Hadronic Molecule Effective Field Theory for T_{cc}^{+}

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26th Quark Confinement and Hadron Spectrum Conference

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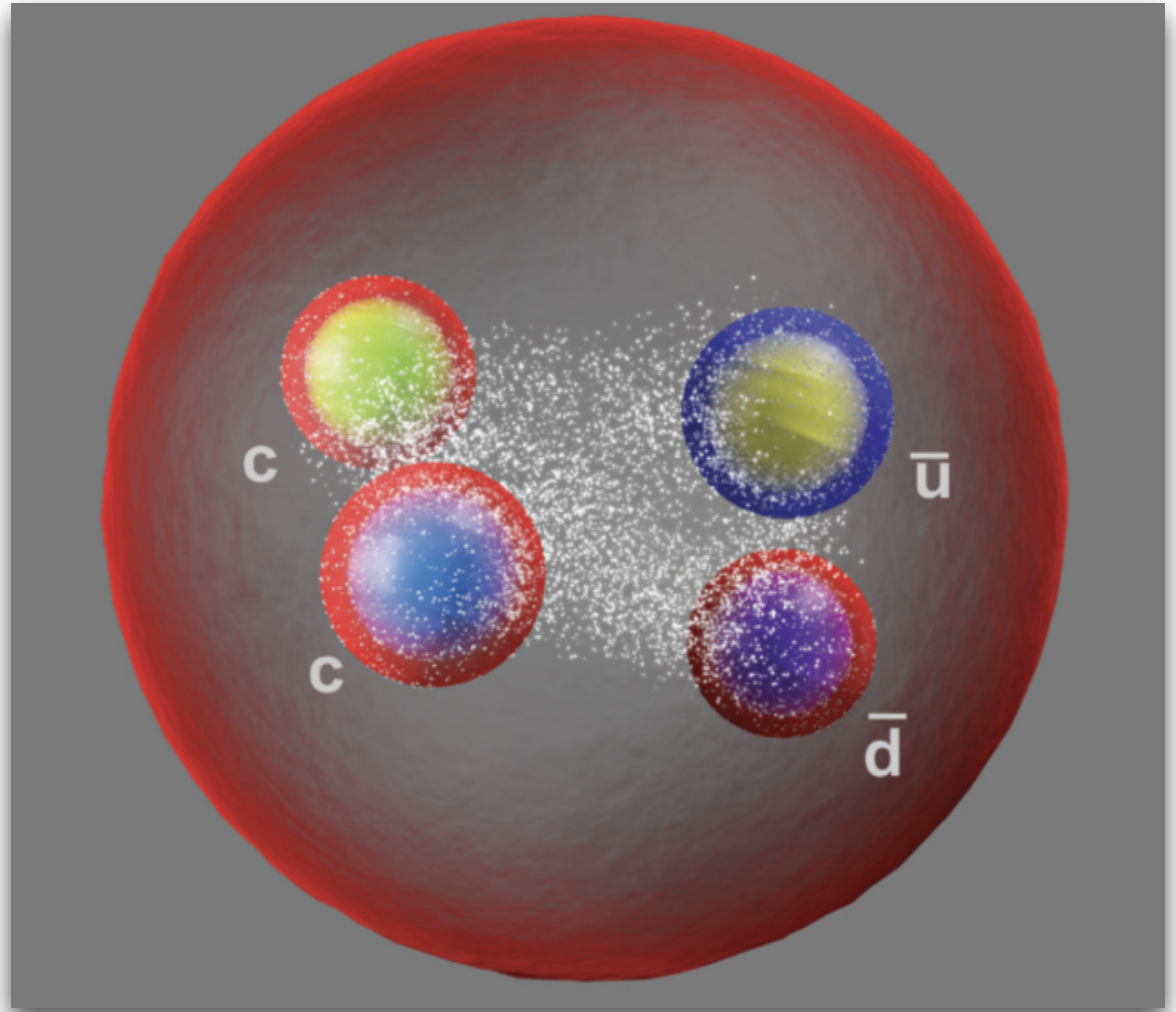
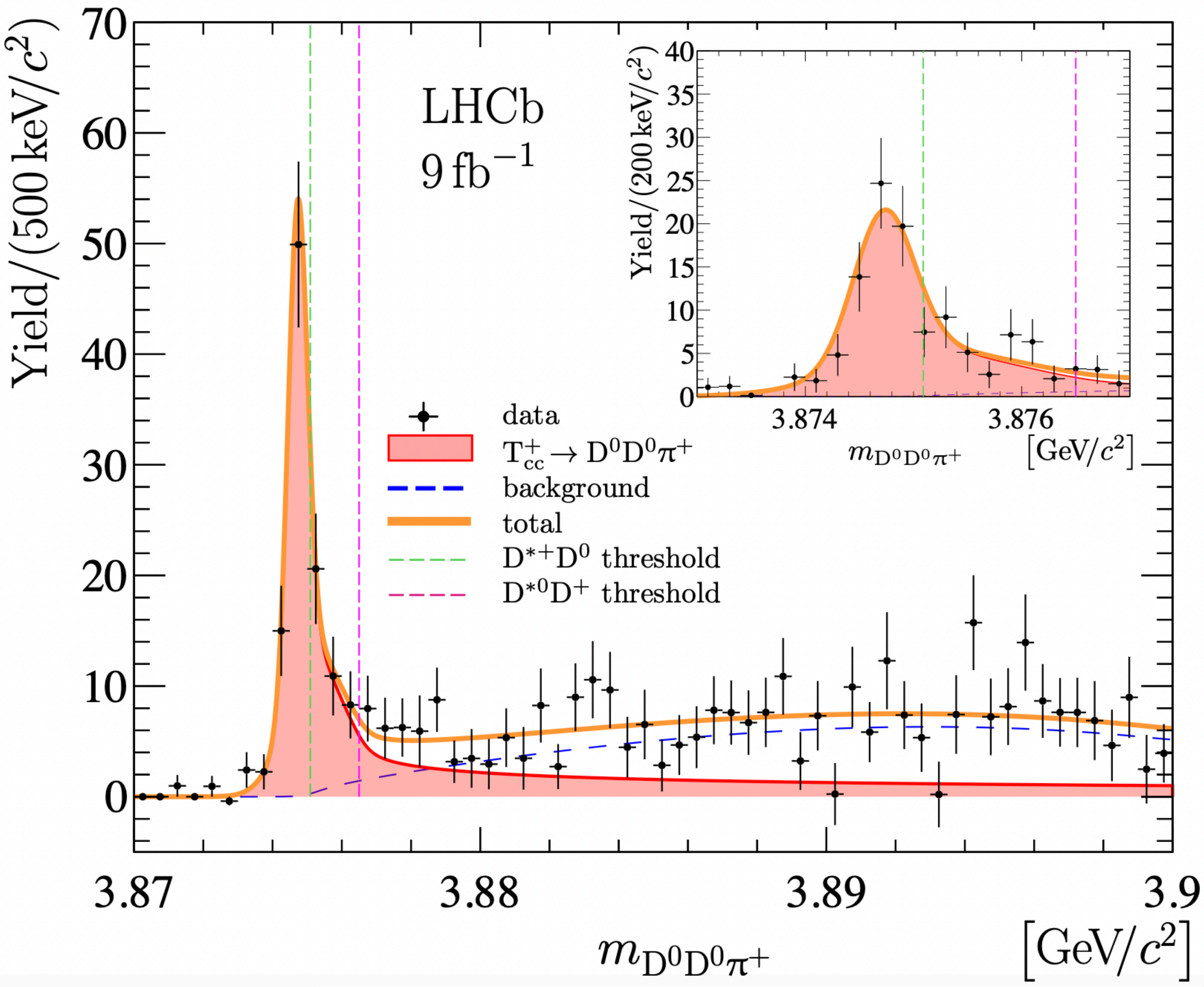
Discovered 7/2021: T_{cc}^{+} doubly charm tetraquark

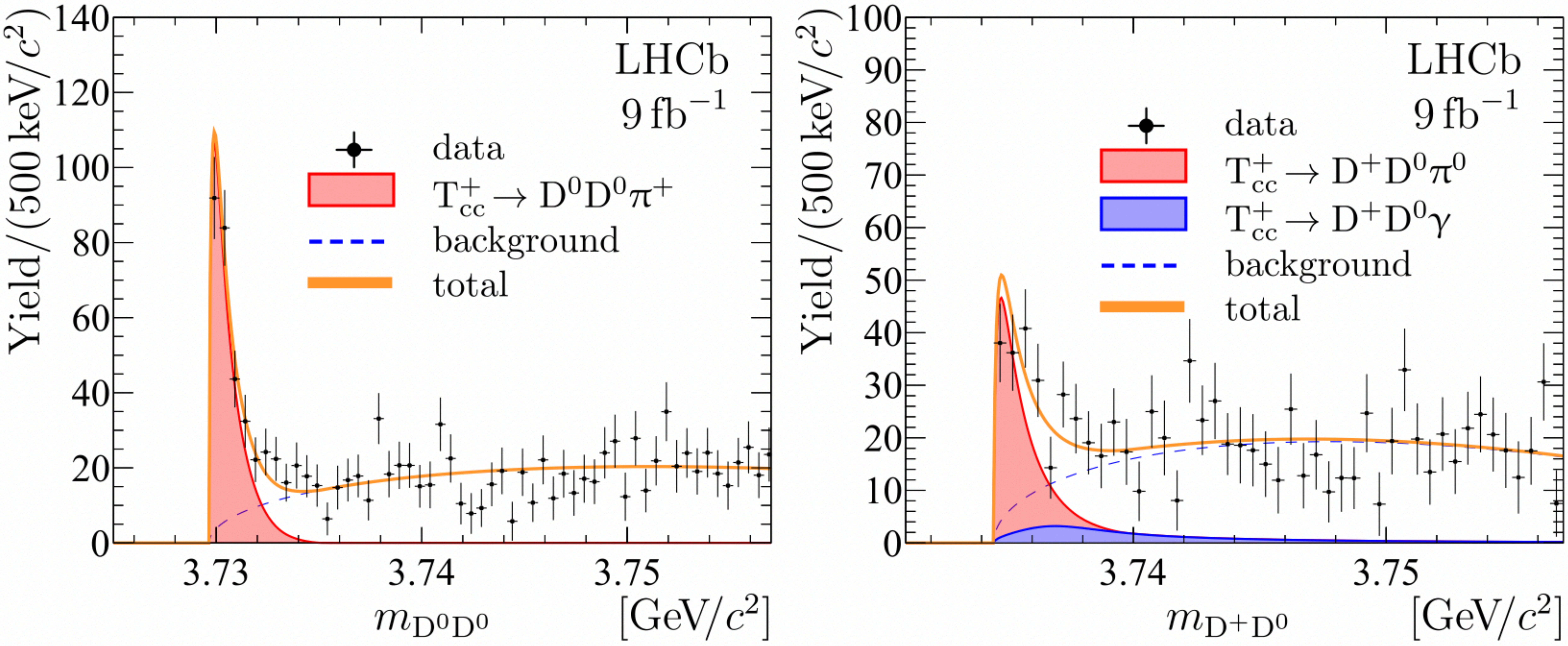


R. Aaij *et. al.* (LHCb), Nature Phys. 18 (2022) 7, 751-754, arXiv 2109.01038 [hep-ex]. (1)

R. Aaij *et. al.* (LHCb), Nature Commun. 13 (2022) 1, 3351, arXiv 2109.01056 [hep-ex]. (2)



$$\delta m_{pole} = -360 \pm 40^{+4}_{-0}\,\text{keV}\,,$$
$$\Gamma_{pole} = 48 \pm 2^{+0}_{-14}\,\text{keV}\,. \quad (2)$$



LHCb
9 fb^{-1}
data
$T_{cc}^{+} \to D^0 D^0 \pi^+$
background
total
Yield / (500 keV/c^2)
$m_{D^0D^0}$ [GeV/c^2]
3.73
3.74
3.75
0
20
40
60
80
100
120
140
LHCb
9 fb^{-1}
data
$T_{cc}^{+} \to D^+ D^0 \pi^0$
$T_{cc}^{+} \to D^+ D^0 \gamma$
background
total
Yield / (500 keV/c^2)
$m_{D^+D^0}$ [GeV/c^2]
3.74
3.75
0
10
20
30
40
50
60
70
80
90
100

T_{cc}^{+} is quite similar to X(3872)

Decays: $X(3872) \to J/\psi\pi^+\pi^-$
$\to D^0\bar{D}^0\pi^0$
$\to D^0\bar{D}^0\gamma$

$X(3872) \to J/\psi\pi^+\pi^-\pi^0$
$\to J/\psi\gamma$ (C=1)
$\to \psi(2S)\gamma$
$\to \chi_{c1}\pi^0$

angular distributions in $J/\psi\pi^+\pi^-$ require $J^{PC} = 1^{++}$

LHCb, PRL 110 (2013) 222001
arXiv:1302.6269 [hep-ex]

S-wave coupling to $D\bar{D}^* + \bar{D}D^*$

$$\frac{Br[X(3872) \to J/\psi\pi^+\pi^-\pi^0]}{Br[X(3872) \to J/\psi\pi^+\pi^-]} = 0.8 \pm 0.3$$

X(3872) is mixed state
w/ I=0 and I=1?

Extremely Close to Threshold:

$$m_X = 3871.65 \pm 0.06\,\mathrm{MeV}$$

$$m_{D^0} = 1864.84 \pm 0.05\,\mathrm{MeV}$$

$$m_{D^{*0}} = 2006.85 \pm 0.05\,\mathrm{MeV}$$ (from PDG)

$$m_X - m_{D^0} - m_{D^{*0}} = -0.04 \pm 0.09\,\mathrm{MeV}$$

Universality: $\psi_{D^0 D^{*0}} \propto \frac{e^{-r/a}}{r}$ $\quad a \geq 9.5\,\mathrm{fm}$ $\quad B.E. = \frac{1}{2\mu_{DD^*} a^2}$

Long distance physics of X(3872) calculable in terms of scattering length, known properties of D mesons - Effective Range Theory (ERT)

(M. B. Voloshin, E. Braaten, et. al.)

$$m_X - m_{D^+} - m_{D^{*-}} = -8.11\,\mathrm{MeV} \qquad \psi_{D^+D^{*-}} \propto \frac{e^{-r/a_+}}{r} \qquad a_+ = 1.6\,\mathrm{fm}$$

Long distance physics of X(3872) dominated by $D^0\bar{D}^{*0} + c.c.$ state for $r \geq 2\,\mathrm{fm}$

This is true even if short-distance structure of X(3872) is, e.g., I = 0

Motivates low energy EFT description of X(3872) with D^0, D^{*0}, π^0 as relevant d.o.f.

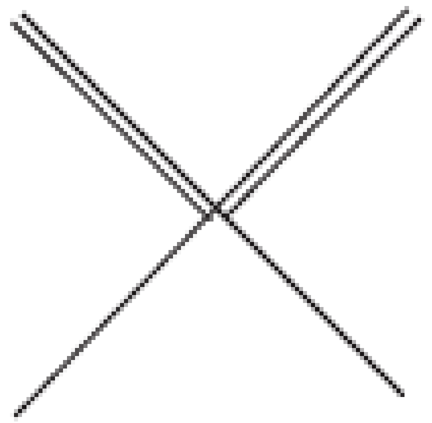
XEFT

S.Fleming, M.Kusunoki, T.M.,
U.van Kolck, PRD76:034006 (2007)

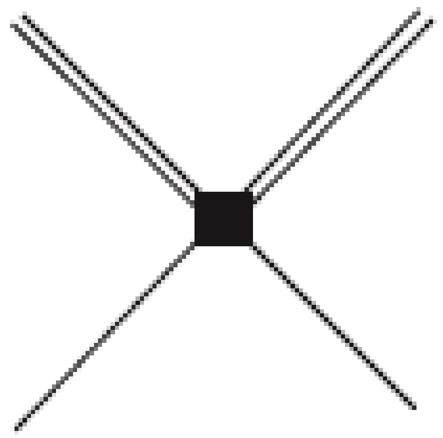
Non-Relativistic Propagators

D $\quad D^*$ $\quad \pi$ $\quad \sim \frac{1}{Q^2}$

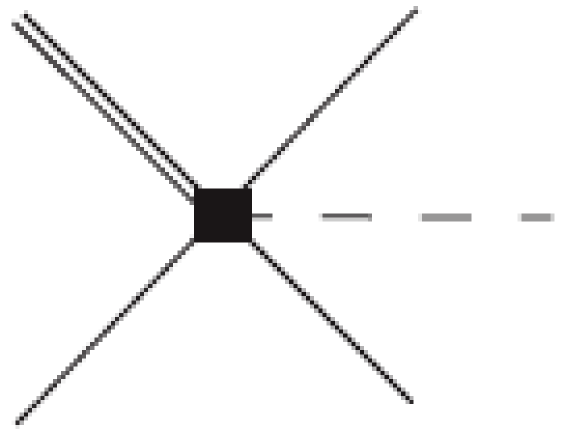
Contact Interactions, Pion Exchange



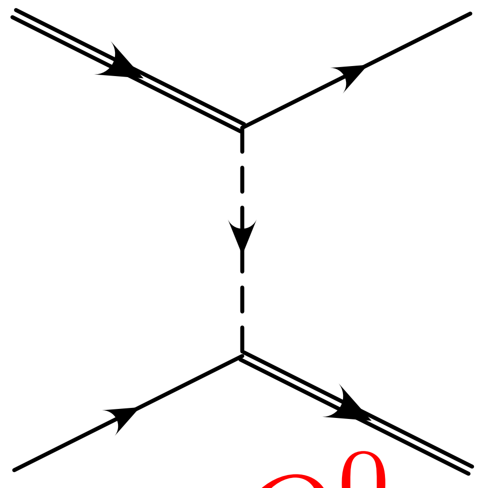
$C_0 \sim Q^{-1}$



$C_2\, p^2 \sim Q^0$



$B_1\, \epsilon \cdot p_\pi \sim Q^{-1}$



$\sim Q^0$

Power Counting

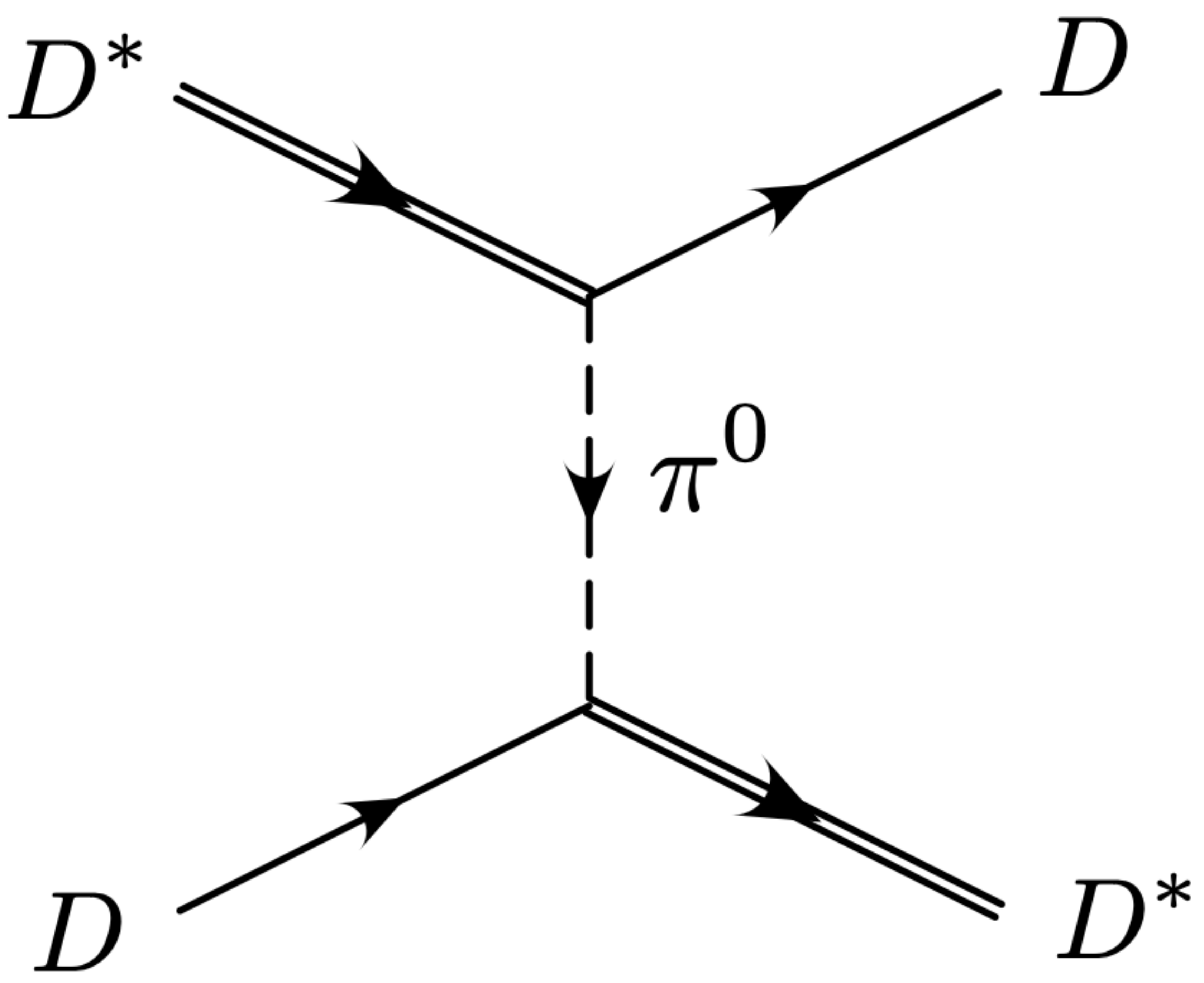
$$p_D \sim p_\pi \sim \mu \sim \gamma \sim Q \qquad \gamma \equiv \sqrt{-2\mu_{DD^*}\mathrm{B.E.}} \leq 34\,\mathrm{MeV}$$

$m_\pi \approx \Delta_H \approx 140\,\mathrm{MeV}$ are large scales in X-EFT

π^0 exchange and the X(3872)

$$\Delta \equiv m_{D^*} - m_D \approx 142\,\mathrm{MeV}$$

$$m_{\pi^0} \approx 135\,\mathrm{MeV}$$



$$\propto \frac{1}{\Delta^2 - \vec{q}^{\,2} - m_\pi^2} = \frac{1}{-\vec{q}^{\,2} + \mu^2}$$

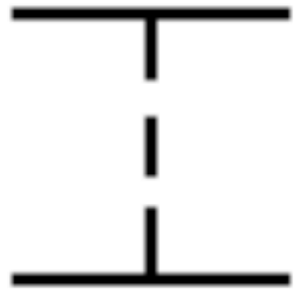
$$\mu^2 \equiv \Delta^2 - m_\pi^2 \approx (44\,\mathrm{MeV})^2$$

μ - new long distance scale

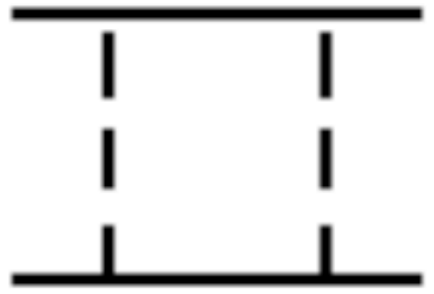
(Torngqvist, Suzuki)

perturbative pions and the X(3872)

nuclear physics: pion exchange in NN scattering



$$= \frac{g_A^2}{2f^2} A\left(\frac{p}{m_\pi}\right),$$



$$= \left(\frac{g_A^2}{2f^2}\right)^2 \frac{M m_\pi}{4\pi} B\left(\frac{p}{m_\pi}\right)$$

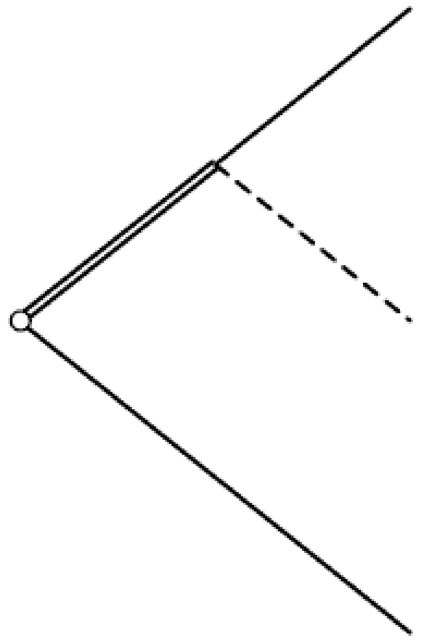
Expansion parameter: $\dfrac{g_A^2 M_N m_\pi}{8\pi f^2} \sim \dfrac{1}{2}$

X(3872) $\quad g_A = 1.25 \rightarrow g \sim 0.5 - 0.7$ $\qquad m_\pi \longrightarrow \mu$

$$\boxed{\frac{g^2 M_D \mu}{8\pi f^2} \sim \frac{1}{20} - \frac{1}{10}}$$

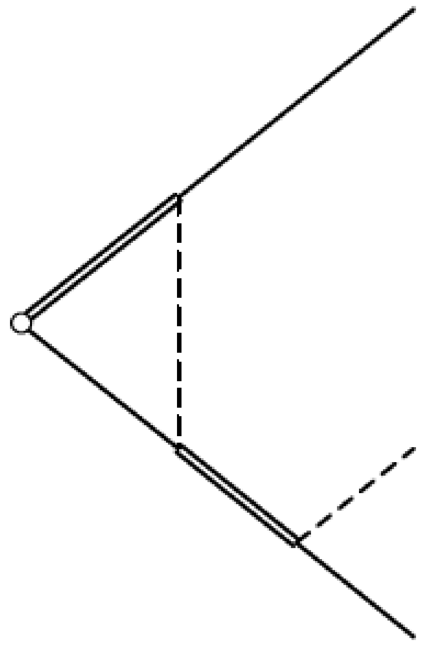
LO - reproduce ERT prediction for $X(3872) \to D^0 \bar{D}^0 \pi^0$

M.B.Voloshin, PLB 579: 316 (2004)



$$\frac{d\Gamma_{\rm LO}}{dp_D^2\, dp_{\bar{D}}^2} = \frac{g^2}{32\pi^3 f_\pi^2} 2\pi\gamma (\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left[\frac{1}{p_D^2+\gamma^2} + \frac{1}{p_{\bar{D}}^2+\gamma^2} \right]^2$$

NLO - range corrections, non-analytic corr. from π^0 exchange



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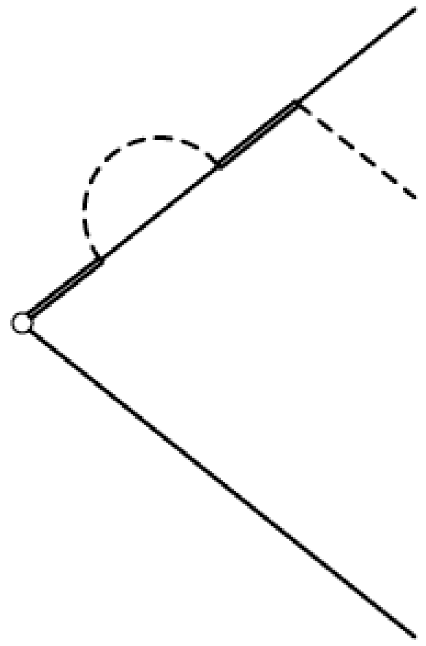
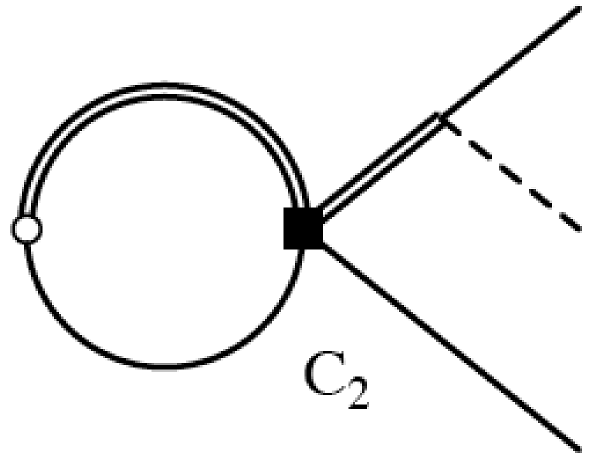
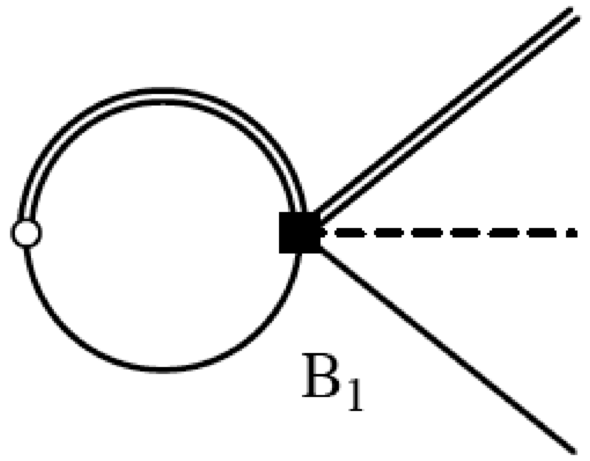


b)

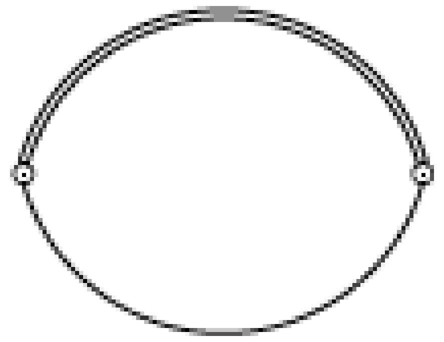


a)

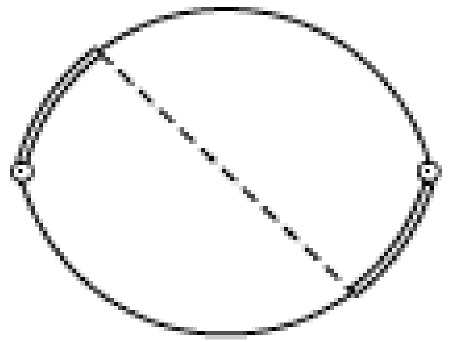


b)

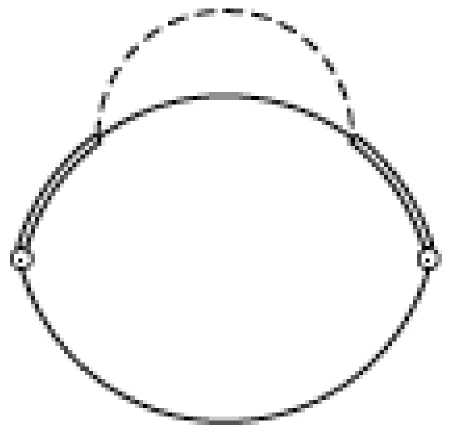
Wavefunction Renormalization



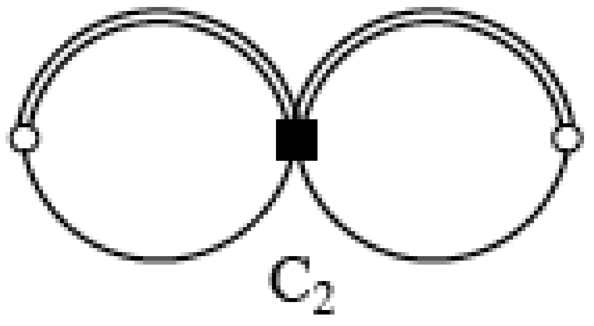
LO



a)



b)

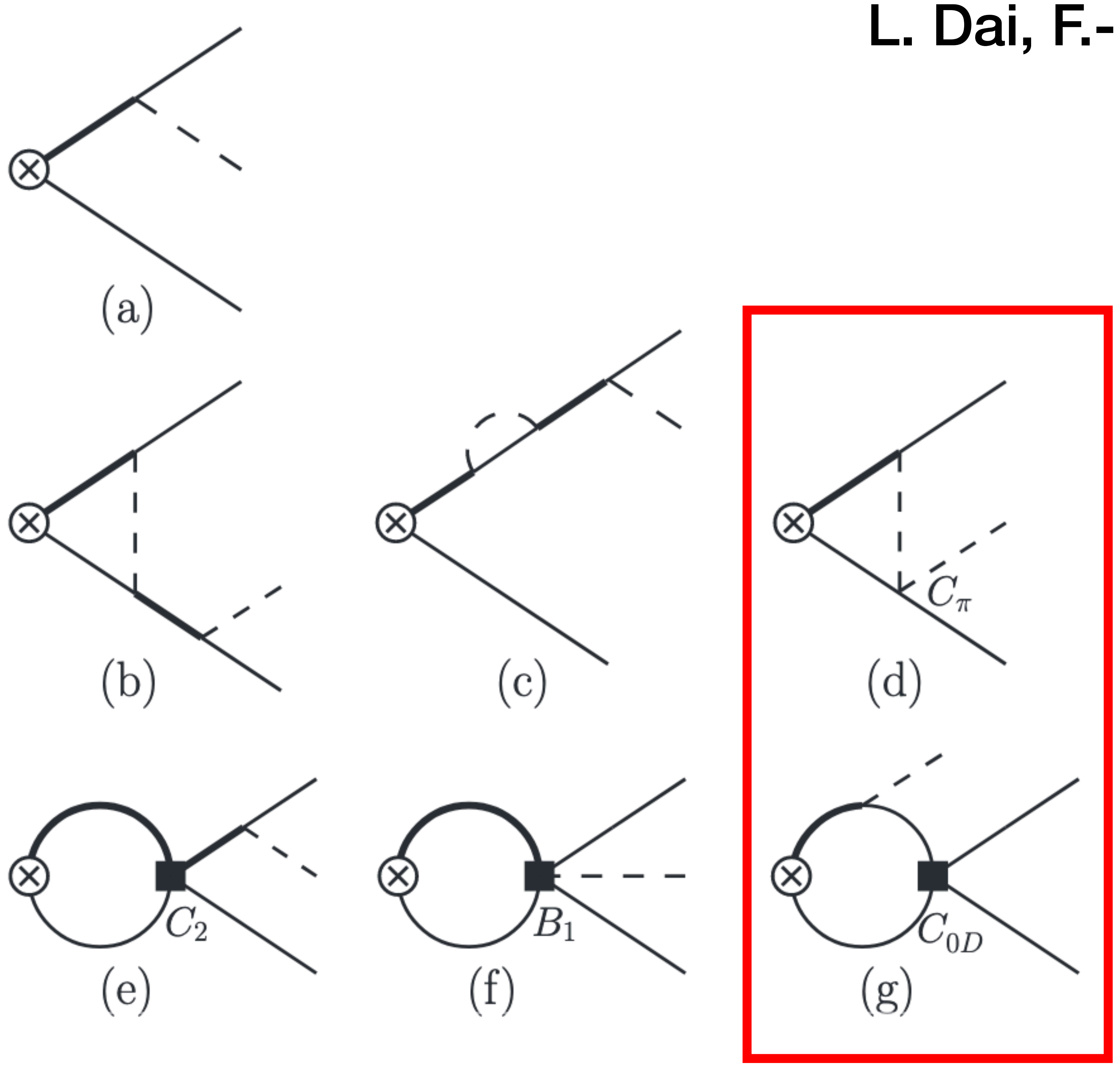


c)

NLO

Revisiting $X(3872) \to D^0\bar{D}^0\pi^0$ in XEFT

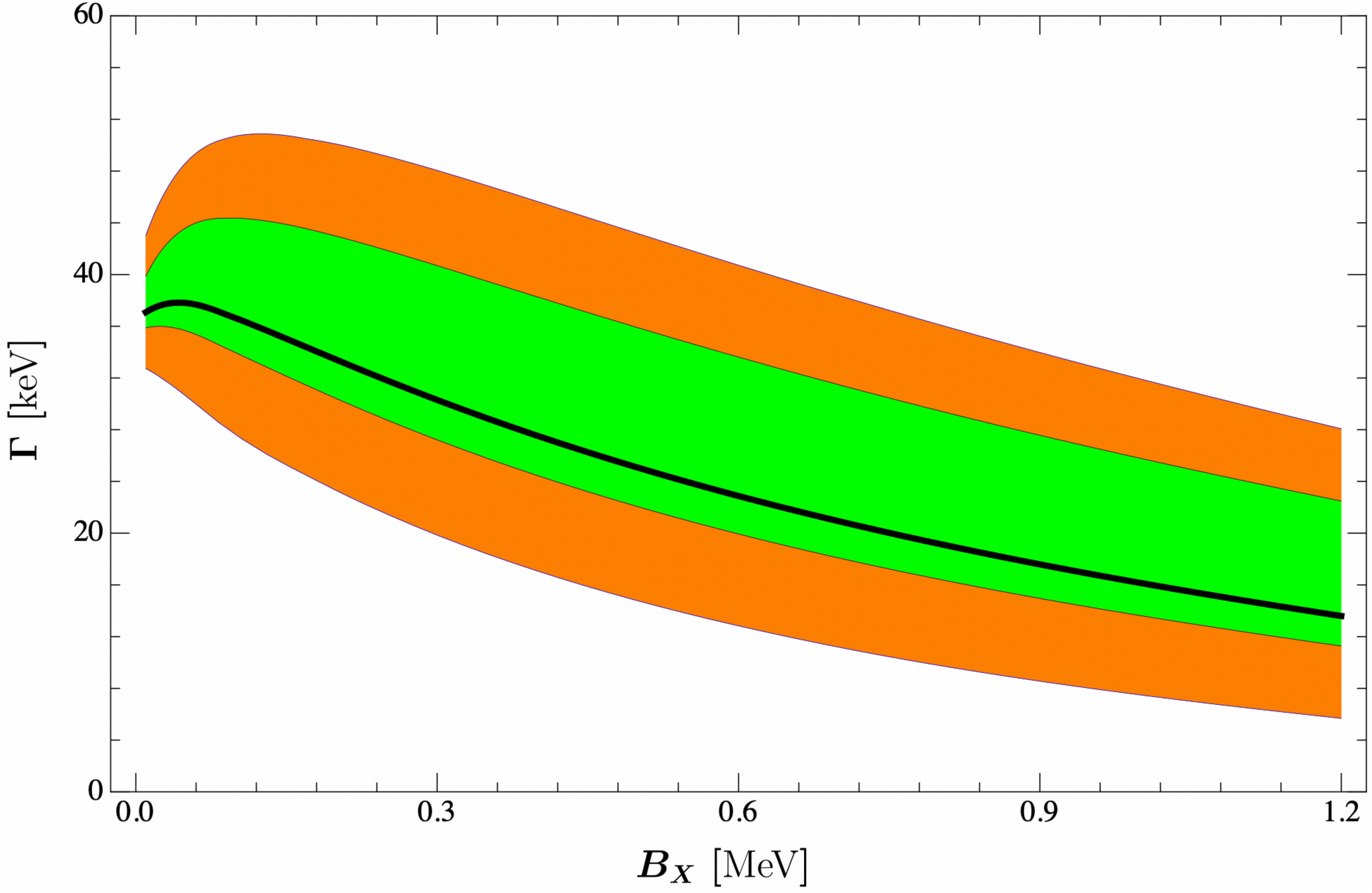
L. Dai, F.-K. Guo, TM, Phys. Rev. D. 101 (2020) 5, 054024, arXiv:1912.04317



πD rescattering

$D\bar{D}$ rescattering

C_π operators had not been known when XEFT as developed, coefficients recently fixed on lattice


60
40
20
0
0.0
0.3
0.6
0.9
1.2
Γ [keV]
B_X [MeV]

Bound on $\Gamma[X(3872)]$

Zero binding energy: $\Gamma[X(3872) \to D^0 \bar{D}^0 \pi^0] = \Gamma[D^{*0} \to D^0 \pi^0]$
$= 36\,\mathrm{keV}$

XEFT + BE < 0.13 MeV: $26\,\mathrm{MeV} < \Gamma[X(3872) \to D^0 \bar{D}^0 \pi^0] < 50\,\mathrm{MeV}$

PDG: $$\frac{\Gamma[X(3872) \to D^0 \bar{D}^0 \pi^0]}{\Gamma[X(3872)]} = 0.49^{+0.18}_{-0.20} \pm 0.16$$

Bound on total width: $\Gamma[X(3872)] < 150 - 200\,\mathrm{KeV}$

TM,Phys.Rev. D92 (2015) no.3, 034019 arXiv:1503.02719

$\chi_{c1}(3872)$ WIDTH

VALUE (MeV)	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
1.19±0.21 OUR AVERAGE			Error includes scale factor of 1.1.		
$1.39 \pm 0.24 \pm 0.10$		15.6k	[1] AAIJ 20AD	LHCB	$pp \to J/\psi \pi^+ \pi^- X$
$0.96^{+0.19}_{-0.18} \pm 0.21$		4.2k	[2] AAIJ 20S	LHCB	$B^+ \to J/\psi \pi^+ \pi^- K^+$

Study of the lineshape of the $\chi_{c1}(3872)$ state arXiv:2005.13419

A study of the lineshape of the $\chi_{c1}(3872)$ state is made using a data sample corresponding to an integrated luminosity of 3 fb^{-1} collected in pp collisions at centre-of-mass energies of 7 and 8\,TeV with the LHCb detector. Candidate $\chi_{c1}(3872)$ and $\psi(2S)$ mesons from b-hadron decays are selected in the $J/\psi\pi^+\pi^-$ decay mode. Describing the {\mbox{lineshape}} with a Breit--Wigner function, the mass splitting between the $\chi_{c1}(3872)$ and $\psi(2S)$ states, Δm, and the width of the $\chi_{c1}(3872)$ state, Γ_{BW}, are determined to be

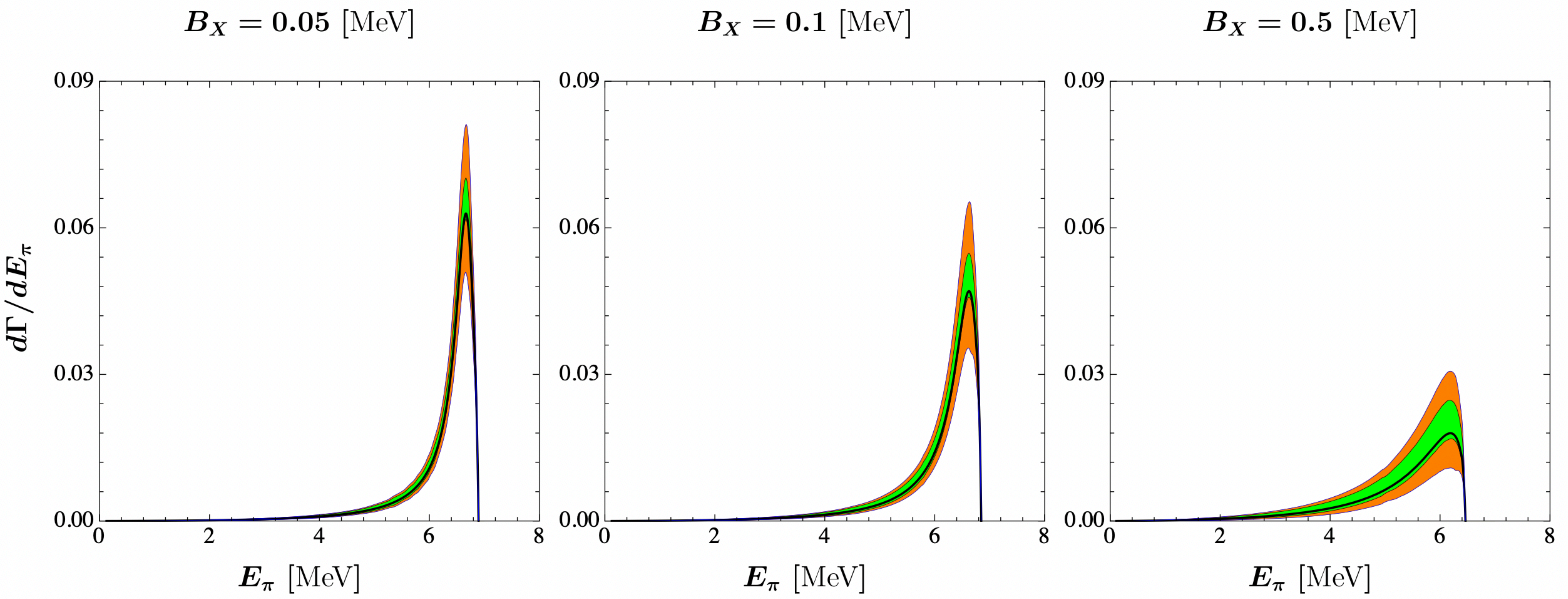
$$\Delta m = 185.598 \pm 0.067 \pm 0.068 \,\text{MeV}\,,$$
$$\Gamma_{\text{BW}} = 1.39 \pm 0.24 \pm 0.10 \,\text{MeV}\,,$$

where the first uncertainty is statistical and the second systematic. Using a Flatté-inspired model, the mode and full width at half maximum of the lineshape are determined to be

$$\text{mode} = 3871.69\,^{+0.00}_{-0.04}\,^{+0.05}_{-0.13}\,\text{MeV}$$
$$\text{FWHM} = 0.22\,^{+0.07}_{-0.06}\,^{+0.11}_{-0.13}\,\text{MeV}.$$

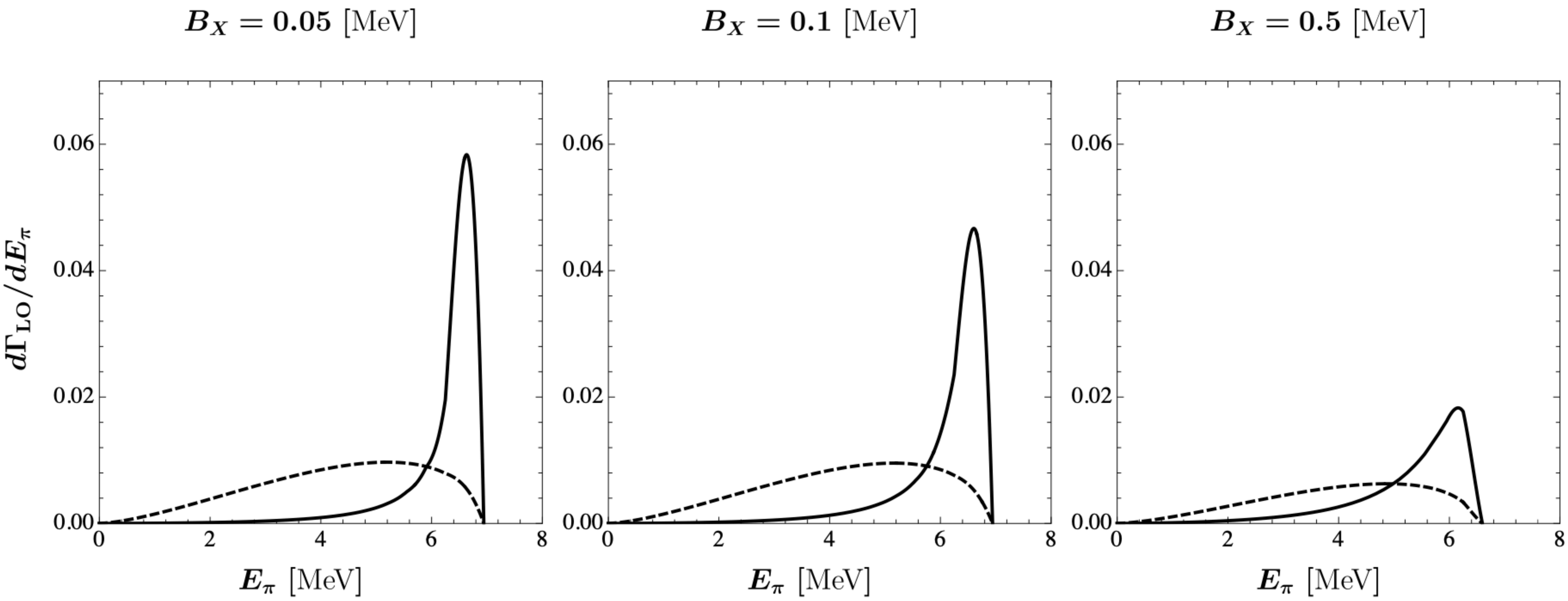
An investigation of the analytic structure of the Flatté amplitude reveals a pole structure, which is compatible with a quasi-bound $D^0\bar{D}^{*0}$ state but a quasi-virtual state is still allowed at the level of 2 standard deviations.

Implications for $\Gamma[X(3872) \to \chi_{cJ}\pi^0]$ TM, Phys.Rev. D92 (2015) no.3, 034019 arXiv:1503.02719



peak location is sensitive to binding energy

insensitive to NLO corrections



dashed line - phase space times p_π^2

Effective Field Theory for T_{cc}^{+}

Heavy Hadron Chiral Perturbation Theory plus contact terms for S-wave D*D scattering

$$\mathcal{L} = H^{*i\dagger}\left(i\partial^0 + \frac{\nabla^2}{2m_{H^*}} - \delta^*\right)H^{*i} + H^\dagger\left(i\partial^0 + \frac{\nabla^2}{2m_H} - \delta\right)H + \frac{g}{f_\pi}H^\dagger\partial^i\pi H^{*i} + \text{h.c.} + \frac{1}{2}H^\dagger\mu_D\vec{B}^i H^{*i} + \text{h.c.} - C_0(H^{*T}\tau_2 H)^\dagger(H^{*T}\tau_2 H) - C_1(H^{*T}\tau_2\tau_a H)^\dagger(H^{*T}\tau_2\tau_a H)\,.$$

$$H = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}, \quad H^{*i} = \begin{pmatrix} D^{*0i} \\ D^{*+i} \end{pmatrix}$$

S. Fleming, R. Hodges, TM, Phys. Rev. D. 104 (2021) 11, 116010, arXiv:2109.02188

Similar to XEFT for X(3872), new feature is coupled channels

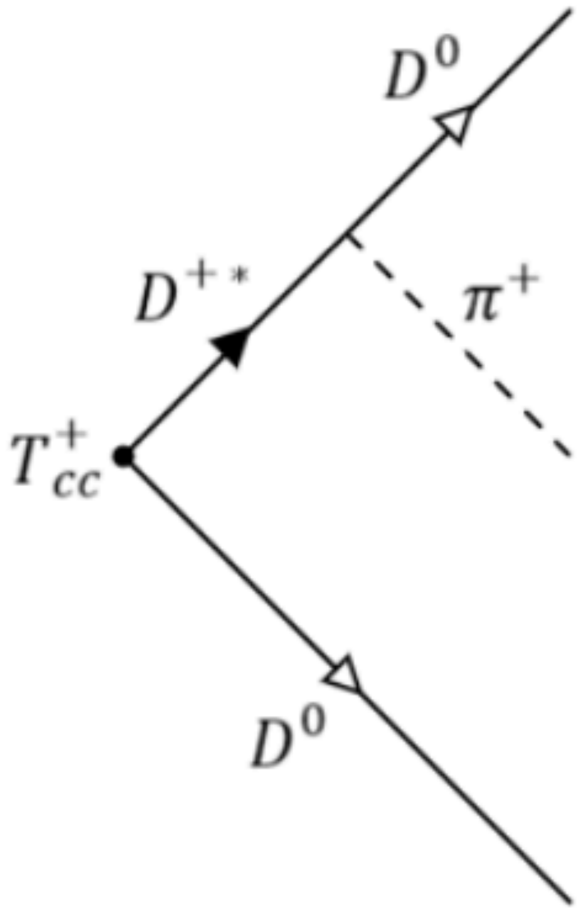
T-Matrix for D*D scattering

$$T = \frac{1}{E + E_T} \begin{pmatrix} g_0^2 & g_0 g_+ \\ g_0 g_+ & g_+^2 \end{pmatrix}$$

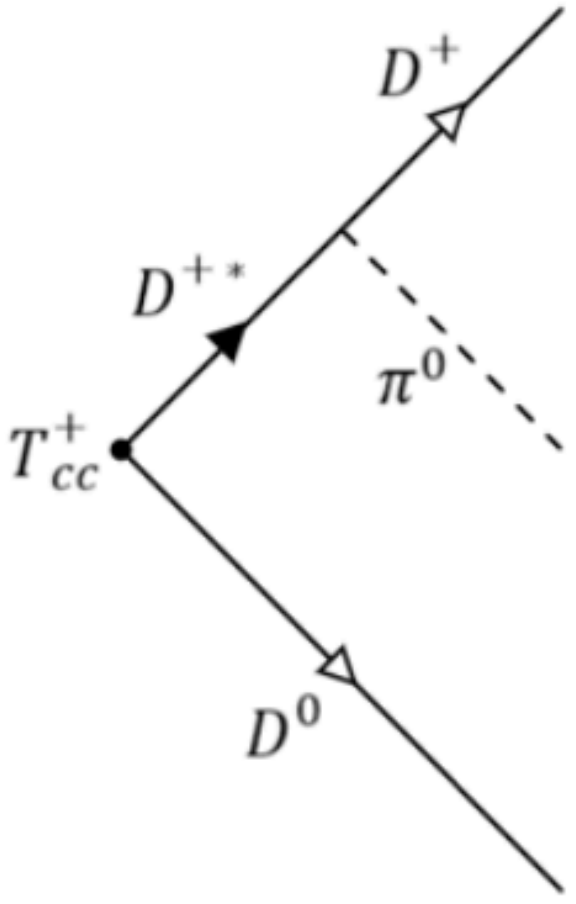
$$g_0^2 \Sigma_0'(-E_T) + g_+^2 \Sigma_+'(-E_T) = 1 \,,$$

Tune interactions to produce pole at T_{cc}^+

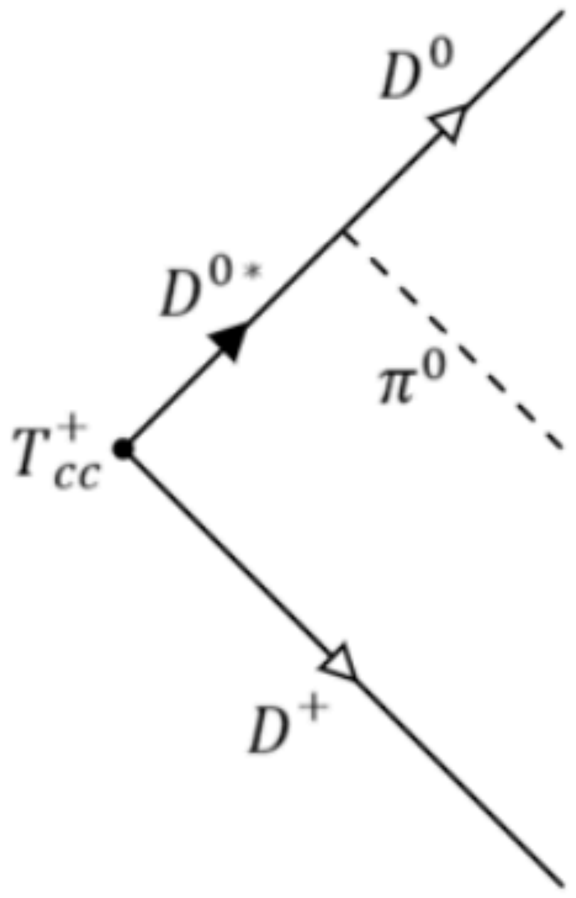
Decay Diagrams



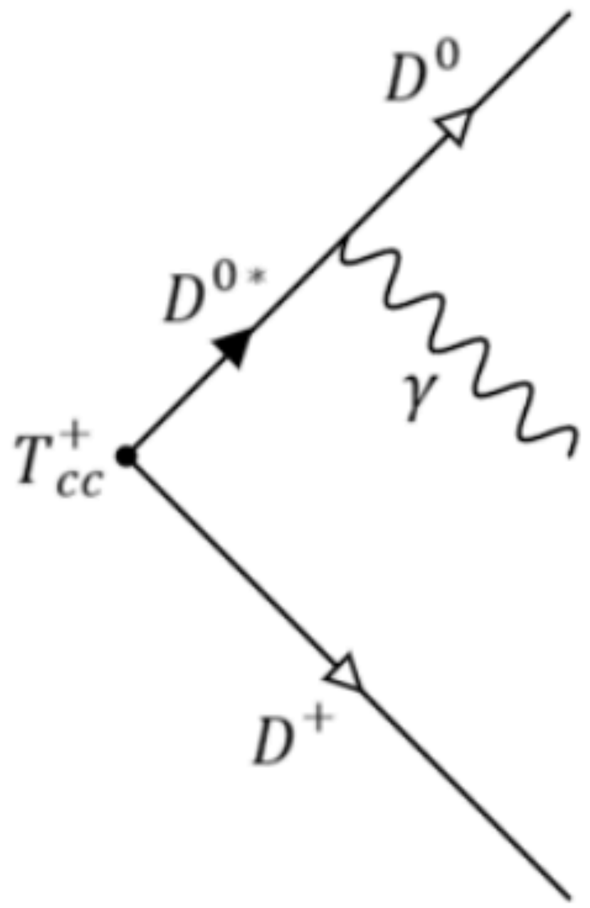
(a)



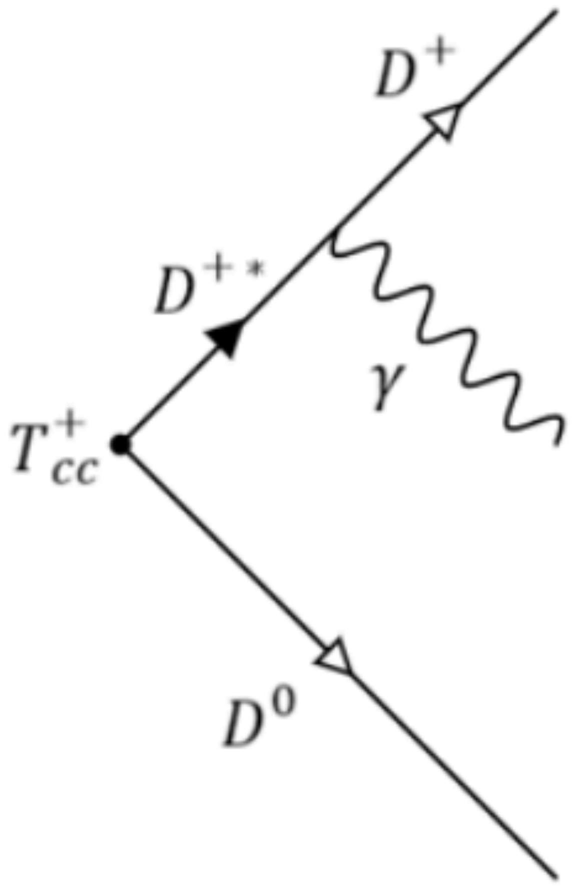
(b)



(c)



(d)



(e)

Decay rate formulae

$$\frac{d\Gamma[T_{cc}^+ \to D^0 D^0 \pi^+]}{dp_{D_1^0}^2 dp_{D_2^0}^2} = c_\theta^2 \frac{g^2}{(4\pi f_\pi)^2} \frac{2\gamma_0 p_\pi^2}{3} \left[\frac{1}{p_{D_1^0}^2 + \gamma_0^2} + \frac{1}{p_{D_2^0}^2 + \gamma_0^2} \right]^2 ,$$

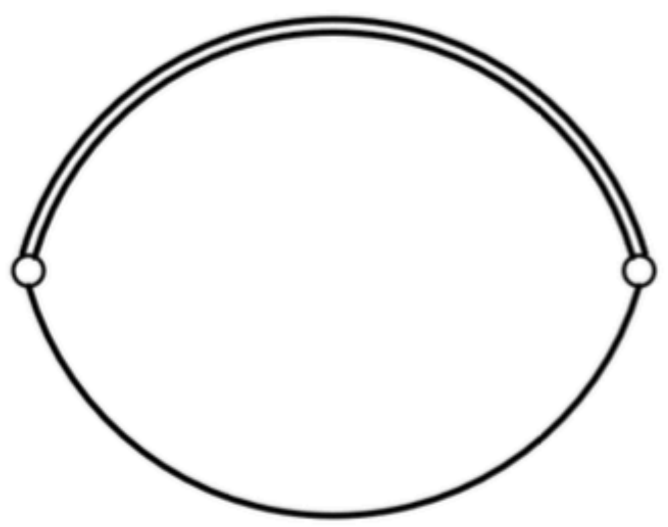
$$\frac{d\Gamma[T_{cc}^+ \to D^+ D^0 \pi^0]}{dp_{D^+}^2 dp_{D^0}^2} = \frac{g^2}{(4\pi f_\pi)^2} \frac{2p_\pi^2}{3} \left[\frac{\sqrt{\gamma_0}\, c_\theta}{p_{D^+}^2 + \gamma_0^2} - \frac{\sqrt{\gamma_+}\, s_\theta}{p_{D^0}^2 + \gamma_+^2} \right]^2 ,$$

$$\frac{d\Gamma[T_{cc}^+ \to D^+ D^0 \gamma]}{dp_{D^+}^2 dp_{D^0}^2} = \frac{E_\gamma^2}{6\pi^2} \left[\frac{\sqrt{\gamma_0}\, c_\theta \mu_{D^0}}{p_{D^+}^2 + \gamma_0^2} - \frac{\sqrt{\gamma_+}\, s_\theta \mu_{D^+}}{p_{D^0}^2 + \gamma_+^2} \right]^2 .$$

$$\gamma_0^2 = 2\mu_0(m_{D^0} + m_{D^{*+}} - m_T)$$
$$\gamma_+^2 = 2\mu_+(m_{D^+} + m_{D^{*0}} - m_T)$$

$$g_0^2 = \frac{\cos^2\theta}{\Sigma_0'(-E_T)} \qquad g_+^2 = \frac{\sin^2\theta}{\Sigma_+'(-E_T)}$$

$$i\Sigma_i(E) =$$



LO Predictions for Decay Rate

	I=0	I=1	$\Gamma_{\max}$
θ	$-32.4°$	$32.4°$	$-8.34°$
$\Gamma[T_{cc}^+ \to D^0 D^0 \pi^+]$	32	32	44
$\Gamma[T_{cc}^+ \to D^+ D^0 \pi^0]$	15	3.8	13
$\Gamma[T_{cc}^+ \to D^+ D^0 \gamma]$	6.1	2.8	1.9
$\Gamma[T_{cc}^+]$	52	38	58

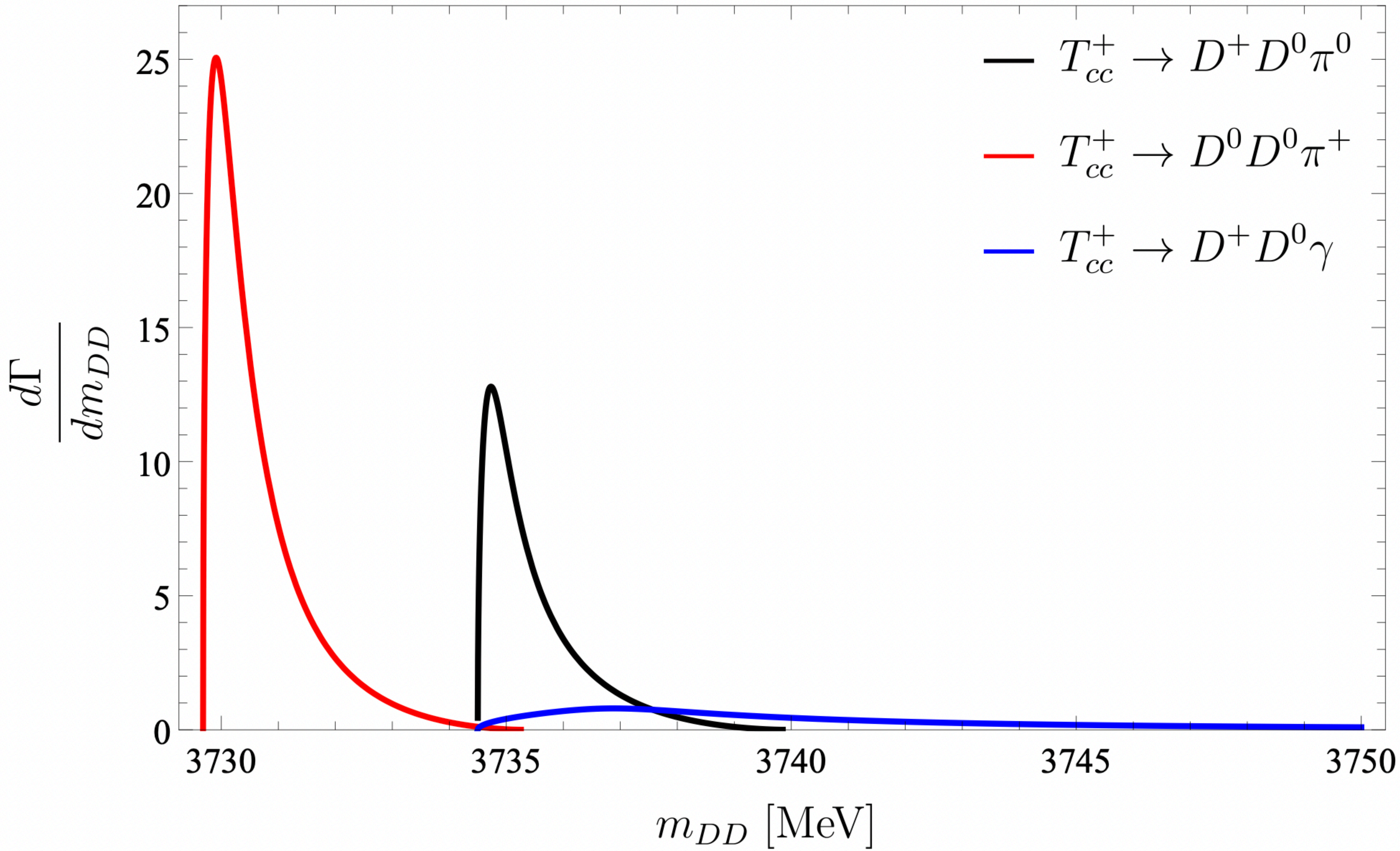
$$I = 0 : g_0 = -g_+$$

$$I = 1 : g_0 = g_+$$

Other Predictions

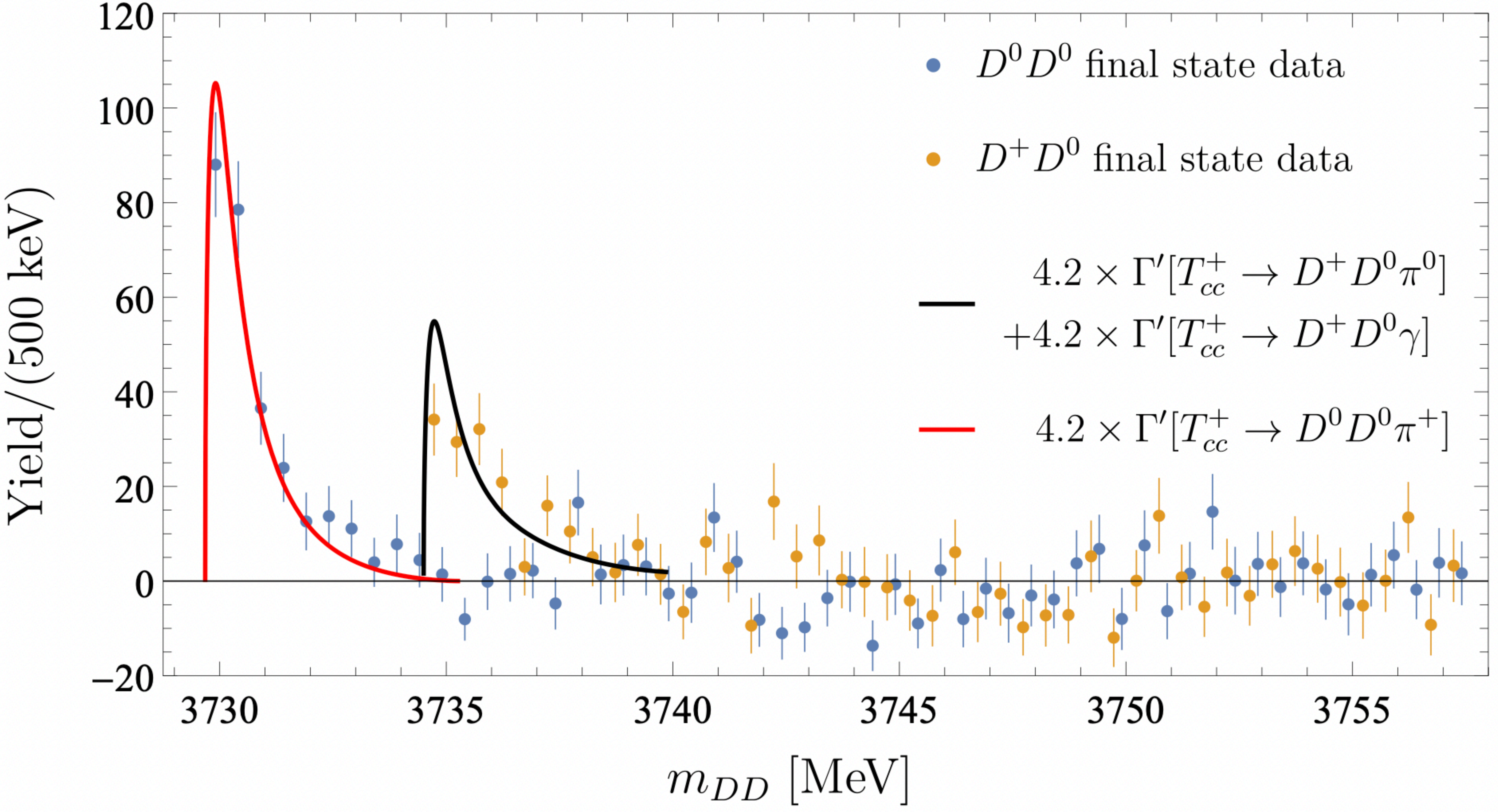
	$\Gamma(T_{cc}^+)$ (keV)
Fleming et al.	**52**
Meng et al.	$46.7^{+2.7}_{-2.9}$
Ling et al.	53
Feijoo et al.	43
Yan & Valderrama	49 ± 16
Albaladejo	77

$$\delta m_{BW} = -273 \pm 61 \pm 5^{+11}_{-14}\,\mathrm{keV}\,,$$
$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\,\mathrm{keV}\,. \quad (1)$$

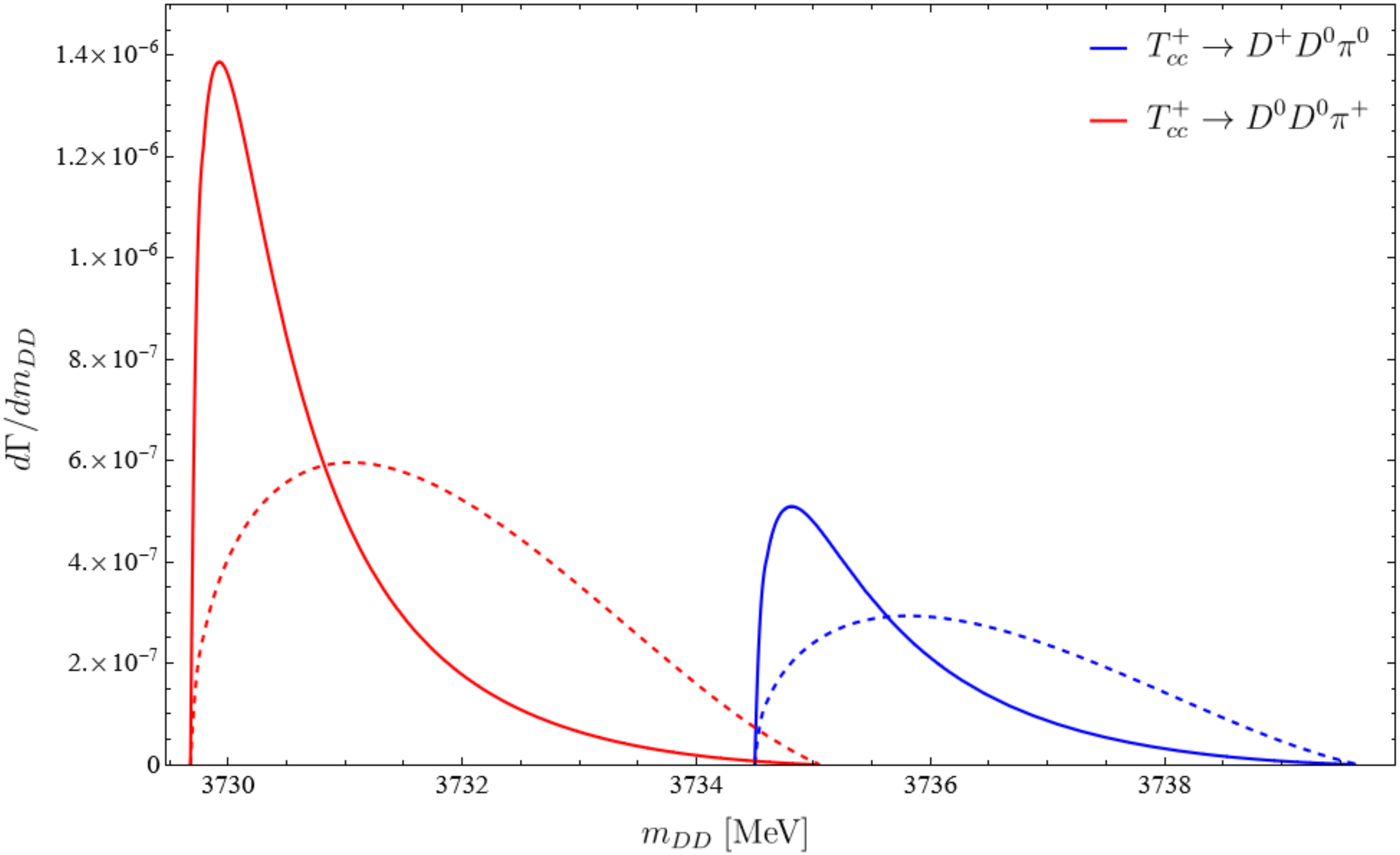


$T_{cc}^+ \to D^+ D^0 \pi^0$
$T_{cc}^+ \to D^0 D^0 \pi^+$
$T_{cc}^+ \to D^+ D^0 \gamma$
$\frac{d\Gamma}{dm_{DD}}$
25
20
15
10
5
0
3730
3735
3740
3745
3750
m_{DD} [MeV]

$d\Gamma/dm_{DD}$ vs. data

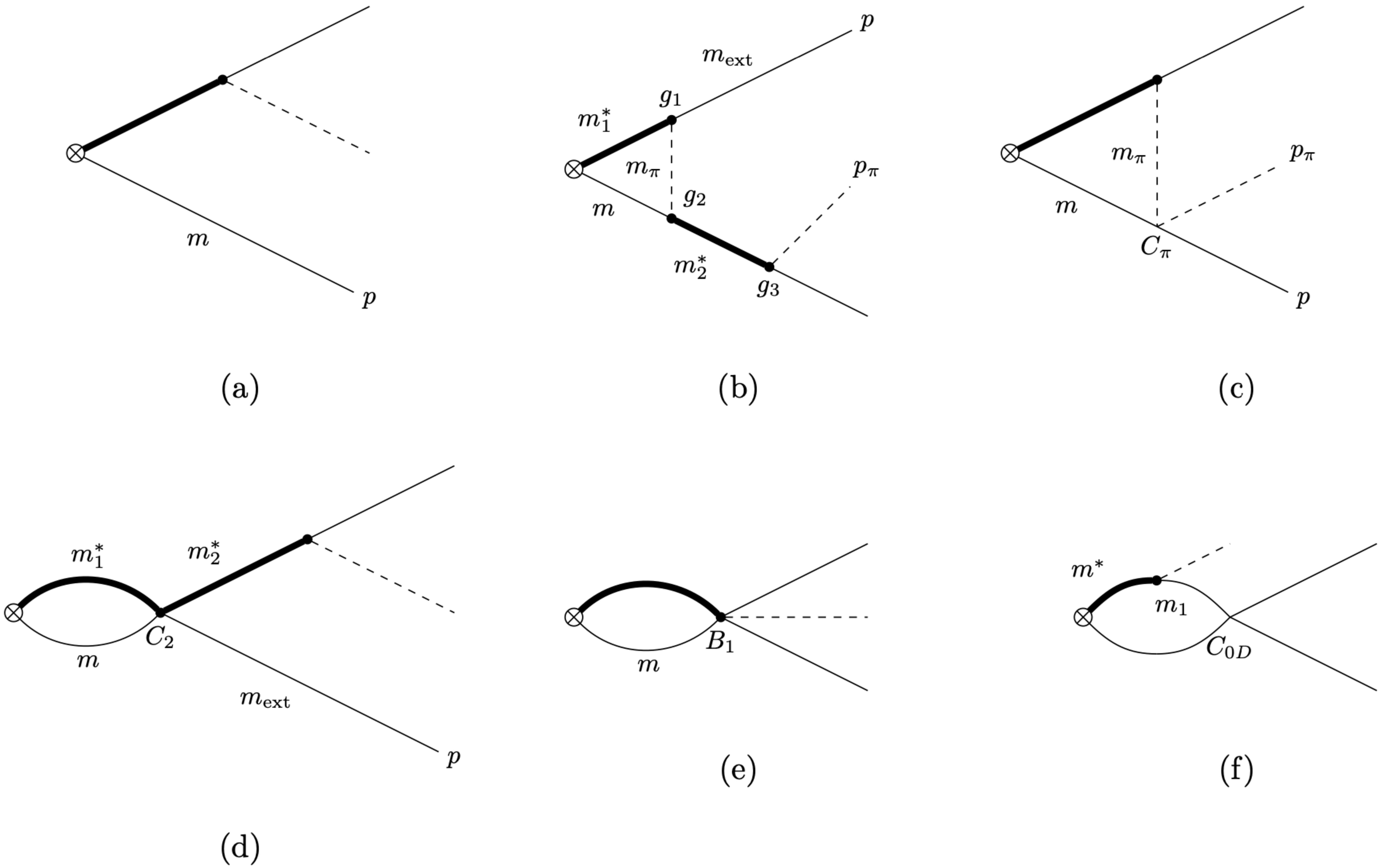


see also M.-L. Du, et.al., Phys. Rev. D, 105 (2022) 1, 014024



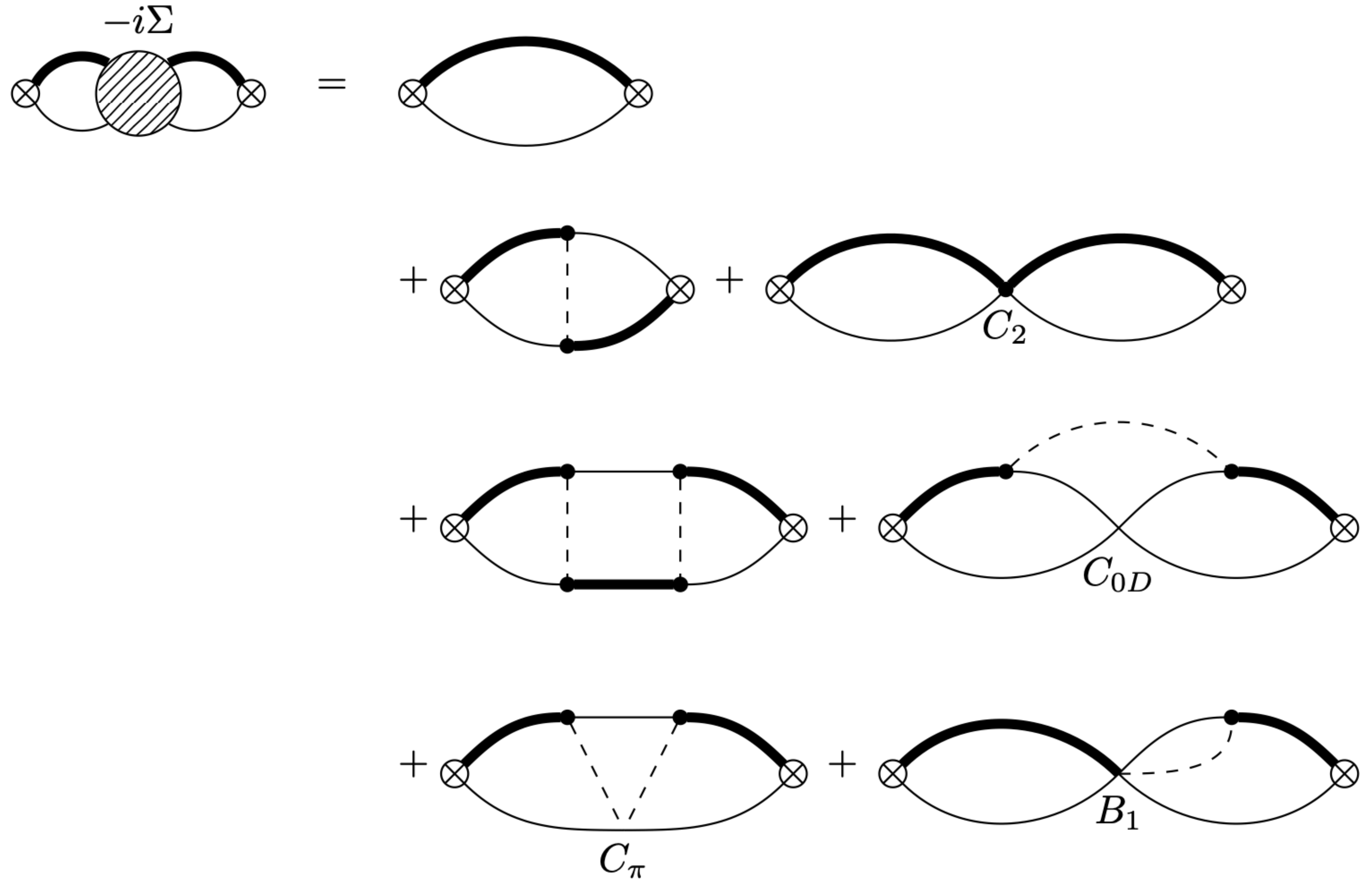
comparison with p_{π}^{2} $\times$ phase space (dashed)

NLO Corrections to Decay Rate - assume I = 0 state



L. Dai, S. Fleming, R. Hodges, TM, Phys. Rev. D 107 (2023) 7, 076001, arXiv:2301.11950 [hep-ph]

$$\Gamma_0 \approx \Gamma^{LO}\left(1 - \frac{\mathrm{Re}\,\Sigma_0'^{NLO}(-E_T)}{\mathrm{Re}\,\mathrm{tr}\,\Sigma'^{LO}(-E_T)}\right) + \frac{2\,\mathrm{Im}\,\Sigma_0^{NLO}(-E_T)}{\mathrm{Re}\,\mathrm{tr}\,\Sigma'^{LO}(-E_T)}$$

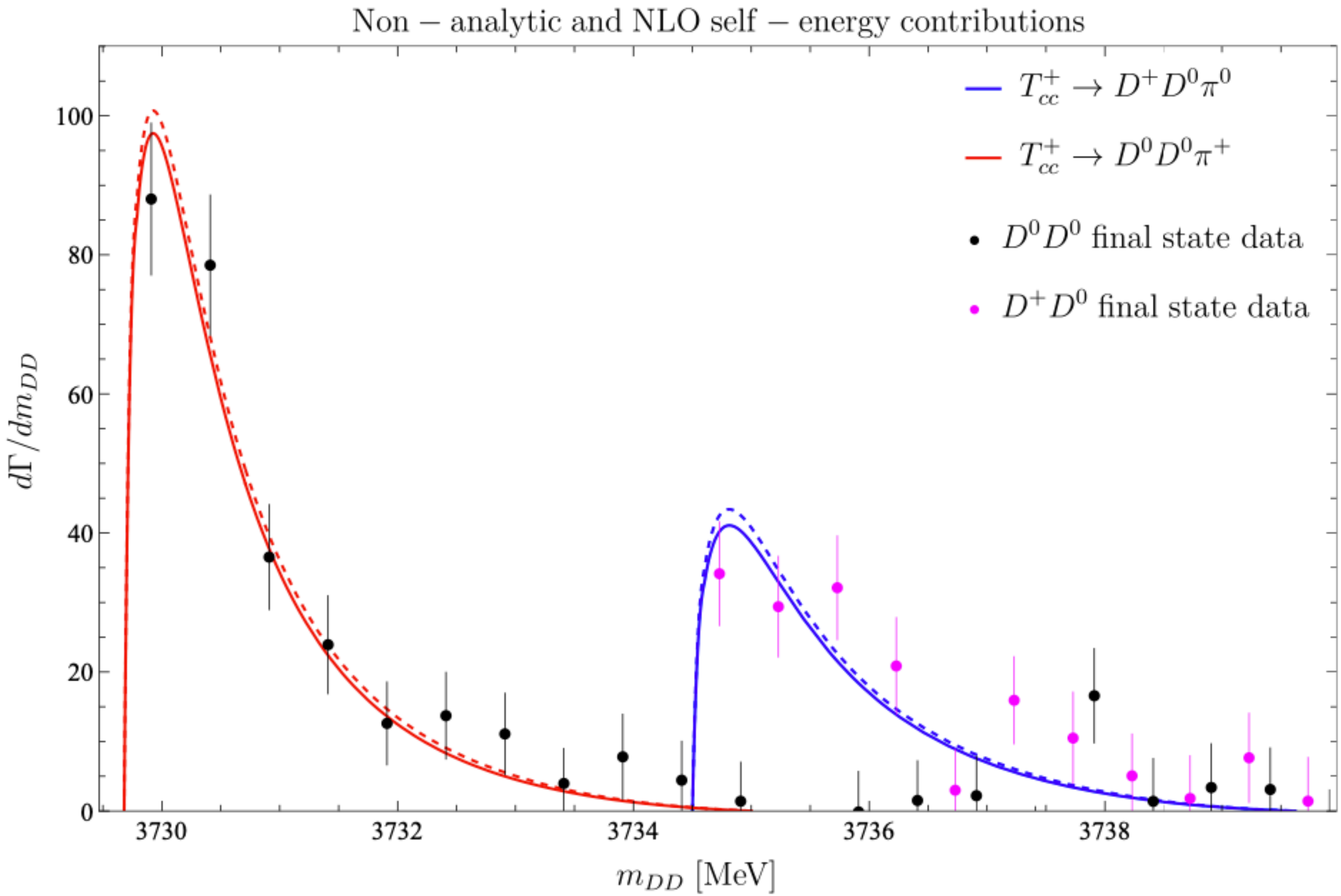


NLO Corrections to Decay Rate

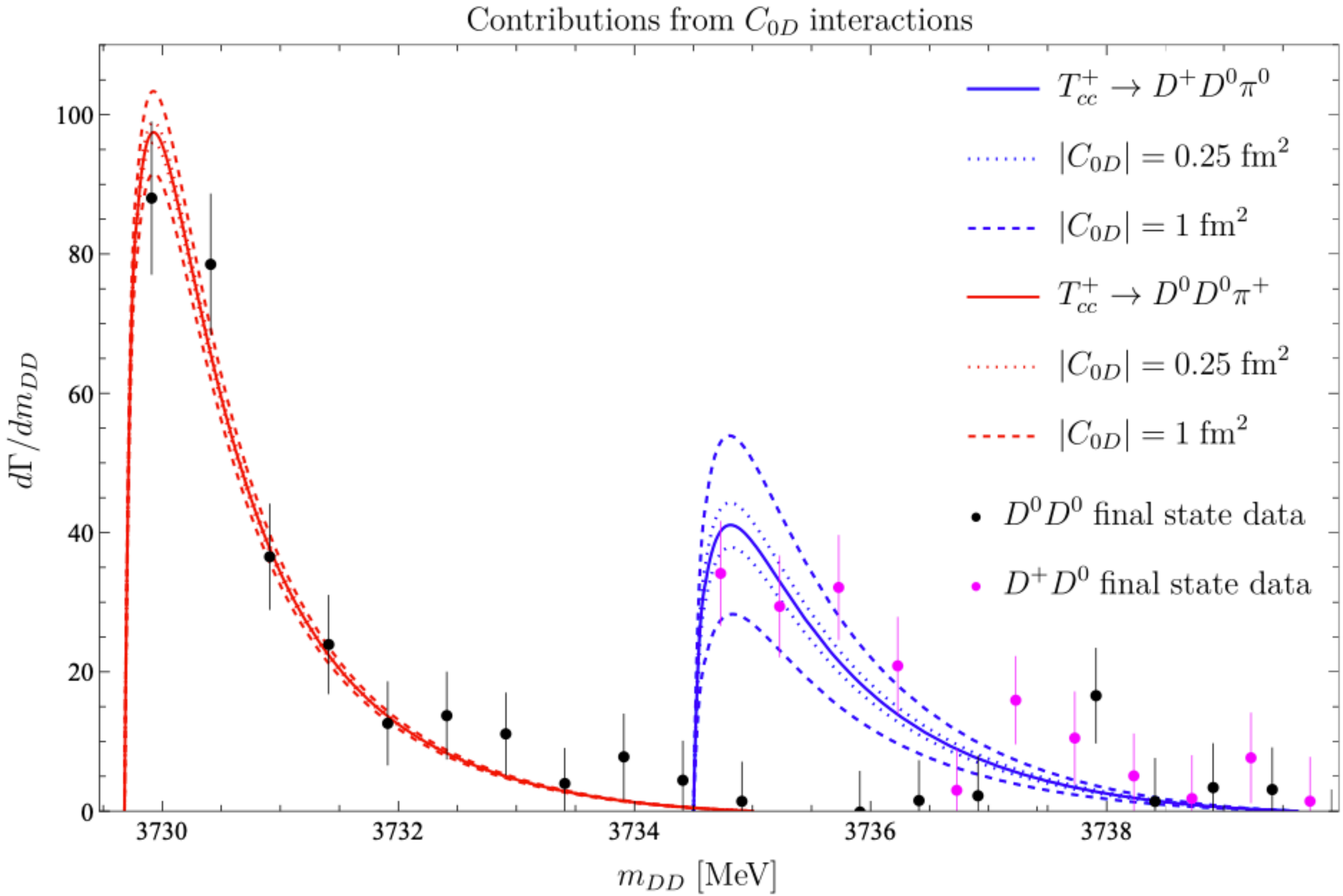
TABLE I: Partial and total widths in units of keV at LO and NLO.

	LO result	NLO lower bound	NLO upper bound
$\Gamma[T_{cc}^+ \to D^0 D^0 \pi^+]$	28	21	44
$\Gamma[T_{cc}^+ \to D^+ D^0 \pi^0]$	13	7.8	21
$\Gamma_{\text{strong}}[T_{cc}^+]$	41	29	66
$\Gamma_{\text{strong}}[T_{cc}^+] + \Gamma_{\text{EM}}^{LO}[T_{cc}^+]$	47	35	72

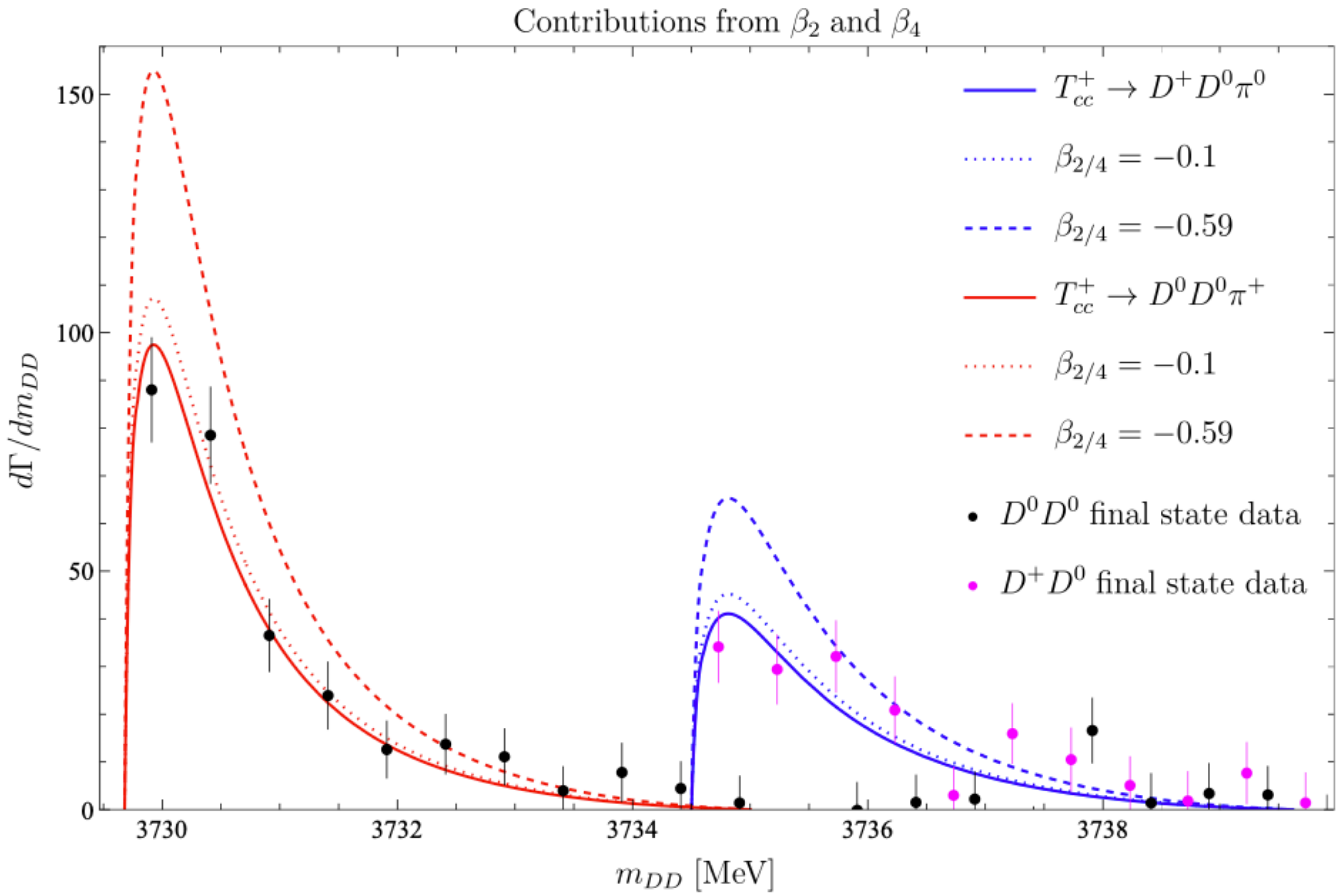
$$\delta m_{pole} = -360 \pm 40^{+4}_{-0}\,\text{keV}\,,$$
$$\Gamma_{pole} = 48 \pm 2^{+0}_{-14}\,\text{keV}\,. \qquad (2)$$



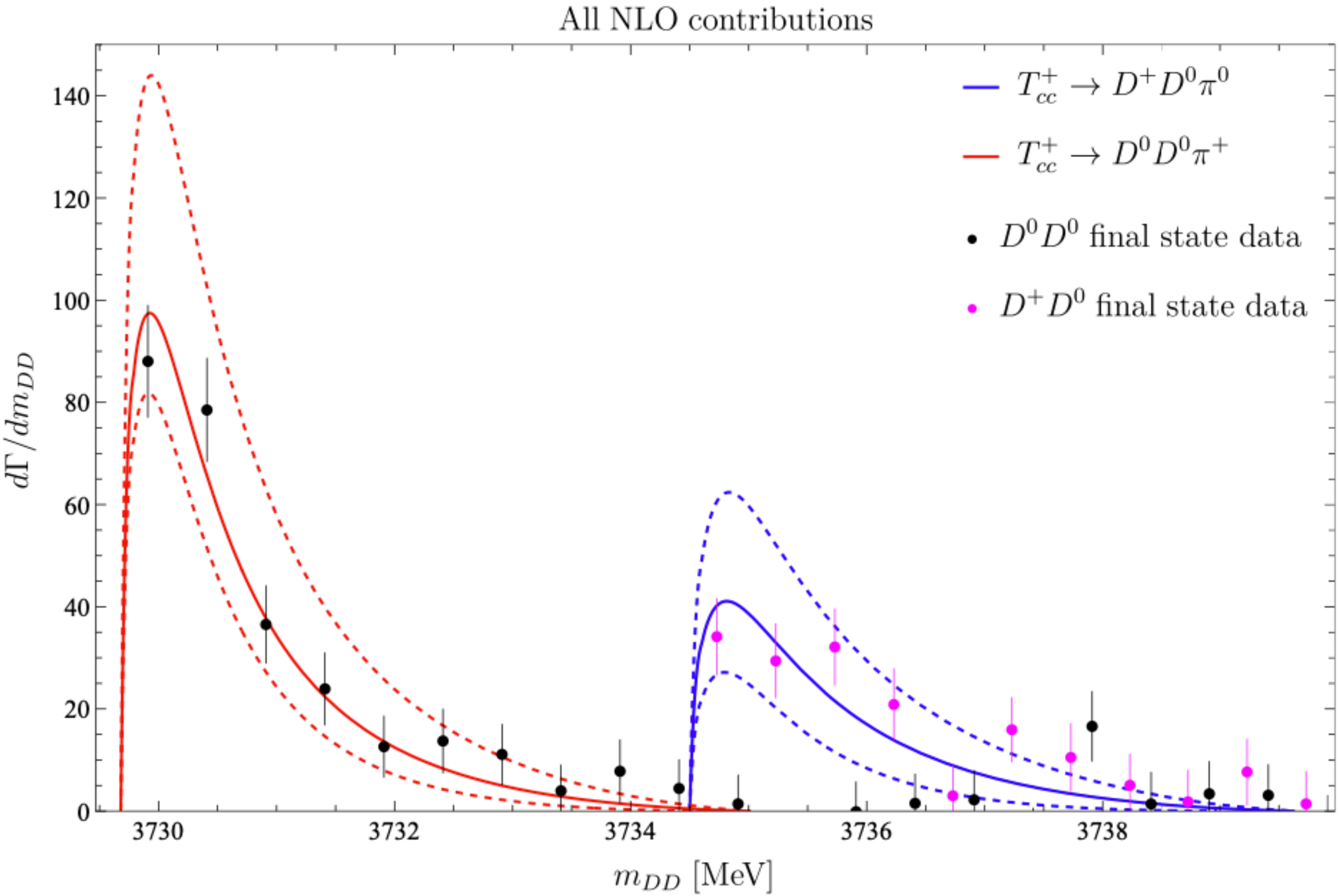
Non − analytic and NLO self − energy contributions
$T_{cc}^{+} \to D^{+}D^{0}\pi^{0}$
$T_{cc}^{+} \to D^{0}D^{0}\pi^{+}$
D^0D^0 final state data
D^+D^0 final state data
$d\Gamma/dm_{DD}$
100
80
60
40
20
0
3730
3732
3734
3736
3738
m_{DD} [MeV]



Contributions from C_{0D} interactions
$T_{cc}^+ \to D^+D^0\pi^0$
$|C_{0D}| = 0.25$ fm^2
$|C_{0D}| = 1$ fm^2
$T_{cc}^+ \to D^0D^0\pi^+$
$|C_{0D}| = 0.25$ fm^2
$|C_{0D}| = 1$ fm^2
D^0D^0 final state data
D^+D^0 final state data
$d\Gamma/dm_{DD}$
m_{DD} [MeV]
0
20
40
60
80
100
3730
3732
3734
3736
3738



Contributions from β_2 and β_4
$T_{cc}^+ \to D^+D^0\pi^0$
$\beta_{2/4} = -0.1$
$\beta_{2/4} = -0.59$
$T_{cc}^+ \to D^0D^0\pi^+$
$\beta_{2/4} = -0.1$
$\beta_{2/4} = -0.59$
D^0D^0 final state data
D^+D^0 final state data
$d\Gamma/dm_{DD}$
m_{DD} [MeV]
0
50
100
150
3730
3732
3734
3736
3738



All NLO contributions
$T_{cc}^+ \to D^+D^0\pi^0$
$T_{cc}^+ \to D^0D^0\pi^+$
D^0D^0 final state data
D^+D^0 final state data
$d\Gamma/dm_{DD}$
m_{DD} [MeV]
0
20
40
60
80
100
120
140
3730
3732
3734
3736
3738

Summary

XEFT: EFT of X(3872) as hadronic molecule

$\Gamma[X(3872) \to D^0 \bar{D}^0 \pi^0]$, bound on $\Gamma[X(3872)]$

energy spectrum of π^0

other XEFT calculations

$\pi^+ X(3872) \to D^* D^*$ E.Braaten, H. Hammer, TM, *Phys.Rev.D* 82 (2010) 034018

$X(3872) D^{(*)} \to X(3872) D^{(*)}$ D. Canham, H. Hammer, R. Springer, *Phys.Rev.D* 80 (2009) 014009

$X(3872) \to \psi(2S)\gamma$

$\psi(4040) \to X(3872)\gamma$

TM, R. Springer, *Phys. Rev. D* 80 (2009) 014009

$\psi(4160) \to X(3872)\gamma$ R. Springer, A Margaryan, *Phys. Rev. D* 88 (2013) 1, 014017

few incisive experimental tests to date….

Summary

excellent agreement w/ LO EFT calculations

$\mathbf{T}_{cc}^{+}$ decays described by XEFT-like theory w/ coupled channels

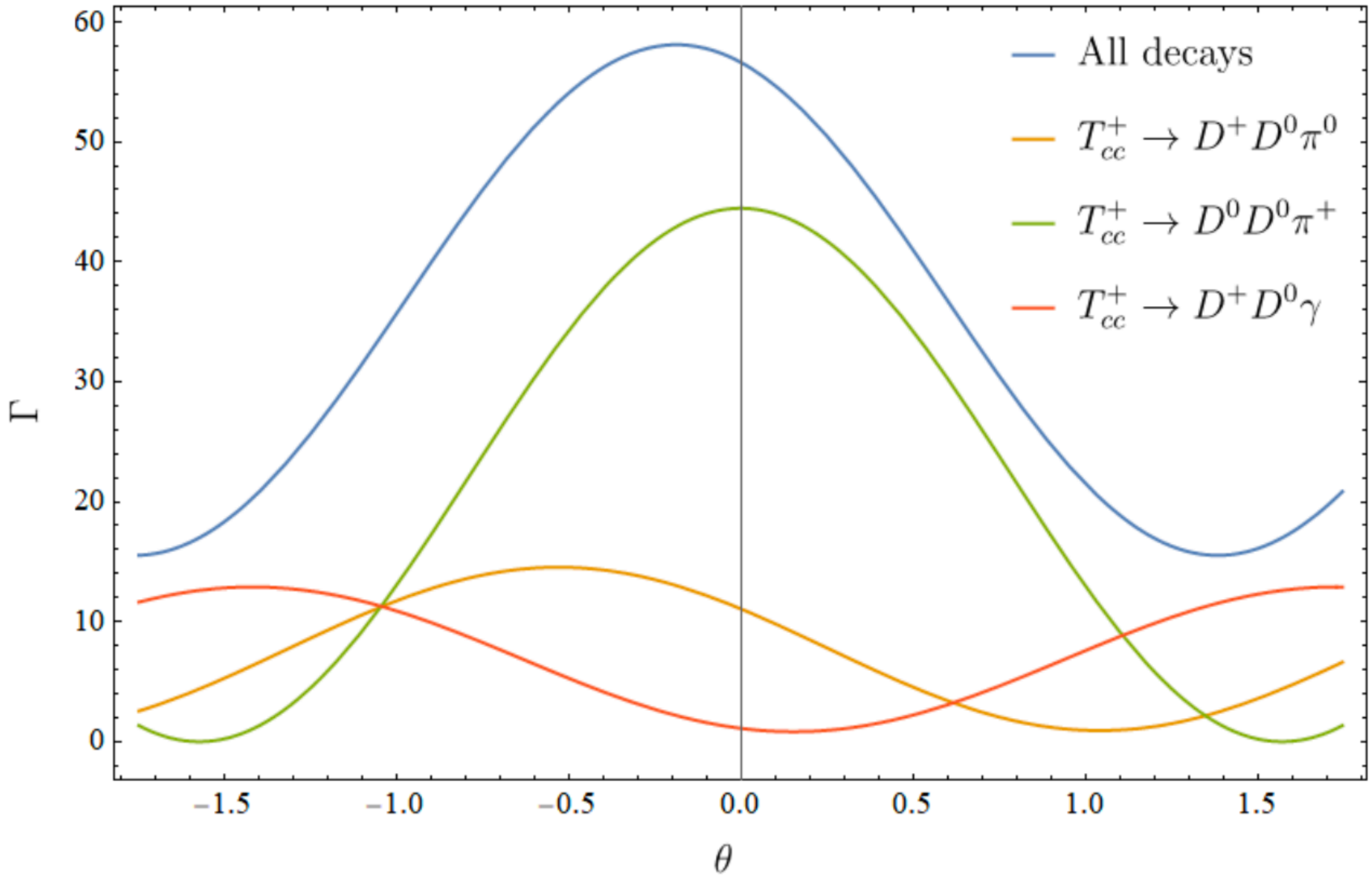
$\Gamma[T_{cc}^{+}], d\Gamma[T_{cc}^{+}]/dm_{DD}$

NLO corrections:	pion loops	negligble
	πD rescattering	negligble
	range corrections	dominant
	DD rescattering	significant

NLO calculations agree with data, could constrain D interactions

would be very interesting to measure $\frac{d\Gamma[X(3872)]}{dm_{D\bar{D}}}$

Extra Slides



this agrees well with the same plot by Meng et al. (2107.14784)