Electromagnetic Properties and Hadron Structure

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CONTENTS

Introduction

Electromagnetic interaction plays important roles in hadron physics

- 1933, Otto Stern measured the magnetic moment of proton;

  $\Longrightarrow$ proton is not a simple point-like particle.

  1960s, deep inelastic scattering of electrons on protons and bound neutrons were measured at SLAC;

  $\Longrightarrow$ proton has more complicated structure.

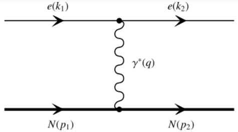
  ……

- Electromagnetic properties are
  - tangled with hadron structure and hadron interaction;
  - important for disclosing the structures of hadrons;
  - able to use perturbation theory

    with small and state-of-the-art electromagnetic couplings.

Electromagnetic form factors

- Electromagnetic form factors can be extracted in both spacelike and timelike regions from the following well-known experiments.



One-photon exchange diagram for elastic scattering, $e + N \to e + N$

116 A. Denig, G. Salmè / Progress in Particle and Nuclear Physics 68 (2013) 113–157

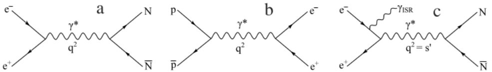


Fig. 1. Diagrammatic representation of the experimental processes used for the measurement of timelike nucleon FF's: (a) e^+e^- annihilation scan experiments; (b) $p\bar{p}$ annihilation; (c) the initial state radiation technique at e^+e^- colliders; in all cases the form factor is measured as a function of the square four-momentum transfer and q^2 of the virtual photon coupling to the baryon pair.

- They are defined by

$$\langle N|J^\mu|N\rangle = \bar{u}(p_2)\,[F_1(q^2)\gamma^\mu - F_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2M}]\,u(p_1) \qquad \text{for nucleon.}$$

  One can obtain magnetic moments, electric radius, and so on with them.
- They help disclose the resonance structures and nonperturbative strong interactions.

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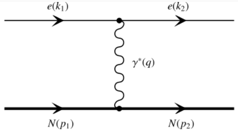

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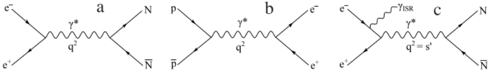

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One can obtain magnetic moments, electric radius, and so on with them.

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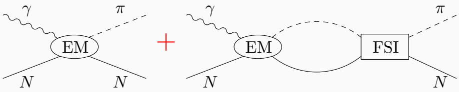
There are more other processes in hadron physics involving electromagnetic interaction.

Pion photoproduction off nucleon with Hamiltonian effective field theory

$\gamma + N \to \pi + N$



$$\mathcal{M}(\gamma N \to \pi N) \sim \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) \otimes \mathcal{M}^{\mathrm{FSI}}(\pi N \to \pi N) + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \eta N) \otimes \mathcal{M}^{\mathrm{FSI}}(\eta N \to \pi N) + \ldots$$

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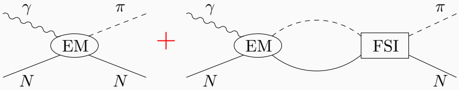

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- γNN etc. couplings are not adjusted.

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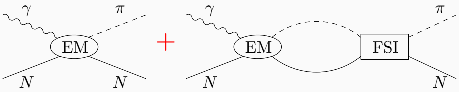

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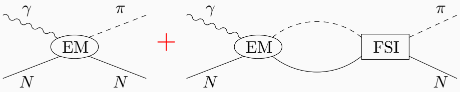

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- It can help understand the structure of nucleon excitations and the interactions of $\pi N/\eta N/\ldots$ at low energies and near the resonance.

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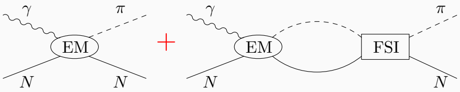

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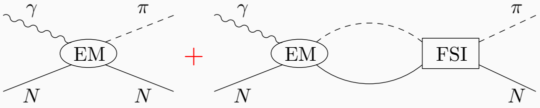
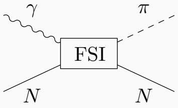
- It can help understand the structure of nucleon excitations and the interactions of $\pi N/\eta N/\ldots$ at low energies and near the resonance.
- It is also the necessities for the photon-nucleus investigation.

The bare triquark core in $N^*(1535)$



- If $N^*(1535)$ has no bare triquark core, it would play roles ONLY in finite state interaction



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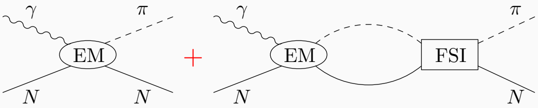

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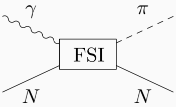
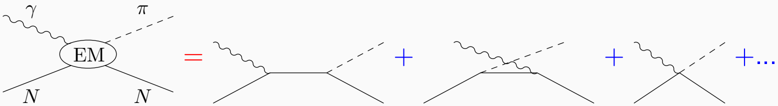


- If with bare core, $N^*(1535)$ also plays roles in electromagnetic potential



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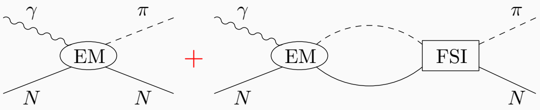

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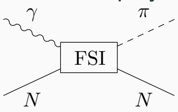

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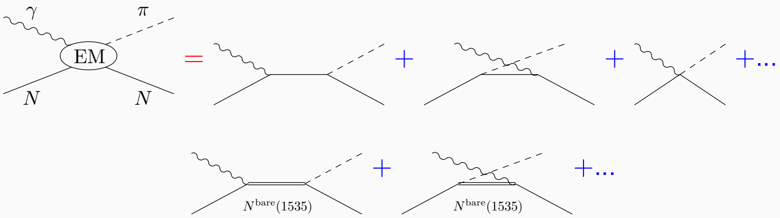
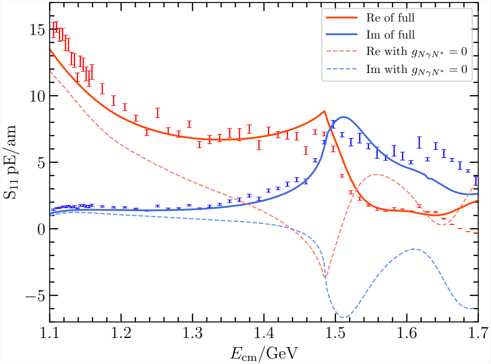


The bare triquark core in $N^*(1535)$ cannot be absent in pion photoproduction



Electric dipole amplitude E_{0+} for $\gamma p \to \pi N$

Y. Zhuge, **Z.-W. Liu**, D. B. Leinweber, A. W. Thomas, arXiv: 2407.05334.

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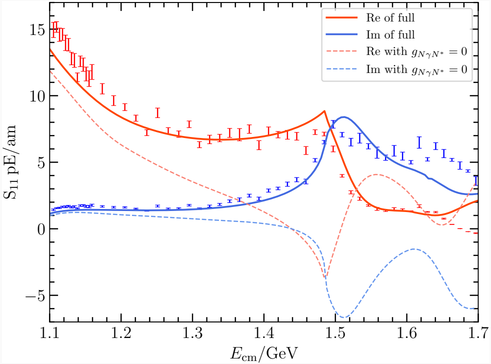

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There are also electroproduction measurements and associated helicity amplitudes which can be used to further analyze the resonances.

Radiative Decays of X(3872) and Molecular Constituents

Structure of the X(3872)

- The X(3872) has been discovered over 20 years.
- The mass is extremely close to the threshold of $D\bar{D}^*$.
- Its structure is still not fully understood.

About theoretical work on $X(3872) \to \gamma J/\psi, \quad \gamma\psi(2S)$,

- the contribution from the $c\bar{c}$ core is relatively clear;
- in the molecule picture,
  the results differ very much with different approaches.

Radiative decays of the X(3872)

$$R_{\gamma\psi} = \frac{\mathcal{B}[X(3872) \to \gamma\psi(2S)]}{\mathcal{B}[X(3872) \to \gamma J/\psi]}$$

has been measured by different collaborations

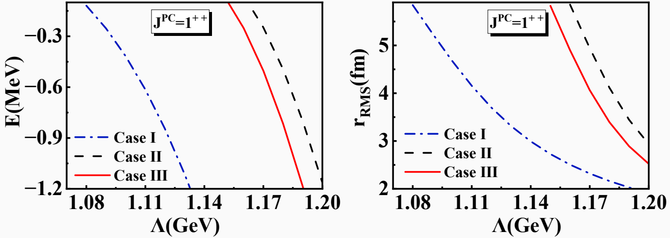
$$R_{\gamma\psi} \begin{cases} = 3.4 \pm 1.4 & [\mathrm{BaBar:2008flx}] \\ = 2.46 \pm 0.64 \pm 0.29 & [\mathrm{LHCb:2014jvf}] \\ < \ 2.1 & [\mathrm{Belle:2011wdj}] \\ = 1.67 \pm 0.21 \pm 0.12 \pm 0.04 & [\mathrm{LHCb:2024tpv}] \\ < \ 0.59 & [\mathrm{BESIII:2020nbj}] \end{cases} .$$

- Ref. [Swanson:2004pp] suggested to measure $R_{\gamma\psi}$ for distinguishing the inner structure.
- Ref. [Guo:2014taa] claimed that the experimental ratio does not contradict the molecule picture.

Effect of Coulomb interaction on the formation of X(3872)

Assuming the X(3872) as the $D\bar{D}^*$ molecule, we solve the binding energy for

- Case I: S-D wave mixing;
- Case II: $+$ isospin breaking from $D^{(*)}$ mass difference;
- Case III: $+$ Coulomb interaction effect.



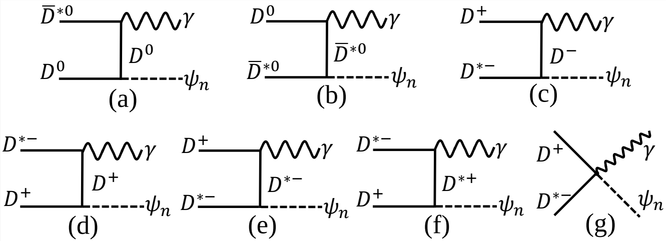
With the promotion of precision of the mass spectrum and corresponding spatial wave function of $D\bar{D}^*$, it makes us reconsider the radiative decay of the X(3872).

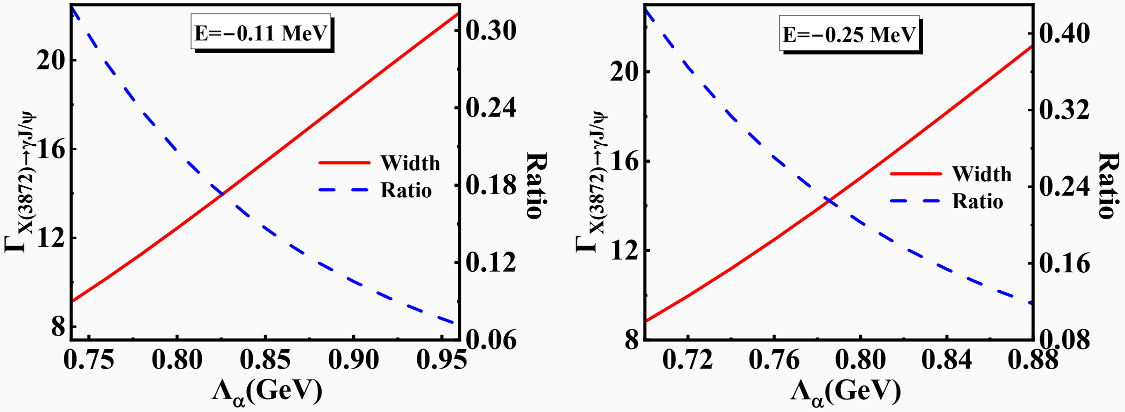
P. Chen, **Z. W. Liu**, Z. L. Zhang, S. Q. Luo, F. L. Wang, J. Z. Wang and X. Liu, Phys. Rev. D 109, no.9, 094002 (2024)

Wave function and radiative decay

We use the explicit wave function of X(3872) to study its radiative decay

$$\mathcal{M}_{X(3872)\to\gamma\psi} \sim \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^{3/2}}\, \hat{\phi}^{X(3872)}_{[D\bar{D}^*]}(\mathbf{p}) \otimes \hat{\mathcal{M}}_{D\bar{D}^*\to\gamma\psi} + \dots$$





The branching ratio $R_{\gamma\psi}$ is less than 1 with pure molecule assumption in our framework, which supports the Belle and BESIII measurements.

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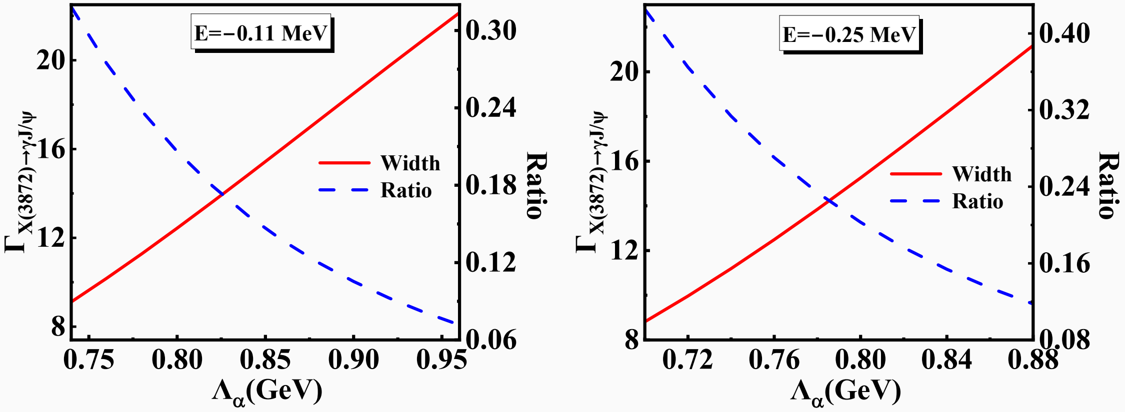

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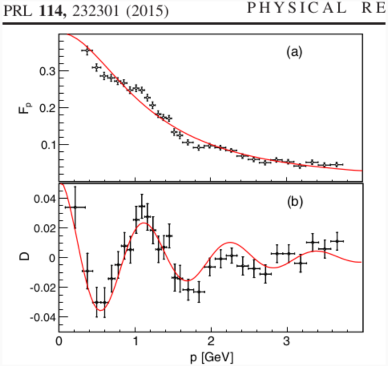
Nucleon Electromagnetic Form Factors and Nucleon Final State Interactions

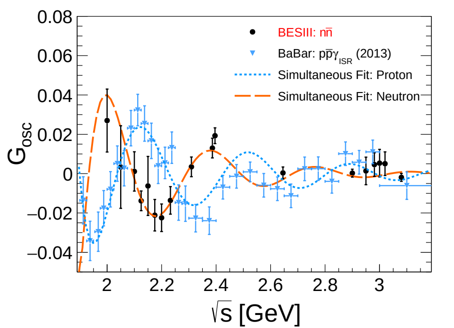
Observation of oscillation in time-like electromagnetic form factors

- Bianconi and Tomasi-Gustafsson first pointed out an unexpected oscillation behavior in the near-threshold region.

- The effective form factors G_{eff} of the nucleons were divided into
  - the main part G^0 describing the main decreasing behavior of the form factor very well
  - the remaining part G^{osc} exhibiting a damped oscillation.



Possible mechanisms



[BESIII:2021tbq, Nature Phys. **17**, 1200]

- vector meson dominance
- cusp effects from coupled channels (baryon-antibaryon channels)
- finite-state interaction
- ...

S. Pacetti et al. / Physics Reports 550–551 (2015) 1–103

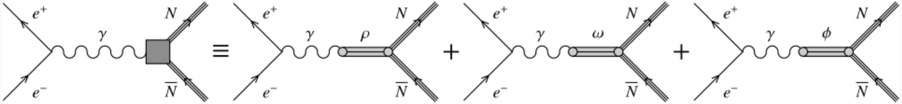


Fig. 15. Feynman diagram of the nucleon FFs in terms of the VMD$_\text{I}$ contributions of the isovector meson ρ and the isoscalar mesons ω and ϕ.

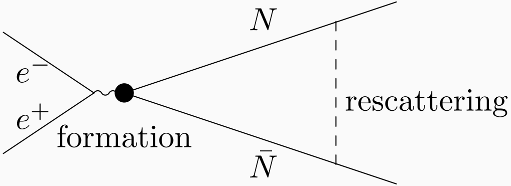
What is mainly focused in our work?

Try to **naturally** explain the **periodicity**.

Separation of formation and rescattering process

- short-range formation process
  - production of $N\bar{N}$
  - $N\bar{N}$ scattering due to annihilation and short-range interaction

  they are strongly tangled since the interaction ranges are similar, $\sim \frac{1}{2m_N}$.



- long-range rescattering process

  the interaction range is $\sim \frac{1}{m_\pi}$

$$\frac{\frac{1}{2m_N}}{\frac{1}{m_\pi}} \approx 14$$

Distorted-wave Born approximation

We can separate the long and short range contributions apart
with classical distorted-wave Born approximation

$$\sigma = \frac{1}{|\mathcal{J}(p)|^2}\sigma_0\,.$$

where Jost function is

$$\mathcal{J}(p) \approx \mathcal{J}_{\ell=0}(p) = \lim_{r\to 0}\frac{j_0(pr)}{\psi_{0,p}(r)}\,.$$

The regular spherical Bessel function $j_0(pr) = \sin(pr)$ is the free radial solution of Schrödinger equation.

With the proper $N\bar{N}$ potential V

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - 2\mu V + p^2\right)\psi_{\ell,p}(r) = 0\,,$$

one can obtain $\psi_{0,p}(r)$.

Reproduction of Sommerfeld factor within our scheme

Sommerfeld factor has been widely used in extracting the form factors of nucleons from the cross sections

$$|G_{\rm eff}(s)| = \sqrt{\frac{3s}{4\pi\alpha^2\beta C(1+2m_N^2/s)}\,\sigma_{e^+e^-\to N\bar{N}}}\,.$$

- It arises from the long-range Coulomb interaction.
- For $e^+e^- \to p\bar{p}$ cross sections

$$C = \left|\frac{1}{\mathcal{S}^2}\right|, \quad \mathcal{S} = \left(\frac{y}{1-e^{-y}}\right)^{-1/2}, \quad y = \frac{\pi\alpha\sqrt{1-\beta^2}}{\beta}.$$

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We can easily reproduce this famous Sommerfeld factor

- with the formalism on the previous page;
- by substituting V with the Coulomb potential;
- by using the non-relativistic approximation $\sqrt{1-\beta^2} \approx 1$ in the near-threshold region.

A toy model

If the $N\bar{N}$ interaction is a simple square-well potential,

$$V(r) = \begin{cases} -V_a & \text{for} \quad 0 \leqslant r < a \\ 0 & \text{for} \quad r \geqslant a \end{cases},$$

we have

$$\psi_{0,p}(r) = \begin{cases} \dfrac{e^{i\delta_0} \sin(p_{in} r)}{\sqrt{\sin^2(p_{in} a) + \frac{p^2}{p_{in}^2} \cos^2(p_{in} a)}} & \text{for} \quad 0 \leqslant r < a \\ e^{i\delta_0} \sin(pr + \delta_0) & \text{for} \quad r \geqslant a \end{cases},$$

where δ_0 is the S-wave phase shift, and

$$p_{in} = \sqrt{p^2 + 2\mu V_a}.$$

We have the long-range factor $|1/\mathcal{J}|^2 > 1$ for the pure attractive interaction

$$|\mathcal{J}(p)| = \sqrt{\frac{p^2}{p_{in}^2} \sin^2(p_{in} a) + \cos^2(p_{in} a)}\,.$$

The long-range factor with the toy model

$$|G_{\text{eff}}(s)| = \frac{1}{|\mathcal{J}|} G_0(s) , \qquad |\mathcal{J}(p)| = \sqrt{\frac{p^2}{p_{in}^2}\sin^2(p_{in}a) + \cos^2(p_{in}a)} .$$

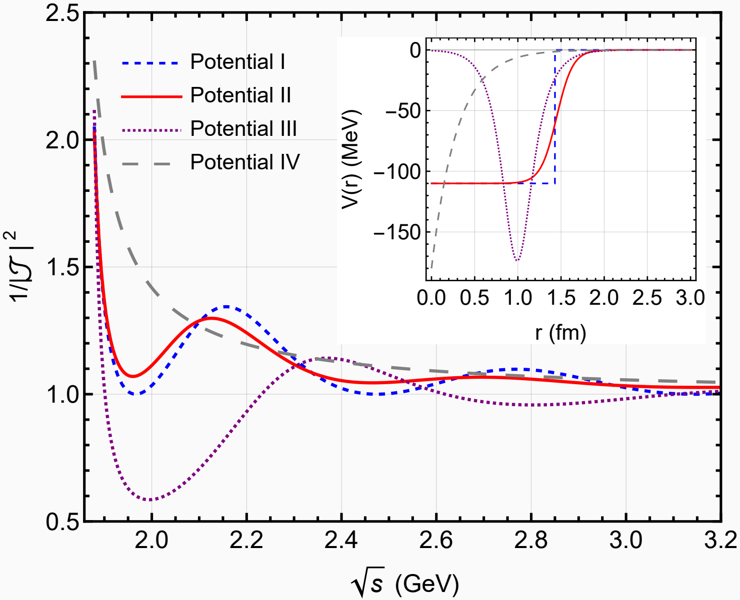
- The energy gaps between the 1st, the 2nd, the 3rd and the 4th minima are:

$$\frac{3\pi^2}{2\mu a^2}, \quad \frac{5\pi^2}{2\mu a^2}, \quad \frac{7\pi^2}{2\mu a^2} .$$

- $\mu = m_N/2$ and the width of square well $a \approx 1/m_\pi$ give the 1st gap $\Delta E_1 \approx 0.6$ GeV. This is close to the value observed in experiment.

- The peaks $|1/\mathcal{J}|^2_{\max} = 1 + 2\mu V_a/p^2$ decrease with the increasing energies.

Different potentials and long-range factors



Description for the effective form factors

To fit the main part, we use the following expression from Ref. [BESIII:2019hdp]

$$G_0(s) = \frac{\mathcal{A}}{(1+s/m_a^2)\,[1-s/(0.71\ \mathrm{GeV}^2)]^2}\,,$$

where $m_a^2 = 7.72$ GeV^2, $\mathcal{A}_p = 9.37$ and $\mathcal{A}_n = 5.8$.

For the oscillatory part,

$$G^{osc}(s) = |G_{\mathrm{eff}}| - G_0 = \left(\frac{1}{|\mathcal{J}|} - 1\right) G_0(s)\,.$$

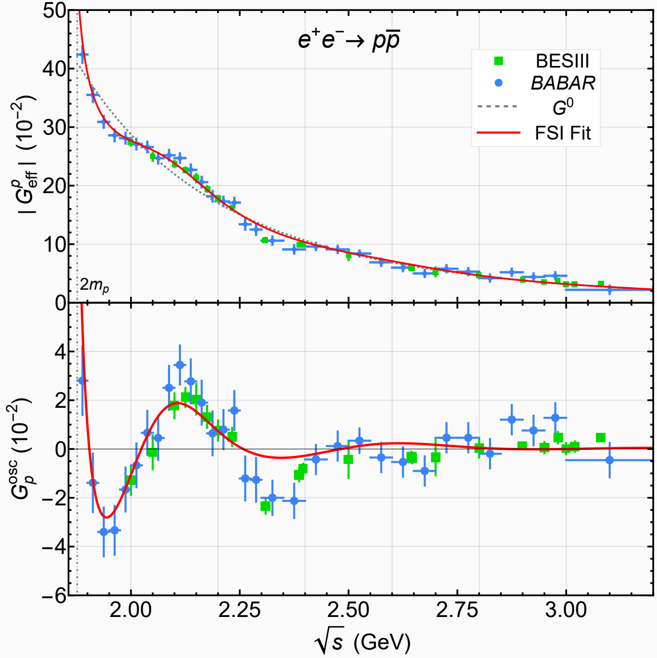
we use the following potential to get the $\mathcal{J}$

$$V(r) = \begin{cases} -V_r & 0 \leqslant r < a_r \\ -V_a & a_r \leqslant r < a \\ 0 & r \geqslant a \end{cases},$$

where $0 < V_r < V_a$ and
we take $a_r = 0.5$ fm.

Table 1: Parameters for $p\bar{p}$ and $n\bar{n}$ potentials.

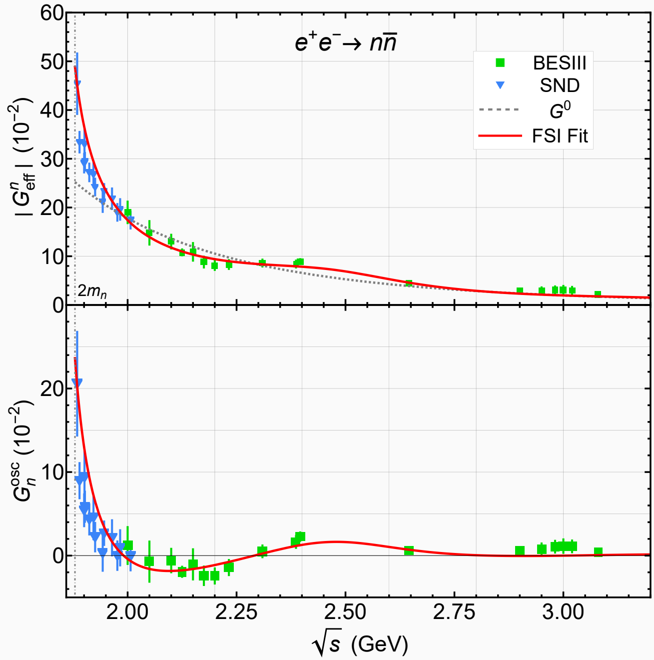
$N\bar{N}$	a_r (fm)	V_r (MeV)	a (fm)	V_a (MeV)
$p\bar{p}$	0.5	50	1.6	90
$n\bar{n}$	0.5	400	1.4	650



← Proton case

- The overall oscillatory behavior is well reproduced by the FSI effect with the interaction range a about $1.4 \sim 1.6$ fm.

R-Q Qian, **Z-W Liu**, X. Cao, X. Liu, Phys. Rev. D 107 (2023) 9, L091502



← Neutron case

R-Q Qian, **Z-W Liu**, X. Cao, X. Liu,

Phys. Rev. D 107 (2023) 9, L091502

Short discussion on the continuum part G_0

The choice of the continuum part G_0

- affects the details of the description of the effective form factors;
- is still not understood very well,

  because the formation process involves complicated hadronization mechanism and other difficulties.

Perhaps one would get better descriptions for $|G_{\mathrm{eff}}|$

by using different G_0 rather than the same as in the experimental article.

Threshold enhancement on cross sections for baryon-antibaryon productions

- The SND measurement observed the enhancement on the neutron cross section just above threshold at $\sqrt{s} - 2m_n \approx 5$ MeV, which contradicts the naive phase space expectation.

  (Ref. [SND:2022wdb])

- Abnormally large cross sections are observed in $e^+e^- \to \Lambda\bar{\Lambda}$ near the threshold $\left(\sqrt{s} - 2m_\Lambda\right) \approx 1$ MeV and possibly $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ at $\left(\sqrt{s} - 2m_{\Lambda_c}\right) \approx 1.58$ MeV.

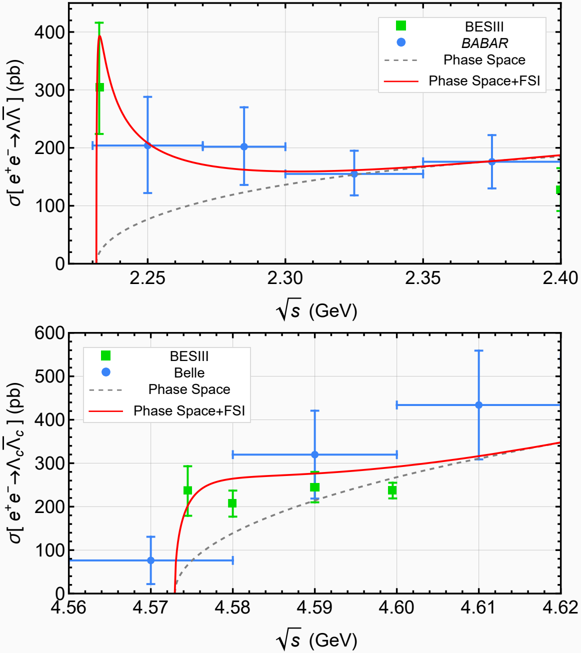
  (Refs.[BESIII:2017hyw,BESIII:2017kqg])

- However, no such phenomenon were found in the $\Xi\bar{\Xi}$ and $\Sigma\bar{\Sigma}$ productions.

  (Refs.[BESIII:2020ktn,BESIII:2021aer,BESIII:2020uqk,BESIII:2021rkn,BESIII:2020uqk])

Our approach can easily provide such an enhancement as seen in the previous figure.

- $1/|\mathcal{J}|_{p\to 0} \to 1/\cos^2\left(\sqrt{2\mu V_a}a\right)$ for an attractive squared-well potential.
- With suitable V_a and a, $1/|\mathcal{J}|_{p\to 0}$ can lead to very large enhancement.


BESIII
BABAR
Phase Space
Phase Space+FSI
$\sigma[e^+e^-\to\Lambda\bar{\Lambda}]$ (pb)
$\sqrt{s}$ (GeV)
0
100
200
300
400
2.25
2.30
2.35
2.40
BESIII
Belle
Phase Space
Phase Space+FSI
$\sigma[e^+e^-\to\Lambda_c\bar{\Lambda}_c]$ (pb)
$\sqrt{s}$ (GeV)
0
100
200
300
400
500
600
4.56
4.57
4.58
4.59
4.60
4.61
4.62

Summary

Summary

We have shown some examples that the electromagnetic properties play important roles in hadron physics.

- About the radiative decay of $X(3872)$,
  - the molecular components contribute to $R_{\gamma\psi}$ around $0.1 \sim 0.4$;
  - this work supports the Belle and BESIII measurements.

- About $e^+e^- \to p\bar{p}, n\bar{n}, \ldots,$
  - we can naturally explain the damped periodicity with the finite state interaction effects which associate with the zero-point wave functions of $p\bar{p}, n\bar{n}$;
  - the threshold enhancement phenomenon can be simultaneously understood with the same mechanism.

Thank you for your attention!