Optimized QCD two-loop correction to exclusive double J/ψ production at B factories

Wen-Long Sang
Southwest University, ChongQing, China

In collaboration with Feng Feng, Yu Jia, Zhewen Mo, Jichen Pan, Jiayue Zhang, based on arXiv:2306.11538

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Outline：

1. Background
2. Outline of calculation
3. Discussion
4. Summary

1. Background

- Exclusive double charmonium production at e^+e^- collider is among the simplest hard exclusive reactions in perturbative QCD, which can be used to testify the factorization

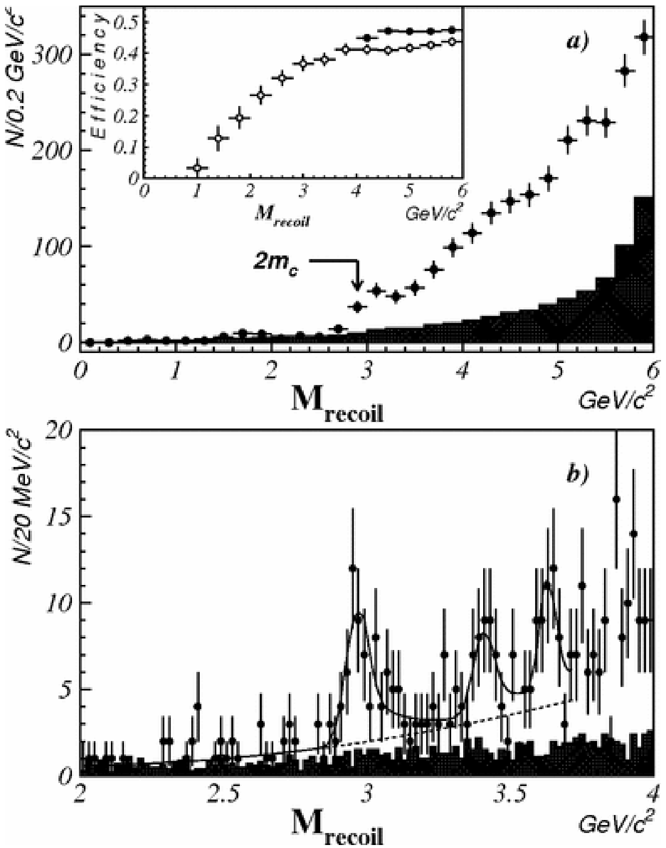
- **Significant attention** has been devoted to the study of double charmonium production at B factories at beginning of this century.

- Considerable effort has been paid to **reduce the discrepancy between experimental measurements and theoretical predictions** for the exclusive double charmonium production; A notable example is for $e^+e^- \to J/\psi + \eta_c$.

1. Background

$$e^+e^- \to J/\psi + \eta_c$$

On experiment side:



$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>4} = 33^{+7}_{-6} \pm 9 \text{ fb} \quad \text{@BELLE(2002)},$$
$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad \text{@BELLE(2004)},$$
$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad \text{@BABAR(2005)},$$

where $\mathcal{B}_{>2}$ signifies that branching fraction of η_c decay into the final states with more than 2 charged tracks

1. Background

$$e^+e^- \to J/\psi + \eta_c$$

There is an abundance of theoretical work on this process, based on various approaches. Below are several studies conduced within the framework of **NRQCD**

Braaten, Lee, PRD(2003)
Liu, He, Chao, PLB(2003)
Zhang, Gao, Chao, PRL(2006) ---- NLO QCD corrections
Bodwin, Kang, Lee, PRD(2006) ---- NLO relativisitc corrections
Bodwin, Kang, Kim, Lee, Yu, hep-ph/0611002
He, Fan, Chao, PRD(2007)
Bodwin, Lee, Yu, PRD(2008)
Gong, Wang, PRD(2008) ---- NLO QCD corrections
Dong, Feng, Jia, PRD(2012) ---- Mixed QCD and relativistic corrections $\mathcal{O}(\alpha_s v^2)$
Li, Wang, CPC(2014)
… …

1. Background $e^+e^- \to J/\psi + \eta_c$

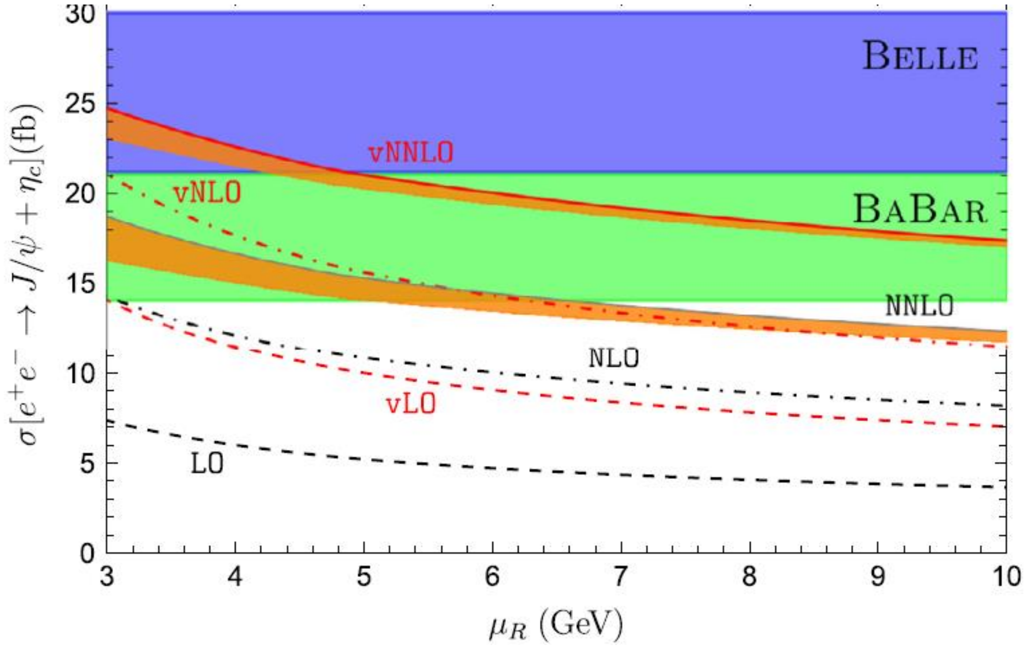
Feng, Jia, Mo, **SWL,** Zhang, arXiv: 1901.08447 (PLB 2024)

Table 2

Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ (in units of fb) at $\sqrt{s} = 10.58$ GeV. We take $\mu_R = \sqrt{s}/2$, and $\mu_\Lambda = 1$ GeV. The first error is obtained by varying m from $= 1.3$ to 1.7 GeV, and the second error is deduced by varying μ_R from $2m$ to $\sqrt{s}$.

LO	vLO	NLO	vNLO	NNLO	vNNLO	Belle	BaBar
$5.05^{+0.92+2.31}_{-0.99-1.49}$	$9.70^{+2.73+4.45}_{-2.79-2.85}$	$10.60^{+2.87+3.74}_{-2.61-2.60}$	$15.25^{+4.69+5.87}_{-4.41-3.96}$	$15.09^{+5.03+3.68}_{-4.16-2.87}$	$20.74^{+8.84+4.00}_{-7.37-3.59}$	$25.6^{+2.8+3.4}_{-2.8-3.4}$	$17.6^{+2.8+1.5}_{-2.8-2.1}$

Despite of considerable uncertainty, the theoretical prediction is consistent with the experimental data.



Note: The two-loop corrections were confirmed by Huang, Gong, Wang, JHEP(2023)

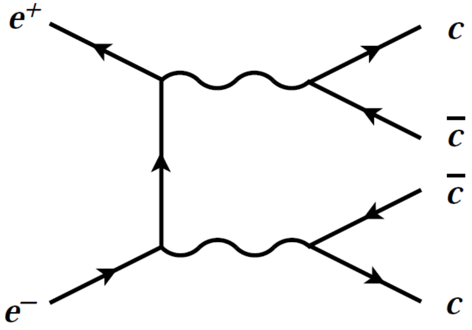
1. Background

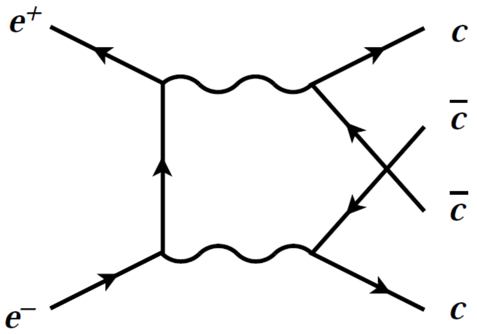
Let us turn to another important double charmonia production at B factory, i.e.,

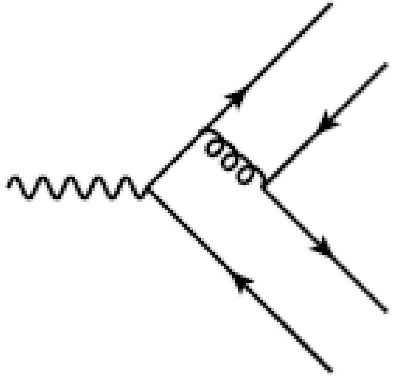
$$e^+e^- \to J/\psi + J/\psi$$

1) Such process has to proceed via e^+e^- annihilating into **two** virtual photons

2) By naive expectation, the production rate is more suppressed due to occurrence of the extra QED coupling constants







$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^4$$

$$\sigma(e^+e^- \to J/\psi + \eta_c) \propto \alpha^2\alpha_s^2$$

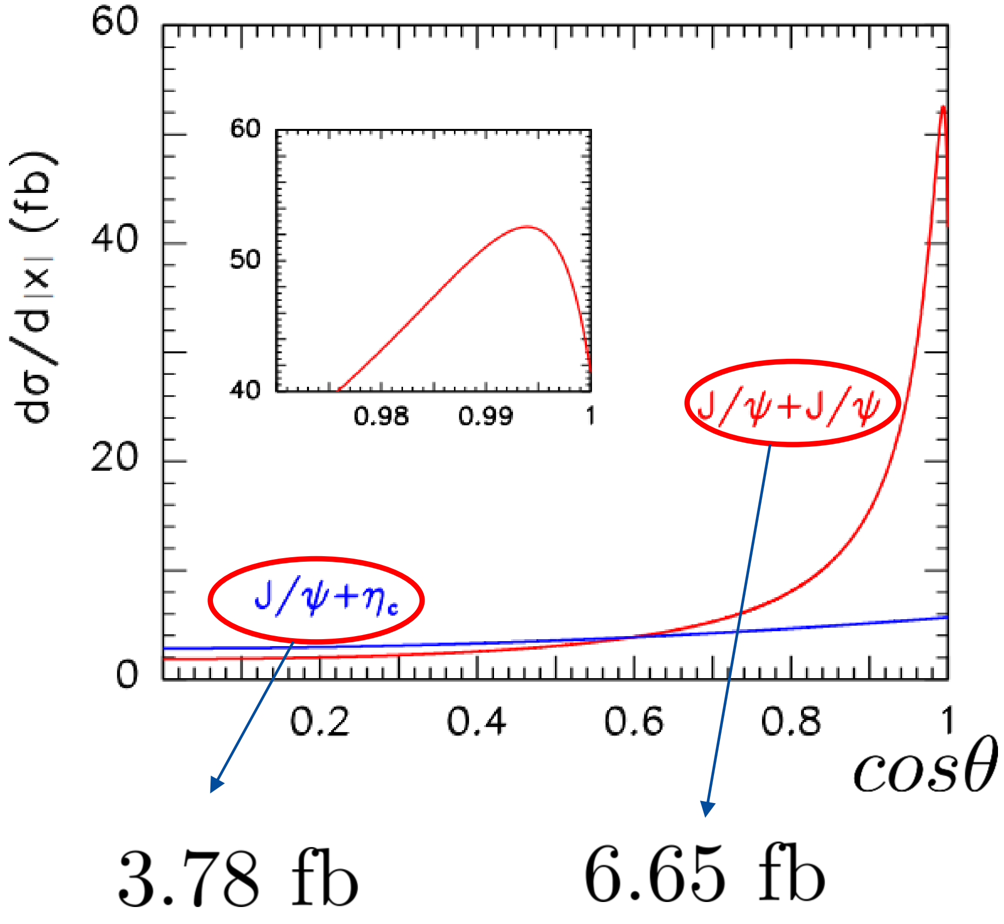
1. Background

Bodwin, Lee and Braaten PRL2003;
Bodwin, Lee and Braaten PRD, 2003 (E 2005)

Unpolarized Cross section (in units of **fb**)

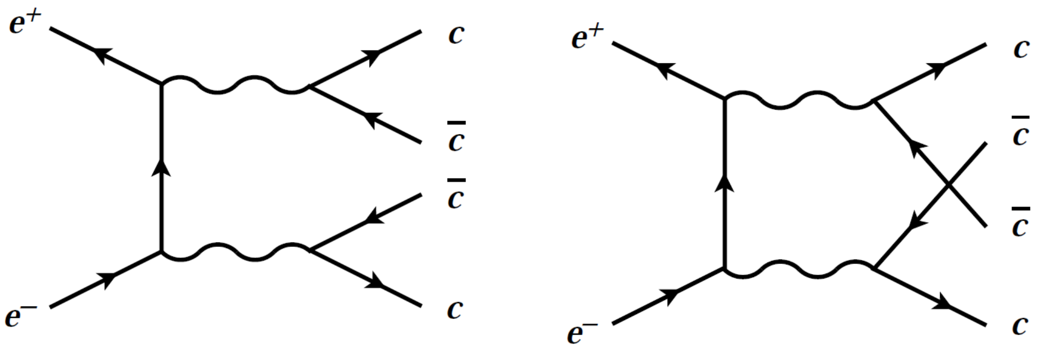
$H_2 \setminus H_1$	J/ψ	$\psi(2S)$
J/ψ	6.65 ± 3.02	5.52 ± 2.50
$\psi(2S)$		1.15 ± 0.52

$H_2 \setminus H_1$	J/ψ	$\psi(2S)$
η_c	3.78 ± 1.26	1.57 ± 0.52
$\eta_c(2S)$	1.57 ± 0.52	0.65 ± 0.22

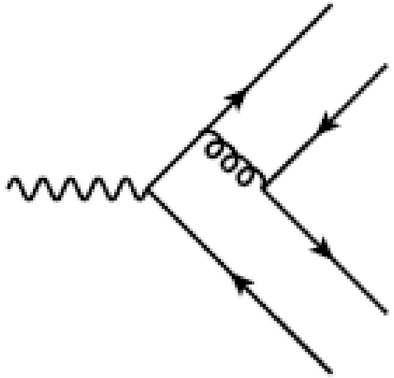


Note the tree-level prediction for $\sigma(J/\psi + J/\psi)$ is even bigger than $\sigma(J/\psi + \eta_c)$

1. Background



$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^4$$



$$\sigma(e^+e^- \to J/\psi + \eta_c) \propto \alpha^2\alpha_s^2$$

How to explain?

1) Enhanced by photon fragmentation (**from photon propagators**) $(s/m_{J/\psi}^2)^4$

2) Enhanced by the propagator of the electron at small azimuthal angle θ

1. Background

PRD(RC) (2004), BELLE K. Abe et al. PRD 2008

Unfortunately, double J/ψ production has not yet been observed at B factories

TABLE I. Summary of the signal yields (N), charmonium masses (M), significances, and cross sections ($\sigma_{\text{Born}} \times \mathcal{B}_{>2}[(c\bar{c})_{\text{res}}]$) for $e^+e^- \to J/\psi(c\bar{c})_{\text{res}}$; $\mathcal{B}_{>2}$ denotes the branching fraction for final states with more than two charged tracks.

$(c\bar{c})_{\text{res}}$	N	M [GeV/c^2]	Signif.	$\sigma_{\text{Born}} \times \mathcal{B}_{>2}$ [fb]
η_c	235 ± 26	2.972 ± 0.007	10.7	$25.6 \pm 2.8 \pm 3.4$
J/ψ	-14 ± 20	fixed	···	<9.1 at 90% CL
χ_{c0}	89 ± 24	3.407 ± 0.011	3.8	$6.4 \pm 1.7 \pm 1.0$
$\chi_{c1} + \chi_{c2}$	10 ± 27	fixed	···	<5.3 at 90% CL
$\eta_c(2S)$	164 ± 30	3.630 ± 0.008	6.0	$16.5 \pm 3.0 \pm 2.4$
$\psi(2S)$	-26 ± 29	fixed	···	<13.3 at 90% CL

TABLE III. Summary of the signal yields (N), significances, and cross sections ($\sigma_{\text{Born}} \times \mathcal{B}_{>0}[(c\bar{c})_{\text{res}}]$) for $e^+e^- \to \psi(2S) \times (c\bar{c})_{\text{res}}$; $\mathcal{B}_{>0}$ denotes the branching fraction for final states containing charged tracks.

$(c\bar{c})_{\text{res}}$	N	Signif.	$\sigma_{\text{Born}} \times \mathcal{B}_{>0}$ [fb]
η_c	36.7 ± 10.4	4.2	$16.3 \pm 4.6 \pm 3.9$
J/ψ	6.9 ± 8.9	···	<16.9 at 90% CL
χ_{c0}	35.4 ± 10.7	3.5	$12.5 \pm 3.8 \pm 3.1$
$\chi_{c1} + \chi_{c2}$	6.6 ± 8.0	···	<8.6 at 90% CL
$\eta_c(2S)$	36.0 ± 11.4	3.4	$16.0 \pm 5.1 \pm 3.8$
$\psi(2S)$	-8.3 ± 8.5	···	<5.2 at 90% CL

Belle finds no evidence for $e^+e^- \to J/\psi + J/\psi$

Why have the experiment found the signal for J/ψ+η_c, but not for J/ψ+J/ψ

1. Background

We have known that both the radiative and relativistic corrections can **significantly** enhance the LO cross section for $J/\psi+\eta_c$

Table 2
Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi+\eta_c]$ (in units of fb) at $\sqrt{s}=10.58$ GeV. We take $\mu_R=\sqrt{s}/2$, and $\mu_\Lambda=1\,\text{GeV}$. The first error is obtained by varying m from $=1.3$ to $1.7\,\text{GeV}$, and the second error is deduced by varying μ_R from $2m$ to $\sqrt{s}$.

LO	vLO	NLO	vNLO	NNLO	vNNLO	BELLE	BABAR
$5.05^{+0.92+2.31}_{-0.99-1.49}$	$9.70^{+2.73+4.45}_{-2.79-2.85}$	$10.60^{+2.87+3.74}_{-2.61-2.60}$	$15.25^{+4.69+5.87}_{-4.41-3.96}$	$15.09^{+5.03+3.68}_{-4.16-2.87}$	$20.74^{+8.84+4.00}_{-7.37-3.59}$	$25.6^{+2.8+3.4}_{-2.8-3.4}$	$17.6^{+2.8+1.5}_{-2.8-2.1}$

How about the corrections for the process $J/\psi+J/\psi$?

1. Background

$e^+e^- \to J/\psi + J/\psi$

Current Research Progress

2002:	Bodwin, Lee, Braaten	NRQCD LO	**8.7** fb
2003:	Bodwin, Lee, Braaten	NRQCD LO	**6.65** fb
2006:	Davier, Peskin, Snyder	VMD	**2.38** fb
2006:	Bodwin, Braaten, Lee, Yu	fragmentation+nonfragmentation	**1.69±0.35** fb
2008:	Gong, Wang	**NRQCD NLO in α_s**	**-3.4—2.3 fb**
2013:	Fan, Lee, Yu	NRQCD NLO in α_s and v^2	**1—1.5** fb

The order-α_s correction is negative and significant!

1) Negative total cross section in some range of renormalization scale
2) How about the **perturbative convergence**? NNLO correction?

To provide useful guidance for experimentalists to search for this channel, it is crucial to present the **precise theoretical prediction**

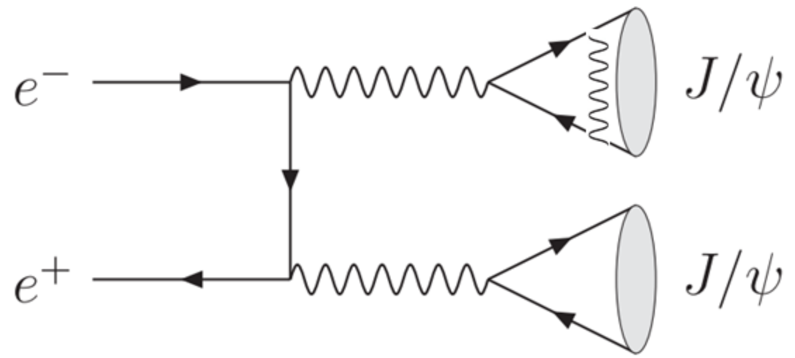
1. Background

$$e^+e^- \to J/\psi + J/\psi$$

Gong, Wang PRL(2008)

m_c (GeV)	μ	$\alpha_s(\mu)$	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
1.5	m_c	0.369	7.409	−2.327	−0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	−3.350	−0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

The main contribution comes from the fragmentation diagrams.



$$\sigma_{\rm NLO}/\sigma_{\rm LO} \approx (1 + f^{(1)}\frac{\alpha_s}{\pi})^4$$

$$\approx 1 - 11\frac{\alpha_s(2m_c)}{\pi} + \mathcal{O}(\alpha_s^2) \approx 1 - 0.88$$

$$\langle J/\psi|\bar{c}\gamma^\mu c|0\rangle = -f_{J/\psi} M_{J/\psi} \varepsilon^{*\mu}_{J/\psi}$$

$$f_{J/\psi} = \sqrt{\frac{2\langle\mathcal{O}\rangle_{J/\psi}}{M_{J/\psi}}}\left(1 + f^{(1)}\frac{\alpha_s}{\pi} + f^{(2)}\frac{\alpha_s^2}{\pi^2} + f^{(3)}\frac{\alpha_s^3}{\pi^3}\cdots\right)$$

$$f^{(1)} = -2C_F \qquad f^{(2)} \approx -43$$

Czarnecki, etc. PRL(98); Beneke, etc. PRL(98)

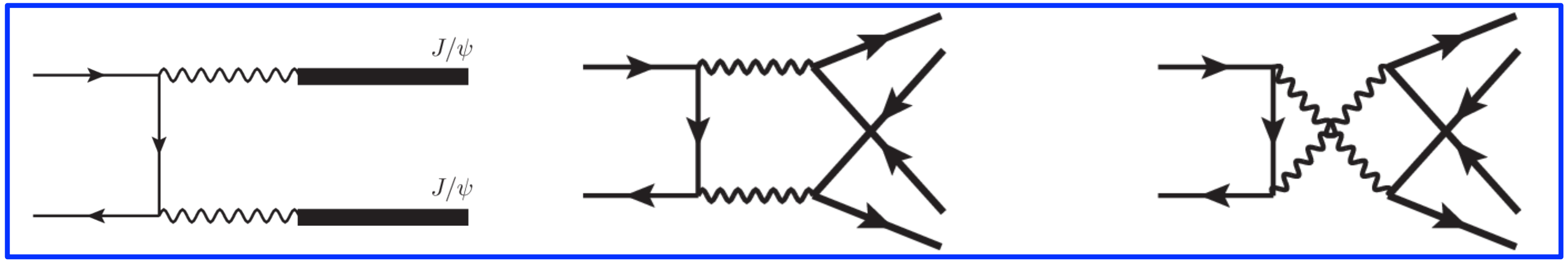
$$f^{(3)} \approx -1736$$

Marquard, etc. PRD(2014); Feng, etc. arXiv:2207.14259

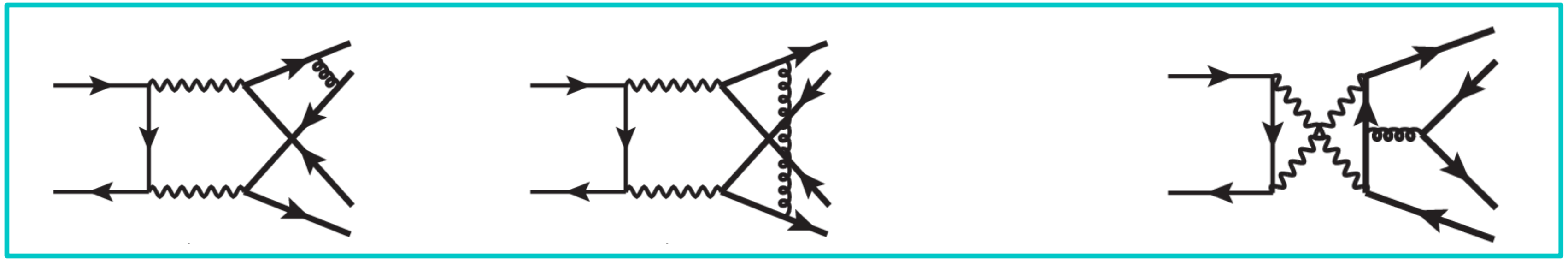
2. Outline of calculation

$e^+e^- \to J/\psi + J/\psi$

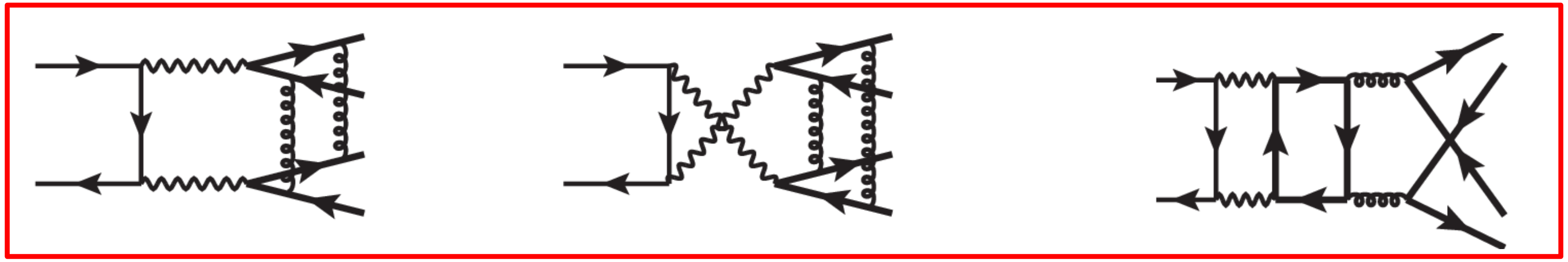
Some typical Feynman diagrams, up to two loop, are illustrated below



LO



NLO



NNLO

2. Outline of calculation

By employing the NRQCD factorization, we have

Short-Distance coefficient (SDC)

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\frac{\beta}{16\pi}\frac{e^8 e_c^4}{4}\,\mathcal{F}\,\frac{|\langle\mathcal{O}\rangle_{J/\psi}|^2}{m_c^2} + \mathcal{O}(v^2)$$

where $\beta = \sqrt{1-4M_{J/\psi}^2/s}$ represents the velocity of the outgoing J/ψ

The NRQCD matrix element is defined via

$$\langle\mathcal{O}\rangle_{J/\psi} \equiv |\langle J/\psi(\lambda)|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}(\lambda)\chi|0\rangle|^2$$

2. Outline of calculation

We can expand the SDC in powers of α_s

$$\mathcal{F} = \mathcal{F}^{(0)}\left[1 + \frac{\alpha_s}{\pi} f^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(f^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2} + 4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2} + f^{(2)}\right)\right]$$

μ_R : renormalization scale

β_0 : one-loop coefficient of the QCD β function

The occurrence of the $\beta_0 \ln\mu_R$ term is constrained by the renormalization group invariance

μ_Λ : factorization scale, the explicit expression is constrained by the NRQCD factorization

Where $\gamma_{J/\psi}$ is the **anomalous dimension of the NRQCD vector current** **(first arises at two loop!)**

At two-loop $\gamma_{J/\psi} = -\frac{\pi^2}{12} C_F(2C_F + 3C_A)$

Czarnecki, Melnikov, PRL(1998)
Beneke, Signer, Smirnov, PRL(1998)

The μ_Λ dependence in the SDC can be canceled by that in the NRQCD matrix element

2. Outline of calculation

$$\mathcal{F}=\mathcal{F}^{(0)}\left[1+\frac{\alpha_s}{\pi}f^{(1)}+\left(\frac{\alpha_s}{\pi}\right)^2\left(f^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}+4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2}+f^{(2)}\right)\right]$$

For convenience, we refer to this fixed-order perturbative expansion as the **traditional NRQCD factorization approach**.

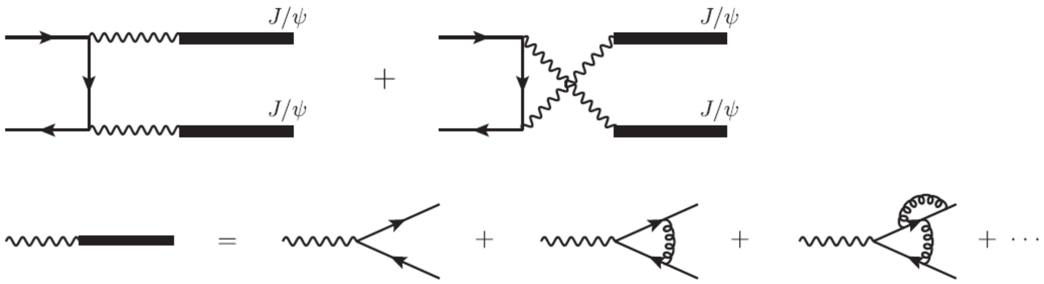
As previously mentioned and as will be evident in the subsequent discussion, the two-loop corrections $f^{(2)}$ are anticipated to be both negative and substantial.

It will cause the perturbative expansion break down.

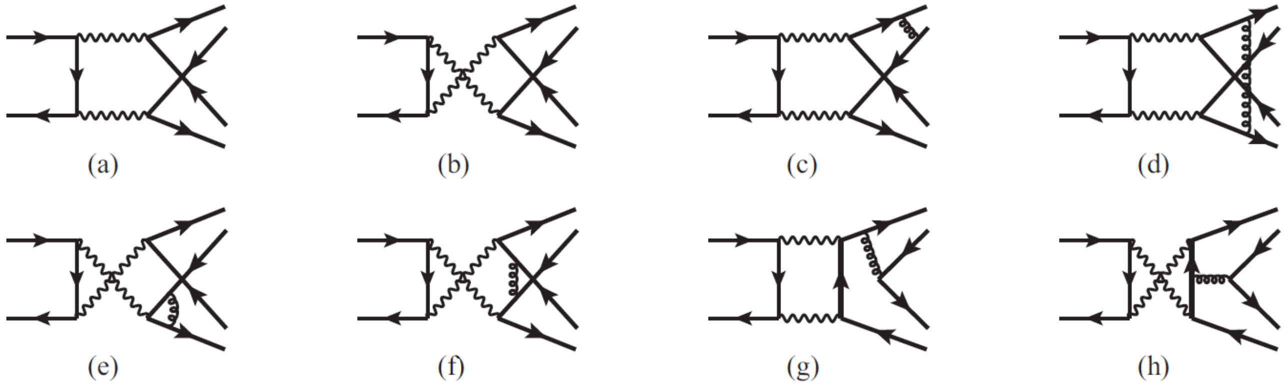
How can we address this issue?

2. Outline of calculation

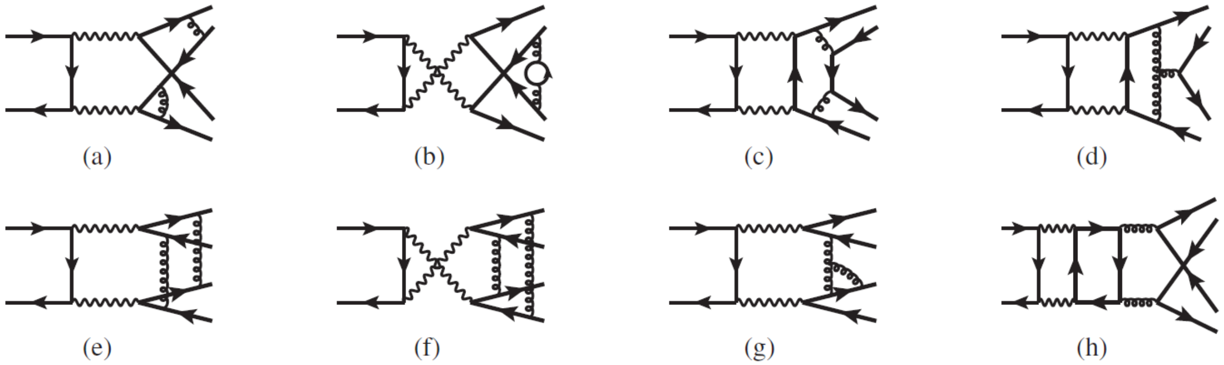
We split the Feynman diagrams into the fragmentation and non-fragmentation pieces



Fragmentation part



LO+NLO



NNLO

Non-fragmentation part

$$\mathcal{M} = \mathcal{M}_{\rm fr} + \mathcal{M}_{\rm nfr}$$

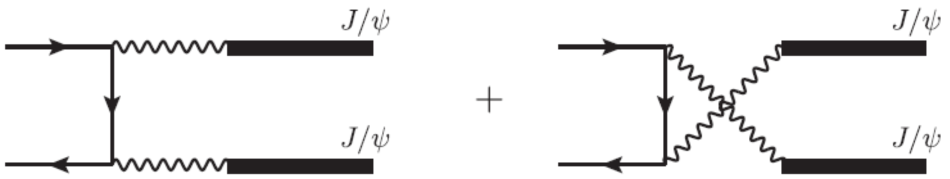
Total amplitude

Note both amplitudes are gauge-invariant

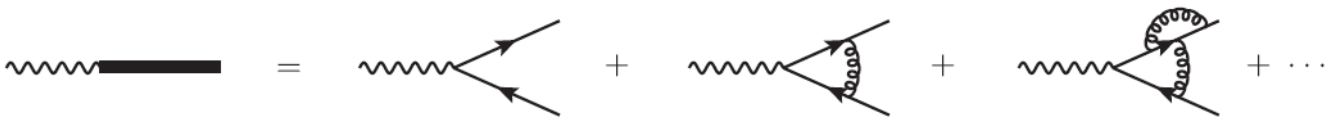
2. Outline of calculation

$$e^+e^- \to J/\psi + J/\psi$$

Special treatment for the fragmentation part



Davier, Peskin, Snyder, hep-ph/0606155;
Bodwin, Lee, Braaten and Yu, PRD(2006)



The $\gamma^* \to J/\psi$ can be expressed in term of the decay constant (also refer to **VMD**)

$$\langle J/\psi|\bar{c}\gamma^\mu c|0\rangle = -f_{J/\psi}M_{J/\psi}\varepsilon^{*\mu}_{J/\psi} \quad \Rightarrow \quad g_{\gamma^*\to J/\psi} = e_c M_{J/\psi} f_{J/\psi}$$

$f_{J/\psi}$ can be derived through $$f_{J/\psi} = \left(\frac{3M_{J/\psi}}{4\pi e_c^2\alpha^2}\Gamma[J/\psi \to l^+l^-]\right)^{1/2}$$

Through this treatment, it implied that **we have resummed an infinite towers of perturbative and relativistic corrections** to **all orders**.

2. Outline of calculation

The differential cross section reads:

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\frac{\beta}{16\pi}\frac{1}{4}\sum_{\rm spin}|\mathcal{M}_{\rm fr}+\mathcal{M}_{\rm nfr}|^2\,.$$

SDCs

So we get

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\frac{\beta}{16\pi}\frac{e^8 e_c^4}{4}\left[\mathcal{C}_{\rm fr} f_{J/\psi}^4 + \mathcal{C}_{\rm int} f_{J/\psi}^2 \frac{\langle\mathcal{O}\rangle_{J/\psi}}{m_c} + \mathcal{C}_{\rm nfr}\left(\frac{\langle\mathcal{O}\rangle_{J/\psi}}{m_c}\right)^2\right]$$

Fragmentation interference Non-fragmentation

In our work, we refer to this treatment as the optimized NRQCD approach

2. Outline of calculation

$$\mathcal{C}_{\rm fr} = \frac{8\left(\left(t^2+u^2\right)\left(tu - M_{J/\psi}^4\right) + 4stuM_{J/\psi}^2\right)}{t^2u^2M_{J/\psi}^4}$$

Davier, Peskin, Snyder, hep-ph/0606155;
Bodwin, Lee, Braaten and Yu, PRD(2006)

According to NRQCD, the other two SDCs can be parameterized **in LO in *v* but through α_s^2** as

$$\begin{aligned}
\mathcal{C}_{\rm int} &= \mathcal{C}_{\rm int}^{(0)}\left[1 + \frac{\alpha_s}{\pi}\hat{c}_{\rm int}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}_{\rm int}^{(1)} + 2\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2} + \hat{c}_{\rm int}^{(2)}\right) + \cdots\right] \\
\mathcal{C}_{\rm nfr} &= \mathcal{C}_{\rm nfr}^{(0)}\left[1 + \frac{\alpha_s}{\pi}\hat{c}_{\rm nfr}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}_{\rm nfr}^{(1)} + 4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2} + \hat{c}_{\rm nfr}^{(2)}\right) + \cdots\right]
\end{aligned}$$

2. Outline of calculation

After integrating over $\cos\theta$ from 0 to 1, we obtain the cross section of fragmentation part, and the LO cross section of the other two parts.

$$\sigma_{\mathrm{fr}} = \frac{32\pi^3 e_c^4 \alpha^4 f_{J/\psi}^4}{M_{J/\psi}^4} \frac{1}{s} \left[\frac{4+(1-\beta^2)^2}{1+\beta^2} \ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta \right]$$

$$\sigma_{\mathrm{int}}^{(0)} = -\frac{16\pi^3 e_c^4 \alpha^4 f_{J/\psi}^2 \langle \mathcal{O} \rangle_{J/\psi}}{3 m_c^3 s^2} \left[(5-\beta^2)(1-\beta^2)^2 \ln\left(\frac{1+\beta}{1-\beta}\right) + 22\beta - \frac{40}{3}\beta^3 + 2\beta^5 \right],$$

$$\sigma_{\mathrm{nfr}}^{(0)} = \frac{2048\pi^3 \alpha^4 e_c^4 |\langle \mathcal{O} \rangle_{J/\psi}|^2}{45 m_c^2 s^3} \beta \left(10 - \frac{20}{3}\beta^2 + \beta^4 \right)$$

In contrast with the fragmentation part that asymptotically scales as **$1/s$**, the interference part of the cross section exhibits a **$1/s^2$** asymptotic decrease, while the non-fragmentation part exhibits a **$1/s^3$** scaling.

Caused by the photon fragmentation

2. Outline of calculation

$$\mathcal{C}_{\rm int} = \mathcal{C}^{(0)}_{\rm int}\left[1+\frac{\alpha_s}{\pi}\hat{c}^{(1)}_{\rm int}+\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}^{(1)}_{\rm int}+2\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2}+\hat{c}^{(2)}_{\rm int}\right)+\cdots\right]$$

$$\mathcal{C}_{\rm nfr} = \mathcal{C}^{(0)}_{\rm nfr}\left[1+\frac{\alpha_s}{\pi}\hat{c}^{(1)}_{\rm nfr}+\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}^{(1)}_{\rm nfr}+4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2}+\hat{c}^{(2)}_{\rm nfr}\right)+\cdots\right]$$

Our task remains to compute $\hat{c}^{(1)}_{\rm int}$, $\hat{c}^{(1)}_{\rm nfr}$ from one loop corrections, and $\hat{c}^{(2)}_{\rm int}$, $\hat{c}^{(2)}_{\rm nfr}$ from two loop corrections.

Since multiple external legs are involved in the Feynman diagrams, the two loop computations turn out to be quite challenging.

2. Outline of calculation

- We adopt the **Feynman gauge** for our computation and utilize the dimensional regularization to regularize both UV and IR divergences.

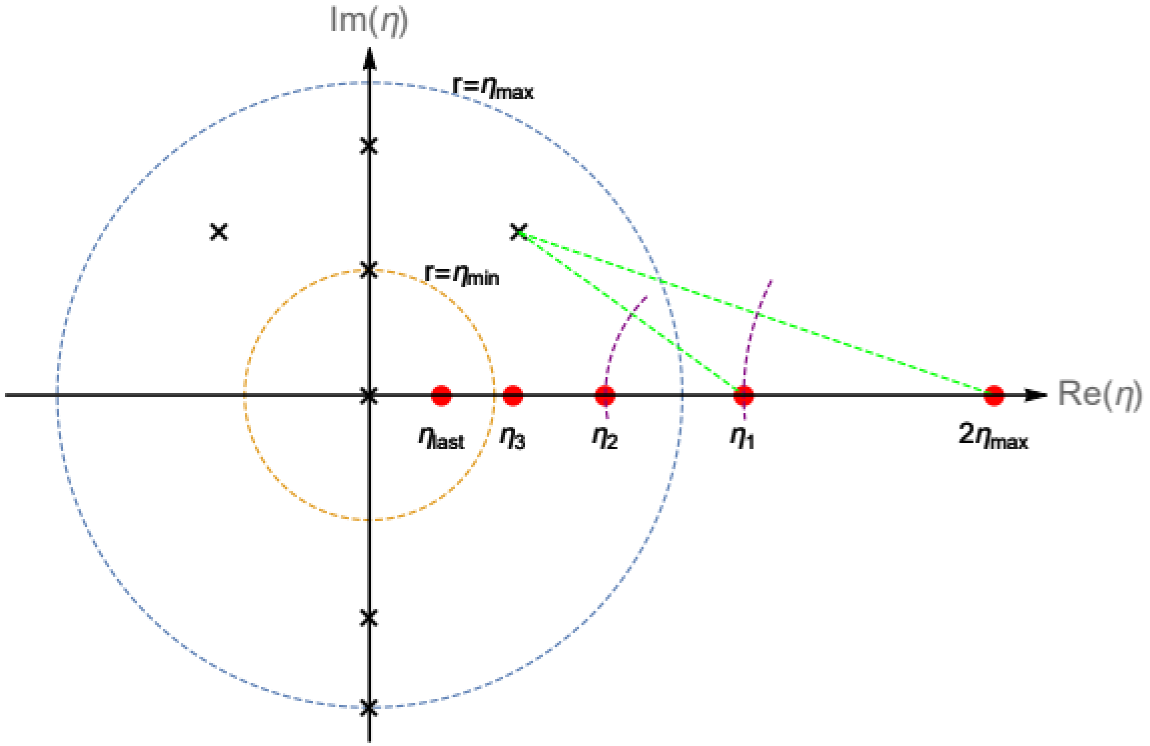
- We neglect the **relative momentum** in each $c\bar{c}$ pair prior to carrying out the loop integration, which amounts to directly extracting the NRQCD SDCs from the hard loop region

"Method of region" : Beneke, Smirnov, hep-ph/9711391

- After integration-by-parts (IBP) reduction, we end up with about **2400** two-loop master integrals (MIs), which are numerically computed by the package "AMFLow" (auxiliary mass flow)

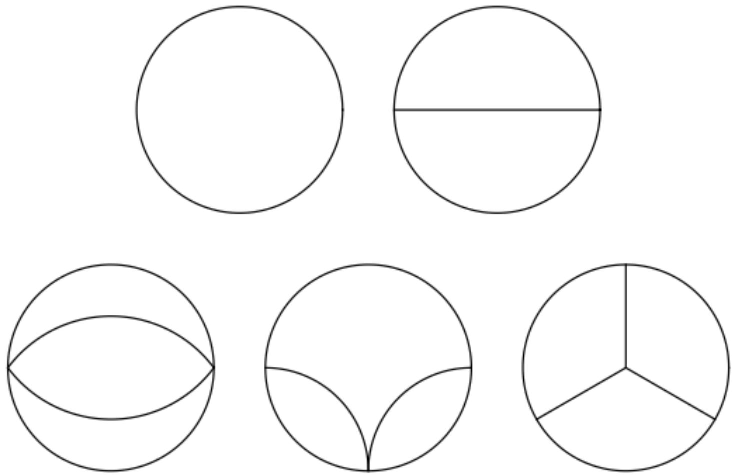
2. Outline of calculation

➢AMFlow
X. Liu, Y. Q. Ma, C. Y. Wang, arXiv:1711.09572
X. Liu, Y. Q. Ma, arXiv:2107.01864
Z. F. Liu, Y. Q. Ma, arXiv:2201.11637(PRL)
Z. F. Liu, Y. Q. Ma, arXiv:2201.11669(Package)



$$I(D;\{\nu_\alpha\};\eta) \equiv \int \prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{\mathrm{i}\pi^{D/2}} \prod_{\alpha=1}^{N} \frac{1}{(\mathcal{D}_\alpha + \mathrm{i}\eta)^{\nu_\alpha}}$$

$$\frac{\partial}{\partial\eta}\vec{I}(\eta) = A(\eta)\vec{I}(\eta)$$



2. Outline of calculation

$\sqrt{s} = 10.58$ GeV and $m_c = 1.5$ GeV

Our main results

$\cos\theta$	$\mathcal{C}_{\rm fr}$ (GeV^{-4})	$\mathcal{C}^{(0)}_{\rm int}$ (GeV^{-4})	$\hat{c}^{(1)}_{\rm int}$	$\hat{c}^{(2)}_{\rm int}$	$\mathcal{C}^{(0)}_{\rm nfr}$ (GeV^{-4})	$\hat{c}^{(1)}_{\rm nfr}$	$\hat{c}^{(2)}_{\rm nfr}$
0.999	4.163	-0.334	-3.62	-71.75	0.006	-7.42	$-143.174+42.974=-100.20$
0.970	3.646	-0.242	-1.34	-76.57	0.007	-6.33	$-146.117+37.424=-108.69$
0.872	1.573	-0.193	-0.73	-80.64	0.008	-5.07	$-152.144+25.321=-126.82$
0.775	0.988	-0.176	-1.27	-81.77	0.010	-5.11	$-155.633+19.124=-136.51$
0.677	0.722	-0.164	-1.85	-82.00	0.011	-5.49	$-157.716+15.969=-141.75$
0.531	0.522	-0.152	-2.58	-81.67	0.012	-6.15	$-159.349+14.092=-145.26$
0.384	0.422	-0.143	-3.12	-81.08	0.012	-6.73	$-160.032+13.777=-146.26$
0.287	0.383	-0.139	-3.38	-80.71	0.012	-7.03	$-160.222+13.898=-146.32$
0.140	0.350	-0.135	-3.63	-80.31	0.012	-7.32	$-160.324+14.160=-146.16$
0	0.340	-0.133	-3.70	-80.17	0.012	-7.41	$-160.341+14.271=-146.07$

Table 1: Numerical values of various SDCs through $\mathcal{O}(\alpha_s^2)$ for ten different values of $\cos\theta$.

3. Discussion

Main input parameters

$$\sqrt{s} = 10.58\ \text{GeV} \qquad M_{J/\psi} = 3.0969\ \text{GeV}, \qquad m_c = 1.5\ \text{GeV}$$

Note, we have taken $M_{J/\psi} = 2m_c$ in computing the inteference and non-fragmentation contributions.

$f_{J/\psi} = 403$ MeV Determined by the decay width of $J/\psi \to e^+e^-$

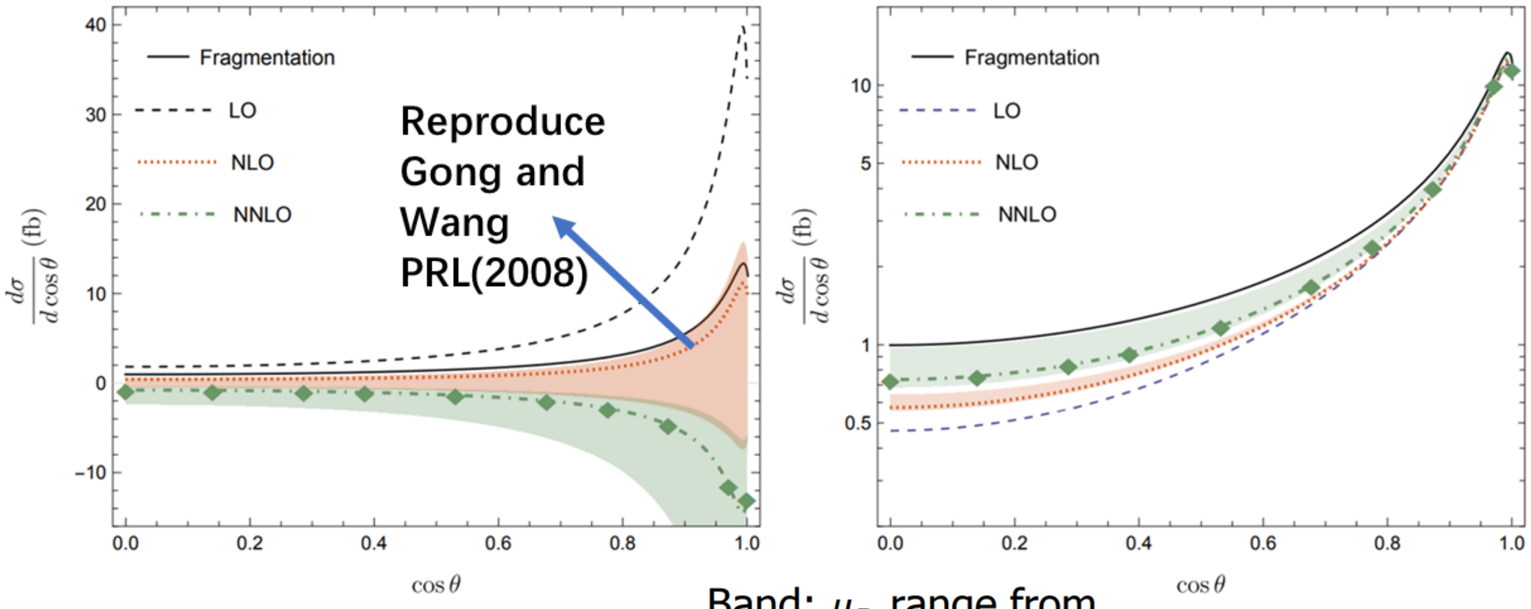
$$\langle \mathcal{O} \rangle_{J/\psi}(\mu_\Lambda = 1\ \text{GeV}) \approx \frac{3}{2\pi} R^2_{J/\psi}(0) \approx 0.387 \text{GeV}^3,$$

we choose to use $R^2_{J/\psi}(0) = 0.81\,\text{GeV}^3$ from Buchmüller-Tye (BT) potential model

Eichten and Quigg, PRD(1995)

3. Discussion

Our main results



Traditional NRQCD

Band: μ_R range from m_c to $\sqrt{s}$, default value takes at $\sqrt{s}/2$

Optimized NRQCD

Differential cross sections for e+e−→J/ψ+J/ψ against cosθ at various perturbative accuracy from traditional NRQCD and our improved NRQCD approach.

- Both NLO and NNLO corrections are positive!
- Exhibit **decent convergence** behavior!
- When J/ψ production plane is nearly collinear to e+e−($\theta \to 0$), fragmentation contribution dominates!
- As θ deviates from 0, corrections from **non-fragmentation amplitude start to play non-negligible role**!

3. Discussion

σ (fb)	Fragmentation	LO	NLO	NNLO
Optimized NRQCD	2.52	1.85	$1.93^{+0.05}_{-0.01}$	$2.13^{+0.30}_{-0.06}$
Traditional NRQCD		6.12	$1.56^{+0.73}_{-2.95}$	$-2.38^{+1.27}_{-5.35}$

Integrated cross section of $e^+e^-\to J/\psi+J/\psi$ at various perturbative accuracy. The uncertainties are estimated by varying μ_R from m_c to $\sqrt{s}$

- To date the Belle and the Belle2 experiments have accumulated about 1500 fb^{-1} data, so we expect about **3105~3645** exclusive double J/ψ events.
- Taking into account $Br(J/\psi\to l^+l^-)$=12%, **about 45~52 four-lepton events from double J/ψ can be produced**.
- Assuming 40% reconstruction efficiency, we expect about **18~21** signal events may be reconstructed.
- With the designed **50 ab^{-1} integrated luminosity at Belle2**, it seems that the observation prospects of exclusive double J/ψ production **is promising in the foreseeable future**.

3. Discussion

See Wang's talk in "Paralle C3b1"

After our work, a study on the same topic was conducted by Wang's group , employing a different approach

X.-D. Huang, B. Gong, R.-C. Niu, H.-M. Yu, J.-X. Wang, JHEP 2024

$$\sigma_{\mathrm{NNLO}} = 1.76^{+2.42}_{-1.66}\mathrm{fb}$$

Next-to-next-to-leading-order QCD corrections to double J/ψ production at the B factories

Xu-Dong Huang,[a,b] Bin Gong,[a,b] Rui-Chang Niu,[a,b] Huai-Min Yu[c] and Jian-Xiong Wang[a,b]

[a] *Institute of High Energy Physics, Chinese Academy of Sciences, 19B Yuquan Road, Shijingshan District, Beijing, 100049, P.R. China*
[b] *University of Chinese Academy of Sciences, Chinese Academy of Sciences, 19A Yuquan Road, Shijingshan District, Beijing, 100049, P.R. China*
[c] *School of Physics, Peking University, Beijing 100871, P.R. China*

E-mail: huangxd@ihep.ac.cn, twain@ihep.ac.cn, niuruichang@ihep.ac.cn, yuhm@stu.pku.edu.cn, jxwang@ihep.ac.cn

JHEP02(2024)055

ABSTRACT: In this paper, we study the next-to-next-to-leading-order (NNLO) QCD corrections for the process $e^+e^- \to J/\psi + J/\psi$ at the B factories. By including the NNLO corrections, the cross section turns negative due to the poor convergence of perturbative expansion. Consequently, to obtain a reasonable estimation for the cross section, the square of the amplitude up to NNLO is used. In addition, the contributions from the bottom quark and the light-by-light part, which are usually neglected, are also included. The final cross section is obtained as $1.76^{+2.42}_{-1.66}$ fb at a center-of-mass energy of $\sqrt{s} = 10.58\,\mathrm{GeV}$. Our result for total cross section and differential cross section could be compared with precise experimental measurement in future at the B factories.

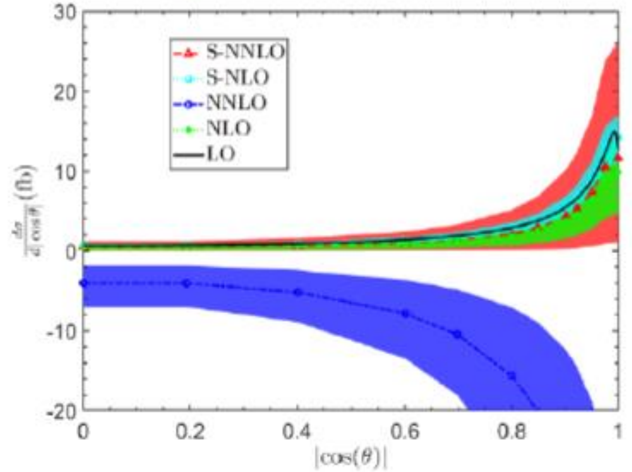


Figure 3. The differential cross section of $e^+e^- \to J/\psi + J/\psi$ as function of $|\cos\theta|$ at various perturbative order, and the bands are obtained by varying the renormalization scale μ_R within the range of $[2m_c, \sqrt{s}]$.

$\sigma(fb)$	LO	NLO	NNLO	S-NLO	S-NNLO
$\mu_R = 2m_c$	2.29	0.61	−21.10	1.83	0.12
$\mu_R = \sqrt{s}/2$	2.29	1.54	−11.97	2.37	1.76
$\mu_R = \sqrt{s}$	2.29	2.25	−5.27	2.84	4.17

4. Summary

- NNLO prediction for double J/ψ production at B factories in traditional NRQCD approach exhibits poor perturbative convergence, leading to unphysical negative cross section.

- In the **optimized NRQCD** approach, we split the amplitude into photon fragmentation and non-fragmentation pieces. Both NLO and NNLO corrections are positive and exhibit a **reasonable convergence** pattern.

- The NNLO prediction for the total cross section is $2.13^{+0.30}_{-0.06}$ fb at CM energy 10.58 GeV in the optimized approach . With the projected integrated luminosity of 50 ab^{-1}, the **prospect to observe this exclusive process at Belle 2 experiment appears to be bright**.

Thank you for your attention！

Backup slide

3. Discussion

Different values for the matrix element are taken in literature

$$\langle \mathcal{O} \rangle_{J/\psi}(\mu_\Lambda = 1\ \text{GeV}) \approx \frac{3}{2\pi} R^2_{J/\psi}(0) \approx 0.387\text{GeV}^3,$$

$$\langle \mathcal{O} \rangle_{J/\psi} = 0.335 \pm 0.024\ \text{GeV}^3,$$ Bodwin, Lee, Braaten, PRL(2003)

$$\langle \mathcal{O} \rangle_{J/\psi} = 0.482 \pm 0.049\ \text{GeV}^3,$$ Bodwin, Braaten, Lee, Yu, PRD(2006)

$$\langle \mathcal{O} \rangle_{J/\psi} = 0.457\ \text{GeV}^3,$$ Gong, Wang, PRL(2008)

$$\langle \mathcal{O} \rangle_{J/\psi} = 0.440^{+0.067}_{-0.055}\ \text{GeV}^3,$$
$$\langle \mathcal{O} \rangle_{J/\psi} = 0.436^{+0.065}_{-0.054}\ \text{GeV}^3,$$ Fan, Lee, Yu, PRD(20013)