Near-threshold Heavy-Strange mesons from Lattice QCD and Hamiltonian Effective Field Theory

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The XVIth Quark Confinement and the Hadron Spectrum Conference
2024.08.21

Outline

- Background
- Hamiltonian Effective Field Theory (HEFT): quark & hadron degrees of freedom
- Study $D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$
- Predict $B_{s0}(5730)$, $B_{s1}(5770)$
- Summary

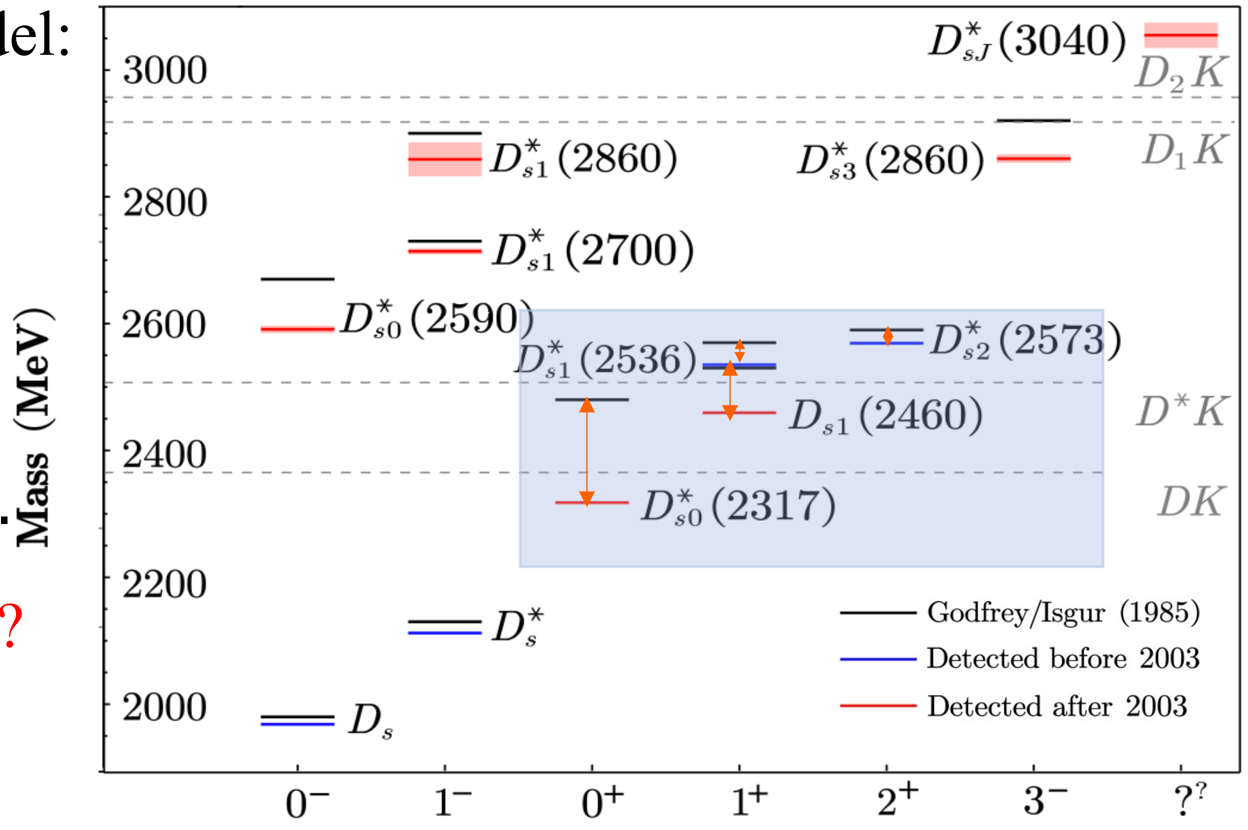
Puzzles of P-wave D_s mesons

- P-wave excited $c\bar{s}$ mesons in quark model:

$$S_{c\bar{s}} = 0,\ J^P = 1^+$$
$$S_{c\bar{s}} = 1,\ J^P = 0^+, 1^+, 2^+$$

- $D_{s1}^*(2536)$ & $D_{s2}^*(2573)$: $m_{exp} \sim m_{the}$.
- $D_{s0}^*(2317)$ & $D_{s1}(2460)$: $m_{exp} < m_{the}$?
- Close to the $D^{(*)}K$ threshold.

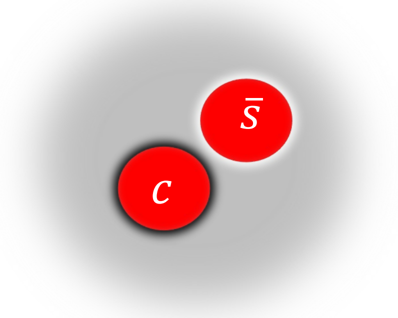
Coupled-channel effects will contribute



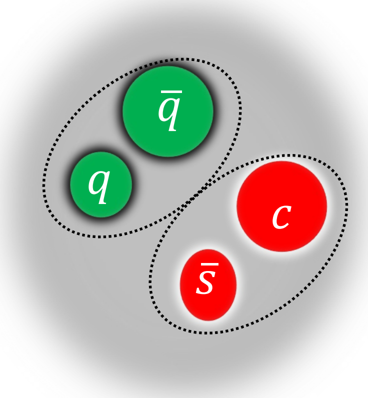
Lu *et al*,. Phys. Rept. 1019, 1-149, (2023)

$D_{s0}^{*}(2317)$& $D_{s1}(2460)$

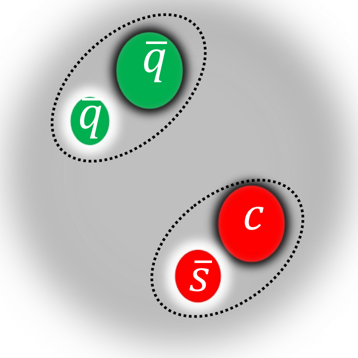
- Various theoretical models: quenched quark model, tetraquark, molecule, …



Dressed $c\bar{s}$ meson



Compact $cq\bar{s}\bar{q}$



$D^{(*)}K$ molecule

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……

Puzzles of P-wave D_s mesons

- Quark model successfully described other D_s mesons: $D_{s1}(2536)$ & $D_{s2}^*(2573)$.
  - Existence of $c\bar{s}$ meson
- Excited hadrons: create $\bar{q}q$ pair from the vacuum
  - Existence of hadronic channels $D^{(*)}K$
- Coupled-channel effect: extremely important for near-threshold states.
- $D_{s0}^*(2317)$ & $D_{s1}(2460)$ lie closely to the $D^{(*)}K$ channels.

$c\bar{s}$ + $D^{(*)}$K + **coupled-channel effects** to study **4** P-wave D_s states.

Hamiltonian Framework

➢ The Hamiltonian $H = H_0 + H_I,$

bare $c\bar{s}$ meson

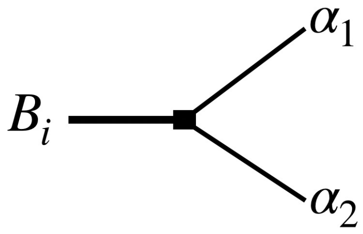
$D^{(*)}K$ channel

- Non-interacting $$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

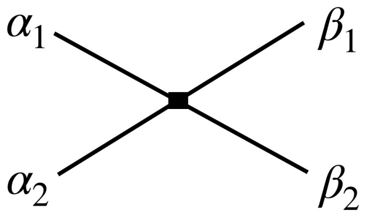
$m_B, E_\alpha(\vec{k})$ → Well established quark model

- Interacting $H_I = g + v$

bare state core -> channel:



channel -> channel:



$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\} \quad v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Hamiltonian Framework

- The Hamiltonian $H = H_0 + H_I,$

  bare $c\bar{s}$ meson $D^{(*)}K$ channel

  - Non-interacting $H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$

$m_B, E_\alpha(\vec{k})$ → Well established quark model

  - Interacting $H_I = g + v$

→ Lack of experimental data for 2→2 scattering

- Energy levels in lattice QCD as an Alternative

finite volume —Hamiltonian Effective Field Theory (HEFT)→ $\boldsymbol{H_I}$ infinite volume

Hamiltonian Effective Field Theory (HEFT)

- In the finite volume, the momentum is discretized as

$$k_n = 2\pi\sqrt{n}/L, \quad n = n_x^2 + n_y^2 + n_z^2, \quad n = 0,1,2,\ldots$$

Continuous	$\int d\vec{k}$	and	$\lvert\alpha(\vec{k}_\alpha)\rangle$	and	$\langle\beta(\vec{k}_\beta)\lvert\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓		↓		↓
Discrete	$\sum_i \left(\frac{2\pi}{L}\right)^3$	and	$\left(\frac{2\pi}{L}\right)^{-3/2}\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha$	and	${}_\beta\langle\vec{k}_j,-\vec{k}_j\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

$$H_0 = \sum_{i=1,n} \lvert B_i\rangle m_i \langle B_i\rvert + \sum_{\alpha,i} \lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \langle\vec{k}_i,-\vec{k}_i\rvert$$

$$H_I = \sum_j \left(\frac{2\pi}{L}\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[\lvert\vec{k}_j,-\vec{k}_j\rangle_\alpha\, g_{i,\alpha}^{+} \langle B_i\rvert + \lvert B_i\rangle g_{i,\alpha}\ {}_\alpha\langle\vec{k}_j,-\vec{k}_j\rvert\right]$$

$$+ \sum_{i,j} \left(\frac{2\pi}{L}\right)^3 \sum_{\alpha,\beta} \lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha\, v_{\alpha,\beta}\ {}_\beta\langle\vec{k}_j,-\vec{k}_j\rvert$$

J.M.M. Hall, et al. Phys. Rev. D 87, 094510.
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Hamiltonian Effective Field Theory (HEFT)

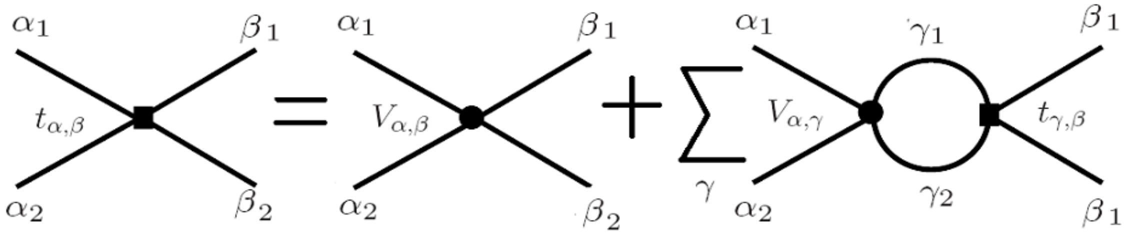
- Discrete Schrödinger Eq. to determine $\hat{g}$, $\hat{v}$

$$(H_0+H_\mathrm{I})|\Psi\rangle = \underline{E}|\Psi\rangle$$

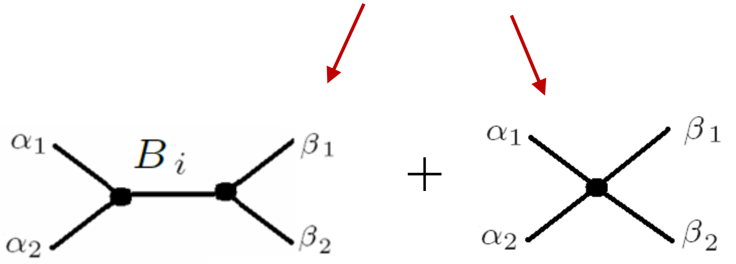
Energy levels in lattice QCD

- T Matrix: pole position, phase shift,..

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\,\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma 1}^2+k_\gamma^2}-\sqrt{m_{\gamma 2}^2+k_\gamma^2}+i\varepsilon}$$



$g_{i,\alpha}^{*}\dfrac{1}{E-m_i}g_{i,\beta}$



$v_{\alpha,\beta}$

Hamiltonian Effective Field Theory (HEFT)

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   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
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4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
   J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young
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   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
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   Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.1, 014506
7. **Nucleon resonance structure in the finite volume of lattice QCD**
   Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas   Phys.Rev. D95 (2017) no.11, 114507
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   Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W.Thomas   Phys.Rev. D97(2018) no.9, 094509
9. **Kaonic Hydrogen and Deuterium in Hamiltonian Effective Field Theory**
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10. **Partial Wave Mixing in Hamiltonian Effective Field Theory**
   Yan Li, Jia-jun Wu, Curtis D. Abell, Derek B. Leinweber, Anthony W. Thomas Phys.Rev. D101(2020) no.11,114501
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   Yan Li, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas   Phys.Rev. D103(2021) no.9, 094518
12. **Regularisation in Nonperturbative Extensions of Effective Field Theory**
   Curtis D. Abell, Derek B. Leinweber, Anthony W. Thomas, Jia-jun Wu   arXiv: 2110.14113
13. **Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold Ds States**   Zhi Yang, Guang-Juan Wang, Jia-jun Wu, Shi-lin Zhu, Makoto Oka   Phys.Rev.Lett.128(2020),112001

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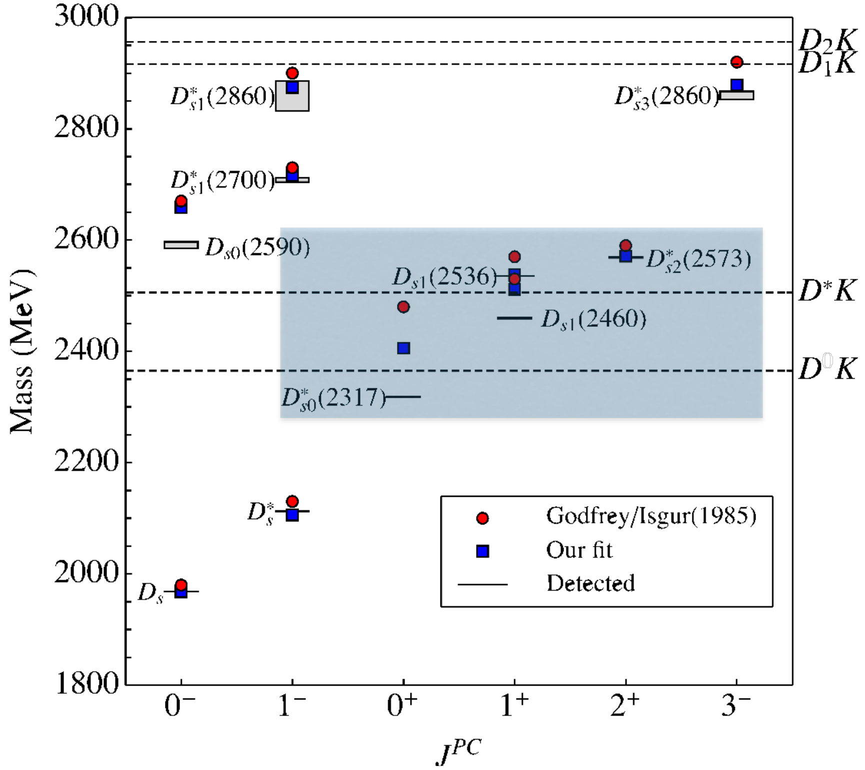
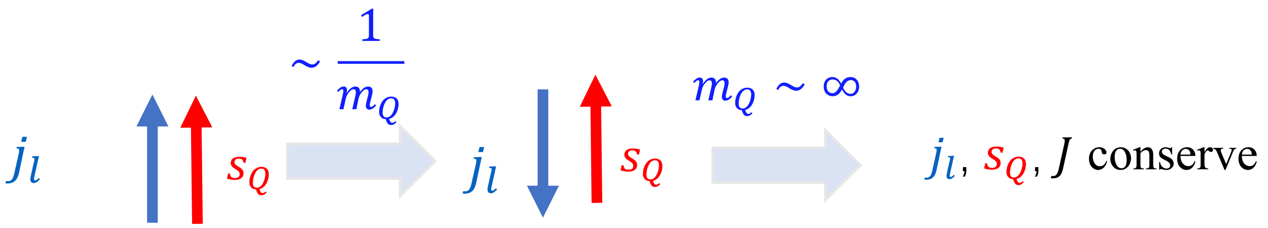
Quark model: bare $c\bar{s}$ state



Z. Yang, G.-J. Wang, J.J. Wu, S.-l. Zhu, M. Oka, Phys. Rev. Lett. 128. 112001

- Mass and wave functions of bare $c\bar{s}$ from GI model
- Good heavy quark spin symmetry



	$c\bar{s}$ core		channel	
	$B(\vert{}^{2S+1}L_J\rangle)$	B(mass)	α	L
$D^*_{s0}(2317)$	$\vert{}^3P_0\rangle$	2405.9	DK	S
$D^*_{s1}(2460)$	$0.68\vert{}^1P_1\rangle - 0.74\vert{}^3P_1\rangle = -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D^*_{s1}(2536)$	$-0.74\vert{}^1P_1\rangle - 0.68\vert{}^3P_1\rangle = -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D^*_{s2}(2573)$	$\vert{}^3P_2\rangle$	2571.2	DK, D^*K	D

0^+ $\quad |\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_0$

1^+ $\quad \phi_s = |\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_1$

→ S-wave $D^{(*)}K$

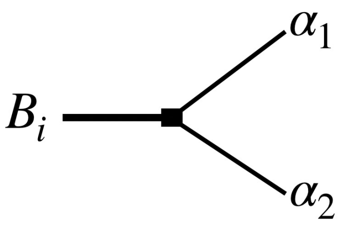
1^+ $\quad \phi_d = |\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_1$

2^+ $\quad |\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_2$

→ D-wave $D^{(*)}K$

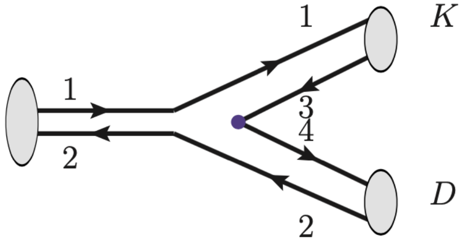
$H_I = g + v$

g: $\bar{c}s \to D^{(*)}K$



$^3P_0$ model

at quark level



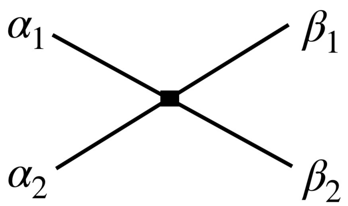
$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

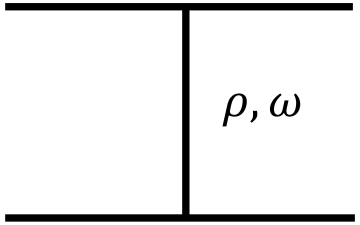
F

Undetermined γ & Λ'

P.G. Ortega, et al. Phys. Rev. D 94, 074037.

v: $D^{(*)}K - D^{(*)}K$





$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{PPV} + \mathcal{L}_{VVV} \\ &= ig_v \mathrm{Tr}(\partial^\mu P[P, V_\mu]) + ig_v \mathrm{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu]),\end{aligned}$$

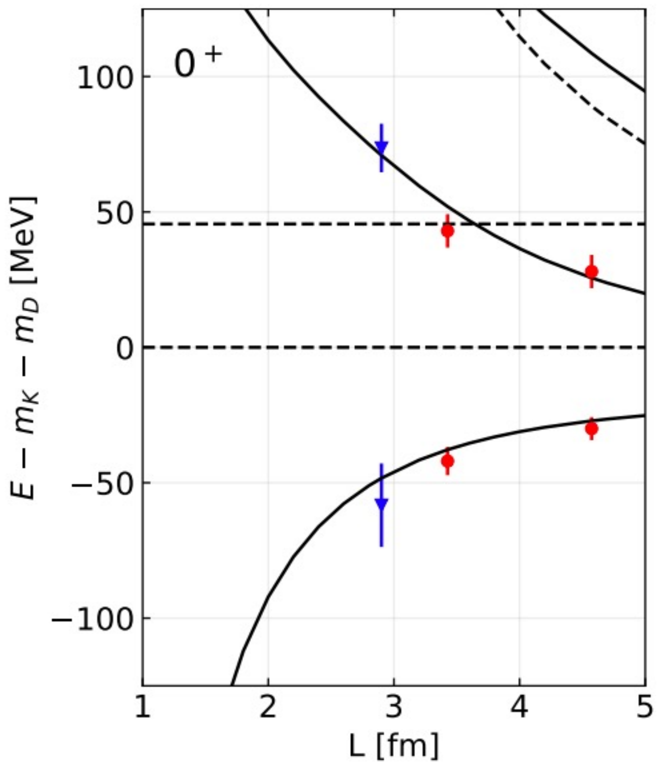
$$\mathcal{V}(l, l'S, j) = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{2E_D^i 2E_D^f 2E_K^i 2E_K^f}}\, 2\pi \int dcos\theta V^v(\vec{p}_f, \vec{p}_i)\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$$

Form factor

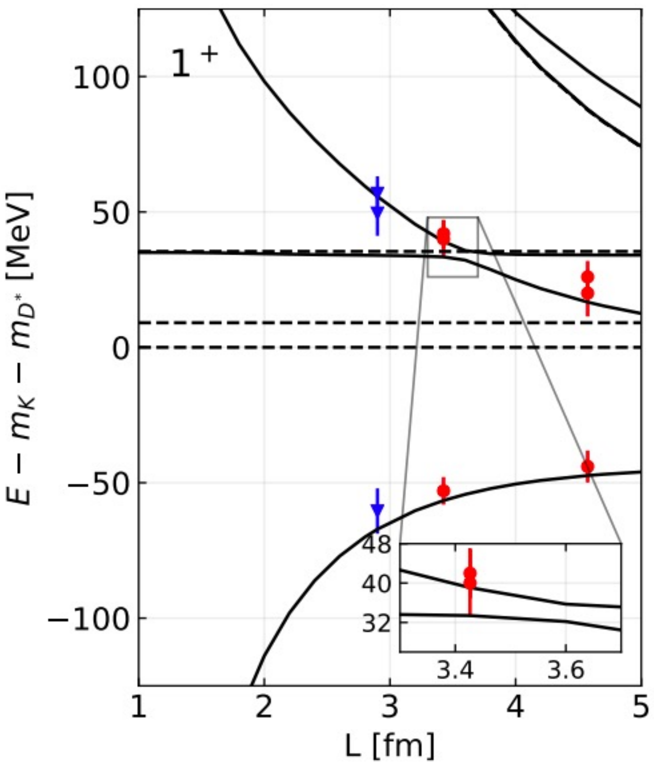
Undetermined $g_c = g_{VDD}\, g_{VKK}(g_{VD^*D^*}\, g_{VKK})$

Fitting the Lattice data

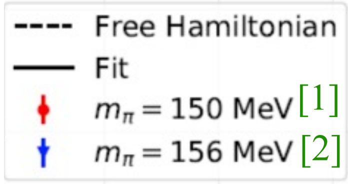
- Fit to the lattice energy levels in the 0^+ and 1^+ sector: χ^2/d.o.f =0.95 with $\Lambda = 1.0$ GeV.



$D_{s0}(2317)$



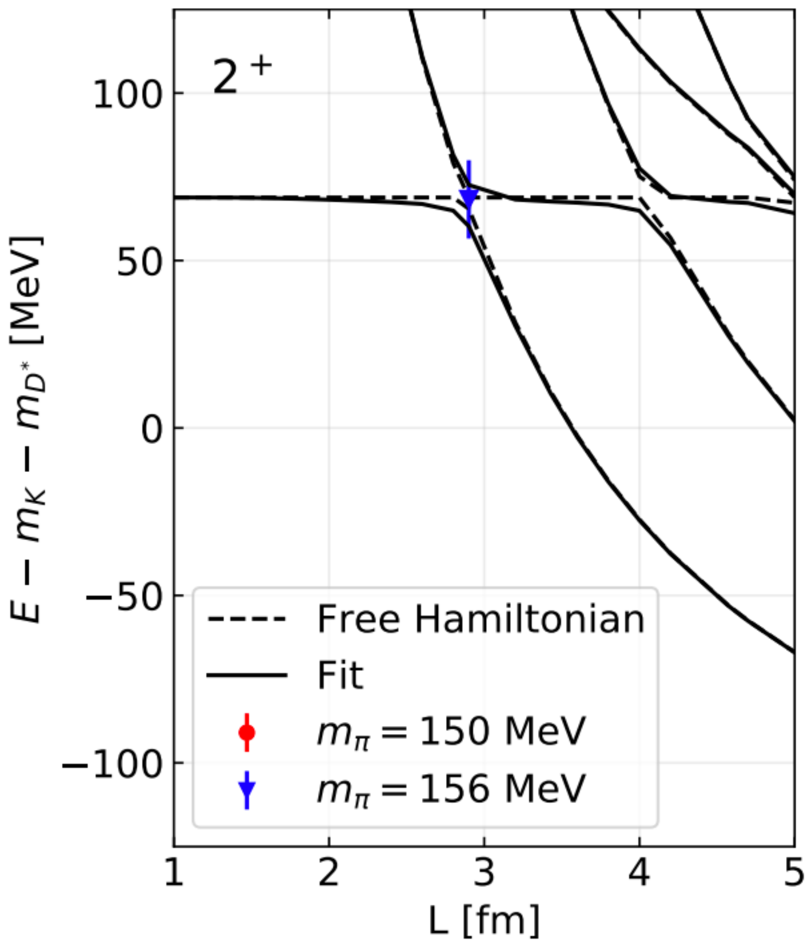
$D_{s1}(2460)$ & $D_{s1}(2536)$



Parameters	g_c	Λ' [GeV]	γ
Best fit	$4.2^{+2.2}_{-3.1}$	$0.323^{+0.033}_{-0.031}$	$10.3^{+1.1}_{-1.0}$
Ref. [3]		0.84	6.5
Ref. [4]	-	-	6.9
Ref. [5]	18.2/9.8	-	-
Ref. [6]	8.4	-	-

[1] G. Bali, et al. (RQCD Collaboration) Phys. Rev. D 96, 074501 (2017).
[2] C. Lang, et al. Phys. Rev. D 90, 034510 (2014) .
[3] P. Ortega, et al. Phys. Rev. D 94 , 074037 (2016).
[4] S. Godfrey, et al. Phys .Rev. D 93 (2016) 3, 034035.
[5] C. W. Shen, et al. Phys. Rev. D 100, 056006 (2019).
[6] Z.W. Lin, et al. Phys. Rev. C 61, 024904 (2000).

Prediction I: $D_{s2}(2573)$



1. $J^P = 2^+$: $c\bar{s}$ core + D-wave D^*K + D-wave DK.

2. The energy levels in the finite volume are predicted.

3. Mass and components: almost pure $c\bar{s}$ mesons.

Results

- Different mass shifting pattern:

	$P(c\bar{s})[\%]$	$P(D^{(*)}K)[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	68.0	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	47.6	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	1.8	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	4.1	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

$D_{s0}^*(2317)$ & $D_{s1}(2460)$: S-wave $D^{(*)}K$

→ Sizable mass shift & mixing

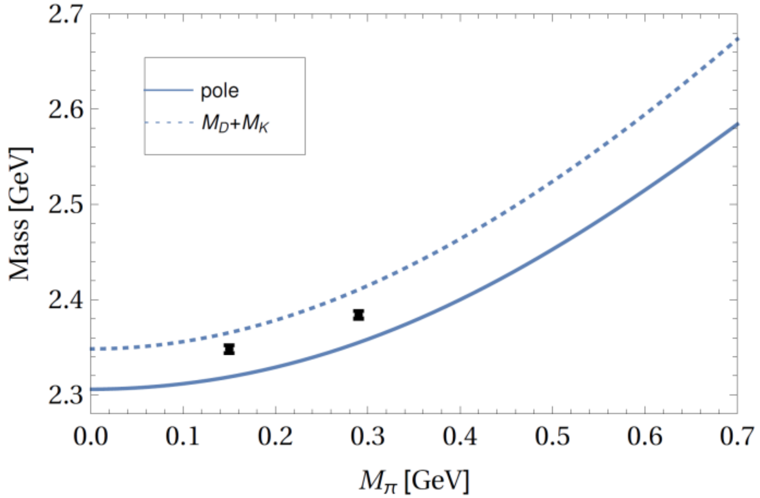
$D_{s1}(2536)$ & $D_{s2}(2573)$: D-wave $D^{(*)}K$

→ Small mass shift & tiny mixing

Prediction II: m_π- dependence

- DK molecule: Tends to become larger with larger m_π.

- Latest lattice results in G. Bali et al., PRD96(2017)074501



curves: prediction in Du et al., EPJC77(2017)728

- Bare state ($c\bar{s}$): Tends to become stable with larger m_π.

"…for the lower lying pseudoscalar and vector D_s meson masses which decrease by 3 MeV (from 1980(1) MeV at m_π = 290 MeV to 1977(1) at m_π = 150 MeV) and 7 MeV (from 2101(1) MeV to 2094(1) MeV), respectively, hinting that the 0+ and 1+ states may have a more complicated internal structure."

G. Bali et al., PRD96(2017)074501

Prediction II: m_π- dependence

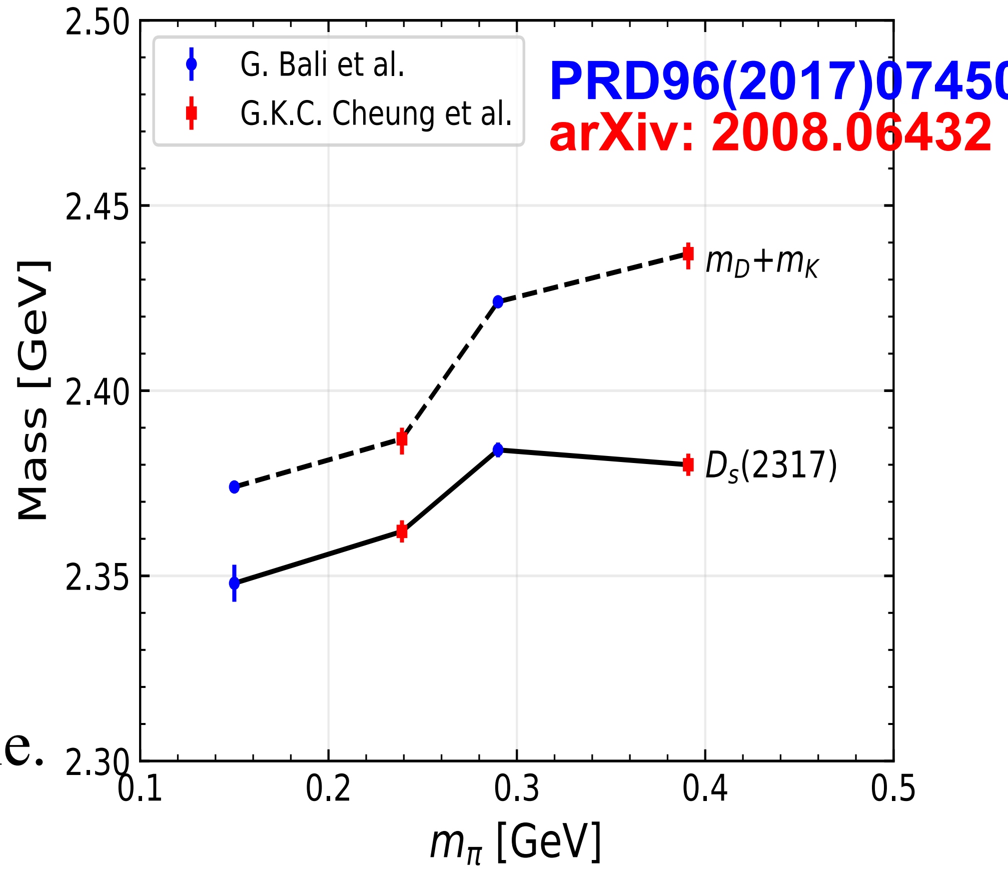
➢ Mass of $D_{s0}^*(2317)$ finally tends to become stable with increasing m_π.

- m_π ↗ , m_{DK} ↗ , $m_{c\bar{s}}$ → stable
- $m_{DK} < m_{c\bar{s}}$

\- $D_{s0}^*(2317)$: dominated by mainly $c\bar{s}$, increasing

- $m_{DK} >> m_{c\bar{s}}$:

\- $D_{s0}^*(2317)$ is mainly $c\bar{s}$. $M_{D_{s0}^*(2317)}$ tends to be stable.



Extension to P-wave B_s mesons

Z. Yang, G.-J. Wang, J.J. Wu, S.-l. Zhu, M. Oka, JHEP 01 058,1-20 (2023)

- Quark model

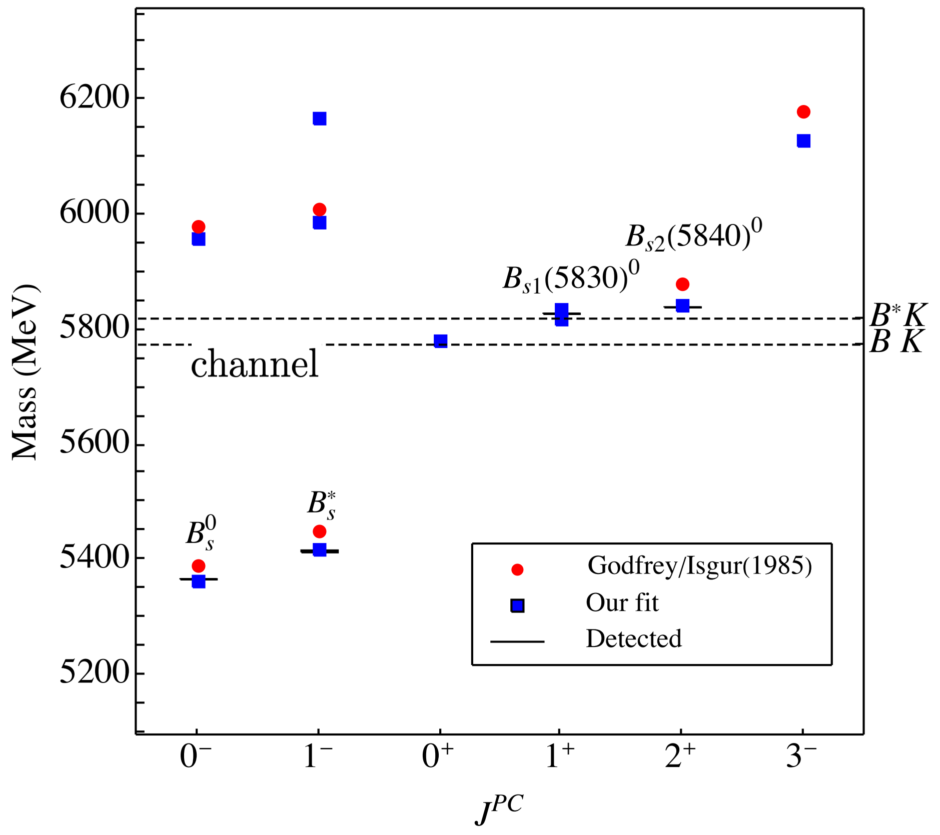
$S_{\bar{b}s} = 0,\ J^P = 1^+$

$S_{\bar{b}s} = 1,\ J^P = 0^+, 1^+, 2^+$

- Absence of 0^+ and lower 1^+ B_s states

$b\bar{s}$ cores | channel

	$b\,(\vert ^{2S+1}L_J\rangle)$	$b\,(\text{mass})$	α	L
B^*_{s0}	$\vert ^3P_0\rangle$	5780.9	$B\bar{K}$	S
B^*_{s1}	$-0.74\,\vert ^1P_1\rangle + 0.67\,\vert ^3P_1\rangle$	5818.5	$B^*\bar{K}$	S, D
	$= 0.98\,\phi_s - 0.22\,\phi_d$			
$B^{*\prime}_{s1}$	$0.67\,\vert ^1P_1\rangle + 0.74\,\vert ^3P_1\rangle$	5835.6	$B^*\bar{K}$	S, D
	$= 0.22\,\phi_s + 0.98\,\phi_d$			
$B^{*\prime}_{s2}$	$\vert ^3P_2\rangle$	5842.7	$B\bar{K},\ B^*\bar{K}$	D

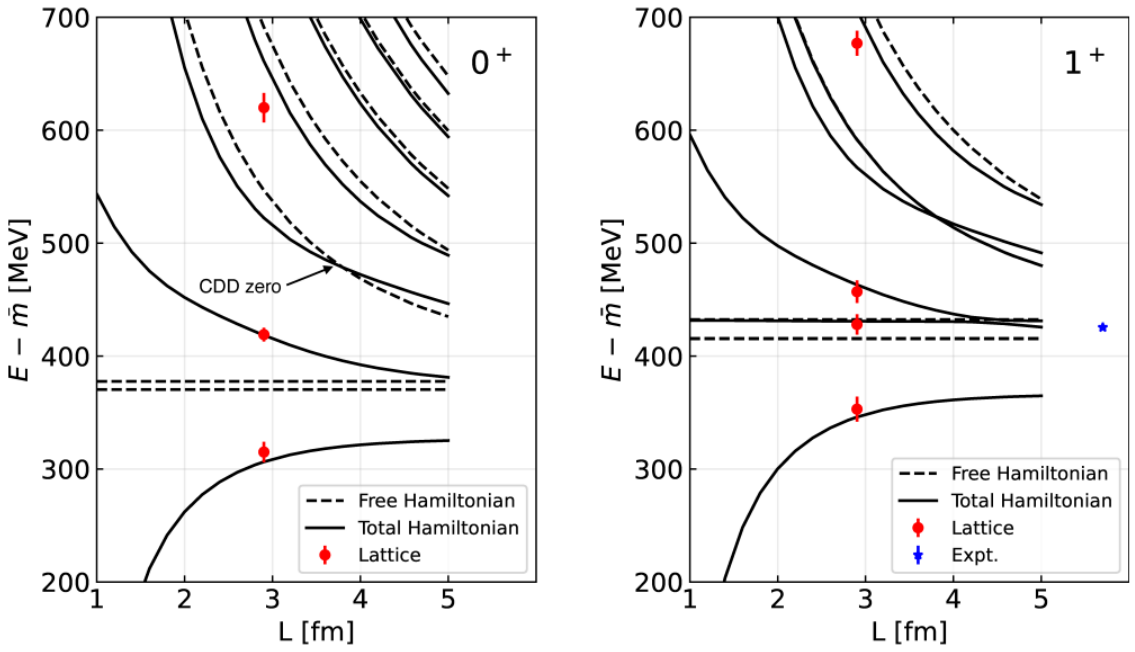


- 0^+ and lower 1^+ B_s-S wave $B^{(*)}\bar{K}$ channels → Sizable mass shift & mixing
- Higher 1^+ and 2^+ B_s-D wave $B^{(*)}\bar{K}$ channels → Small mass shift & tiny mixing

Extension to P-wave B_s mesons

➢ Heavy quark flavor symmetry: Using Previous Parameters

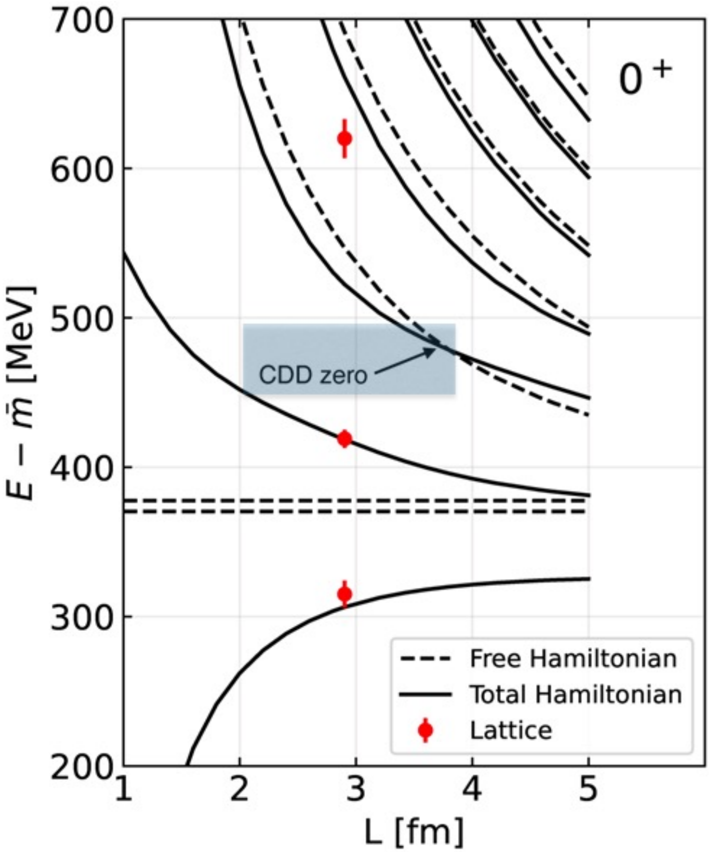
Postprediction, not a fit!

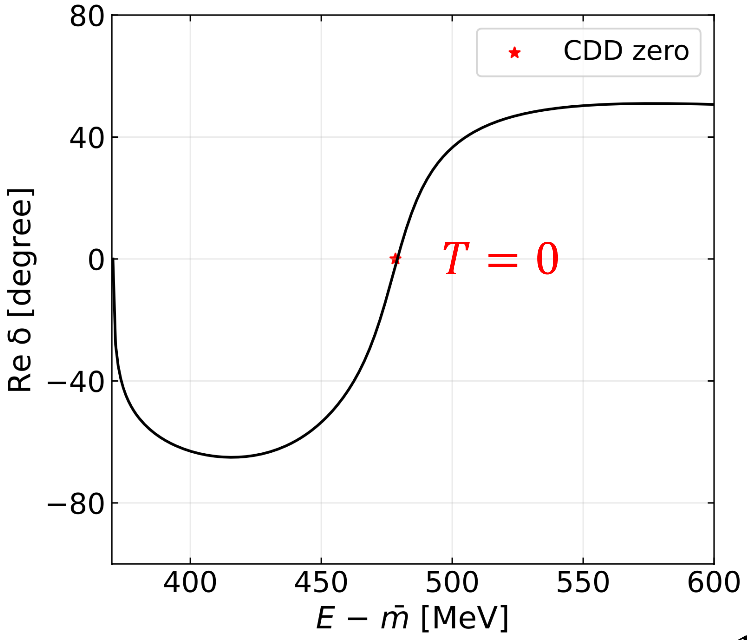


C.B. Lang , et al. Phys. Lett. B 750 (2015) 17

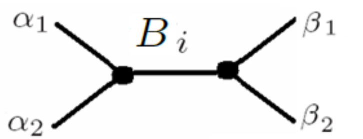
J^P		0^+	1^+
mass [MeV]	rel. quark model [65]	5804	5842
	rel. quark model [66]	5833	5865
	rel. quark model [67]	5830	5858
	nonrel. quark model [68]	5788	5810
	quark model (KKMT) [94]	5719	5765
	LO $\chi-$SU(3) [19]	5643	5690
	Bardeen, Eichten, Hill [95]	5718 ± 35	5765 ± 35
	LO UChPT [25, 26]	5725 ± 39	5778 ± 7
	NLO UHMChPT [31]	$5696\pm20\pm30$	$5742\pm20\pm30$
	NLO UHMChPT [96]	5720^{+16}_{-23}	5772^{+15}_{-21}
	HQET + ChPT [69]	5706.6 ± 1.2	5765.6 ± 1.2
	Covariant ChPT [70]	5726 ± 28	5778 ± 26
	local hidden gauge [71]	$5475.4\sim5457.5$	$5671.2\sim5663.6$
	heavy meson chiral unitary [72]	5709 ± 8	5755 ± 8
	lattice QCD [97]	$5752\pm16\pm5\pm25$	$5806\pm15\pm5\pm25$
	lattice QCD [93]	$5713\pm11\pm19$	$5750\pm17\pm19$
	this work	$5730.2^{+2.4}_{-1.5}$	$5769.6^{+2.4}_{-1.6}$
$P(\bar{b}s)$[%]	heavy meson chiral unitary [72]	$48.2\pm1.5/54.2\pm1.1$	$50.3\pm1.4/51.7\pm1.3$
	this work	$54.7^{+5.2}_{-4.1}$	$56.7^{+4.6}_{-3.7}$

A CDD Zero

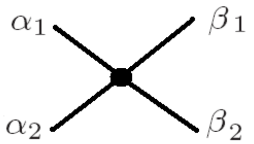




$$\bar{m} = \frac{1}{4}\left(m_{B_s} + 3m_{B_s^*}\right) = 5403.3 \text{ MeV}$$



$$g^*_{i,\alpha}\frac{1}{E-m_i}g_{i,\beta}$$



$$v_{\alpha,\beta}$$

- The CDD zero indicates there are two mechanisms which will cancel at this energy.
- Give a new method to search CDD zero: LQCD.

Summary

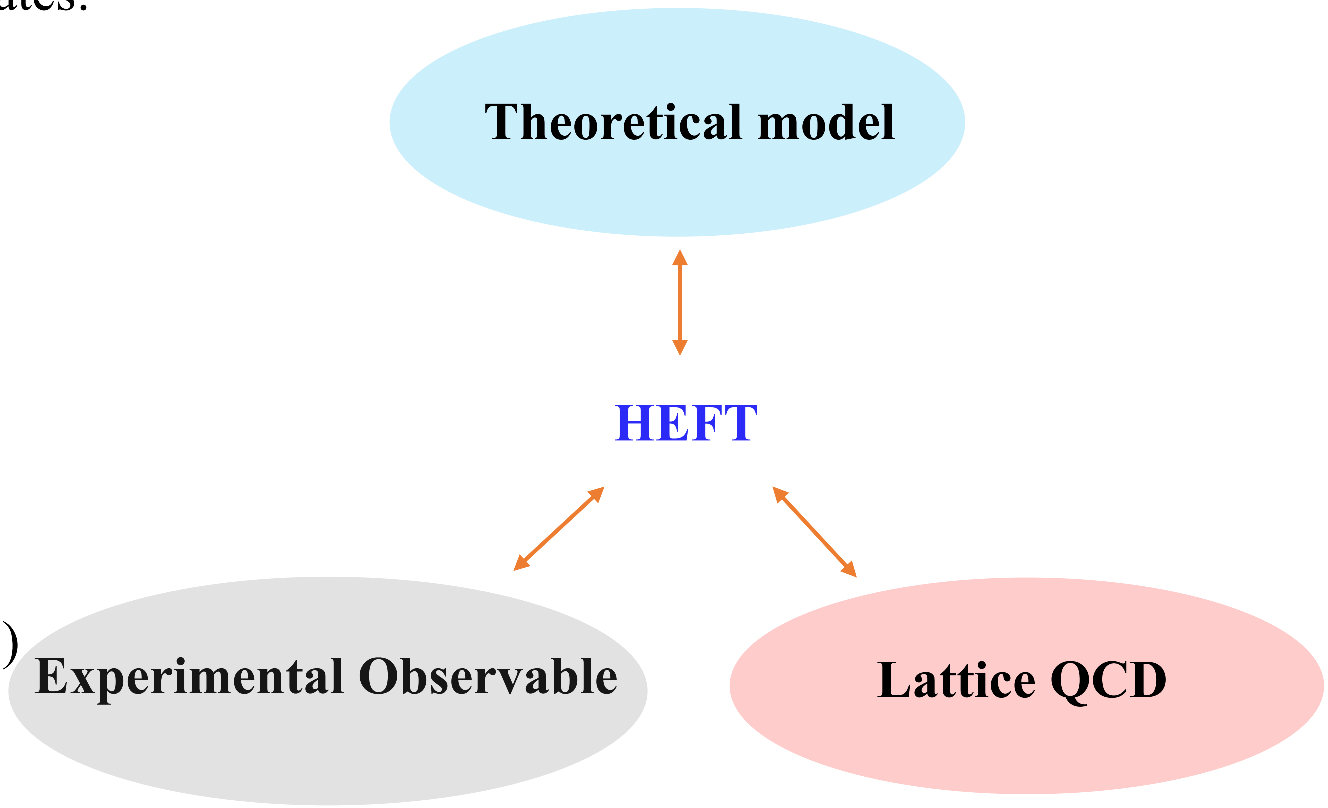
- Quark model + coupled channel effect +HEFT & Lattice QCD: Near-threshold states.

- Investigation of inner structures of P-wave D_s states:
  $D_{s0}(2317)$ [$c\bar{s}$-DK(S-wave)],
  $D_{s1}(2460)$ [$c\bar{s}$-$D^{(*)}$K (S-wave)],
  $D_{s1}(2536)$ [$c\bar{s}$-$D^{(*)}$ K(D-wave)],
  $D_{s2}(2573)$ [$c\bar{s}$-$D^{(*)}$ K(D-wave)].

- Prediction of the $B_{s0}(5730)$ and $B_{s1}(5770)$.

- $\boldsymbol{q\bar{q}}$ and **hadron interactions are** always there.

- Extension to other near-threshold states: X(3872)



Jia-Jun Wu
Parallel talk at C: Heavy quarks
(12:30-13:00, Aug. 22)

Thank you for your attention!

Backup slides

Quark model: bare $c\bar{s}$ state

The relativized quark model:

$$H = H_0 + V$$

$$H_0 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

$$V = G_{\text{eff}}(r) + S_{\text{eff}}(r)$$

$$G_{\text{eff}}(r) = \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2} \tilde{G}(r) \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2}$$

$$+ \left[\frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{G}_{11}^{\text{so}(v)}}{\partial r} + \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial \tilde{G}_{22}^{\text{so}(v)}}{\partial r} + \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}}{m_1 m_2} \frac{1}{r} \frac{\partial \tilde{G}_{12}^{\text{so}(v)}}{\partial r}\right]$$

$$+ \frac{2\mathbf{S}_1 \cdot \mathbf{S}_2}{3 m_1 m_2} \nabla^2 \tilde{G}_{12}^{c} - \left[\frac{\mathbf{S}_1 \cdot \hat{r} \mathbf{S}_2 \cdot \hat{r} - \frac{1}{3} \mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\right] \left[\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right] \tilde{G}_{12}^{t}$$

$$S_{\text{eff}}(r) = \tilde{S}(r) - \frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{S}_{11}^{\text{so}(s)}}{\partial r} - \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial \mathbf{S}_{22}^{\text{so}(s)}}{\partial r}$$

Godfrey, Isgur, Phys. Rev. D 32,189 (1985)

- Relativized Modification:

- Relativistic kinematic energy

- Energy dependence

$$\omega_{ij} = 1 + \frac{p_i p_j}{E_i E_j} \qquad \rho_{ij} = \frac{m_i m_j}{E_i E_j}$$

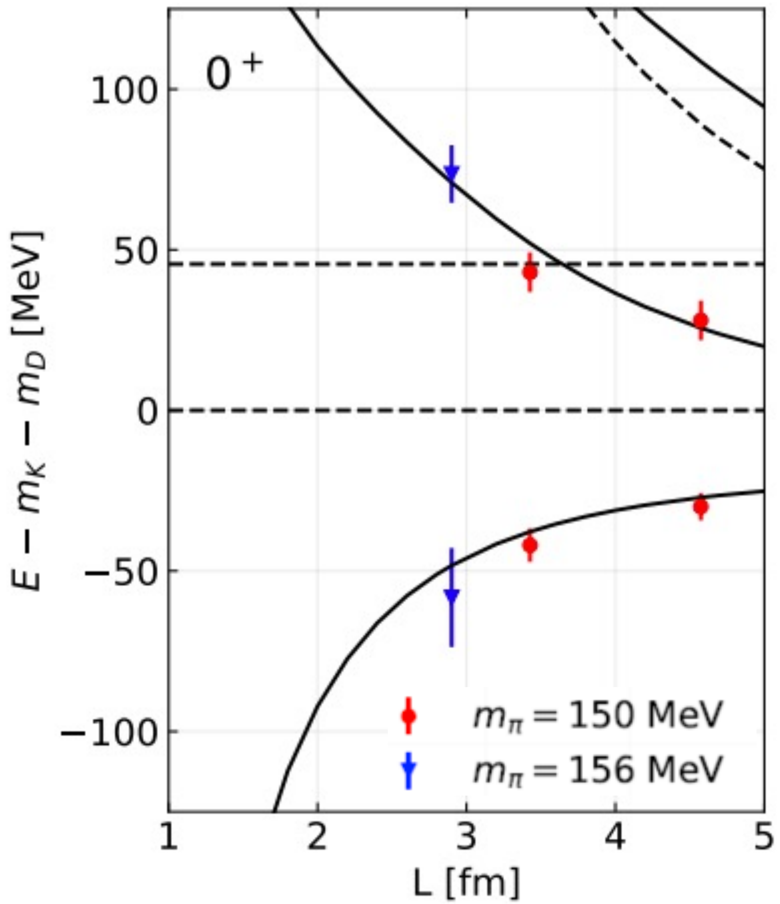
- Mass & wavefunction:

$$H|B\rangle = m_B\, |B\rangle$$

- Input of fit:

All the well-established mesons far away from two-meson thresholds as input (from π to Υ).

Results: $D_{s0}^{*}(2317)$



1. $J^P = 0^+$: $c\bar{s}\ (^3P_0)$ + S-wave DK.

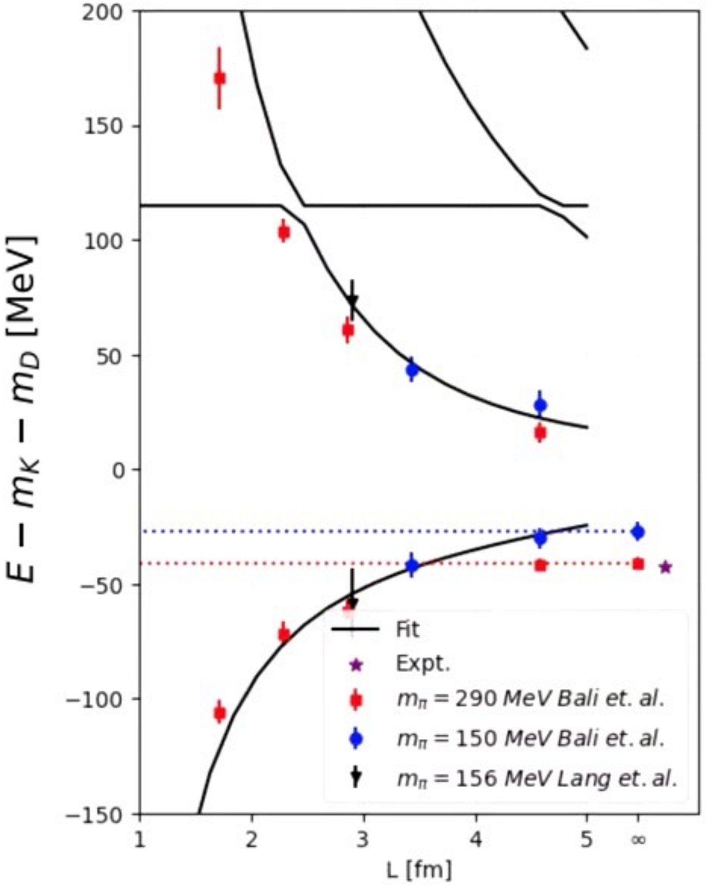
2. $M^{th}_{D_{s0}(2317)}(2338.9^{+2.1}_{-2.7}) > M^{exp}(2317.8)$ due to larger Lattice data.

3. A mixture of the bare $c\bar{s}$ & DK component: ($L = 4.57$ fm)

$$\mathrm{P}(DK) = 68.0\%, P(\bar{c}s) = 32.0\%$$

Coupled-channel effect: $D_{s0}^*(2317)$

➢ Without the coupling between the $c\bar{s}$ and DK: $g = 0$.




```
m = 7
vy_temp = get_eigenvalue_for_L_range(fit2.args, 5, 45, m)
vy_temp = (vy_temp[:,1:5] - threshold*ones(m,5))*1000
```

7×5 Array{Float64,2}:

-31.2307	15.2889	99.9988	114.901	182.443
-2.12962	5.35116	20.1357	36.946	49.5472
-0.15231	4.82955	10.0105	16.3477	22.0089
-0.0363223	2.9096	5.84284	9.10952	12.2431
-0.0136705	1.87382	3.75011	5.75603	7.7183
-0.00650106	1.29737	2.59473	3.95439	5.29493
-0.00356646	0.948922	1.89754	2.88051	3.85355

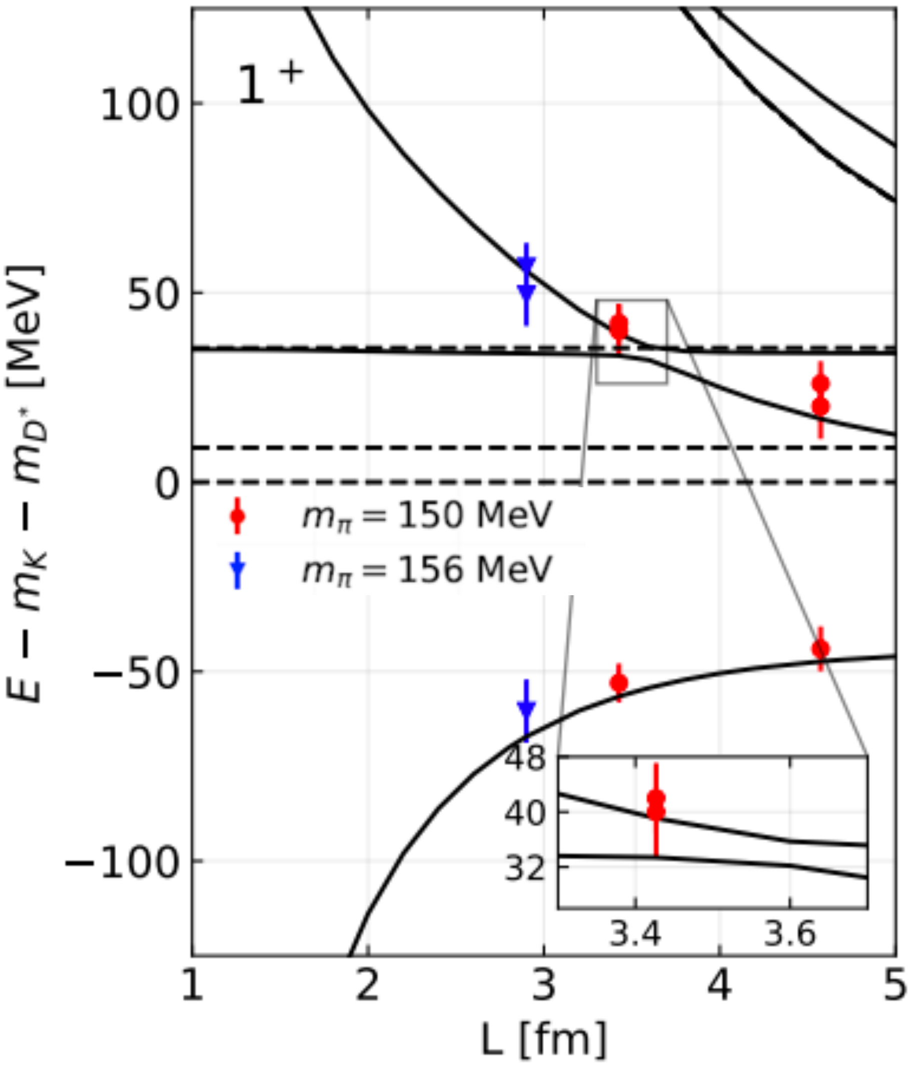
L: 5 → 45 fm

1. The behavior of the third state around $L \in (3,4)$ fm is quite different with the previous one.

2.When $L \to \infty$, the lowest state is approaching the scattering state and is not the $D_s(2317)$ state.

Coupled-channel effect is essential & bare $s\bar{c}$ core is unsubstituted for $D_{s0}^*(2317)$.

Results: $D_{s1}^{*}(2460)$ & $D_{s1}^{*}(2536)$



$J^P = 1^+$: D_{s1} (2460)&D_{s1}(2536)

1. $J^P = 1^+$: two $c\bar{s}$ cores + S-wave D$^*K$ + D-wave D$^*K$.

2. Three energy levels.

3. $D_{s1}^{*}(2460)$: the lowest state.

4. A special cross at L =3.5 fm: well approved by lattice QCD.

- The dropping line: lowest excited D^*K channel, $E_{kin} \sim \left(\frac{2\pi}{L}\right)^2$.

- The flat line represents the $D_{s1}^{*}(2536)$.