

Quark mass dependence of charmed mesons

R. Molina, F. Gil, A. Giachino, and Pan-Pan Shi



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Motivation: Heavy quark symmetry and exotic states

Heavy quark spin and flavor symmetries

- Heavy quark symmetries arises when $m_Q \gg \Lambda_{QCD}$. $Q\bar{q}$ system, $\Delta p \simeq \Lambda_{QCD} \rightarrow \Delta v = \frac{\Delta p}{m_Q} \rightarrow 0$ in the limit $m_Q \rightarrow \infty$. The heavy quark behaves as a static source.
- The heavy quark interacts with gluons through the chromoelectric charge. Spin dependent interactions, $\mu^c \propto 1/m_Q$. Heavy Quark Spin Symmetry (HQSS) and Heavy Quark Flavor Symmetry (HQFS)
Isgur, Nathan and Wise, Mark B. PLB89, PLB90, PRL91, Manohar, Aneesh V. and Wise, Mark B., "Heavy Quark Physics"
- SU(2) spin symmetry, heavy hadrons organize into doublets with approximately similar mass and it is possible to work in this basis

$$|S_H, L, J\rangle \quad \text{HQSS basis} \quad (1)$$

- Doublets

$$(D, D^*) \quad (\eta_c, J/\psi) \quad (D_s, D_s^*) \cdots \quad (2)$$

$Q\bar{q}$ -like systems

' $D_{(s)}$ mesons, $j_l = L \pm \frac{1}{2}$

- $L = 0$: $J^P = 0^-, 1^-$ (D, D^*), (D_s, D_s^*)
- $L = 1$: Two Doublets:
 - $j_l^P = \frac{1}{2}^+$. $J^P = (0, 1)^+$. $D^*(2300)$, $D_1(2420)$?
 - $j_l^P = \frac{3}{2}^+$. $J^P = (1, 2)^+$. $D_1(2430)$, $D_2^*(2460)$?
 - $j_l^P = \frac{1}{2}^+$. $J^P = (0, 1)^+$. $D_{s0}(2317)$, $D_{s1}(2460)$?
 - $j_l^P = \frac{3}{2}^+$. $J^P = (1, 2)^+$. $D_{s1}(2536)$, $D_{s2}^*(2573)$?
- However, there are some puzzles:
 - Exp. masses and widths of the $D_{s0}(2317)$ and $D_{s1}(2460)$ not compatible with $q\bar{q}$ expectations. Role of the DK, D^*K channel.
 - Masses of the D_s counterparts are expected to be 100 higher since $m_s/m_d \simeq 20$. [Godfrey, Isgur, Kokoski, PRD79,85](#) However,

$$B(D_{s0}(2317))_{DK} \simeq B(D_{s1}(2460))_{D^*K} \simeq 40 \text{ MeV} \quad (3)$$

- Measurement of the $D_0^*(2300)$ mass varies from 2300 – 2400 MeV. Possible two pole structure related to $D\pi, D_s\bar{K}$ channels. Not confirmed yet by LQCD (see C. Thomas talk on Monday).

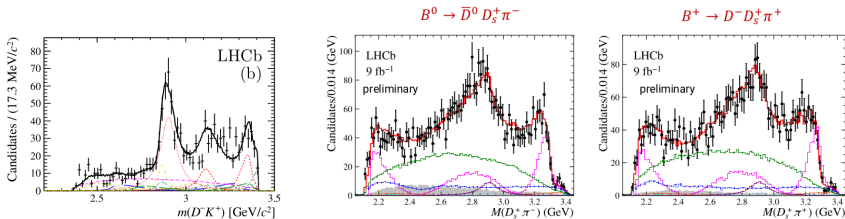
LHCb (2020, 2022)

T_{cs} states $J^P = 0^+, 1^-$ decaying to $D\bar{K}$, and $T_{c\bar{s}}(2900) (0^+)$, decaying to $D_s^+\pi^-$ and $D_s^+\pi^+$, $\sim cs\bar{q}\bar{q}, c\bar{s}qq, q = u, d$.

$$T_{cs,0}(2866) : M = 2866 \pm 7, \quad \Gamma = 57.2 \pm 12.9 \text{ MeV},$$

$$T_{cs,1}(2900) : M = 2904 \pm 5, \quad \Gamma = 110.3 \pm 11.5 \text{ MeV}$$

$$T_{c\bar{s}}(2900) : M = 2908 \pm 11 \text{ MeV}, \quad \Gamma = 136 \pm 23 \text{ MeV}$$



R. Aaij et al. (LHCb Collaboration), PRL20, PRD20, PRD23

Flavour exotic states

Molina, Branz, Oset, PRD10

HGF-Interaction of vector mesons

| C, S | Channels | $I[J^P]$ | \sqrt{s} | $\Gamma_A (\Lambda = 1400)$ | $\Gamma_B (\Lambda = 1200)$ | State | \sqrt{s}_{exp} | Γ_{exp} |
|--------|---|------------|---|-----------------------------|-----------------------------|-------------------------------|-------------------------|-----------------------|
| 1, -1 | $D^* \bar{K}^*$ | $0[0^+]$ | 2848 | 23 | 59 | $X_0(2866)$ or $T_{CS}(2900)$ | 2866 | 57 |
| | | $0[1^+]$ | 2839 | 3 | 3 | | | |
| | | $0[2^+]$ | 2733 | 11 | 36 | | | |
| 1, 1 | $D^* K^*, D_S^* \omega$ $D_S^* \phi$ | $0[0^+]$ | 2683 | 20 | 71 | $D_{S2}(2573)$ | 2572 | 20 |
| | | $0[1^+]$ | 2707 | 4×10^{-3} | 4×10^{-3} | | | |
| | | $0[2^+]$ | 2572 | 7 | 23 | | | |
| 1, 1 | $D^* K^*, D_S^* \rho$ | $1[0^+]$ | Cusp structure around $D_S^* \rho, D^* K^*$ | | | new $T_{C\bar{S}}(2900)$ | 2908 | 136 |
| 1, 1 | | $1[1^+]$ | Cusp structure around $D_S^* \rho, D^* K^*$ | | | | | |
| 1, 1 | | $1[2^+]$ | 2786 | 8 | 11 | | | |
| 2, 0 | $D^* D^*$ | $0[1^+]$ | 3969 | 0 | 0 | | | |
| 2, 1 | $D^* D_S^*$ | $1/2[1^+]$ | 4101 | 0 | 0 | | | |

Updates: Dai, Molina, Oset, PRD22, $B_{D^* D^*} \simeq 5$ MeV. Cusp at the $D^* D_S^*$ threshold. $T_{CS}/T_{C\bar{S}}$, Molina, Oset, PLB20, PRD23

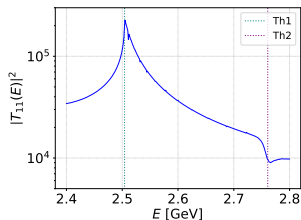
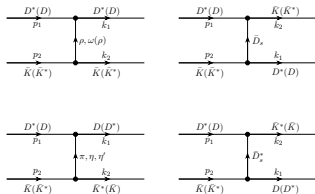
Sign of a new resonance below the $D^* D^*$ threshold from LQCD

Whyte, Wilson, Thomas, 2405.15741 (2024)

Flavour exotic states

New prediction of states in the coupled-channel system $D^* \bar{K} - D \bar{K}^*$

| | single channel | | coupled channel | |
|------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Pole [MeV] | 2482.35 - 0.04 <i>i</i> | 2760.18 - 7.27 <i>i</i> | 2486.46 - 0.04 <i>i</i> | 2761.27 - 5.22 <i>i</i> |
| RS | (-) | (+) | (-, +) | (-, +) |
| Channel | Coupling g_i [GeV] | | | |
| $D^* \bar{K}(S)$ | 0.00 - 4.10 <i>i</i> | - | 0.00 + 3.96 <i>i</i> | 0.05 - 0.42 <i>i</i> |
| $D^* \bar{K}(D)$ | 0.00 - 0.00 <i>i</i> | - | 0.00 - 0.00 <i>i</i> | 0.26 - 0.09 <i>i</i> |
| $D \bar{K}^*(S)$ | - | 1.89 - 0.52 <i>i</i> | -0.09 + 2.24 <i>i</i> | 1.94 - 0.92 <i>i</i> |
| $D \bar{K}^*(D)$ | - | -0.00 - 0.00 <i>i</i> | 0.12 + 0.20 <i>i</i> | -0.01 - 0.01 <i>i</i> |



Quark mass dependence of exotic states

Heavy meson field: $P_I^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)})$

$$H_I^{(Q)} = \frac{(1 + \not{v})}{2} (P_{I\mu}^{*(Q)} \gamma^\mu - P_I^{(Q)} \gamma_5) . \quad (4)$$

They transform as a doublet under heavy quark spin symmetry and like an anti-triplet under $SU(3)_V$ flavour symmetry,

$$H_I^{(Q)} \rightarrow S_Q (H^{(Q)} U^\dagger)_I, \quad \bar{H}_I^{(Q)} \rightarrow (U \bar{H}^{(Q)})_I S_Q^\dagger \quad (5)$$

At leading order, the chiral lagrangian reads (Jenkins, NPB94),

$$\mathcal{L}^0 = -i \text{Tr} \bar{H}_V (v \cdot \partial) H_V + i \text{Tr} \bar{H}_V H_V (v \cdot V) + 2g \text{Tr} \bar{H}_V H_V (S_{IV} \cdot A) \quad (6)$$

with $g^2 = 0.55$. Relevant lagrangian for one-calculation,

$$\begin{aligned} \mathcal{L}_V^\xi = & \sigma \text{Tr} M_\xi \text{Tr} \bar{H}_V H_V + a \text{Tr} \bar{H}_V H_V M_\xi + b \text{Tr} \bar{H}_V H_V M_\xi M_\xi + c \text{Tr} M_\xi \text{Tr} \bar{H}_V H_V M_\xi + d \text{Tr} M_\xi M_\xi \text{Tr} \bar{H}_V H_V \\ & - \frac{\Delta}{8} \text{Tr} \bar{H}_V \sigma^{\mu\nu} H_V \sigma_{\mu\nu} - \frac{\Delta^{(a)}}{8} \text{Tr} \bar{H}_V \sigma^{\mu\nu} H_V \sigma_{\mu\nu} M_\xi - \frac{\Delta^{(\sigma)}}{8} \text{Tr} M_\xi \text{Tr} \bar{H}_V \sigma^{\mu\nu} H_V \sigma_{\mu\nu} , \end{aligned} \quad (7)$$

(a, b, c, d, σ) Respect HQSS $\mathcal{O}(1)$; $(\Delta, \Delta^{(\sigma)}, \Delta^{(a)}) \mathcal{O}(1/m_Q)$; $\frac{m_B^* - m_B}{m_D^* - m_D} \sim \frac{m_D}{m_B}$

Quark mass dependence of the $D(D^*)$ mesons

Heavy Hadron Chiral Perturbation Theory (HH χ PT)

E. Jenkins, NPB412 (1994); Gil-Domínguez, Molina PLB (2023)

$$\frac{1}{4}(D + 3D^*) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} (\gamma_a^{(X)} - \lambda_a^{(X)} \alpha_a) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + c_a$$
$$(D^* - D) = \Delta + \sum_{X=\pi,K,\eta} (\gamma_a^{(X)} - \lambda_a^{(X)} \Delta) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + \delta c_a$$

$\mu = 770$ MeV; $g^2 = 0.55$ MeV (Decay of the D^* meson)

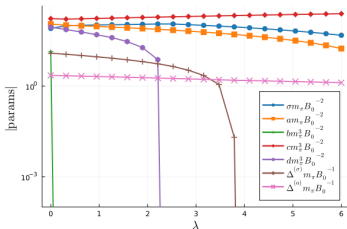
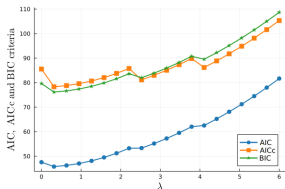
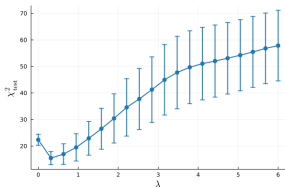
$$\left. \begin{aligned} \frac{1}{4}(D + 3D^*) &= m_H + f(\sigma, a, b, c, d) \\ (D^* - D) &= \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)}) \end{aligned} \right\} \begin{array}{l} 9 \text{ parameters, but different collaborations/scale} \\ \text{settings, } 7 + 2 \times 7 = 21 \text{ parameters, } \sim 80 \text{ data} \\ \text{points} \end{array}$$

ETMC, PACS, HSC, CLS, RQCD, S.Prelovsek, MILC

$D(D^*$ quark mass dependence

LASSO + information criteria;

$$\chi_P^2 = \chi^2 + \lambda \sum_i^n |p_i|; \quad \text{Data} = \text{Training (70\%)} + \text{Test (30\%)} \quad (8)$$



Some of the parameters
are not relevant
Plots for ETMC data
analysis

$D(D^*)$ quark mass dependence

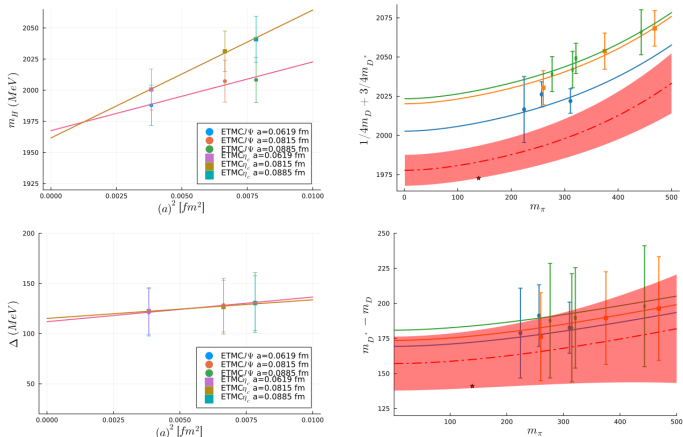


Figure 1: Extrapolation to the physical point of the ETMC data.

$$m_H = m'_H + r_H a^2, \quad \Delta = \Delta' + r_\Delta a^2.$$

Global analysis

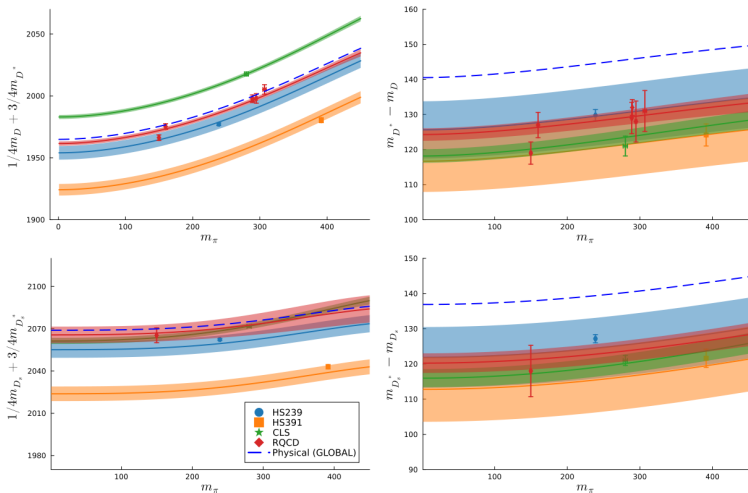
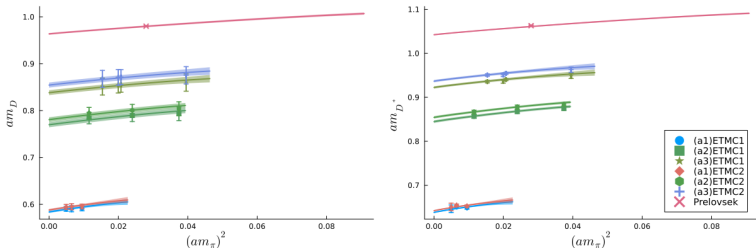


Figure 2: Results of the D , D^* , D_s and D_s^* meson masses for the global analysis of some collaborations.

Global analysis



| Fit | $\frac{\sigma m_\pi}{B_0^2} \cdot 10^{-4}$ | $\frac{am_\pi}{B_0^2} \cdot 10^{-4}$ | $\frac{bm_\pi^3}{B_0^2} \cdot 10^{-10}$ | $\frac{cm_\pi^3}{B_0^2} \cdot 10^{-10}$ | $\frac{dm_\pi^3}{B_0^2} \cdot 10^{-10}$ | $\frac{\Delta(\sigma) m_\pi}{B_0} \cdot 10^{-5}$ | $\frac{\Delta(a) m_\pi}{B_0} \cdot 10^{-5}$ |
|------------------------|--|--------------------------------------|---|---|---|--|---|
| ETMC _(I) | 0.89 ± 0.08 | 2.08 ± 0.13 | -0.55 ± 0.21 | 6.51 ± 0.20 | 3.39 ± 0.18 | 5.78 ± 3.43 | -2.16 ± 2.27 |
| ETMC _(II) | 1.02 ± 0.01 | 2.02 ± 1.18 | - | 6.16 ± 0.24 | 3.24 ± 0.27 | 5.88 ± 3.85 | -2.17 ± 2.24 |
| MILC _(I) | 3 ± 22 | 3 ± 18 | 7 ± 108 | 2 ± 55 | 1.56 ± 7.08 | 4 ± 255 | 36 ± 478 |
| MILC _(II) | 1.74 ± 0.40 | 2.40 ± 0.73 | - | 5.61 ± 1.66 | - | - | - |
| PACS _(I) | 0.90 ± 0.24 | 2.62 ± 0.20 | -2.38 ± 1.81 | 7.03 ± 1.24 | 3.53 ± 0.74 | 3.60 ± 0.16 | -0.76 ± 0.44 |
| PACS _(II) | 1.22 ± 0.02 | 2.36 ± 0.03 | - | 5.40 ± 0.03 | 4.50 ± 0.03 | 3.59 ± 0.16 | -0.75 ± 0.44 |
| GLOBAL _(I) | 1.84 ± 0.01 | 2.54 ± 0.02 | 0.57 ± 0.05 | 4.47 ± 0.04 | 2.34 ± 0.03 | 2.70 ± 0.21 | -1.43 ± 0.47 |
| GLOBAL _(II) | 1.17 ± 0.01 | 2.58 ± 0.02 | - | 4.89 ± 0.05 | 2.27 ± 0.03 | 2.56 ± 0.20 | -1.47 ± 0.46 |

Table 1: (I) "before LASSO" and (II) "after LASSO".

Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Predictions: Cleven, Guo, Hanhart, Meissner (2011); Liu, Orginos, Guo, Meissner, (2013); **Previous analyses of LQCD data:** Martínez Torres, Oset, Prelovsek, A. Ramos (2015), Yao, Du, Guo, and Meißner, (2015); Albaladejo, Fernandez-Soler, Nieves, Ortega, (2018). **LQCD data:**

| Col. | $a(\text{fm})$ | $L(\text{fm})$ | $m_\pi(\text{MeV})$ | $m_{D_s}(\text{MeV})$ |
|------------------|-------------------------------|----------------|---------------------|-----------------------|
| HSC (2021) | 0.12 | | | |
| | $a_t^{-1} = 6079 \text{ MeV}$ | 3.8 | 239 | 1967 |
| | $a_t^{-1} = 5667 \text{ MeV}$ | 1.9 – 2.9 | 391 | 1951 |
| RQCD (2017) | 0.071 | 4.5 | 150 | 1977 |
| PACS-CS (2014) | 0.0907 | 2.9 | 156 | 1809 |
| Prelovsek et al. | 0.1239 | 2.0 | 266 | 1657 |

Table 2: Charm quark mass settings, $m_{D_s}^{phys} = 1968.35 \pm 0.07 \text{ MeV}$.

Formalism in the finite volume

Infinite volume (Two-meson loop)

$$G = G^{\text{co}}(E) = \int_{q < q_{\text{max}}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{2M_i}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon}$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i|^2}$.

Finite volume (Doring, Meißner, Oset, Rusetsky (2011))

$$\vec{q}_i = \frac{2\pi}{L} \vec{n}_i; \quad T \longrightarrow \tilde{T}; \quad G(E) \longrightarrow \tilde{G}(E),$$

$$\tilde{G}(E) = \frac{1}{L^3} \sum_{\vec{q}_i} I(E, \vec{q}_i); \quad I(E, \vec{q}_i) = \frac{\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(E)^2 - (\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i))^2}$$

$$\begin{aligned} \tilde{G} &= G^{\text{DR}} + \lim_{q_{\text{max}} \rightarrow \infty} \left(\frac{1}{L^3} \sum_{q < q_{\text{max}}} I(E, \vec{q}) - \int_{q < q_{\text{max}}} \frac{d^3 q}{(2\pi)^3} I(E, \vec{q}) \right) \\ &\equiv G^{\text{DR}} + \lim_{q_{\text{max}} \rightarrow \infty} \Delta G, \end{aligned}$$

Martinez Torres, Dai, Koren, Jido and Oset (2012)

Formalism in the finite volume

- **Bethe-Salpeter** equation in finite volume, **One-channel case**

$$\tilde{T}^{-1} = V^{-1} - \tilde{G}$$

- Energy levels in the box in the presence of interaction V correspond to the condition

$$\det(I - V\tilde{G}) = 0$$

- One-channel-amplitude in infinite volume T

$$T = (\tilde{G}(E_i) - G(E_i))^{-1}.$$

- Phase shift and scattering parameters:

$$\tan\delta = -k/(8\pi E\Delta G); \quad p\cot\delta = \frac{1}{a} + \frac{1}{2}r_0p^2.$$

(Similar to the Lüscher condition (1986) but relativistic.)

Doering, Meißner, Oset, Rusetsky (2012)

$\vec{q}_1, \vec{q}_2 = \vec{P} - \vec{q}_1$, $s \equiv W^2 = (P^0)^2 - \vec{P}^2$, and \vec{q}^* the momenta in the CM frame

$$\int \frac{d^3 \vec{q}^*}{(2\pi)^3} I(|\vec{q}^*|) \rightarrow \tilde{G}(P) = \frac{1}{L^3} \frac{\sqrt{s}}{P^0} \sum_{\vec{n}} I(|\vec{q}^*(\vec{q})|).$$

$$\vec{q}_{1,2}^* = \vec{q}_{1,2} + \left[\left(\frac{\sqrt{s}}{P^0} - 1 \right) \frac{\vec{q}_{1,2} \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_{1,2}^{*0}}{P^0} \right] \vec{P}; \text{ with } \vec{q} = \frac{2\pi}{L} (n_x, n_y, n_z), \vec{P} = \frac{2\pi}{L} (N_x, N_y, N_z).$$

$$\tilde{T}_{lm,l'm'} = V_l \delta_{ll'} \delta_{mm'} + \sum_{l''m''} V_l \tilde{G}_{lm,l''m''} \tilde{T}_{l''m'',lm}$$

$$\det(\delta_{ll'} \delta_{mm'} - V_l \tilde{G}_{lm,l'm'}) = 0$$

Irreducible representations for asymmetric boxes and boost $\vec{P} = \frac{2\pi}{L}(0, 0, 1)$,

$$I = L = 0 \longrightarrow A^+ : -1 + V_0 G_{00,00} = 0$$

$$I = L = 1 \longrightarrow A_2^- : -1 + V_1 G_{10,10} = 0; E^- : -1 + V_1 G_{11,11} = 0$$

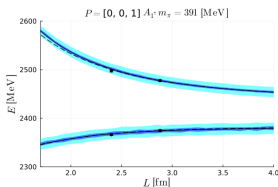
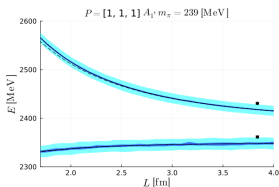
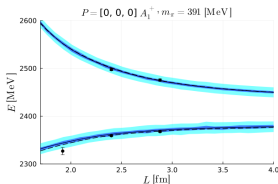
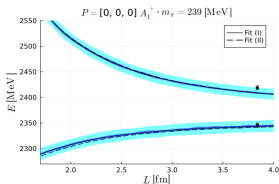
Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Potential $V(s)$ (consistent with HQSS)

$$V(s) = V_{DK}(s) + V_{\text{ex}}(s); \quad V_{DK} = -\frac{s-u}{2f^2}; \quad \text{Fit I : } V_{\text{ex}} = 0;$$

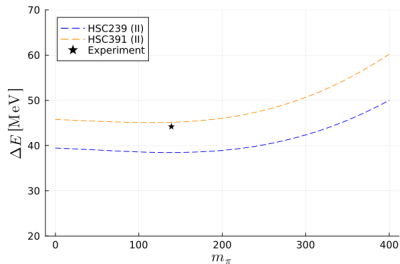
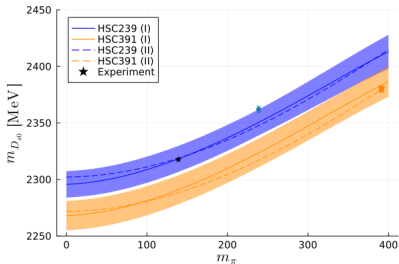
$$\text{Fit II : } V_{\text{ex}} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2}; \quad V_{c\bar{s}}(s) = -\frac{c}{f} \sqrt{M_D m_{c\bar{s}}} \frac{s + m_K^2 - M_D^2}{\sqrt{s}}$$

Fitting parameters: $a = a_1 + a_2 m_\pi^2$ (subtraction constant), $m_{c\bar{s}}$, c .



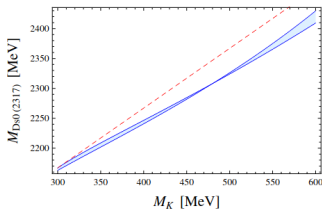
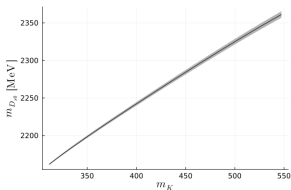
Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Pion and kaon mass dependence

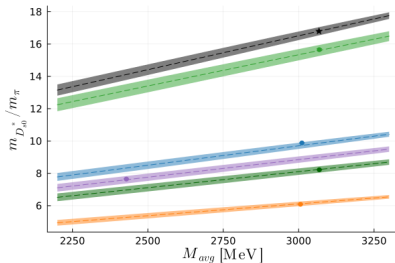
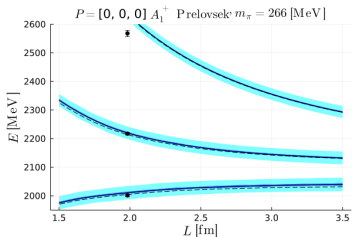
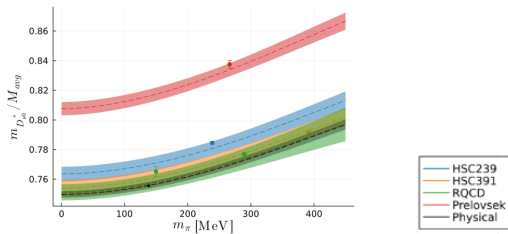
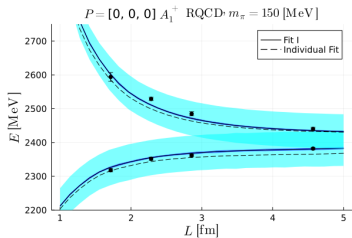


$m_{\pi} = 236 \text{ MeV}$; $a_t^{-1} = 5.667 \text{ GeV}$; $a_t M_{\eta_C} = 0.2412$, $M_{\eta_C} = 2986 \text{ MeV}$; $m_{\pi} = 391 \text{ MeV}$; $a_t^{-1} = 6.079 \text{ GeV}$; $a_t M_{\eta_C} = 0.2735$;

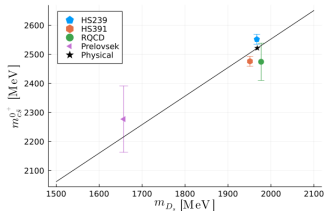
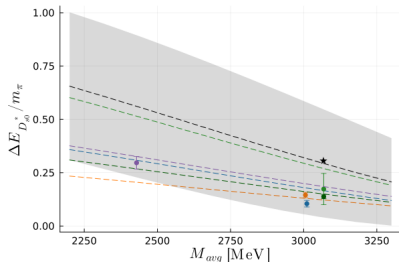
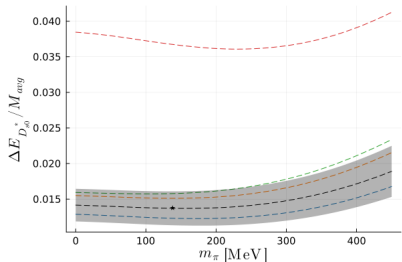
$M_{\eta_C} = 2963 \text{ MeV}$; Similar trends than in Cleven, Guo, Hanhart, Meißner (2010)



Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$



Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$



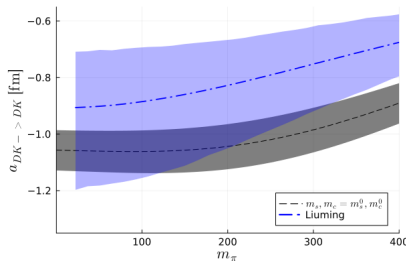
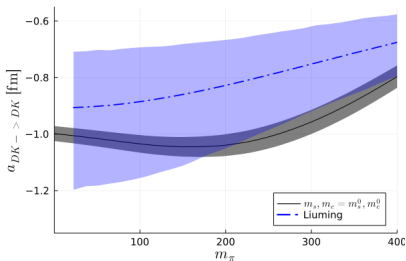
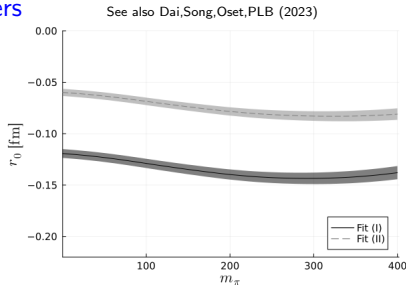
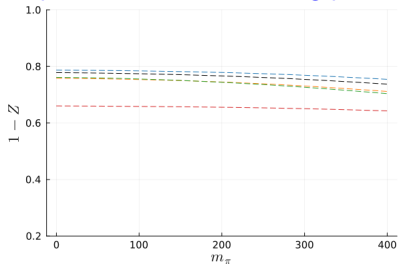
| | c | a_1^{0+} | a_1^{1+} | $a_2[(m_\pi^0)^{-2}]$ |
|--------|-----------|------------|------------|-----------------------|
| Fit I | - | $-1.87(1)$ | $-2.05(2)$ | $-0.040(2)$ |
| Fit II | $1.05(1)$ | $-1.34(1)$ | $-1.44(1)$ | $-0.002(1)$ |

| | $m_{c\bar{s}}^{0+}$ | $m_{c\bar{s}}^{1+}$ |
|-----------|---------------------|---------------------|
| HSC239 | $2552(26)$ | - |
| HSC391 | $2476(25)$ | - |
| RQCD | $2475(25)$ | $2553(25)$ |
| Prelovsek | $2277(23)$ | $2356(23)$ |

$m_{D_s} - m_{D_s^0} \simeq m_{c\bar{s}} - m_{c\bar{s}^0}$. Compatible with Albaladejo, Nieves, PRD18

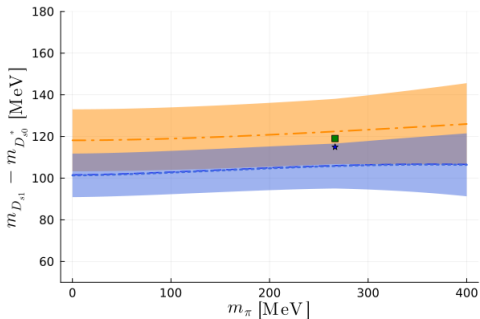
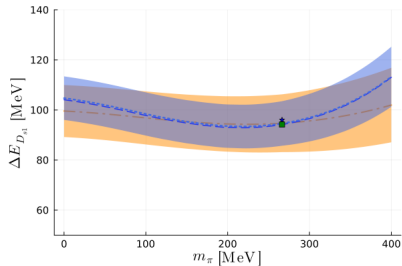
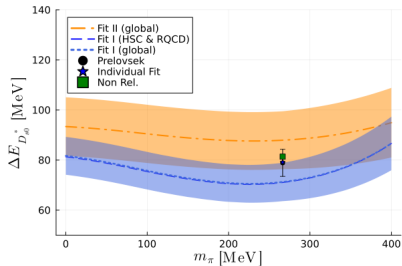
Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Compositeness and Scattering parameters



Comparison with Liu, Orginos, Guo, Meissner (2013)

Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$



Decay width of the $D_{s0}^*(2317)$ to $D_s^+ \pi^0$

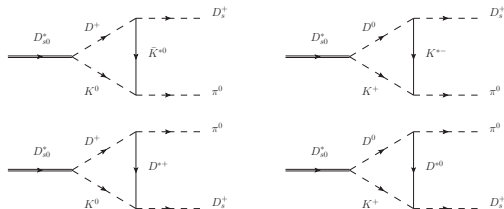


Figure 3: Feynman diagrams of the $D_{s0}^{*0} \rightarrow D_s^+ \pi^0$.

$\pi^0 - \eta$ mixing

$$\tilde{\pi}^0 = \pi^0 \cos \tilde{\epsilon} + \eta_8 \sin \tilde{\epsilon}$$

$$\tilde{\eta} = -\pi^0 \sin \tilde{\epsilon} + \eta_8 \cos \tilde{\epsilon}$$

with $\eta_8 = \frac{2\sqrt{2}}{3}\eta - \frac{1}{3}\eta'$

$$g_X = \frac{g_{DK}^{(I=0)}}{\sqrt{2}}$$

$$\Gamma_X = |\vec{p}_f| \frac{|t|^2}{8\pi m_X^2} \quad (9)$$

$$\Gamma_{D_{s0}^*} = 128 \pm 40 \text{ KeV}$$

H. L. Fu et al., 120_{-4}^{+18} KeV,
EPJA58(2022), M. Cleven,

green state: $\Gamma = 7.83_{-1.55}^{+1.97}$ KeV

M. Han et al., CPC23

Results for the T_{cc}

Preliminary (including only vector meson exchange)

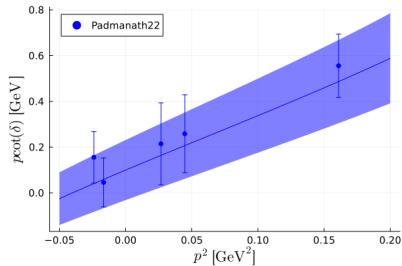
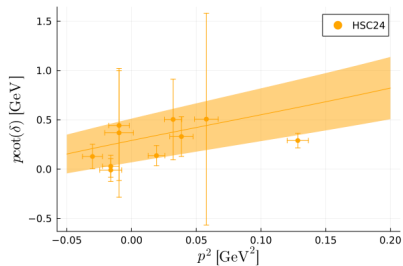
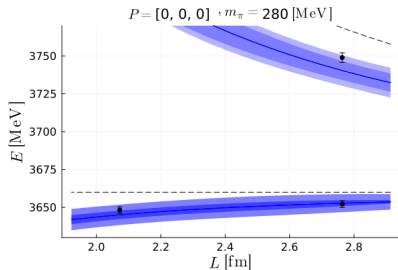
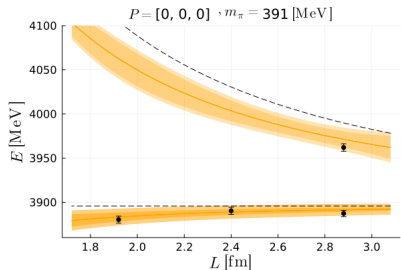
$$V_{\lambda,\lambda'}^{\rho(I=0)}(p, p') = -g^2 \frac{(p_1 + p_3)_\mu (p_2 + p_4)^\mu}{t - m_\rho^2} \epsilon_{\lambda,\nu}(p_1) \epsilon_{\lambda'}^{*\nu}(p_3) \quad (10)$$

and $g = g_1 + g_2 m_\pi^2$. Quark mass dependence of m_ρ , Molina, Elvira, JHEP20

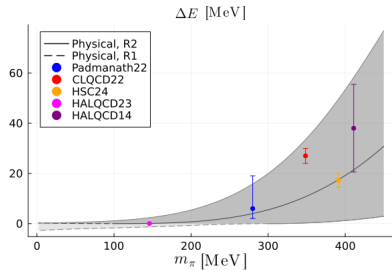
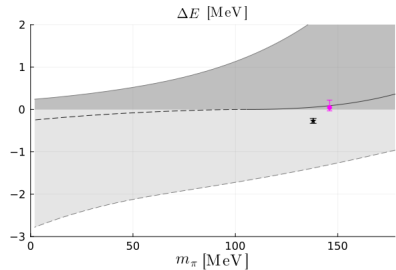
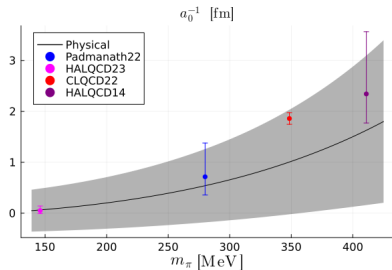
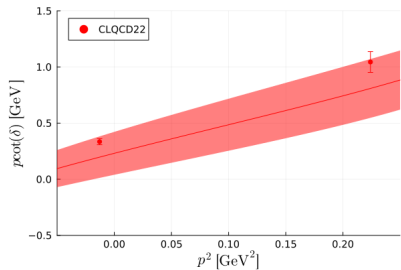
| Col. | a | L | m_π |
|-----------------------|-------|-----------|---------|
| Padmanath22,Collins24 | 0.086 | 2 – 3 | 280 |
| HSC24 | 0.120 | 1.9 – 2.9 | 391 |
| CLQCD Chen22 | 0.152 | 2.4 | 349 |

| Col. | a | m_π | a_0^{-1} |
|----------|--------|---------|------------|
| HALQCD23 | 0.0846 | 146 | 0.05 |
| HALQCD14 | 0.0907 | 411 | 2.34 |

Results for the T_{cc}



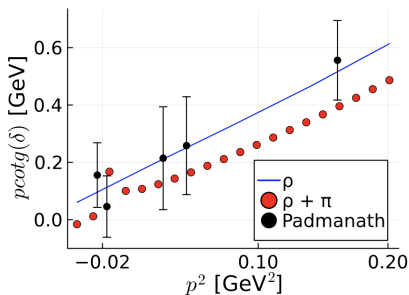
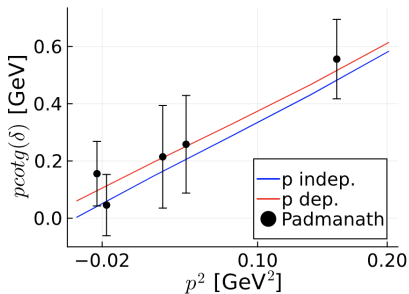
Results for the T_{cc}



Results for the T_{cc}

Preliminary momentum dependent BS equation including $\pi + \rho$

$$V_{\lambda,\lambda'}^{\pi(I=0)}(p,p') = \frac{3}{4} g_{D^*D\pi}^2 \frac{e^{u/\Lambda^2}}{u - m_\pi^2} (2p_4 - p_1)_\mu \epsilon_{\lambda}^\mu(p_1) (2p_2 - p_3)_\nu \epsilon_{\lambda'}^{*\nu}(p_3)$$



Conclusions

Conclusions

- The masses of the low-lying charmed mesons are well described by HHChPT, PLB 843, 137997 (2023)
- The combination of LQCD with EFT's is a usefool tool to extract the properties or esonances with high accuracy
- The study of the pion mass dependence of the $D_{s0}(2317)$ and $D_{s1}(2460)$ supports the DK and D^*K molecular picture ($1 - Z \simeq 0.7 - 0.8$), Phys.Rev.D 109, 096002 (2024)
- The results of the LQCD simulation for the T_{cc} are compatible with a **virtual state** that is transitioning into a **bound state** for a pion mass close to physical and with the vector meson exchange being dominant