Three-Nucleon Forces with Symmetry Preserving Regulator

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RUB

In collaboration with Evgeny Epelbaum

LENPIC

Low Energy Nuclear Physics International Collaboration

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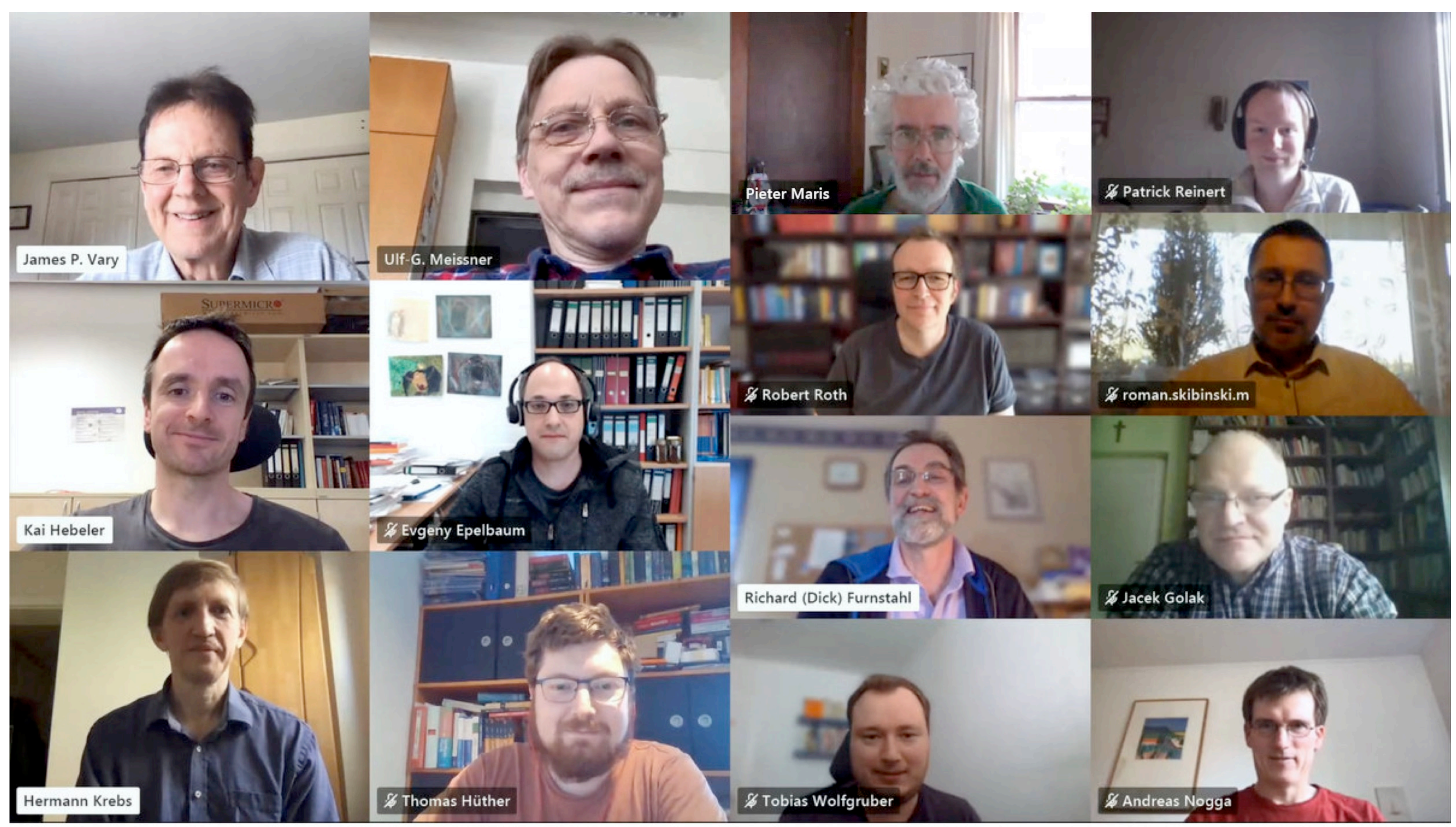
(LENPIC Collaboration)

LENPIC aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents

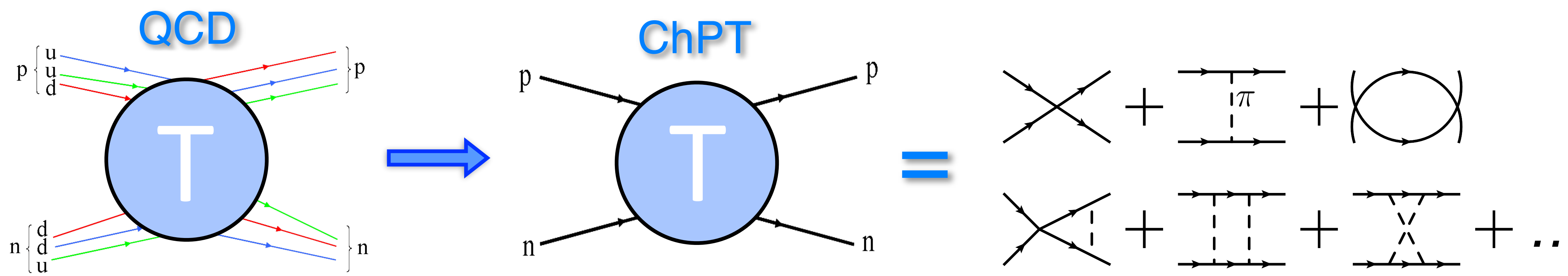
LENPIC



Outline

- Nuclear forces in chiral EFT
- Selected observables in three-nucleon (3N) sector
- 3N forces (3NF) at N^3LO
  - Gradient flow regularization
  - Nuclear forces within path-integral formalism
  - Long range part of 3NF

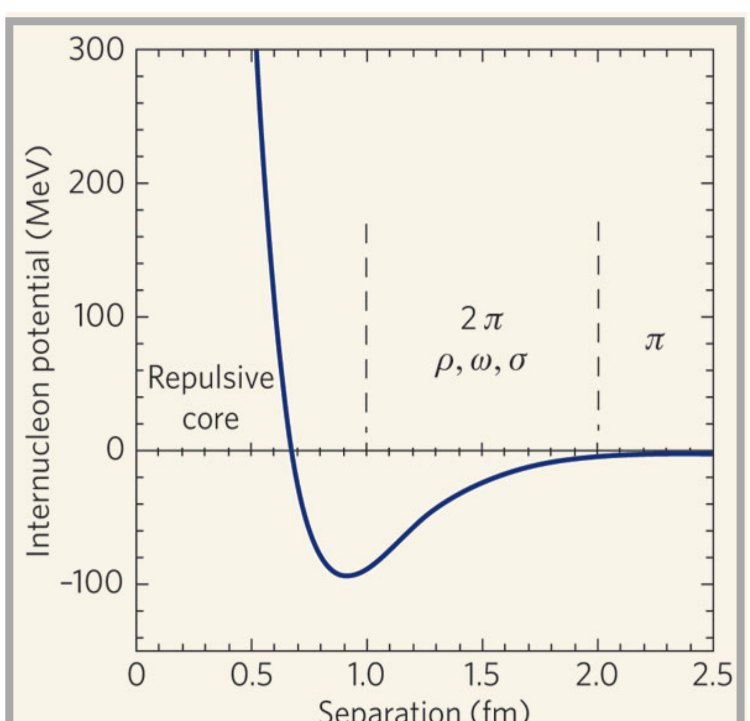
From QCD to nuclear physics

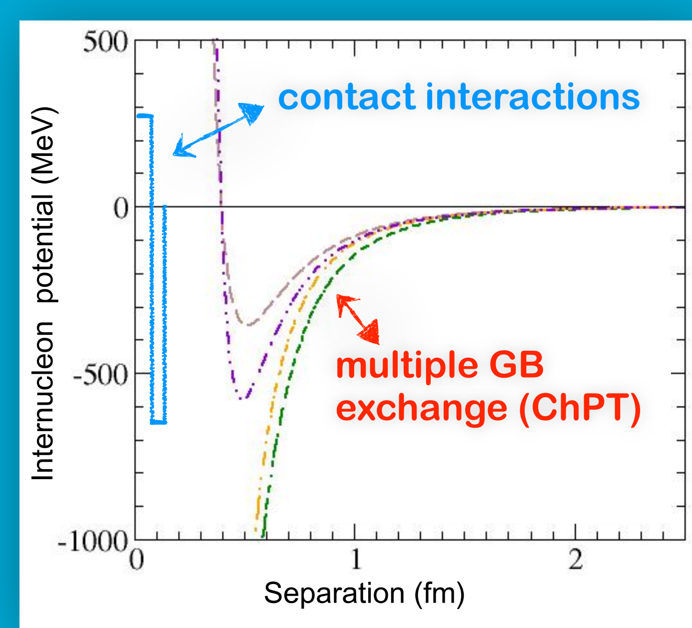


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

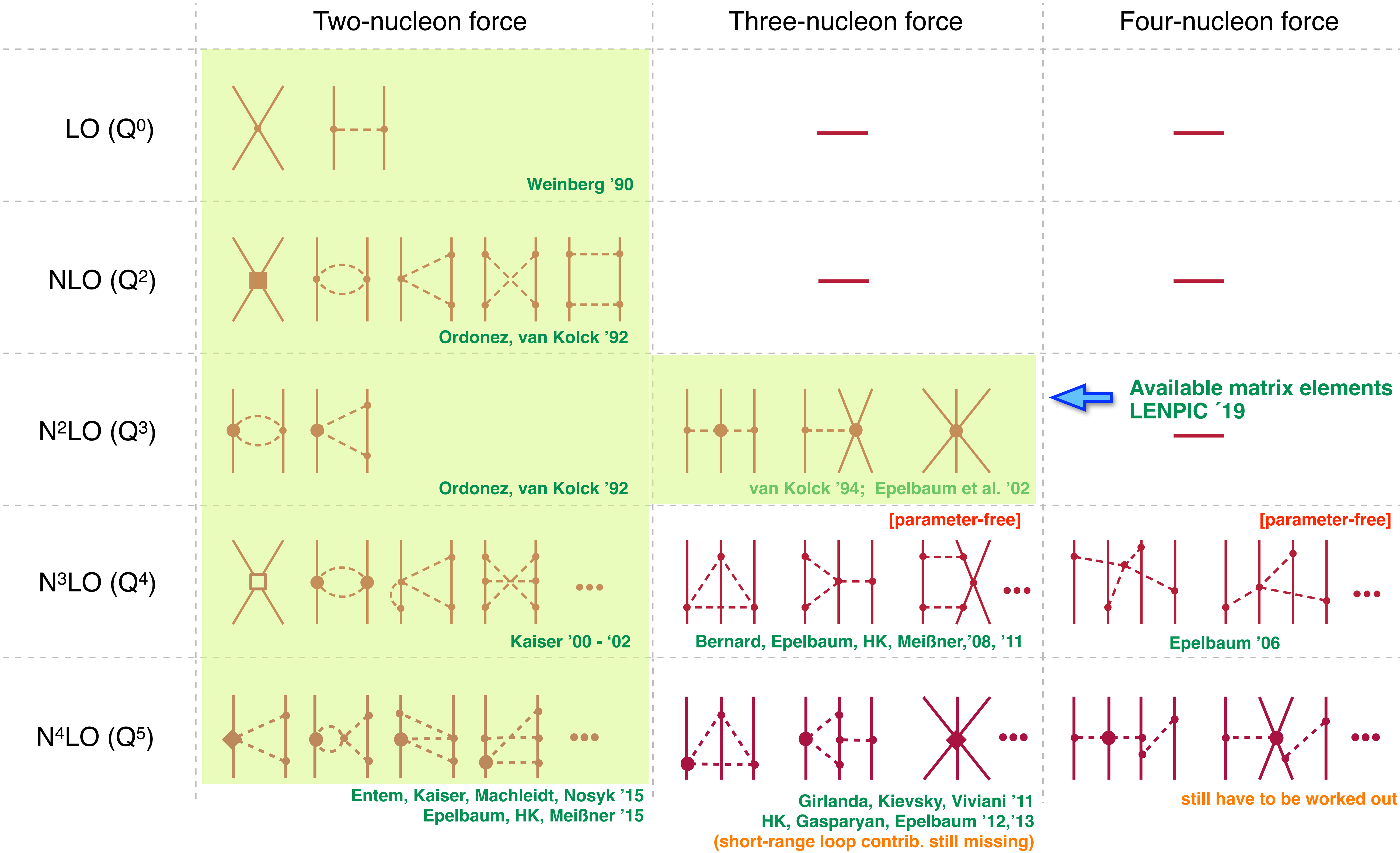
Weinberg '91





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

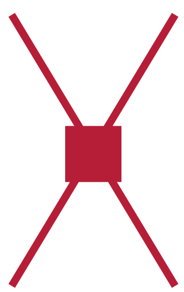
Chiral Expansion of the Nuclear Forces



Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N^2LO [Q^3]: no new terms
N^3LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)
N^4LO [Q^5]: + 5 IB operators
N^4LO$^+$ [Q^6]: + 4 operators (F-waves)

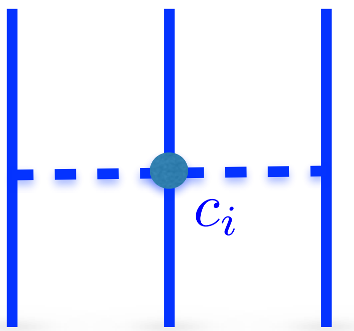
# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

Summary on NN

- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data
  χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

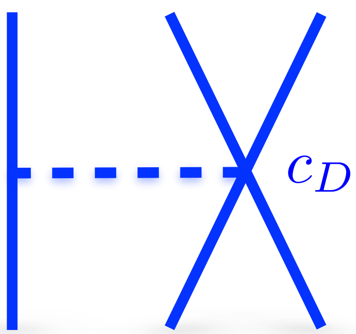
Three-Nucleon Force at N^{2}LO

Epelbaum et al. EPJA56 (2020) 92; Maris et al. PRC103 (2021) 054001



c_i´s are extracted from solutions of Roy-Steiner equation in pion-nucleon scattering: Hoferichter et al. PRL115 (2015) 192301

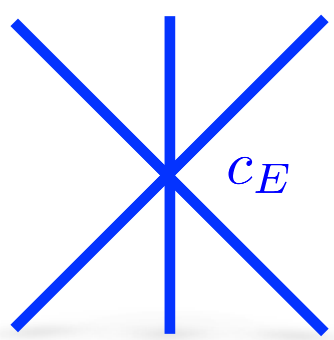
$$c_1 = -0.74\,\mathrm{GeV}^{-1} \qquad c_3 = -3.61\,\mathrm{GeV}^{-1} \qquad c_4 = 2.44\,\mathrm{GeV}^{-1}$$



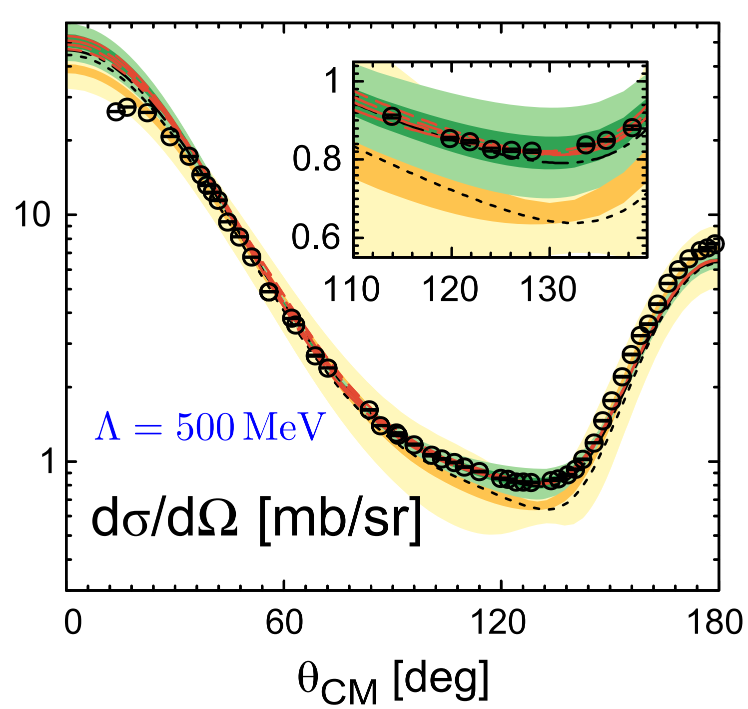
Requirement to reproduce ^{3}H correlates c_D & c_E

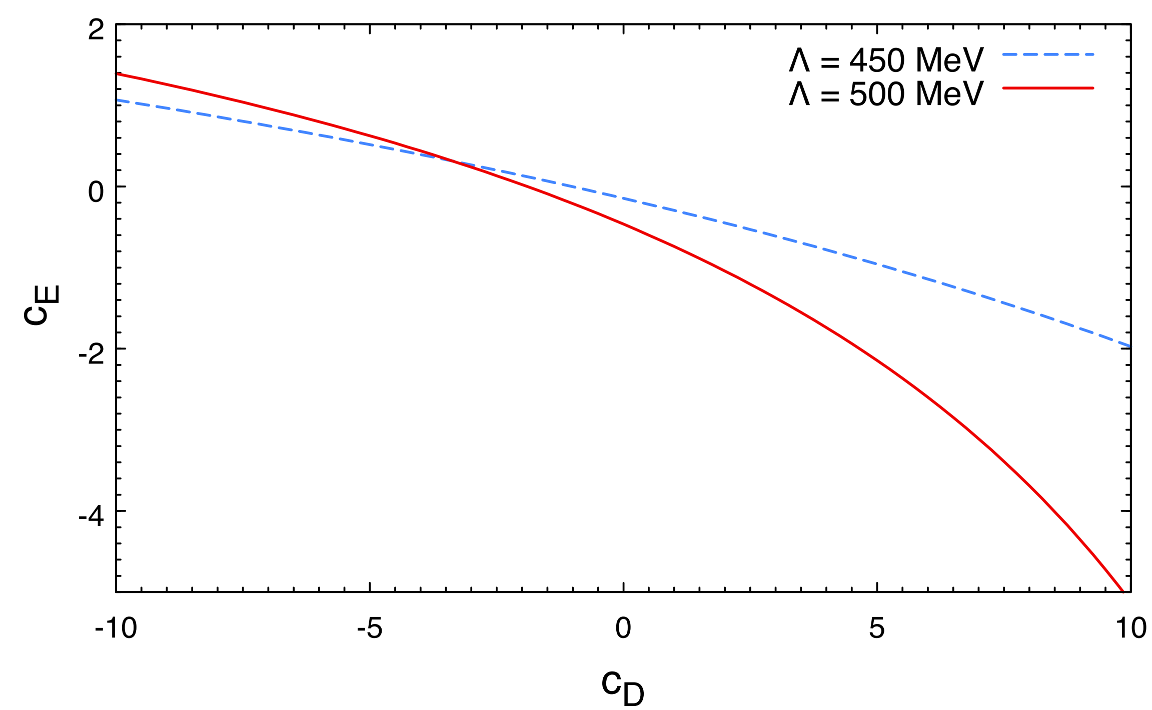
c_D is fitted to the minimum of Nd-scattering cross section at $E_{\mathrm{lab}}^N = 70\,\mathrm{MeV}$

Sekiguchi et al. PRC65 (2002) 034003



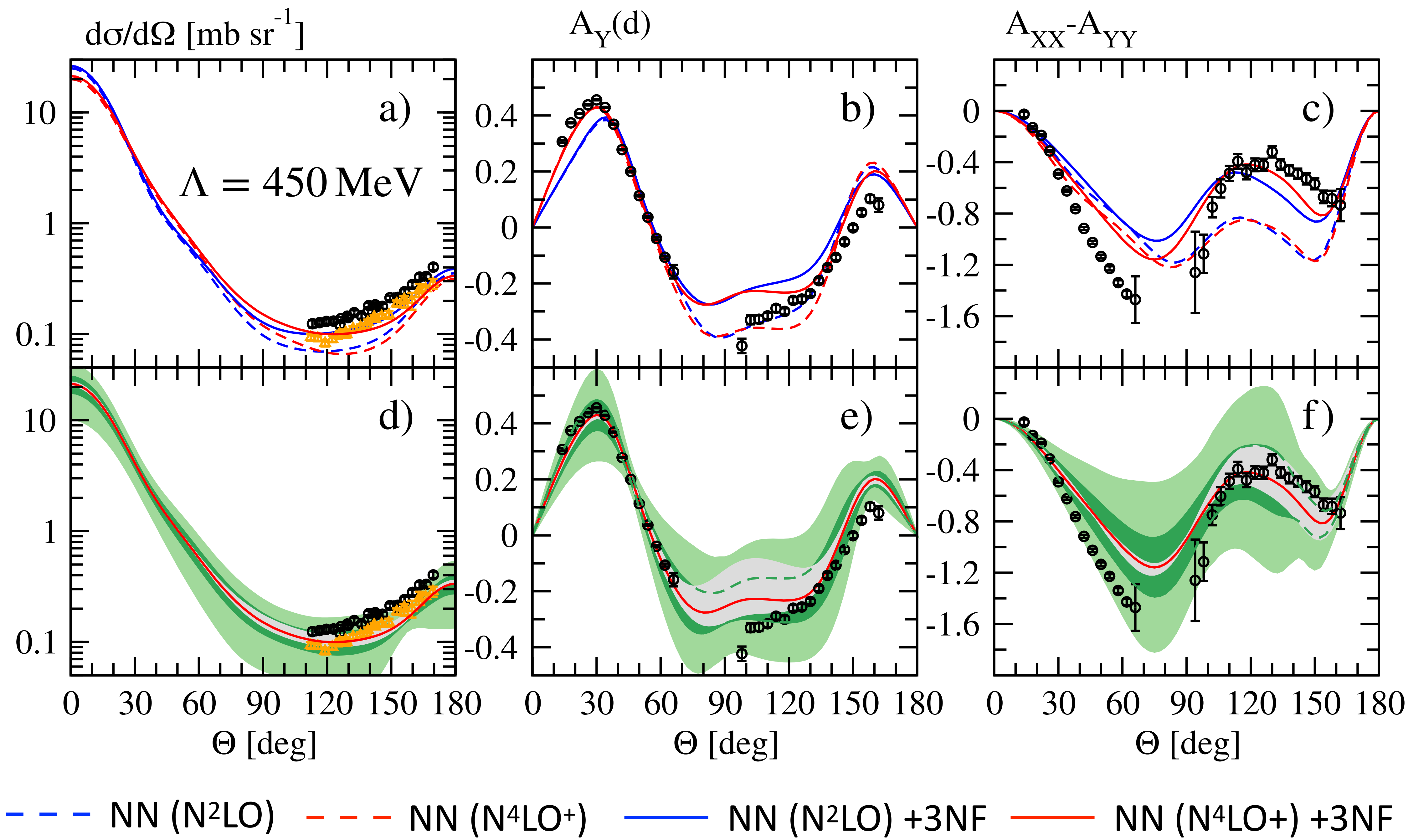
$$c_D = 2.485, \quad c_E = -0.528 \quad \text{for} \quad \Lambda = 450\,\mathrm{MeV}$$
$$c_D = -1.626, \quad c_E = -0.063 \quad \text{for} \quad \Lambda = 500\,\mathrm{MeV}$$





Nd Scattering

Differential cross section and selected analyzing powers of elastic Nd scattering at E_N = 200 MeV



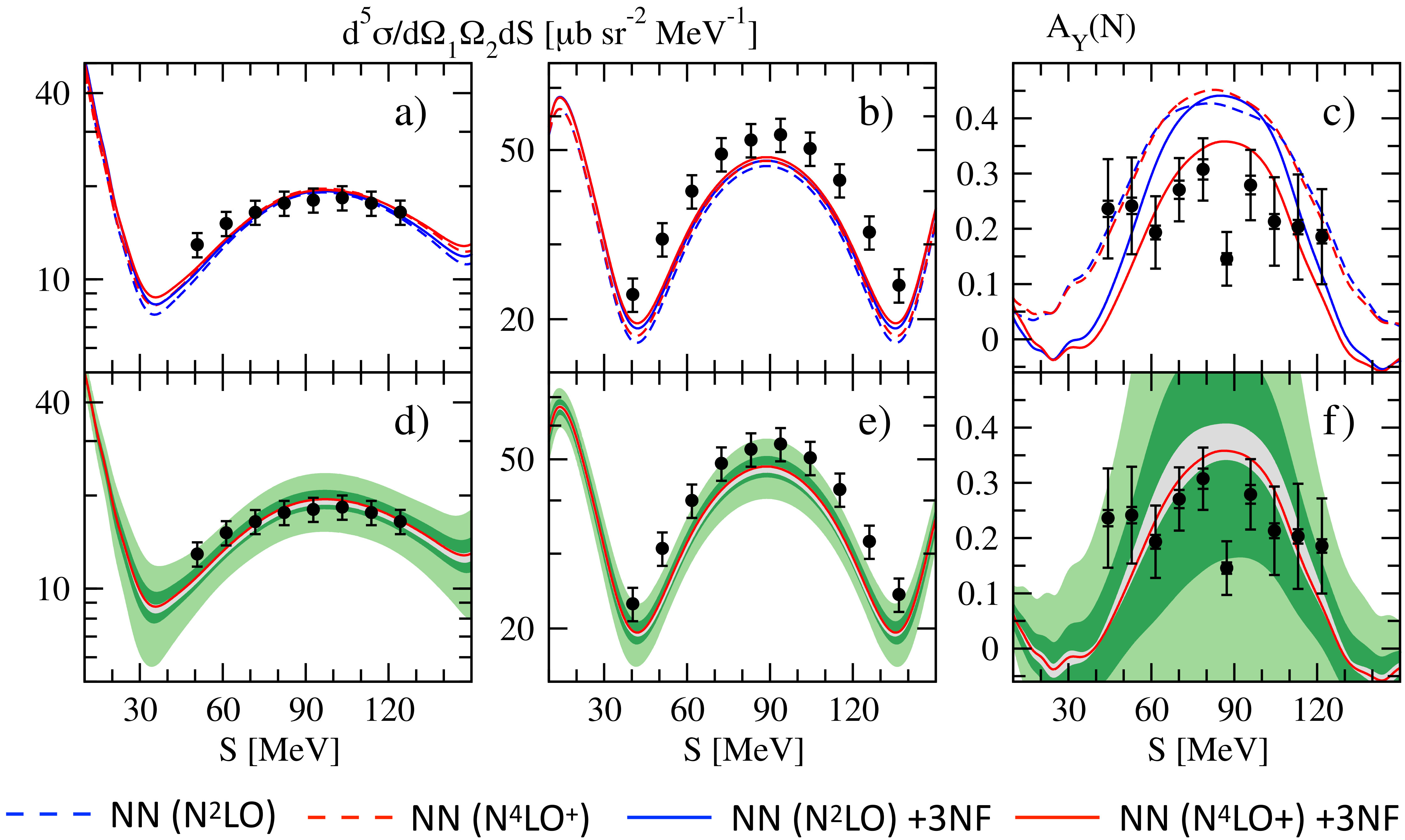
Light (dark) green band correspond to truncation errors at 95% (68%) DoB

- Similar to phenomenological forces $d\sigma/d\Omega$ is slightly underestimated

Nd Breakup Scattering

Maris et al. PRC106 (2022) 6

Differential cross section and selected analyzing powers of Nd breakup at E_N = 135 MeV

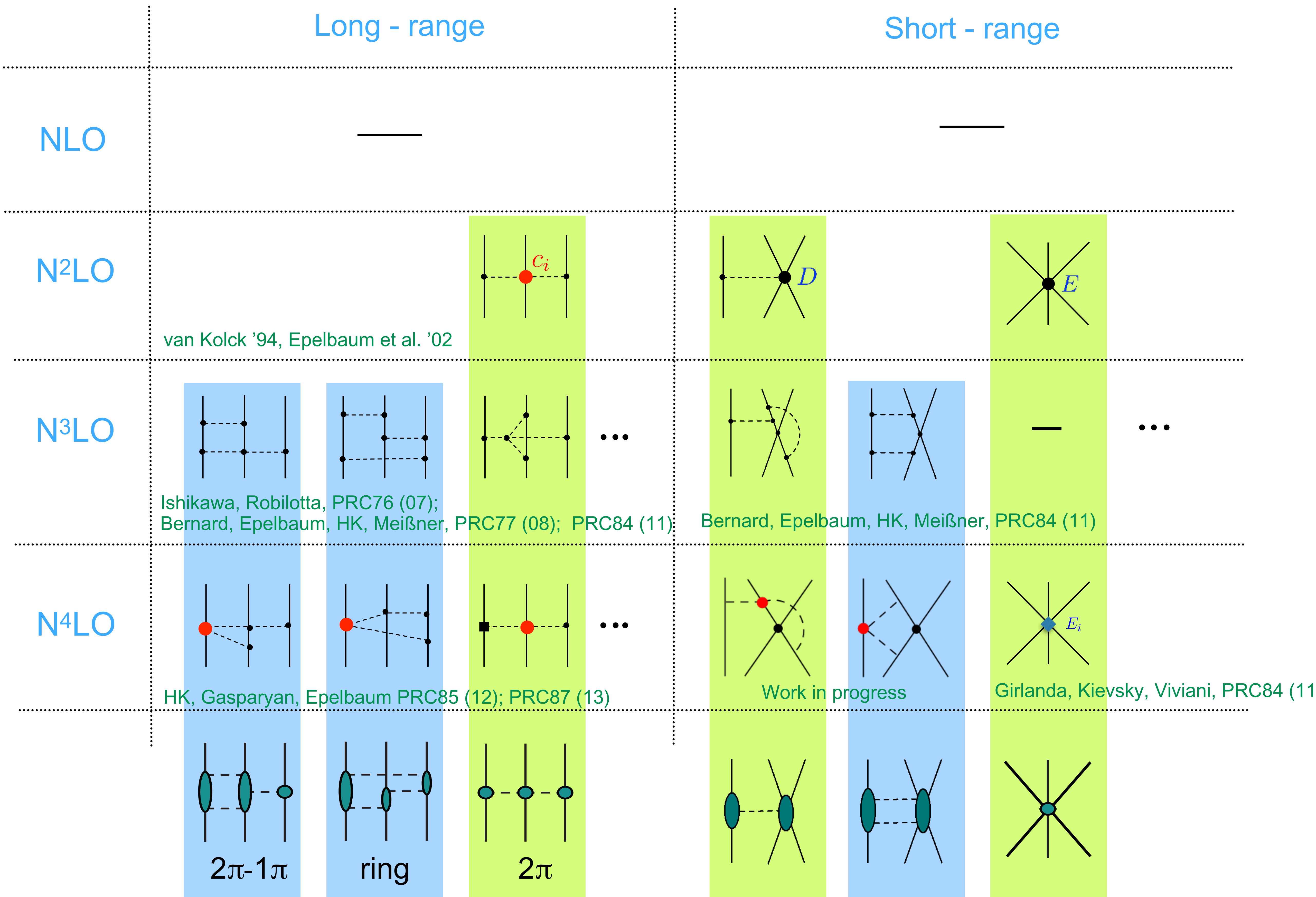


Light (dark) green band correspond to truncation errors at 95% (68%) DoB

Satisfactory description of 3N data leaving room for corrections from higher order

Three-Nucleon Forces at N^3LO

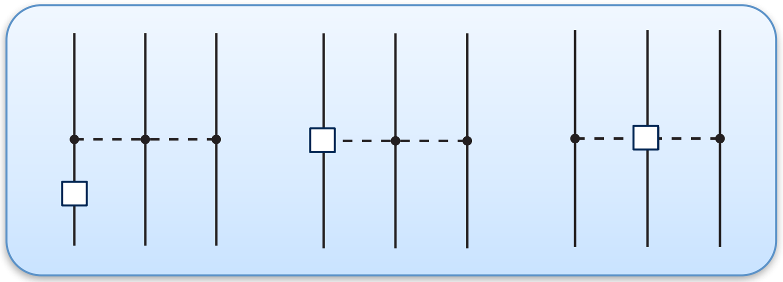
3NF up to N^4LO



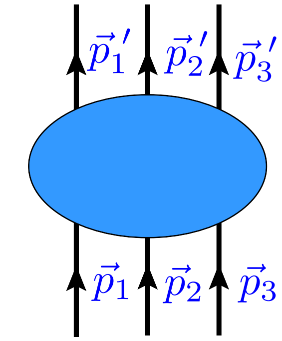
Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, Front. in Phys. 8 (2020) 98



← 1/m - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i\frac{g_A^2}{32mF_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\tau_1\cdot(\tau_2\times\tau_3)(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

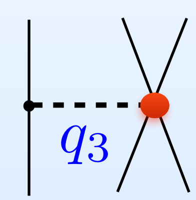
Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

First iteration with OPE NN potential

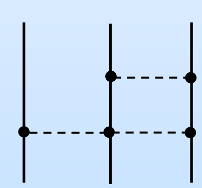
$$V_{2\pi,1/m}^{g_A^2,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{1\pi}^{Q^0,\Lambda}+V_{1\pi}^{Q^0,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2}+\ldots$$

No such D-like term in chiral Lagrangian →



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi}$ =



+ … is finite

Gradient-Flow Equation (GFE)

Yang-Mills gradient flow in QCD: **Lüscher, JHEP 04 (2013) 123**

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \quad \text{with} \quad B_\mu|_{\tau=0} = A_\mu \quad \& \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT: **HK, Epelbaum, arXiv:2312.13932**

(Proposed in various talks by D. Kaplan for nuclear forces)

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \quad \text{with} \quad w = \sqrt{W} \quad \text{and} \quad \mathrm{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Properties under Chiral Transformation

Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}^{(1)}_{\pi N}, \ldots, \mathscr{L}^{(4)}_{\pi N}$: $U \to W$

$$\mathscr{L}^{(1)}_{\pi N} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D^0_w + g\, w \cdot S \Big) N$$

Chiral transformation: by induction, one can show

$$U \to R U L^\dagger \longrightarrow W \to R W L^\dagger$$

- Regularized pion fields transform under τ - independent transformations

- Nucleon fields transform in τ - dependent way

$$N \to K N, \quad K = \sqrt{L U^\dagger R^\dagger} R \sqrt{U} \longrightarrow N \to K_\tau N, \quad K_\tau = \sqrt{L W^\dagger R^\dagger} R \sqrt{W}$$

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\tau \cdot \phi(1 - \alpha\phi^2) - \frac{\phi^2}{2}\left[1 + \left(\frac{1}{4} - 2\alpha\right)\phi^2\right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty} \frac{1}{F^n}\phi_b^{(n)}$$

In the absence of external sources we have

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(1)}(x,\tau) = 0, \quad \phi_b^{(1)}(x,0) = \pi_b(x)$$

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(3)}(x,\tau) = (1-2\alpha)\partial_\mu\phi^{(1)} \cdot \partial_\mu\phi^{(1)}\phi_b^{(1)} - 4\alpha\partial_\mu\phi^{(1)} \cdot \phi^{(1)}\partial_\mu\phi_b^{(1)}$$
$$+ \frac{M^2}{2}(1-4\alpha)\phi^{(1)} \cdot \phi^{(1)}\phi_b^{(1)}, \quad \phi_b^{(3)}(x,0) = 0$$

Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q,\tau) = \int d^4x\, e^{iq\cdot x}\phi_b^{(n)}(x,\tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2+M^2)}\tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}(2\pi)^4\delta(q - q_1 - q_2 - q_3)\int_0^\tau ds\, e^{-(\tau-s)(q^2+M^2)}e^{-s\sum_{j=1}^3(q_j^2+M^2)}$$
$$\times \left[4\alpha\, q_1 \cdot q_3 - (1-2\alpha)q_1 \cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\pi}(q_1) \cdot \tilde{\pi}(q_2)\tilde{\pi}_b(q_3)$$

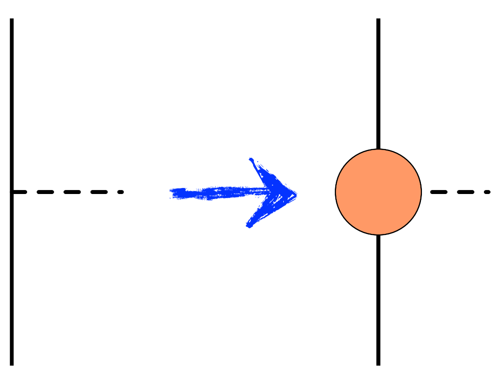
Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Regularization for Nuclear Forces

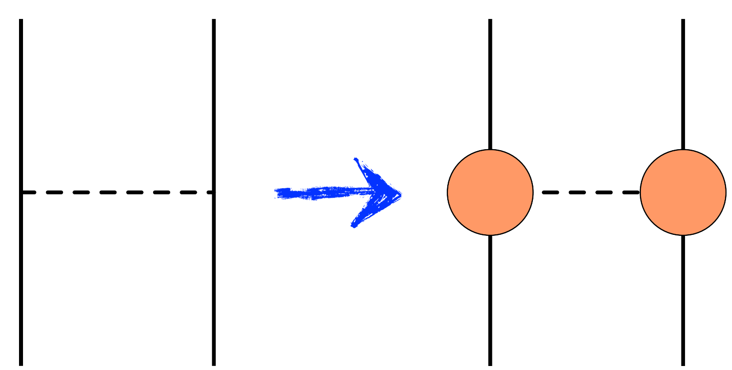
To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathscr{L}_{\pi}^{(2)}$ & $\mathscr{L}_{\pi}^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathscr{L}_{\pi N}^{(1)} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D_w^0 + g\, w \cdot S \Big) N$$



$\sim e^{-\tau(q^2+M^2)}$



$\sim e^{-2\tau(q^2+M^2)} \dfrac{1}{q^2+M^2}$

For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Conceptional Challenge

$$\phi^c = e^{-\tau(-\partial^2+M^2)}\pi^c + \ldots = e^{-\tau(-\partial_0^2-\vec{\nabla}^2+M^2)}\pi^c + \ldots$$

Appearance of second and higher order in time-derivatives of pion fields

→ Canonical quantization of the regularized theory becomes difficult (Ostrogradski - approach, Constrains, …)

- Unitary transformation (UT) approach can not be used any more

→ Use path-integral (PI) quantization

Canonical Quantization of QFT		Path-Integral Quantization of QFT
Hamiltonian & Hilbert space	⟷	Lagrangian & action
Creation/annihilation operators		Summation over all classical paths
Time-ordered perturbation theory		Loop expansion & Feynman rules

- PI approach was a natural choice in pionic and single-nucleon sector

Gasser, Leutwyler, Annals Phys. 158 (1984) 142;
Bernard, Kaiser, Kambor, Meißner, Nucl. Phys. B 388 (1992) 315

Nuclear Forces in PI Formulation

In two - and more - nucleon sector Weinberg used canonical quantization language

Weinberg Nucl. Phys. B 362 (1991) 3

> In using old-fashioned perturbation theory we must work with the Hamiltonian rather than the Lagrangian. The application of the usual rules of canonical quantization to the leading terms in (1) and (9) yields the total

Formulation where we can work with the Lagrangian: HK, Epelbaum, arXiv:2311.10893

Derivation steps of nuclear forces in PI formulation

Inspired by: Friar et al. Phys. Rev. C 70 (2004) 044001, Borasoy et al. EPJA 31 (2007)105

- Perform a perturbative loop expansion in pion fields

  → Non-instant NN, 3N & 4N interactions

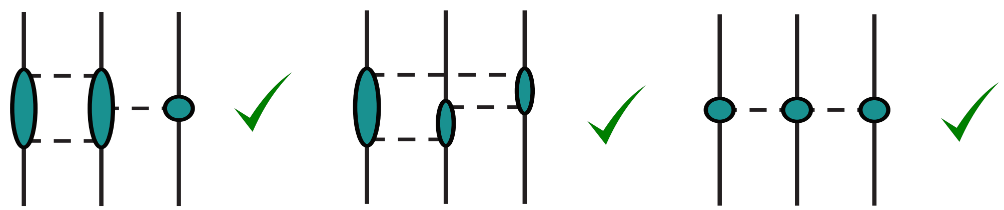
- Perform non-local nucleon field redefinitions to bring all non-instant interactions into instant form

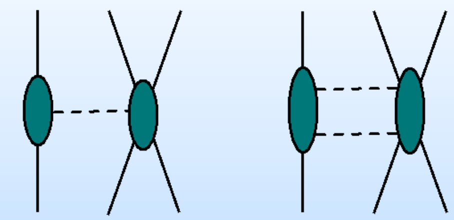
- Due to non-locality of nucleon-field redefinitions we get functional determinants

  → $\det\left(\frac{\delta(N'^{\dagger}, N')}{\delta(N^{\dagger}, N)}\right)$ include loop corrections to nuclear forces

Status Report on 3N at N^3LO

- UT and PI approaches lead to the same 3NF & 4NF up to N^4LO within dim. reg.
- We calculated all long-range contributions to 3NF & 4NF at N^3LO



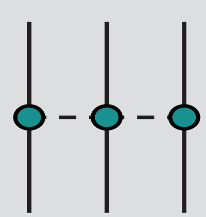
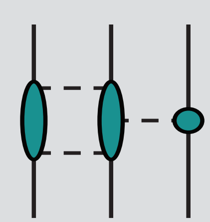
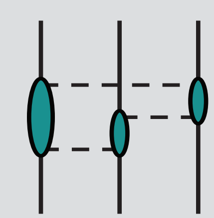


being finalized

3NF's are given in terms of integrals over Schwinger parameters

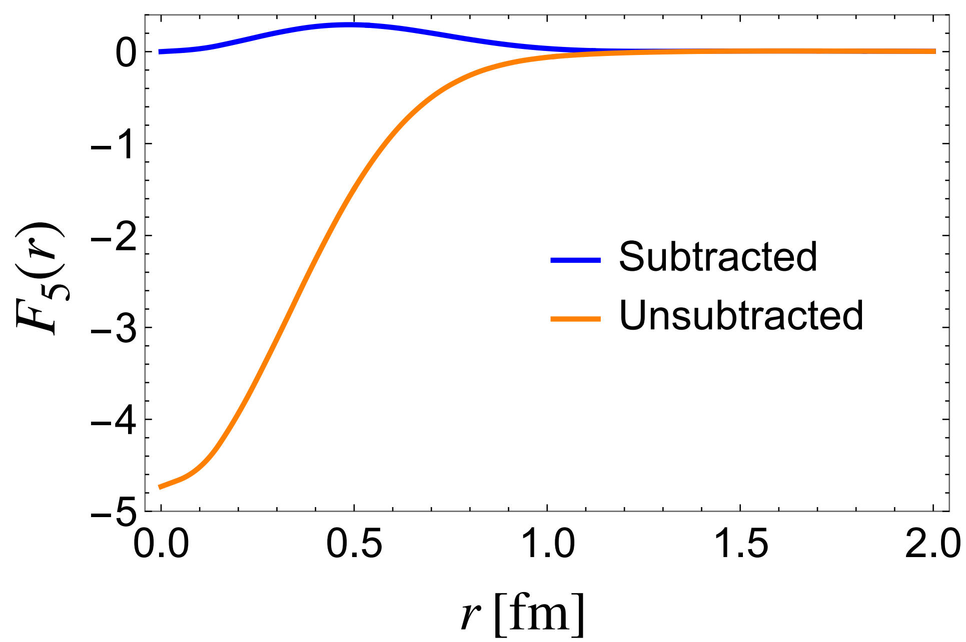
$$V_{3N}^{2\pi-1\pi} = \tau_1\cdot\tau_2\times\tau_3\,\vec{q}_1\cdot\vec{\sigma}_1\times\vec{\sigma}_2\,\vec{q}_3\cdot\vec{\sigma}_3\,\frac{e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{q_3^2+M_\pi^2}\left(-\frac{g_A^4}{F_\pi^6}\frac{q_1}{2048\pi}\int_0^\infty d\lambda\,\mathrm{erfi}\left(\frac{q_1\lambda}{2\Lambda\sqrt{2+\lambda}}\right)\frac{\exp\left(-\frac{q_1^2+4M_\pi^2}{4\Lambda^2}(2+\lambda)\right)}{2+\lambda}+\dots\right)+\dots$$

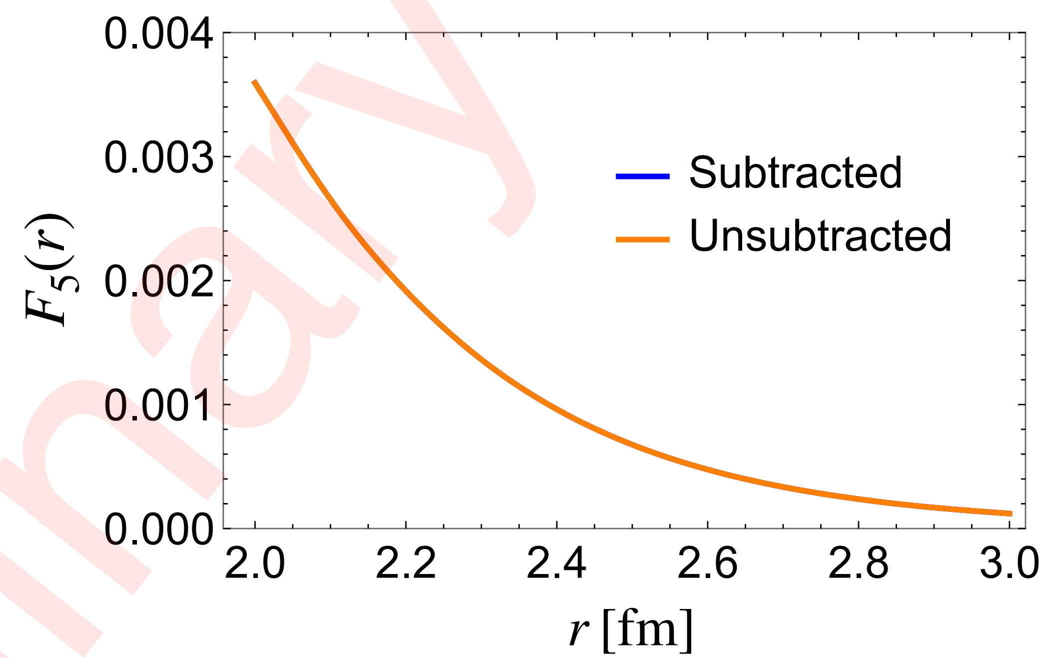
Dimension of integrals over Schwinger parameters depends on topology

Space			
Momentum	2	1	3
Coordinate	4	1	0

Selected Profile Functions

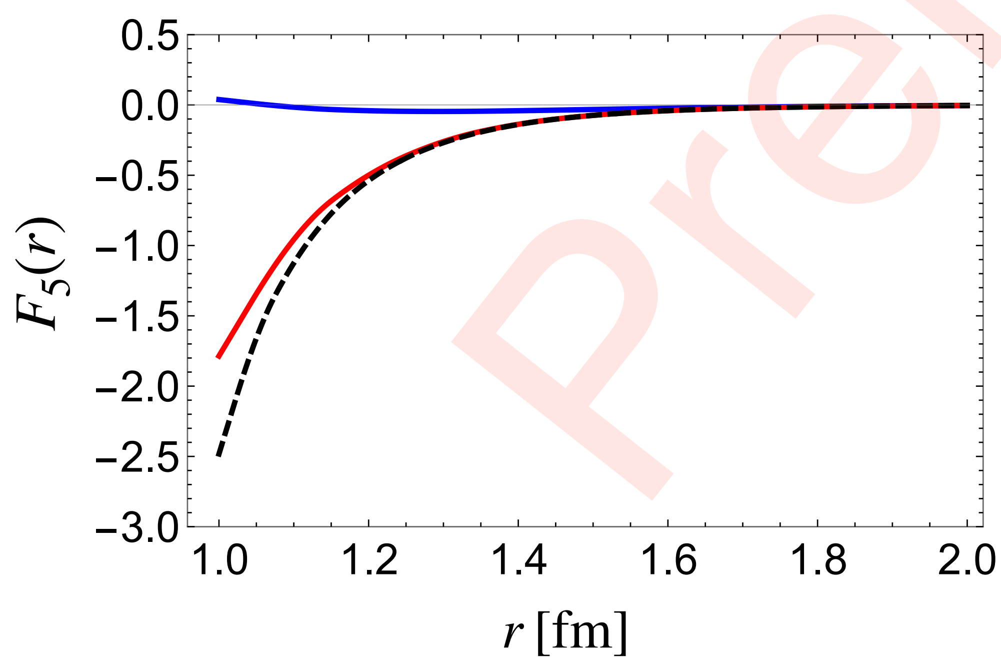
$V_{3N}^{\rm ring} = F_1(r_{12}, r_{23}, r_{13}) + \ldots + \tau_2 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 F_5(r_{12}, r_{23}, r_{13}) + \ldots$ $\qquad F_5(r) = F_5(r, r, r)$ [MeV]

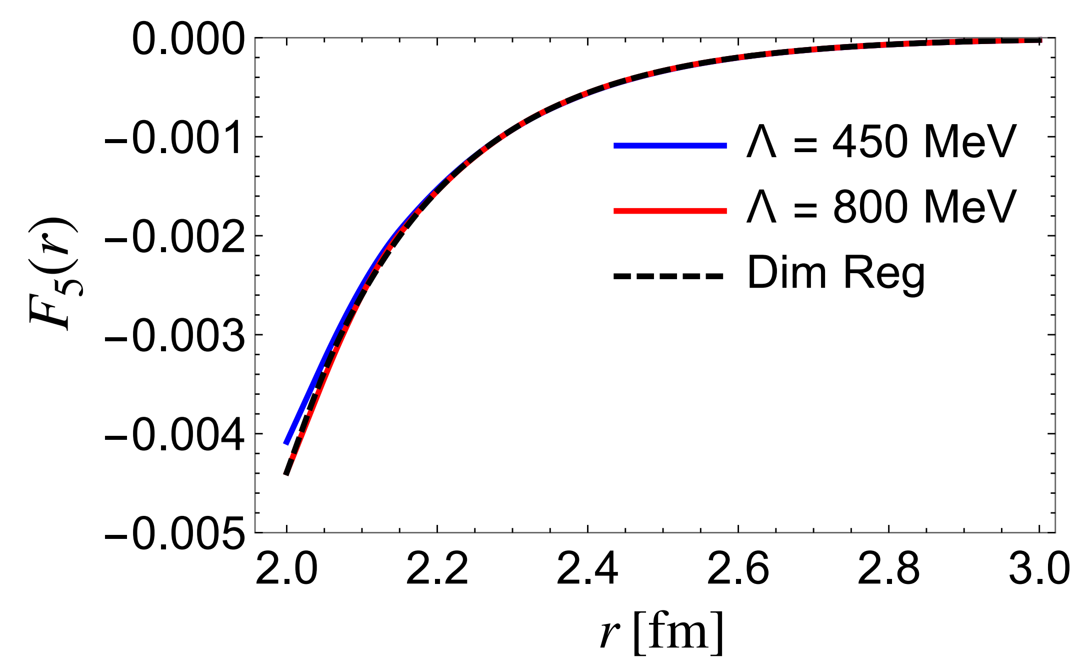




By construction: subtracted & unsubtracted forces differ in the short-range region

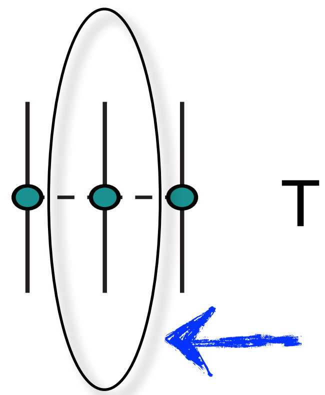
At $\Lambda \to \infty$ regularized 3NF reproduce dim. reg. results from **Bernard et al. PRC77 (08)**





Preliminary

Homework



- TPE topology includes pion-nucleon amplitude as a subprocess

  Pion-nucleon amplitude with gradient-flow regulator depends on c_i's

  Fit c_i's to pion-nucleon sub-threshold coefficients which are determined from Roy-Steiner equation

  Calculation of pion-nucleon scattering with gradient-flow regulator required

- PWD is computationally more expensive, due to higher dimension of integrals over Schwinger parameters

Summary on 3N

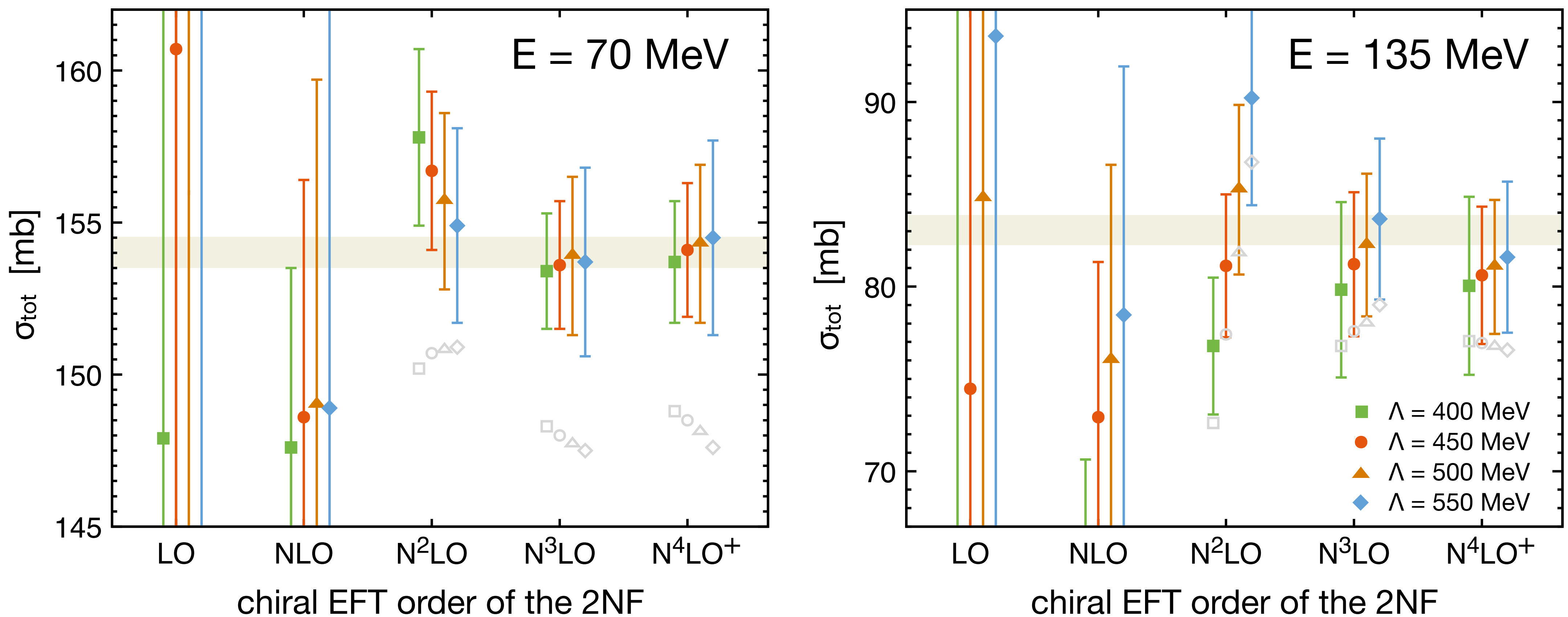
- Gradient flow regularization preserves chiral symmetry
- Path-integral approach for derivation of nuclear forces
- Long-range part of 3NF at N^3LO has been calculated

Outlook

- Short-range part of 3NF at N^3LO
- Partial wave decomposition
- Symmetry preserving regularized nuclear currents

Neutron-Deuteron Scattering at N^{4}LO$^+$

Maris et al. PRC106 (2022) 6; Maris et al. PRC103 (2021) 054001



Error bar from Bayesian analysis: 68% DoB **Epelbaum et al. EPJA56 (2020) 92**
Furnstahl et el. PRC92 (2015) 2, 024005

$$X = X_{\rm ref}\left(c_0 + c_2 Q^2 + c_3 Q^3 + \dots\right) \qquad Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\rm eff}}{\Lambda_b}\right) \qquad \Lambda_b = 650\,{\rm MeV}$$

- Similar to phenomenological potentials NN only from N^{2}LO on underestimate $\sigma_{\rm tot}$
- N^{2}LO 3NF increases the total cross section bringing the calculations in agreement with the data

A = 3 & 4 Nuclei

Faddeev and Yakubovsky equations in momentum space: **Nogga et al. PRC65 (2002) 054003**

All angular momenta ≤ 5 of the two-body subsystem are taken into account

Numerical accuracy $\sim 1\,\mathrm{keV}$ reached for $A = 3$ binding energies and expectation values

Maris et al. PRC 106 (2022) 064002

		Λ	E	$\langle V_{3NF} \rangle$
^{3}H	LO	450	−12.22	—
	NLO		−8.515	—
	N^{2}LO		−8.483	−0.459
	N^{3}LO		−8.483	−0.520
	N^{4}LO		−8.483	−0.430
	N^{4}LO$^+$		−8.483	−0.460
^{3}H	LO	500	−12.52	—
	NLO		−8.325	—
	N^{2}LO		−8.482	−0.660
	N^{3}LO		−8.483	−0.724
	N^{4}LO		−8.483	−0.628
	N^{4}LO$^+$		−8.484	−0.672
Expt. ^{3}H			−8.482	—

		Λ	E	$\langle V_{3NF} \rangle$
^{3}He	LO	450	−11.34	—
	NLO		−7.751	—
	N^{2}LO		−7.734	−0.452
	N^{3}LO		−7.737	−0.509
	N^{4}LO		−7.739	−0.423
	N^{4}LO$^+$		−7.740	−0.452
^{3}He	LO	500	−11.63	—
	NLO		−7.574	—
	N^{2}LO		−7.739	−0.641
	N^{3}LO		−7.738	−0.705
	N^{4}LO		−7.743	−0.615
	N^{4}LO$^+$		−7.744	−0.658
Expt. ^{3}He			−7.718	—

Attractive contribution of 3NF brings E to its physical value

Point Coulomb interaction has been included for pp system in ^{3}He calc

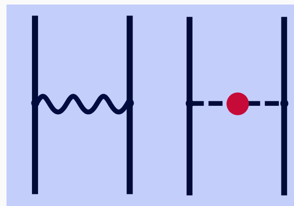
Isospin-breaking in the Nuclear Force

Have been employed in **Reinert, HK, Epelbaum, EPJA 54 (2018) 88**

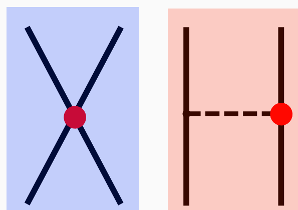
Parameter-free: depend on δM_π, δm = 1.29 MeV and $(\delta m)^{QCD}$ = -1.87(16) MeV **[Gasser, Leutwyler, Rusetsky '21]**

Depend on 3 πN coupling constants + 3 IB contact terms in p-waves

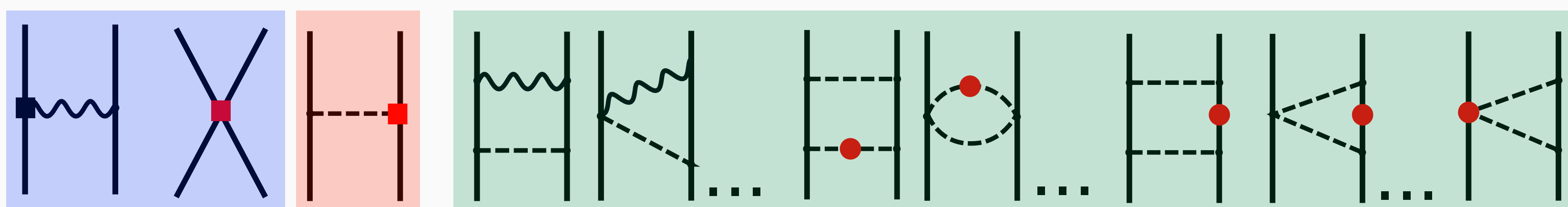
NLO



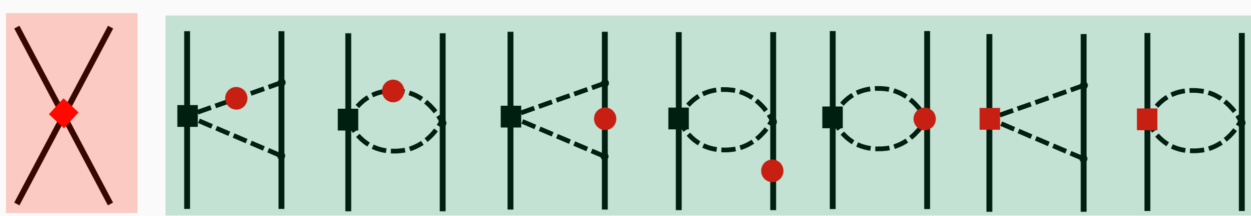
N^2LO

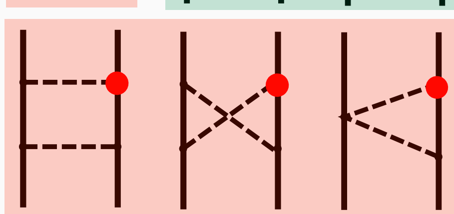


N^3LO



N^4LO





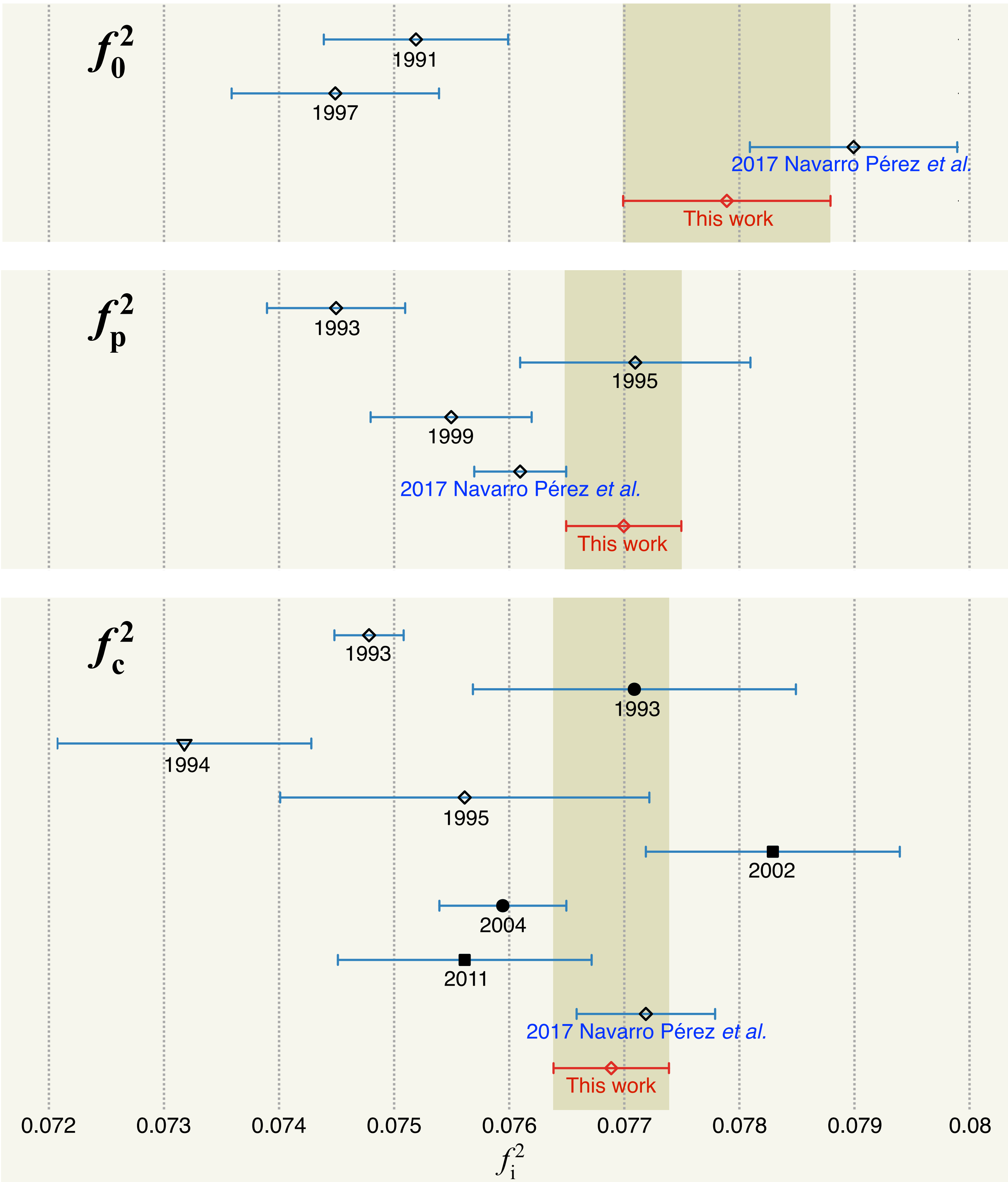
van Kolck et al.'98; Friar et al. '99,'03,'04; Niskanen '02; Epelbaum, Meißner '05

Away from the isospin limit, one introduces 3 πN coupling constants:

$$f_{\pi^0 \mathrm{pp}} = \frac{M_{\pi^\pm}\, g_{\pi^0 \mathrm{pp}}}{2\sqrt{4\pi}\, m_\mathrm{p}} \qquad f_{\pi^0 \mathrm{nn}} = \frac{M_{\pi^\pm}\, g_{\pi^0 \mathrm{nn}}}{2\sqrt{4\pi}\, m_\mathrm{n}} \qquad f_{\pi^\pm \mathrm{pn}} = \frac{M_{\pi^\pm}\, g_{\pi^\pm \mathrm{pn}}}{\sqrt{4\pi}\,(m_\mathrm{p} + m_\mathrm{n})}$$

$$\begin{aligned} f_0^2 &= -f_{\pi^0 \mathrm{nn}}\, f_{\pi^0 \mathrm{pp}} \\ f_p^2 &= f_{\pi^0 \mathrm{pp}}\, f_{\pi^0 \mathrm{pp}} \\ 2 f_c^2 &= f_{\pi^\pm \mathrm{pn}}\, f_{\pi^\pm \mathrm{pn}} \end{aligned}$$

Determination of πN constants



Reinert, HK, Epelbaum PRL126 (2021) 092501

$$f_0^2 = 0.0779(9)(1.3)$$
$$f_p^2 = 0.0770(5)(0.8)$$
$$f_c^2 = 0.0769(5)(0.9)$$

statistical **and systematic** errors due to the EFT truncation, choice of E_{max} and data selection

uncertainty in the subleading πN LECs

No evidence for charge dependence of the πN coupling constants

Our f_c^2 value is consistent with the extractions from the πN system

f_c^2 corresponds to $g_{\pi NN} = 13.23 \pm 0.04$

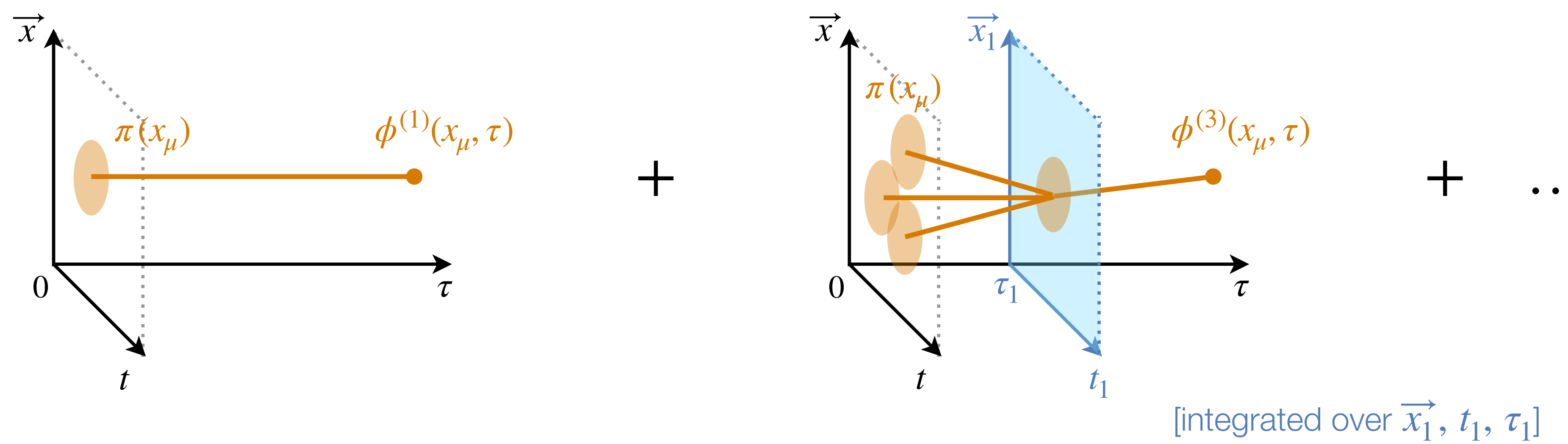
Pionic hydrogen exp. at PSI **Hirtl et al.´21**

$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D}: \quad g_{\pi NN} = 13.10 \pm 0.10$

$\Gamma_{1s}^{\pi H}: \quad g_{\pi NN} = 13.24 \pm 0.10$

Iterative solution in Coordinate Space

$\phi(x_\mu, \tau) =$



Light-shaded area visualizes smearing in Euclidean space of size $\sim \sqrt{2\tau}$

Solid line stands for Green-function:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right] G(x-y, \tau - s) = \delta(x-y)\delta(\tau - s)$$

$$G(x,\tau) = \theta(\tau) \int \frac{d^4 q}{(2\pi)^4} e^{-\tau(q^2+M^2)} e^{-i\, q\cdot x}$$

$$\phi^{(1)}_b(x,\tau) = \int d^4 y\, G(x-y,\tau)\pi_b(y)$$

$$\phi^{(3)}_b(x,\tau) = \int_0^\tau ds \int d^4 y\, G(x-y,\tau-s)\Big[(1-2\alpha)\partial_\mu \phi^{(1)}(y,s)\cdot\partial_\mu\phi^{(1)}(y,s)\phi^{(1)}_b(y,s) - 4\alpha\, \partial_\mu \phi^{(1)}(y,s)\cdot\phi^{(1)}(y,s)\partial_\mu\phi^{(1)}_b(y,s) + \frac{M^2}{2}\phi^{(1)}(y,s)\cdot\phi^{(1)}(y,s)\phi^{(1)}_b(y,s)\Big]$$

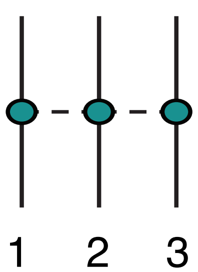
Subtraction Scheme

Choice of the short-range scheme

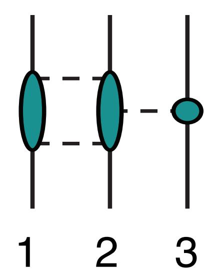
- NN case: local part of NN force vanishes if distance between nucleons vanishes

  → leads to natural size of LECs

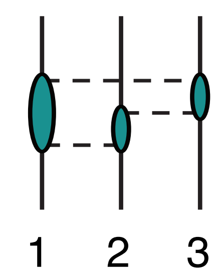
- 3N case: vanishing of the local part of 3NF is topology dependent



$= 0$ if $r_{12} = 0$ or $r_{23} = 0$

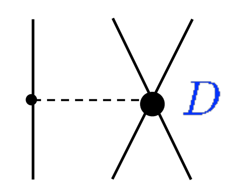


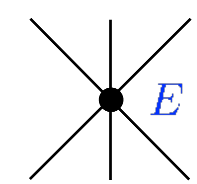
$= 0$ if $r_{12} = 0$



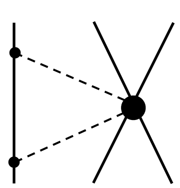
$= 0$ if $r_{12} = r_{23} = 0$

Can be achieved by adjustment of D- and E-like terms:





Vanishing of 3NF for any $r_{ij} = 0$ would require inclusion of two-pion-contact terms



Appear first at N^5LO and are expected to be small