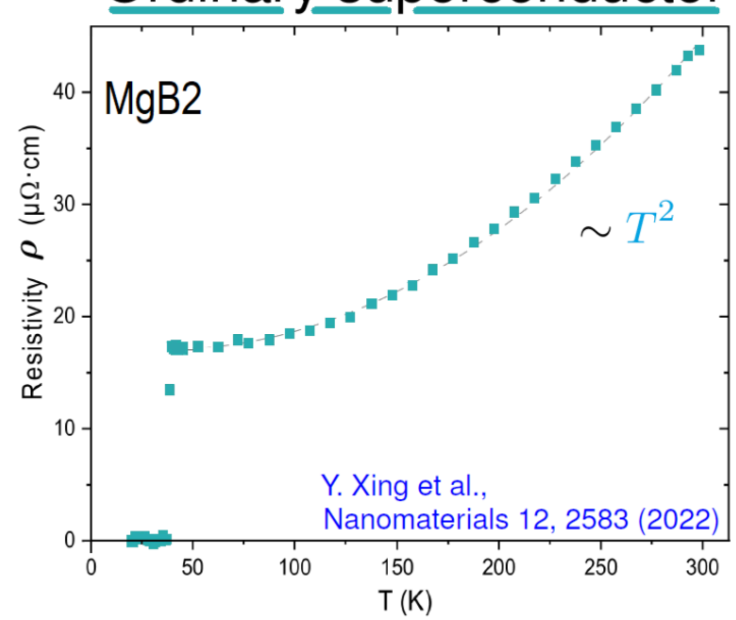
- Ordinary vs. High-T_c superconductors:

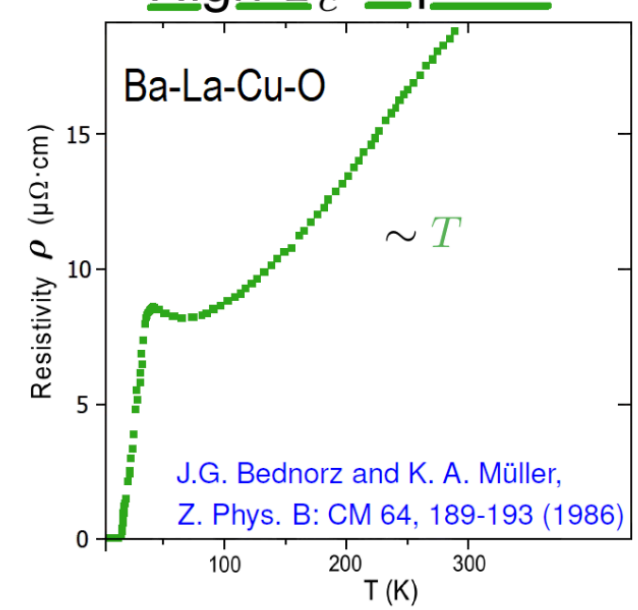
Fermi liquid

Ordinary superconductor



Y. Xing et al., Nanomaterials 12, 2583 (2022)

High-T_c cuprate



J.G. Bednorz and K. A. Müller, Z. Phys. B: CM 64, 189-193 (1986)

Strange Metals

- Strange resistivity $\rho \propto T$ in high-T_c cuprates:

Non-Fermi liquid, aka strange metals

- Ordinary vs. High-T_c superconductors:

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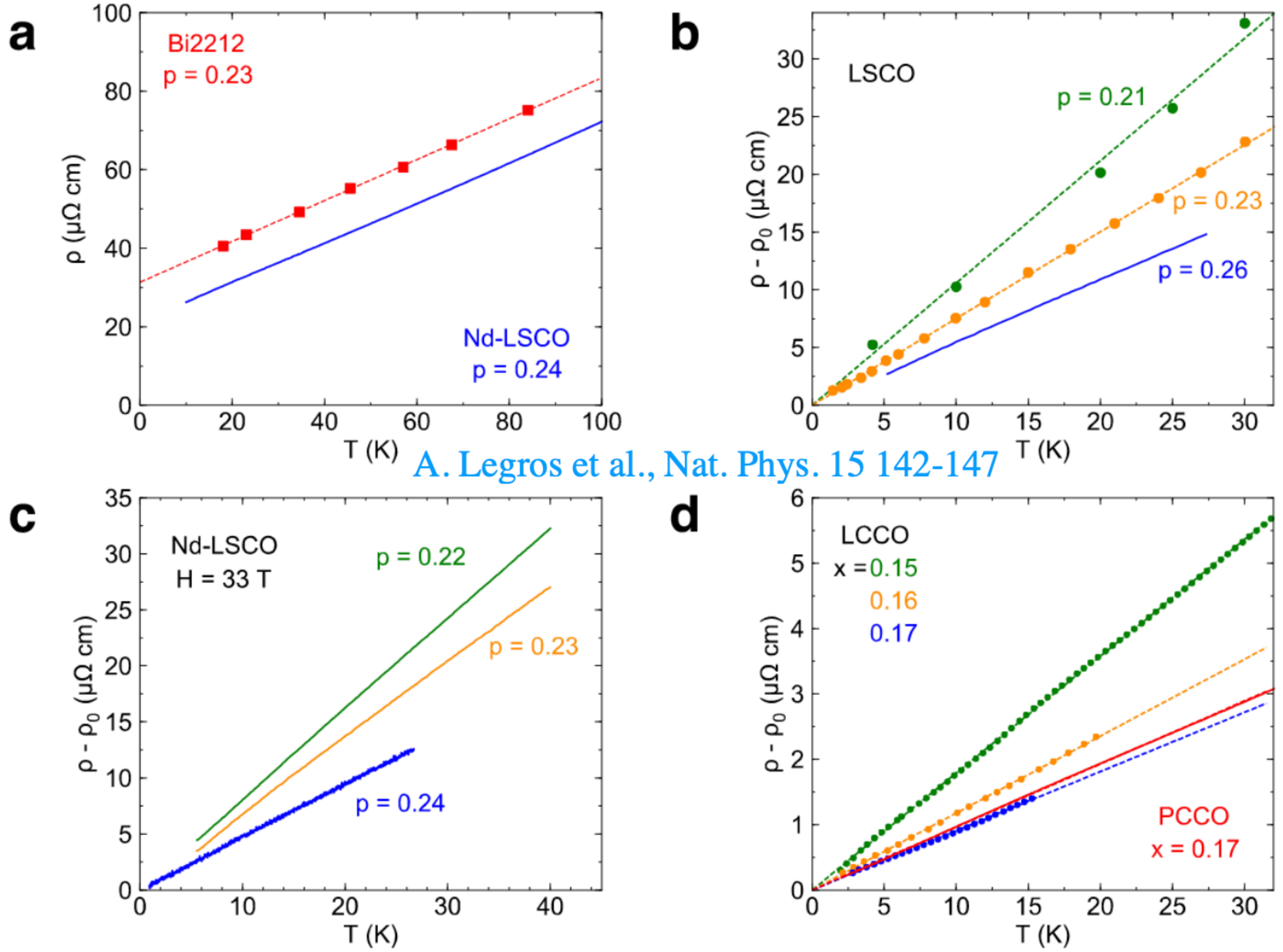

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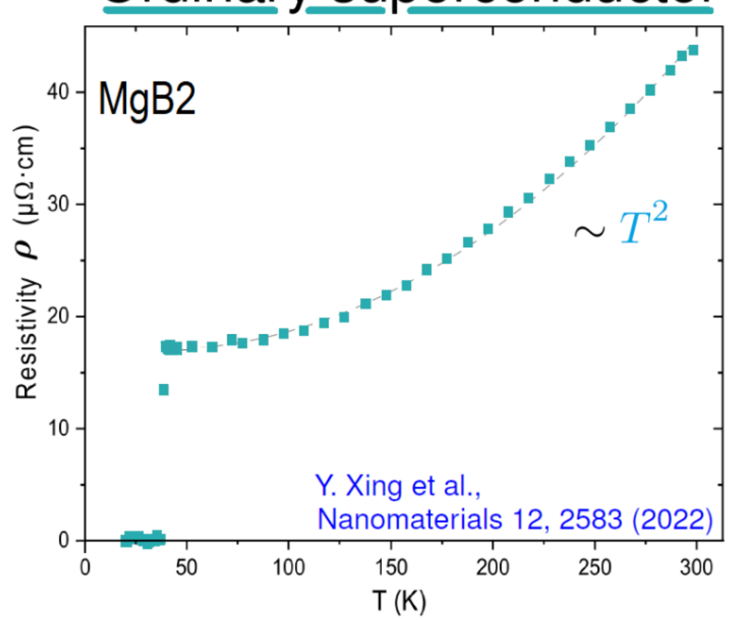

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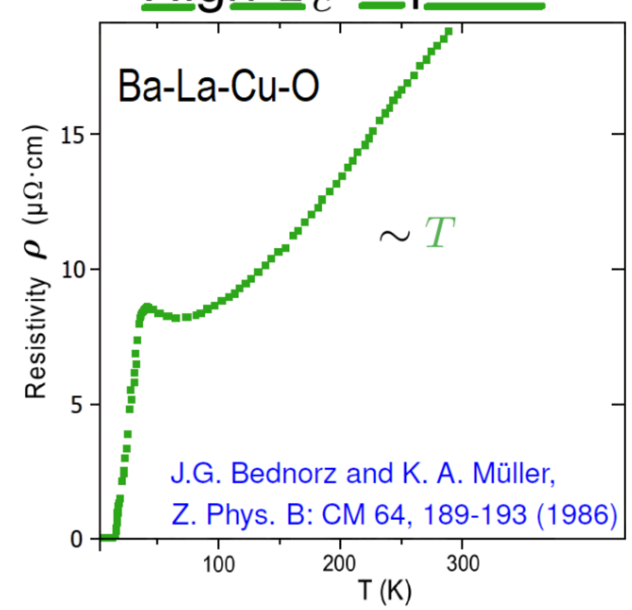

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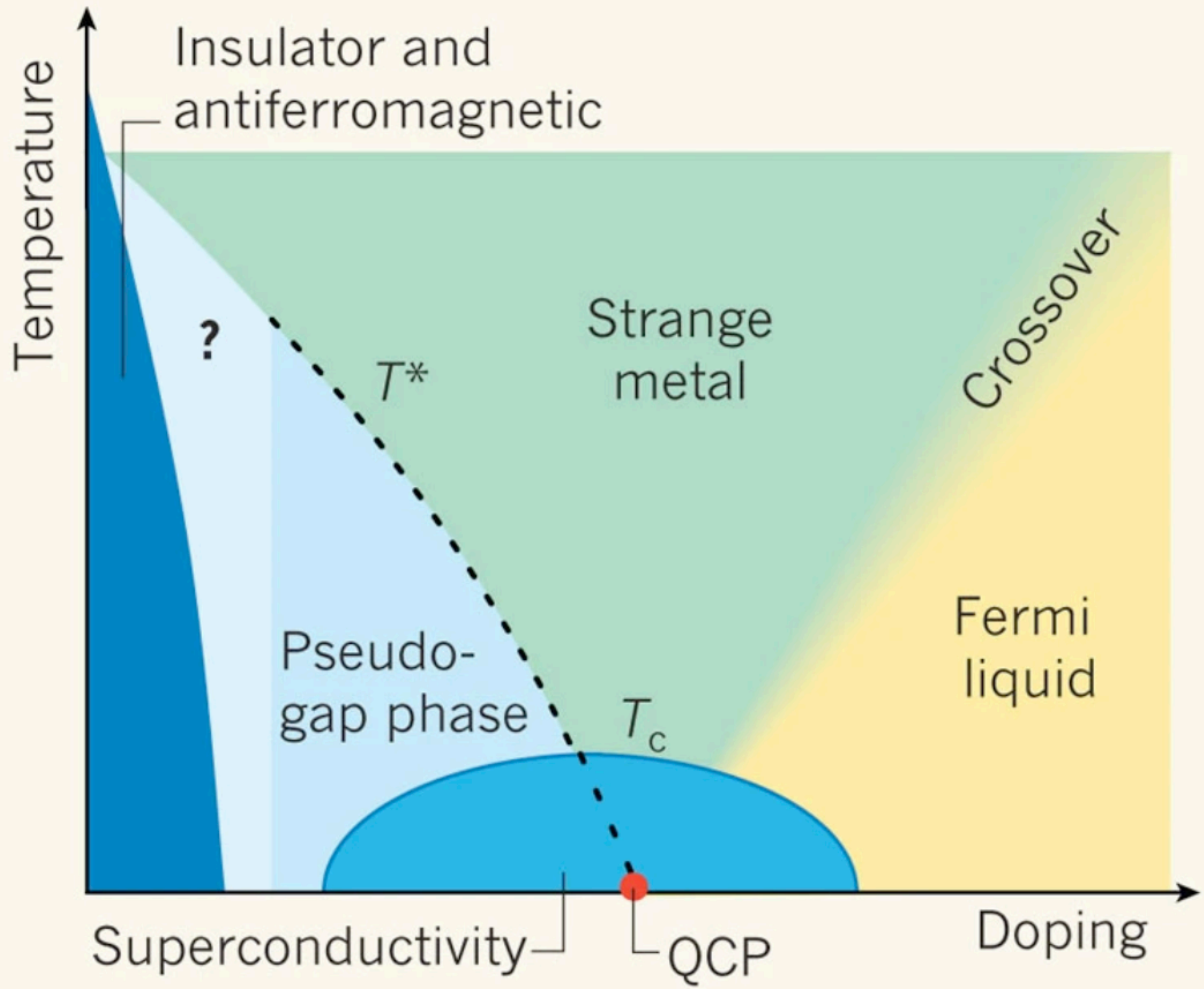

Source: www.nature.com/articles/468184a

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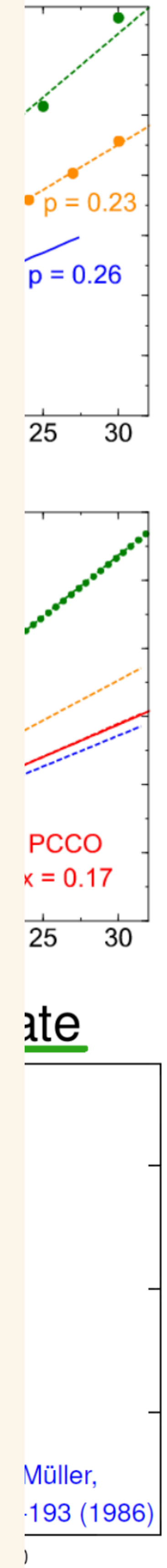

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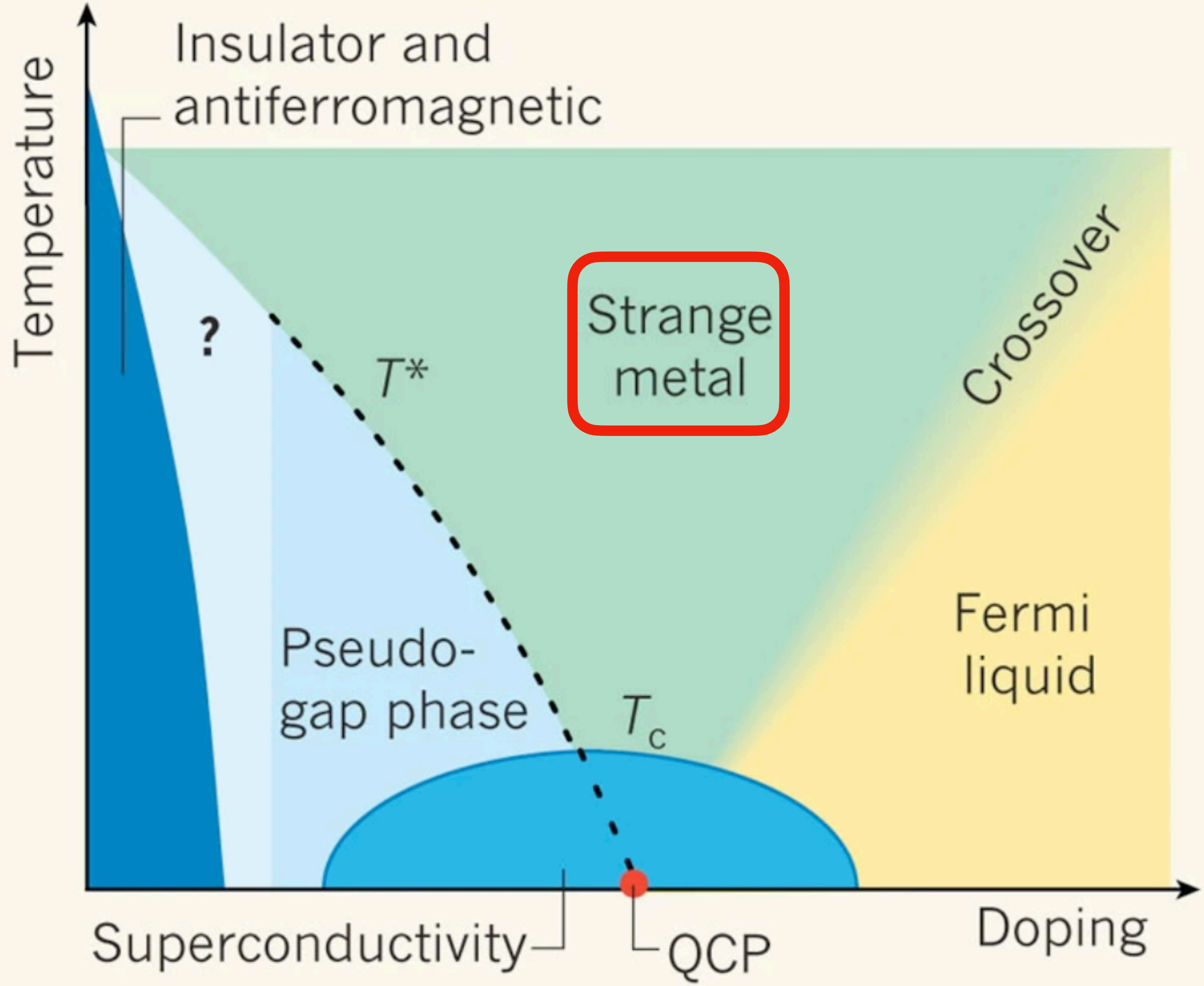

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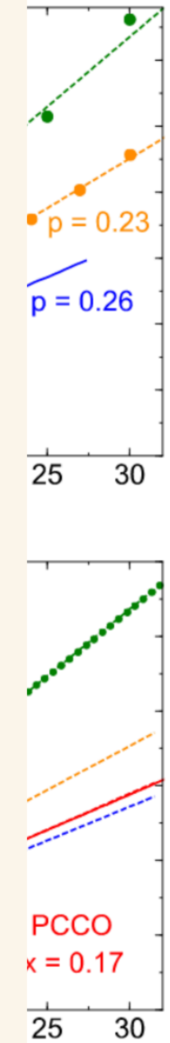

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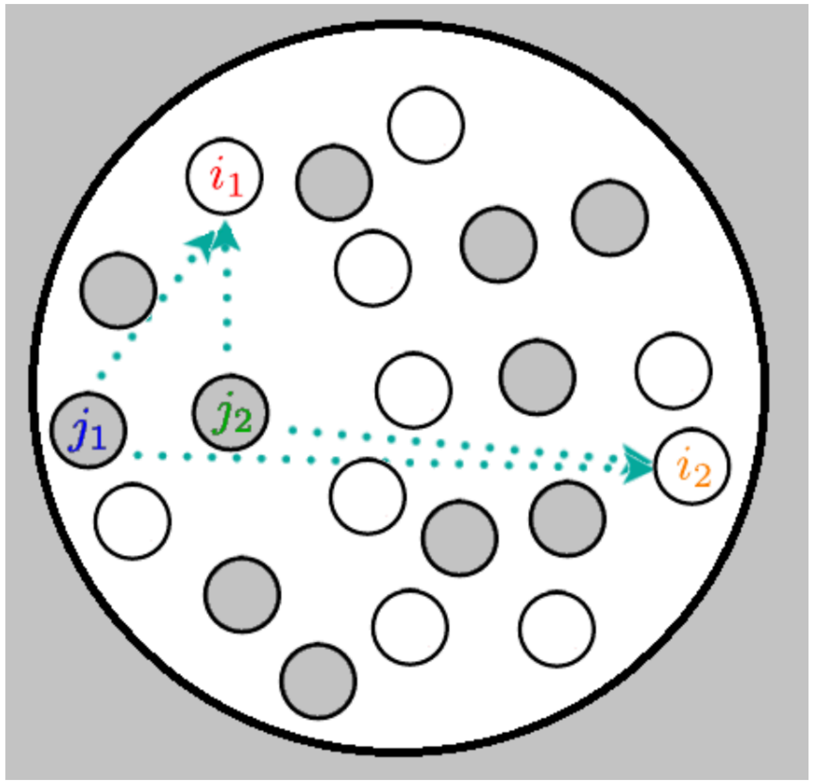
Müller,
193 (1986)

Sachdev-Ye-Kitaev Model



$$\mathcal{H}_4 = \sum_{\substack{1\leq i_1<i_2\leq N \\ 1\leq j_1<j_2\leq N}} \underbrace{J^{i_1 i_2}_{j_1 j_2}}_{\text{(random coupling)}} \underbrace{c^\dagger_{i_1} c^\dagger_{i_2} c_{j_2} c_{j_1}}_{\text{4-sites}}$$

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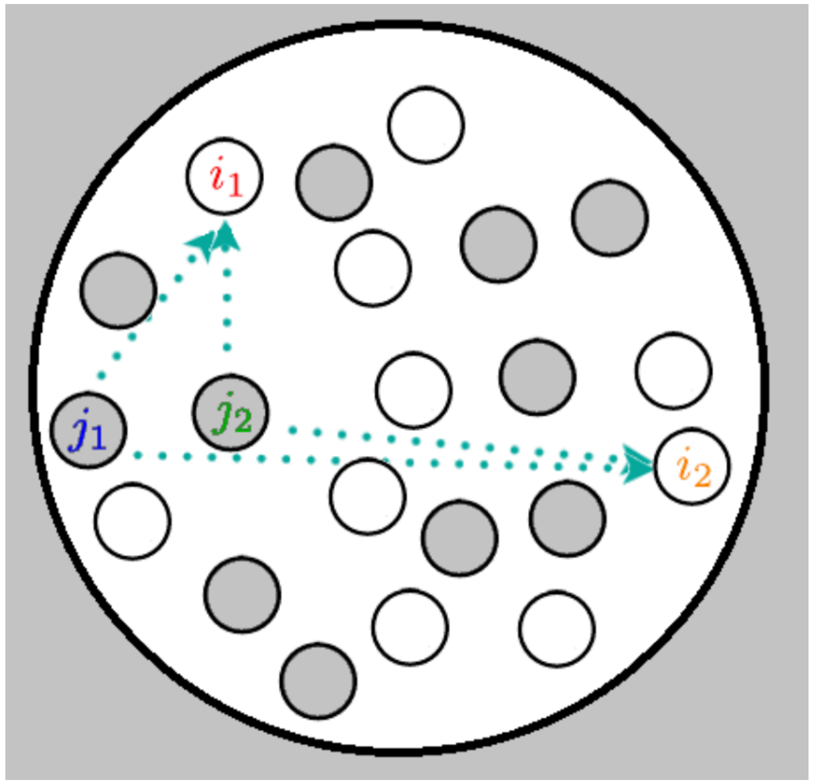

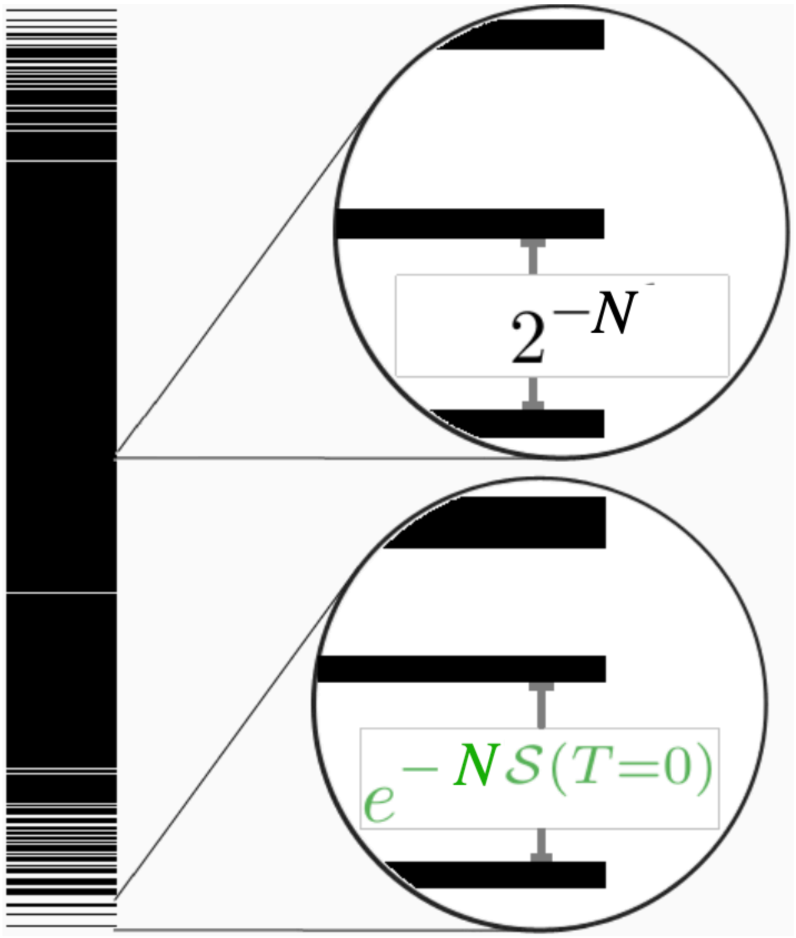

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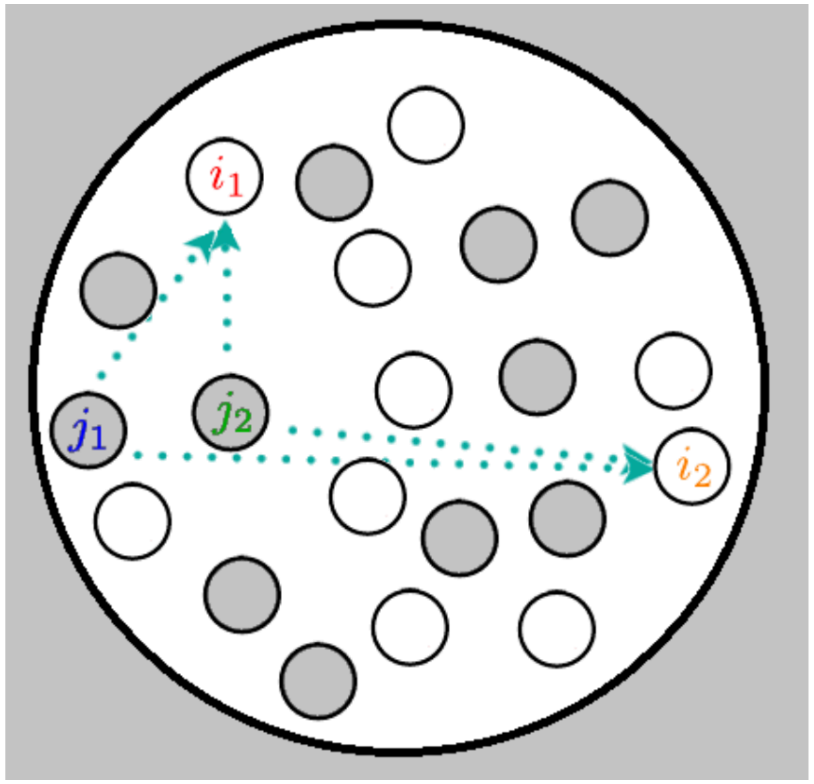

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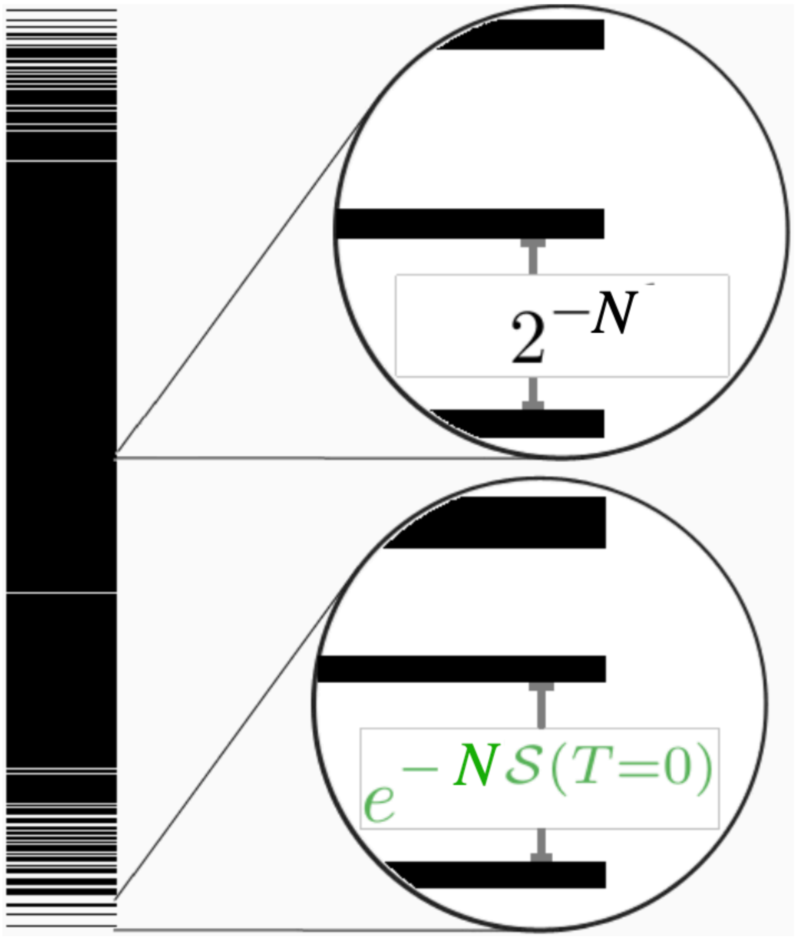

$\Longrightarrow$ **No quasiparticles**

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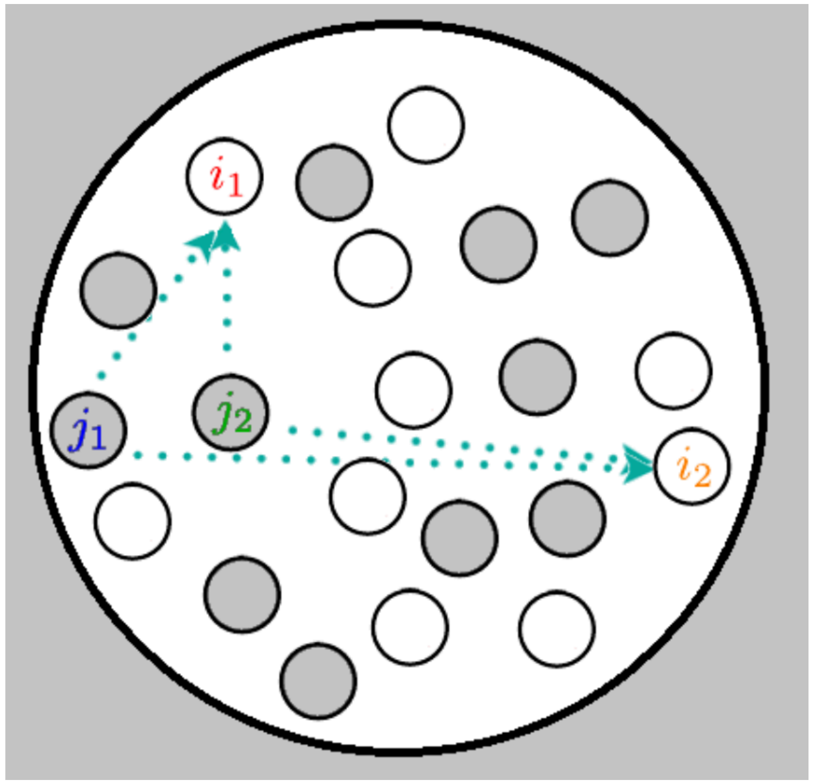

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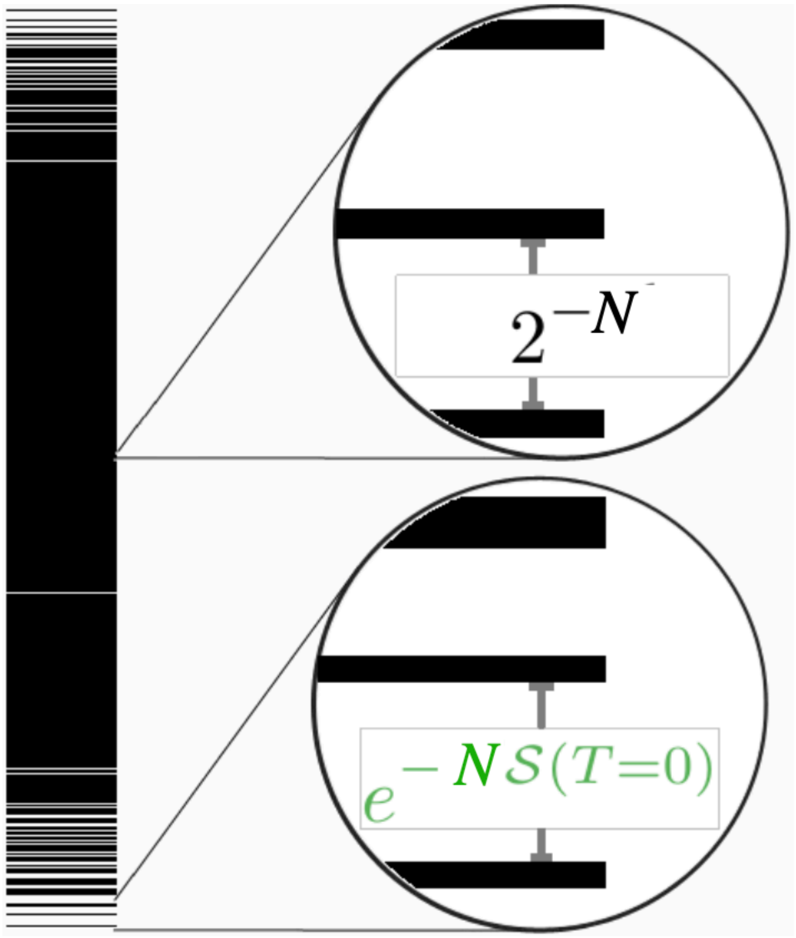
Properties:

- Non-integrable
- Saturates Maldacena-Shenker-Stanford bound:
  $\lambda_{\text{Lyapunov}} = \lambda_{\text{Max}} = 2\pi \underbrace{k_B T/\hbar}_{\text{Universal Planckian rate}}$
- $N \to \infty$: Analytically solvable!
- Conserved $U(1)$ charge:

$$\mathcal{Q} = \frac{1}{N}\sum_i \left\langle c^\dagger_i c_i - 1/2 \right\rangle$$



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S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Kitaev, unpublished
Y. Gu, et. al., JHEP 02 (2020) 157
W. Fu, PhD Thesis (Harvard University, 2018)

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Sachdev-Ye-Kitaev Model

	Fermi-liquid	SYK model
Energy level spacing	$\frac{1}{N}$	$e^{-\alpha N}$
Quasiparticles	Yes	No
Equilibration rate $\tau_{\mathbf{eq}}^{-1}$	$\alpha^2 T^2$	$\approx 1.\frac{k_B T}{\hbar}$
Electric resistivity	T^2	T

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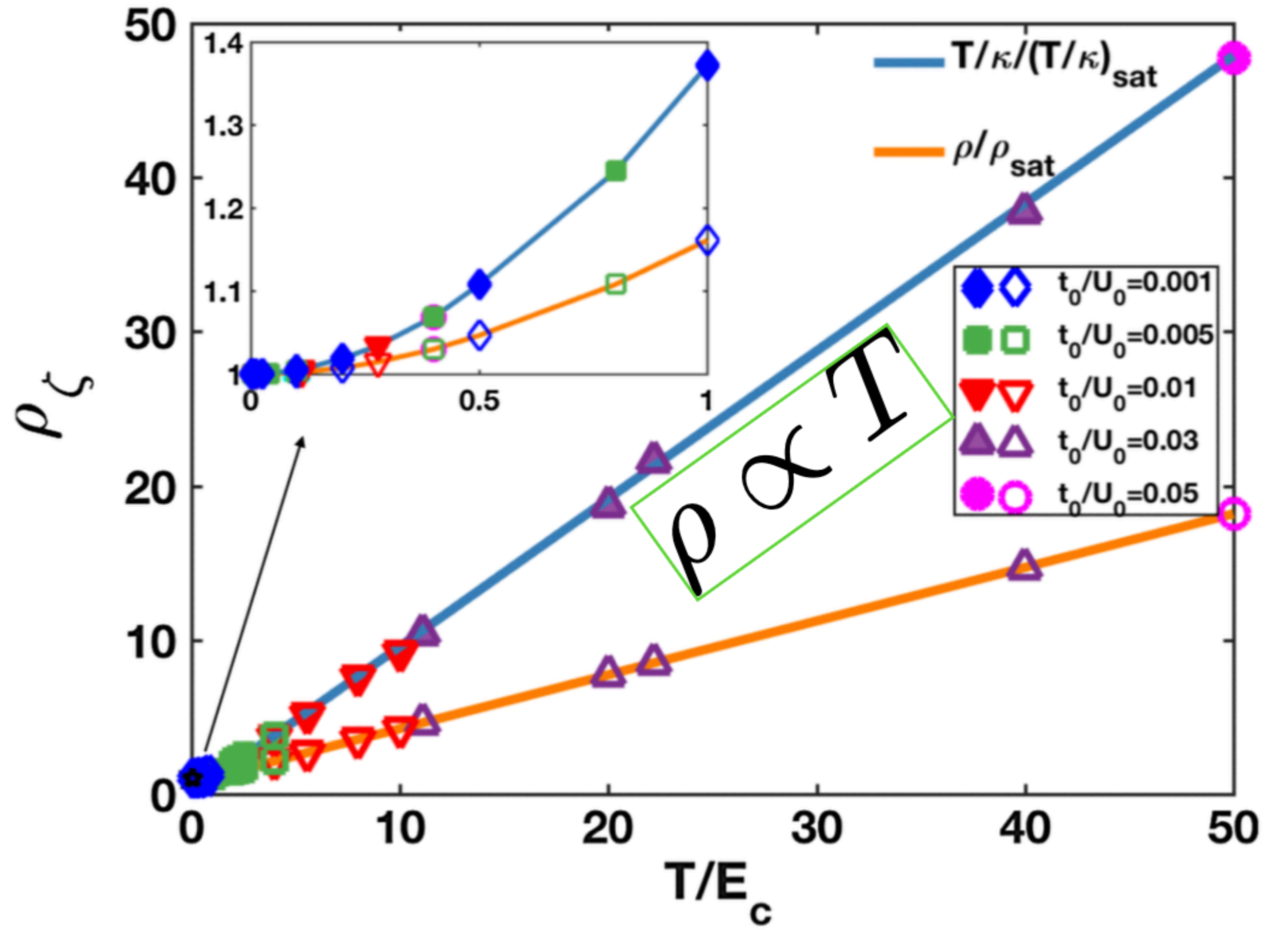

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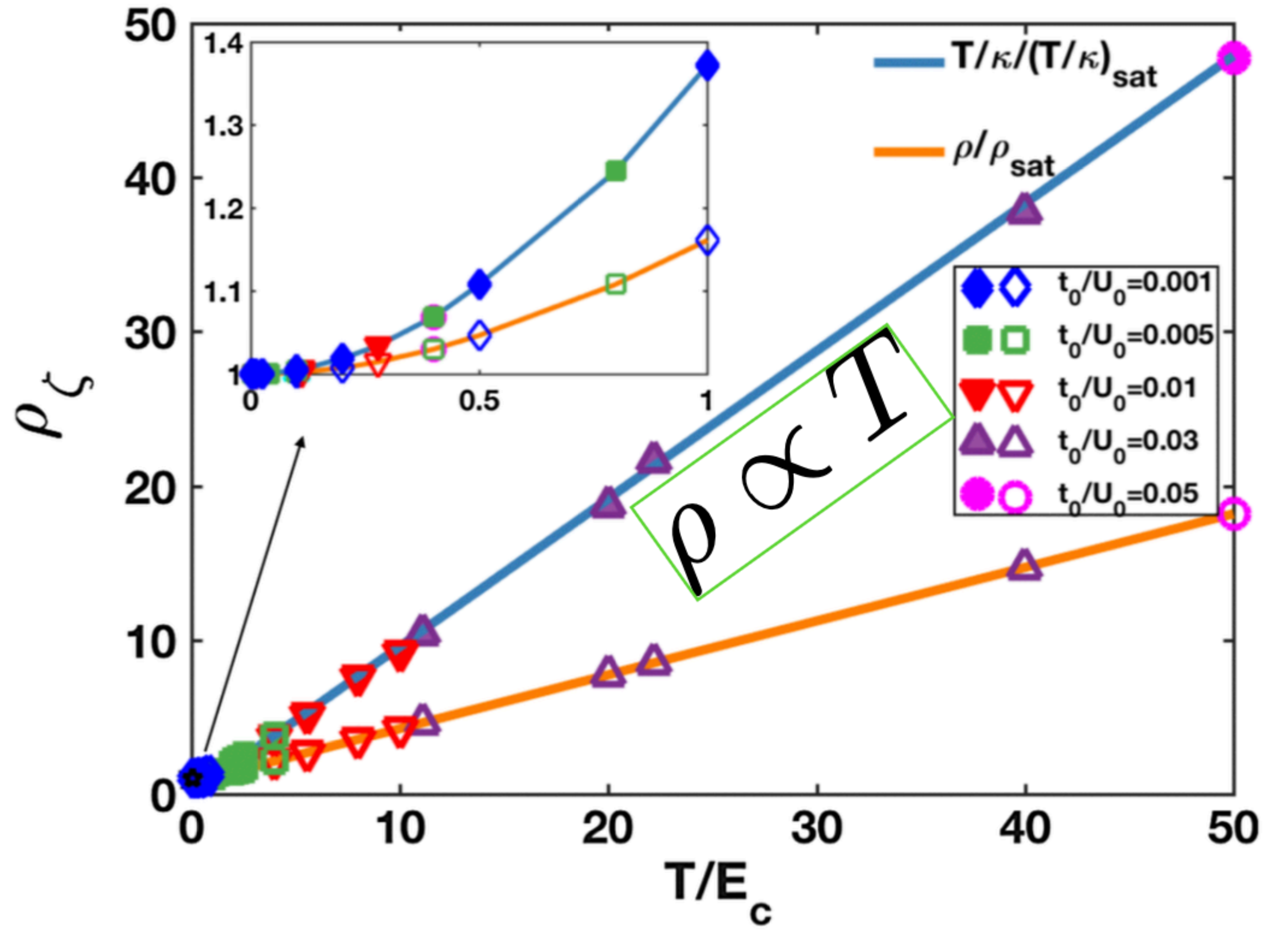

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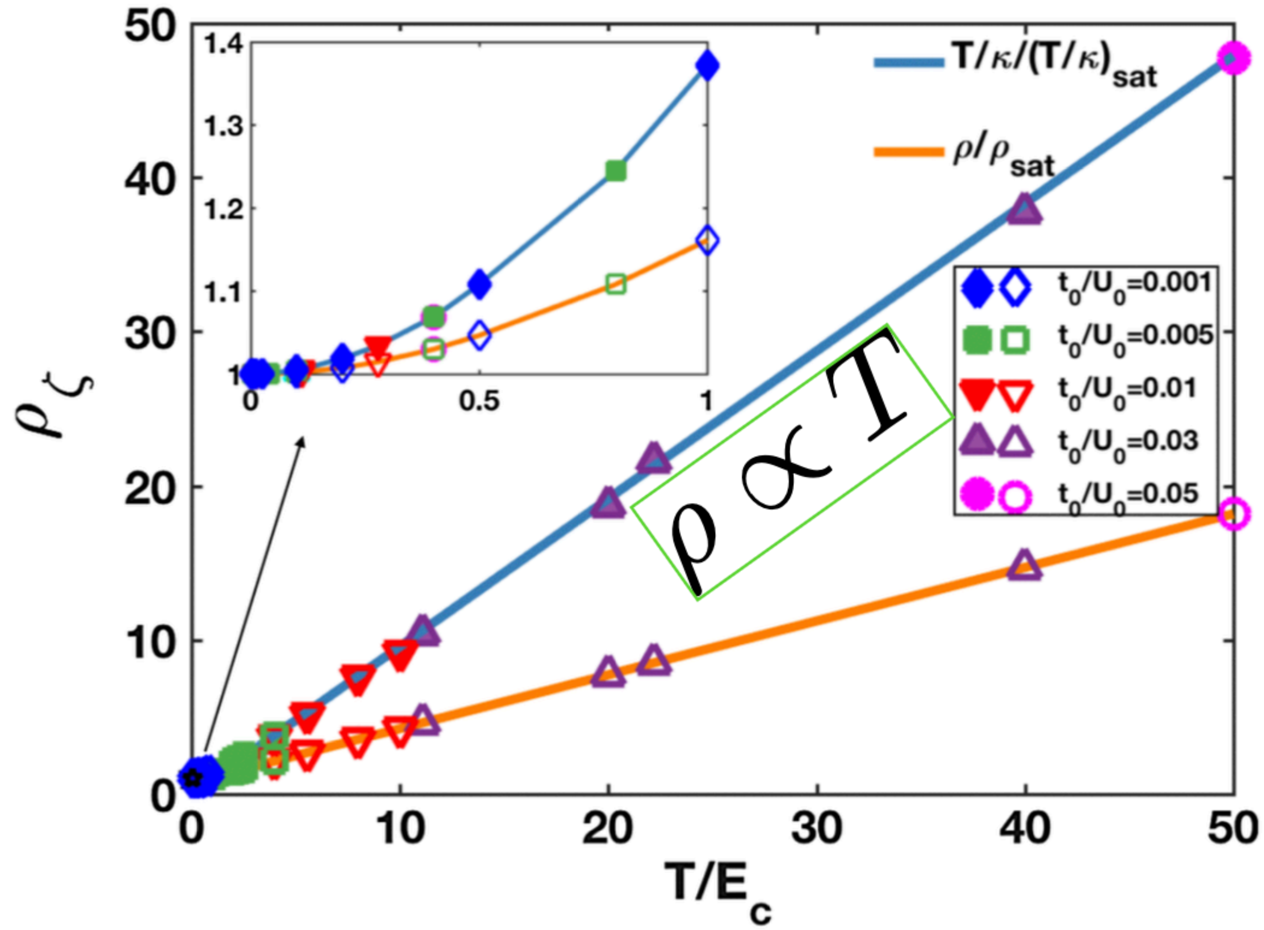

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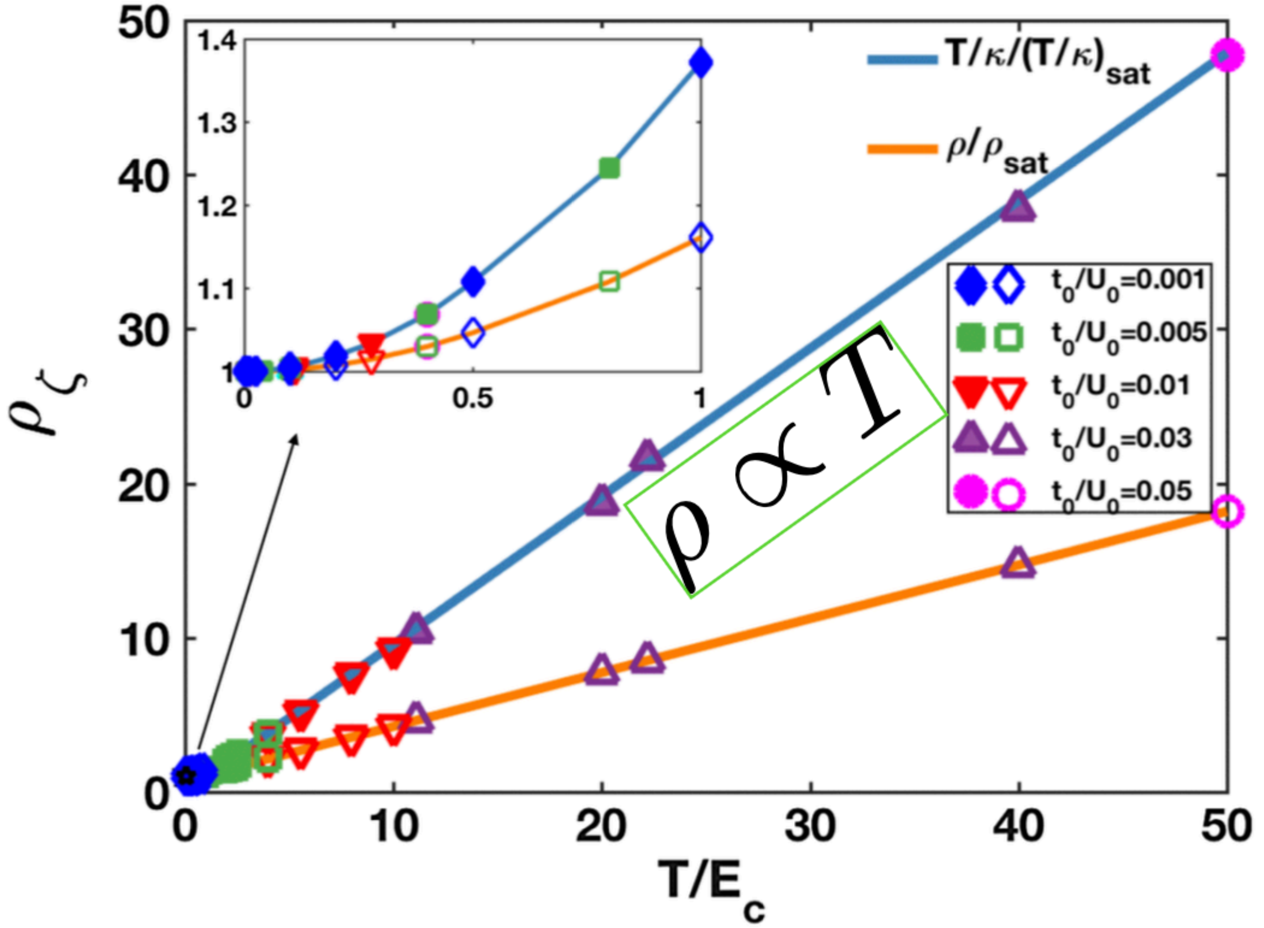

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SYK Model = A model for strange metals!

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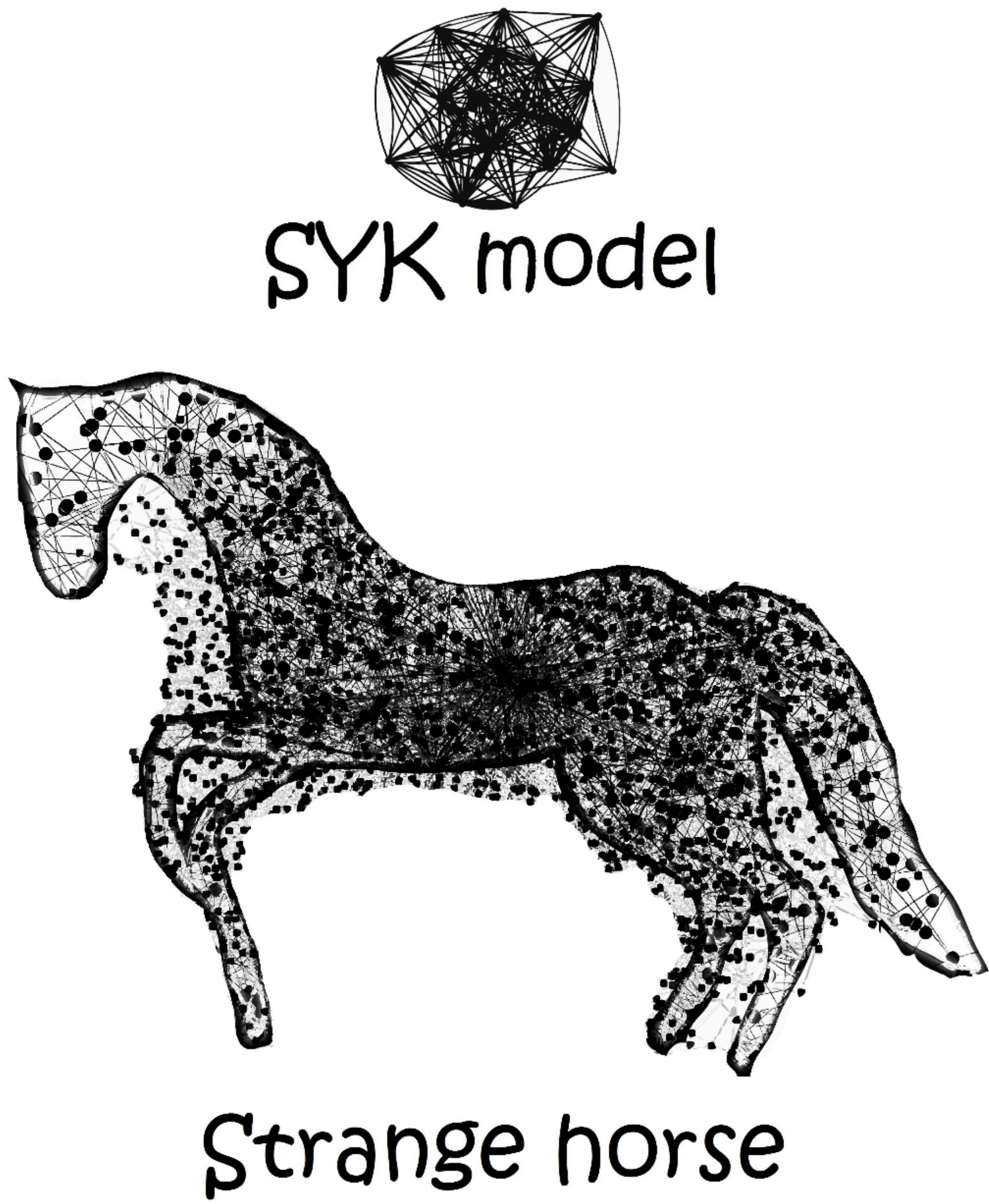

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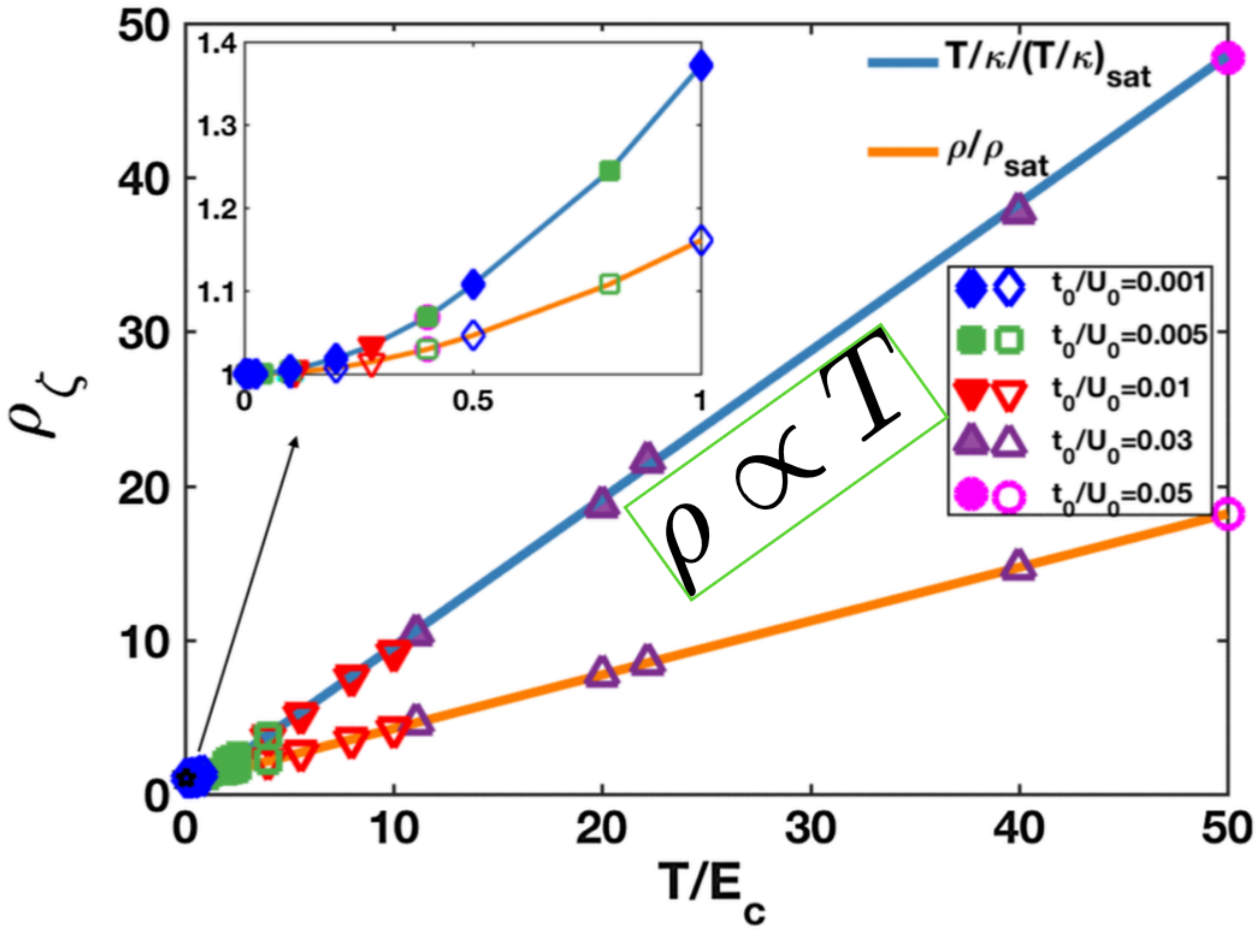

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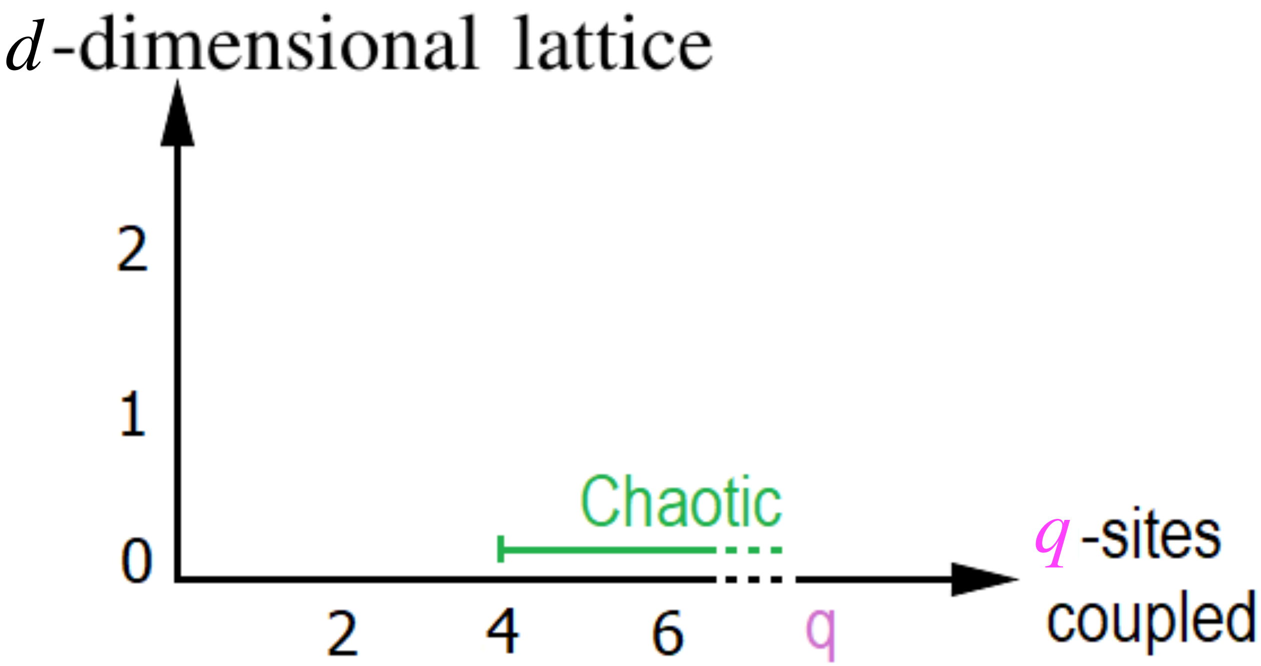
Generalization



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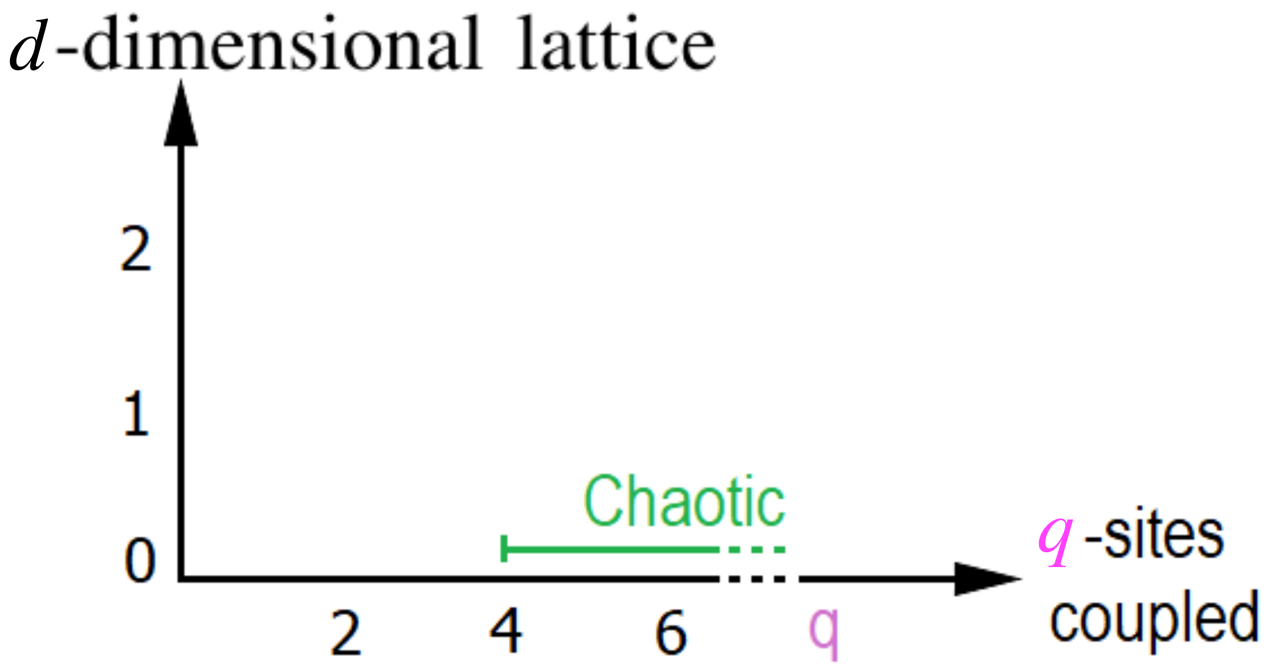

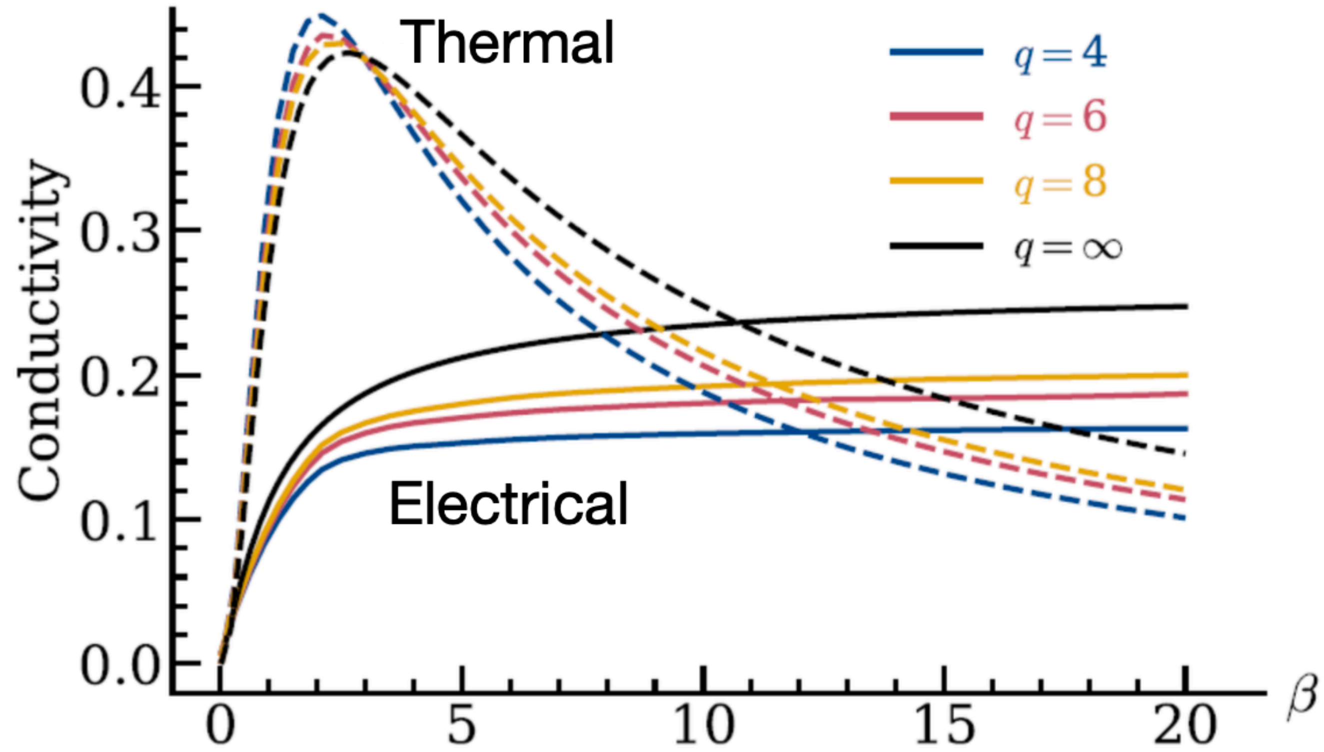

Zanoci and B. Swingle, Phys. Rev. B 105, 235131 (2022)

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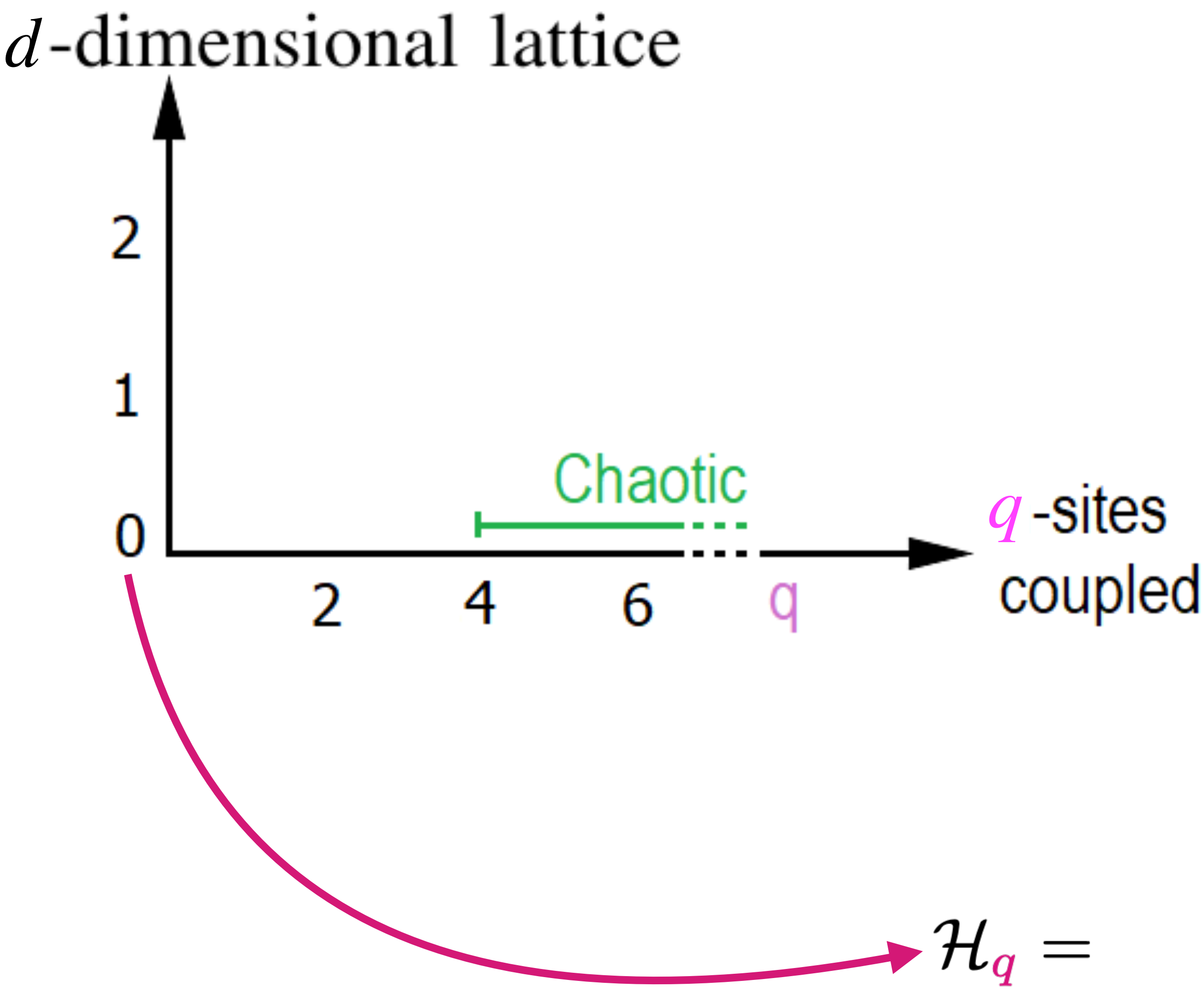

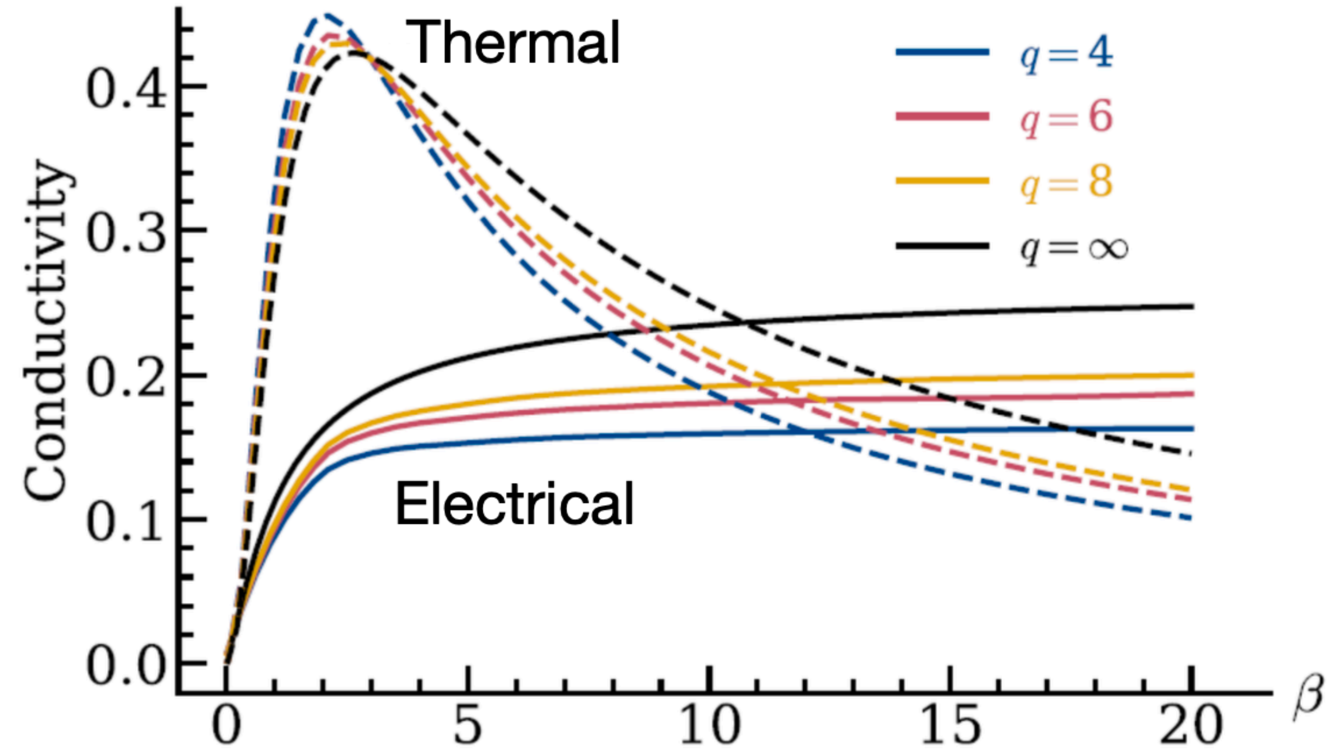

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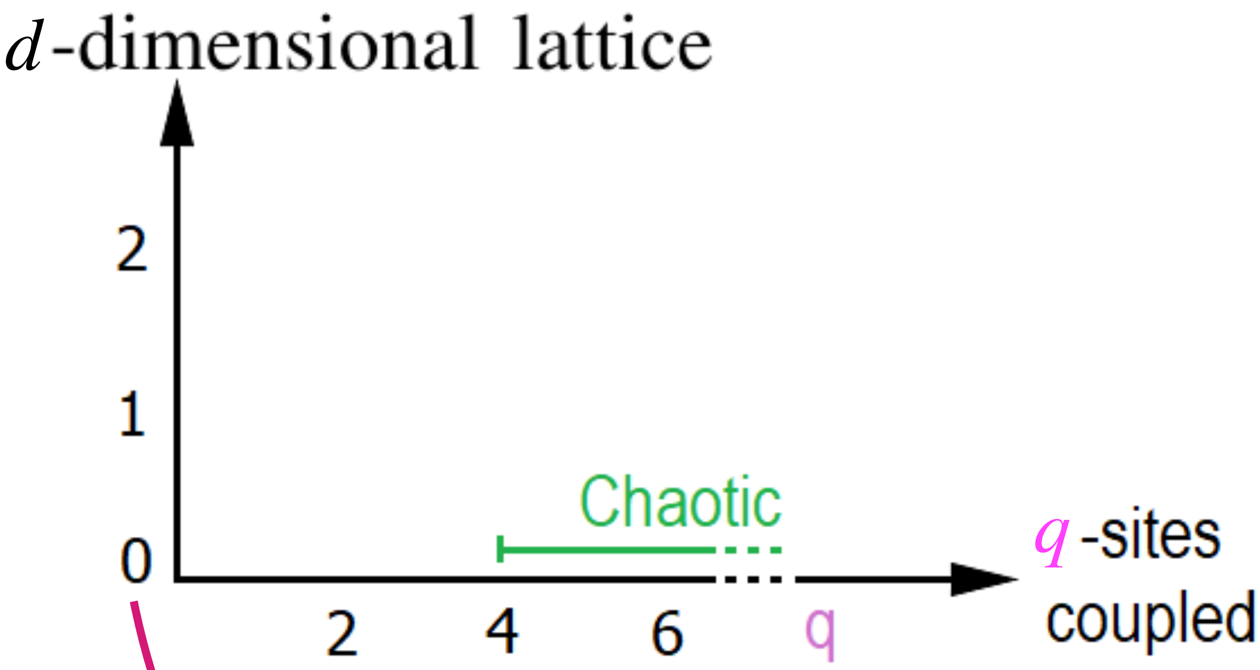

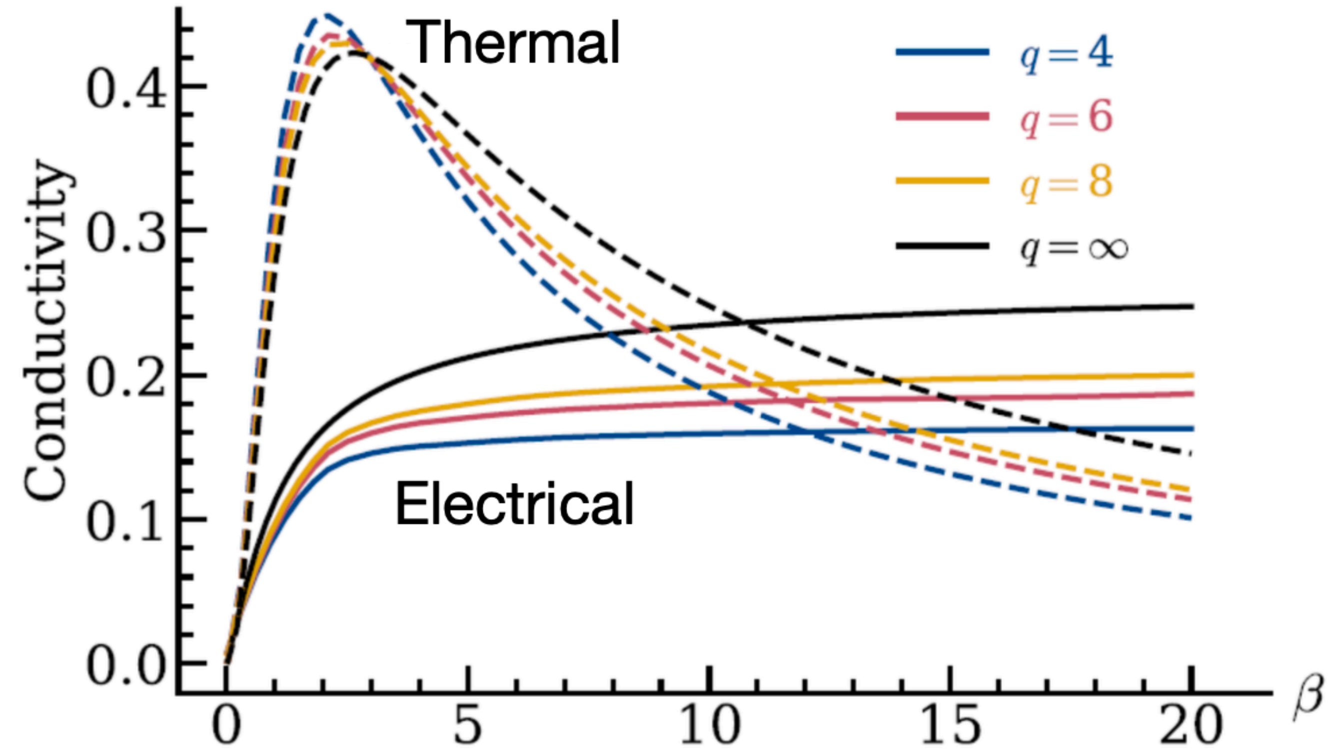

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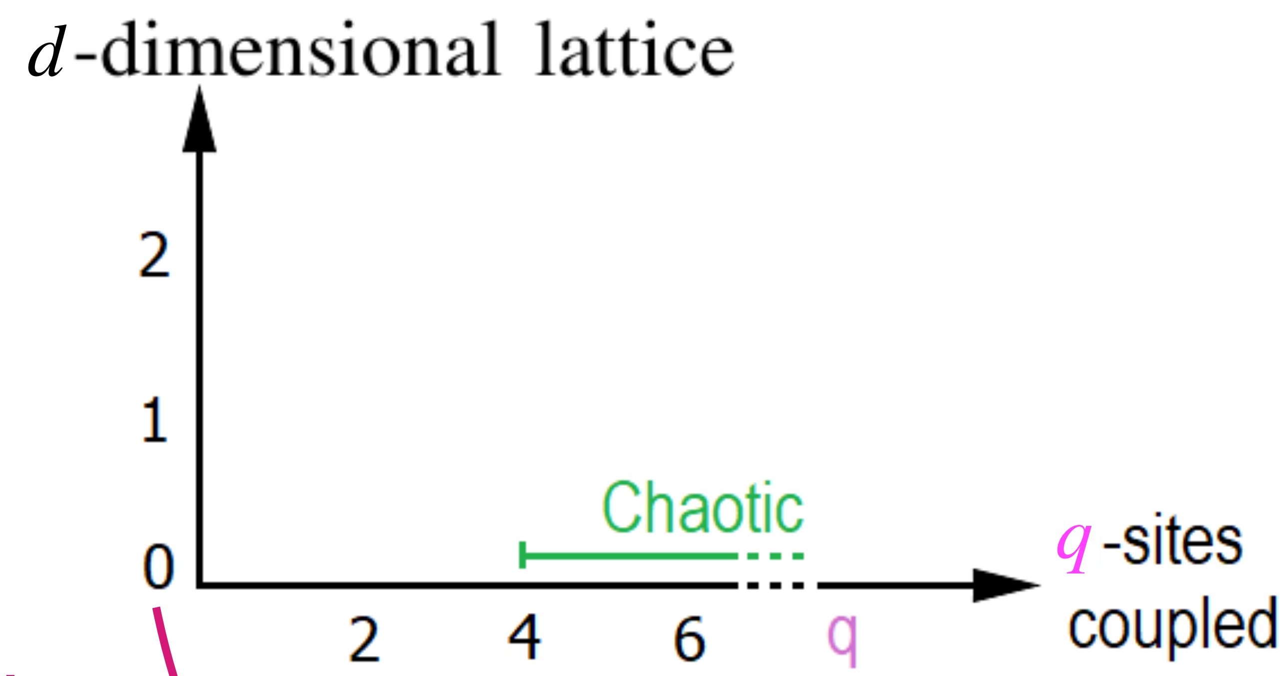
Combining SYK models:

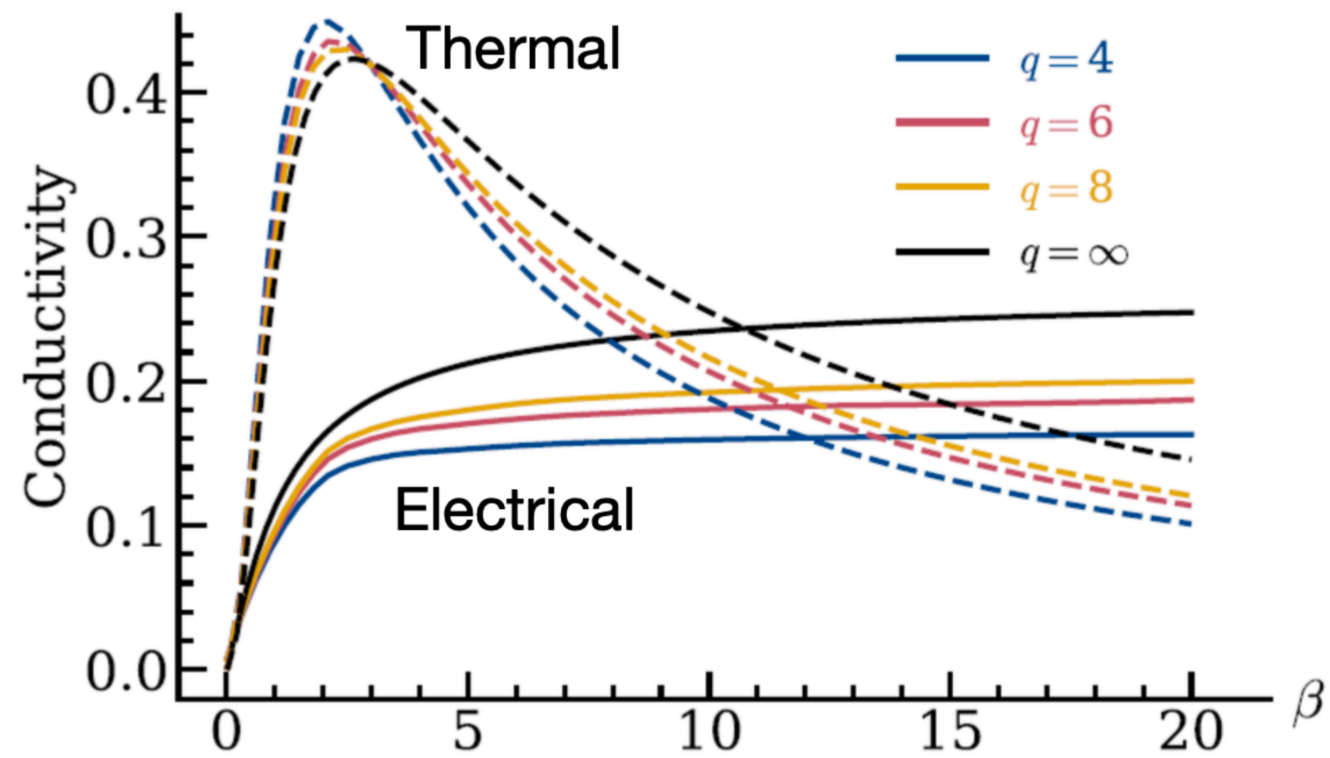
$$\mathcal{H} = \sum_{q=1}^{\infty} \underbrace{J_{2q}(t)}_{q\text{-site coupling strength}} \mathcal{H}_{2q}$$

retains solvability!

J. Maldacena and D. Stanford, PRD 94, 106002 (2016)

Generalization

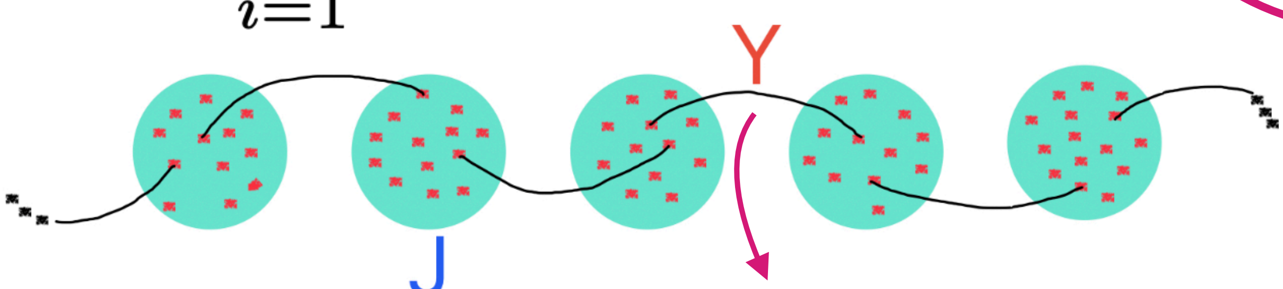




Zanoci and B. Swingle, Phys. Rev. B 105, 235131 (2022)

$$\mathcal{H}_q = \sum_{\text{over indices}} \underbrace{J^{i_1 \cdots i_{q/2}}_{j_1 \cdots j_{q/2}}}_{\text{random coupling}} \underbrace{c^\dagger_{i_1} \cdots c^\dagger_{i_{q/2}} c_{j_{q/2}} \cdots c_{j_1}}_{q-\text{ Sites}}$$

$$\mathcal{H} = \sum_{i=1}^{L} \left(\mathcal{H}_{q,i} + \mathcal{H}_{\text{hopping },i}\right)$$



✳Quadratic 2−body hopping

✳κq−body hopping where $\kappa = \left\{\frac{1}{2}, 1, 2\right\}$

PRB 107, 235114 (2023)
arXiv:2312.14644v2 [hep-th]
arXiv:2407.20733 [cond-mat.str-el]

Combining SYK models:

$$\mathcal{H} = \sum_{q=1}^{\infty} \underbrace{J_{2q}(t)}_{q\text{-site coupling strength}} \mathcal{H}_{2q}$$

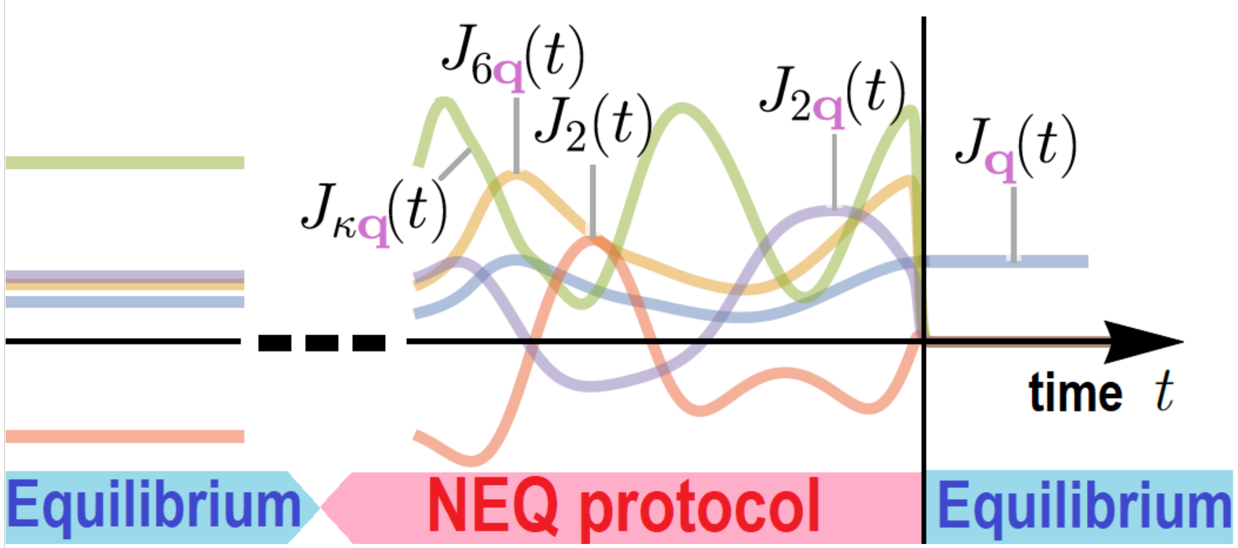
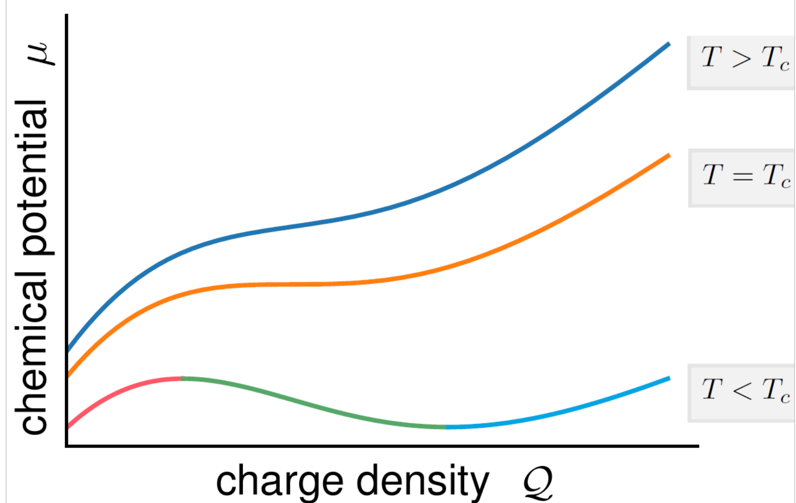
retains solvability!

J. Maldacena and D. Stanford, PRD 94, 106002 (2016)

Dynamics of a Dot at Large-q

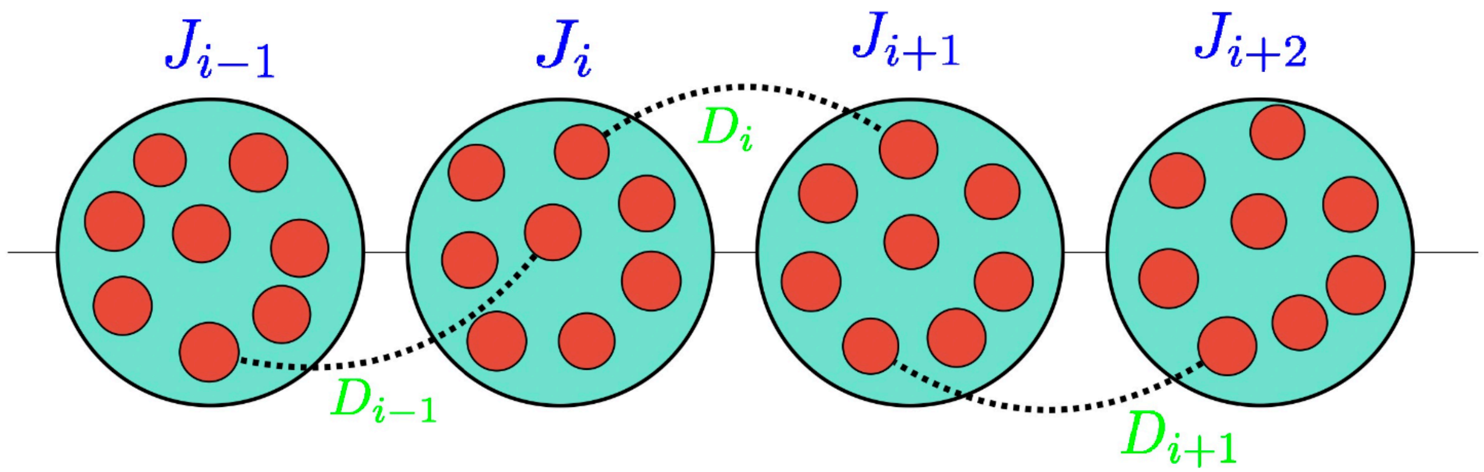
MODEL: $$\mathcal{H}(t) = \sum_{\ell} K_{\ell q}(t)\mathcal{H}_{\ell q} \xrightarrow{t>0} \mathcal{H}_q$$

J. C. Louw and S. Kehrein, PRB 105, 075117 (2022)
A. Eberlein, et. al. Phys. Rev. B 96, 205123 (2017)

Thermalization	Transport	Thermodynamics
 Instantaneously thermalizes!	Zero-dimensional dot ($t > 0$) No transport!	Mean-field SYK phase transition  van der Waals universality class

Dynamics of a Chain at Large-q

MODEL:



$$\mathcal{H}(t) = \sum_{i=1}^{L} \mathcal{H}_{q,i} \xrightarrow{t>0} \mathcal{H} = \sum_{i=1}^{L} (\mathcal{H}_{q,i} + \mathcal{H}_{\text{hopping}\ ,i})$$

RJ and J. C. Louw, PRB 107, 235114 (2023)
J. C. Louw, L. v. Manen and RJ, arXiv:2312.14644v2 [hep-th]

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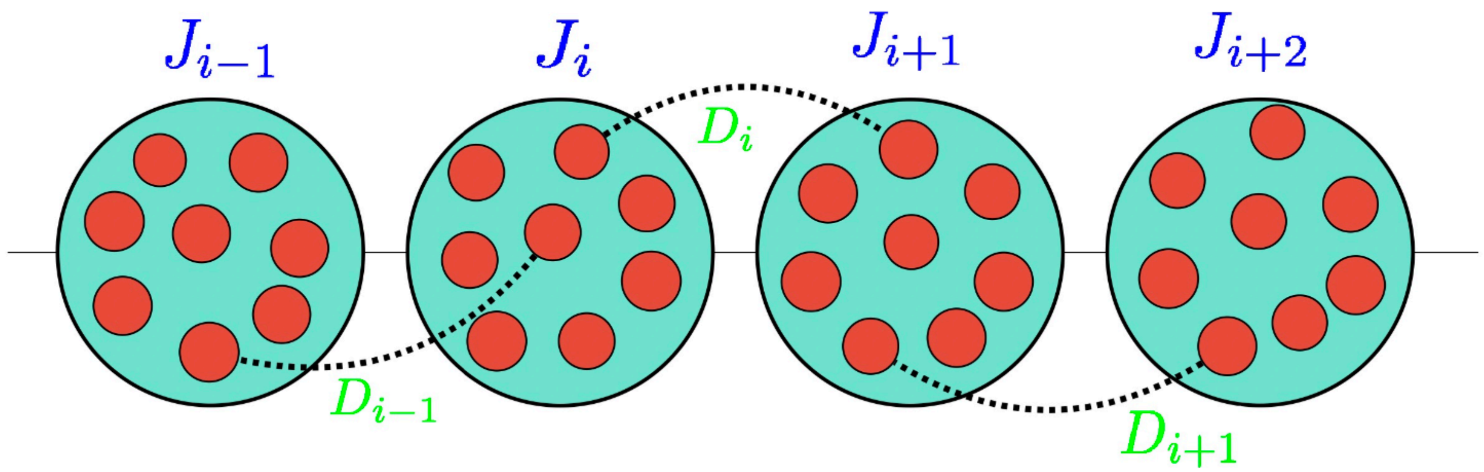

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RJ and J. C. Louw, PRB 107, 235114 (2023)
J. C. Louw, L. v. Manen and RJ, arXiv:2312.14644v2 [hep-th]

Thermalization	Transport	Thermodynamics
DOES NOT instantaneously thermalize! $\Downarrow$ Finite equilibration rate	Nonequilibrium charge $\mathcal{Q}_i$ transport for 2-body hopping: $\Downarrow$ $\ddot{\mathcal{Q}}_i(t) = \frac{4}{q}\left[\mathcal{Q}_{i-1}(t) - 2\mathcal{Q}_i(t) + \mathcal{Q}_{i+1}(t)\right]$	Uniformly coupled & $q/2$-body hopping: van der Waals universality class (again)

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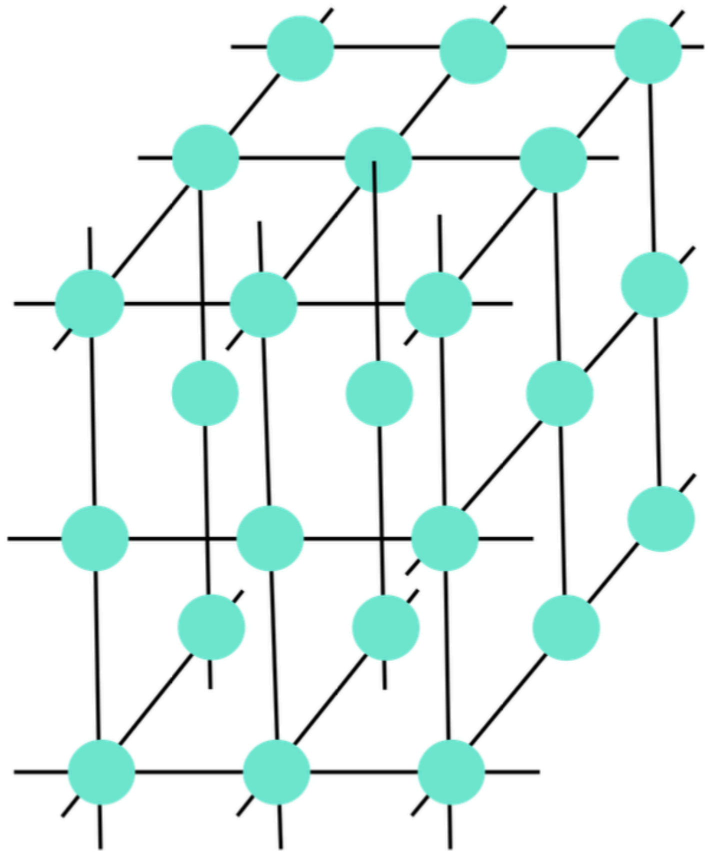
Higher Dimensional Lattices

MODEL:



$$\mathcal{H}(t) = \underbrace{\sum_{x\in\Lambda} \mathcal{H}_x(t)}_{\text{On-site SYK dots}} + \underbrace{\sum_{\langle x,x'\rangle\in\Lambda} \mathcal{H}_{x\to x'}(t)}_{\text{Nearest-neighbor } r\text{-body hopping}}$$

RJ and J. C. Louw, PRB 107, 235114 (2023)

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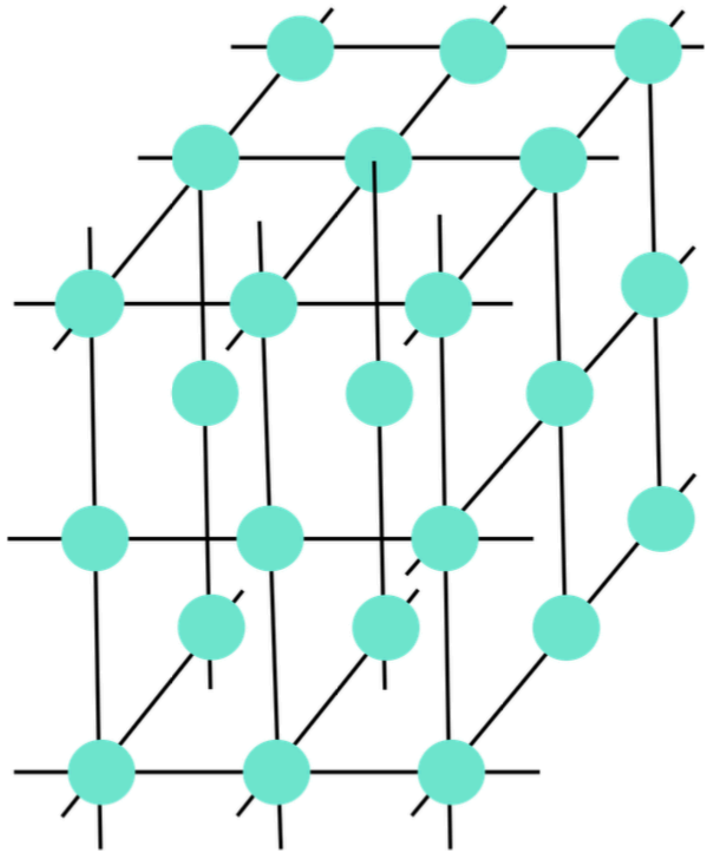

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RJ and J. C. Louw, PRB 107, 235114 (2023)

Thermalization	Transport	Thermodynamics
DOES NOT instantaneously thermalize! $\Downarrow$ Finite equilibration rate	Nonequilibrium charge $\mathcal{Q}_i$ transport $\Downarrow$ Preserves closed-form relation for charge dynamics	FUTURE RESEARCH

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Coincidence?

Large−q complex SYK model: $\mathcal{H} = \mathcal{H}_q = \underbrace{\sum}_{\text{over indices}} \underbrace{J^{i_1 \cdots i_{q/2}}_{j_1 \cdots j_{q/2}}}_{\text{random coupling}} \underbrace{c^\dagger_{i_1} \cdots c^\dagger_{i_{q/2}} c_{j_{q/2}} \cdots c_{j_1}}_{q-\text{ Sites}}$

J. C. Louw and S. Kehrein, PRB 107, 075132 (2023)

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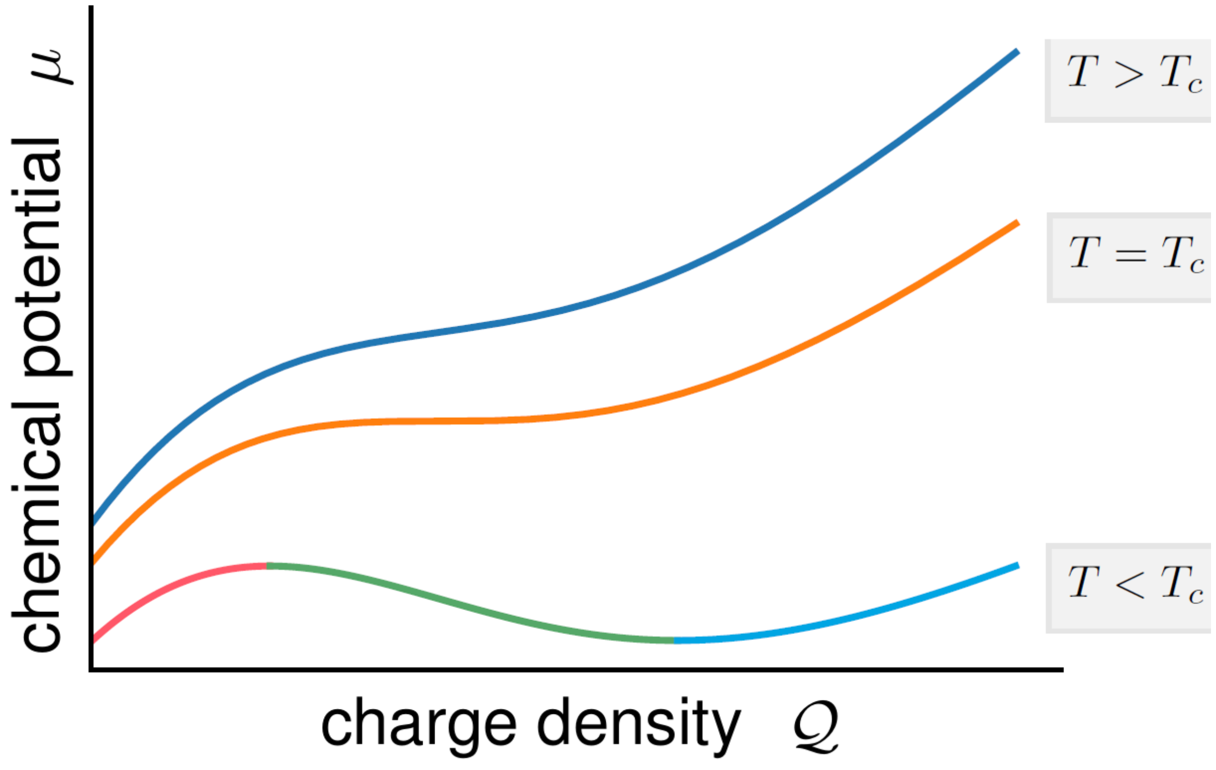

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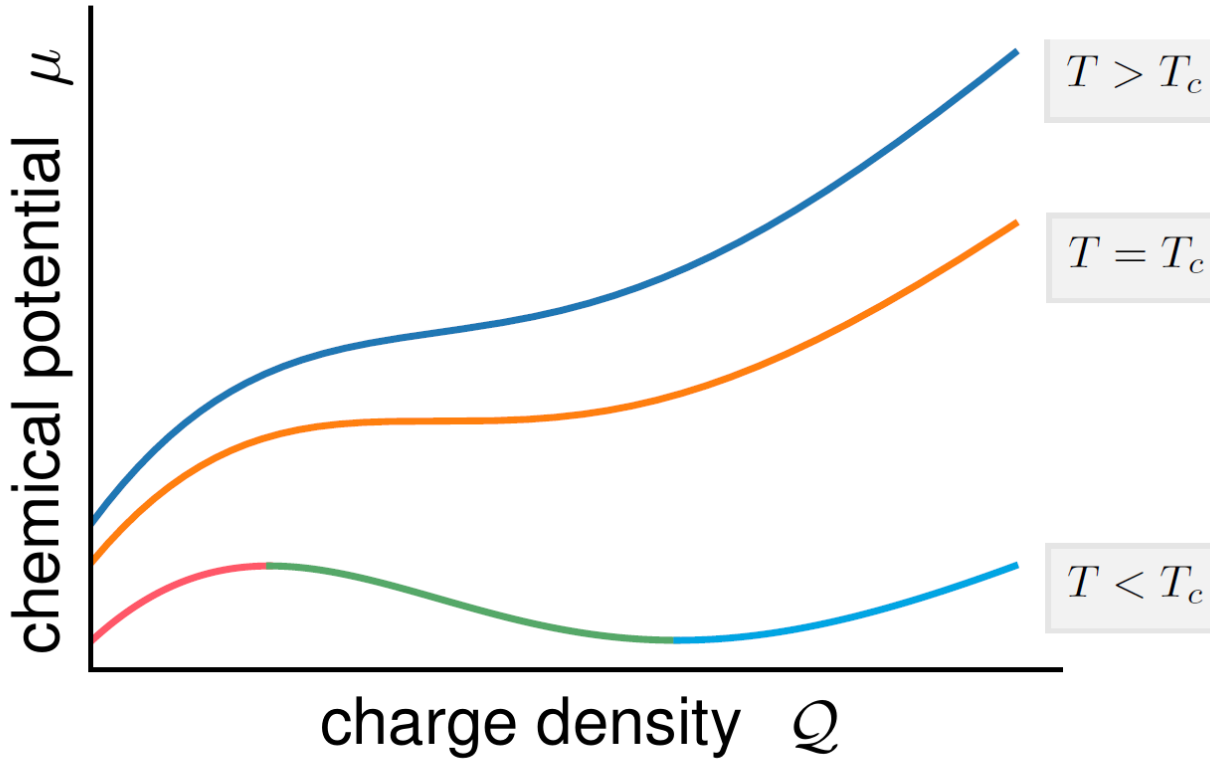

Critical exponents:

α	β	γ	δ
0	$\frac{1}{2}$	1	3

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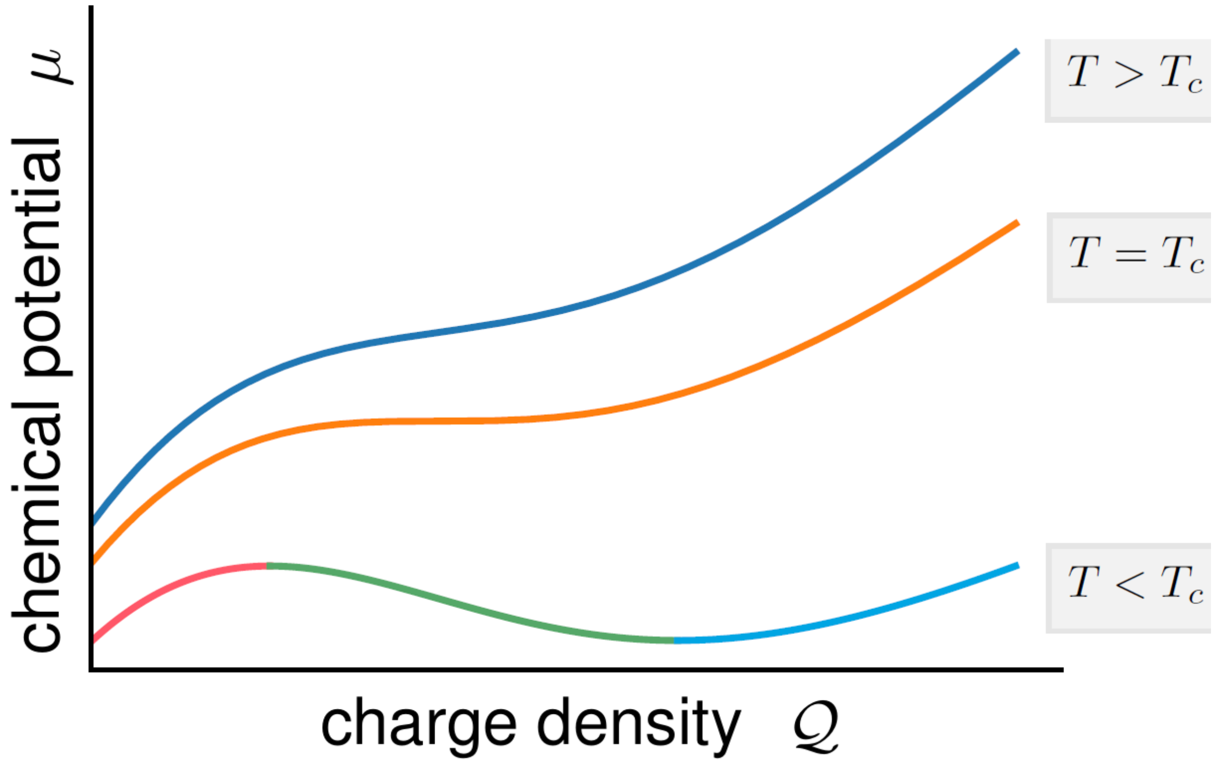

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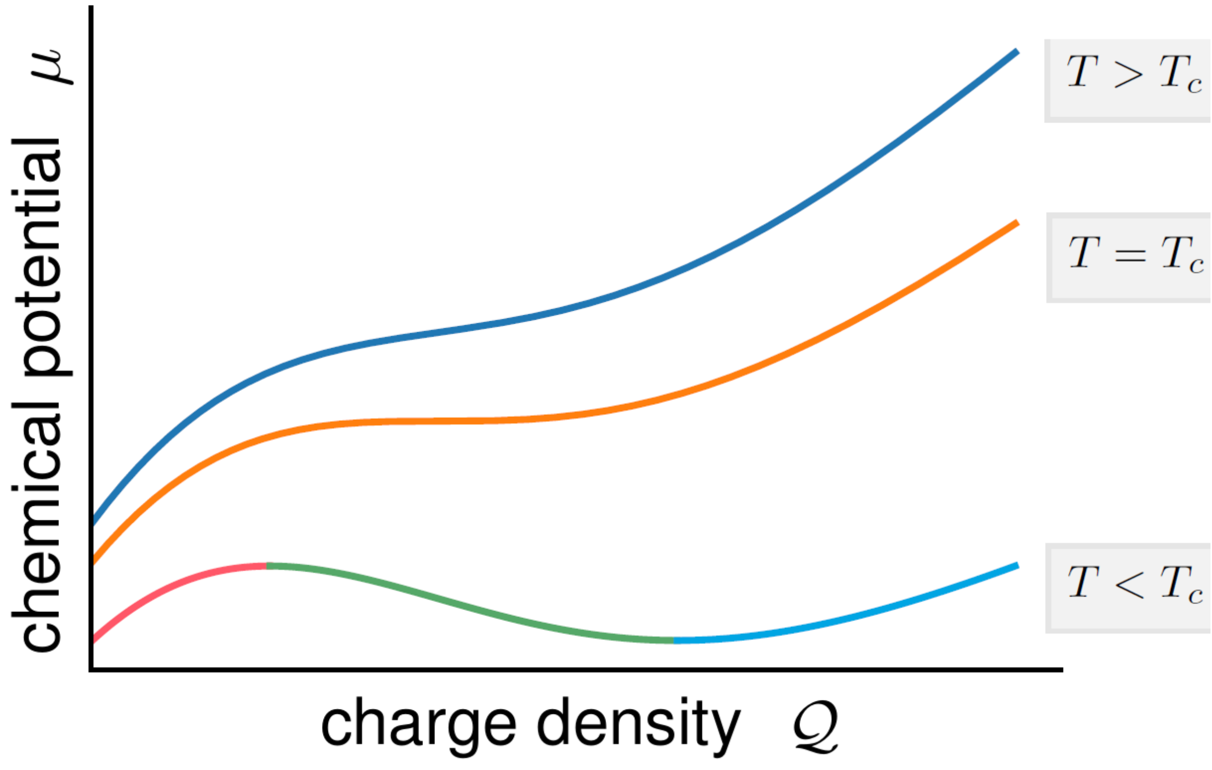

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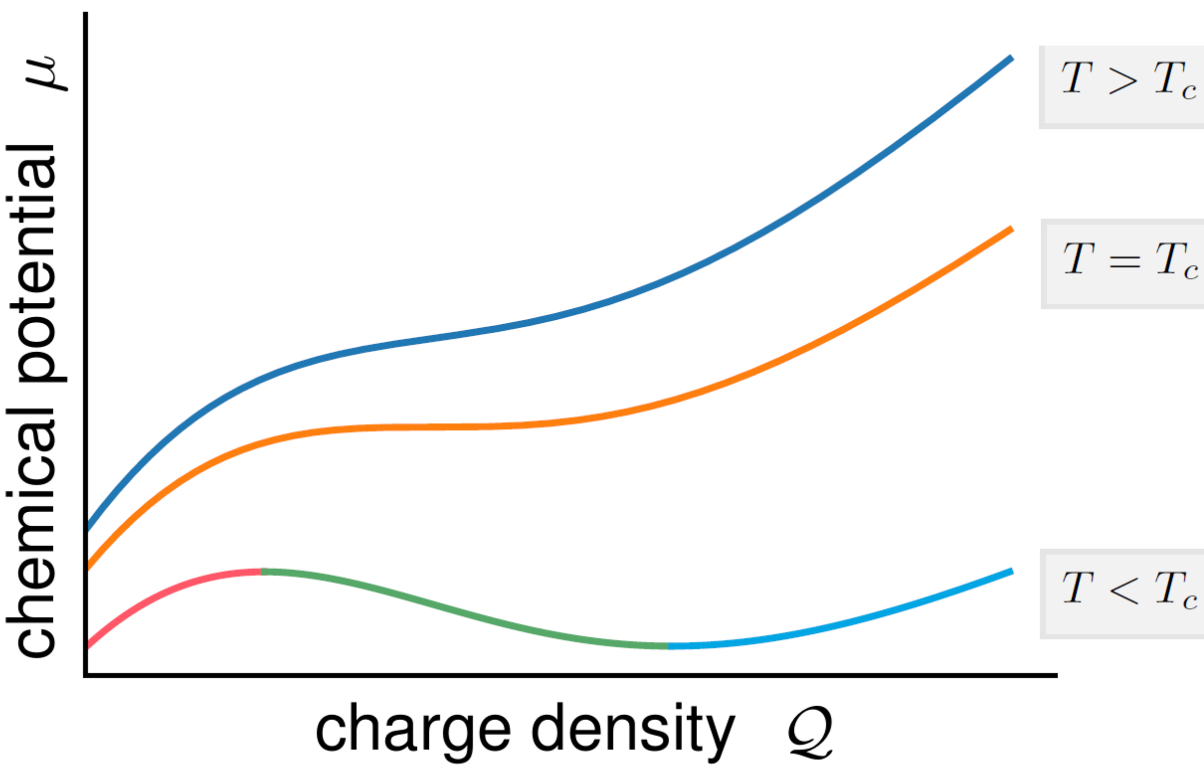

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Ginzburg-Landau (mean-field)

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True for all large−q SYK? $\mathcal{H} = \sum_{k>0} \mathcal{K}_{kq} \mathcal{H}_{kq}$

RJ, S. Kehrein, J. C. Louw, arXiv:2407.20733 [cond-mat.str-el]

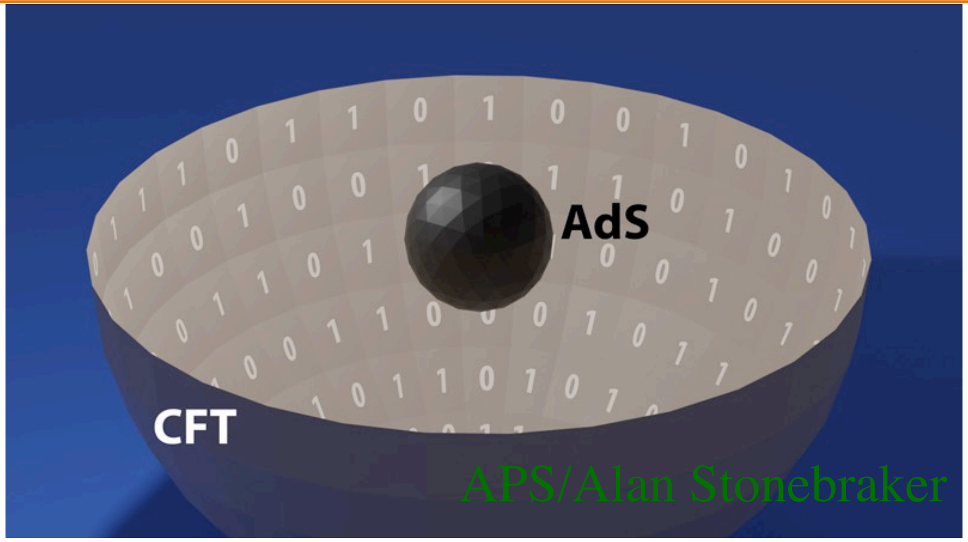
Holography $\Rightarrow$ Black Holes

Holography:

$$\text{Gravity}_d \overset{\text{dual}}{\longleftrightarrow} \text{Quantum}_{d-1}$$

$$\text{Coupling Strength } \frac{1}{J} \overset{\text{dual}}{\longleftrightarrow} \text{Coupling Strength } J$$

J. M. Maldacena, Adv.Theor.Math.Phys.2:231-252,1998



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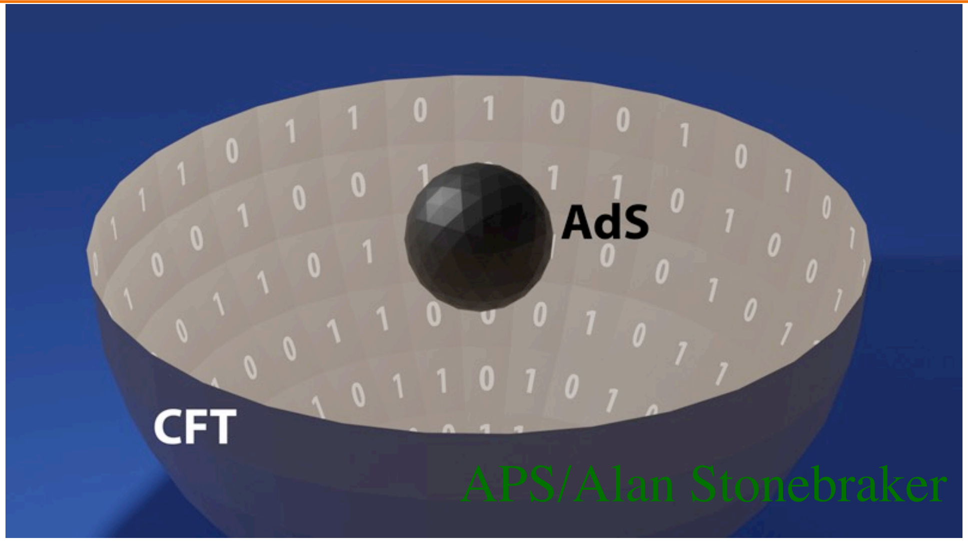

SYK_q – dot and $\text{SYK}_q + \text{SYK}_{q/2}$ – uniform chain

- Dual to deformed JT gravity
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J. C. Louw and S. Kehrein, PRB 107, 075132 (2023)
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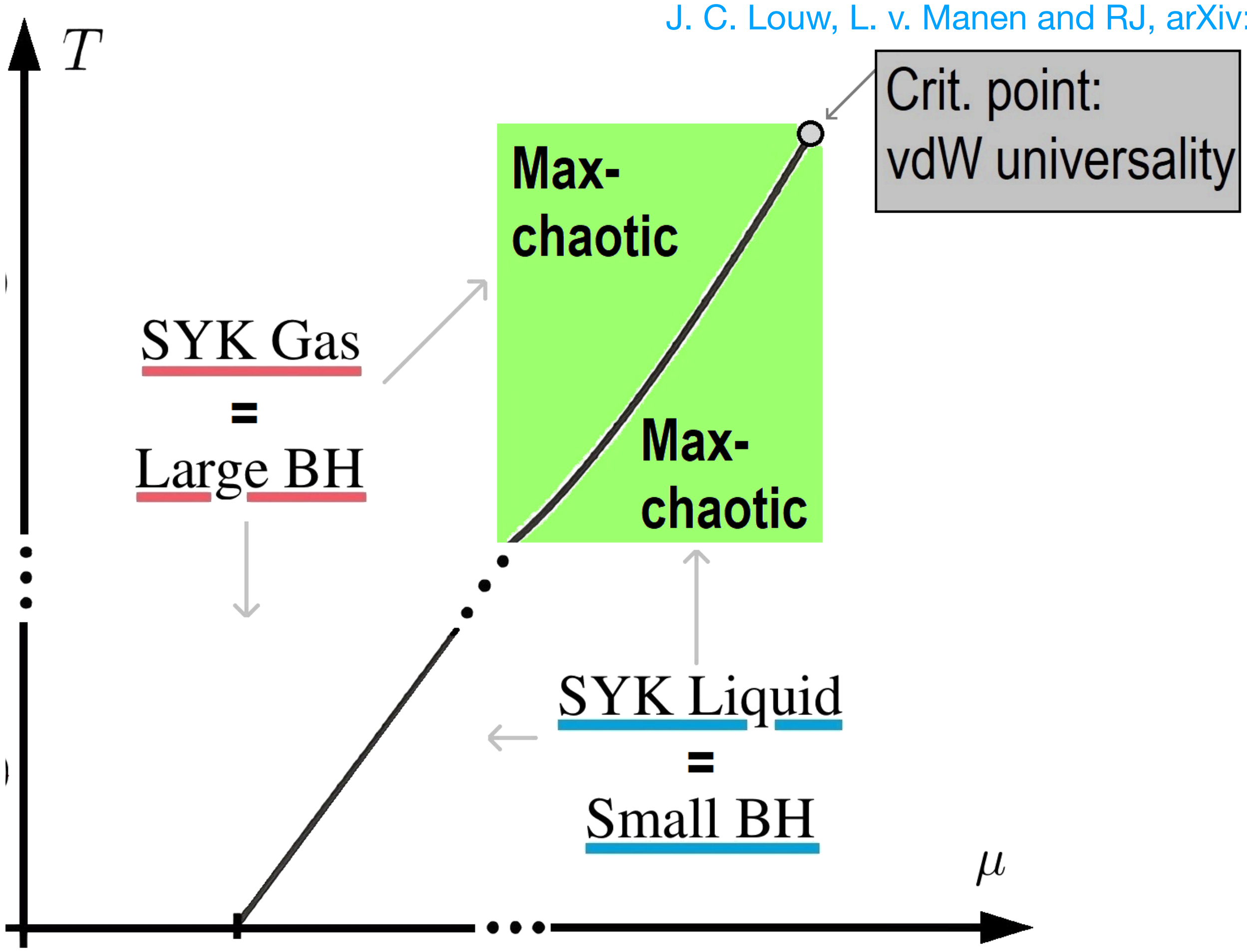

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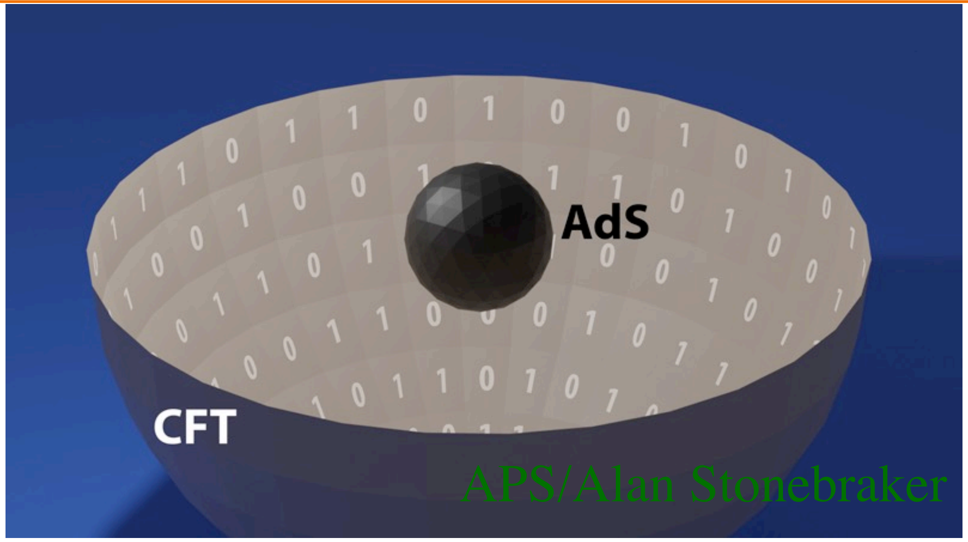

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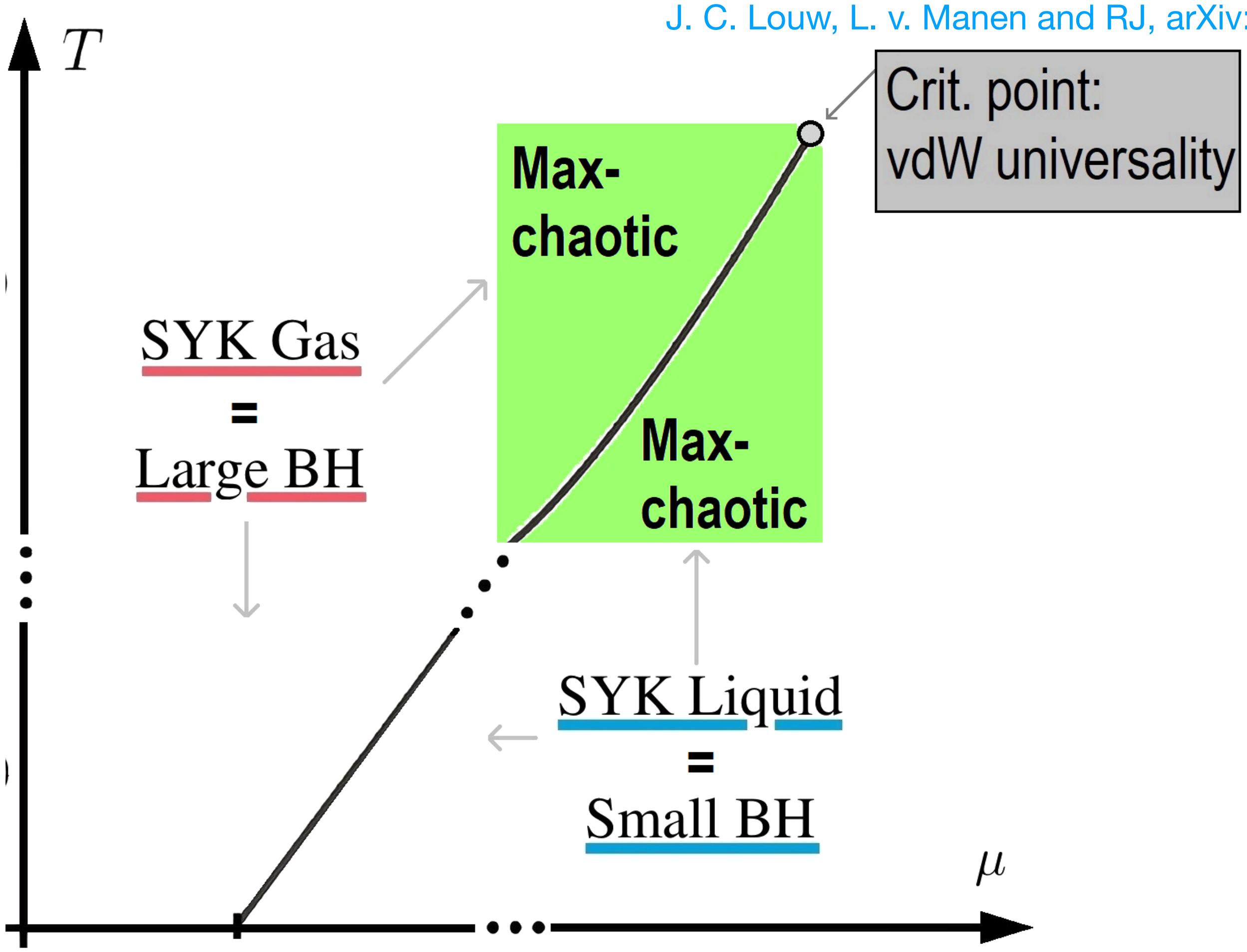

Black Holes = SYK ?

- Maximally chaotic
- $\tau_{\text{eq}}^{-1} \approx 1 \cdot k_B T/\hbar$
- van der Waals universality

Y.-Q. Lei and X.-H. Ge, PRD 105, 084011 (2022)
C.V. Vishveshwara, Nature 227, 936 (1970)
G. Carullo, et. al, PRL 126, 161102 (2021)

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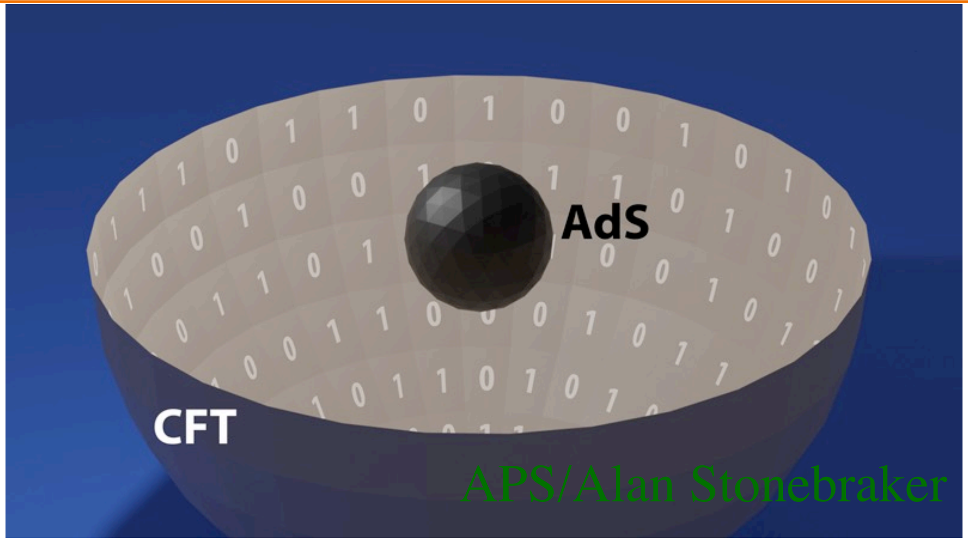

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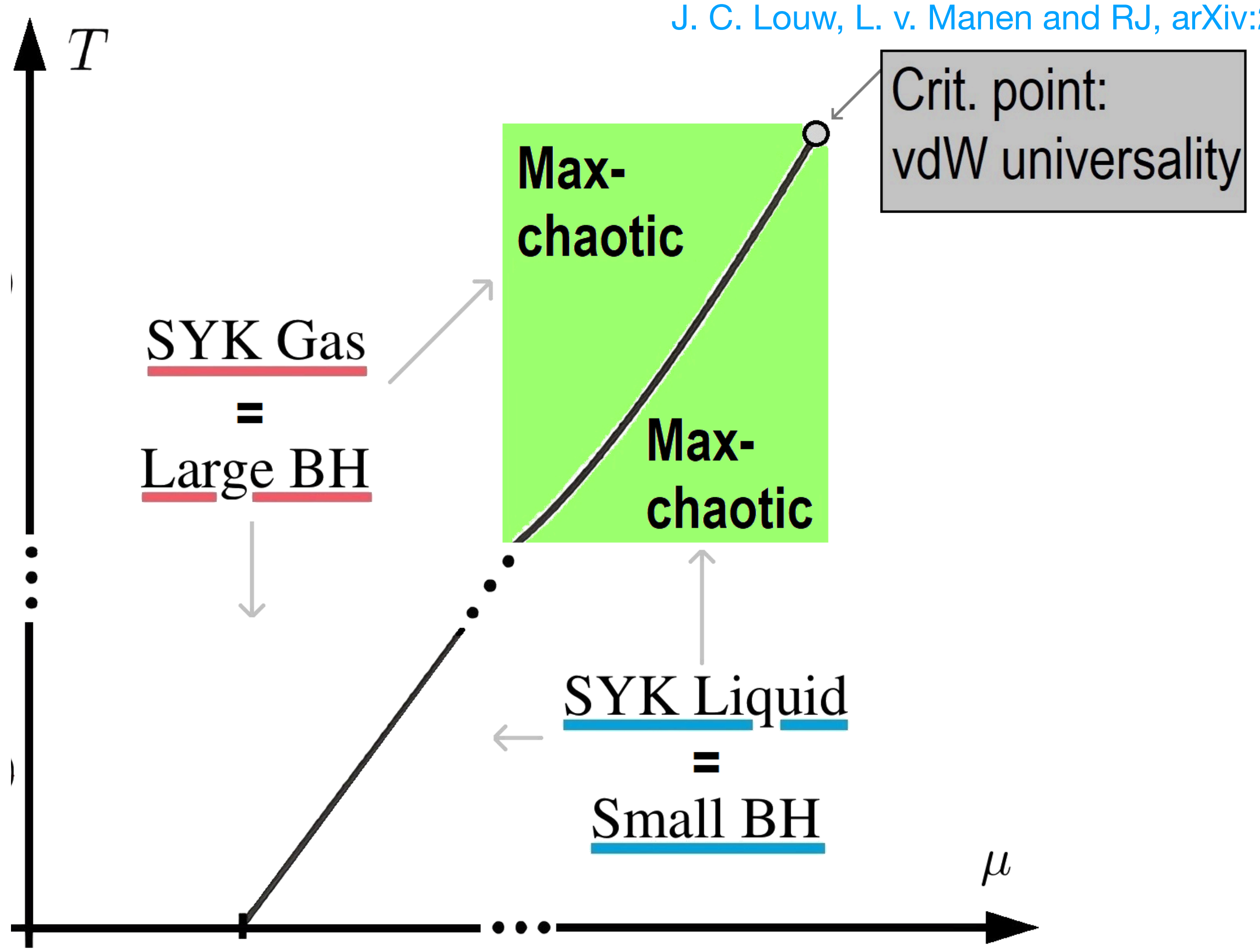

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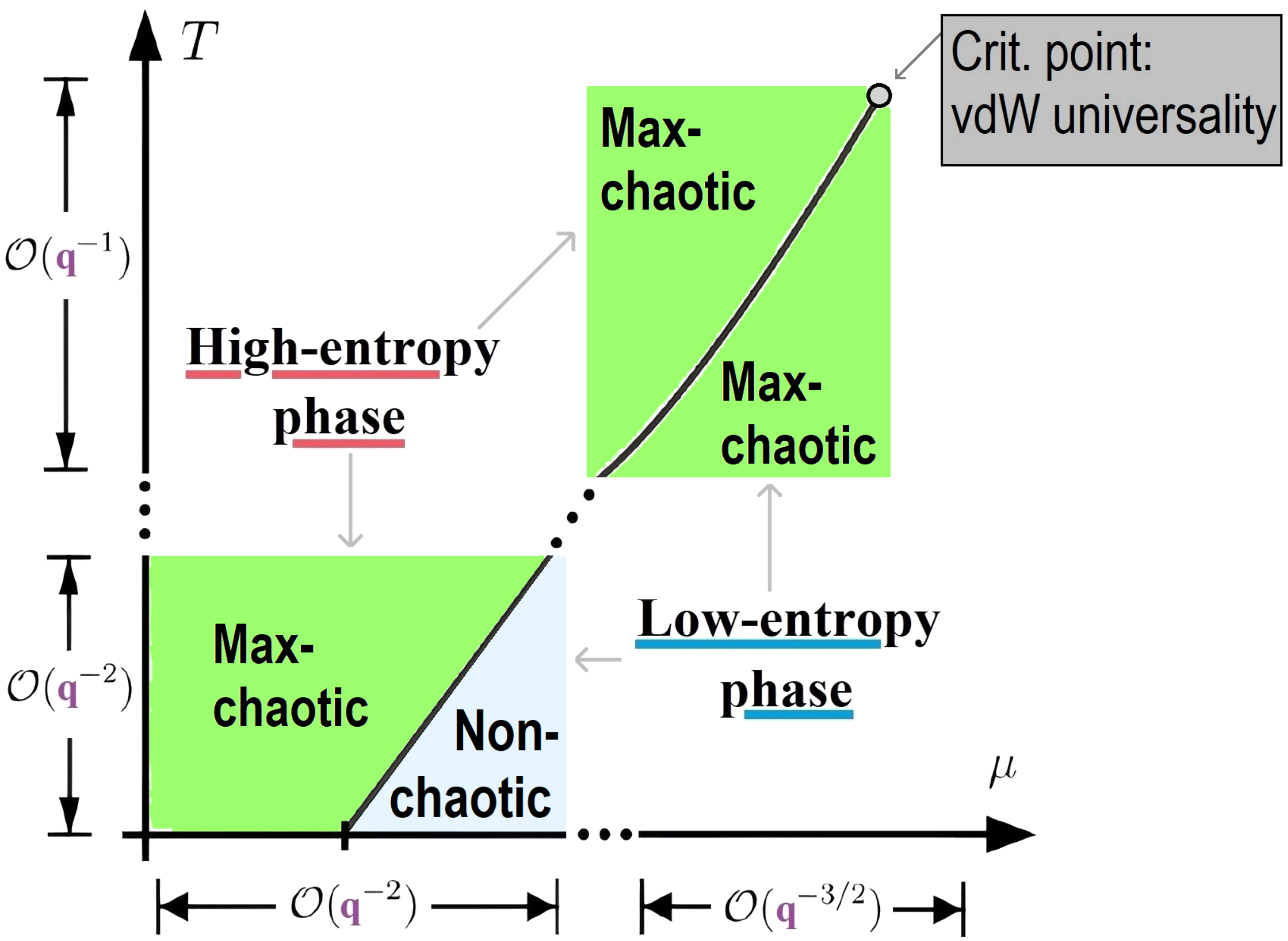
SYK Dimension > 1+1

FUTURE
RESEARCH

Relation with Hawking-Page Transition

SYK	Charged Black Holes



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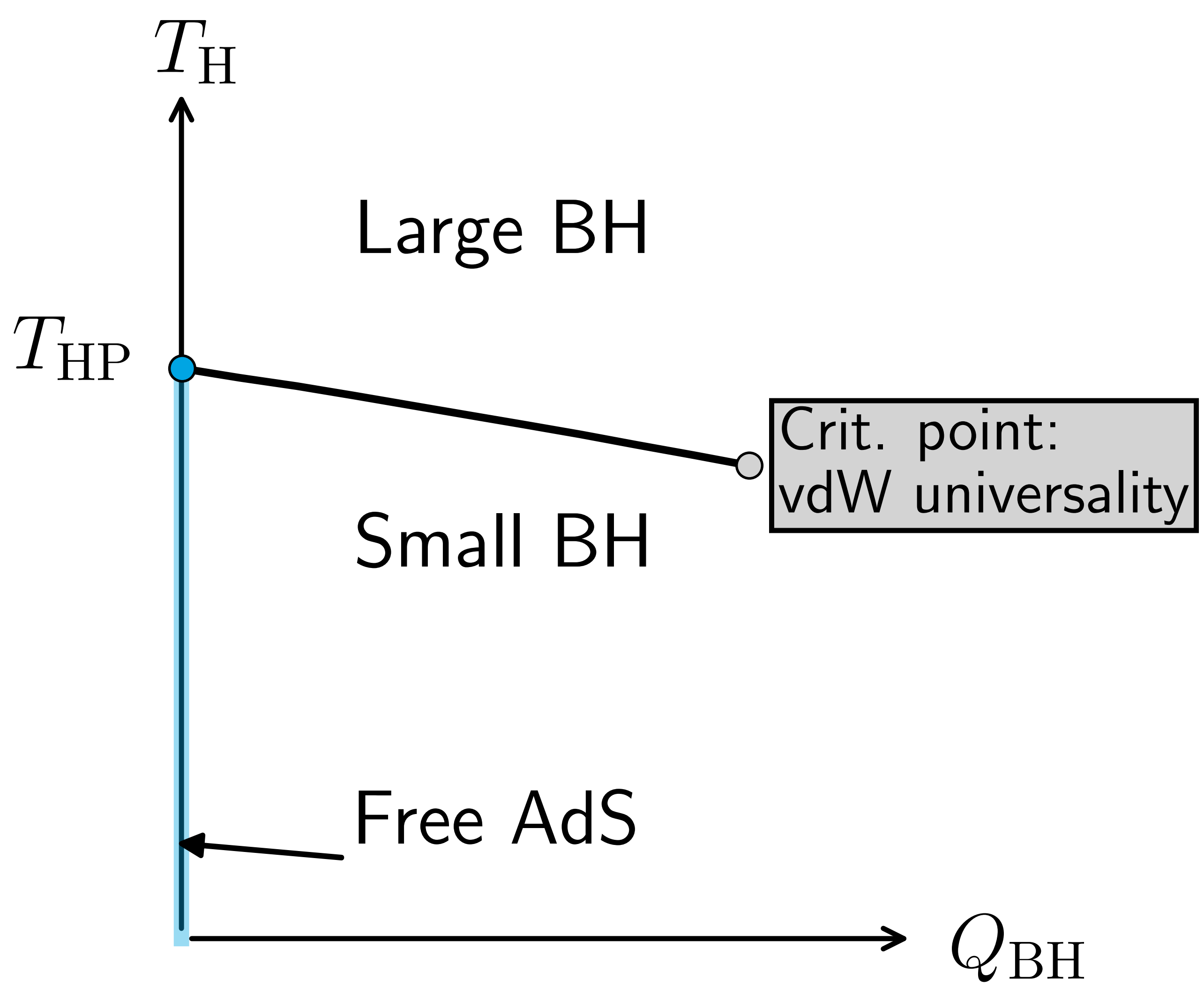

D. Kubiznak, R.B. Mann, JHEP 07 (2012) 033

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- Future research:
  (i) finding thermodynamic properties, &
  (ii) holographic dual with Hawking-Page type transition of $d > 1$ sheets
  (iii) Lyapunov & critical exponents for general coupled SYK models

Summary

- High-T_c cuprates violate Fermi liquid (~ quasiparticle) behavior at low temperature

- Non-fermi liquid (or strange metals) are without quasiparticles; Sachdev-Ye-Kitaev model and its generalizations are candidates for strange metals

- Analytical solvability (in large-N and large-q) despite being chaotic & non-integrable

- $d = 0$ SYK dot:
  (i) instantaneously thermalizes, &
  (ii) belongs to van der Waals universality class

- $d \geq 1$ SYK lattices with nearest-neighbor quadratic hopping:
  (i) closed form non-equilibrium charge transport independent of coupling strengths, &
  (ii) does not instantaneously thermalize

- $d = 1$ uniformly coupled SYK chain with nearest-neighbor $q/2-$hopping:
  (i) belongs to van der Waals universality class as $d = 0$ case, &
  (ii) saturates the maximum bound of chaos (MSS) as $d = 0$ case

- Future research:
  (i) finding thermodynamic properties, &
  (ii) holographic dual with Hawking-Page type transition of $d > 1$ sheets
  (iii) Lyapunov & critical exponents for general coupled SYK models

Thank you!

Hope you had a nice nap!

Backup Slides

Majorana SYK Model

[S. Sachdev, J. Ye, Phys. Rev. Lett. 70 (1993) 3339-3342; A. Kitaev, unpublished]

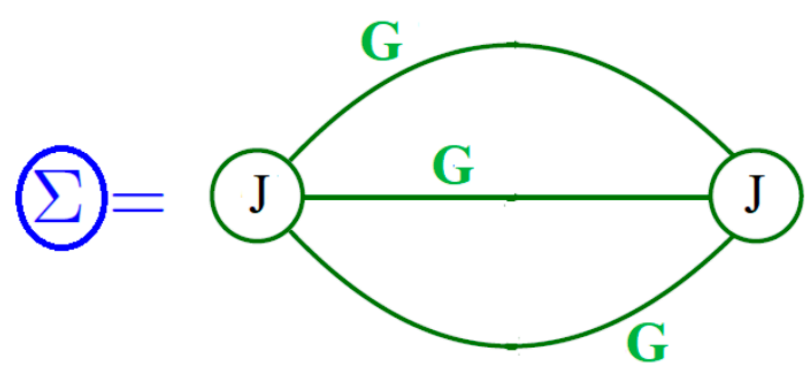
Majorana SYK Model

$$\mathcal{H} = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \qquad (\{\chi_i, \chi_j\} = \delta_{ij})$$

Random Antisymmetric Couplings $\longrightarrow$ Gaussian Ensemble:

$$\Rightarrow \overline{J_{ijkl} J_{mnop}} = \frac{3! J^2}{N^3} \delta_{im} \delta_{jn} \delta_{ko} \delta_{lp}, \qquad \overline{J_{ijkl}} = 0.$$

Large-N Limit:



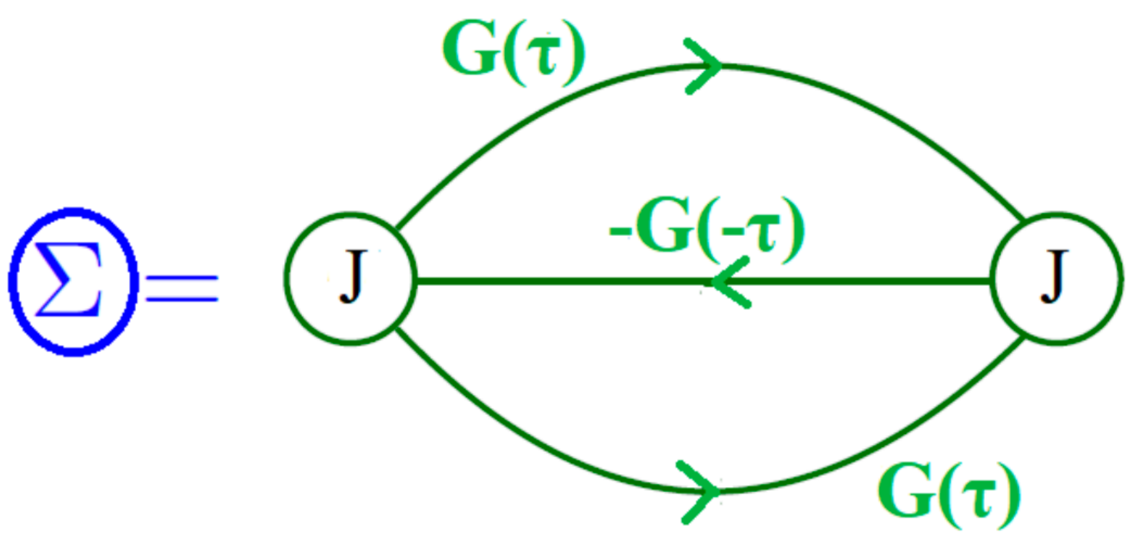
Schwinger-Dyson Equations

$$\Sigma(\tau_1, \tau_2) = J^2 G(\tau_1, \tau_2)^3, \qquad \frac{1}{G(i\omega)} = \frac{1}{G_0(i\omega)} - \Sigma(i\omega)$$

[J. Maldacena, D. Stanford, Phys. Rev. D 94, 106002 (2016)]

Large-N Limit in Complex SYK Model

$q = 4$:



Schwinger-Dyson Equations $\forall\, q$

$$\Sigma(\tau) = (-1)^{\frac{q}{2}} J^2 G(\tau)^{\frac{q}{2}} G(-\tau)^{\frac{q}{2}-1}, \quad \frac{1}{G(i\omega)} = \frac{1}{G_0(i\omega)} - \Sigma(i\omega) - \mu$$

Can be derived from the effective classical action:

$$S_{\text{eff.}}[G,\Sigma] = N\Big[-\log\det\left[(\partial_{\tau'} - \mu)\,\delta(\tau - \tau') - \Sigma(\tau,\tau')\right] + \int d\tau d\tau' \left(G\Sigma - \frac{J^2}{(q/2)}\left(-G(\tau,\tau')G(\tau',\tau)\right)^{q/2}\right)\Big]$$

Large-q Limit

$q \to \infty$:

Strong coupling limit; Emergent ($\approx$) conformal symmetry

Analytically solvable Schwinger-Dyson equations

Scale invariant $G_c(\tau, \tau') \propto \frac{1}{(\tau-\tau')^{2\Delta}}$ is a solution if $\Delta = \frac{1}{q}$

Generate another solution for arbitrary $f(\tau)$:

$$G_c \longrightarrow G_{c,f}(\tau, \tau') = \left[f'(\tau) f'(\tau')\right]^{\Delta} G_c\left(f(\tau), f(\tau')\right)$$

Conformal symmetry allows finite temperature extrapolation:

$$G_f = \left[\frac{\pi}{\beta \sin \frac{\pi\tau}{\beta}}\right]^{2\Delta} \qquad \left(\Delta = \frac{1}{q}, f(\tau) = \tan\frac{\pi\tau}{\beta}\right)$$

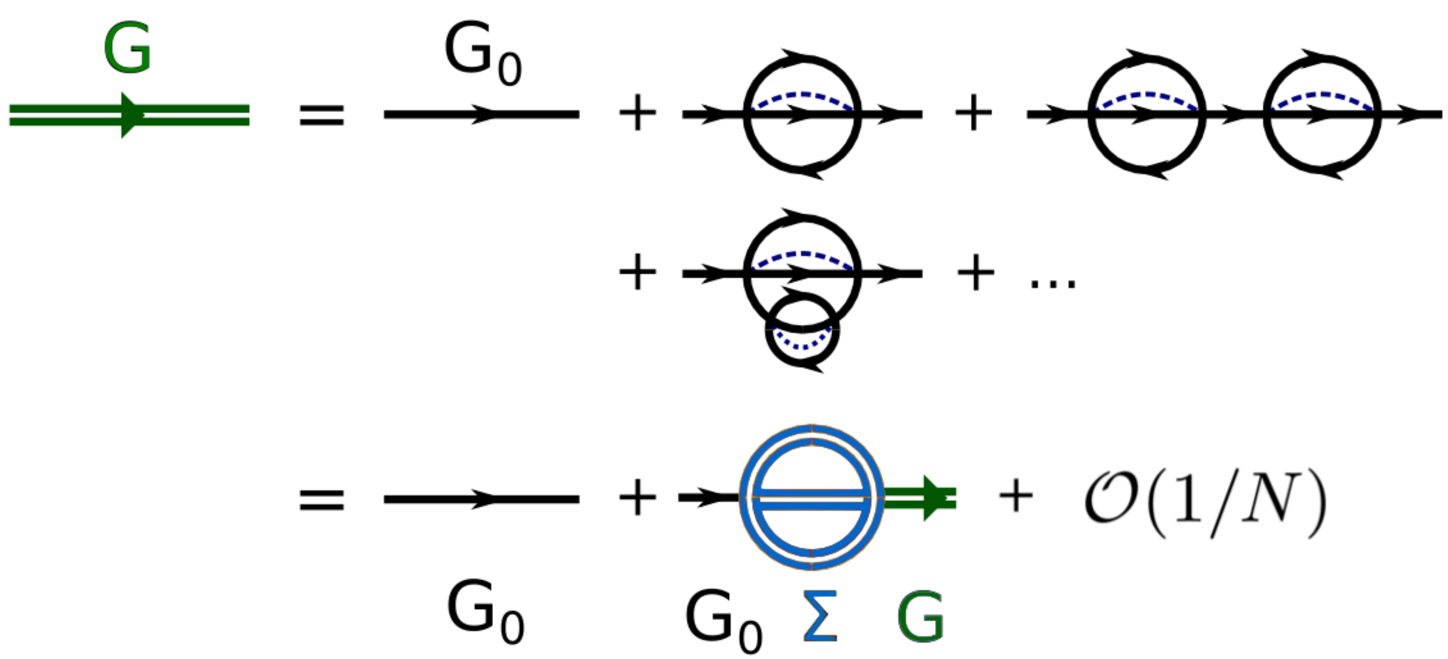
Large-q Expansion:

$G(\tau) = G_0(\tau)\left[1 + \frac{g(\tau)}{q}\right]$ where $\partial_\tau^2 g(\tau) = 2J^2 \frac{q}{2^{q-1}} e^{g(\tau)}$

Solvable Green's Functions

$$\mathcal{H}_4 = J \sum X^{i_1 i_{4/2}}_{j_1 j_{4/2}} c^\dagger_{i_1} c^\dagger_{i_{4/2}} c_{j_{4/2}} c_{j_1}$$

Dyson's equation: $\dot{G}(\tau) = -\int \mathrm{d}t\, \Sigma(t) G(\tau - t)$



Self energy

$$\Sigma(t) = 2J^2 G(t) G(-t) G(t)$$
$$= -2\left(J[-2G(t)2G(-t)]^{4/2-1}\right) G(t)/4$$

- Charge density

  $\mathcal{Q} = G(0^\pm) \pm 1/2$

- Energy density $\frac{2}{4}\dot{G}(0^+)$

- Effective interaction

$$\mathcal{J} = J[\underbrace{-2G(0^+)}_{1-2\mathcal{Q}}\, \underbrace{2G(0^-)}_{1+2\mathcal{Q}}]^{\frac{4-2}{4}}$$

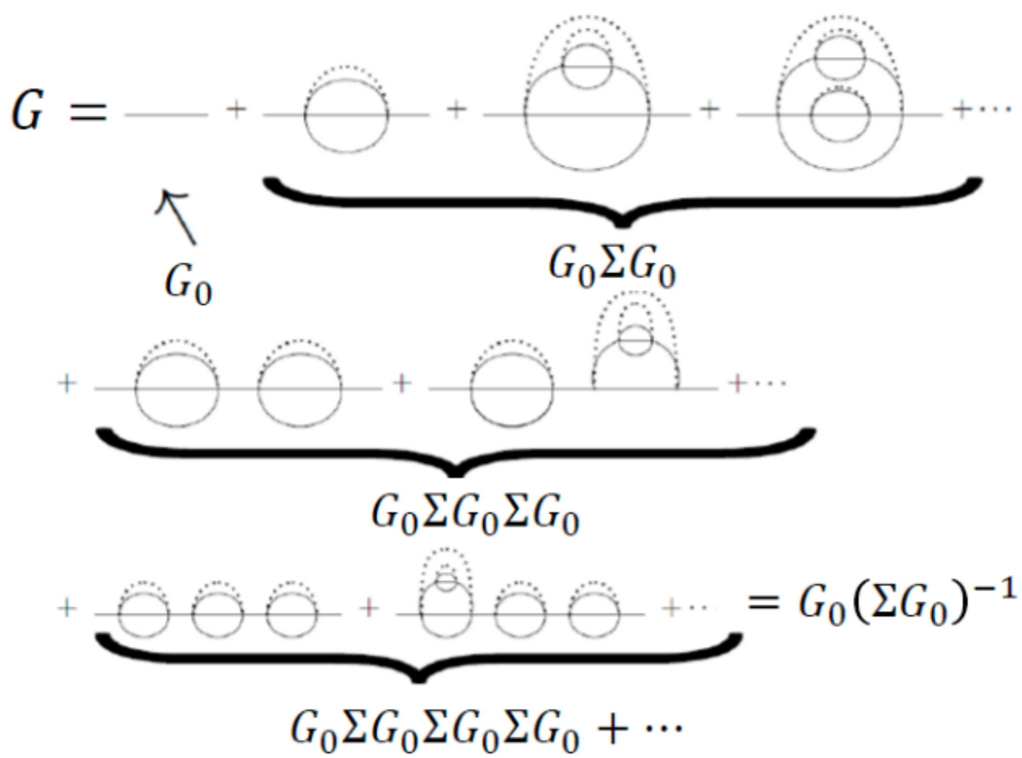
Diagrams

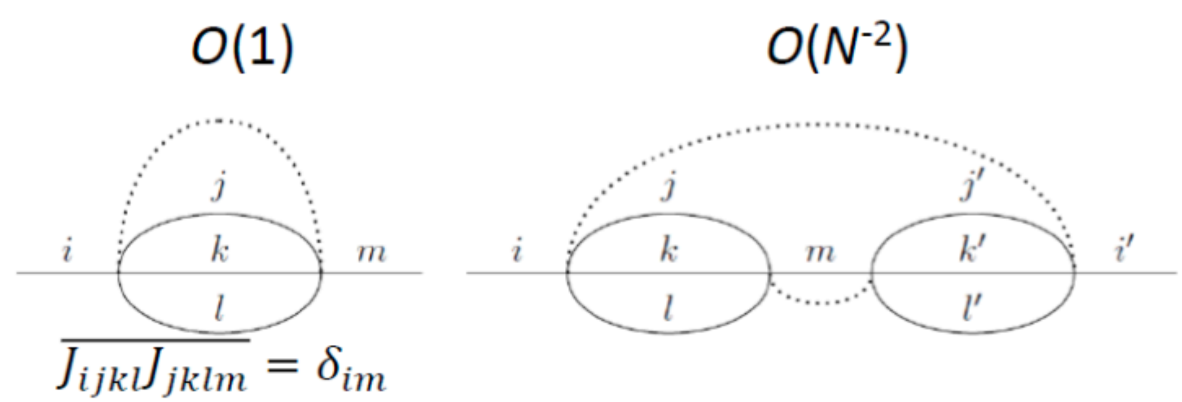
The SYK model

Figures from [I. Danshita, MT, and M. Hanada: Butsuri **73**(8), 569 (2018)]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

Analytically solvable in $N \gg 1$ limit





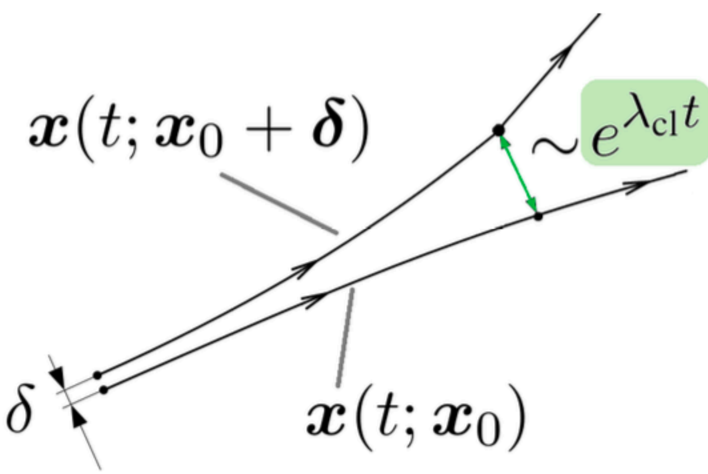
Only "melon-type" diagrams survive

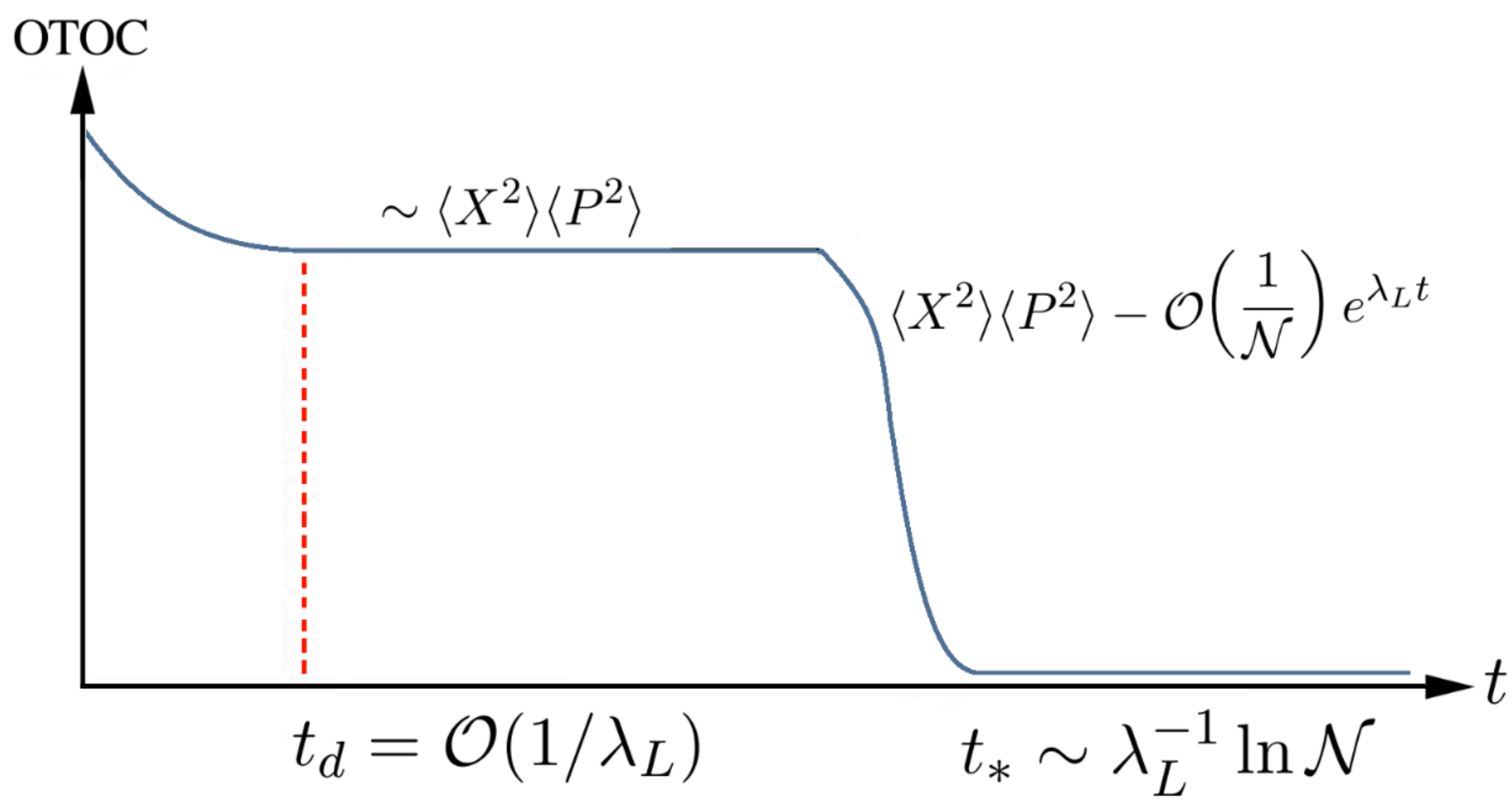
$$G(i\omega)^{-1} = i\omega - \Sigma(i\omega) \qquad \Sigma = J^2 G^3$$

Chaos

$$\left|\frac{\partial x^i(t)}{\partial x^j(0)}\right| = |\{x^i(t), p^j(0)\}_{\text{PB}}| \sim e^{\lambda_{\text{cl.}} t}$$

$$C(t) = \left\langle |X(t), P(0)|^2 \right\rangle = \text{TOC}(t) - 2\Re \underbrace{\left\langle (X(t)P)^2 \right\rangle}_{\text{OTOC}}$$



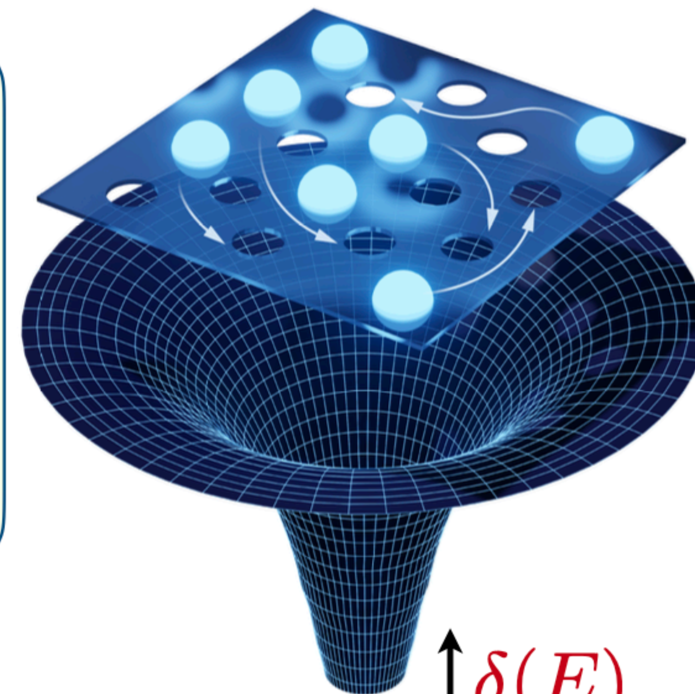


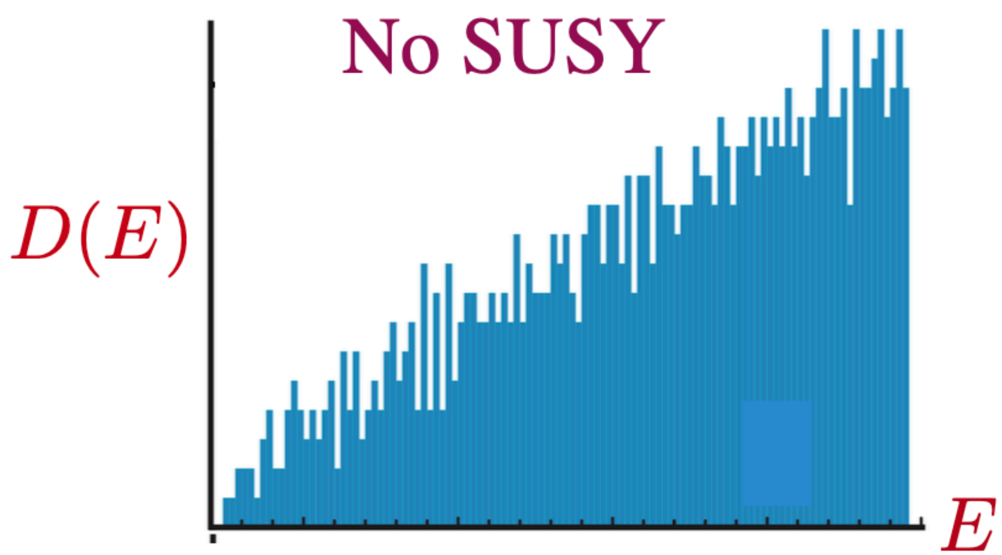
SYK & Black Holes Density of States

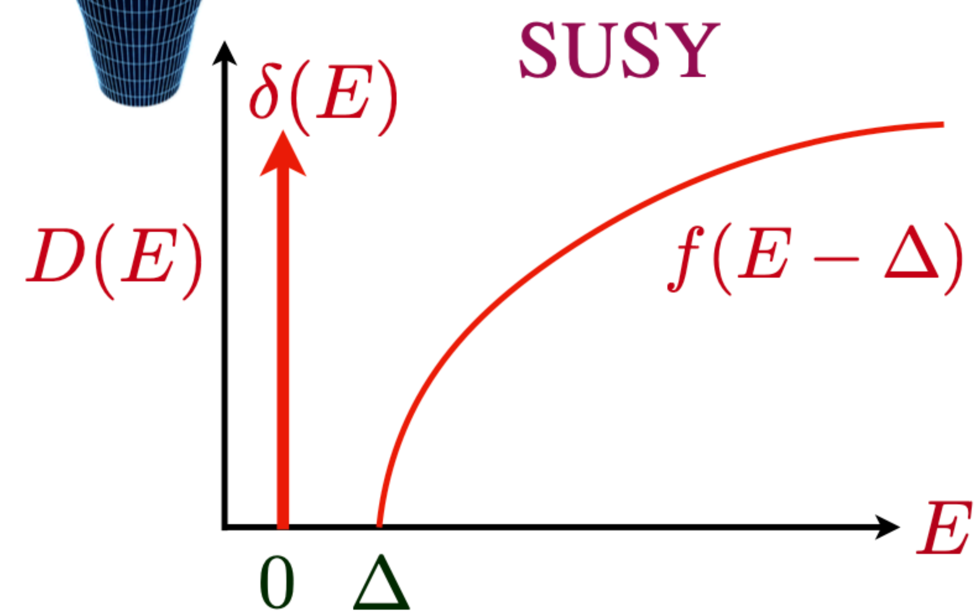
S. Sachdev, arXiv:2304.13744

$$D_{SYK}(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left(\sqrt{2N\gamma E}\right)$$

$$D_{BH}(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \times \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$





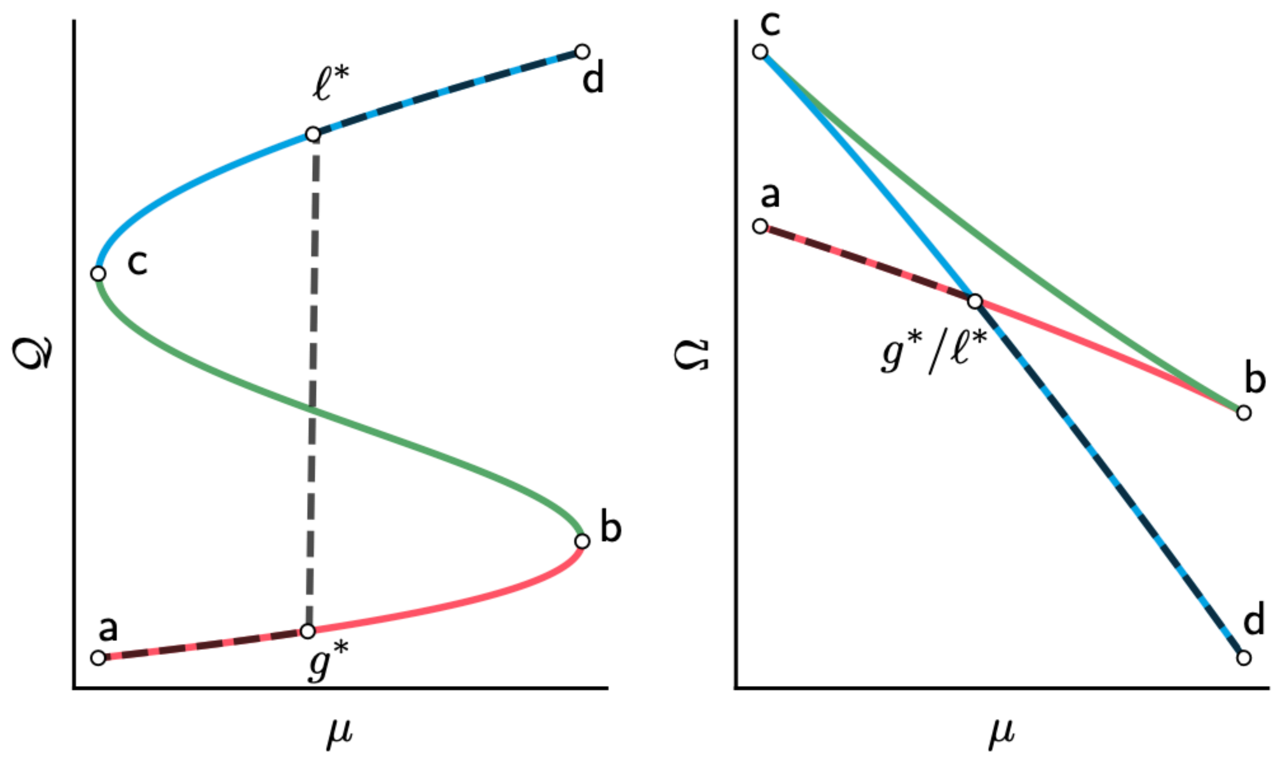


<u>Comparison of many-body densities of SYK models and charged black holes with & without SUSY.</u>

- Black holes and SYK models without SUSY do not have delta function, nor a gap, but an exponentially dense spacing of levels down to E = 0.
- Both black holes and SYK models with sufficient low energy SUSY have an energy gap Δ, above a delta function.

Thermodynamically Preferred Phase

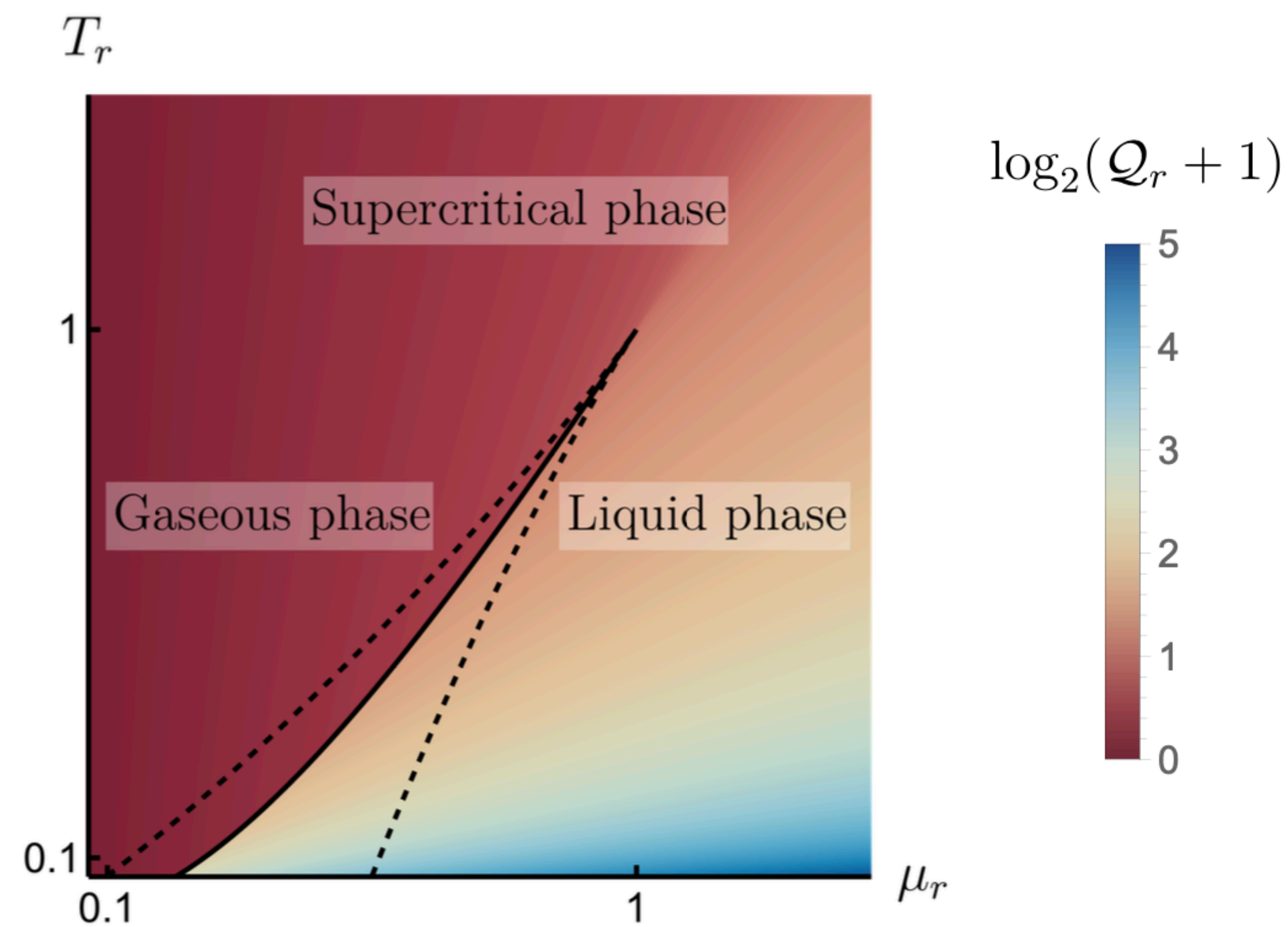
Thermally preferred : $\min_{\text{sols.}}\{\tilde{\Omega}\}$



For some $\tilde{T} < \tilde{T}_c$

Phase diagram

J. C. **Louw** and S. **Kehrein**, PRB 107, 075132 (2023) :



in terms of reduced variables

$$T_r = \tilde{T}/\tilde{T}_{\text{c}},\ \mu_r = \tilde{\mu}/\tilde{\mu}_{\text{c}},\ \mathcal{Q}_r = \tilde{\mathcal{Q}}/\tilde{\mathcal{Q}}_{\text{c}}$$

Equation of State

$$\mu(\mathcal{Q}) = \underbrace{2T\tanh^{-1}(2\mathcal{Q})}_{\text{non-int}} + \underbrace{q^{-1}4\mathcal{Q}\mathcal{J}(\mathcal{Q})\sin(\pi v/2)}_{\text{int}}$$

$$\underbrace{\mu(\mathcal{Q})}_{q^{-3/2}\tilde{\mu}(\tilde{\mathcal{Q}})} = q^{-3/2}4\tilde{\mathcal{Q}}[\underbrace{qT}_{\tilde{T}}[1+\mathcal{O}(1/q)] + \mathcal{J}(\mathcal{Q})\underbrace{\sin(\pi v/2)}_{1+\mathcal{O}(1/q)}]$$

Lyapunov exponent $\lambda = v\lambda_{\text{max}}(T)$

- Maximally chaotic:

$$v = 1 - \mathcal{O}(T/\mathcal{J})$$

- Nonchaotic:

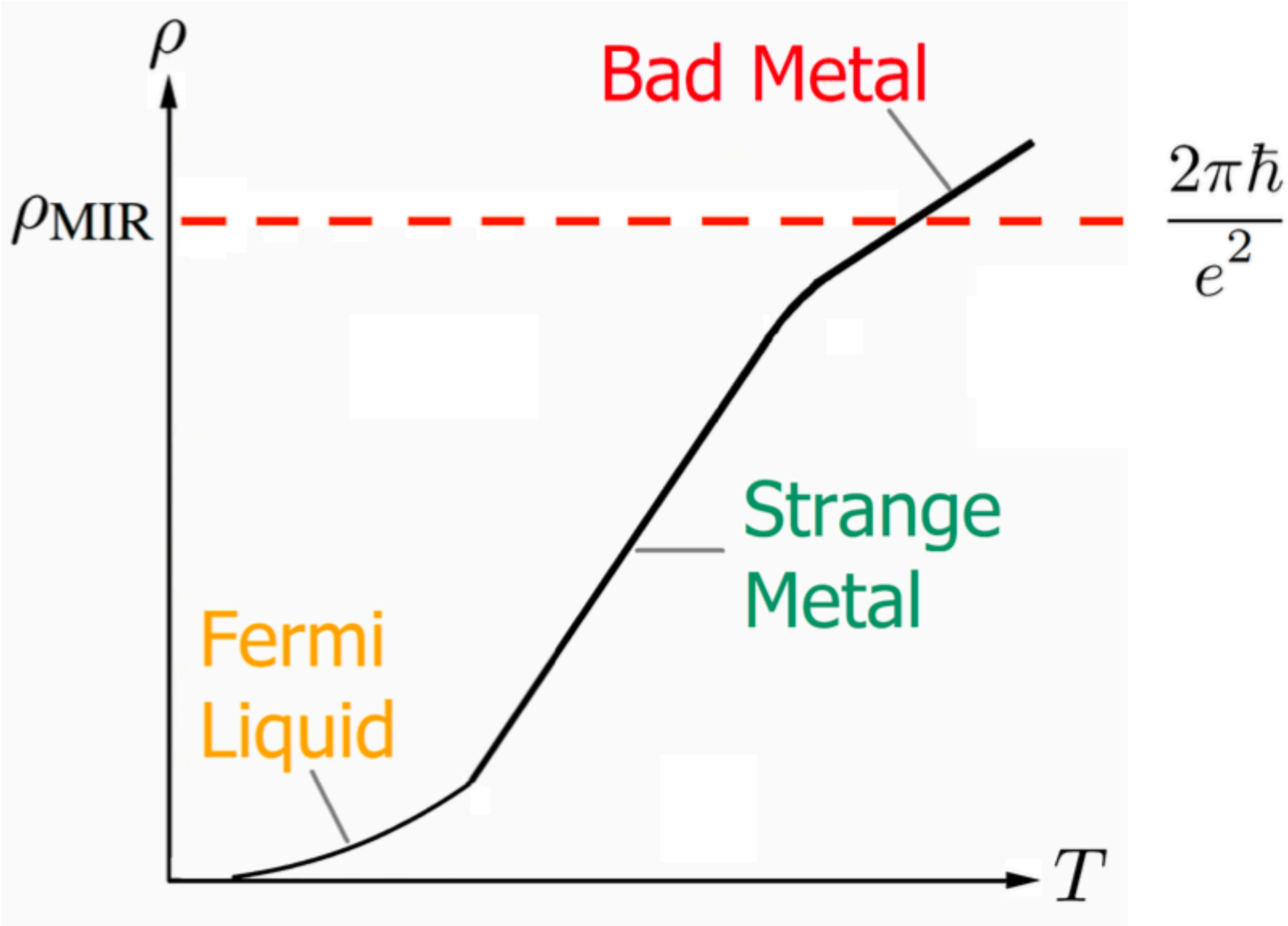
$$v = 0 + \mathcal{O}(\mathcal{J}/T)$$

Effective coupling

$$\mathcal{J}(\mathcal{Q}) = e^{(q-2)\frac{\ln[1-4\mathcal{Q}^2]}{4}}J$$
$$= Je^{-(q-2)[\mathcal{Q}^2+\mathcal{O}(\mathcal{Q}^4)]}$$
$$= Je^{-\tilde{\mathcal{Q}}^2+\mathcal{O}(1/q)},$$

with $\mathcal{Q} = \tilde{\mathcal{Q}}q^{-1/2}$

Mott-Ioffe-Regel Limit



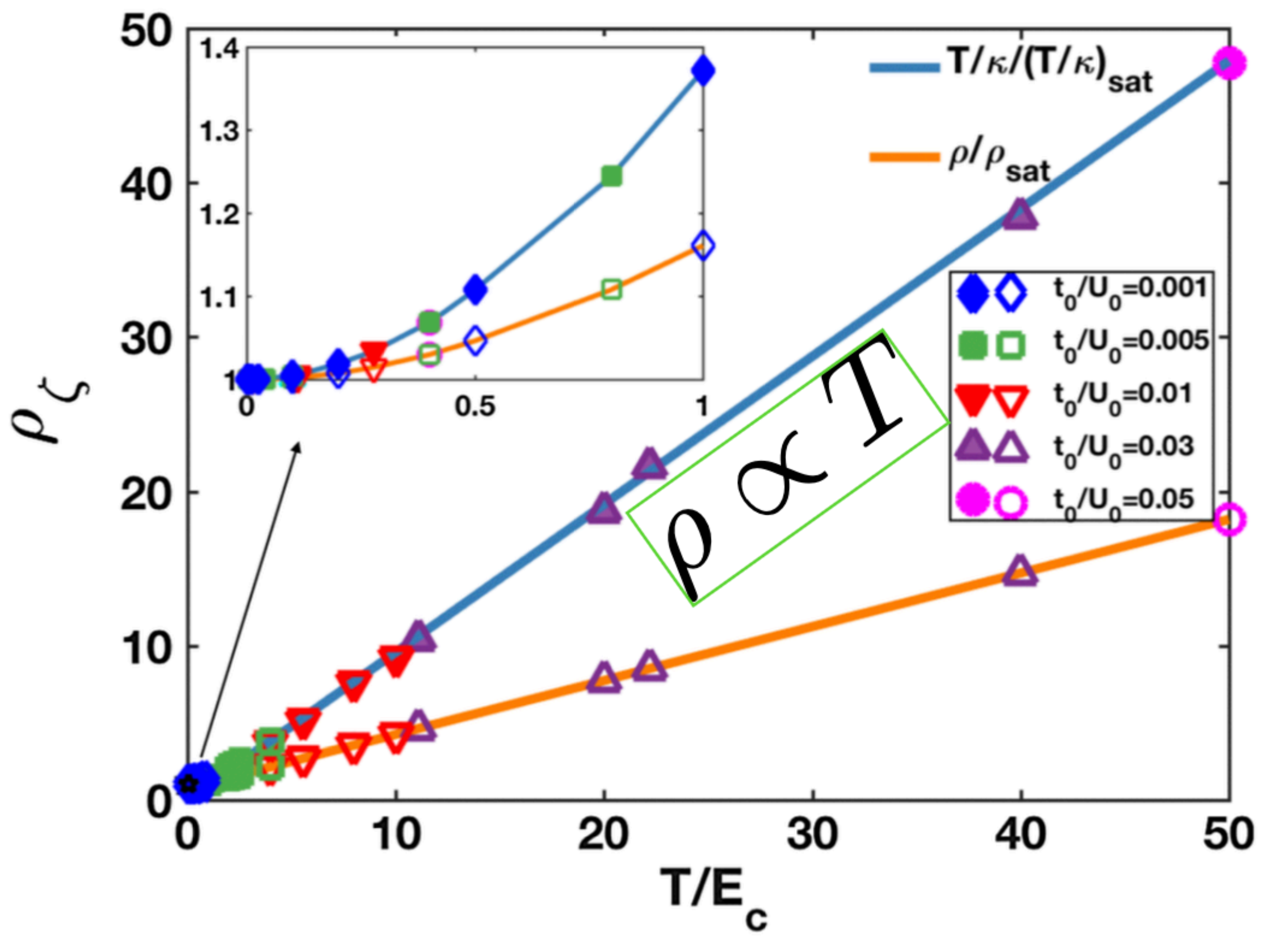
Bad Sheet

SYK sheet is actually a bad metal

$$\rho \sim \rho_{\mathrm{MIR}} \frac{k_{\mathrm{B}} T}{t^2 / J_4}$$

when

$$1 \ll \frac{k_{\mathrm{B}} T}{t^2 / J_4} \ll \frac{J_4^2}{t^2}$$



Xue-Yang Song, et. al., PRL 119, 216601 (2017)

See:

RJ, S. Kehrein, J. C. Louw, arXiv:2407.20733 [cond-mat.str-el]

for true strange metal behavior!