

Spectator Effects in heavy hadron decays from HQET

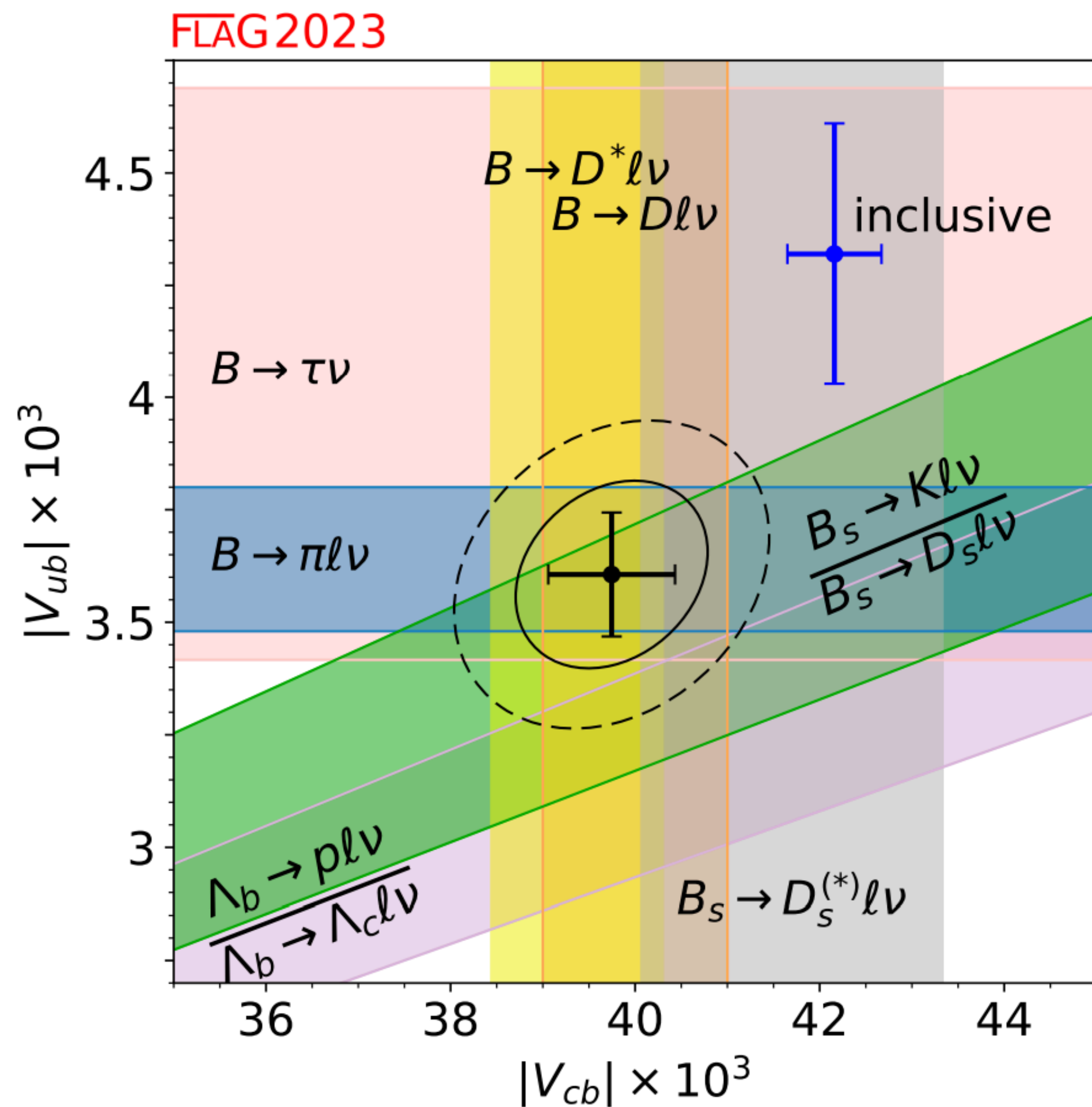
Joshua Lin

with

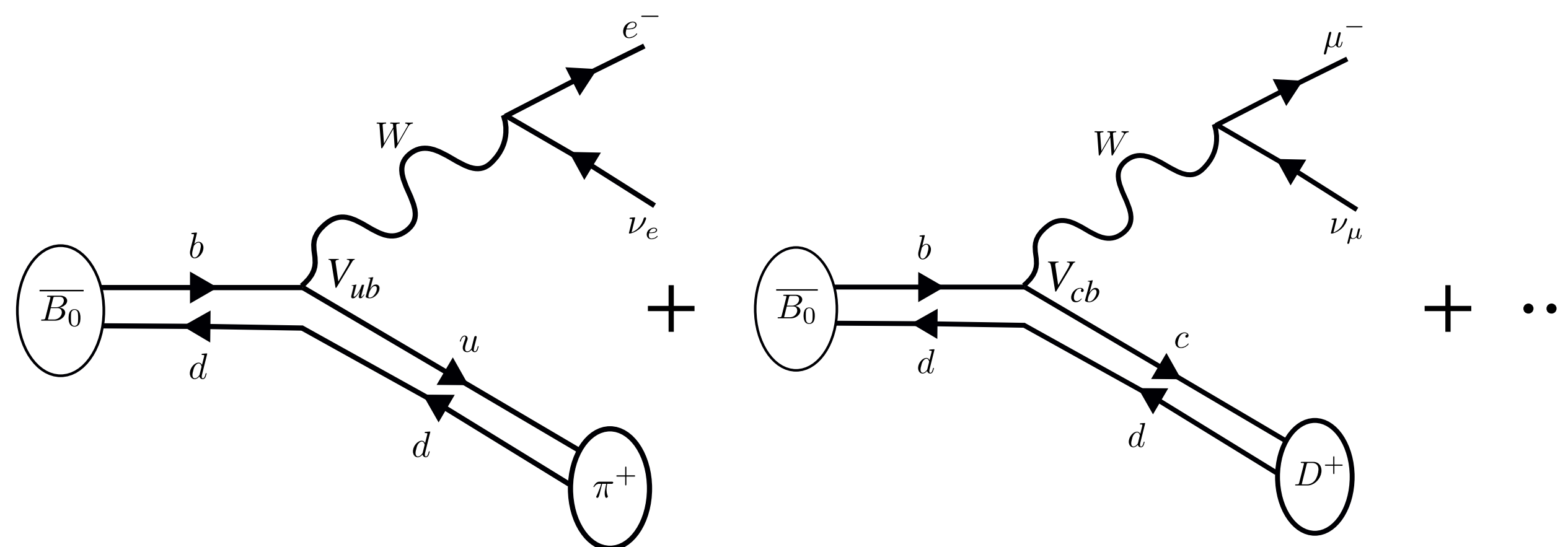
Will Detmold

Stefan Meinel

Inclusive/exclusive tensions



► Exclusive measurements (shaded bands) from a single decay channel



► Inclusive measurement from summing over all (semileptonic) channels. This is quite difficult theoretically!

► Persistent 3σ -tension.

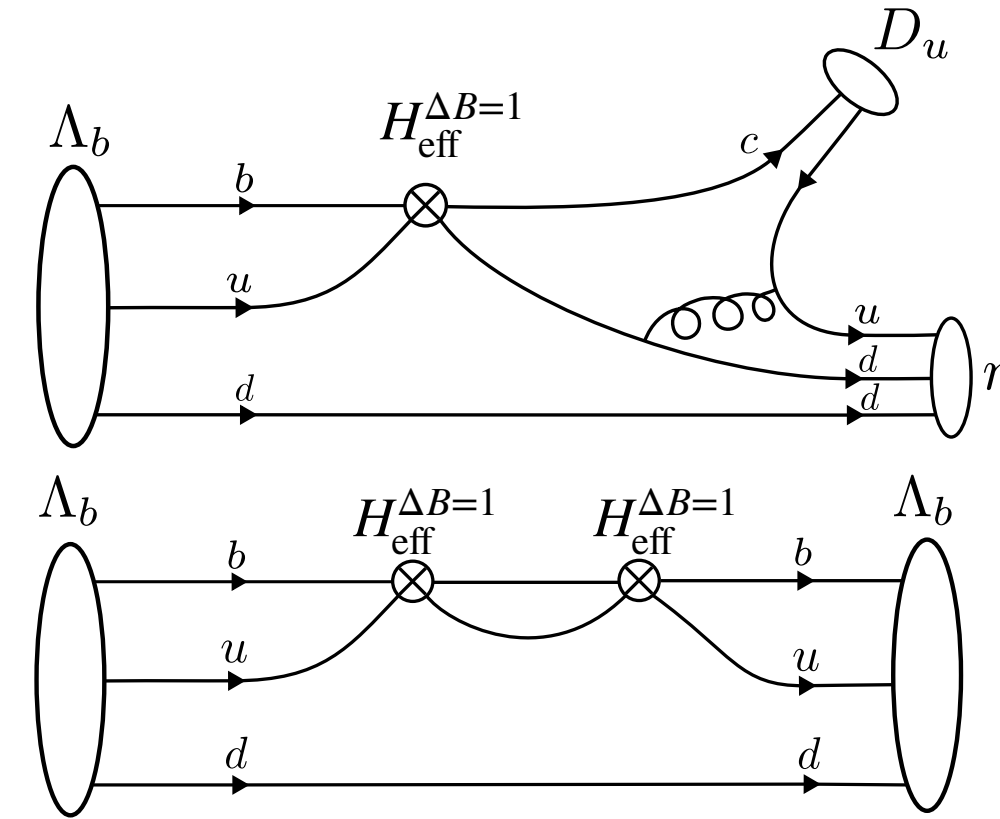
Theory predictions for Inclusive Lifetimes

3

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^* V_{us} (C_1 Q_1 + C_2 Q_2) + \dots V_{cb}^* Q_l^c + \dots \right]$$

↓ Optical Theorem

$$\Gamma(B \rightarrow X) = \frac{1}{2m_B} \langle B | \hat{T} | B \rangle = \frac{1}{2m_B} \langle B | \text{Im } i \int d^4x T (H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0)) | B \rangle$$



August 2024

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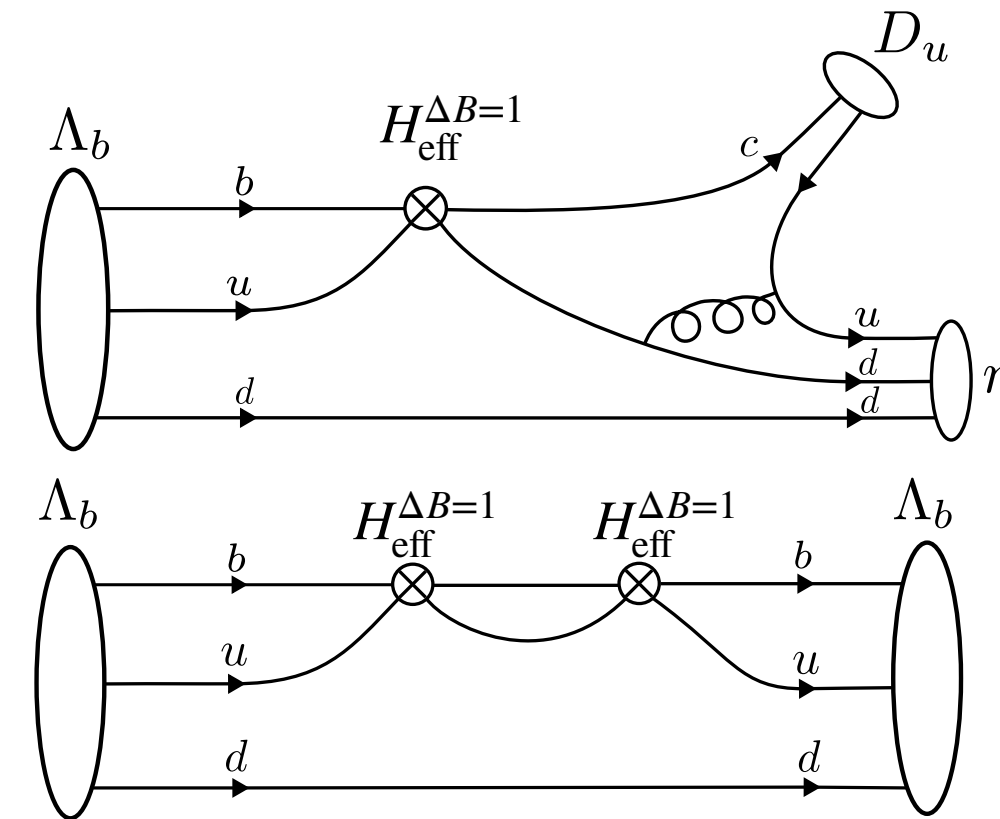
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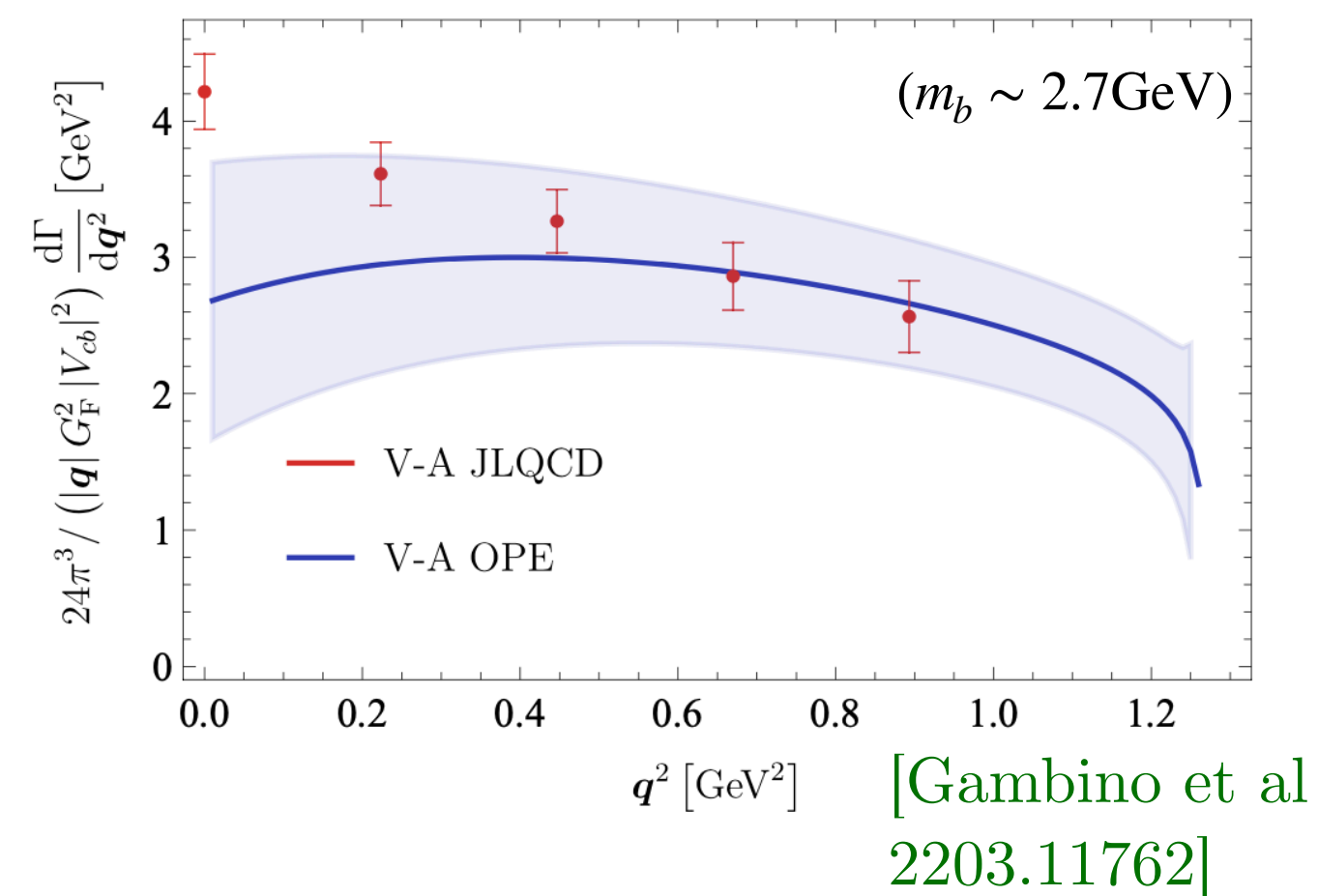
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Spectral Reconstruction methods



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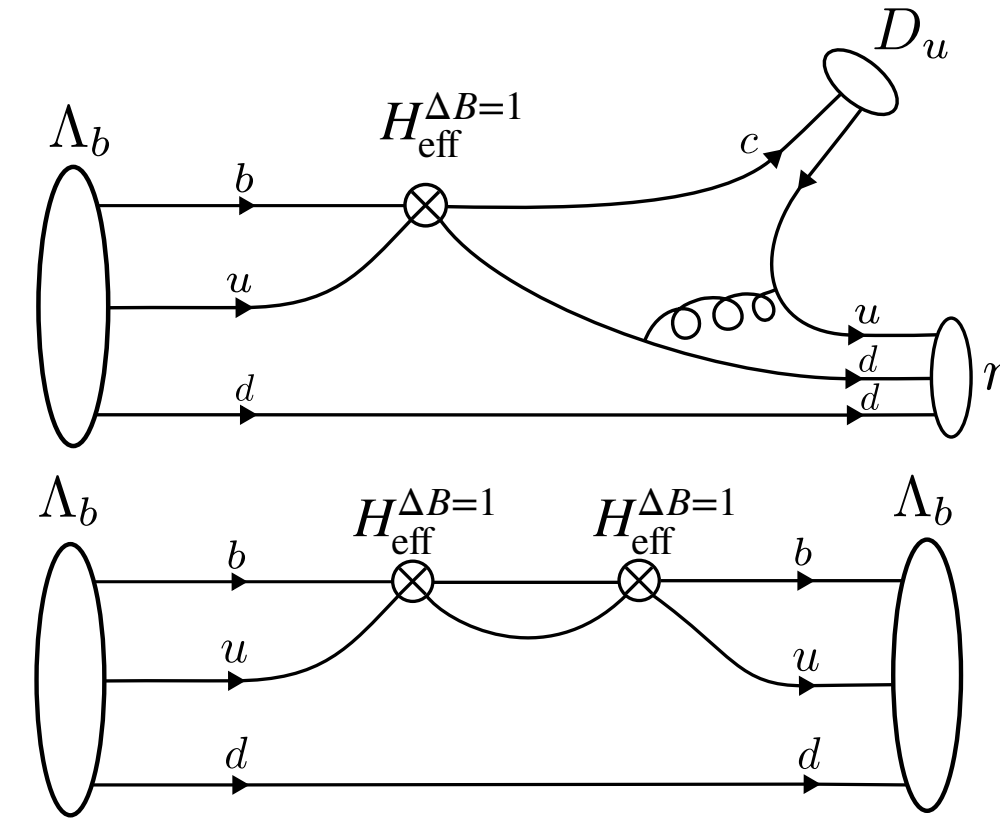
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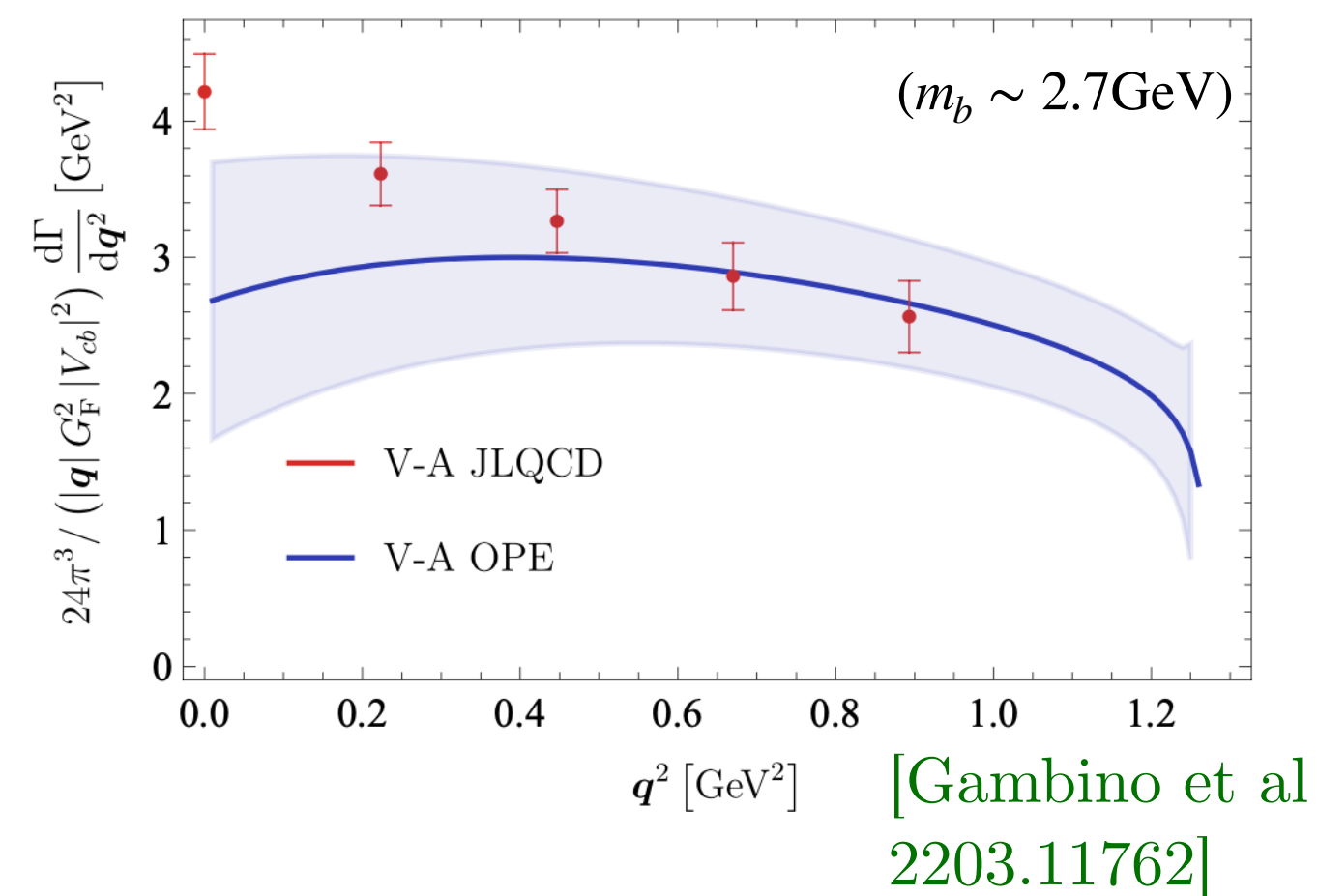
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Spectral Reconstruction methods



Operator Product Expansion

$$\hat{T} = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left[c^{(3)} \bar{b}b + \frac{1}{m_b^2} c^{(5)} g_s \bar{b} \sigma^{\mu\nu} G_{\mu\nu} b + \frac{1}{m_b^3} \sum_k c_k^{(6)} O_k^{(6)} + O(1/m_b^4) \right]$$

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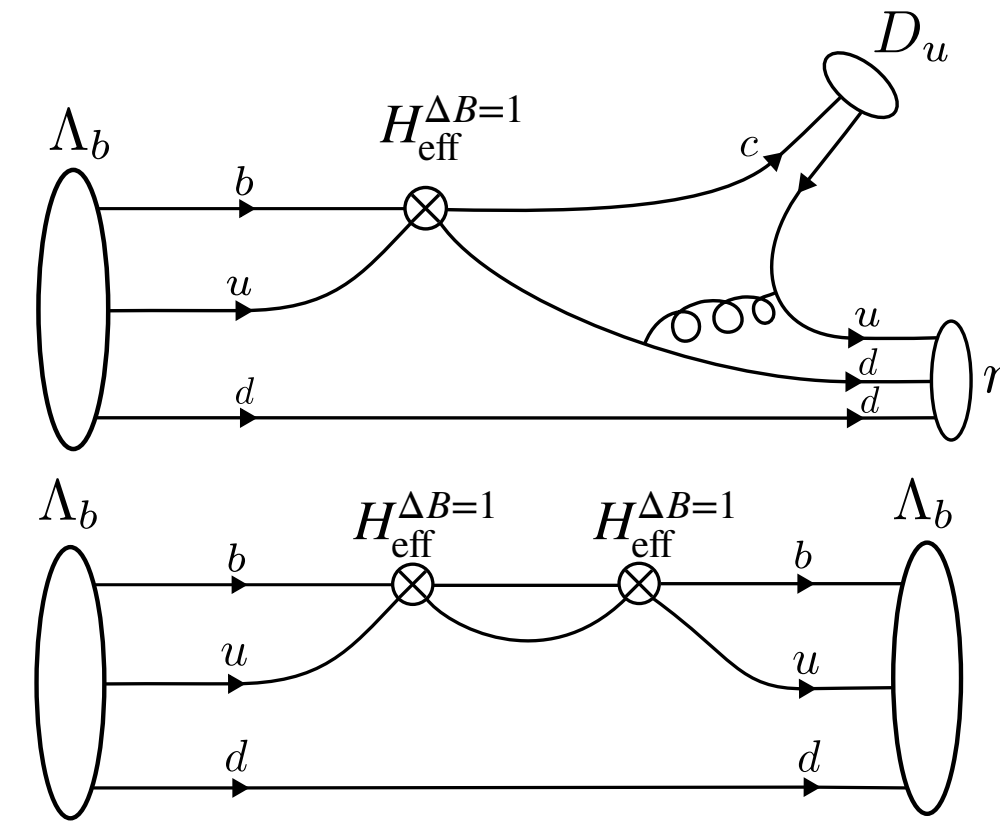
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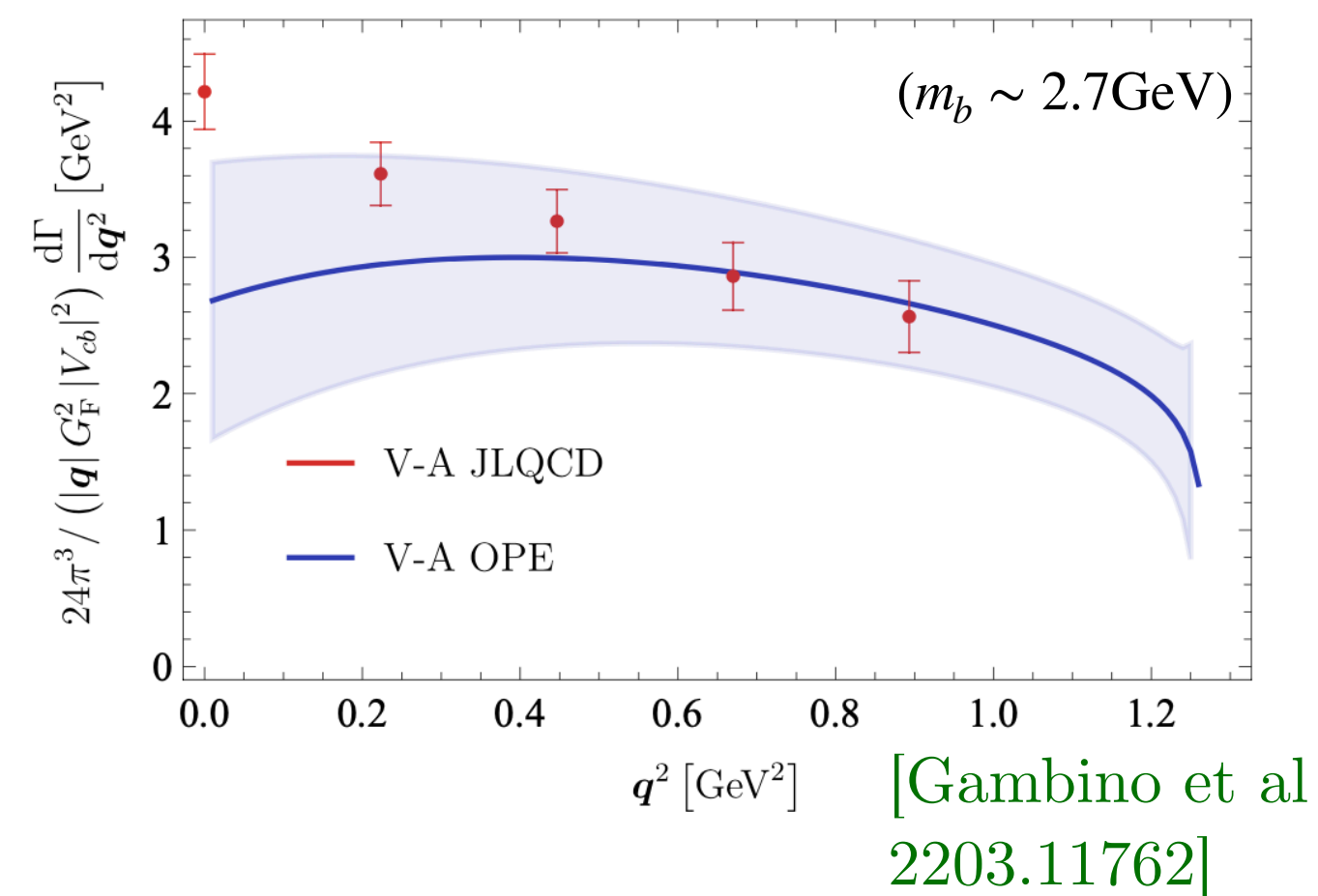
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Gradient Flow Schemes

[Black et al, 2310.18059]

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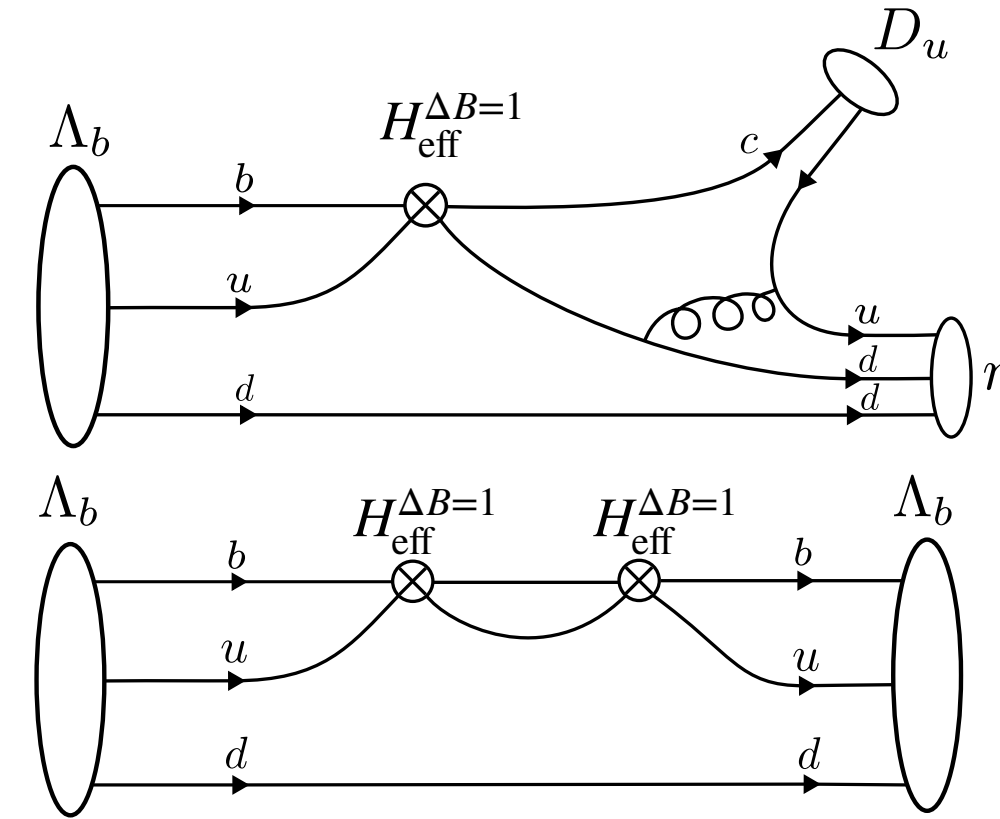
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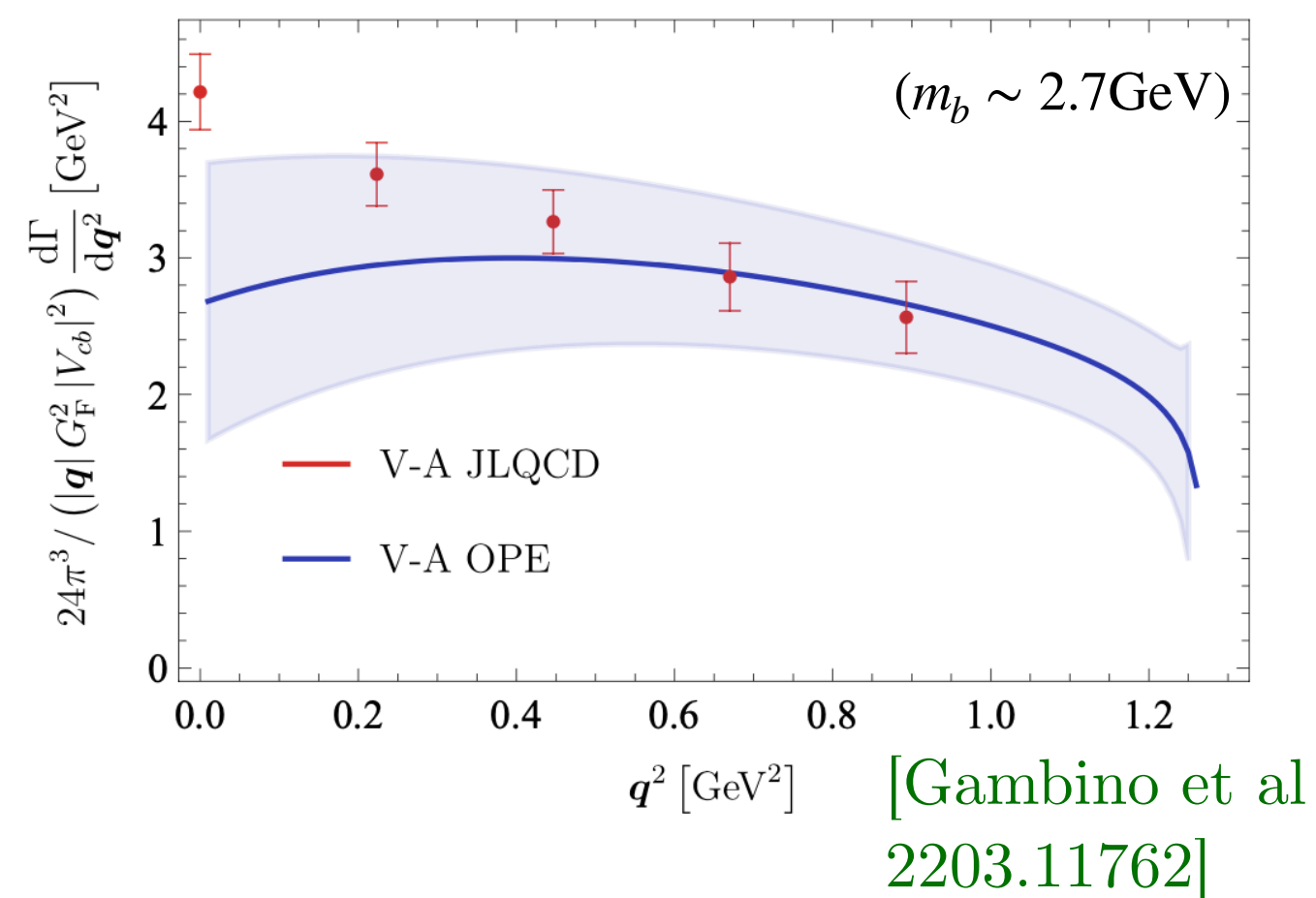
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Fits to physical parameters
as functions of m_b

[Gambino et al, 1704.06105]

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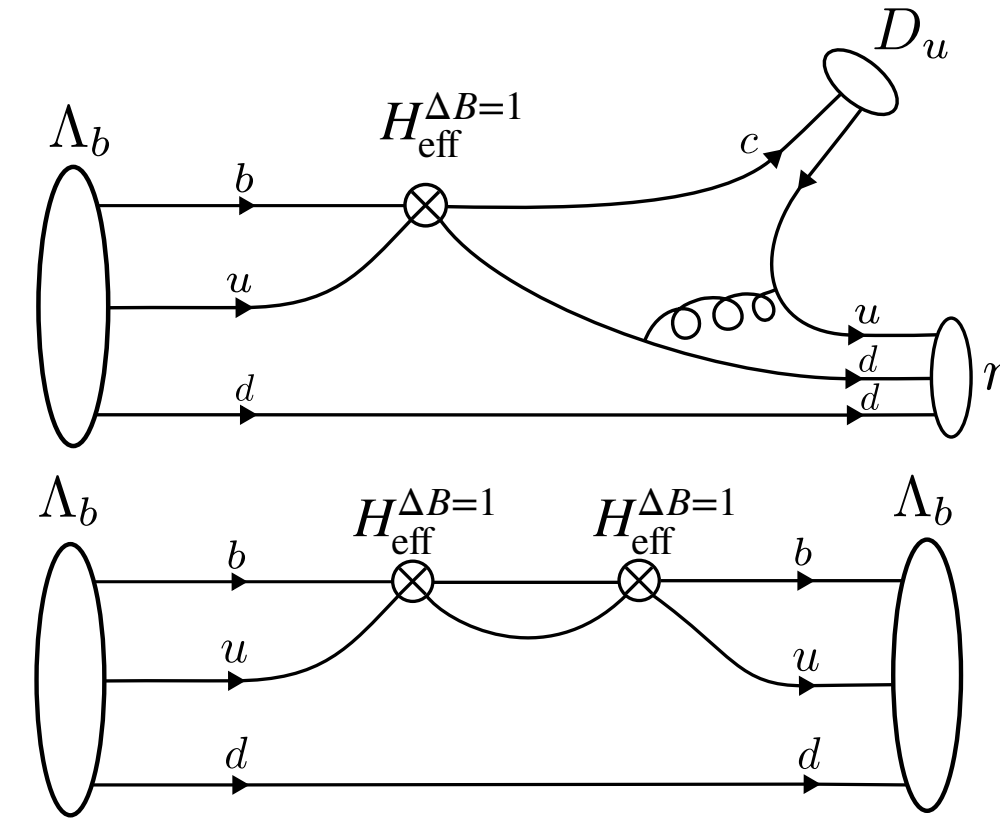
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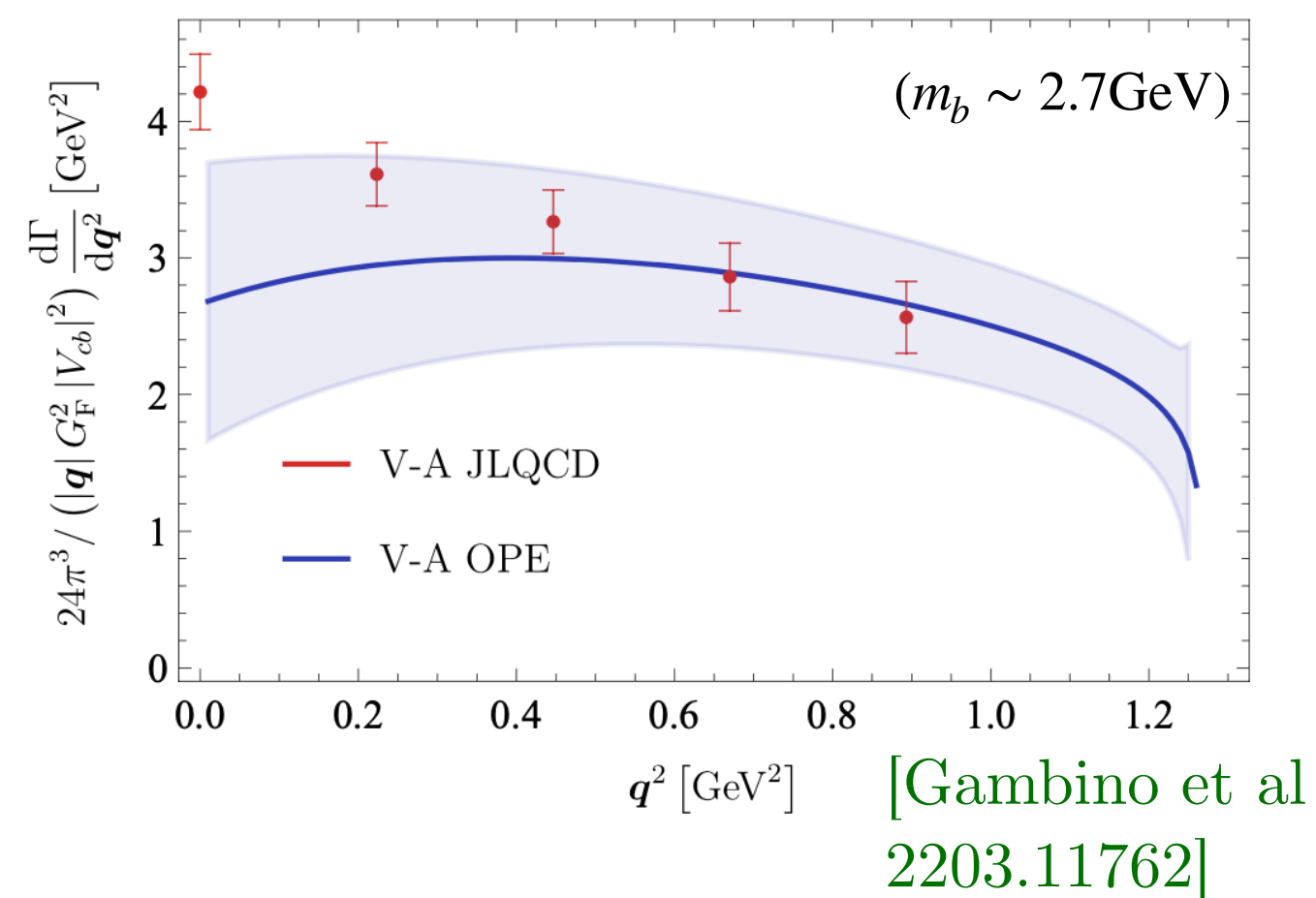
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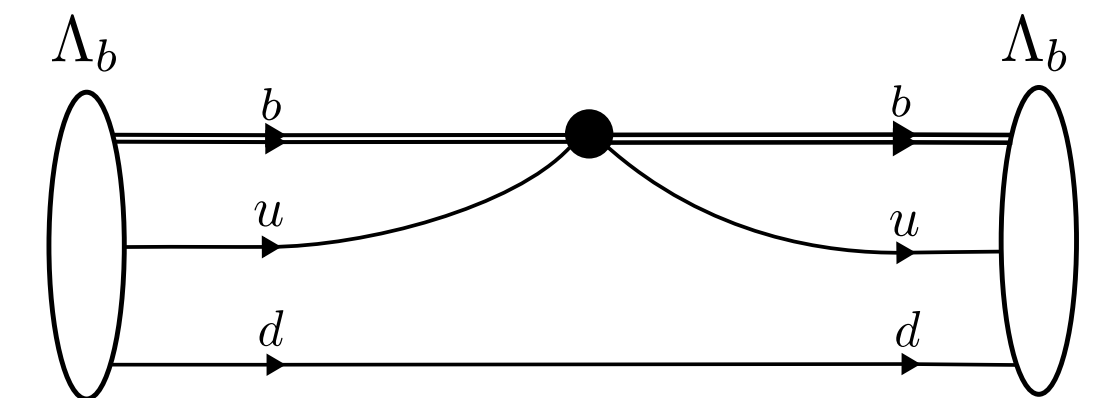
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Heavy Quark Effective Theory



[Pierro, Sachrajda 9805028]

Now!

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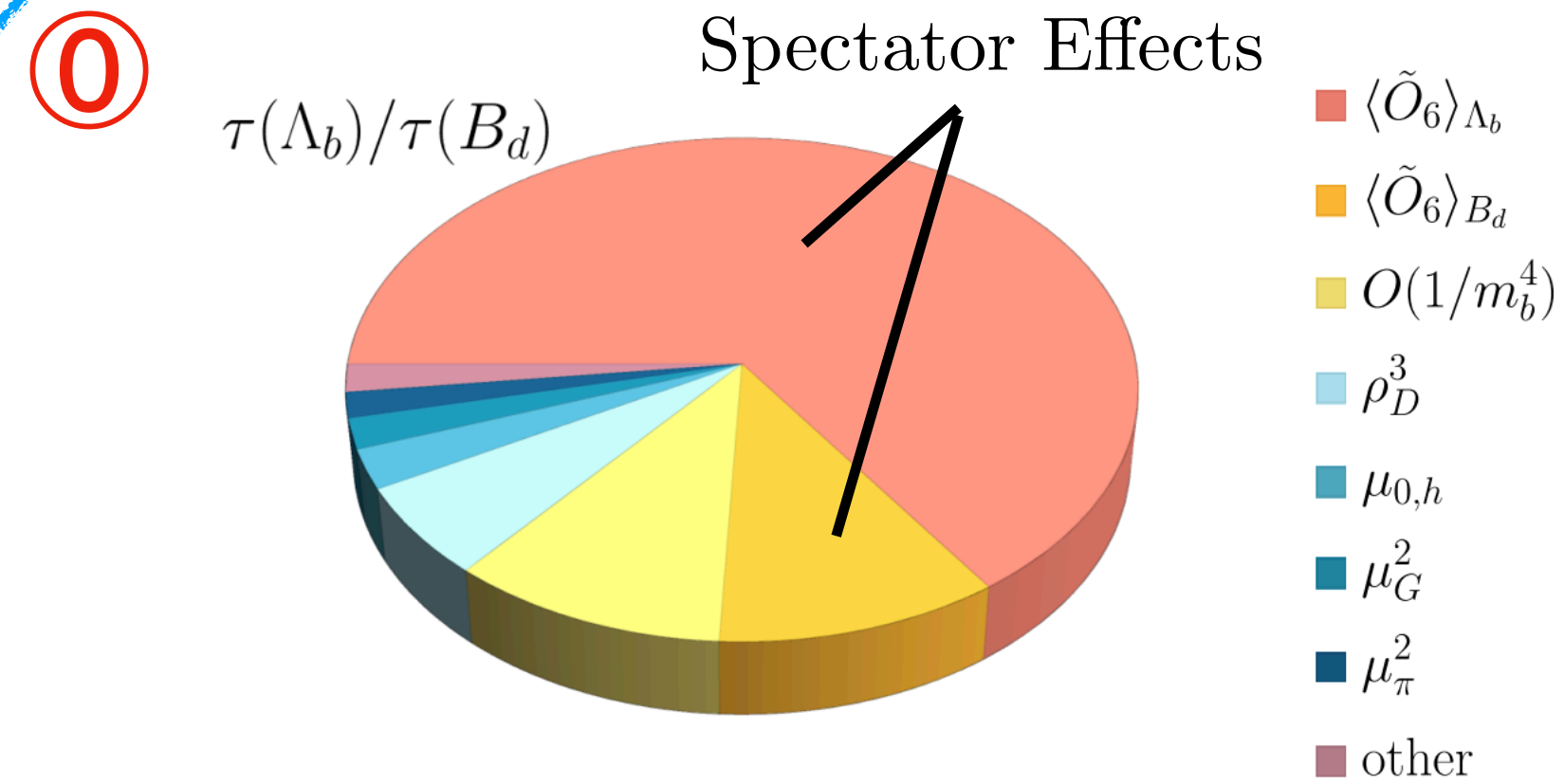


Fig. 9 Composition of the theoretical error in the HQE prediction of the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$.

“Here, clearly, future lattice QCD evaluations of these [dim-6] matrix elements would be highly desirable.”

[J. Albrecht et al, 2402.04224]

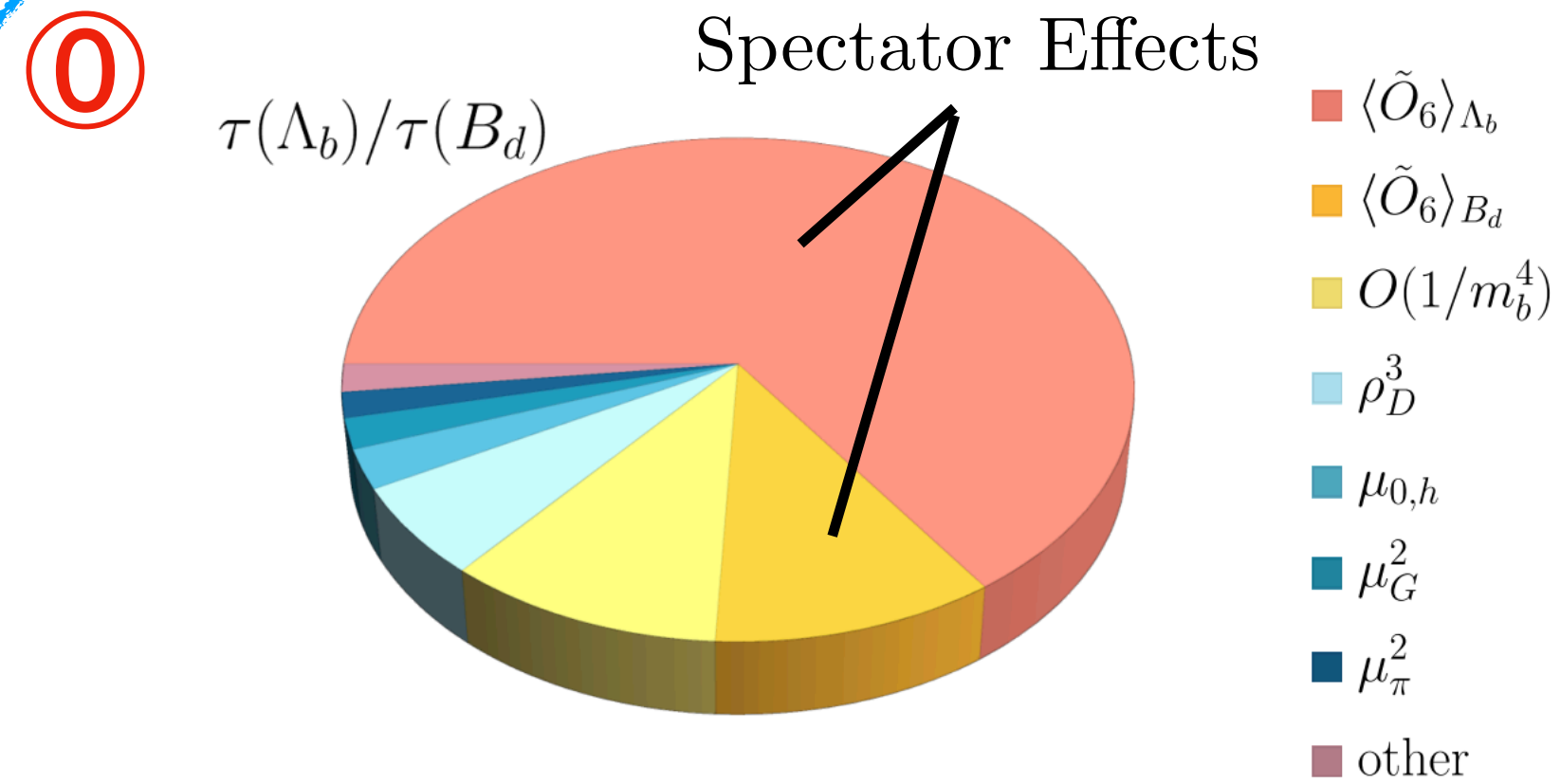
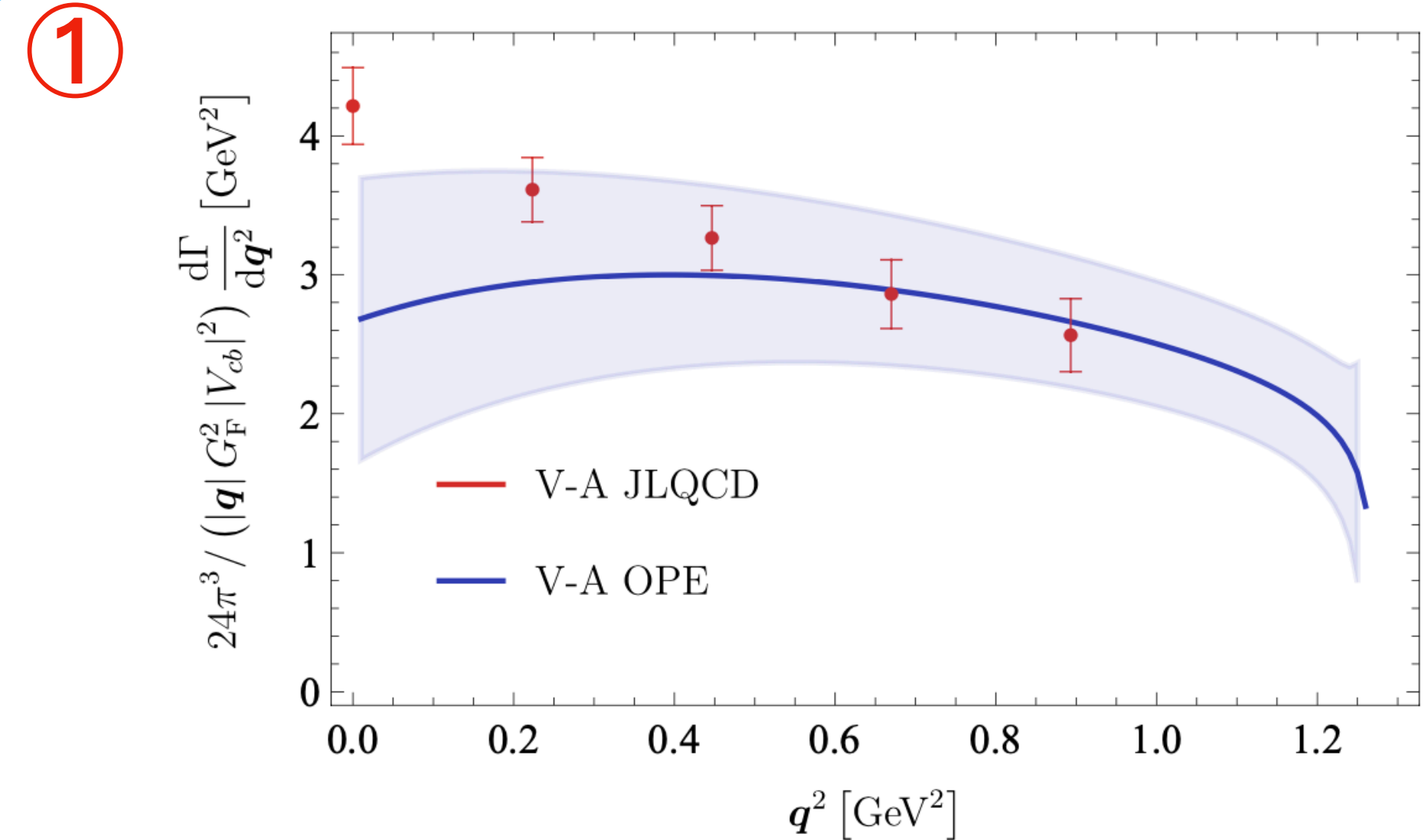


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For newer methods (new systematics) it’s good to compare against time-tested methods

[Gambino et al 2203.11762]

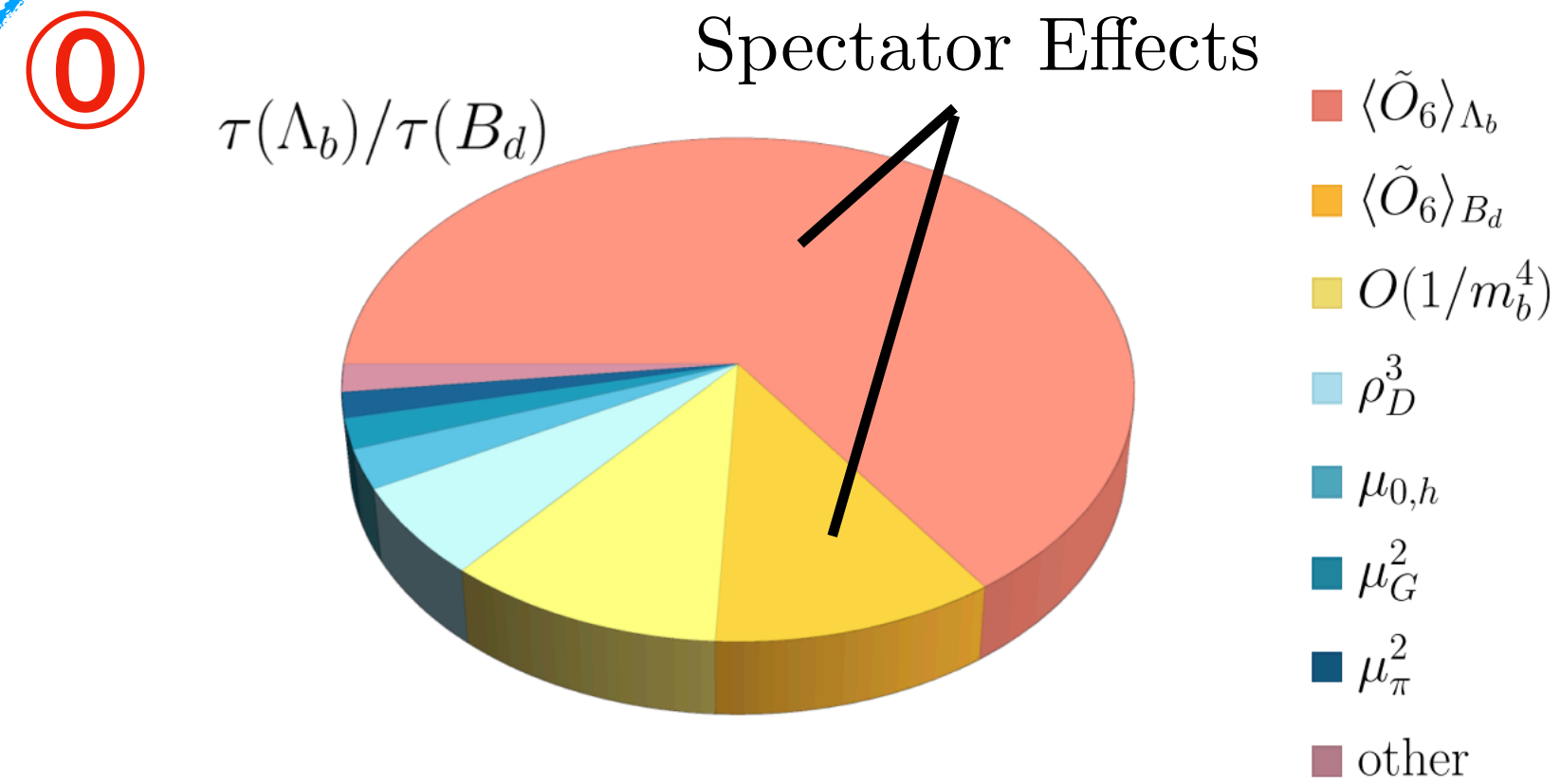
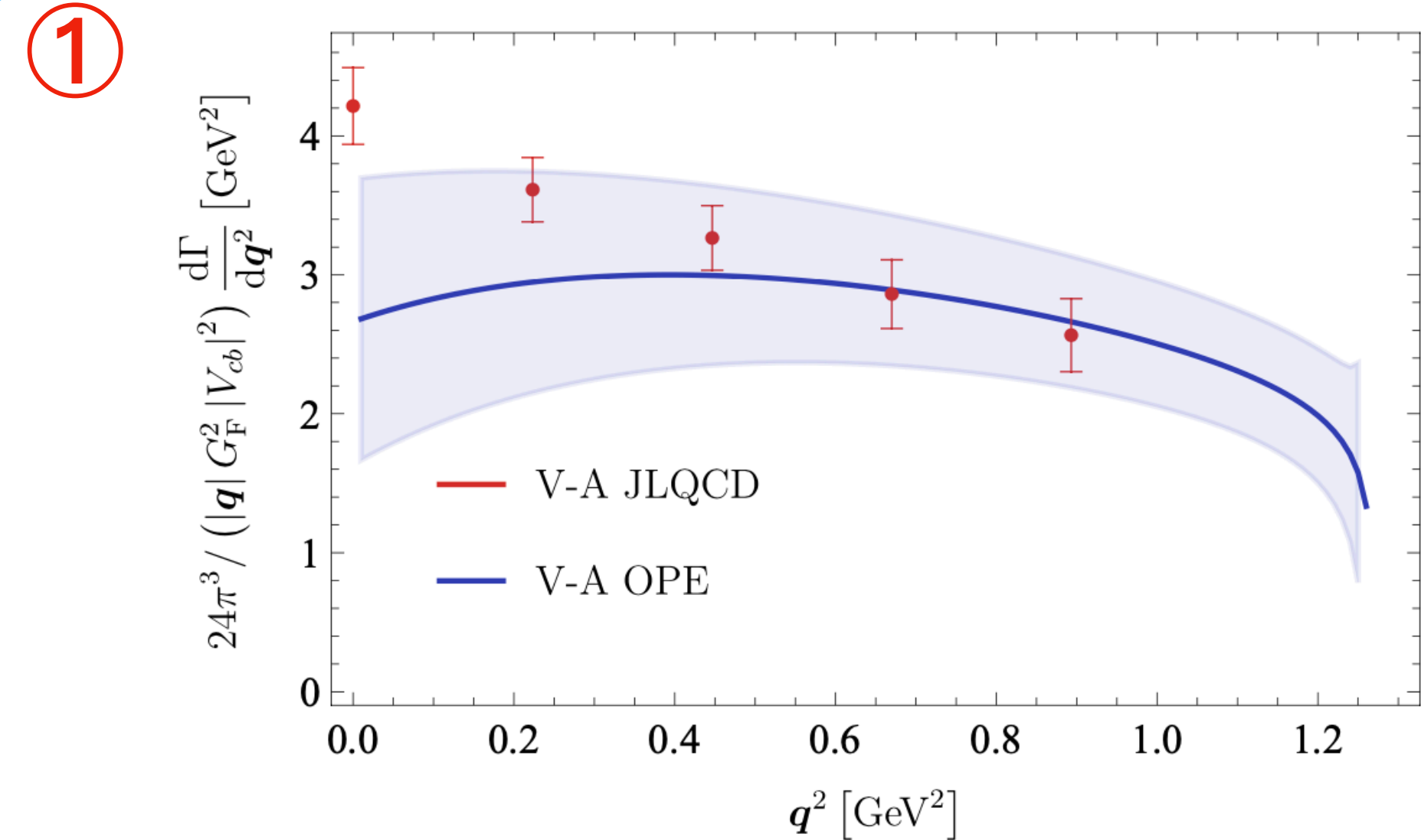


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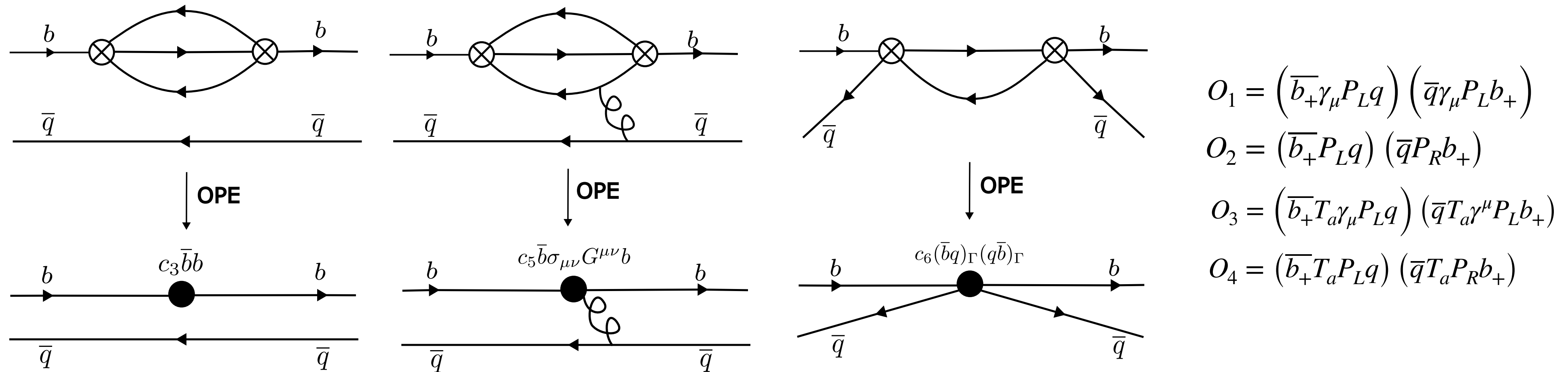
② Experimental measurements of the inclusive decay rates are performed by fits to OPE matrix elements

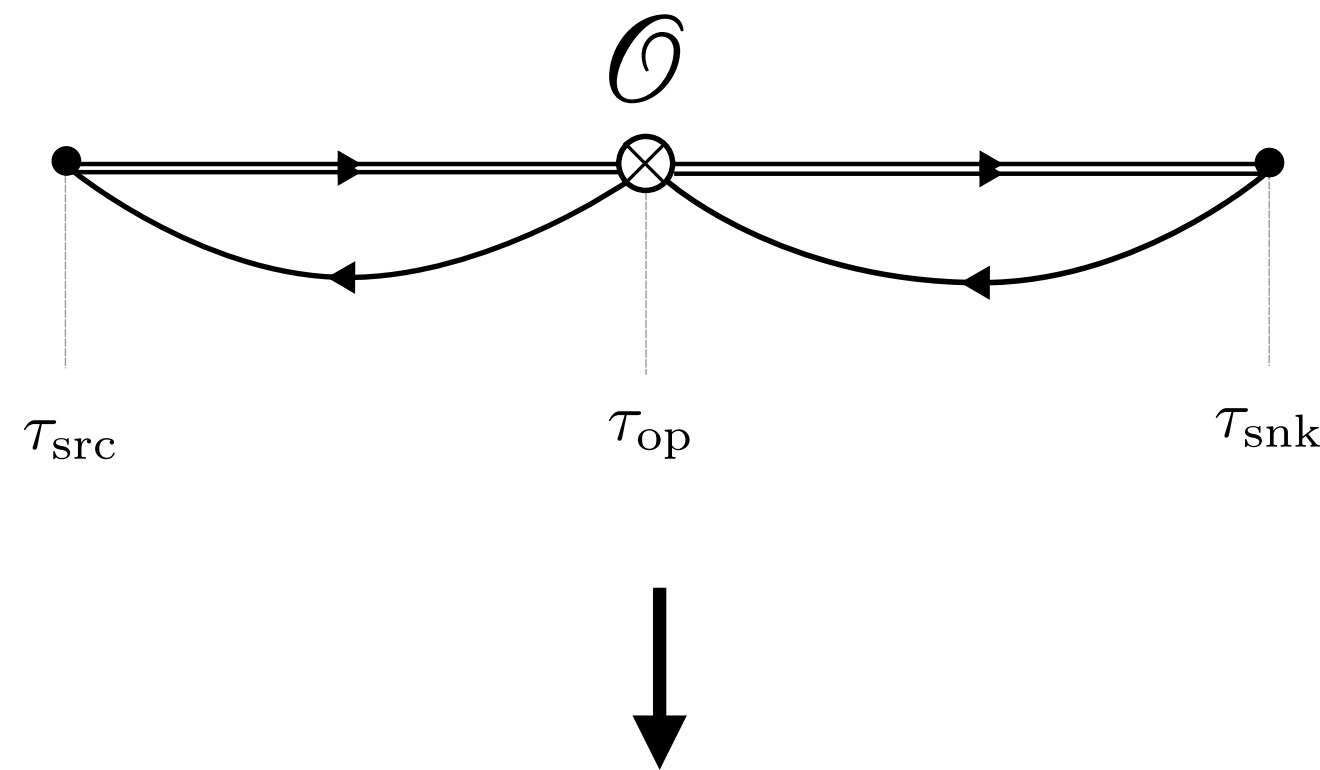
[HFLAV, 1909.12524]

[Review: Neubert Sachrajda, hep-ph/9603202]

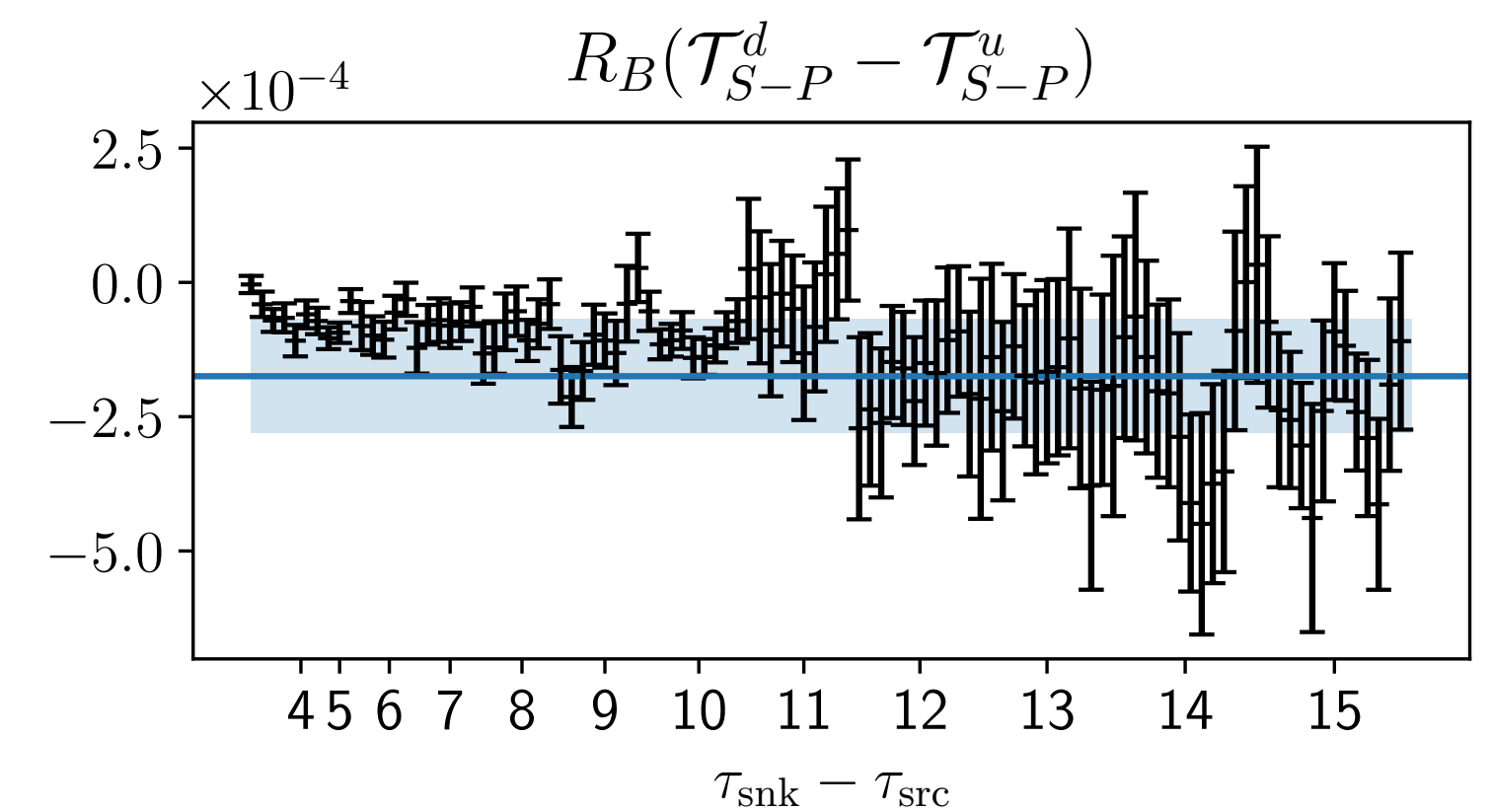
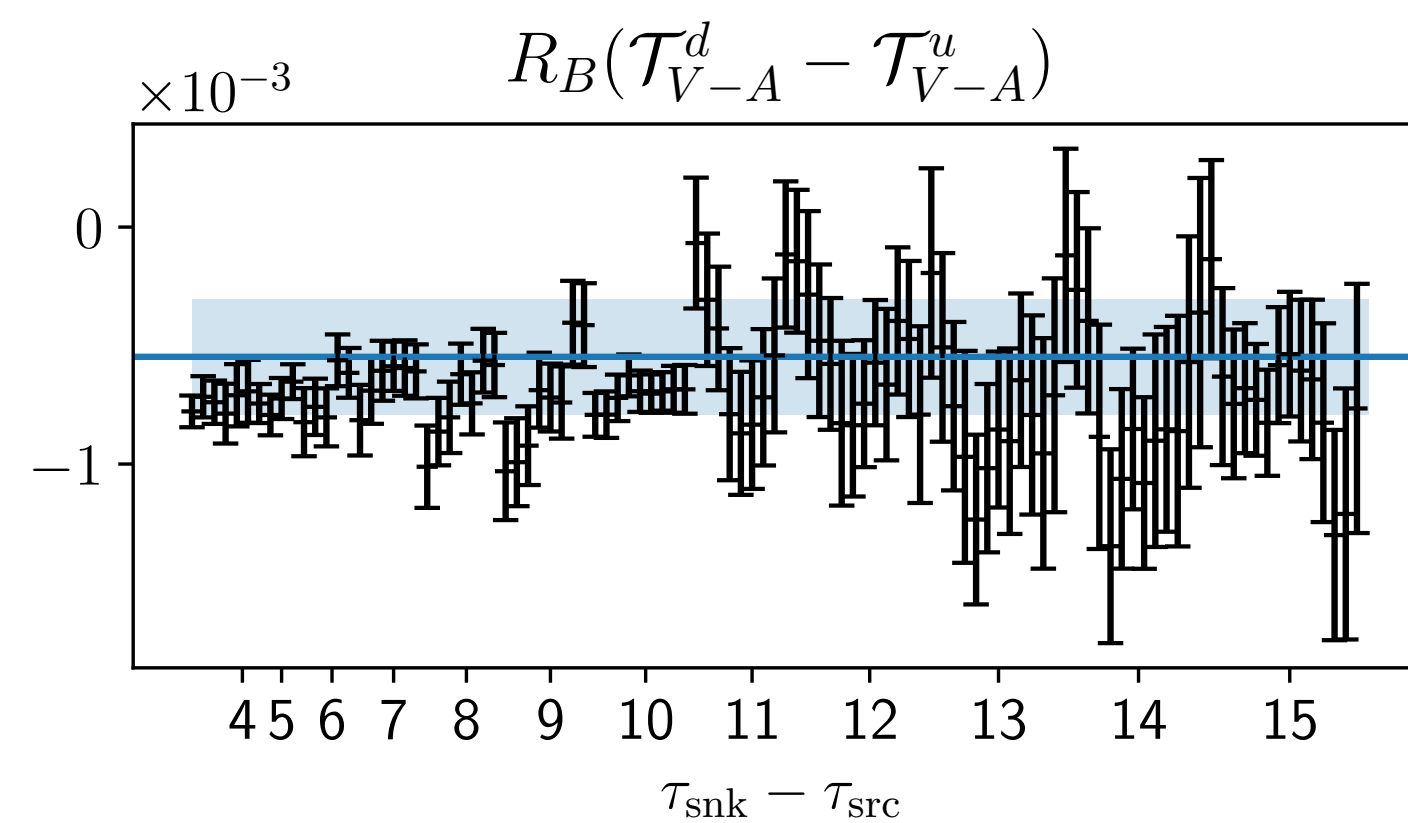
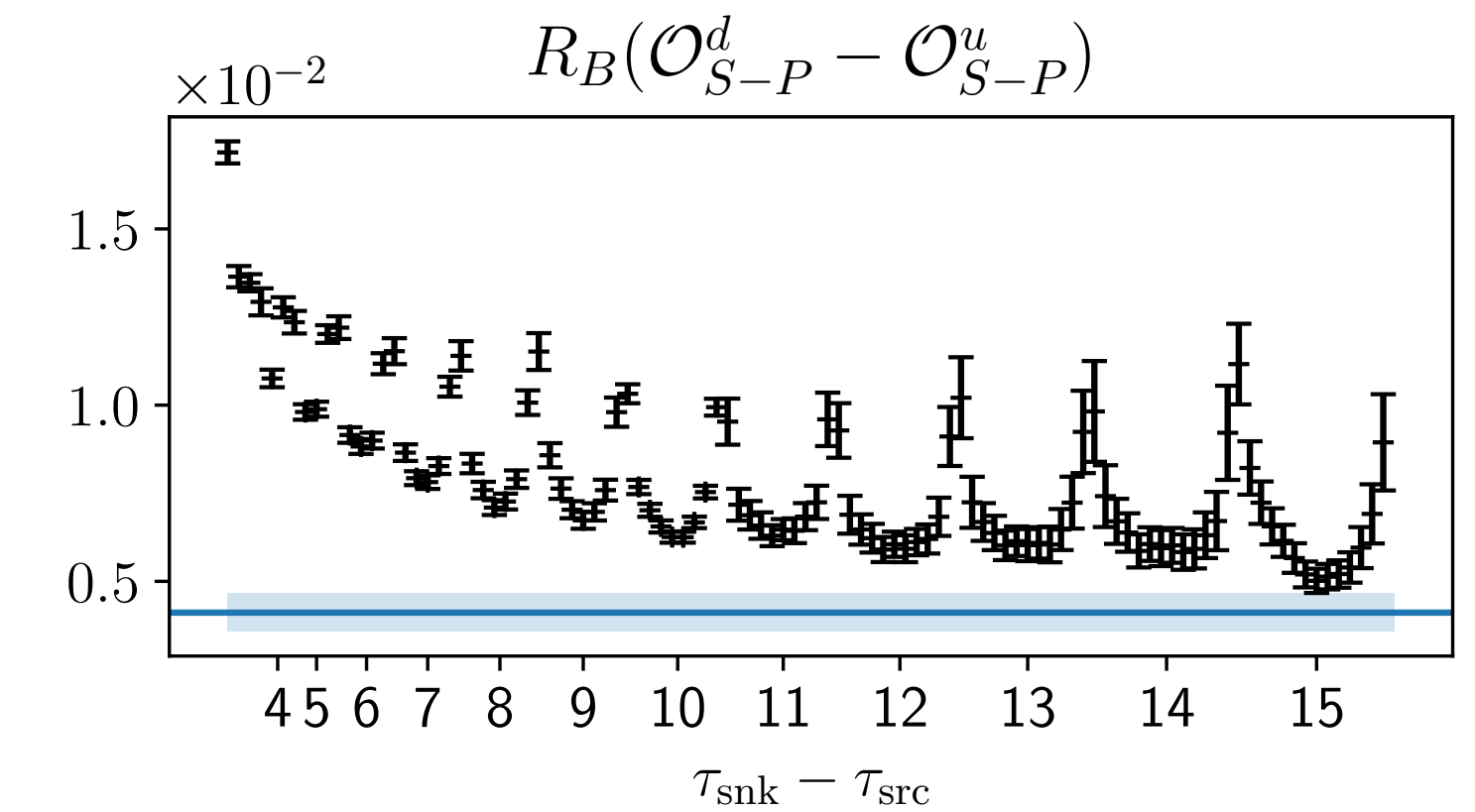
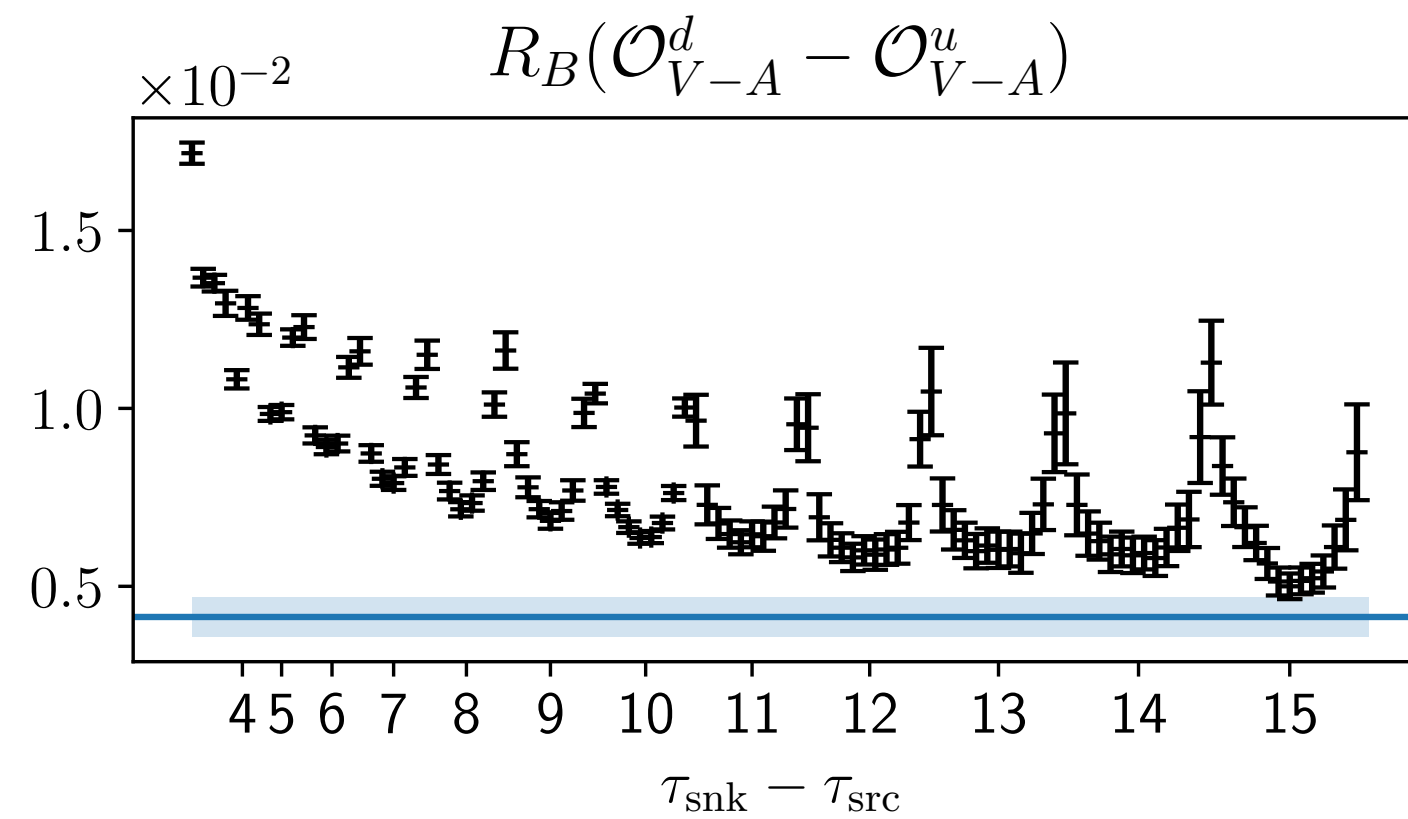
$$\Gamma = \frac{G_F^2 m_b^5}{192 \pi^3} V_{cb}^2 \left[c_3 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right) + 2c_5 \left(\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right) + c_6 \left(\frac{1}{m_b^3} \frac{\langle B | (\bar{b}_+ q)_\Gamma (\bar{q} b_+)_\Gamma | B \rangle}{M_B} + \dots \right) \right]$$

- μ_π^2, μ_G^2 from masses of hadrons, [I.I. Bigi, Th. Mannel, N. Uraltsev, hep-ph/1105.4574]



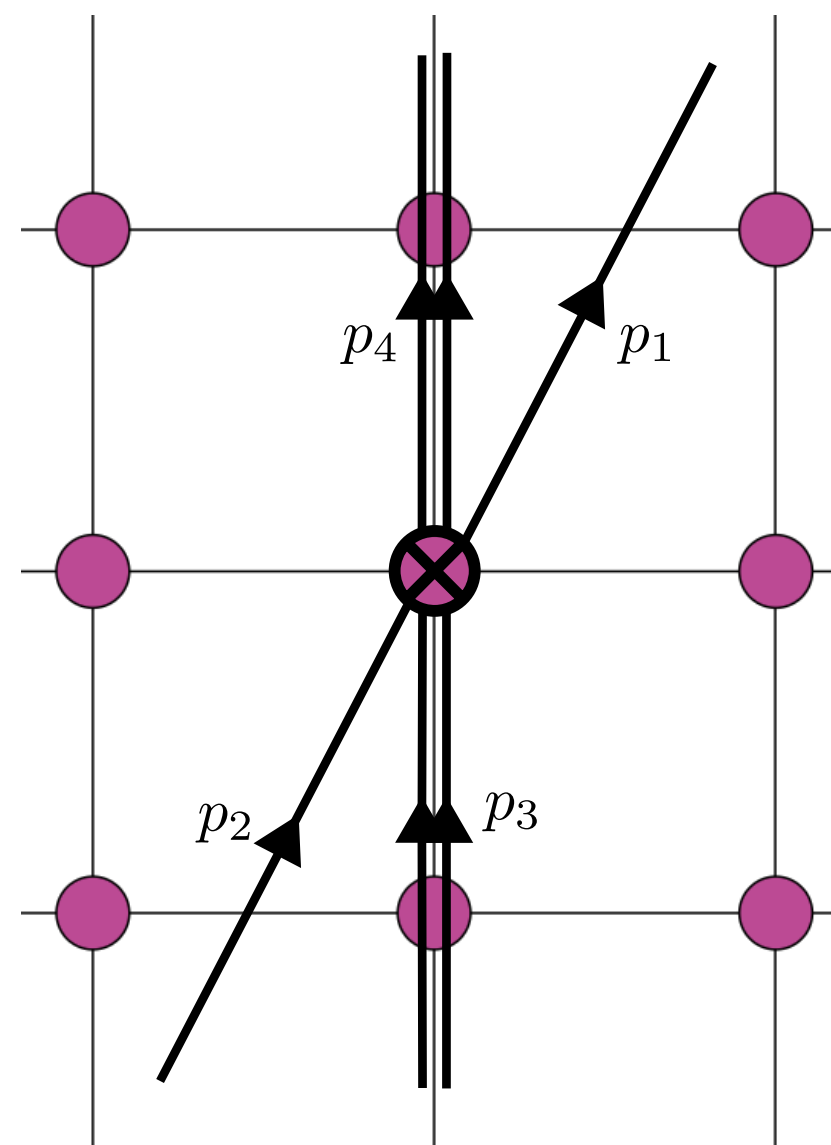


- Correlation functions computed for Wilson flow times $a^{-2}t \in \{0.5, 1.0, 2.0\}$
- Coupled two-point and three-point fits on various fitting ranges, AIC to choose how many excited states to include on each fitting range



[J.Lin, W. Detmold, S. Meinel: 2212.09275]

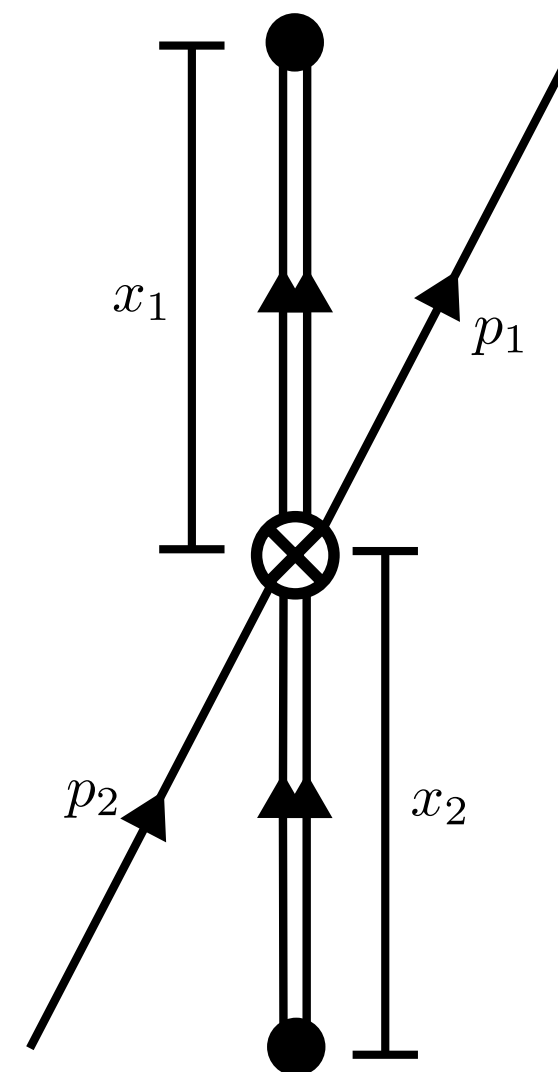
Lattice Perturbation Theory



- ▶ [Ishikawa et al, 1101.1072]
Potentially poor convergence properties

Nonperturbative Renormalization

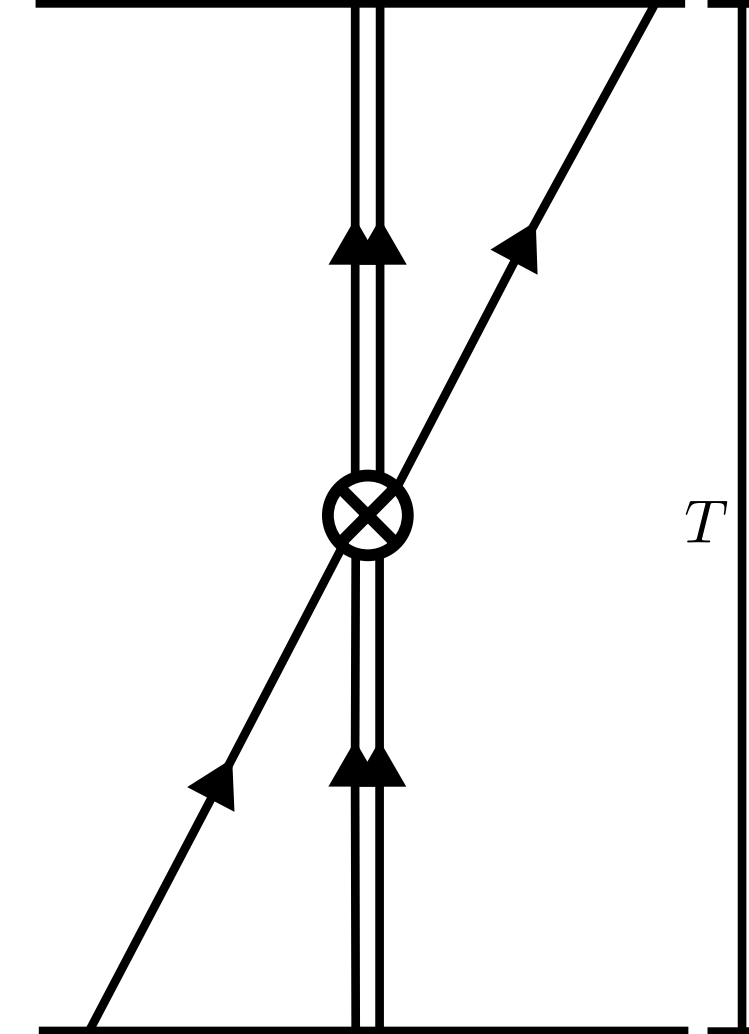
RI-xMOM



- ▶ Long history for light 4q operators (RIMOM)
- ▶ Gauge-fixed

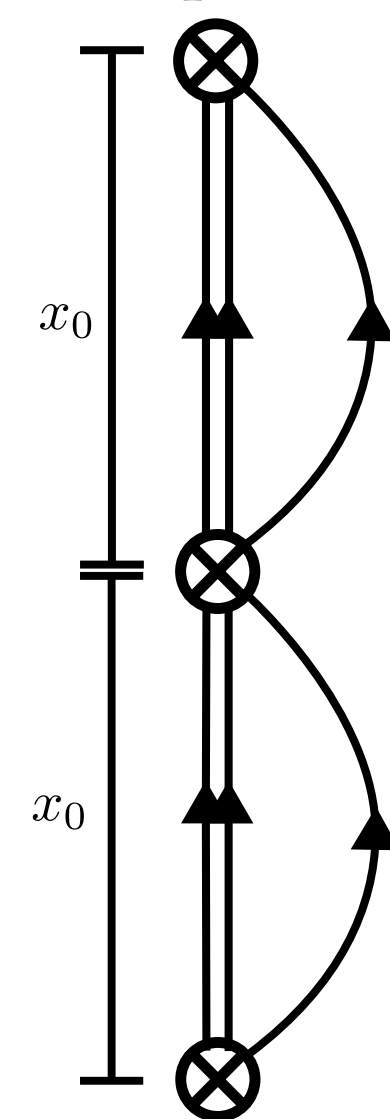
Schrodinger-Functional

Dirichlet Boundary



- ▶ Requires additional ensembles + step scaling

X-Space



- ▶ 4q HQET operators [Lin et al, 2404.16191]
- ▶ 4q light operators [Constantinou et al, 2406.08065]



b-Hadron Lifetimes



EFT MEs



Bare MEs



LQCD ensemble

* Requires three-point renormalization condition:

$$\frac{\langle J_\alpha(x_0) \mathcal{O}_j^{(X)}(\mu = x_0^{-1}, 0) J_\alpha^\dagger(-x_0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(-x_0) \rangle} = \frac{\langle J_\alpha(x_0) \mathcal{O}_j^{(0)}(0) J_\alpha^\dagger(-x_0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(-x_0) \rangle} \Bigg|_{\text{Tree Value}} =: T_{j,\alpha}$$

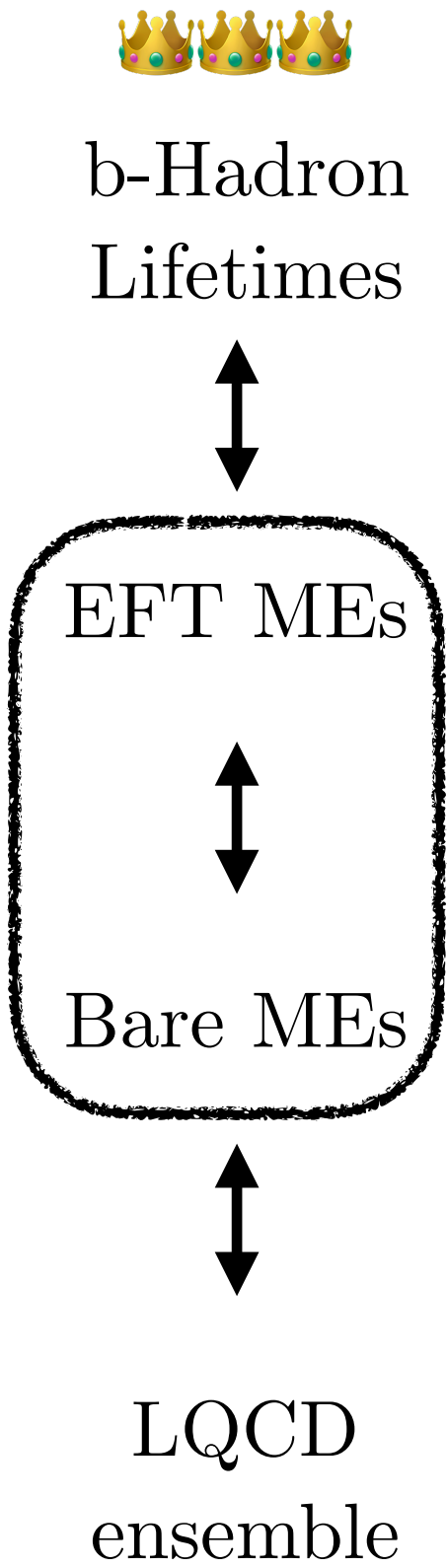
Measured on the lattice \rightarrow $M_{i,\alpha} = \frac{\langle J_\alpha(x_0) \mathcal{O}_i^{(0)}(0) J_\alpha^\dagger(-x_0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(-x_0) \rangle} = Z_{ij}(\mu = x_0^{-1}) \frac{\langle J_\alpha(x_0) \mathcal{O}_j^{(X)}(\mu = x_0^{-1}, 0) J_\alpha^\dagger(-x_0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(-x_0) \rangle} = Z_{ij}(x_0) T_{j,\alpha}$

$$Z_{ij} = M_{i,\alpha} T_{\alpha,j}^{-1}$$

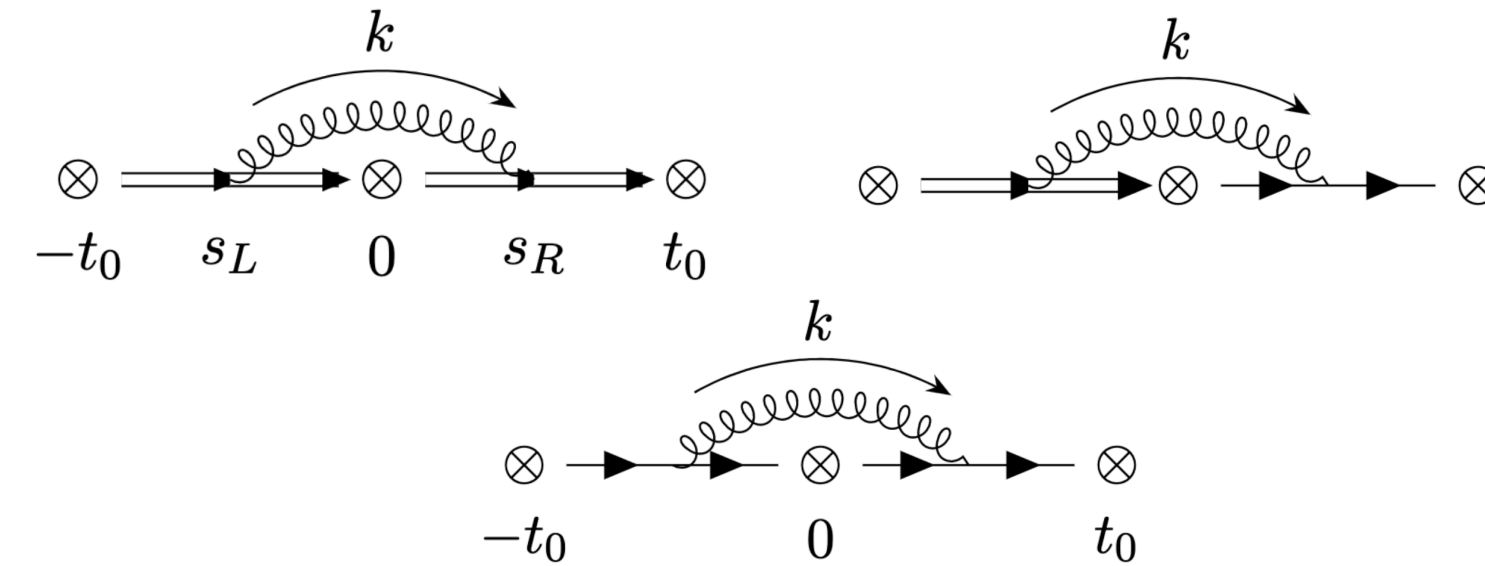
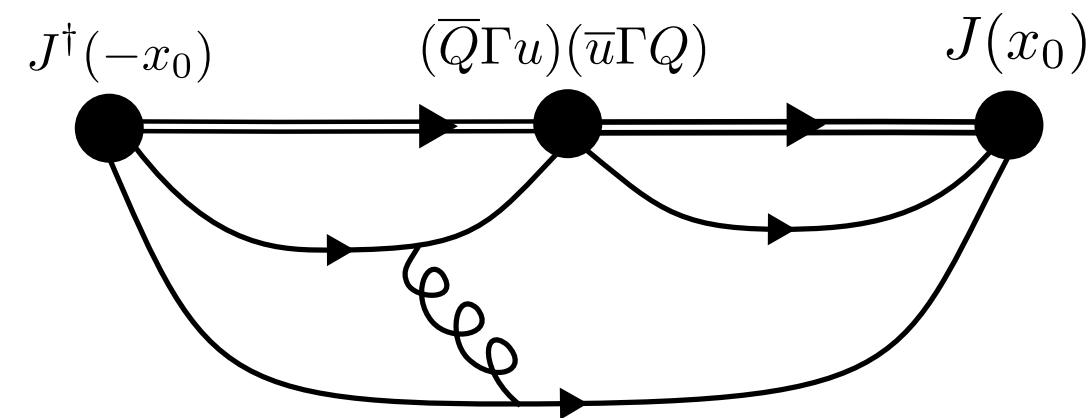
- * Ratios are nice because Z-factors of source cancels, and linear divergence of Wilson line cancels.
- * Sources need to be chosen so that the tree level matrix is invertible, otherwise don't have enough constraints on the Z-matrix. Need baryonic sources, otherwise the colour-mixed operators will have vanishing matrix elements.

$$O_q^3 = \left(\bar{Q} T_a \gamma_\mu P_L q \right) \left(\bar{q} T_a \gamma^\mu P_L Q \right) \quad O_q^4 = \left(\bar{Q} T_a P_L q \right) \left(\bar{q} T_a P_R Q \right)$$

Rough schematic of the calculation



Source-sink pairs: $(H_f^-, H_f^-), (H_{f,i}^{*-}, H_{f,i}^{*-}), (\Lambda_1, \Sigma_{2,0}), (\Lambda_2, \Sigma_{1,0}), (\Sigma_{1,\alpha}, \Sigma_{1,\alpha}), (\Sigma_{2,\alpha}, \Sigma_{2,\alpha}), (\Sigma_{1,\alpha,i}^*, \Sigma_{1,\alpha,i}^*), (\Sigma_{2,\alpha,i}^*, \Sigma_{2,\alpha,i}^*)$



$$\int \frac{d^d p_L}{(2\pi)^d} \frac{d^d p_R}{(2\pi)^d} \frac{d^d k}{(2\pi)^d} \frac{e^{i p_L x_L - i p_R x_R} p_R^\alpha (p_R - k)^\beta (p_L - k)^\rho p_L^\delta}{(-p_L^2)(-(p_L - k)^2)(-(p_R - k)^2)(-p_R^2)(-k^2)}$$

$$\int \frac{d^d p_L}{(2\pi)^d} \frac{d^d p_R}{(2\pi)^d} \frac{d^d k}{(2\pi)^d} \frac{e^{i x_L p_L - i x_R p_R} p_R^\alpha (p_R - k)^\beta}{(-p_R^2)(-(p_R - k)^2)(-k^2)(v \cdot (p_L - k))(v \cdot p_L)}$$

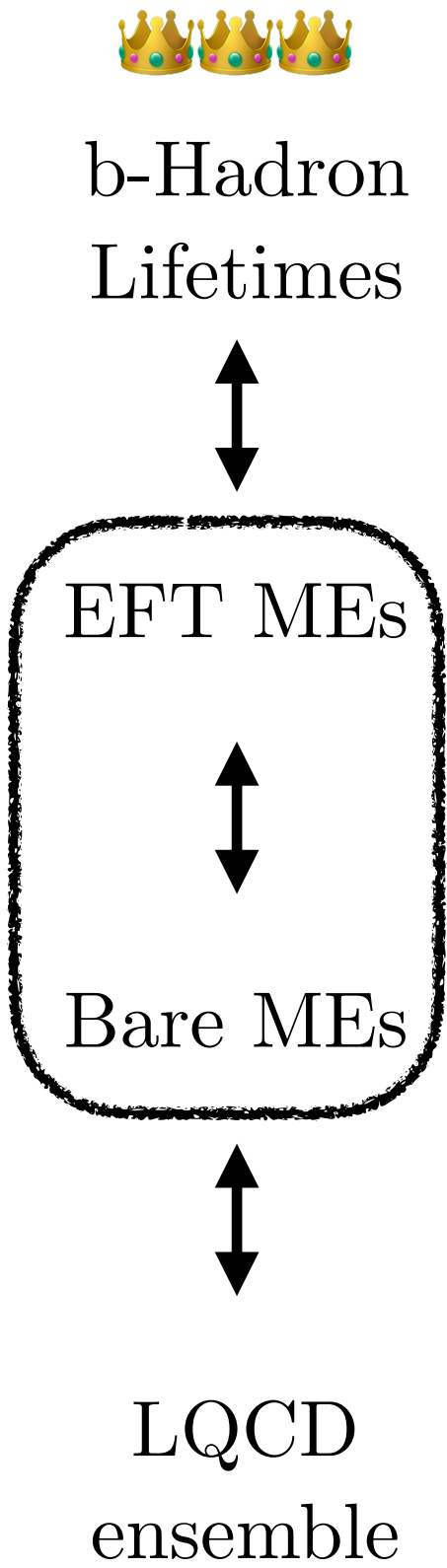


Integration by parts

$$\int d^d k \left(\frac{\partial}{\partial k_\mu} \cdot p_\mu \right) \circ f(k) = 0$$

Two-loop master integrals

[J.Lin, W. Detmold, S. Meinel: 2404.16191]



Four-quark operators determining spectator effects in inclusive lifetimes

$$\left(\beta := e^{2\gamma_E} \frac{\mu^2 t^2}{16} \right)$$

$$O_1^f := (\bar{Q} \gamma_\mu P_L q_f) (\bar{q}_f \gamma_\mu P_L Q),$$

$$O_2^f := (\bar{Q} P_L q_f) (\bar{q}_f P_R Q),$$

$$O_3^f := (\bar{Q} \gamma_\mu P_L T^A q_f) (\bar{q}_f \gamma_\mu P_L T^A Q),$$

$$O_4^f := (\bar{Q} P_L T^A q_f) (\bar{q}_f P_R T^A Q),$$

$$C_{ij, n \in \{1,2,3,4\}}^{(\overline{\text{MS}}; X)} := \sum_k Z_{ik}^{(\overline{\text{MS}})} (Z^{(X)})_{kj, n \in \{1,2,3,4\}}^{-1} = \mathbb{1} +$$

$$\alpha_S(\mu) \begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ 0 & \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{1}{9\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ -\frac{1}{36\pi} & -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

Four-quark operators determining B-mixing

$$O_1 := (\bar{Q}_+ P_L q) (\bar{Q}_- P_L q),$$

$$O_2 := (\bar{Q}_+ P_L T^A q) (\bar{Q}_- P_L T^A q),$$

$$O_3 := (\bar{Q}_+ P_L q) (\bar{Q}_- P_R q),$$

$$O_4 := (\bar{Q}_+ P_L T^A q) (\bar{Q}_- P_R T^A q).$$

$$C_{\{O_1, O_2\}}^{(\overline{\text{MS}}, X)} = \mathbb{1} + \frac{\alpha_S}{\pi} \begin{pmatrix} \frac{7\log(\beta)}{9} + \frac{4\pi^2}{9} + \frac{23}{9} & \frac{2\log(\beta)}{3} - \frac{\pi^2}{3} + \frac{4}{3} \\ \frac{4\log(\beta)}{27} - \frac{2\pi^2}{27} + \frac{8}{27} & \frac{5\log(\beta)}{9} + \frac{5\pi^2}{9} + \frac{19}{9} \end{pmatrix}$$

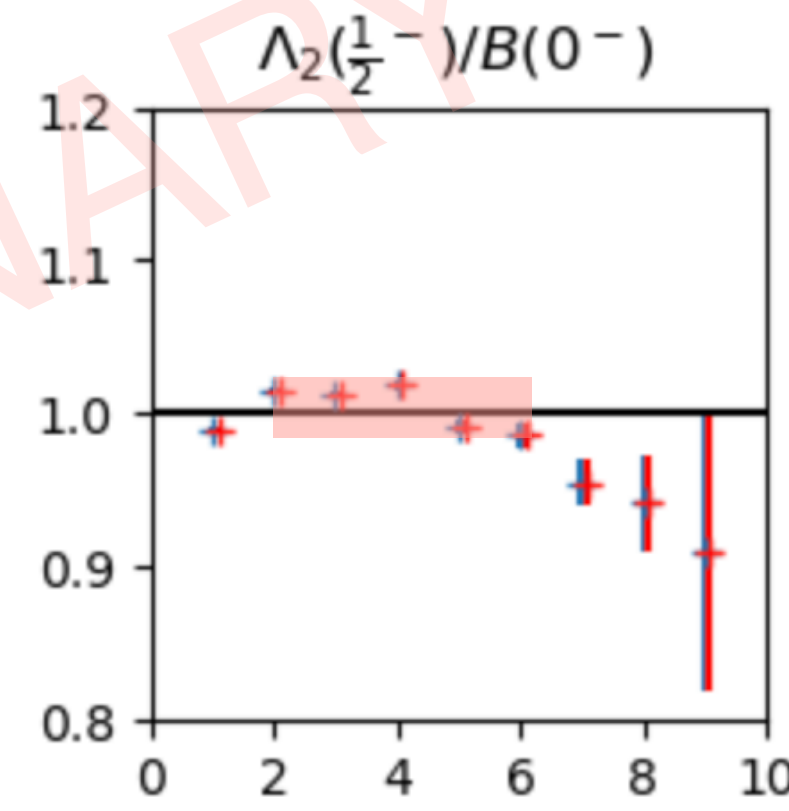
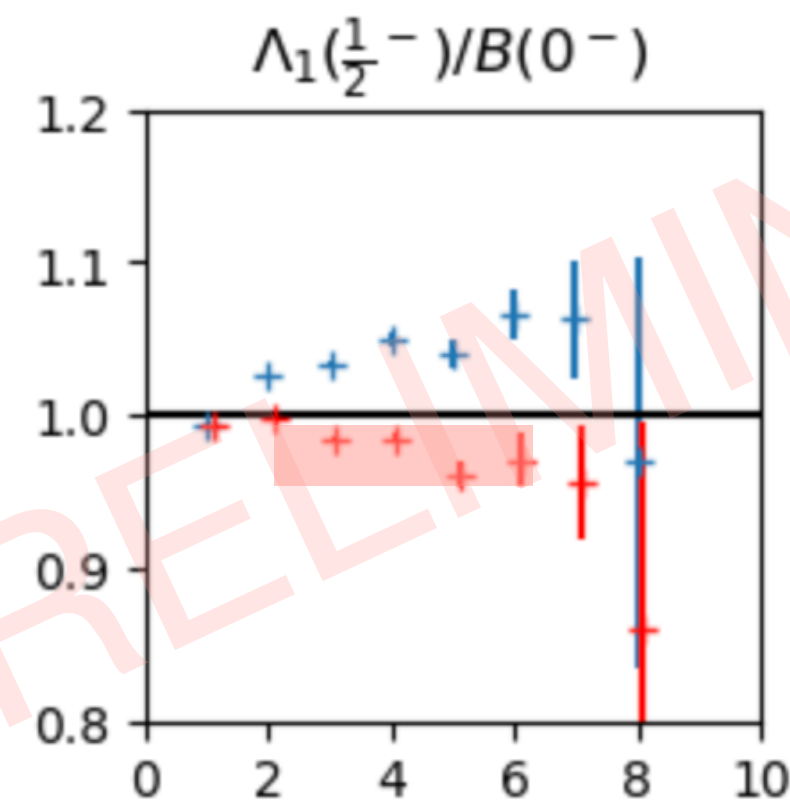
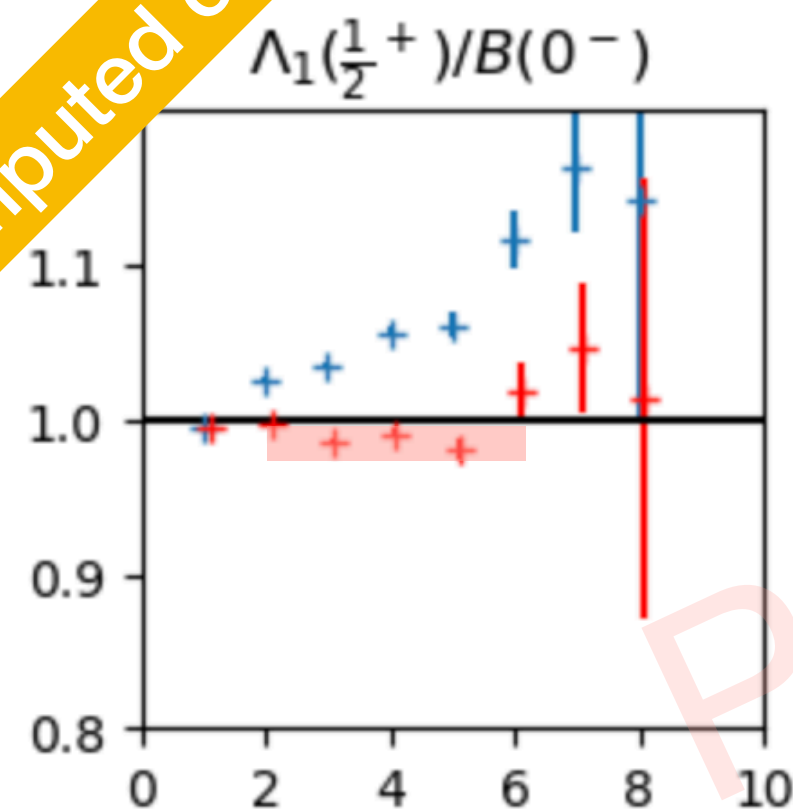
$$C_{\{O_3, O_4\}}^{(\overline{\text{MS}}, X)} = \mathbb{1} + \frac{\alpha_S}{\pi} \begin{pmatrix} \log(\beta) + \frac{4\pi^2}{9} + \frac{25}{9} & \frac{3\log(\beta)}{4} - \frac{\pi^2}{3} + \frac{7}{6} \\ \frac{\log(\beta)}{6} - \frac{2\pi^2}{27} + \frac{7}{27} & \frac{3\log(\beta)}{4} + \frac{5\pi^2}{9} + \frac{43}{18} \end{pmatrix}$$

Window Problem (I) : Two-point

- ▶ Window Problem where $\sqrt{x^2} \ll \Lambda_{\text{QCD}}^{-1}$ is required to control perturbation-theory errors appearing in the perturbative matching to $\overline{\text{MS}}$, and $a \ll \sqrt{x^2}$ is required to control discretisation artifacts.
- ▶ RBC/UKQCD 2+1f Domain Wall Fermion, Iwasaki action configurations were used [hep-lat/1411.7017],

Name	Volume	1/a (GeV)	$am_{u,d}, am_s$	N
24I	$24^3 \times 64(\times 16)$	1.785(5)	0.005, 0.04	228
32I	$32^3 \times 64(\times 16)$	2.383(9)	0.004, 0.03	239
32IF	$32^3 \times 64(\times 16)$	3.148(17)	0.0047, 0.0186	97

Computed on 32I



- ▶ Ratios of two-point functions cancel the static quark self-energy divergence. Bare ratios of Z-factors shown in blue

- ▶ One-loop run values are shown in red, with approximate plateaus for separations $2 \leq t \leq 6$ shaded

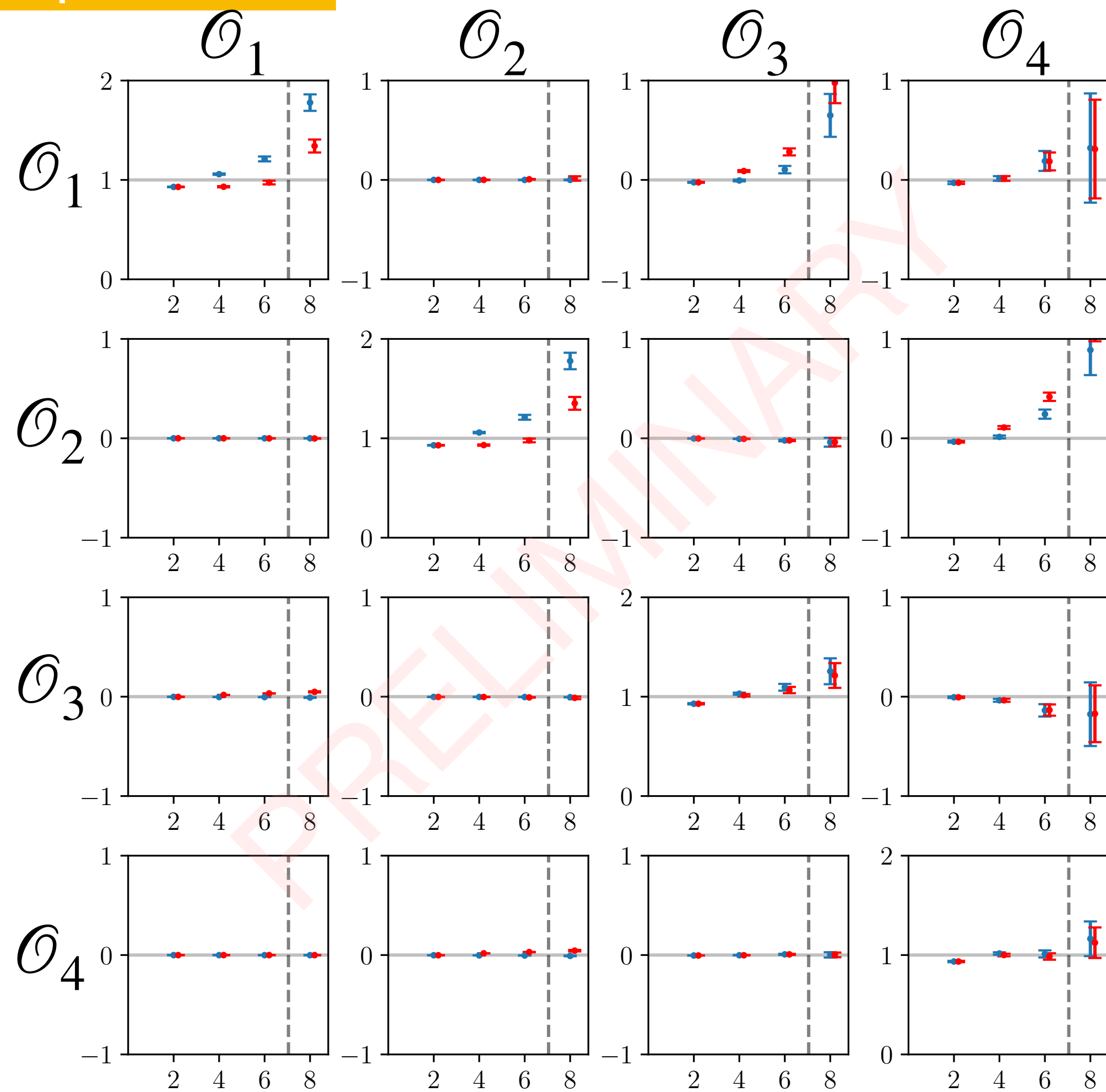
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Window Problem (II) : Four quark operators

12

Computed on 321



- ▶ Blue datapoints : $Z_{ij}(\mu^2 = (t_{\text{snk}} - t_{\text{src}})^{-2})$
- ▶ Red datapoints : Renormalization constants RG-run to a common scale using $O(\alpha_S)$ anomalous-dimension.
- ▶ Dashed vertical line : $\Lambda_{\text{QCD}}^{-1}$, where perturbation theory should break down.
- ▶ As $\Delta t \rightarrow 0$, $Z \rightarrow$ identity matrix.
- ▶ Some non-plateau behaviour in off-diagonal renormalization constants
 - ▶ May require $O(\alpha_S^2)$ -anomalous dimensions, or $O(a)$ -improvement of lattice operators

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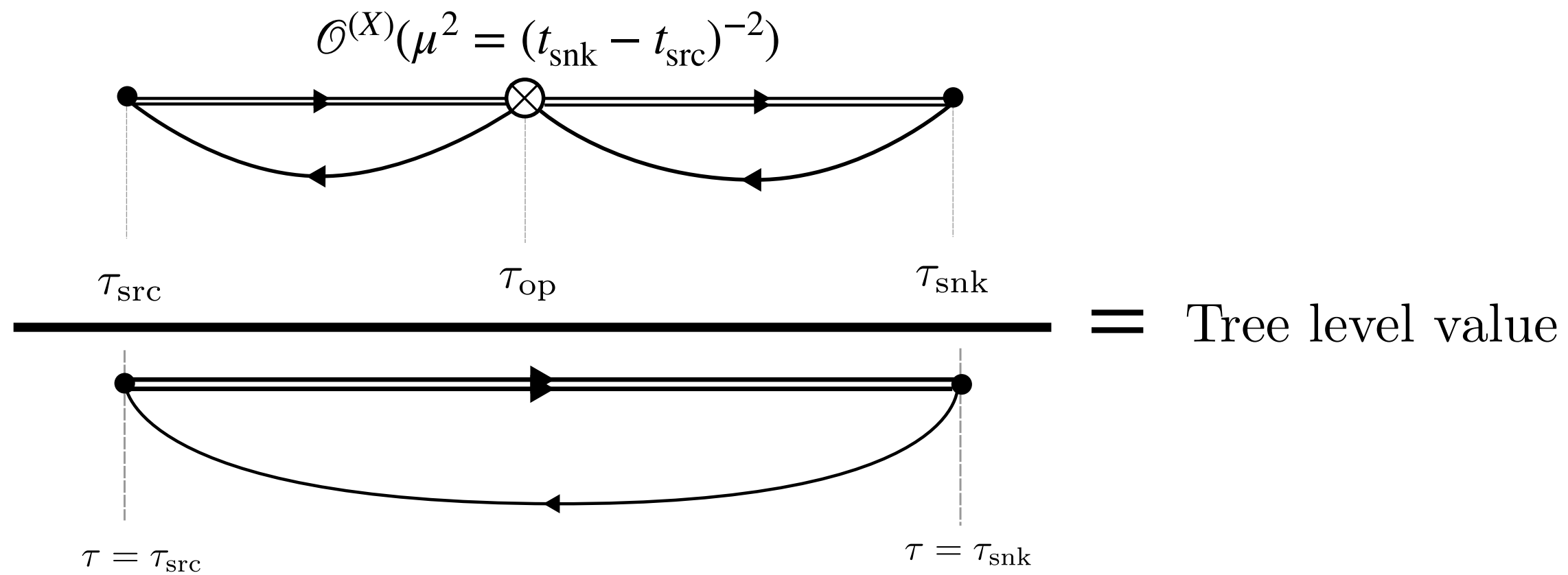
- ✓ Matching between X-space and $\overline{\text{MS}}$ computed for spectator-effect four-quark operators and B-meson mixing four-quark operators in Heavy Quark Effective Field Theory.
- ✓ Window problem investigated, approximate plateaus found.



- Quantification of errors:
 - Continuum Limit, chiral extrapolations
 - $O(a)$ mixing with dimension-7 operators, $O(\alpha_s^2)$ -anomalous dimensions

X-space schemes

BACKUP



$$C_{ij,n \in \{1,2,3,4\}}^{(\overline{\text{MS}}; X)} := \sum_k Z_{ik}^{(\overline{\text{MS}})} (Z^{(X)})_{kj,n \in \{1,2,3,4\}}^{-1} = \mathbb{1} +$$

$$\alpha_S(\mu) \begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ 0 & \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{1}{9\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ -\frac{1}{36\pi} & -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

$$(\beta = e^{2\gamma_E} \frac{\mu^2 t^2}{16})$$

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

- Due to the mixing between the four different operators, four nondegenerate sets of source/sink interpolators are needed to determine the entire mixing matrix.
- Different choices of source/sink sets lead to different renormalization conditions.

n	J_n	K_n
1	H_f^-	H_f^-
2	$H_{f,i}^{*-}$	$H_{f,i}^{*-}$
3	Λ_1	$\Sigma_{2,0}$
4	$\Sigma_{1,\alpha}$	$\Sigma_{1,\alpha}$

$$SU(2)_h \begin{cases} H_f^-(0^-) : \bar{q}_f \gamma_5 Q \\ H_{f,i}^{*-}(1^-) : \bar{q}_f \gamma_i Q \end{cases}$$

$$\Lambda_1 \left(\frac{1}{2}^+ \right) : \epsilon^{abc} [q^{aT} \tau^A C \gamma_5 q^b] Q^c,$$

$$SU(2)_h \begin{cases} \Sigma_{1,\alpha} \left(\frac{1}{2}^+ \right) : \epsilon^{abc} [q^{aT} \tau_\alpha^S C \gamma_i q^b] \gamma_i \gamma_5 Q^c, \\ \Sigma_{1,\alpha,i}^* \left(\frac{3}{2}^+ \right) : \epsilon^{abc} [q^{aT} \tau_\alpha^S C \gamma_i q^b] Q^c - \frac{1}{3} \epsilon^{abc} [q^{aT} \tau_A C \gamma_j q^b] \gamma_i \gamma_j Q^c, \end{cases}$$

$$SU(2)_h \begin{cases} \Sigma_{2,\alpha} \left(\frac{1}{2}^+ \right) : \epsilon^{abc} [q^{aT} \tau_\alpha^S C \gamma_0 \gamma_i q^b] \gamma_i \gamma_5 Q^c, \\ \Sigma_{2,\alpha,i}^* \left(\frac{3}{2}^+ \right) : \epsilon^{abc} [q^{aT} \tau_\alpha^S C \gamma_0 \gamma_i q^b] Q^c - \frac{1}{3} \epsilon^{abc} [q^{aT} \tau_\alpha^S C \gamma_0 \gamma_j q^b] \gamma_i \gamma_j Q^c, \end{cases}$$

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Master integrals and blocks

BACKUP

* Master Integrals look like:

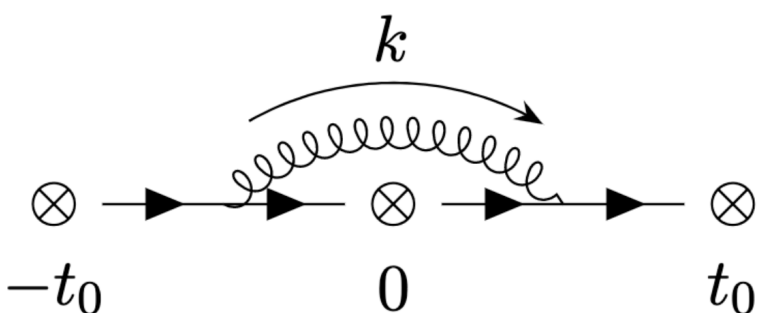
$$T_{LL}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(-p_L^2)^{n_1} (-p_R^2)^{n_2} (-(p_L - p_R)^2)^{n_3}} \quad (\text{agrees with [M.Costa et al. hep-lat/2102.00858]})$$

$$= \frac{-\Gamma(\frac{d}{2} - n_1) \Gamma(d - n_1 - n_2 - n_3)}{\Gamma(n_2) \Gamma(n_3) \Gamma(\frac{d}{2}) 4^{n_1+n_2+n_3} \pi^d} (-x_R^2)^{-d+n_1+n_2+n_3} \int_0^1 dx (1-x_1)^{-\frac{d}{2}+n_1+n_2-1} x_1^{-\frac{d}{2}+n_1+n_3-1} {}_2F_1\left(\frac{d}{2} - n_1, d - n_1 - n_2 - n_3, \frac{d}{2}, \frac{-(x_L - x_1 x_R)^2}{x_1(1-x_1)x_R^2}\right)$$

$$T_{HH}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(v \cdot p_L)^{n_1} (v \cdot p_R)^{n_2} (-(p_L - p_R)^2)^{n_3}}$$

$$= \frac{1}{4^{n_3} \pi^{\frac{d}{2}}} \frac{\Gamma(\frac{d}{2} - n_3)}{\Gamma(n_3) \Gamma(n_1 + n_2)} {}_2F_1\left(n_1, d - 2n_3, n_1 + n_2, \frac{v \cdot (x_L - x_R)}{v \cdot x_L}\right) (-iv \cdot (x_R - x_L))^{-1+n_1+n_2} (-(v \cdot x_L)^2)^{-\frac{d}{2}+n_3} \delta_{\perp}(v \cdot (x_L - x_R)) \theta(v \cdot (x_R - x_L) > 0)$$

* Building blocks look like:



$$= g^2 (\mu^2)^{\frac{4-d}{2}} (-t_0^2)^{8-\frac{7d}{4}} \left[\left(\frac{-1}{32\pi^6 \epsilon} + \frac{10 \log 2 - 3 \log \pi - 3\gamma_E}{64\pi^6} \right) (\not{k})(\not{k}) + \left(\frac{-1}{32\pi^6 \epsilon} + \frac{3 + 4 \log 2 - 6 \log \pi - 6\gamma_E}{128\pi^6} \right) (\gamma_{\mu})(\gamma_{\mu}) + \left(\frac{1}{128\pi^6 \epsilon} + \frac{-1 - 2 \log 2 + 3 \log \pi + 3\gamma_E}{256\pi^6 \epsilon} \right) (\gamma_{\alpha} \gamma_{\beta})(\gamma_{\alpha} \gamma_{\beta}) \right]$$

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b-Hadron
Lifetimes



EFT MEs



Bare MEs



LQCD
ensemble

- * In any dimensional-regularisation scheme, you have to contend with evanescent operators:

$$\begin{aligned}
 & (\bar{Q}_a \Gamma_L q_b) (\bar{q}_c \Gamma_R Q_d) \\
 & = \int \frac{d^d k}{(2\pi)^d} \frac{-i}{k^2 + i\epsilon} \left(\frac{1 + \psi}{2} \Gamma_L \frac{i}{(\not{p} - \not{k}) + i\epsilon} (ig\gamma_\mu T^A) \right)_{ab} \\
 & \quad \times \left((ig\gamma_\mu T^A) \frac{i}{(\not{p} - \not{k}) + i\epsilon} \Gamma_R \frac{1 + \psi}{2} \right)_{cd} \\
 & = \frac{\alpha_S}{8\pi\epsilon} \left(\frac{1 + \psi}{2} \Gamma_L \gamma^\mu \gamma^\nu T^A \right)_{ab} \left(T^A \gamma^\nu \gamma^\mu \Gamma_R \frac{1 + \psi}{2} \right)_{cd} + O(\epsilon^0)
 \end{aligned}$$

- * These are operators that vanish at $d=4$, but still have finite contribution to any renormalisation scheme

$$E_1 := (\bar{Q} \gamma_\mu P_L \gamma_\alpha \gamma_\beta q) (\bar{q} \gamma_\beta \gamma_\alpha \gamma_\mu P_L Q) - 4O_1,$$

$$E_2 := (\bar{Q} P_L \gamma_\alpha \gamma_\beta q) (\bar{q} \gamma_\beta \gamma_\alpha P_R Q) - 4O_2,$$

$$E_3 := (\bar{Q} \gamma_\mu P_L \gamma_\alpha \gamma_\beta T^A q) (\bar{q} \gamma_\beta \gamma_\alpha \gamma_\mu P_L T^A Q) - 4O_3,$$

$$E_4 := (\bar{Q} P_L \gamma_\alpha \gamma_\beta T^A q) (\bar{q} \gamma_\beta \gamma_\alpha P_R T^A Q) - 4O_4.$$

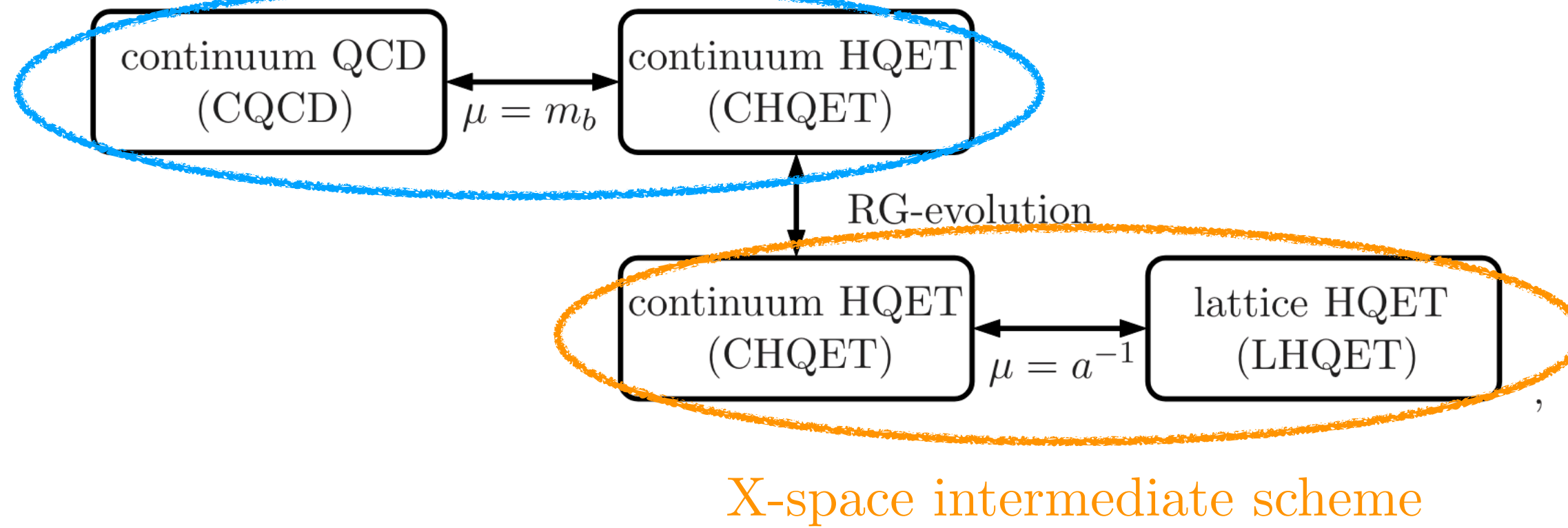
- * For regularisation independent schemes, you need to match evanescent-subtracted operators (as continuum evanescent operators have no analogue in lattice discretisations)

Position-Space Scheme

BACKUP

► [Ishikawa et al, 1101.1072] Two-step matching procedure

Heavy Quark Expansion



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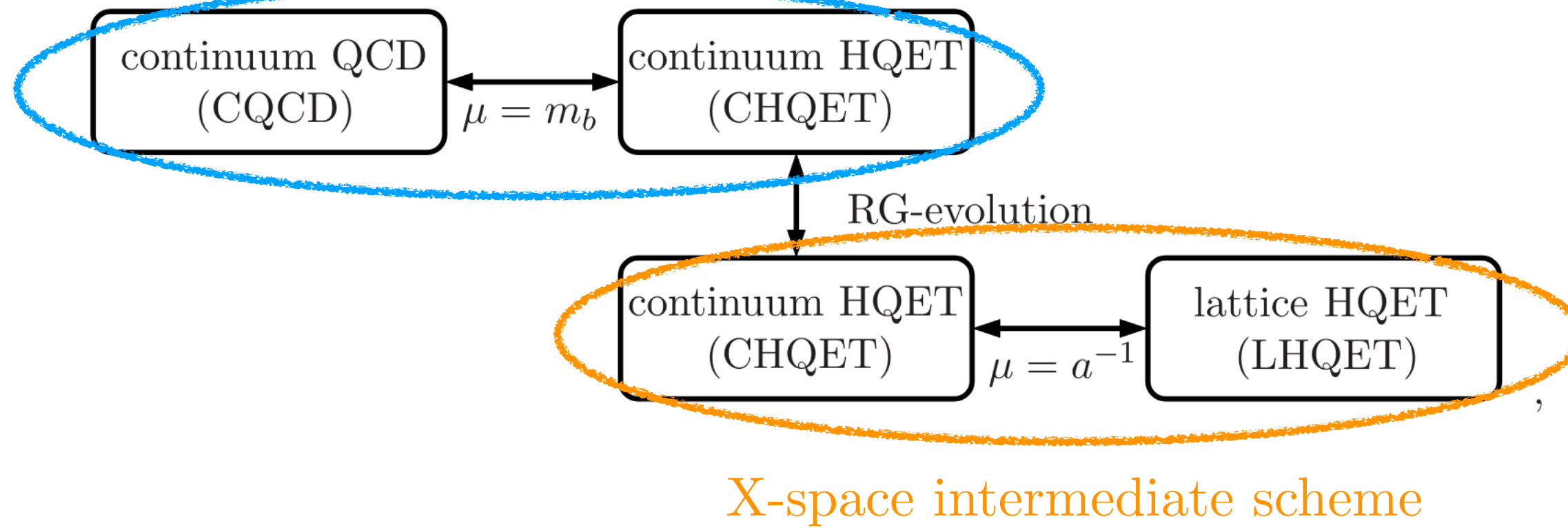
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Position-Space Scheme

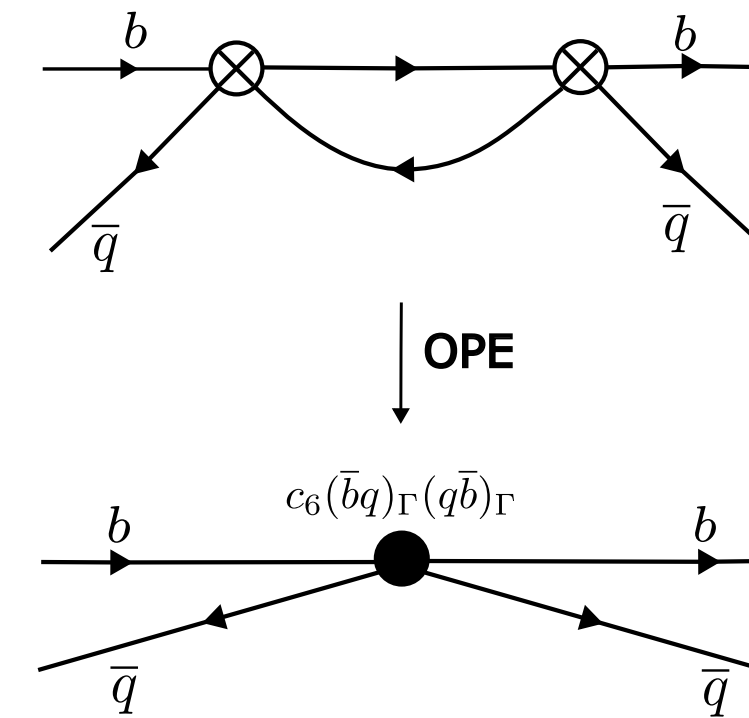
BACKUP

- [Ishikawa et al, 1101.1072] Two-step matching procedure

Heavy Quark Expansion



- Dominant $O(1/m_b^3)$ contribution come from phase-space enhanced Spectator Effects where light quarks contribute



$$O_1 = (\bar{b}_+ \gamma_\mu P_L q) (\bar{q} \gamma_\mu P_L b_+)$$

$$O_2 = (\bar{b}_+ P_L q) (\bar{q} P_R b_+)$$

$$O_3 = (\bar{b}_+ T_a \gamma_\mu P_L q) (\bar{q} T_a \gamma^\mu P_L b_+)$$

$$O_4 = (\bar{b}_+ T_a P_L q) (\bar{q} T_a P_R b_+)$$

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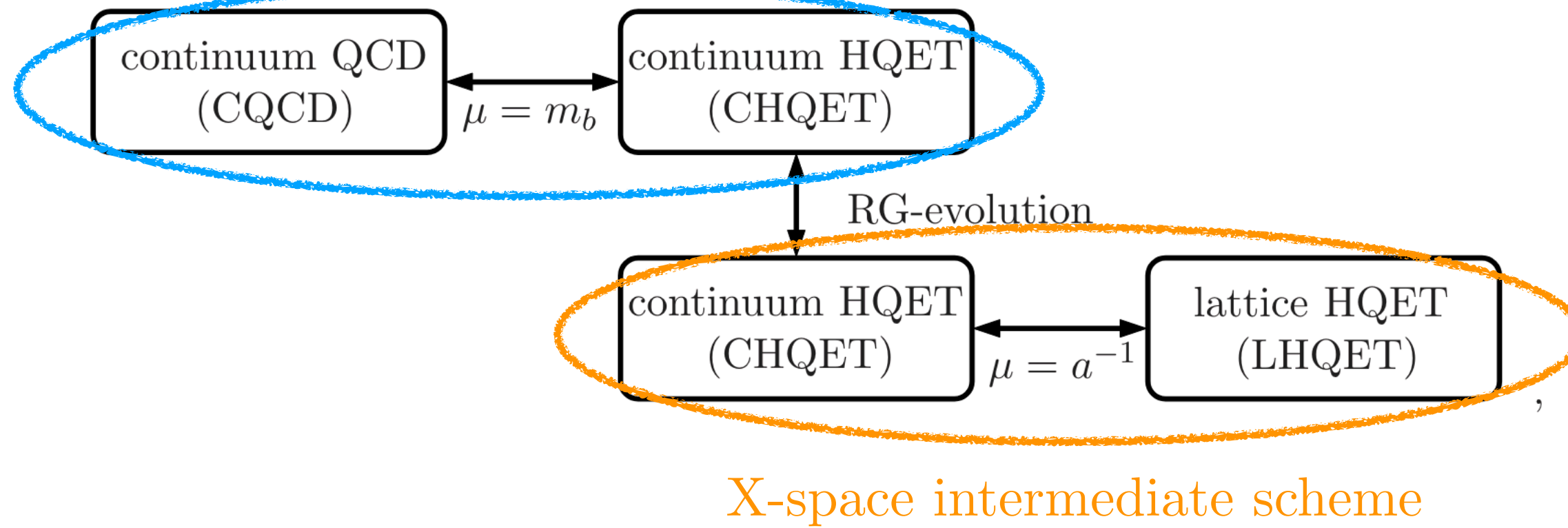
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Position-Space Scheme

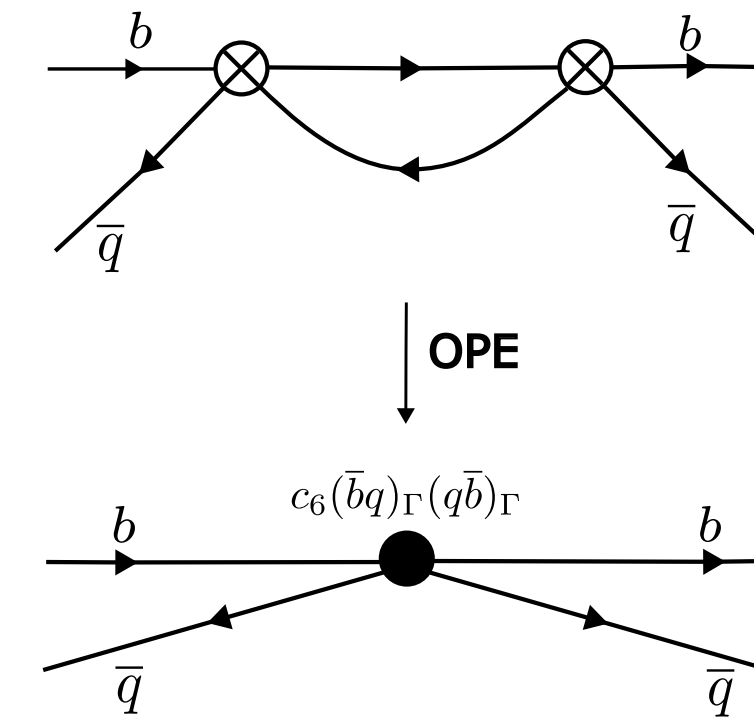
BACKUP

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Heavy Quark Expansion



- Dominant $O(1/m_b^3)$ contribution come from phase-space enhanced Spectator Effects where light quarks contribute

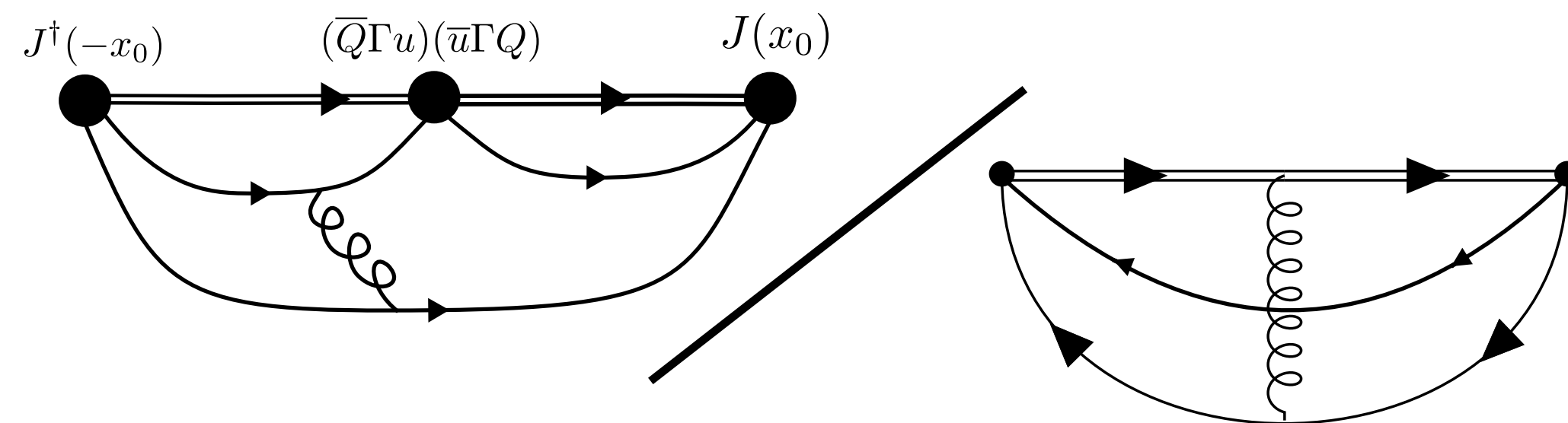


$$O_1 = (\bar{b}_+ \gamma_\mu P_L q) (\bar{q} \gamma_\mu P_L b_+)$$

$$O_2 = (\bar{b}_+ P_L q) (\bar{q} P_R b_+)$$

$$O_3 = (\bar{b}_+ T_a \gamma_\mu P_L q) (\bar{q} T_a \gamma^\mu P_L b_+)$$

$$O_4 = (\bar{b}_+ T_a P_L q) (\bar{q} T_a P_R b_+)$$



- The static action suffers from a power-divergent mixing of the kinetic operator $\bar{h}D_0h$ with a mass term $O(\alpha_s)\bar{h}h/a$
- This is cancelled in appropriate ratios!

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