ER, Haxton, & McElvain, PRL **130**, 131901 (2023)
Haxton, **ER**, McElvain, & Ramsey-Musolf, PRC **107**, 035504 (2023)
Haxton, McElvain, Menzo, **ER**, & Zupan, hep-ph/2406.13818

LA-UR-24-28908

Los Alamos NATIONAL LABORATORY

Effective Theory Predictions for $\mu \to e$ Conversion

Evan Rule | XVIth Quark Confinement | August 20, 2024

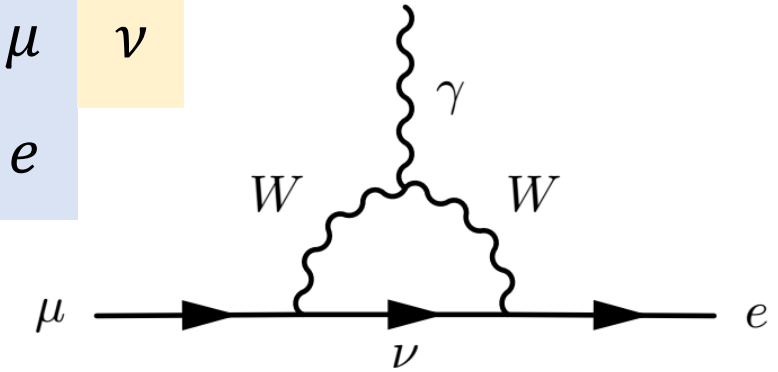
Berkeley UNIVERSITY OF CALIFORNIA

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

NSF

μ ν

e



Theory Predicts: $\mathrm{BR}(\mu^+ \to e^+\gamma) \approx 10^{-4}$

Feinberg, Phys. Rev. **110**, 1482 (1958)

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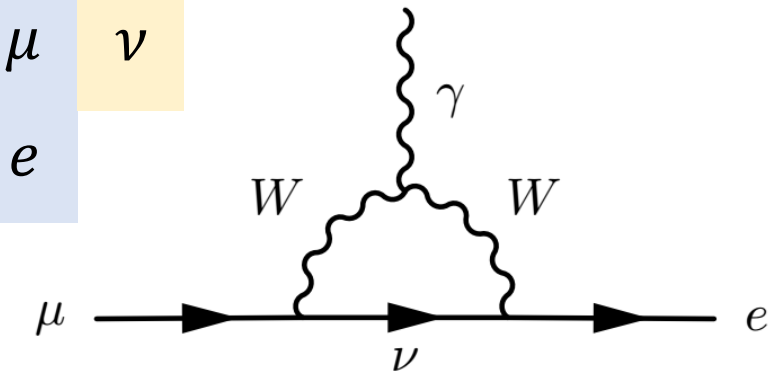
μ ν

e



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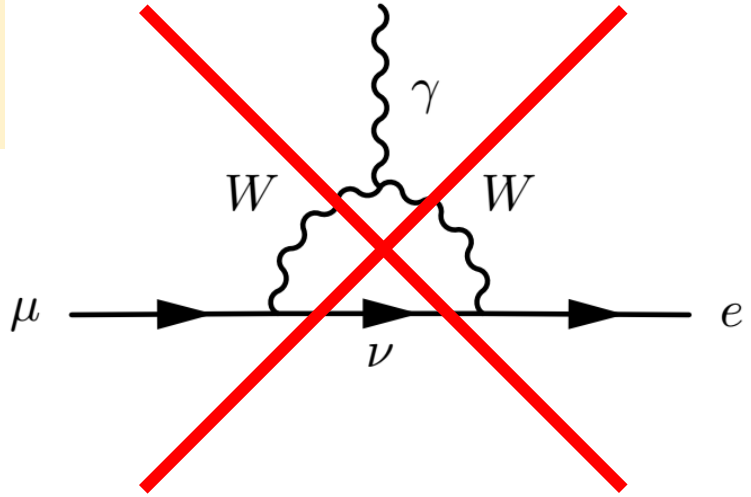
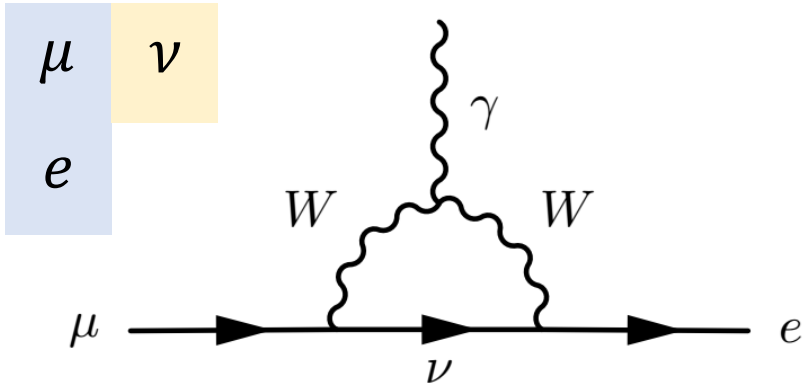
2 Neutrino Hypothesis

Pontecorvo, Zh. Eksp. Teor. Fiz. **37**, 1751–1757 (1959).

μ ν_μ

e ν_e





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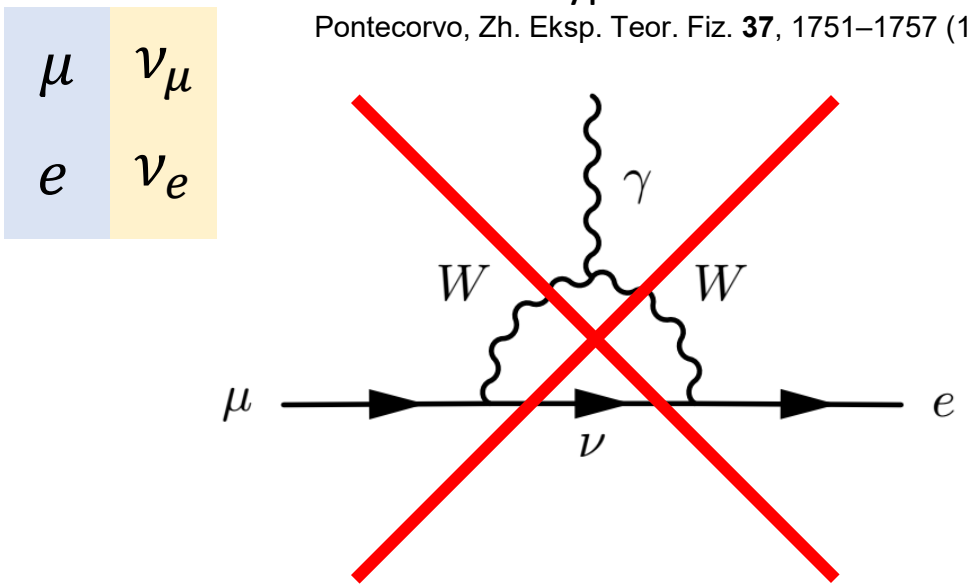
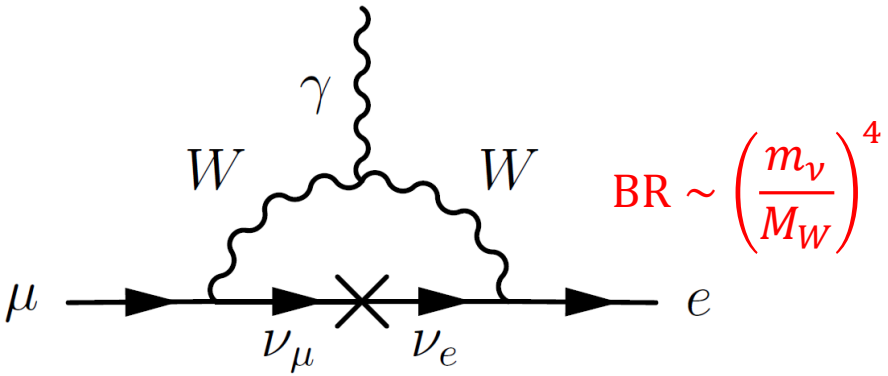


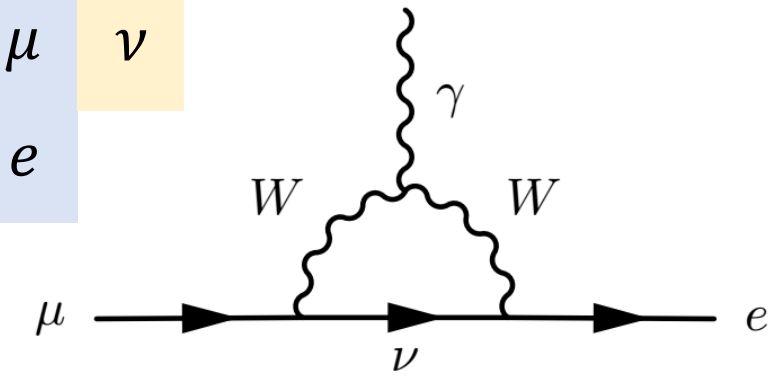


$$\mathrm{BR} \sim \left(\frac{m_\nu}{M_W}\right)^4$$

Super-K, SNO

- Neutrino flavor oscillations → lepton flavor not conserved
- Charged lepton flavor violation (CLFV) extremely suppressed

μ	ν
e	



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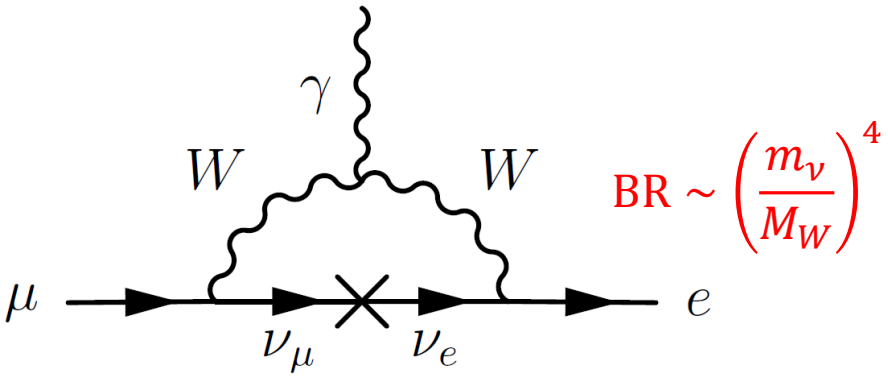

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μ	ν_μ
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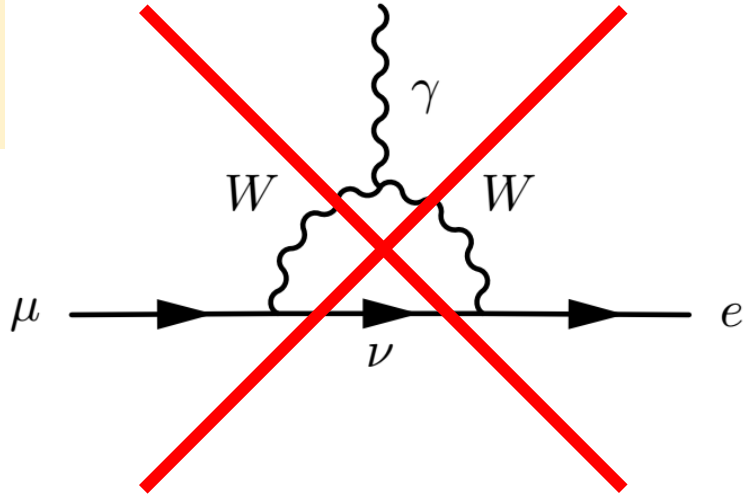
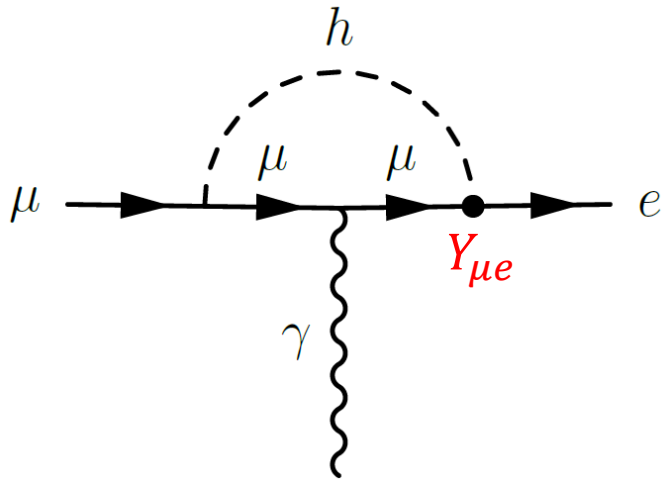




CLFV limits constrain BSM theories

Collider Constraints

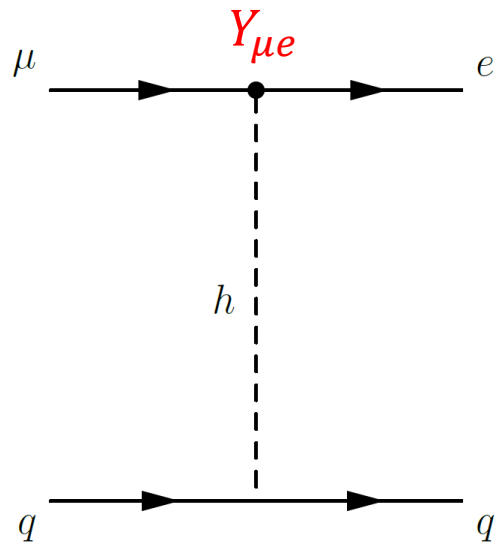
Process	BR Limit	CL	Experiment
$Z \to e\tau$	5.0×10^{-6}	95%	ATLAS
$Z \to \mu\tau$	6.5×10^{-6}	95%	ATLAS
$Z \to e\mu$	7.5×10^{-7}	95%	ATLAS
$h \to e\tau$	2.2×10^{-3}	95%	CMS
$h \to \mu\tau$	1.5×10^{-3}	95%	CMS
$h \to e\mu$	6.1×10^{-5}	95%	ATLAS
$B^+ \to K^+\mu^-\tau^+$	2.8×10^{-5}	90%	BaBar
$B^+ \to K^+e^-\mu^+$	6.4×10^{-9}	90%	LHCb
$D^+ \to K^+e^+\mu^-$	7.5×10^{-8}	90%	LHCb
$\tau^- \to e^-\gamma$	3.3×10^{-8}	90%	BaBar
$\tau^- \to \mu^-\gamma$	4.2×10^{-8}	90%	Belle
$\tau^- \to e^-e^+e^-$	2.7×10^{-8}	90%	Belle

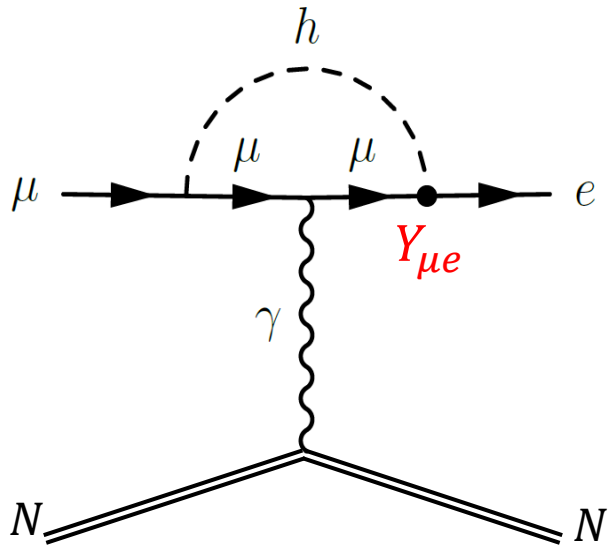
Stopped Muon Constraints

Process	BR Limit	CL	Experiment
$\mu^+ \to e^+\gamma$	4.2×10^{-13}	90%	MEG + MEG II
$\mu^+ \to e^+e^-e^+$	1.0×10^{-12}	90%	SINDRUM
$\mu^- + \mathrm{Cu} \to e^- + \mathrm{Cu}$	1.6×10^{-8}	90%	SINDRUM II
$\mu^- + {}^{32}\mathrm{S} \to e^- + {}^{32}\mathrm{S}$	7×10^{-11}	90%	SIN
$\mu^- + \mathrm{Pb} \to e^- + \mathrm{Pb}$	4.6×10^{-11}	90%	SINDRUM II
$\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$	6.1×10^{-13}	90%	SINDRUM II
$\mu^- + \mathrm{Au} \to e^- + \mathrm{Au}$	7.0×10^{-13}	90%	SINDRUM II
$\mu^+ \to e^+\gamma$	6×10^{-14}	90%	MEG II
$\mu^+ \to e^+e^-e^+$	4×10^{-16}	90%	Mu3e
$\mu^- + \mathrm{Al} \to e^- + \mathrm{Al}$	8×10^{-17}	90%	Mu2e
$\mu^- + \mathrm{Al} \to e^- + \mathrm{Al}$	7×10^{-17}	90%	COMET

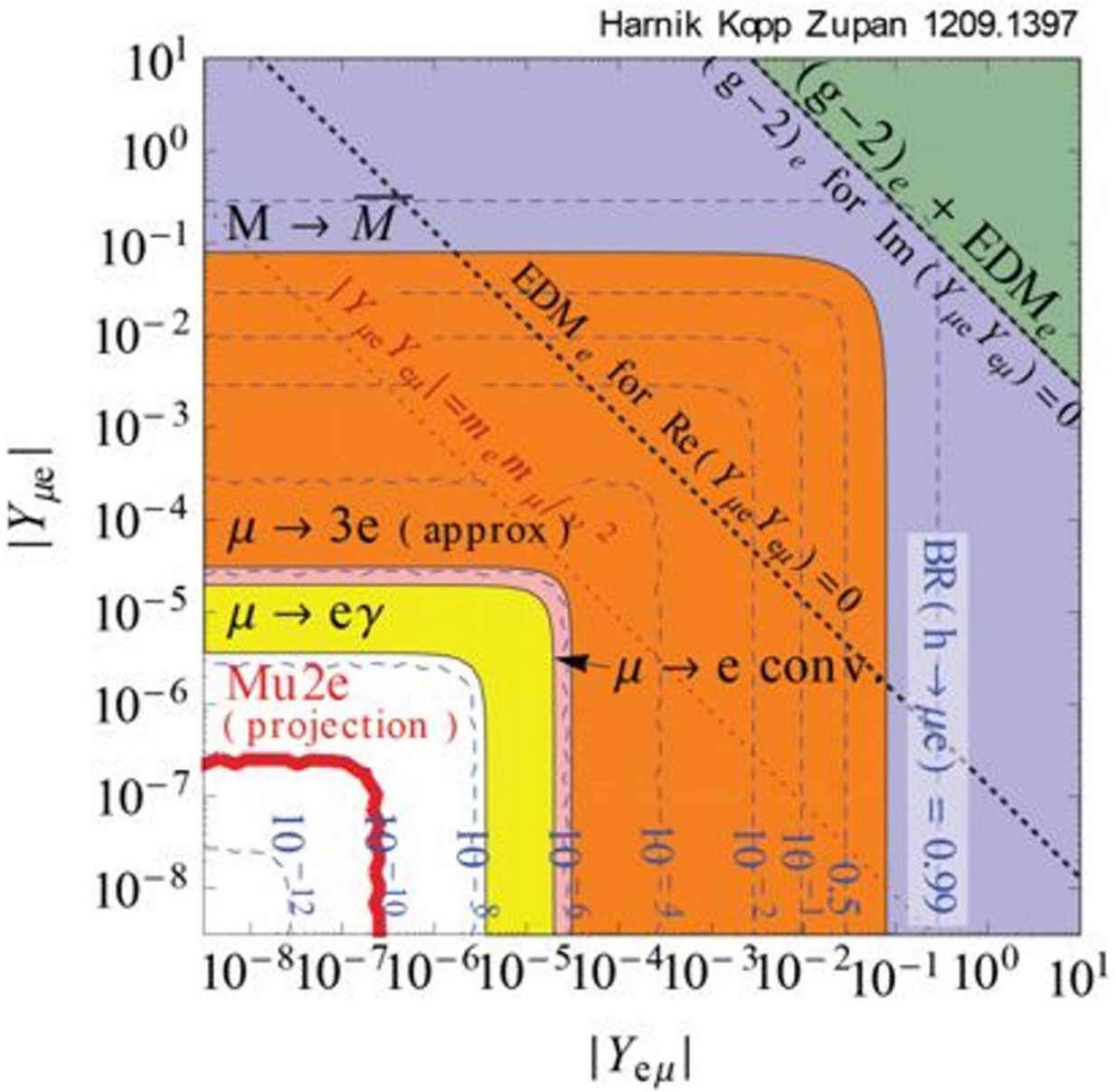
Next-Generation Experiments

Process	BR Limit	CL	Experiment
$h \to e\mu$	6.1×10^{-5}	95%	ATLAS
$\mu^- + \mathrm{Al} \to e^- + \mathrm{Al}$	8×10^{-17}	90%	Mu2e

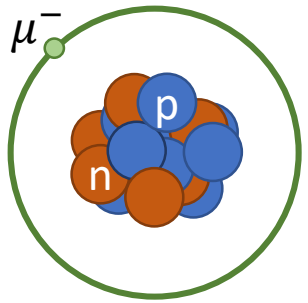




$\mu \to e$ provides **better constraint** than $h \to e\mu, \mu \to e\gamma, \mu \to 3e$



Muon captured
in 1s orbital



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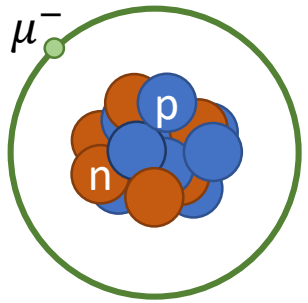
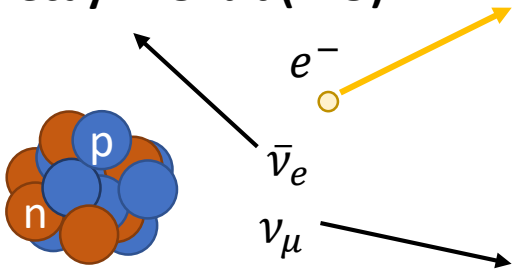


Decay in Orbit (DIO)



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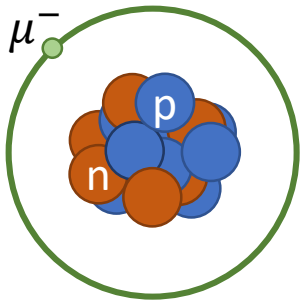

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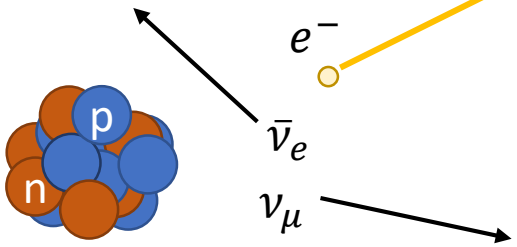
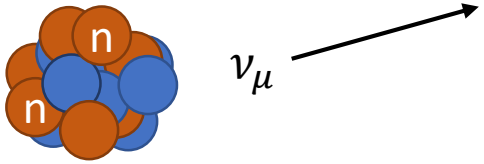


Standard μ Capture



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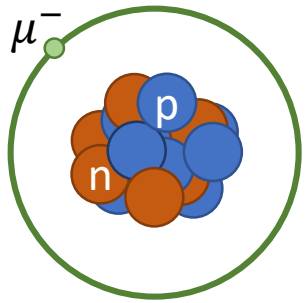

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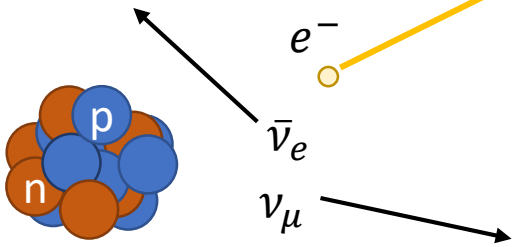

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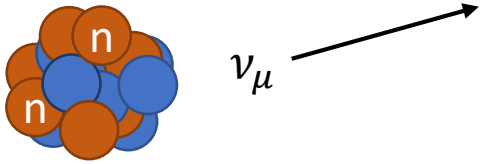
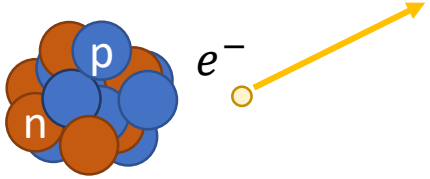


CLFV $\mu \to e$ Conversion



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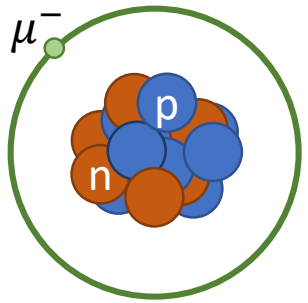

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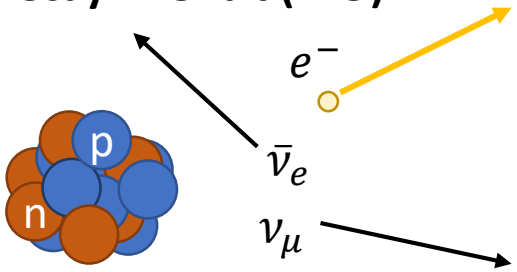

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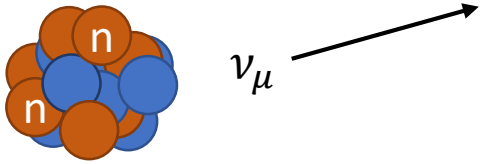

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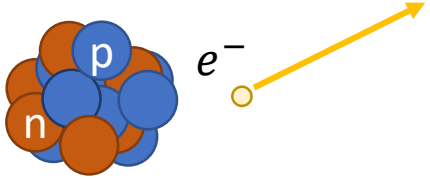
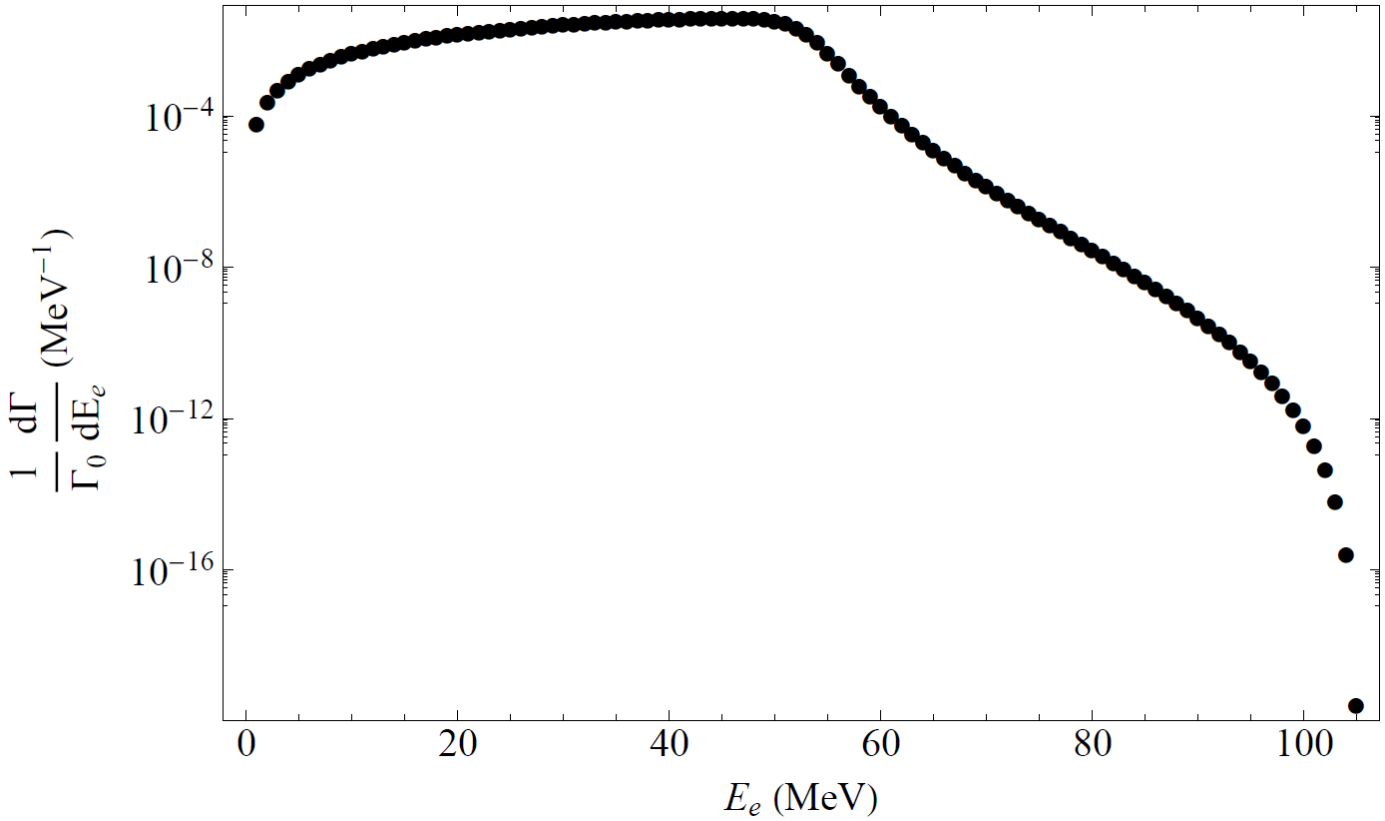


DIO Spectrum (Background)

Figure: Czarnecki, Garcia i Tormo, & Marciano, Phys. Rev. D **84**, 013006 (2011)



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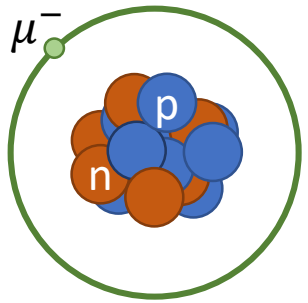

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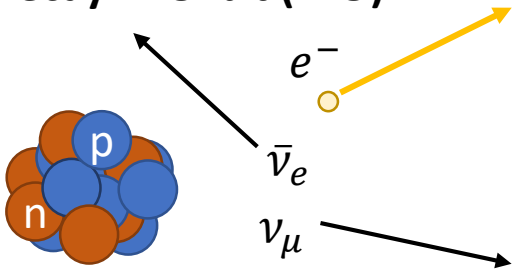

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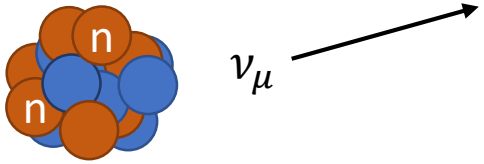

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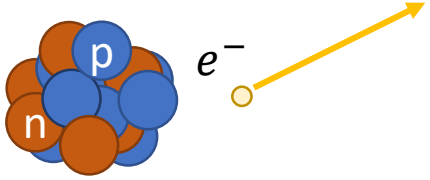

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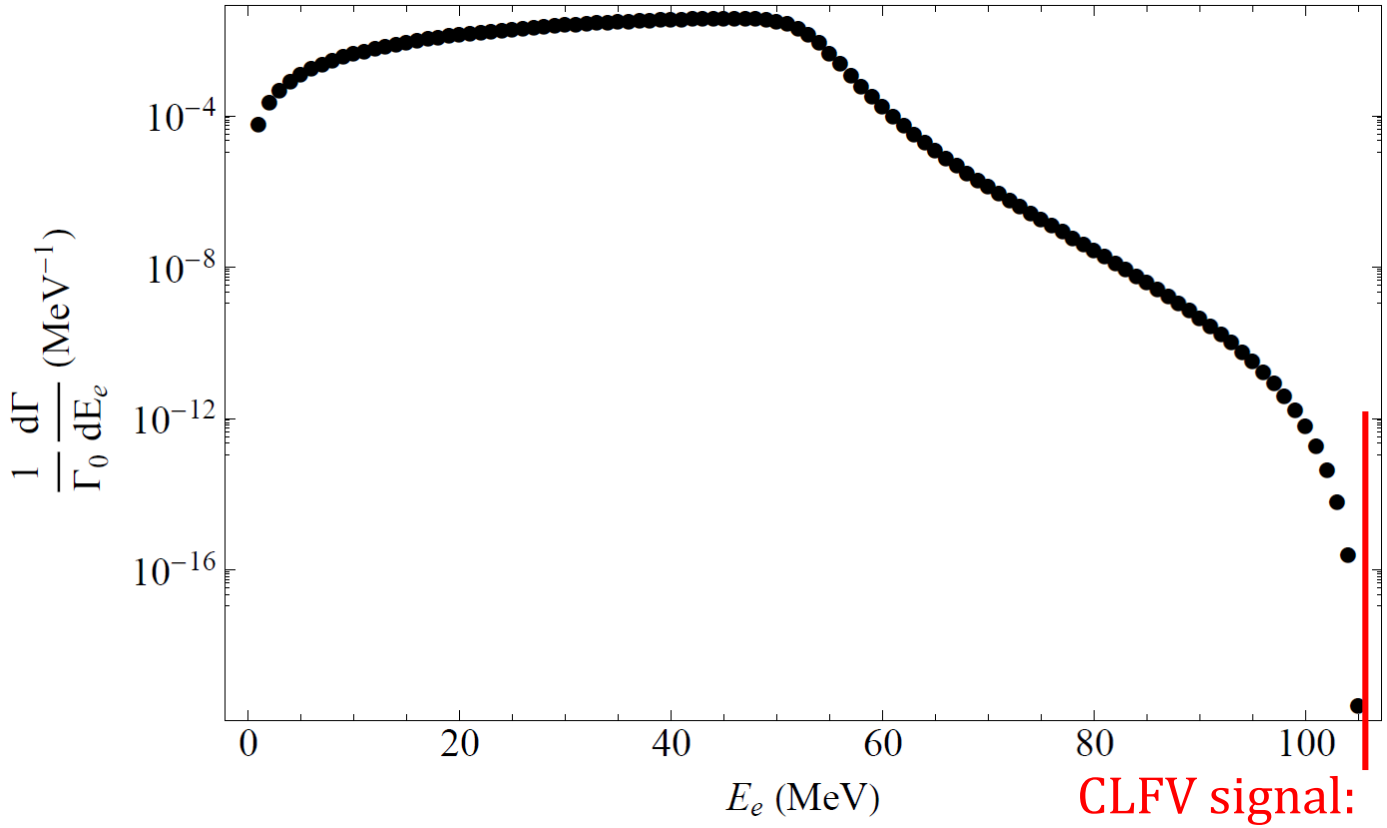
Figure: Czarnecki, Garcia i Tormo, & Marciano, Phys. Rev. D **84**, 013006 (2011)



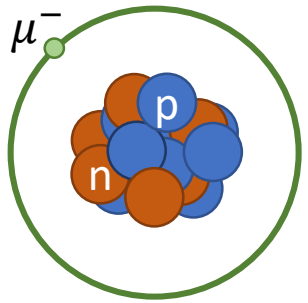
CLFV signal:

$E_e \approx m_\mu - B_\mu$

** Assuming nucleus remains in ground state **

"Elastic conversion"

Muon captured in 1s orbital



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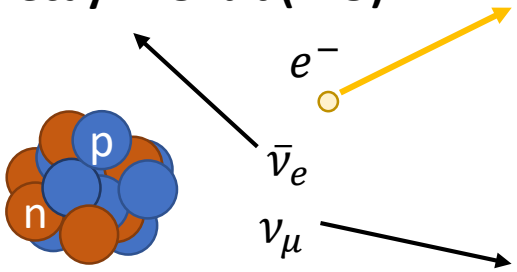

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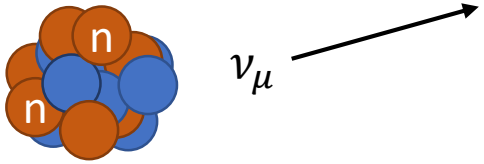

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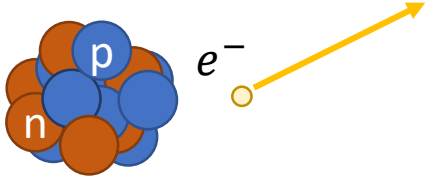

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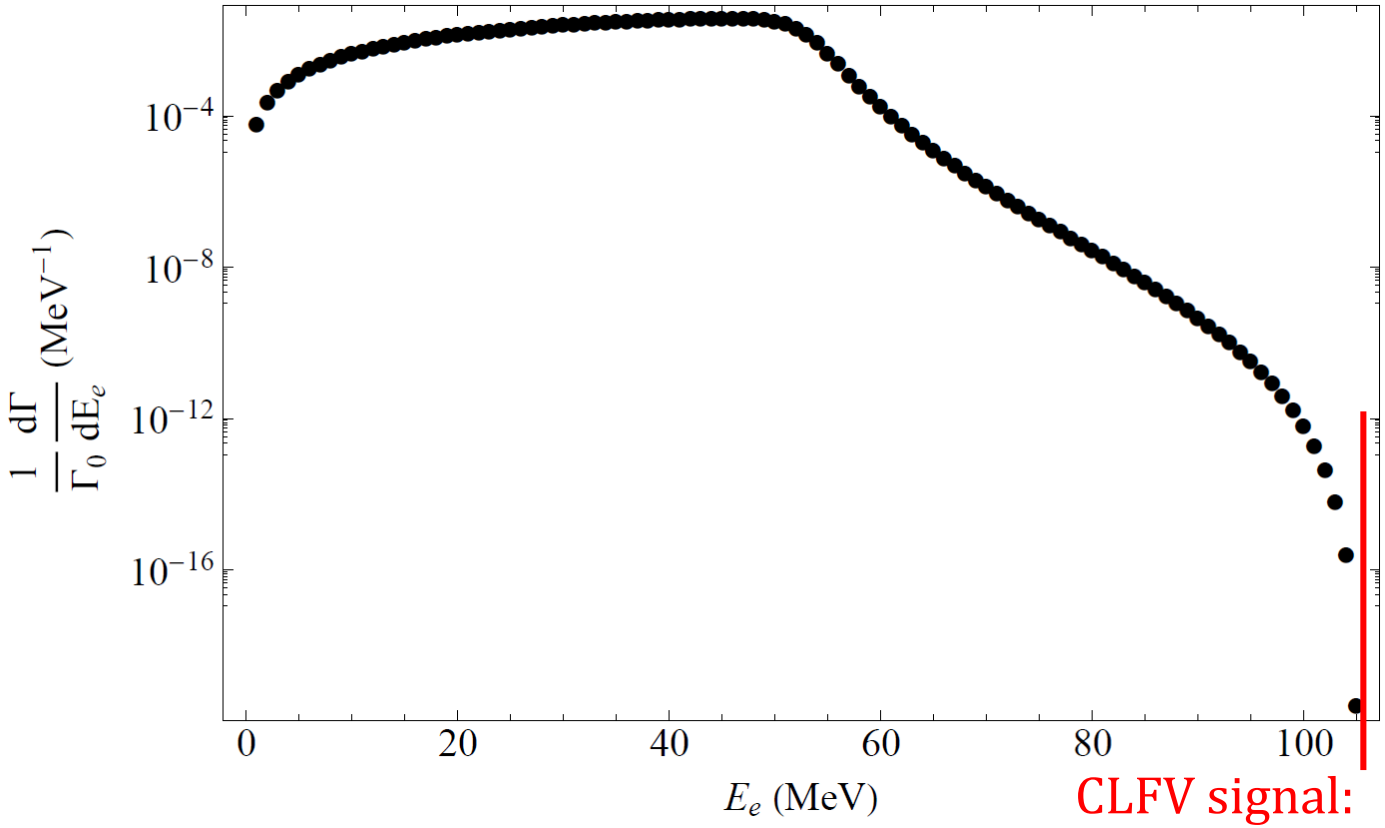

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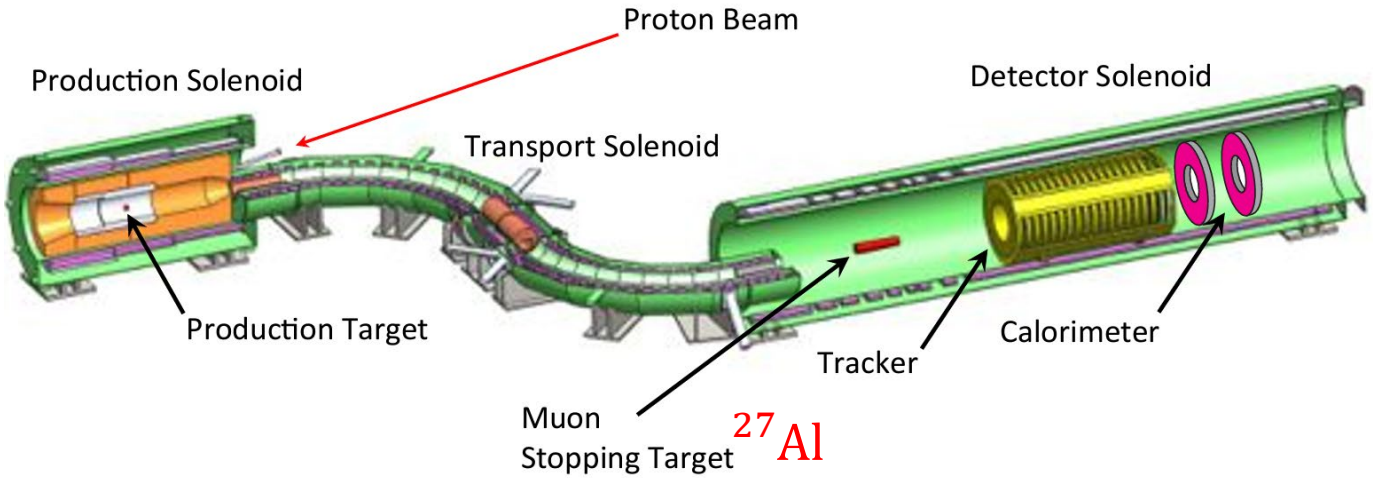
$$\mathrm{BR}\left(\mu^- + (A,Z) \to e^- + (A,Z)\right) \equiv \frac{\Gamma\left(\mu^- + (A,Z) \to e^- + (A,Z)\right)}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

** Assuming nucleus remains in ground state **
"Elastic conversion"

Mu2e Experiment

Figure: Mu2e collaboration, arXiv:1501.05241



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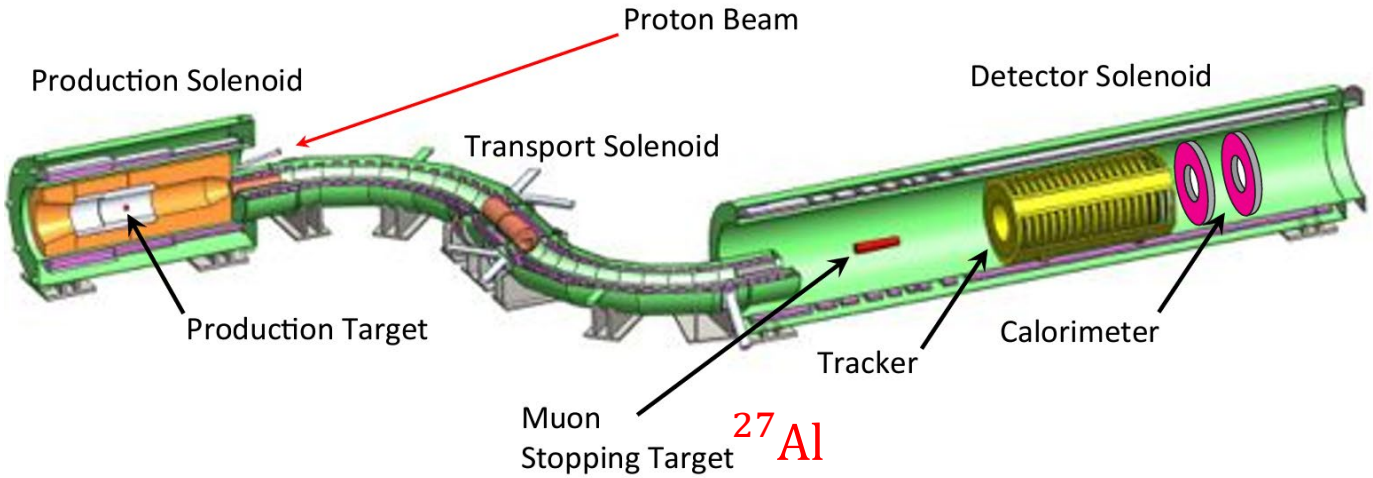

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- Huge Intensity: 10^{18} muons captured over lifetime
- Clean CLFV signal, free of standard-model backgrounds
- Target can be varied to obtain complementary constraints

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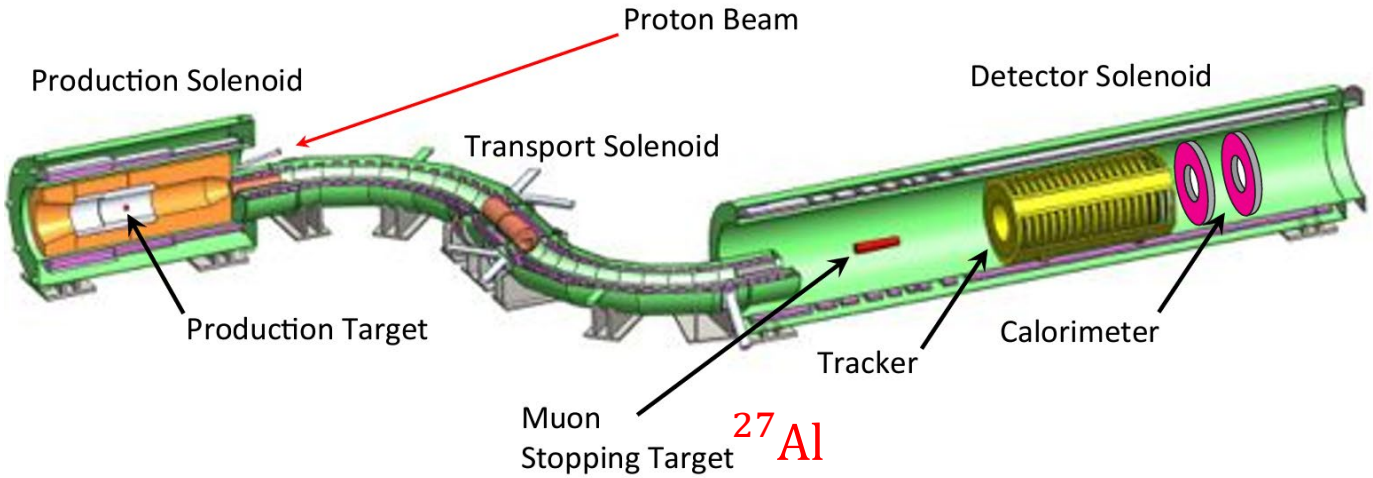

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- CLFV physics filtered by nuclear physics
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  - Parity **P** and time-reversal **T** symmetries
- Large scale separation between experiments and UV theories

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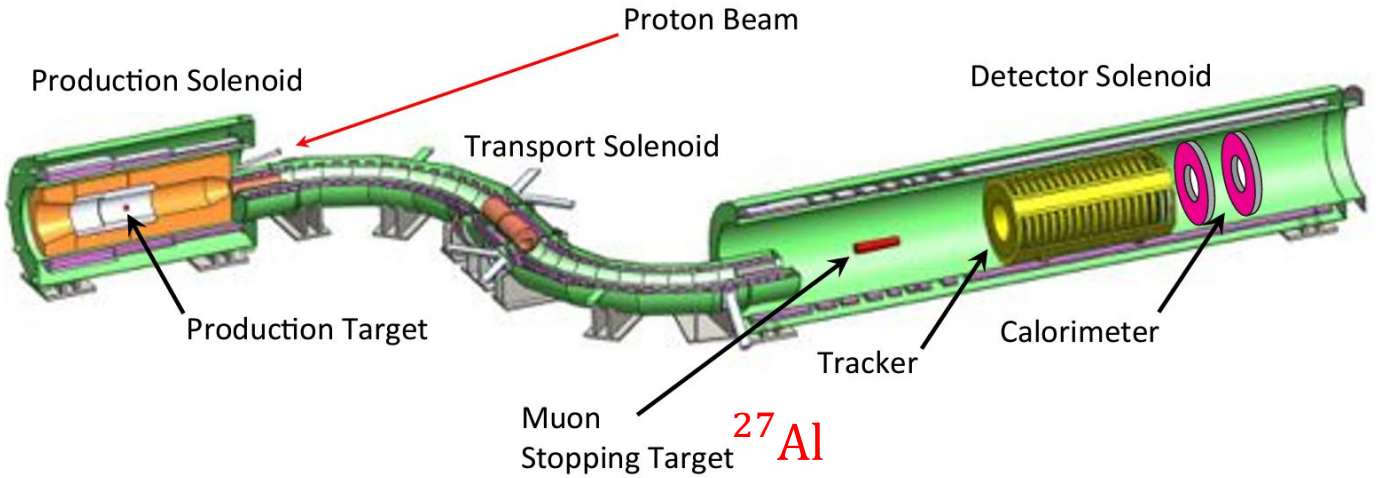

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Positive signal is evidence of new physics

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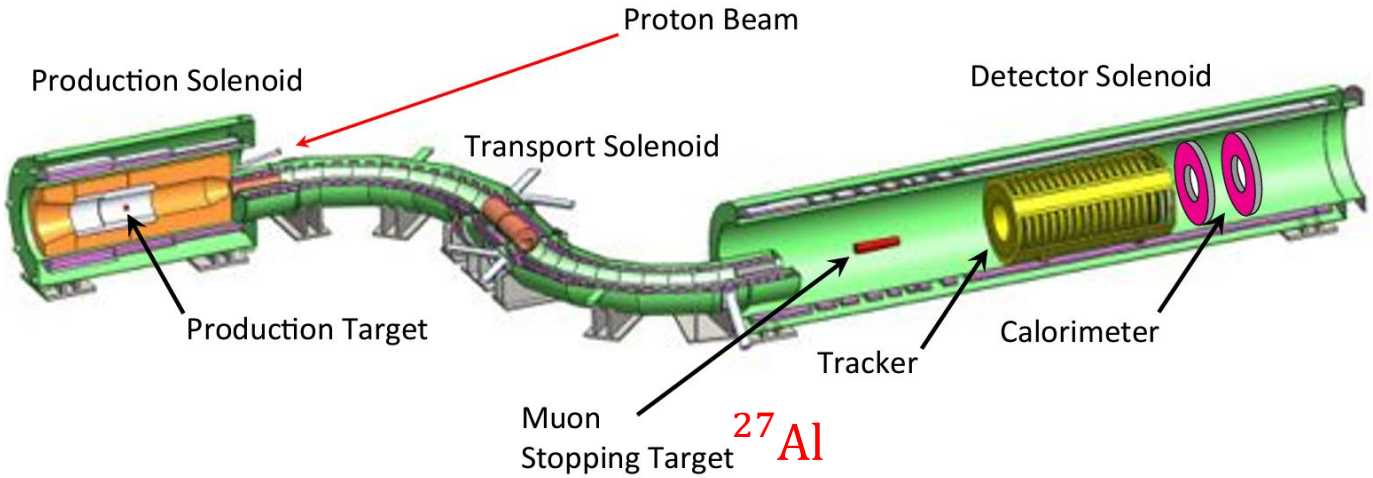

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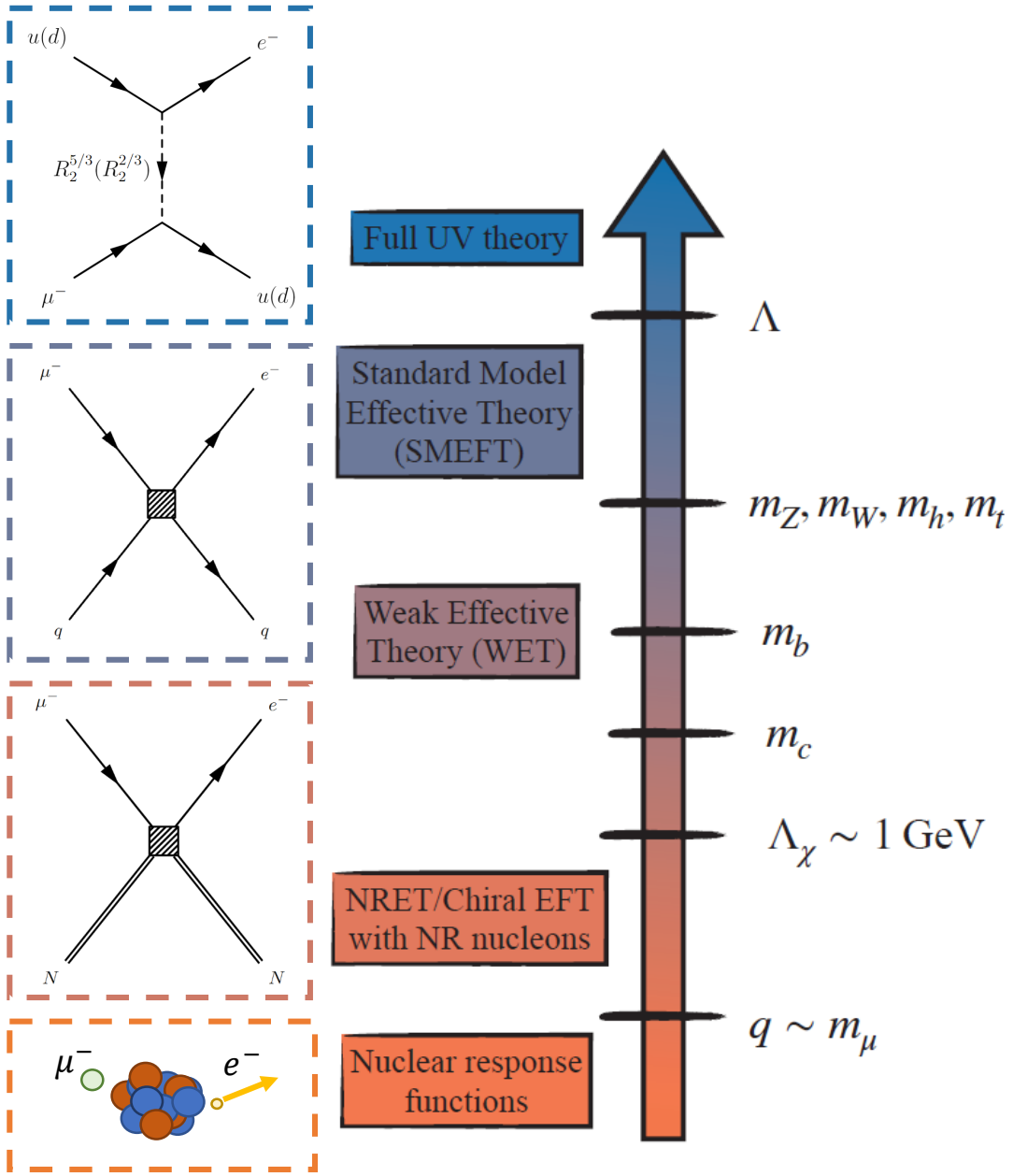
Challenges

- CLFV physics filtered by nuclear physics
- Nuclear ground state restricts operators that contribute
  - Parity **P** and time-reversal **T** symmetries
- Large scale separation between experiments and UV theories

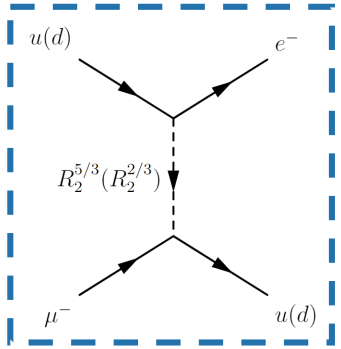
Positive signal is evidence of new physics

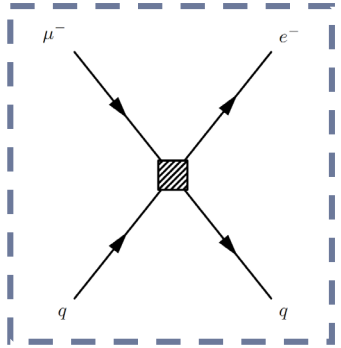
Can we learn anything else about the underlying mechanism?

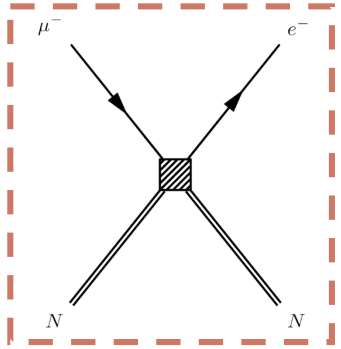
A Tower of EFTs

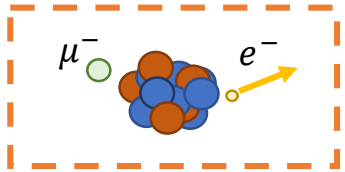


A Tower of EFTs









Full UV theory

Standard Model Effective Theory (SMEFT)

Weak Effective Theory (WET)

NRET/Chiral EFT with NR nucleons

Nuclear response functions

Λ

m_Z, m_W, m_h, m_t

m_b

m_c

$\Lambda_\chi \sim 1\ \text{GeV}$

$q \sim m_\mu$

SMEFT

Jenkins, Manohar, & Trott, JHEP **10** (2013) 087
Jenkins, Manohar, & Trott, JHEP **01** (2014) 035
Alonso, Jenkins, Manohar, & Trott, JHEP **04** (2014) 159
Brivio & Trott, Phys. Rept. **793** (2019) 1-98

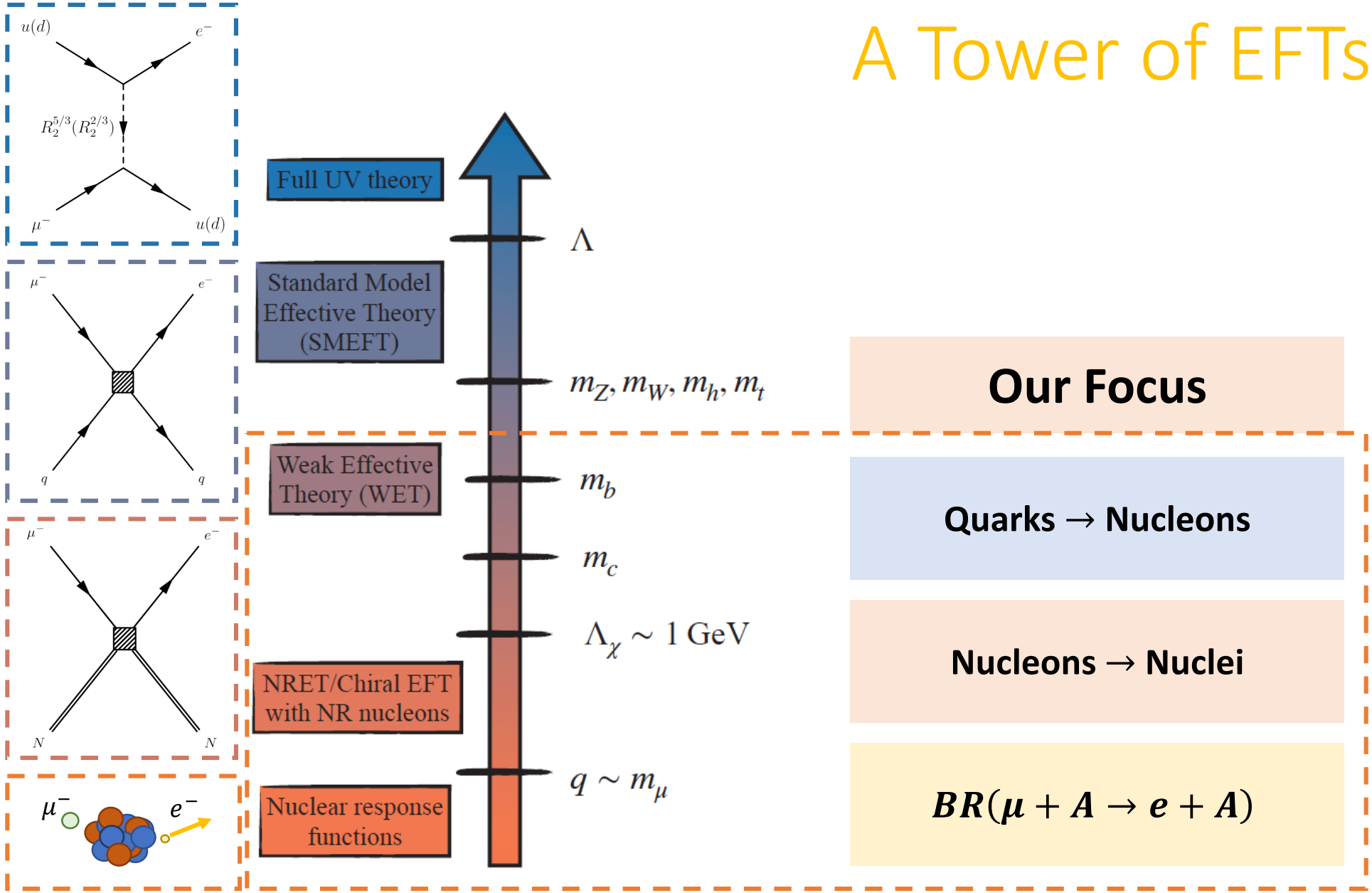
WET

Jenkins, Manohar, & Stoffer, JHEP **03** (2018) 016
Dekens & Stoffer, JHEP **10** (2019) 197

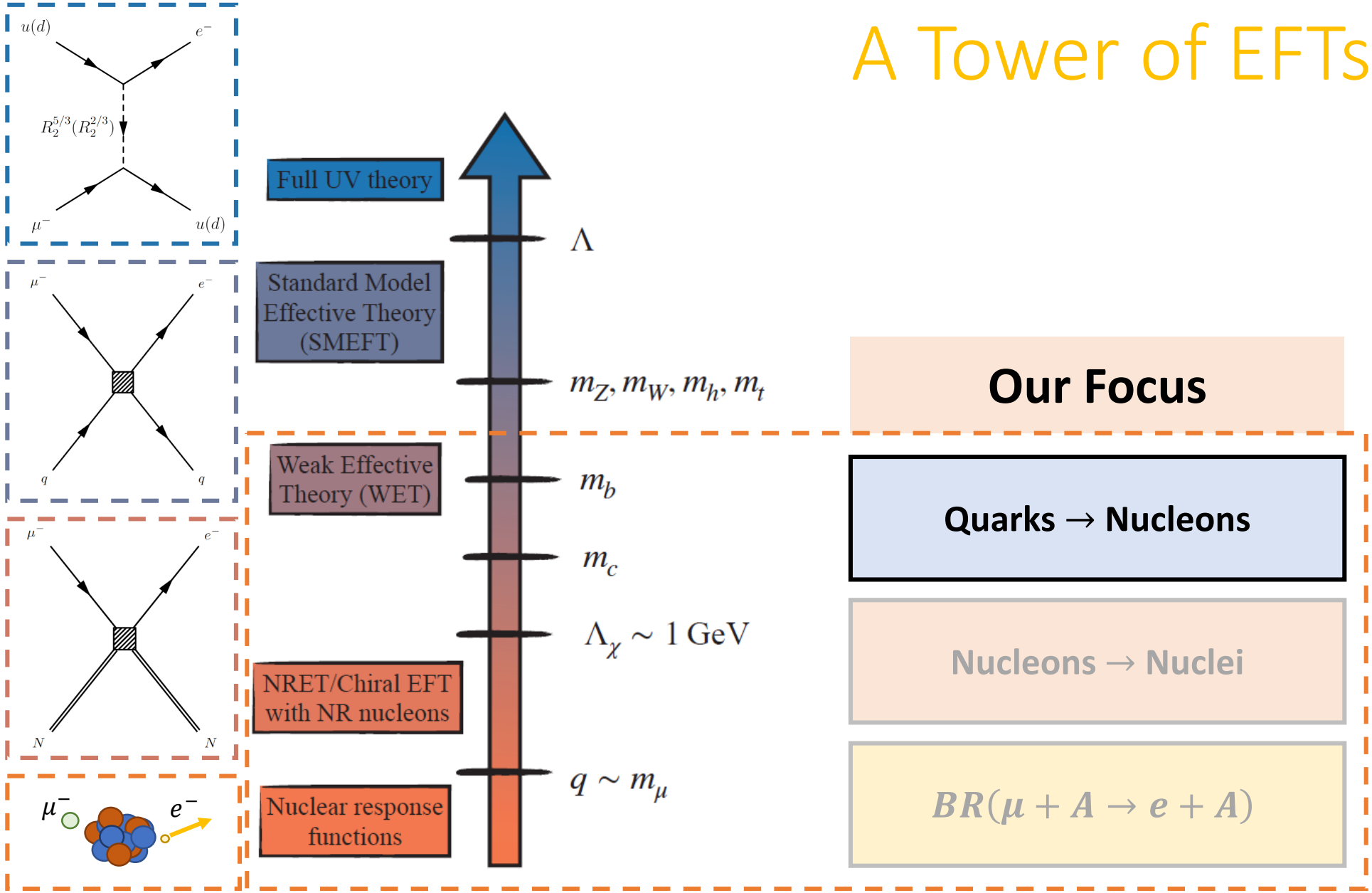
Codes

Wilson: Aebischer, Kumar, & Straub, Eur. Phys. J. C **78** (2018) 1026
DsixTools: Celis, Fuentes-Martin, Vicente, and Virto, Eur. Phys. J. C **77** (2017)
Fuentes-Martin, Ruiz-Femenia, Vicente & Virto, Eur. Phys. J. C **81** (2021)

A Tower of EFTs



A Tower of EFTs

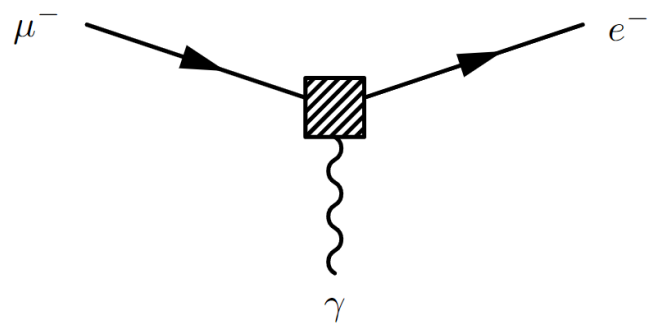


Weak Effective Theory at $\mu = 2$ GeV

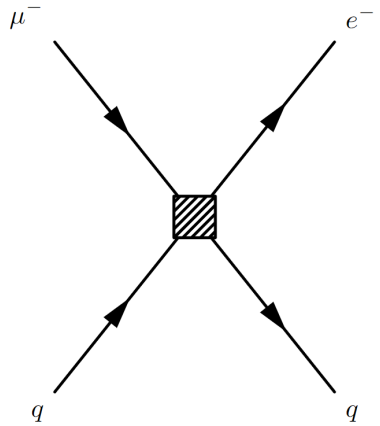
- **CLFV lepton currents:** muon, electron
- **Light quarks:** $q = u, d, s$
- **Massless gauge bosons:** gluon, photon

Weak Effective Theory at $\mu = 2$ GeV

$D = 5$ (2 Operators)

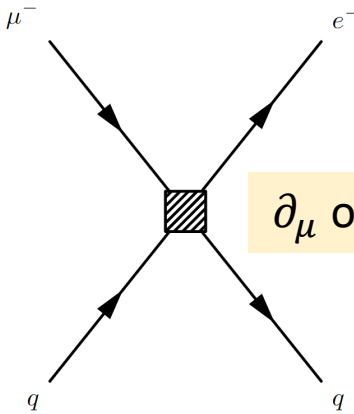


$D = 6$ (10 Operators)



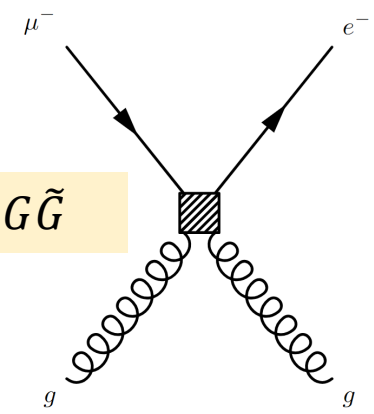
$D = 7$ (16 Operators)

8



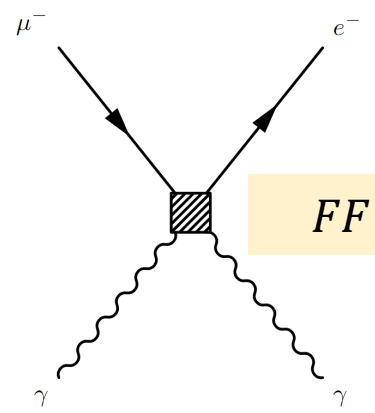
∂_μ on leptons

4



GG or $G\tilde{G}$

4



FF or $F\tilde{F}$

∂_μ required to generate all 16 NRET operators

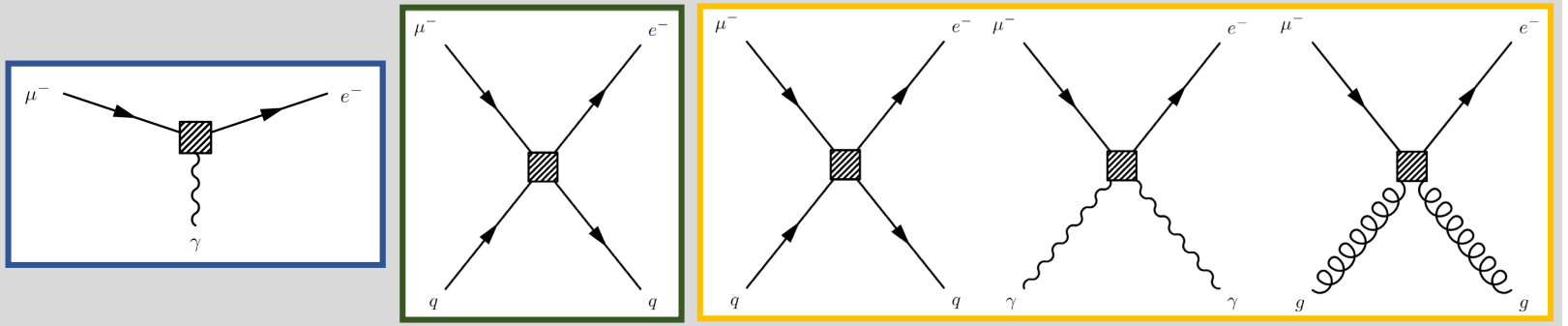
GG generated by coupling to heavy c, b, t quarks

Effective theory matching below 2 GeV

UV Theories

WET

- 28 Operators

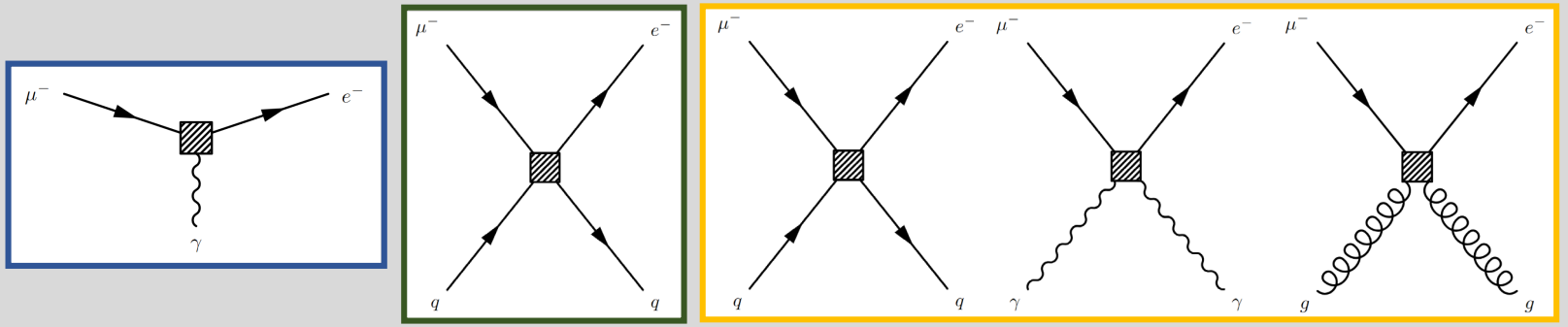


Experiments

Effective theory matching below 2 GeV

UV Theories

WET
- 28 Operators



Form-factor matching + non-relativistic expansion

Examples:

$$\langle N'|m_q \bar{q} q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N \approx F_S^{q/N}(q^2)\,\bar{N}' 1_N N$$

$$\langle N'|m_q \bar{q} i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N \approx F_P^{q/N}(q^2)\, i\frac{\vec{q}}{2m_N}\cdot \bar{N}'\vec{\sigma}_N N$$

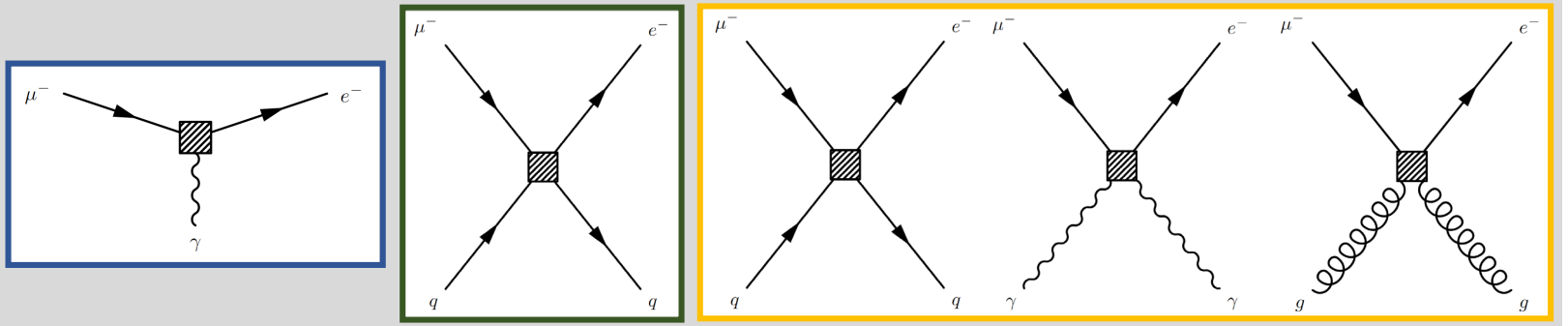
- Form factors evaluated at $q^2 \approx -m_\mu^2$

Experiments

Effective theory matching below 2 GeV

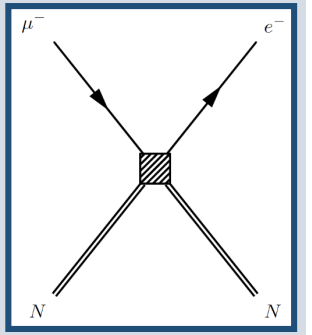
UV Theories

WET
- 28 Operators



NRET
- 16 Operators

	v_N - indep.	v_N - dep.
σ_N - indep.	$\mathcal{O}_1, \mathcal{O}_{11}$	$\mathcal{O}_2, \mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{16}$
σ_N - dep.	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_9, \mathcal{O}_{10}$	$\mathcal{O}_3, \mathcal{O}_7, \mathcal{O}_{12}, \mathcal{O}_{13}, \mathcal{O}_{14}, \mathcal{O}_{15}$

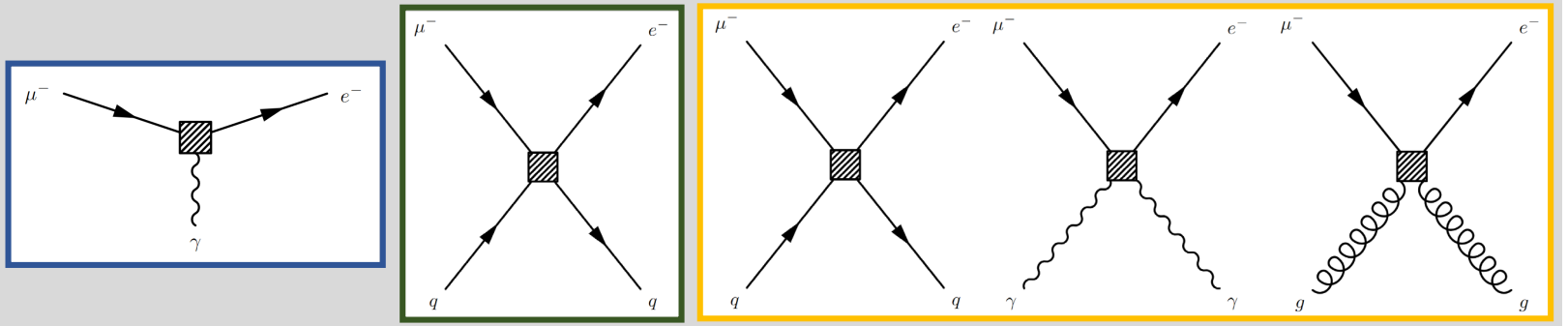


Experiments

Effective theory matching below 2 GeV

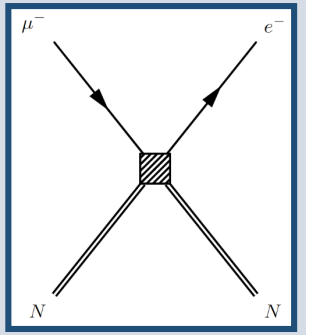
UV Theories

WET
- 28 Operators



NRET
- 16 Operators

	v_N - indep.	v_N - dep.
σ_N - indep.	$\mathcal{O}_1, \mathcal{O}_{11}$	$\mathcal{O}_2, \mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{16}$
σ_N - dep.	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_9, \mathcal{O}_{10}$	$\mathcal{O}_3, \mathcal{O}_7, \mathcal{O}_{12}, \mathcal{O}_{13}, \mathcal{O}_{14}, \mathcal{O}_{15}$



Nuclear embedding

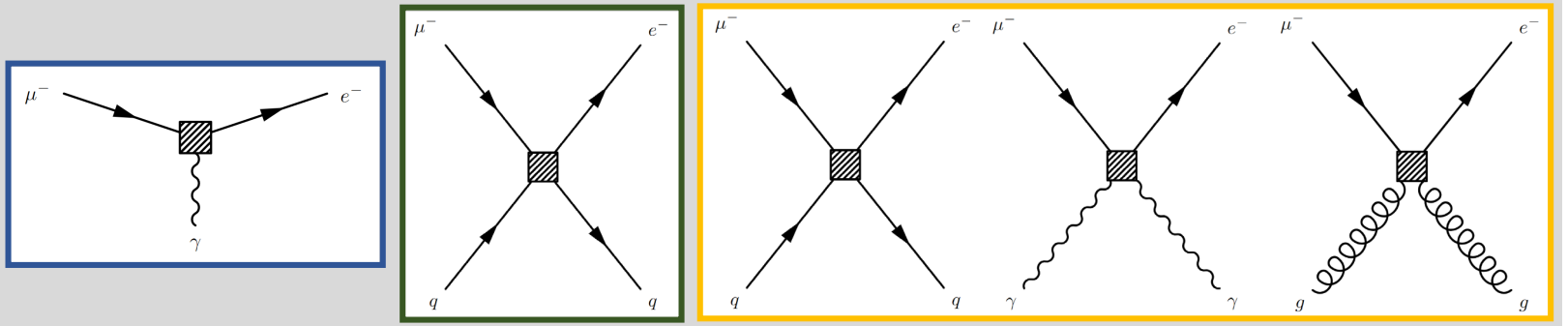
- Choose particular nuclear target
- **Selection rules:** angular momentum, parity, time-reversal

Experiments

Effective theory matching below 2 GeV

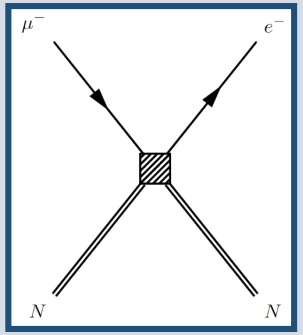
UV Theories

WET
- 28 Operators



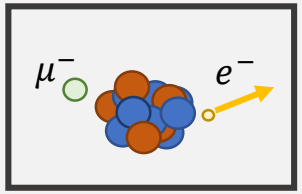
NRET
- 16 Operators

	v_N - indep.	v_N - dep.
σ_N - indep.	$\mathcal{O}_1, \mathcal{O}_{11}$	$\mathcal{O}_2, \mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{16}$
σ_N - dep.	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_9, \mathcal{O}_{10}$	$\mathcal{O}_3, \mathcal{O}_7, \mathcal{O}_{12}, \mathcal{O}_{13}, \mathcal{O}_{14}, \mathcal{O}_{15}$



Nuclear ET (elastic)
- 8 Responses

	v_N - indep.	v_N - dep.
σ_N - indep.	W_M	W_Δ
σ_N - dep.	$W_{\Sigma'}$ $W_{\Sigma''}$	$W_{\tilde{\Phi}'}$ $W_{\Phi''}$ $(W_{M\Phi''}$ $W_{\Delta\Sigma'})$

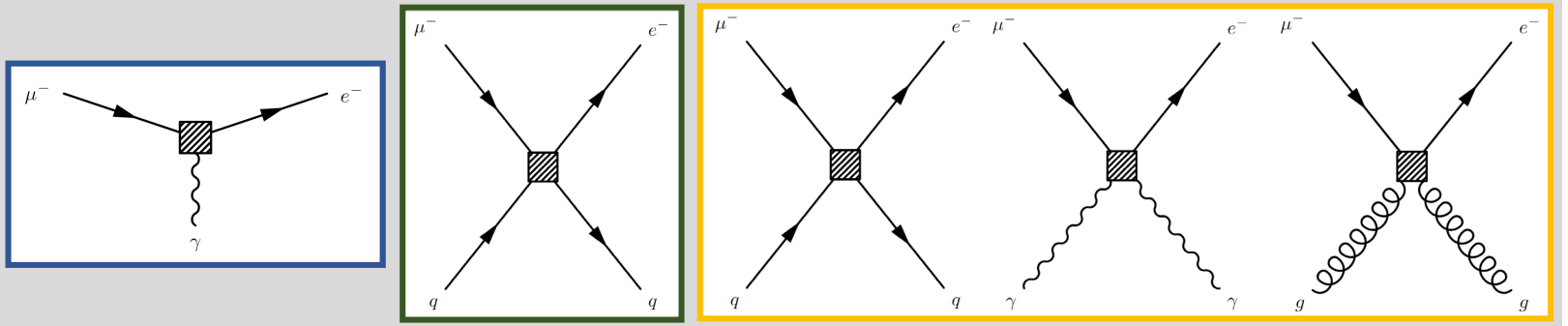


Experiments

Effective theory matching below 2 GeV

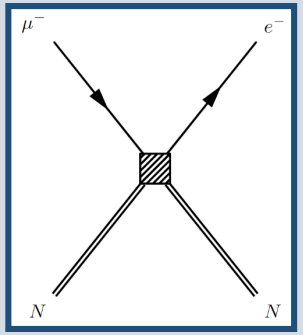
UV Theories

WET
- 28 Operators



NRET
- 16 Operators

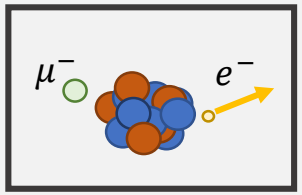
	v_N - indep.	v_N - dep.
σ_N - indep.	$\mathcal{O}_1, \mathcal{O}_{11}$	$\mathcal{O}_2, \mathcal{O}_5, \mathcal{O}_8, \mathcal{O}_{16}$
σ_N - dep.	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_9, \mathcal{O}_{10}$	$\mathcal{O}_3, \mathcal{O}_7, \mathcal{O}_{12}, \mathcal{O}_{13}, \mathcal{O}_{14}, \mathcal{O}_{15}$



Nuclear ET (elastic)
- 8 Responses

	v_N - indep.	v_N - dep.
σ_N - indep.	W_M	W_Δ
σ_N - dep.	$W_{\Sigma'}$ $W_{\Sigma''}$	$W_{\tilde{\Phi}'}$ $W_{\Phi''}$ $(W_{M\Phi''}$ $W_{\Delta\Sigma'})$

8 Constraints from experiments

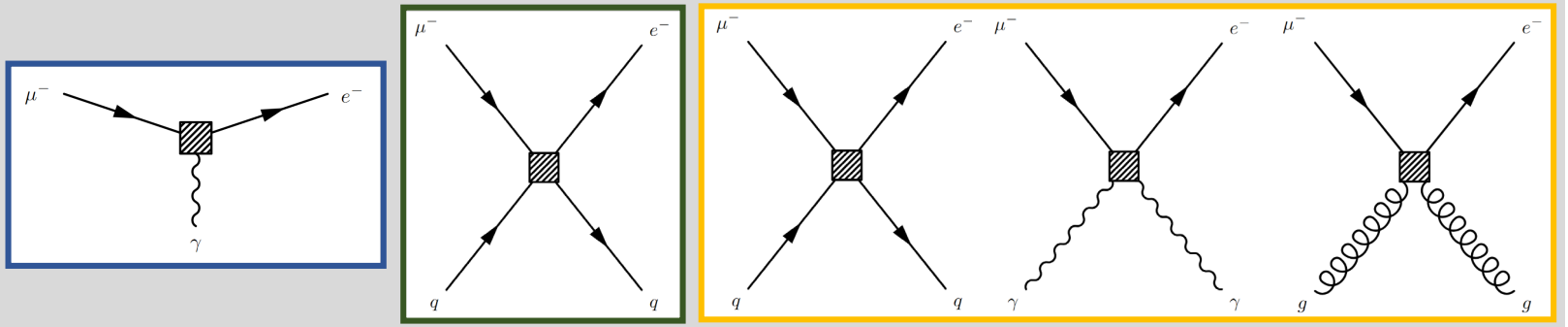


Experiments

Effective theory matching below 2 GeV

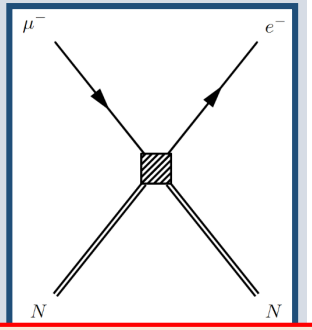
UV Theories

WET
- 28 Operators



NRET
- 16 Operators

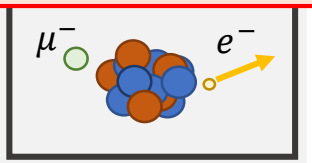
	v_N - indep.	v_N - dep.
σ_N - indep.	$\mathcal{O}_1, \mathcal{O}_{11}$	~~$\mathcal{O}_2$~~, $\mathcal{O}_5, \mathcal{O}_8$, ~~$\mathcal{O}_{16}$~~
σ_N - dep.	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_9, \mathcal{O}_{10}$	$\mathcal{O}_3$, ~~$\mathcal{O}_7$~~, $\mathcal{O}_{12}, \mathcal{O}_{13}$, ~~$\mathcal{O}_{14}$~~, $\mathcal{O}_{15}$



Excluded by nuclear selection rules

Nuclear ET (elastic)
- 6 Responses

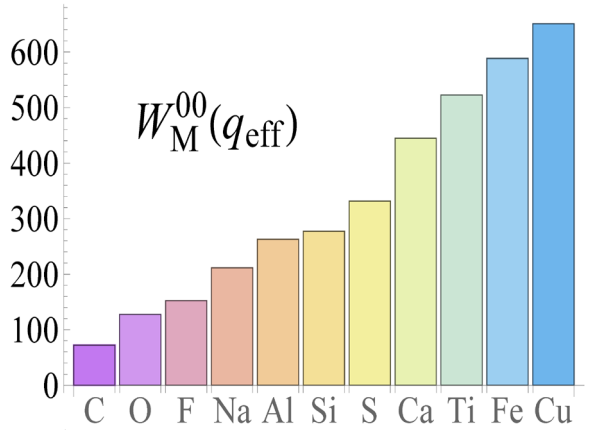
	v_N - indep.	v_N - dep.
σ_N - indep.	W_M	W_Δ
σ_N - dep.	$W_{\Sigma'}$ $W_{\Sigma''}$	$W_{\tilde{\Phi}'}$ $W_{\Phi''}$ $(W_{M\Phi''}$ $W_{\Delta\Sigma'})$

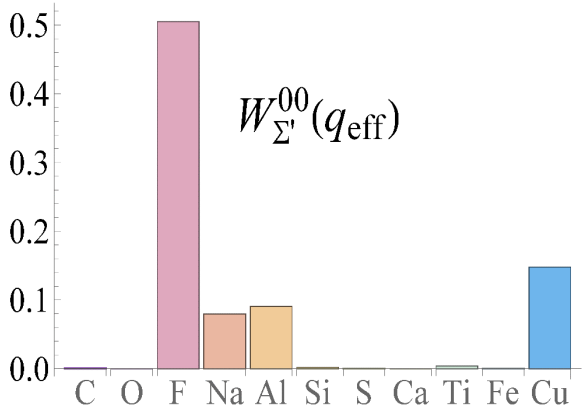


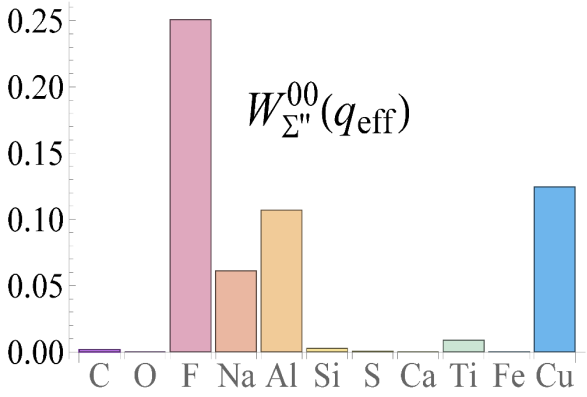
Experiments

Velocity-independent

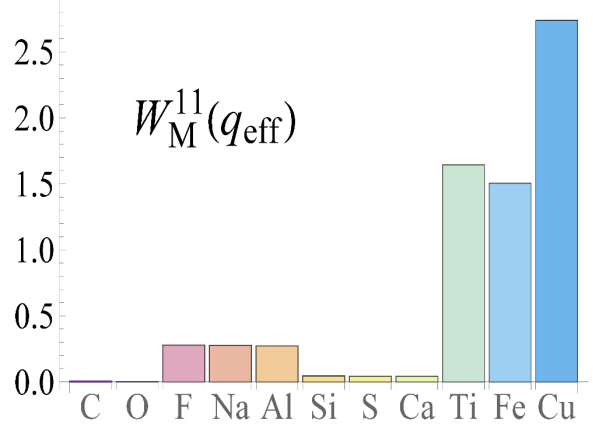
Isoscalar

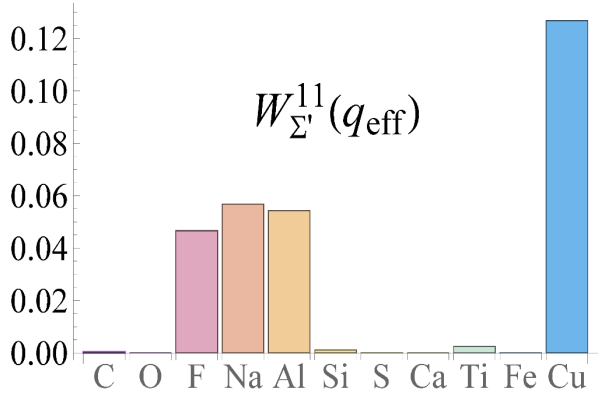


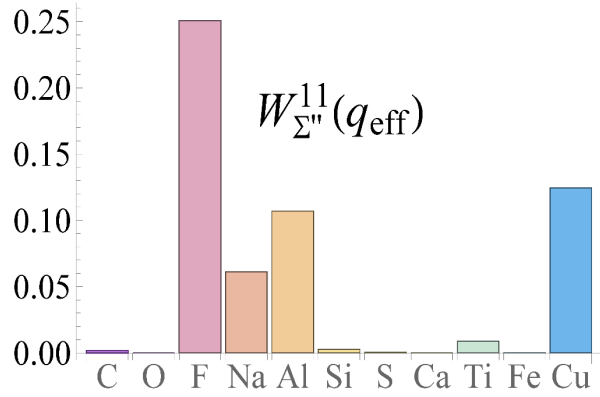




Isovector

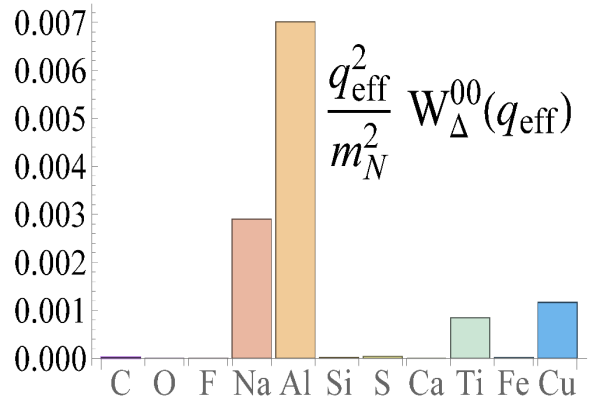


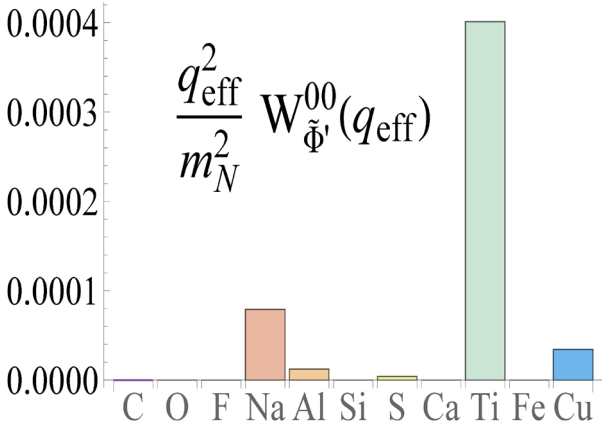


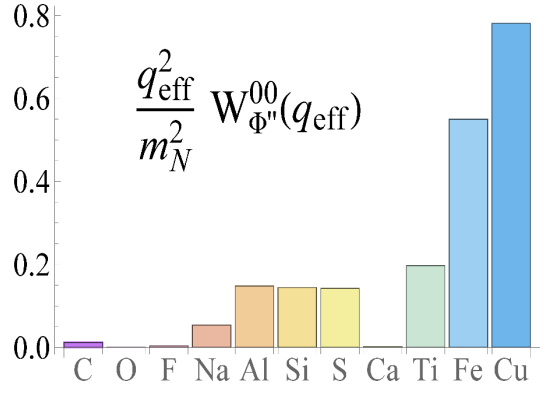


Velocity-dependent

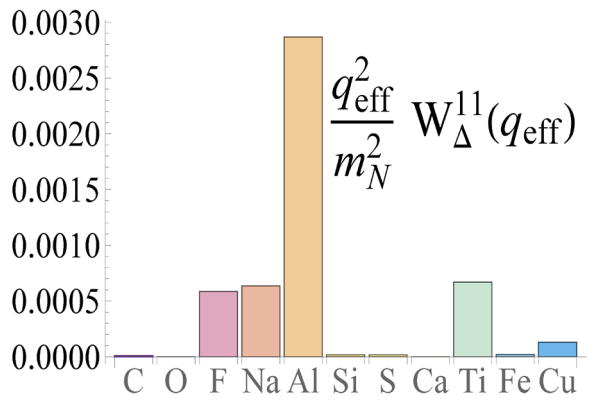
Isoscalar

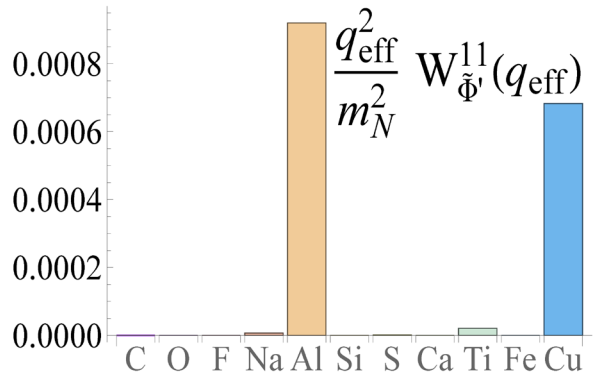


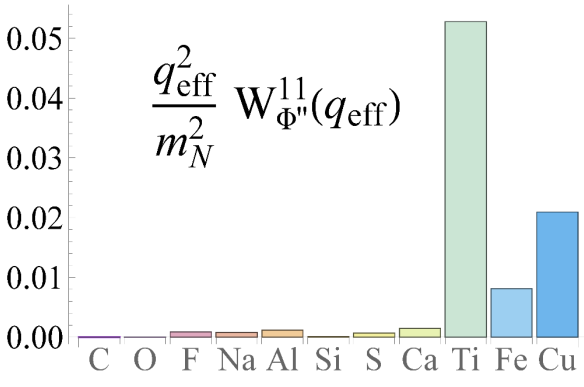




Isovector





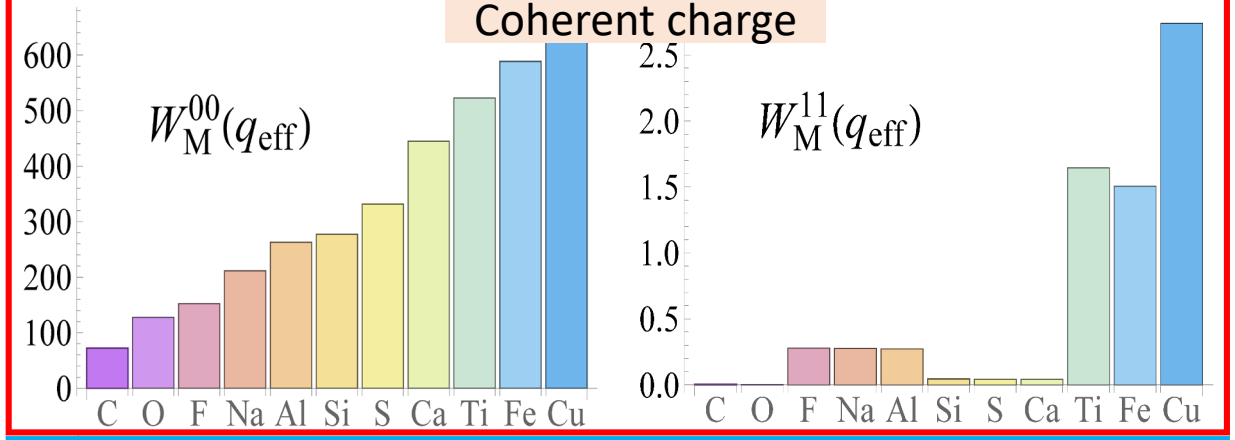


Velocity-independent

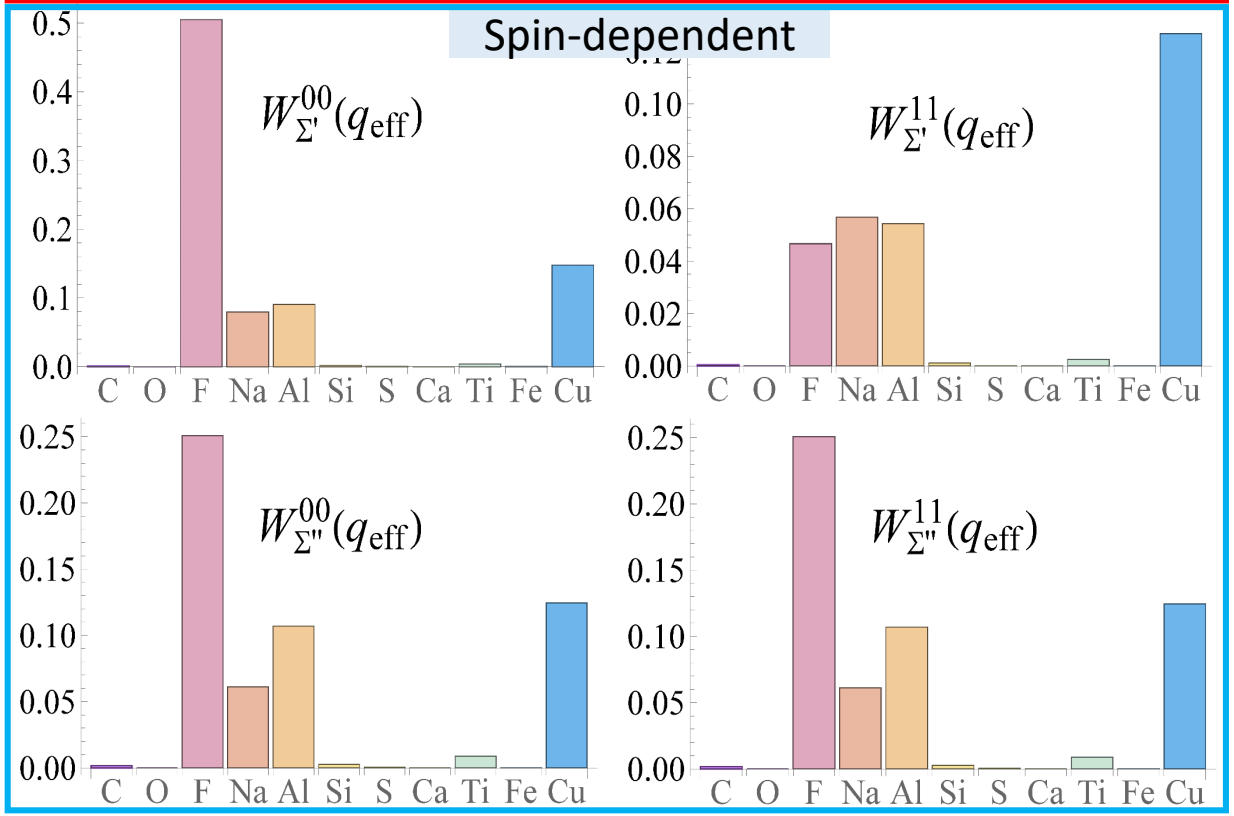
Isoscalar

Isovector

Coherent charge



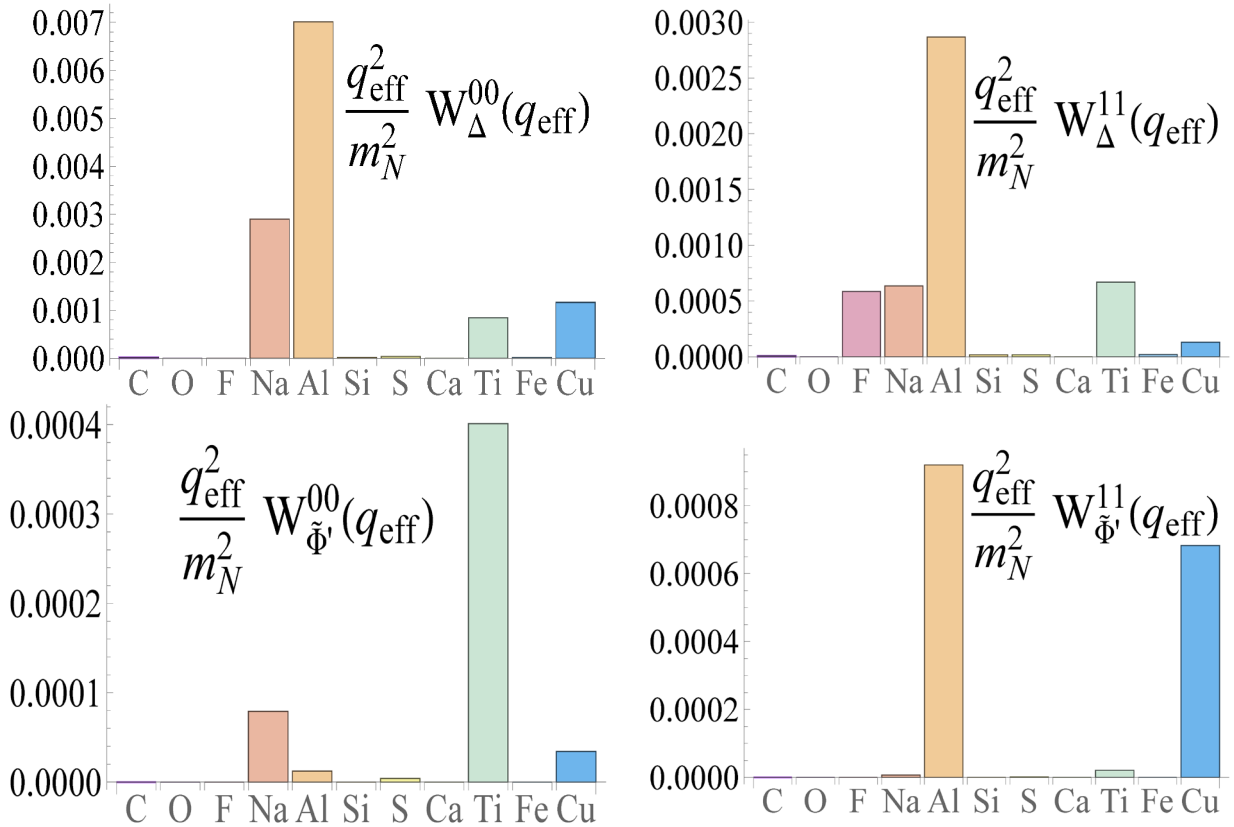
Spin-dependent



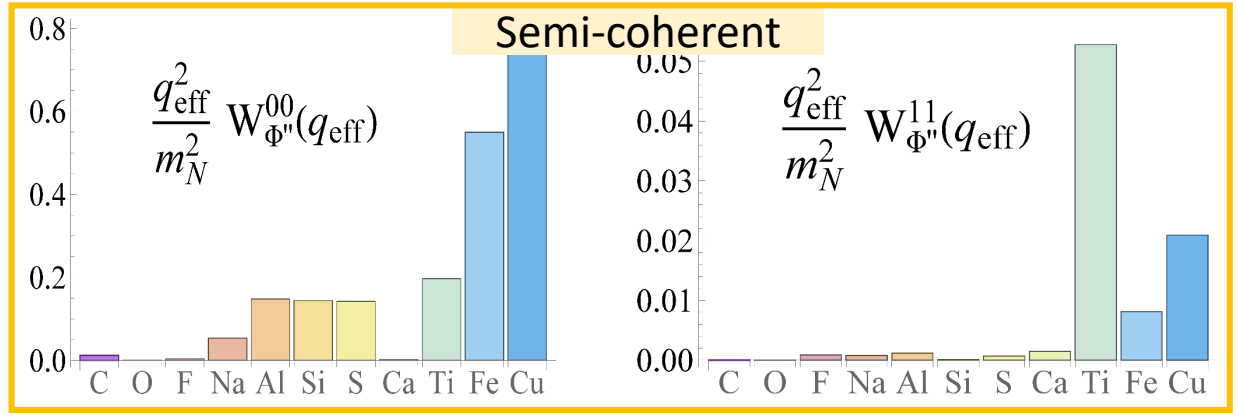
Velocity-dependent

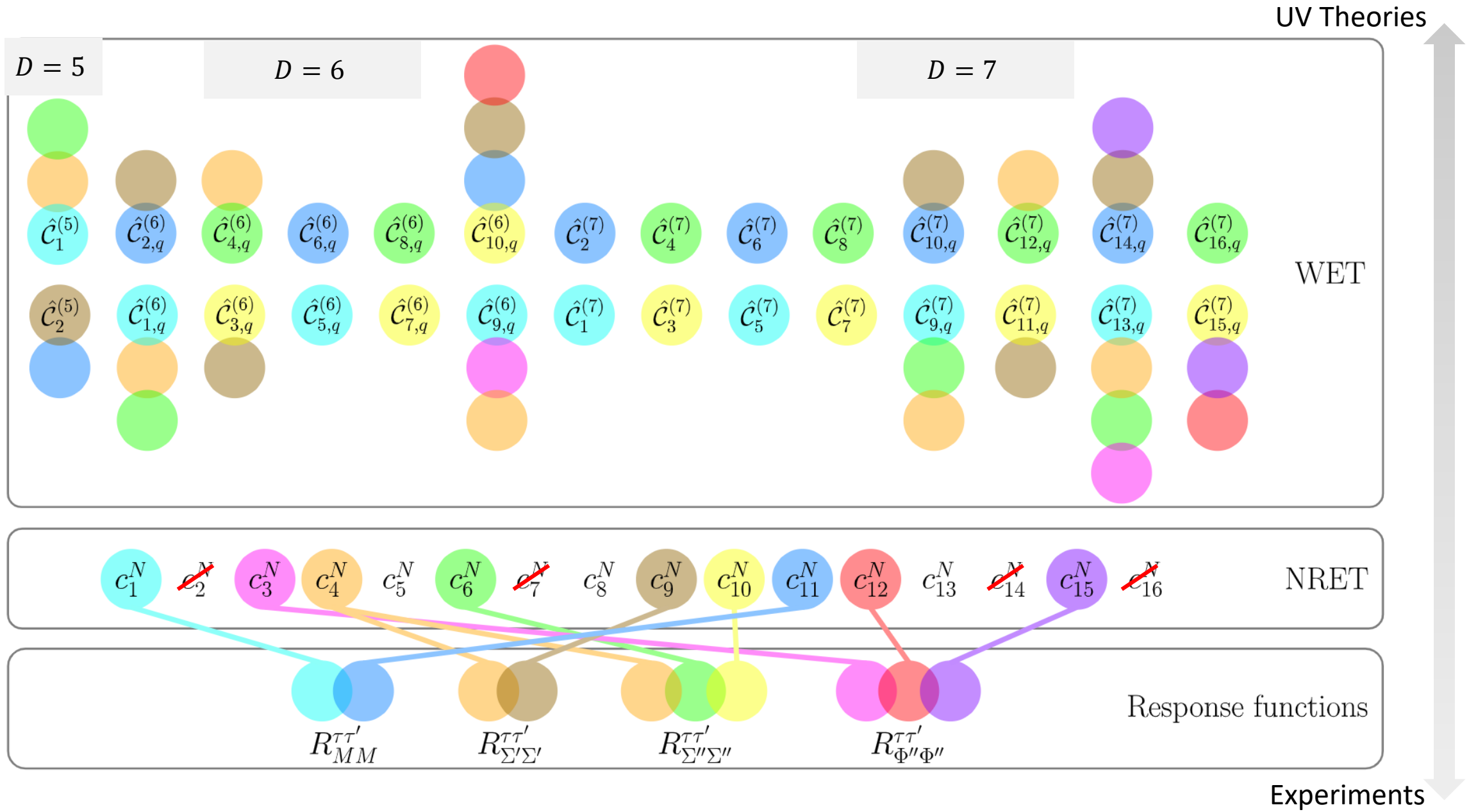
Isoscalar

Isovector

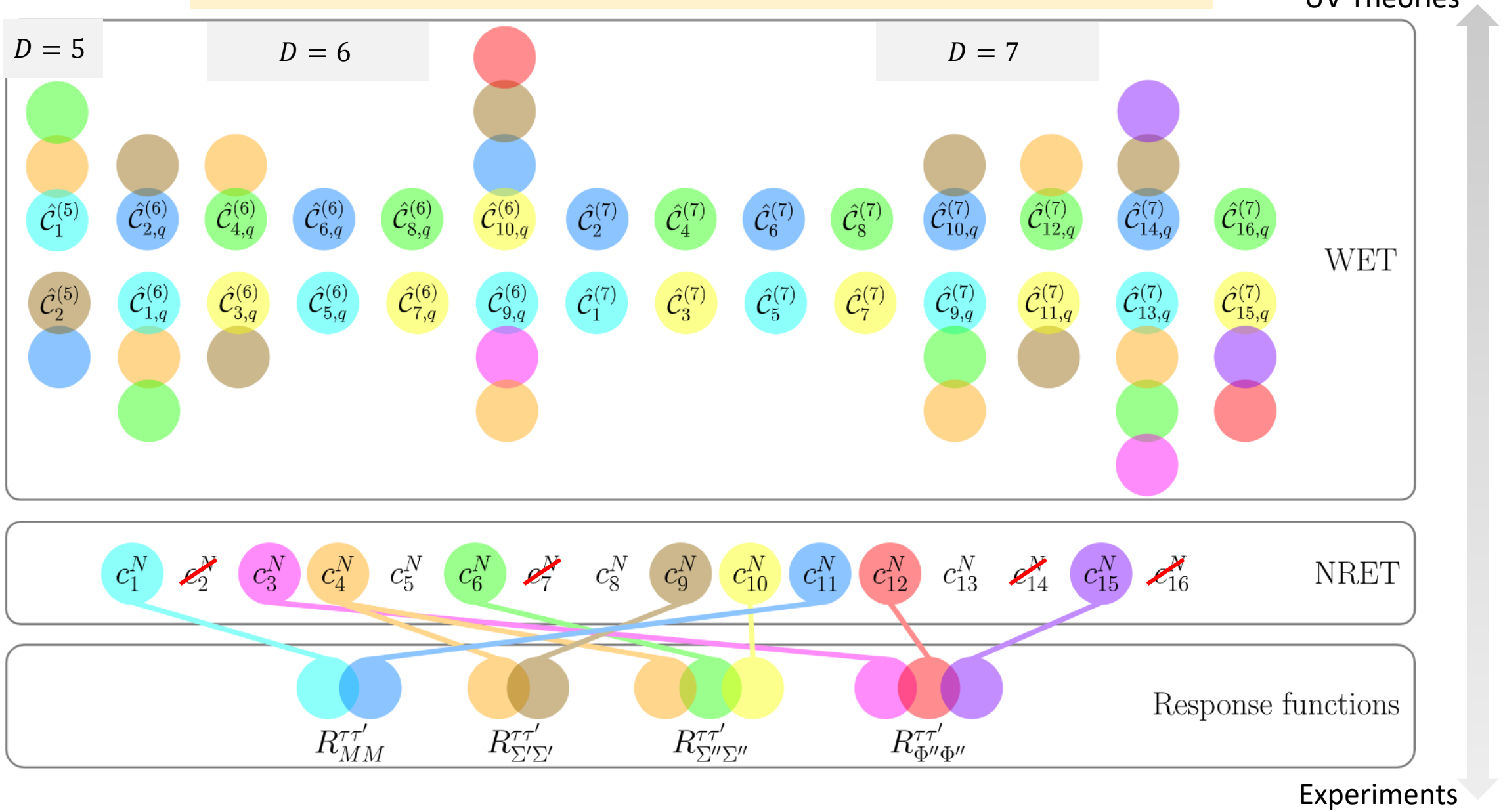


Semi-coherent




UV Theories
$D = 5$
$D = 6$
$D = 7$
$\hat{\mathcal{C}}_1^{(5)}$ $\hat{\mathcal{C}}_2^{(5)}$
$\hat{\mathcal{C}}_{2,q}^{(6)}$ $\hat{\mathcal{C}}_{1,q}^{(6)}$ $\hat{\mathcal{C}}_{4,q}^{(6)}$ $\hat{\mathcal{C}}_{3,q}^{(6)}$ $\hat{\mathcal{C}}_{6,q}^{(6)}$ $\hat{\mathcal{C}}_{5,q}^{(6)}$ $\hat{\mathcal{C}}_{8,q}^{(6)}$ $\hat{\mathcal{C}}_{7,q}^{(6)}$ $\hat{\mathcal{C}}_{10,q}^{(6)}$ $\hat{\mathcal{C}}_{9,q}^{(6)}$
$\hat{\mathcal{C}}_2^{(7)}$ $\hat{\mathcal{C}}_1^{(7)}$ $\hat{\mathcal{C}}_4^{(7)}$ $\hat{\mathcal{C}}_3^{(7)}$ $\hat{\mathcal{C}}_6^{(7)}$ $\hat{\mathcal{C}}_5^{(7)}$ $\hat{\mathcal{C}}_8^{(7)}$ $\hat{\mathcal{C}}_7^{(7)}$
$\hat{\mathcal{C}}_{10,q}^{(7)}$ $\hat{\mathcal{C}}_{9,q}^{(7)}$ $\hat{\mathcal{C}}_{12,q}^{(7)}$ $\hat{\mathcal{C}}_{11,q}^{(7)}$ $\hat{\mathcal{C}}_{14,q}^{(7)}$ $\hat{\mathcal{C}}_{13,q}^{(7)}$ $\hat{\mathcal{C}}_{16,q}^{(7)}$ $\hat{\mathcal{C}}_{15,q}^{(7)}$
WET
c_1^N c_2^N c_3^N c_4^N c_5^N c_6^N c_7^N c_8^N c_9^N c_{10}^N c_{11}^N c_{12}^N c_{13}^N c_{14}^N c_{15}^N c_{16}^N
NRET
$R_{MM}^{\tau\tau'}$ $R_{\Sigma'\Sigma'}^{\tau\tau'}$ $R_{\Sigma''\Sigma''}^{\tau\tau'}$ $R_{\Phi''\Phi''}^{\tau\tau'}$
Response functions
Experiments

All $D \leq 7$ WET operators map to at least one leading-order response function



MuonBridge Code

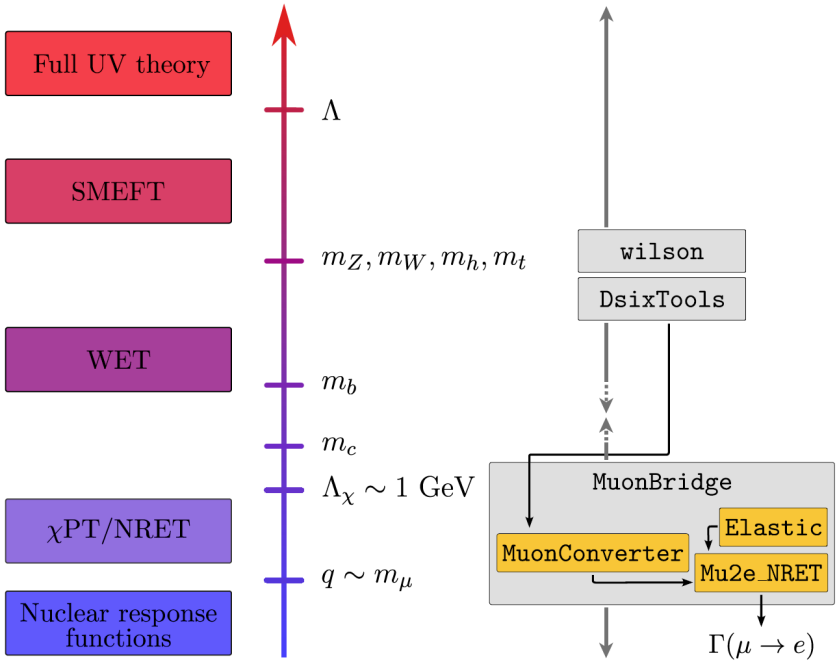
Haxton, McElvain, Menzo, **ER**, & Zupan, 2406.13818

https://github.com/Berkeley-Electroweak-Physics/MuonBridge

Python + Mathematica Tools for $\mu \to e$ Conversion

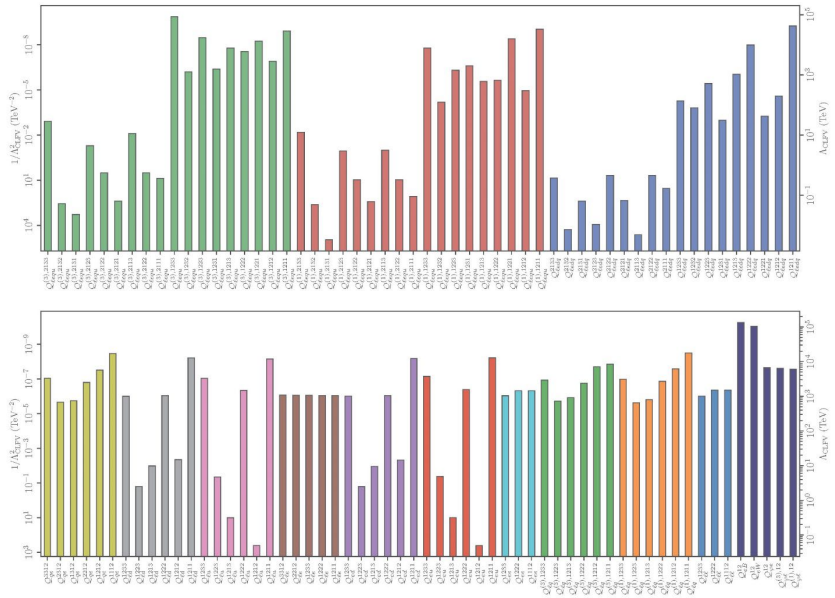
- Connects arbitrary UV physics to experiment

User-specified UV Theory



$\mu \to e$ conversion rate
in chosen nuclear target

Useful for theorists: model building, sensitivity studies



Useful for experimentalists: constraints, design, isotope selection

& Zupan, 2406.13818

loweak-Physics/MuonBridge

MµON BRIDGe

tica Tools for $\mu \to e$ Conversion

rary UV physics to experiment

orists: model building, sensitivity

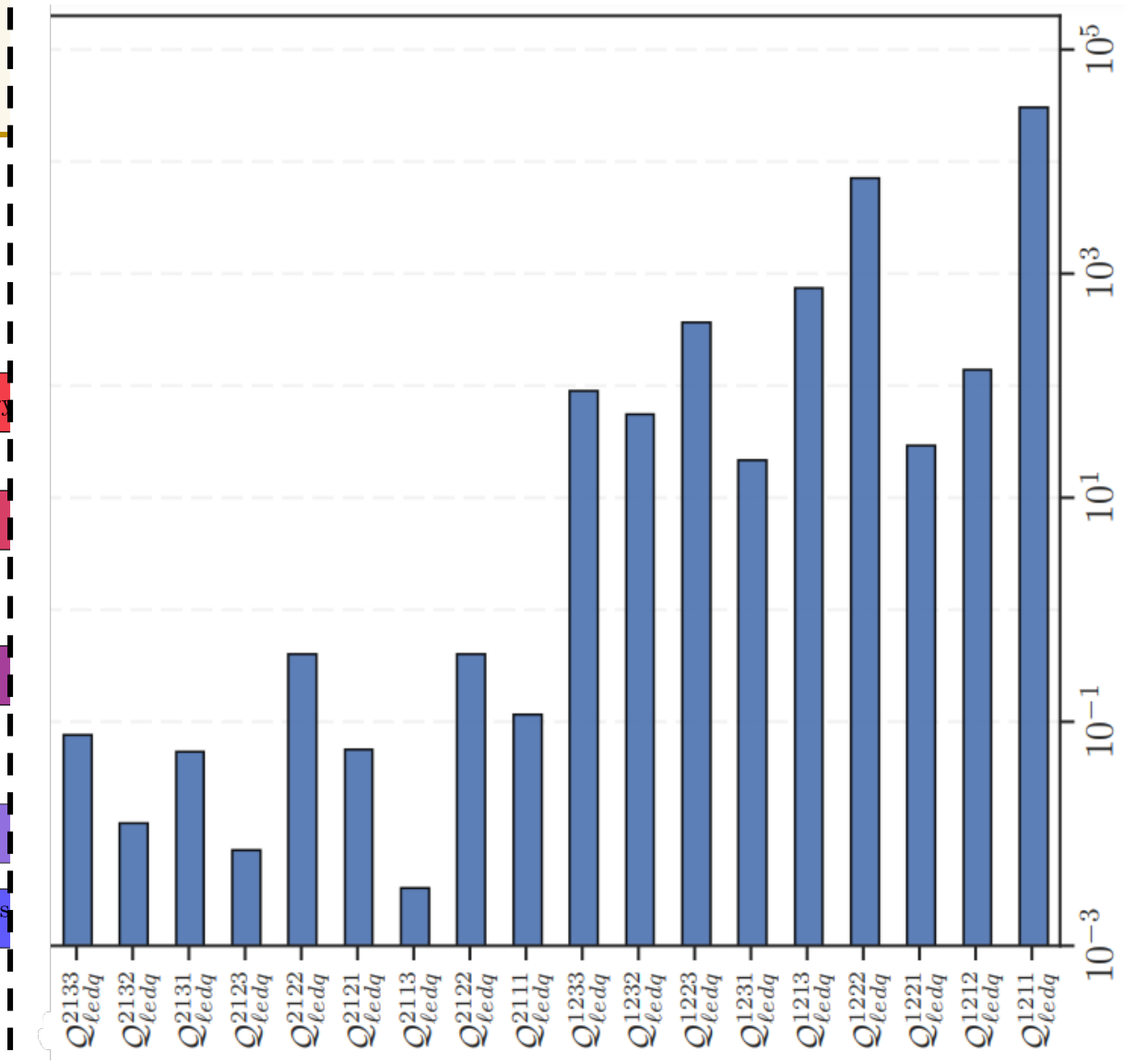
Full UV theory

SMEFT

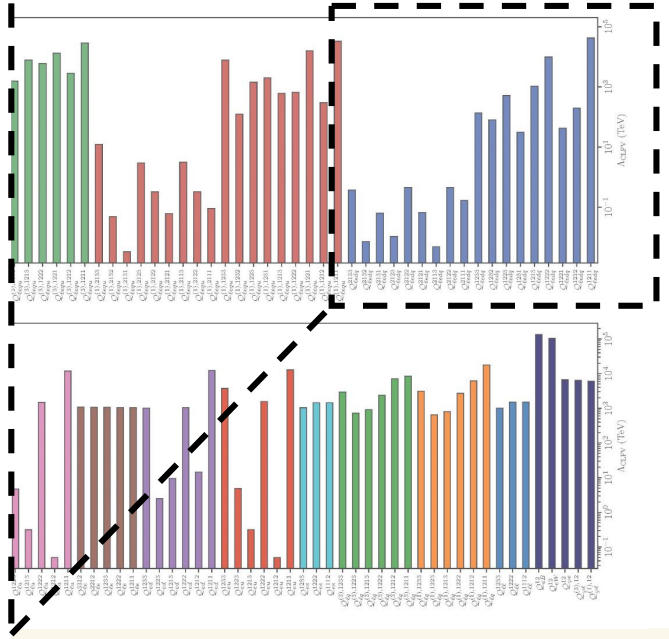
WET

χPT/NRET

Nuclear respons functions



in chosen nuclear target

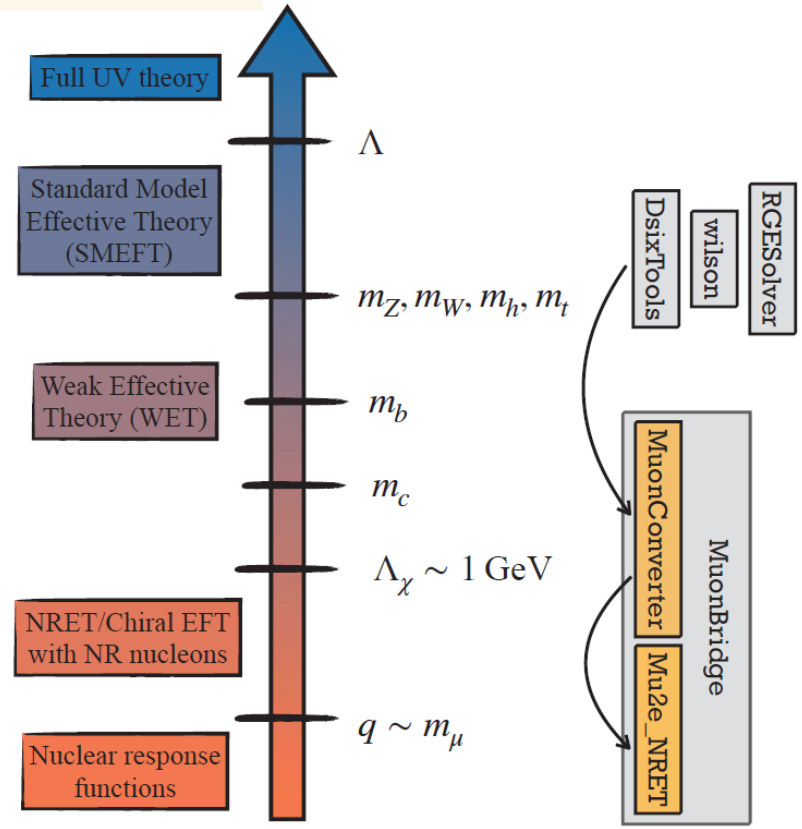


Useful for experimentalists: constraints, design, isotope selection

Summary

https://github.com/Berkeley-Electroweak-Physics/MuonBridge

- **Complete effective theory** description from **UV** to **experiments**
- **Nuclear ET** identifies **six CLFV response functions** + **two interference terms** probed by **elastic $\mu \to e$ conversion**
- Publicly-available **Python** & **Mathematica codes** for $\mu \to e$ effective theory



ER, Haxton, and McElvain, Phys. Rev. Lett. **130**, 131901 (2023)
Haxton, **ER**, McElvain, and Ramsey-Musolf, Phys. Rev. C **107**, 035504 (2023)
Haxton, McElvain, Menzo, **ER**, & Zupan, hep-ph/2406.13818

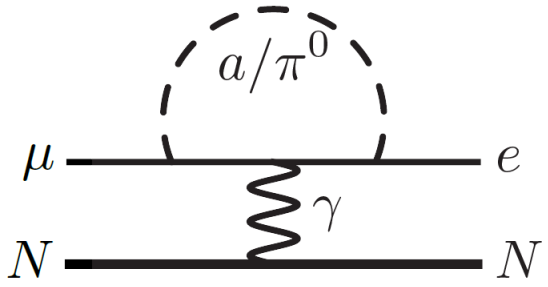
Backup slides

Spin-dependent $\mu \to e$ Conversion

Flavor-violating ALPs

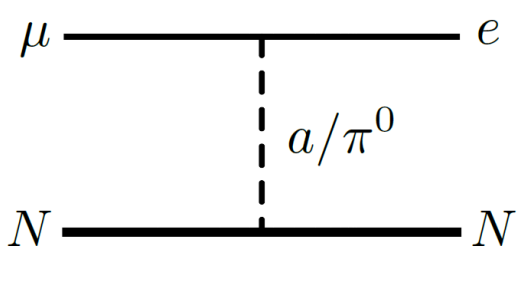
Fuyuto & Mereghetti, arXiv:2307:13076

$$\mathcal{L}^a \supset -2i\frac{a}{f_a}m_\mu a_{\mu\mu}\bar{\mu}\gamma_5\mu - i\frac{a}{f_a}m_\mu\bar{e}\left(v_{e\mu} + a_{e\mu}\gamma_5\right)\mu$$



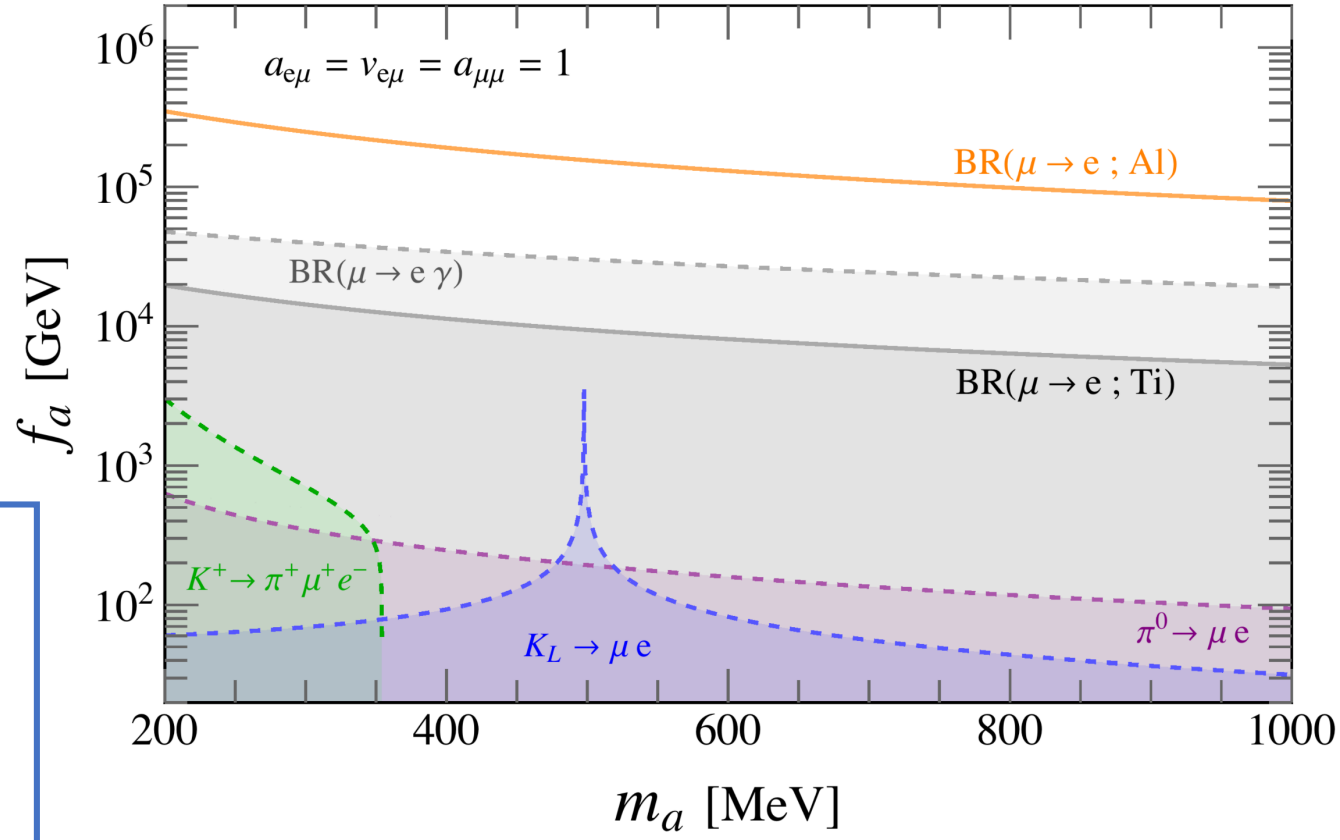
Coherent

$\langle M_0 \rangle$



Spin-dependent

$\langle \Sigma_J'' \rangle$



- Coherent dipole is suppressed by $\approx \left(\frac{\alpha_{EM}}{4\pi}\right)^2$
- Spin-dependent process dominates

Single-nucleon Effective Theory (NRET)

Building blocks:

$1_L,\ 1_N,\ i\hat{q},\ \vec{v}_N,\ \vec{\sigma}_L,\ \vec{\sigma}_N$

16 Single-nucleon CLFV operators

2 nuclear charges, 3 nuclear currents

$\mathcal{O}_1 = 1_L 1_N$
$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\ 1_N$

$\mathcal{O}_7 = 1_L\ \vec{v}_N\cdot\vec{\sigma}_N$
$\mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\ \vec{v}_N\cdot\vec{\sigma}_N$

$\mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N$
$\mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{\sigma}_N$
$\mathcal{O}_9 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N)$
$\mathcal{O}_{10} = 1_L\ i\hat{q}\cdot\vec{\sigma}_N$

$\mathcal{O}'_2 = 1_L\ i\hat{q}\cdot\vec{v}_N$
$\mathcal{O}_5 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N)$
$\mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N$
$\mathcal{O}'_{16} = i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{v}_N$

$\mathcal{O}_3 = 1_L\ i\hat{q}\cdot(\vec{v}_N\times\vec{\sigma}_N)$
$\mathcal{O}_{12} = \vec{\sigma}_L\cdot(\vec{v}_N\times\vec{\sigma}_N)$
$\mathcal{O}'_{13} = \vec{\sigma}_L\cdot[i\hat{q}\times(\vec{v}_N\times\vec{\sigma}_N)]$
$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot(\vec{v}_N\times\vec{\sigma}_N)$

$$\mathcal{L}_{\text{eff}} \sim \sqrt{2}G_F \sum_{i=1}^{16}\sum_{\tau=0,1} \tilde{c}_i^{\tau} O_i t^{\tau}$$

Unknown $\tilde{c}_i^{\tau}$'s contain all CLFV physics

- Target-independent

Coherent Conversion

Dipole:

$$\frac{1}{\Lambda^2} C_{D\alpha} m_\mu \, \bar{e}\sigma^{\lambda\nu} P_\alpha \mu \, F_{\lambda\nu}$$

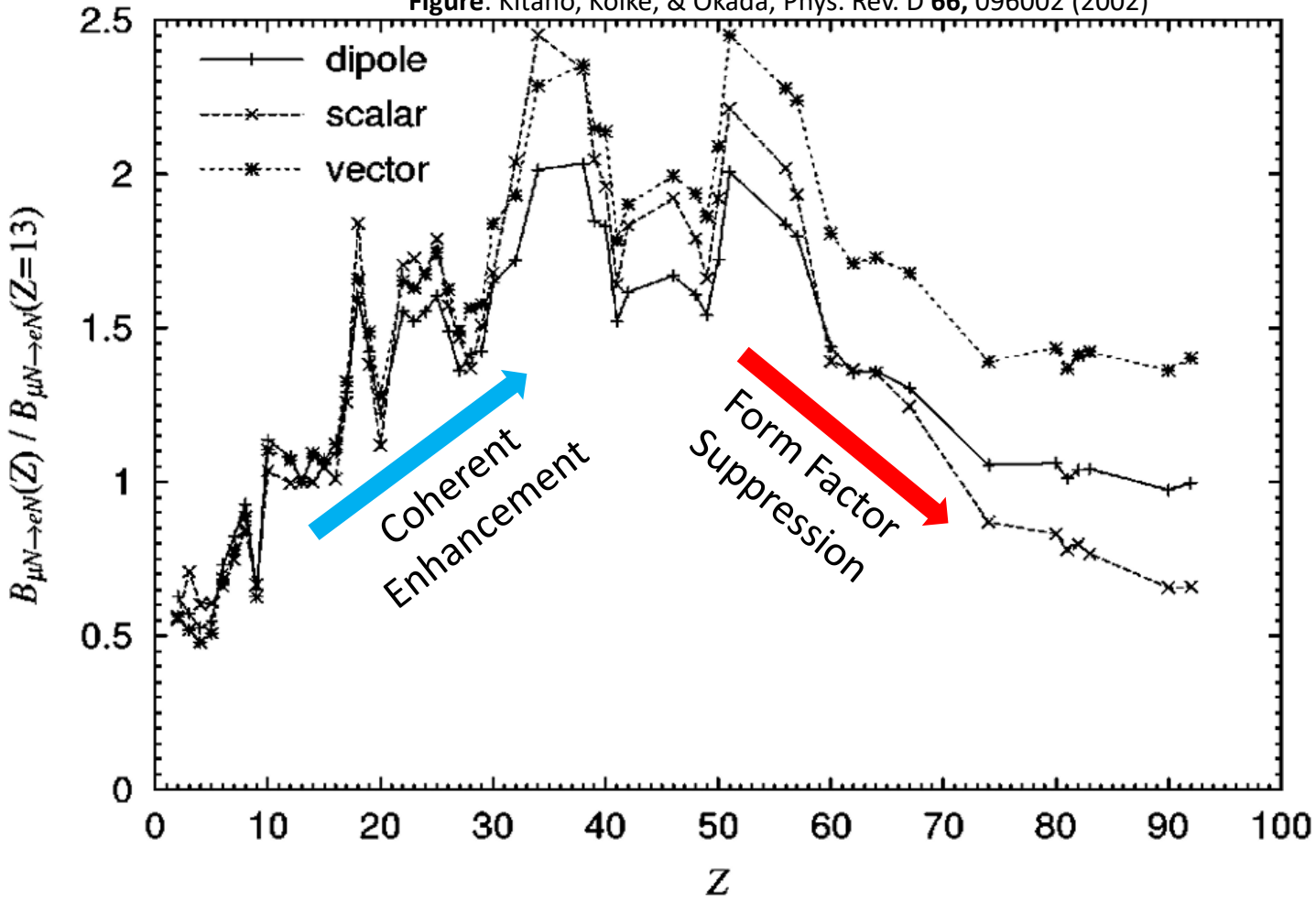
Scalar:

$$\frac{1}{\Lambda^2} \sum_q C_{S\alpha}^{(q)} G_F m_\mu m_q \, \bar{q} q \, \bar{e} P_\alpha \mu$$

Vector:

$$\frac{1}{\Lambda^2} \sum_q C_{V\alpha}^{(q)} G_F m_\mu m_q \, \bar{q}\gamma^\nu q \, \bar{e}\gamma_\nu P_\alpha \mu$$

Figure: Kitano, Koike, & Okada, Phys. Rev. D **66,** 096002 (2002)

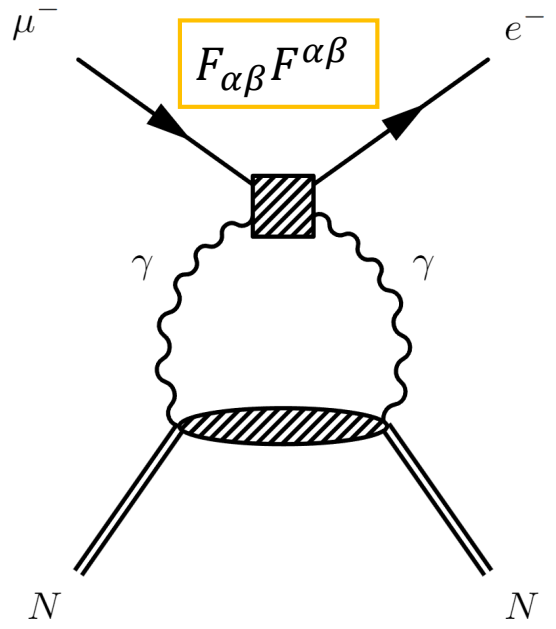


Nuclear operator: isoscalar charge monopole $\boldsymbol{M_0(q)}$

- Rate enhanced by A^2
- Nuclear matrix elements constrained by electron scattering, πN scattering, pionic atoms,…

Rayleigh Operators

CP-even Rayleigh Operator



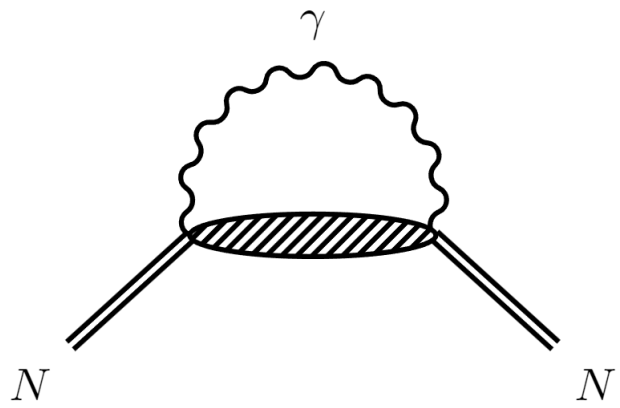
Forward Limit $q \to 0$

$$F_\gamma^N(0) = -\frac{\alpha_{EM}}{3\pi}\delta M_N^\gamma$$

$$F_\gamma^p(0) = 4.7(2.6) \times 10^{-7}\ \text{GeV}$$
$$F_\gamma^n(0) = 1.5(0.5) \times 10^{-6}\ \text{GeV}$$

Isoscalar magnetic polarizability $\beta_p + \beta_n$

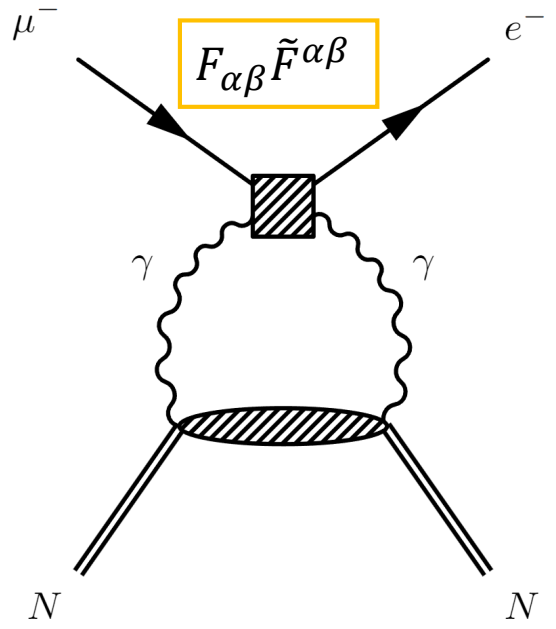
Nucleon EM Self-energy



Walker-Loud, Carlson, & Miller, PRL **108**, 232301 (2012)

Rayleigh Operators

CP-odd Rayleigh Operator

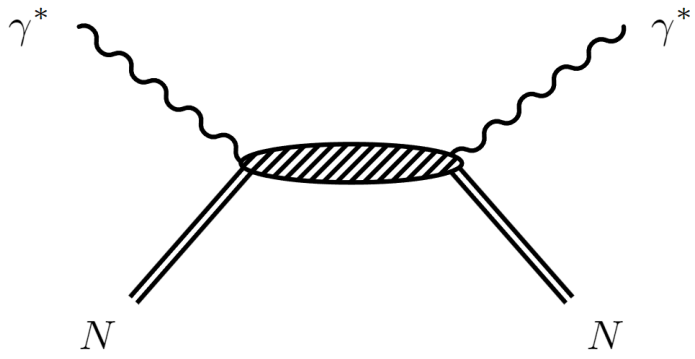


Forward Limit $q \to 0$

Elastic contribution in Born approx.

$$F_{\tilde{\gamma}}^{p}(0) = 3.83(3) \times 10^{-6}\ \text{GeV}$$
$$F_{\tilde{\gamma}}^{n}(0) = -3.9(7) \times 10^{-7}\ \text{GeV}$$

Spin-dependent forward double-virtual Compton Scattering



Lensky et al., Phys. Rev. D **95**, 074001