Doubly charmed tetraquark T_{cc}^{+} from lattice QCD

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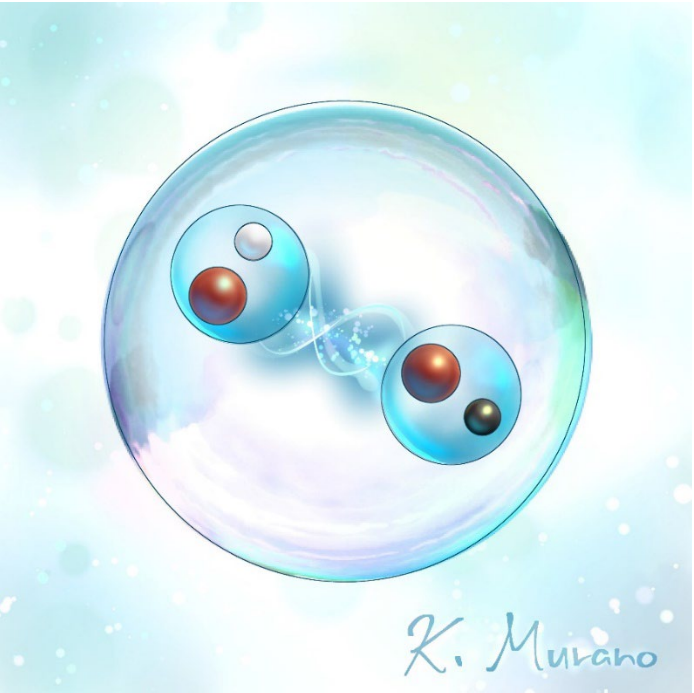
Aug. 22, 2024

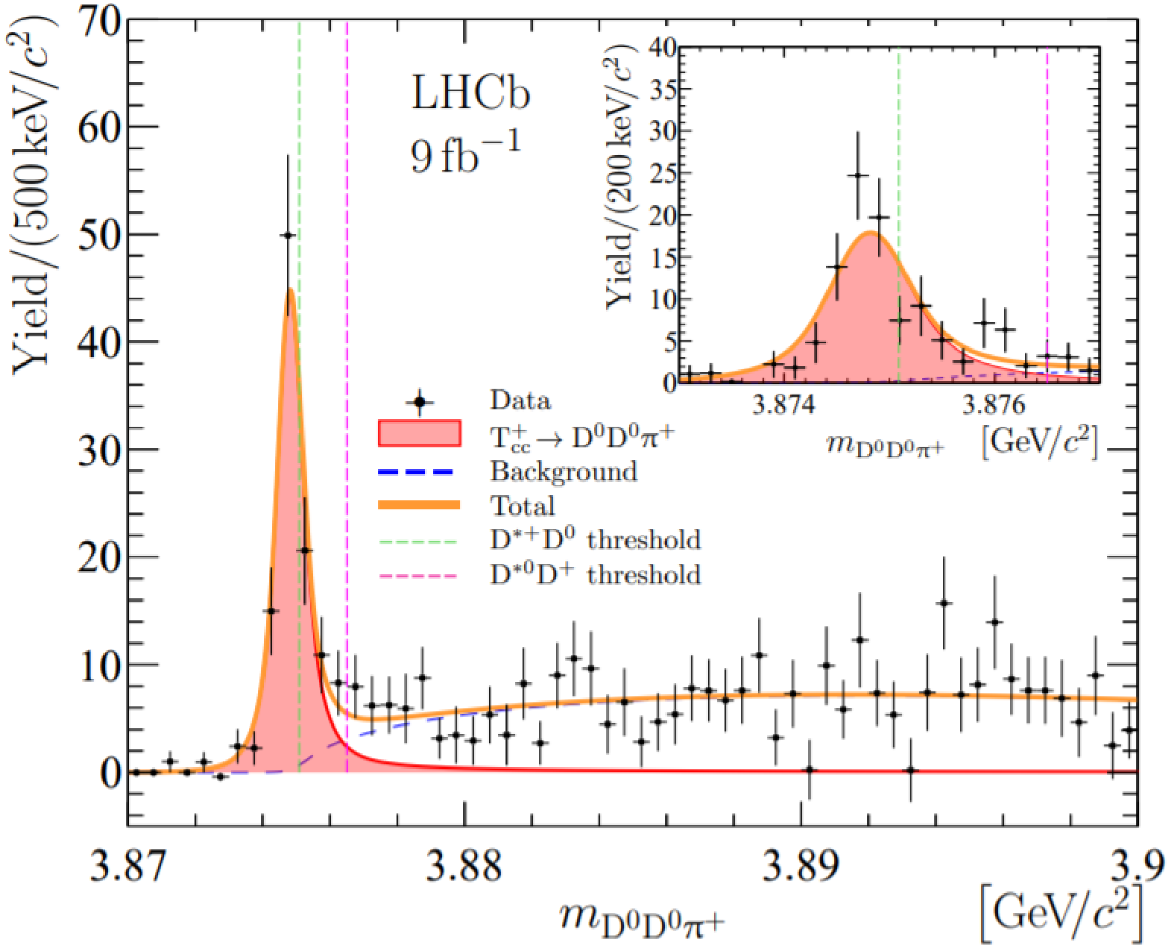
Based on: YL, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, PRL 131, 161901 (2023)



First doubly charmed tetraquark T_{cc}^+

- 2022, LHCb discovered T_{cc}^+ in the $D^0D^0\pi^+$ spectrum

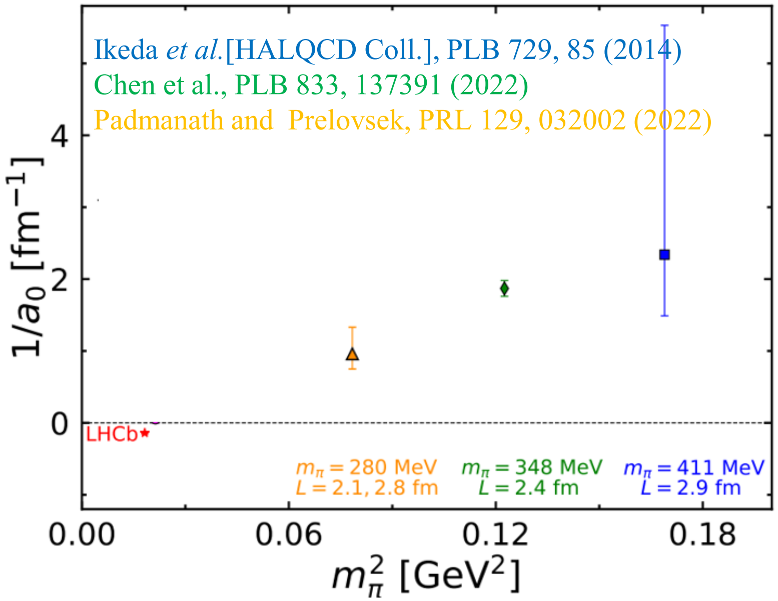
LHCb Coll., Nature Phys. 18, 751 (2022); Nature Comm. 13, 3351 (2022)

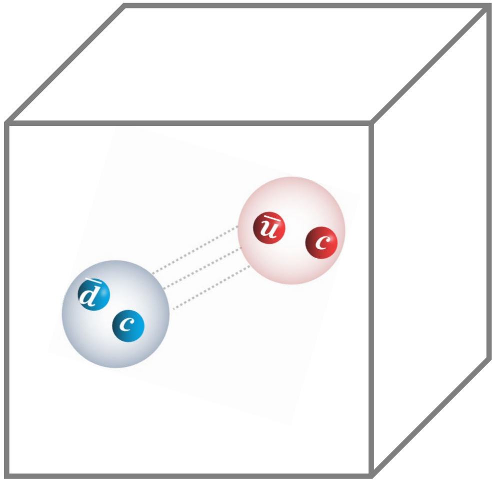


$I(J^P)$	δm_{pole}	Γ_{pole}	$\text{Re}(a_0)$	$\text{Im}(a_0)$
$0(1^+)$	-360±40 keV	48±2 keV	7.16 ± 0.51 fm	-1.85 ± 0.28 fm

T_{cc}^+ from first-principle lattice QCD

- Limited to heavy quark masses ($m_\pi \geq 280$ MeV)





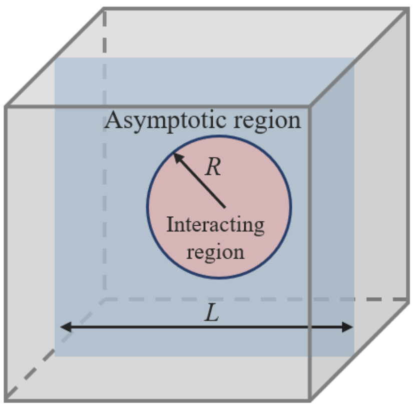
- DD^*-D^*D^* coupled scattering @ m_π =391MeV (Christopher Thomas's talk)
  T. Whyte, D. Wilson, T. Thomas (HadSpec), 2405.15741
- Charm quark dependence @ $m_\pi = 280$ MeV
  S. Collins, A. Nefediev, M. Padmanath, and S. Prelovsek, Phys. Rev. D 109, 094509 (2024)

- Purpose of this talk

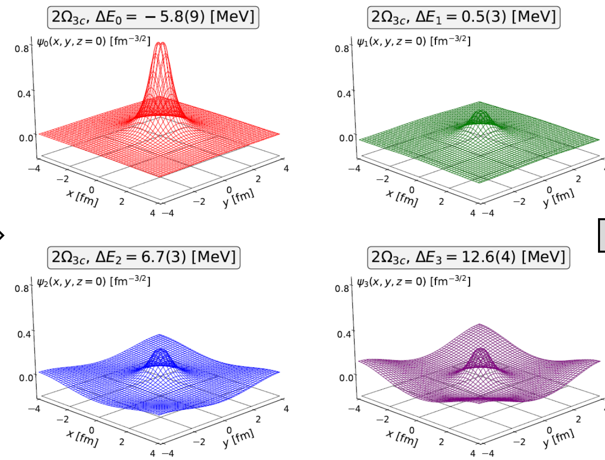
  1. present lattice results with (nearly) physical quark masses
  2. directly compare theoretical and experimental $DD\pi$ mass spectrum

HAL QCD method

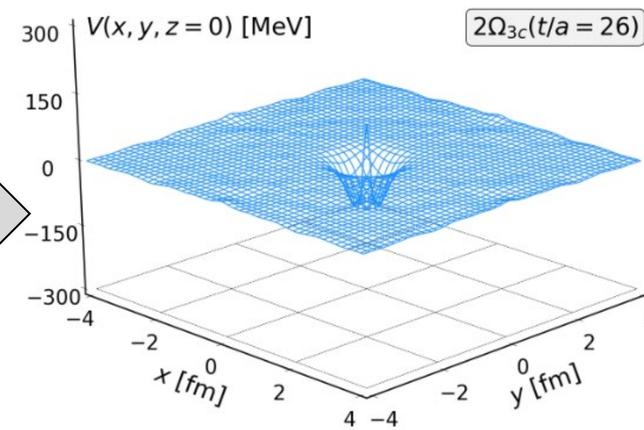
N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)



Lattice simulation



NBS amplitude



Potential

$$\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\boldsymbol{r}, t) = \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\boldsymbol{r}', t)$$

$$R(\boldsymbol{r}, t) = \sum_n a_n \psi_n(\boldsymbol{r}) e^{-\Delta E_n t}$$

(2+1) flavor lattice QCD at m_π =146 MeV

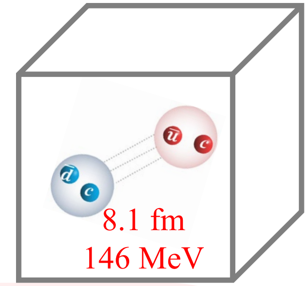
[MeV]

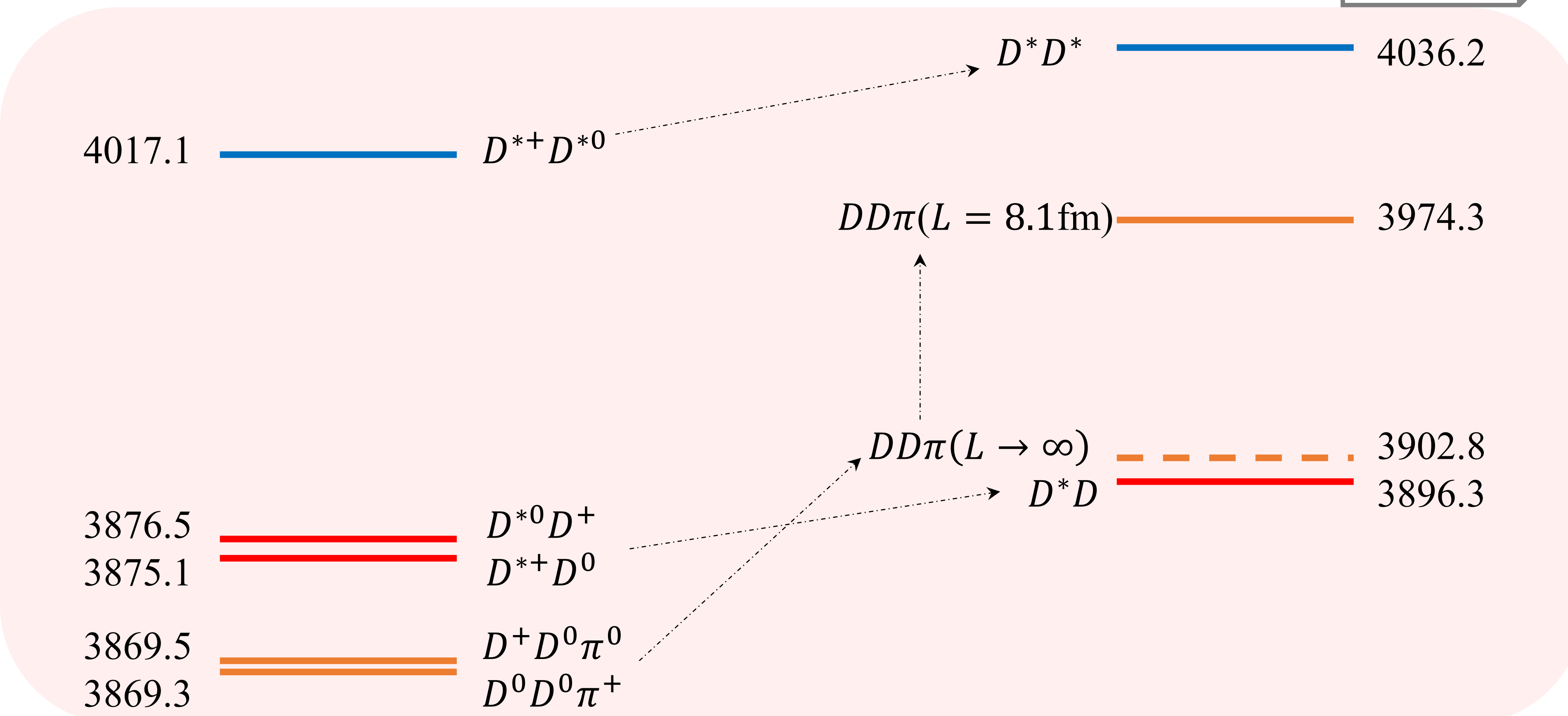
Nature

$\pi^0(134.98)$ $\pi^+(139.57)$
$D^0(1864.84)$ $D^+(1869.66)$
$D^{*0}(2006.85)$ $D^{*+}(2010.26)$

Lattice

$\pi(146.4)$
$D(1878.2)$
$D^*(2018.1)$

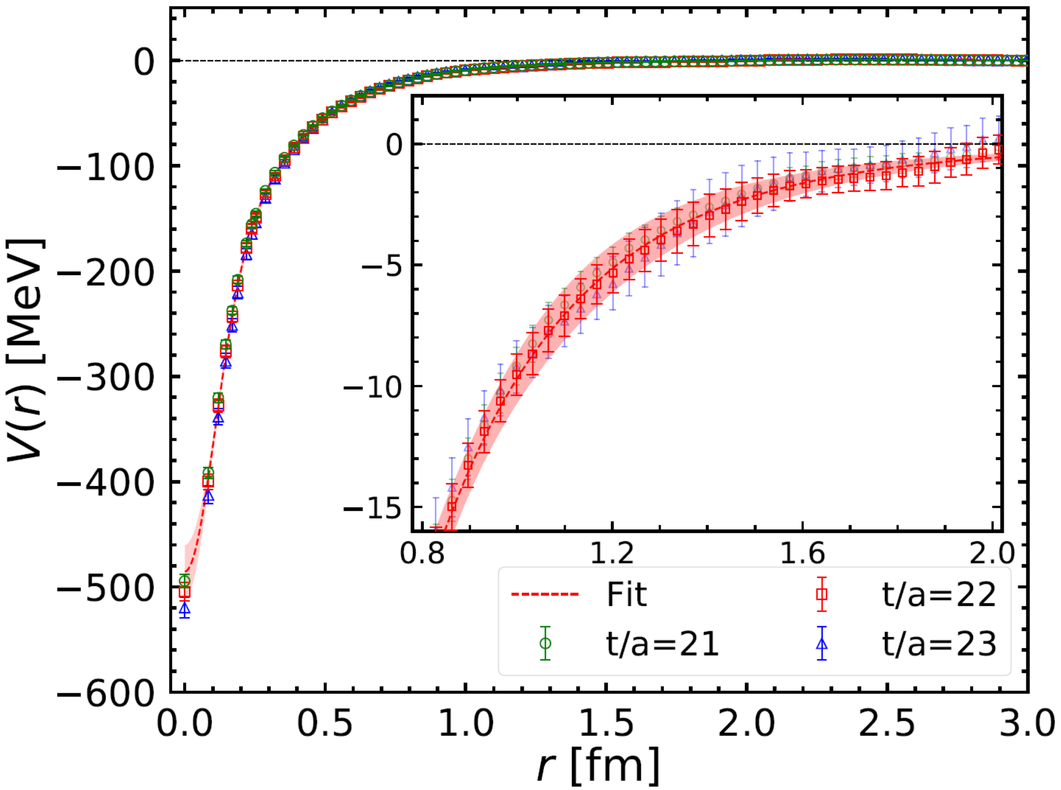




- The lowest energy level of $DD\pi$ (D^*D^*) is around 78 (140) MeV above on the lattice

D^*D interaction

- D^*D potential in the $(I, J^P) = (0, 1^+)$ channel



- Short range: antidiquark-diquark $[\bar{u}\bar{d}]_{3_c, I=J=0}[cc]_{\bar{3}_c, J=1}$

M. Karliner and H. Lipkin, arXiv: 0307243

R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91 232003 (2003)

- Long range: attraction from pion-exchange interaction

Long-range potential

- One-pion exchange

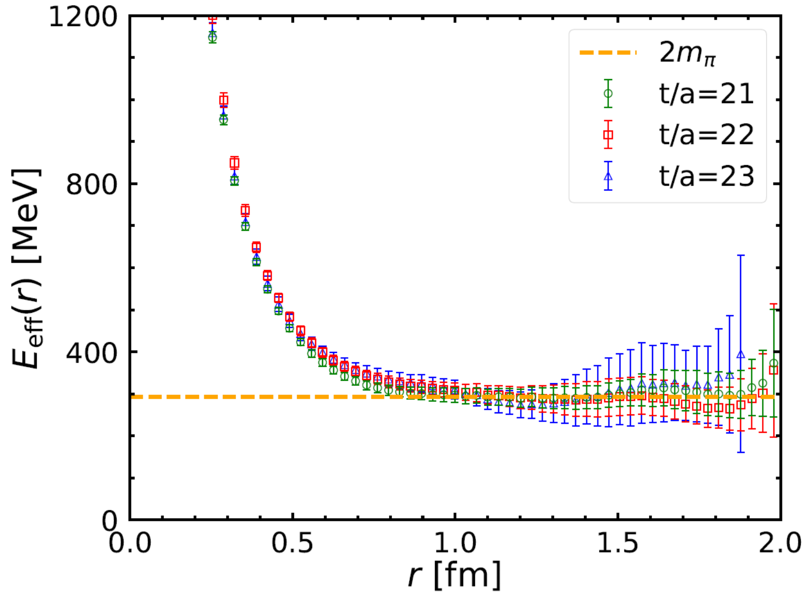
S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D 86, 034019 (2012)
Ning Li, Zhi-Feng Sun, Xiang Liu, and Shi-Lin Zhu, Phys. Rev. D 88, 114008 (2013)

$$V(r) = -\alpha \frac{e^{-\mu r}}{r}, \quad \mu = m_\pi \text{ or } \sqrt{m_\pi^2 - (m_{D^*} - m_D)^2}$$

  - Fail to describe long-range potential (why?)

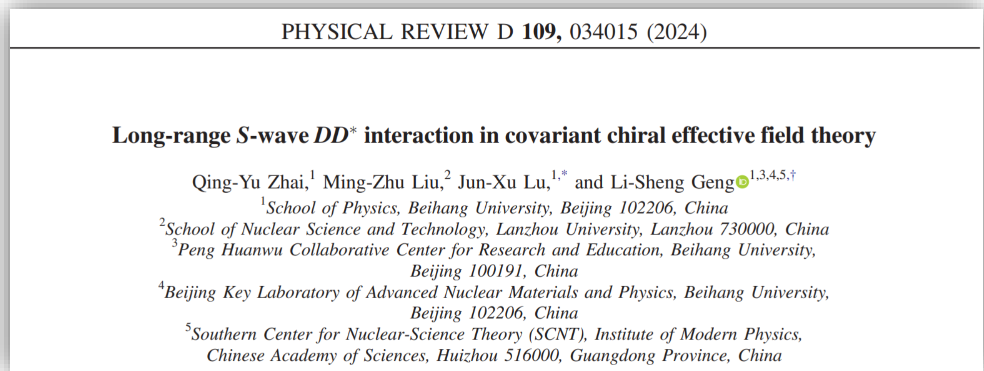
- Two-pion exchange

$$V(r) = -\alpha \frac{e^{-2m_\pi r}}{r^2} \Rightarrow E_{\text{eff}}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r}$$



  - Long range potential ($1 \leq r \leq 2$ fm) is consistent with two-pion exchange

An explanation based on covariant chiral EFT


PHYSICAL REVIEW D **109**, 034015 (2024)

Long-range S-wave DD^* interaction in covariant chiral effective field theory

Qing-Yu Zhai,[1] Ming-Zhu Liu,[2] Jun-Xu Lu,[1,*] and Li-Sheng Geng[1,3,4,5,†]

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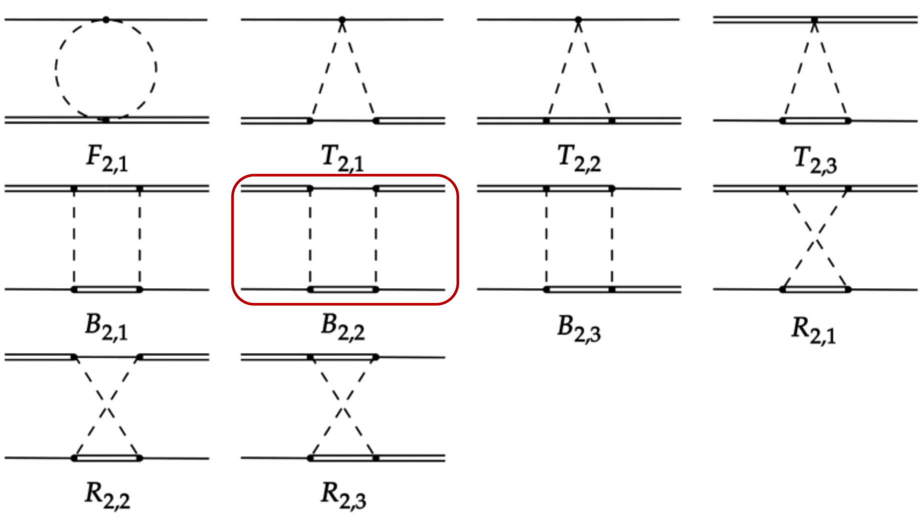
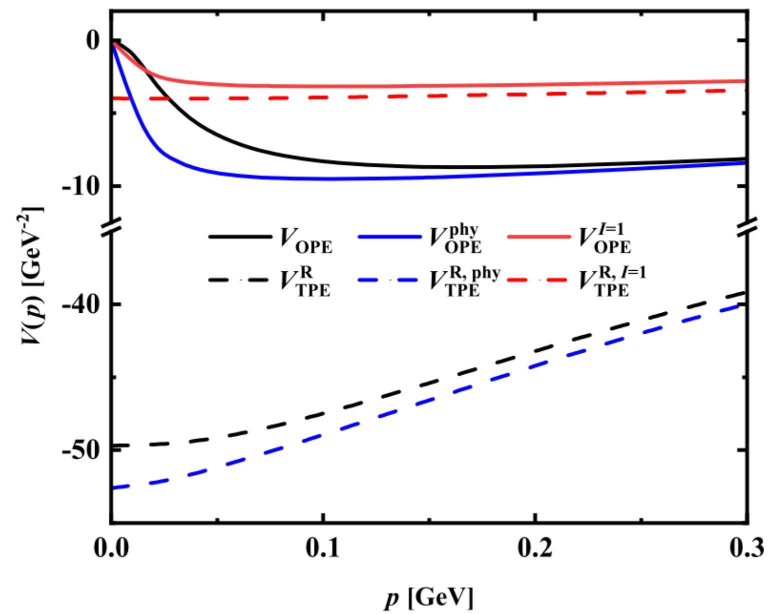
[2]*School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*
[3]*Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China*
[4]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[5]*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China*

- TPE diagrams @NLO



- TPE vs OPE



- Box diagram $B_{2,2}$ play a dominate role due to 4 propagators are almost on-shell
- TPE is much strong than OPE around $p \simeq 0$

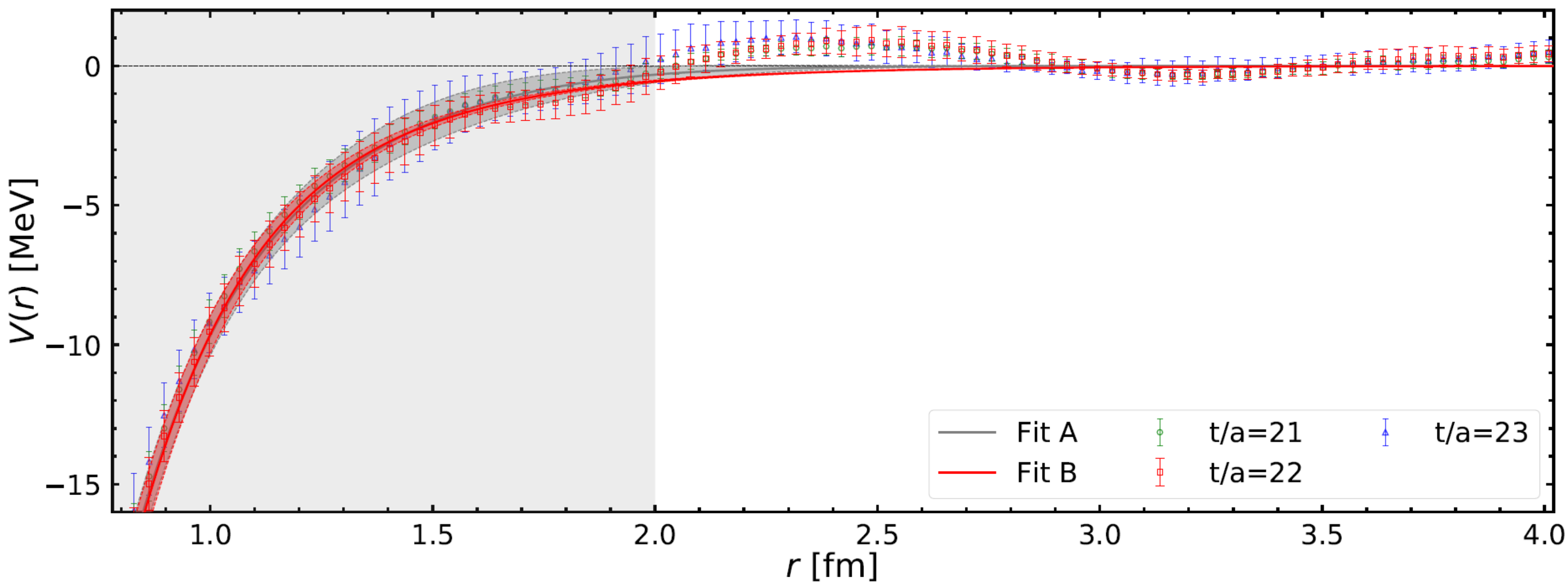
Fit

- Fit A: purely phenomenological fit ($\chi^2/\text{dof} = 1.01$)

$$V_{\text{fit}}(r) = \sum_{i=1,\cdots,4} a_i e^{-(r/b_i)^2}$$

- Fit B: TPE-motivated fit ($\chi^2/\text{dof} = 0.96$)

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3(1 - e^{-(r/b_3)^2})^2 \frac{e^{-2m_\pi r}}{r^2}$$

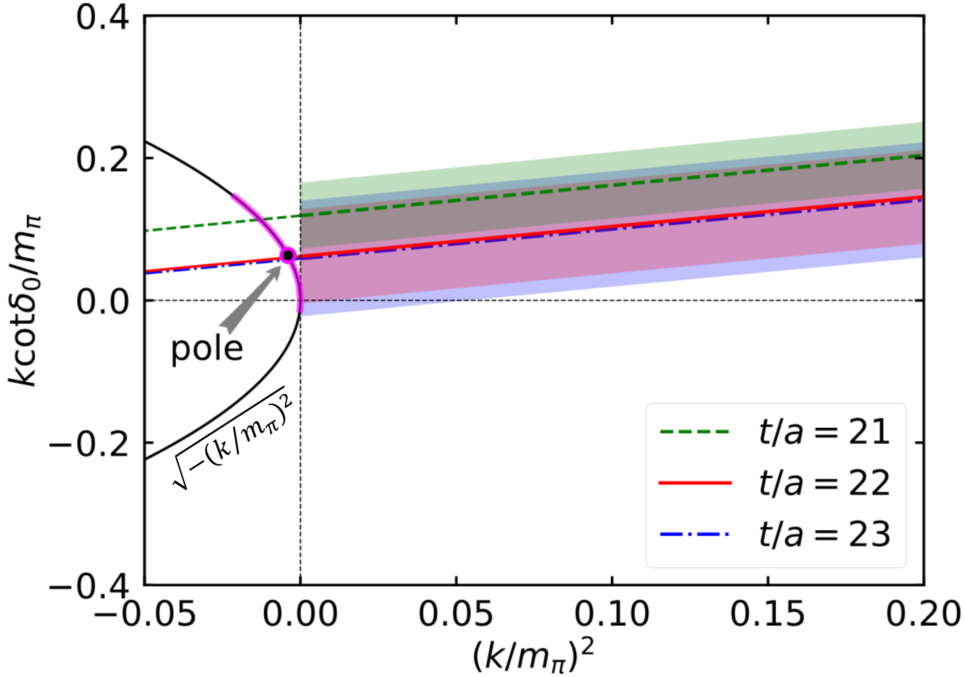


Scattering properties

- Scattering phase shift
  - ERE expansion

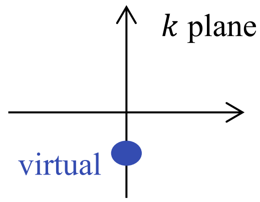
$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2$$

$$S(k) = \frac{k \cot \delta_0 + ik}{k \cot \delta_0 - ik}$$



- Scattering parameters and pole singularities

	m_π (MeV)	146.4
	$1/a_0$ (fm^{-1})	$0.05(5)(^{+2}_{-2})$
	r_{eff} (fm)	$1.12(3)(^{+3}_{-8})$
$k = i\kappa_{\mathrm{pole}}$	κ_{pole} (MeV)	$-8(8)(^{+3}_{-5})$
	E_{pole} (keV)	$-59(^{+53}_{-99})(^{+2}_{-67})$

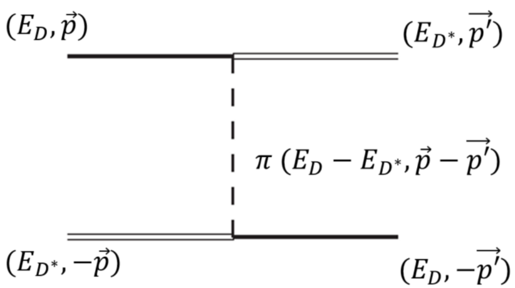


  - T_{cc}^+ appears as a near-threshold virtual state at $m_\pi = 146.4$ MeV

Possible left-hand cut singularity

- Left-hand cut from one-pion exchange

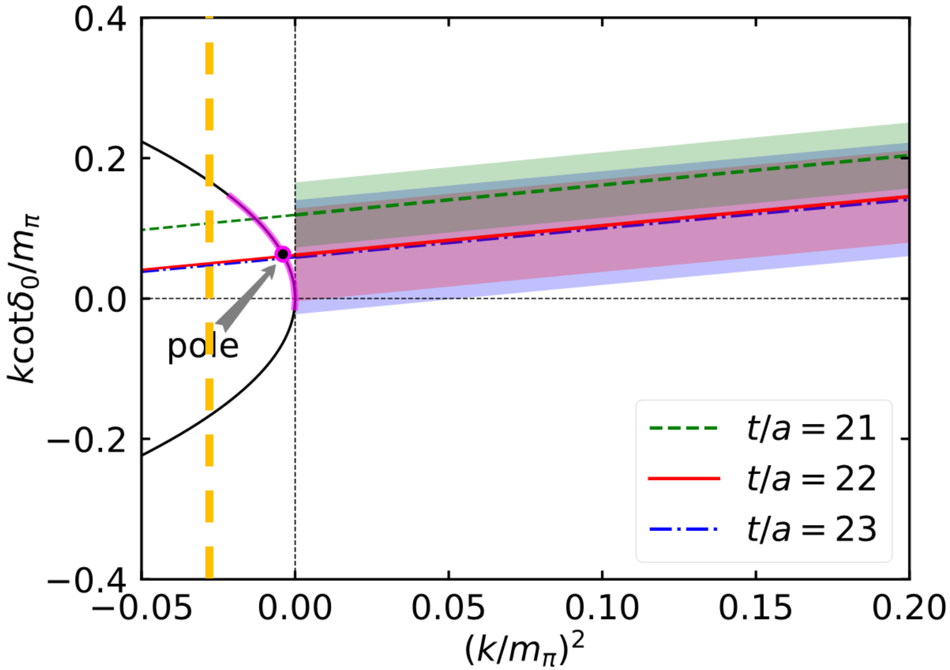
M.-L. Du *et. al.*, Phys. Rev. Lett. 131, 131903 (2023)



$$m_\pi^2 = (E_D - E_{D^*})^2 - (\vec{p} - \vec{p'})^2$$
$$\simeq (\Delta M)^2 - 2p^2(1 - \cos\theta), \quad \Delta M = m_{D^*} - m_D$$
$$p_{1\pi}^2 = \frac{(\Delta M)^2 - m_\pi^2}{2(1 - \cos\theta)} \leq \frac{(\Delta M)^2 - m_\pi^2}{4}$$

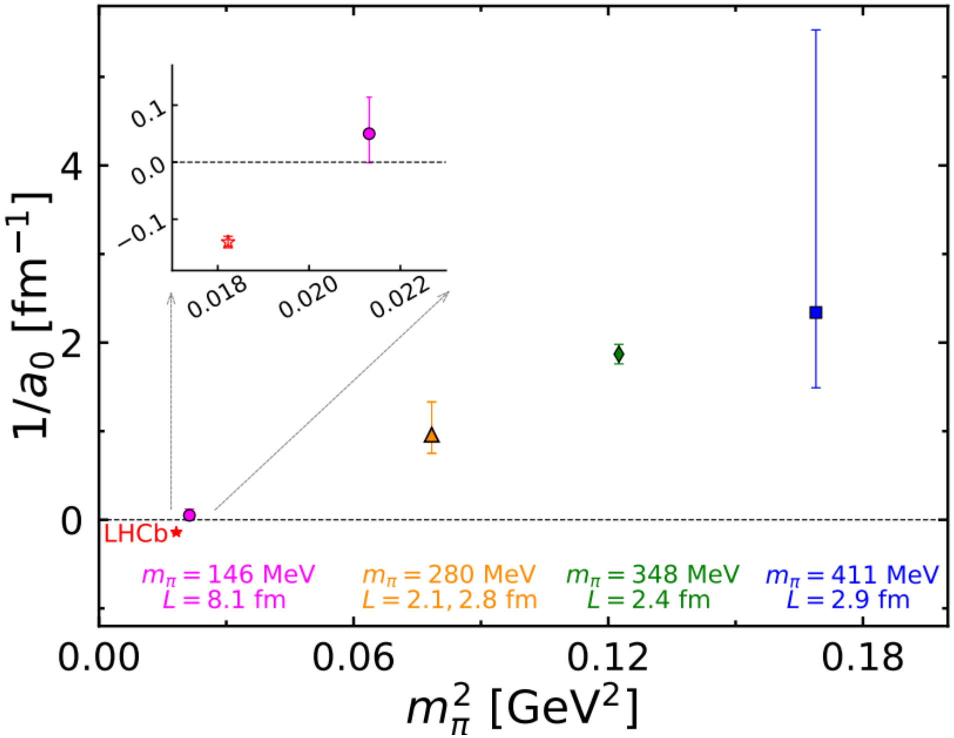
→ Lu Meng 14:00

- Our analysis is safe as the pole is above the branch point of left-hand cut



Comparison

➢ $1/a_0$



Ikeda *et al.*[HALQCD Coll.], Phys. Lett. B 729, 85 (2014)
Chen et al., Phys. Lett. B 833, 137391 (2022)
Padmanath and Prelovsek, Phys. Rev. Lett. 129, 032002 (2022)

- $1/a_0$ from current study with $m_\pi = 146$ MeV is extremely close to LHCb data
- As m_π decreases, LQCD results approach to the LHCb data

Extrapolate to physical point based on TPE

➢ Extrapolation

- Extrapolate TPE interaction to physical point

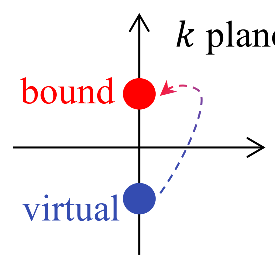
$$V_{\text{fit}}(r; m_\pi = 146 \to 135 \text{ MeV})$$

- Adopt physical values for $m_{D^{*+}}$ and m_{D^0}

- Do NOT consider isospin breaking nor opening of $DD\pi$ channel

➢ Scattering parameters and pole singularities

m_π (MeV)	146.4	135.0
$1/a_0$ (fm^{-1})	$0.05(5)(^{+2}_{-2})$	$-0.03(4)$
r_{eff} (fm)	$1.12(3)(^{+3}_{-8})$	$1.12(3)$
$k = i\kappa_{\text{pole}}$ κ_{pole} (MeV)	$-8(8)(^{+3}_{-5})$	$+5(8)$
E_{pole} (keV)	$-59(^{+53}_{-99})(^{+2}_{-67})$	$-45(^{+41}_{-78})$



- m_π =146 $\to$ 135 MeV, T_{cc}^+ evolves from a near-threshold virtual state into a loosely bound state

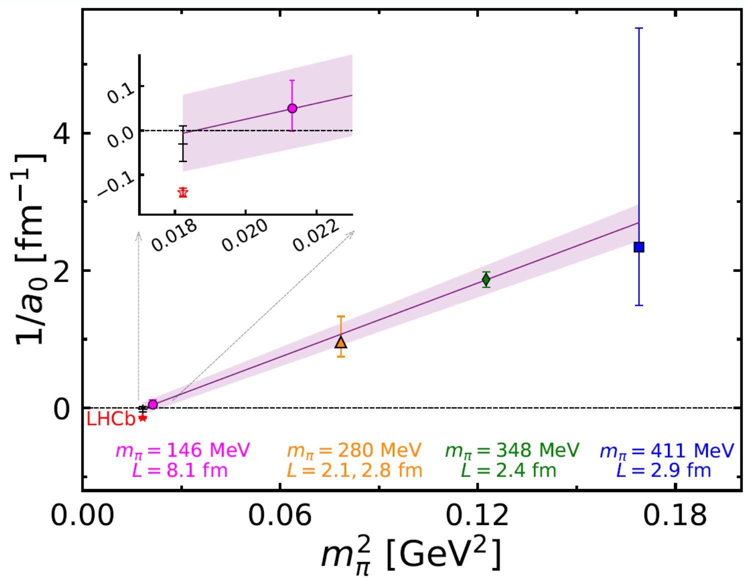
Extrapolation to physical point based on a_0

- Extrapolation
  - A linear fit to four $1/a_0$s from different m_π

$$1/a_0(m_\pi) = c + dm_\pi^2$$

  - Four data from different calculations and posses different systematics

- $1/a_0$



c [fm^{-1}]	d	χ^2/d.o.f.	$1/a_0$ [fm^{-1}]
$-0.33(6)$	$18(1)$ GeV$^{-2}\cdot$ fm^{-1}	0.1	$-0.01(9)$

- $1/a_0$ from two extrapolations are consistent with each other

Construction of $D^0 D^0 \pi^+$ spetrum

- Production amplitude of $D^{*+} D^0$ from a source function P

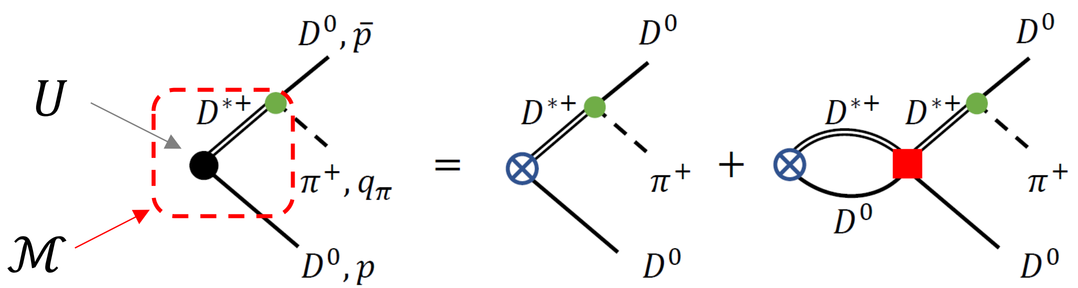
$$U(M,p) = P + \int \frac{d^3q}{(2\pi)^3} T(M,p,q) G(M,q) P$$

PHYSICAL REVIEW D **105,** 014024 (2022)

Coupled-channel approach to T_{cc}^+ including three-body effects

Meng-Lin Du,[1,*] Vadim Baru,[2,3,†] Xiang-Kun Dong,[4,5,‡] Arseniy Filin,[2] Feng-Kun Guo,[4,5,§] Christoph Hanhart,[6,‖] Alexey Nefediev,[7,8,¶] Juan Nieves,[1,**] and Qian Wang[9,10,11,††]

  - For simplicity, consider a pointlike source (constant in p-space, $P = \mathcal{N}$)
  - Only S-wave production at low energies



- Three-body mass spectrum for $D^0 D^0 \pi^+$

$$\mathcal{M}(U \to D^0 D^0 \pi^+) = U(M,p) G(M,p) q_\pi + U(M,\bar{p}) G(M,\bar{p}) \bar{q}_\pi$$

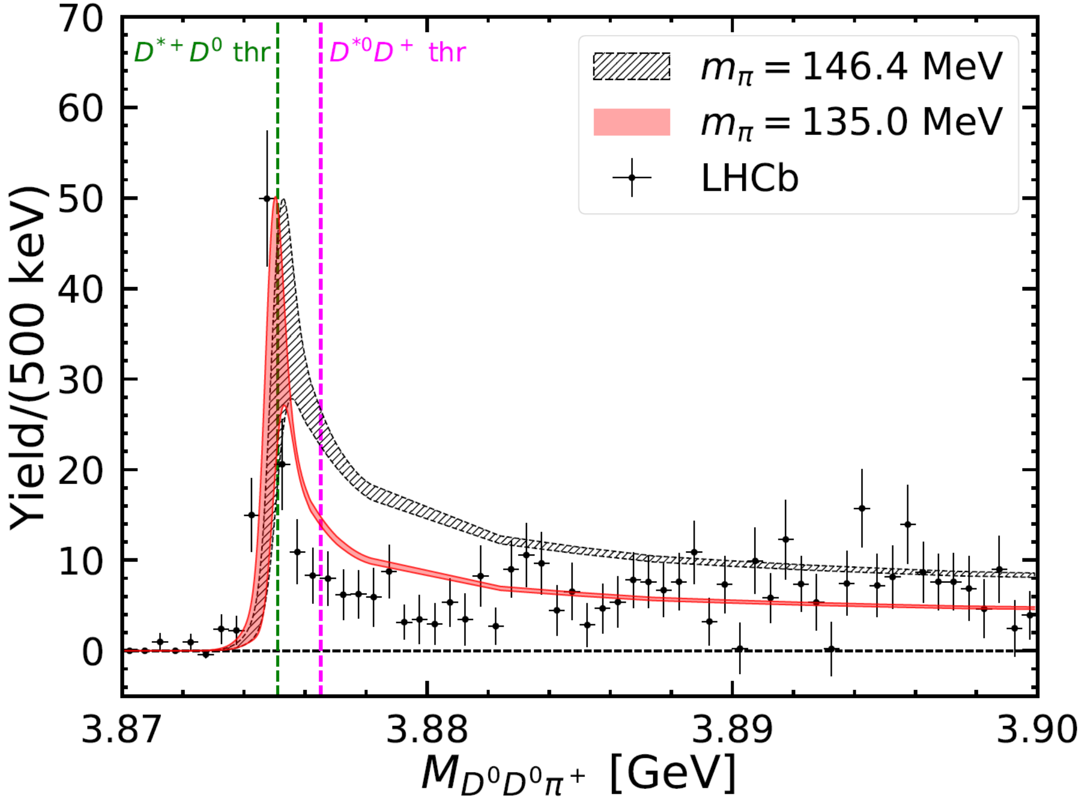
$$\frac{d\mathrm{Br}}{dM} = \mathcal{N}' \int_0^{p_{\max}} p dp \int_{\bar{p}_{\min}}^{\bar{p}_{\max}} \bar{p} d\bar{p} |\mathcal{M}(U \to D^0 D^0 \pi^+)|^2$$

  - Adopt experimental values for m_{D^{*+}, D^0, π^+} and $\Gamma_{D^{*+}}$ in the kinematics to keep the same phase space with the experiment
  - A known energy resolution function needs to considered for comparison w/ exp. data

LHCb Coll., Nature Comm. 13, 3351 (2022)

$D^0D^0\pi^+$ spetrum

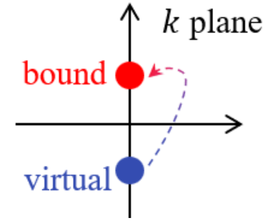
- Results at different m_π



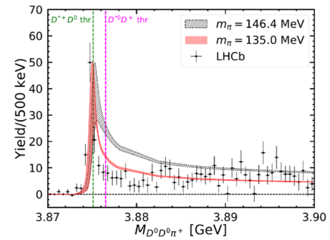
- A peak around $D^{*+}D^0$ threshold
- m_π =146 MeV → 135 MeV, peak position shifts to the left, better description to LHCb data

Summary & Outlook

- What does T_{cc}^{+} look like if m_π^{lat} down to just a few MeV above m_π^{phy}?
  - T_{cc}^{+} appears a near-threshold virtual state
  - T_{cc}^{+} evolves into a loosely bound state as m_π =146→135 MeV



- How far we are from explaining/confirming the experimental results?
  - $1/a_0$ is extremely close to the experimental data
  - LHCb $D^0D^0\pi^+$ spectrum can be explained semiquantitatively



- How to improve our understanding?
  - Physical point simulations, $DD\pi$ coupled channel
  - Dynamic charm quark, isospin breaking, QCD+QED

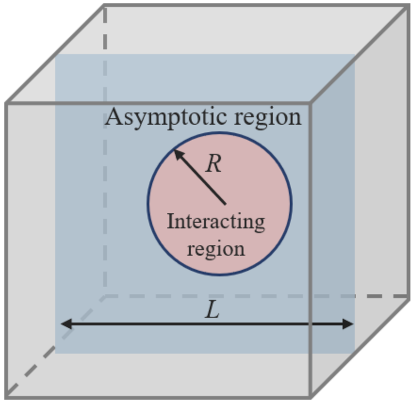
Thanks for your attention!

Fig. courtesy of K. Murano

HAL QCD method

- Nambu-Bethe-Salpeter (NBS) amplitude

$$\psi^k(\boldsymbol{r})e^{-Et} = \langle 0|\hat{D}^*(\boldsymbol{r},t)\hat{D}(\boldsymbol{0},t)|D^*(\boldsymbol{k})D(-\boldsymbol{k});E\rangle$$



  - Asymptotic region: $\psi^k(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$
  - Interacting region: define potential

$$(\nabla^2 + k^2)\psi^k(\boldsymbol{r}) = 2\mu\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')$$

- R correlator (superposition of NBS amplitudes) $R(\boldsymbol{r},t) = \sum_n a_n \psi_n(\boldsymbol{r})e^{-\Delta E_n t}$

$$\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right)R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)$$

  - Derivative expansion: $U(\boldsymbol{r},\boldsymbol{r}') = \sum V_i(\boldsymbol{r})\,\nabla^i\delta(\boldsymbol{r}-\boldsymbol{r}')$

$$V(r) = R(\boldsymbol{r},t)^{-1}\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right)R(\boldsymbol{r},t)$$

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)

Lattice setup

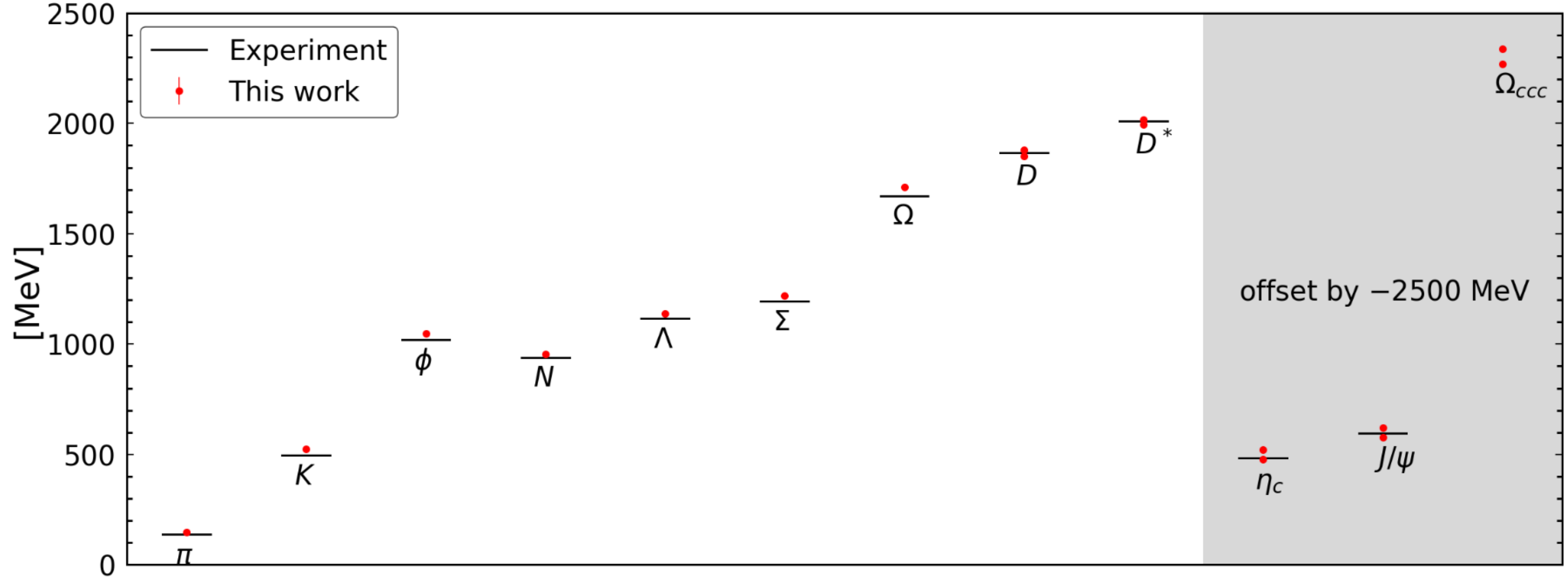
- (2+1)-flavor configuration
  - Iwasaki gauge action
  - $O(a)$-improved Wilson quark action for uds quark
  - Relativistic heavy quark action for c quark

K.-I. Ishikawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2015 075 (2016)
Y. Namekawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2016 125 (2017)

Fugaku supercomputer (440 PFlops)

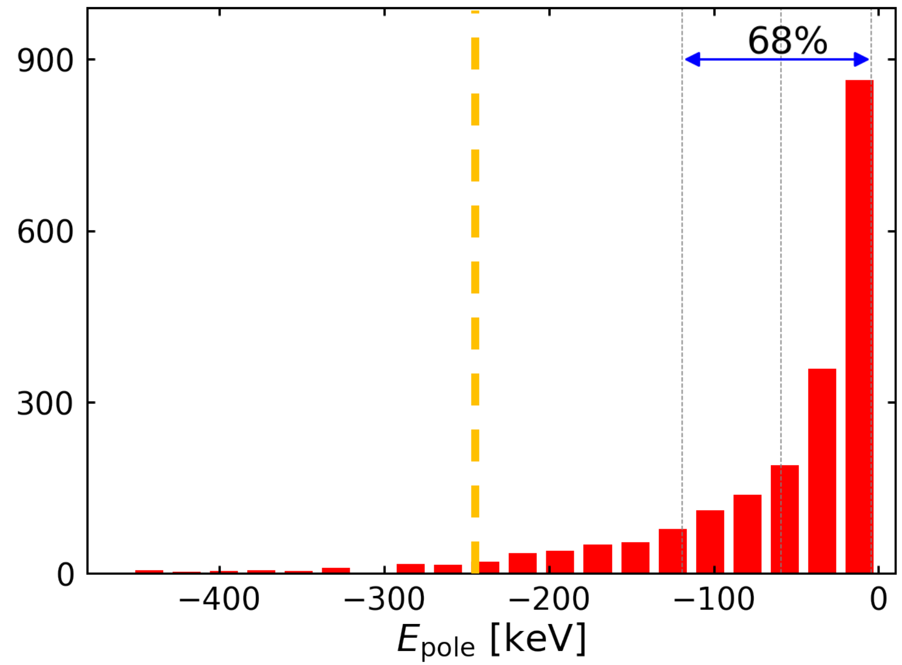
$L^3 \times T$	a [fm]	La [fm]	m_π [MeV]	m_K [MeV]
$96^3 \times 96$	0.0846	8.1	146	525

- Hadron mass



Possible left-hand cut effect on our results

- Our pole position is far from possible singular region



  - $E_{1\pi}^{\rm lhc} = -240$ keV is well below our virtual pole position with a probability of 97%

- The pole positions from solving LS Eq. for T matrix and ERE are same

ERE	***T* matrix**
$E_{\rm pole} = -59(^{+52}_{-99})$ keV	$E_{\rm pole} = -59(^{+53}_{-99})$ keV

  - Our pole position is free from possible left-hand cut issue