

Scattering Amplitudes of Massive Spin-2 Kaluza-Klein States with Matter

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Dark Matter Evidence

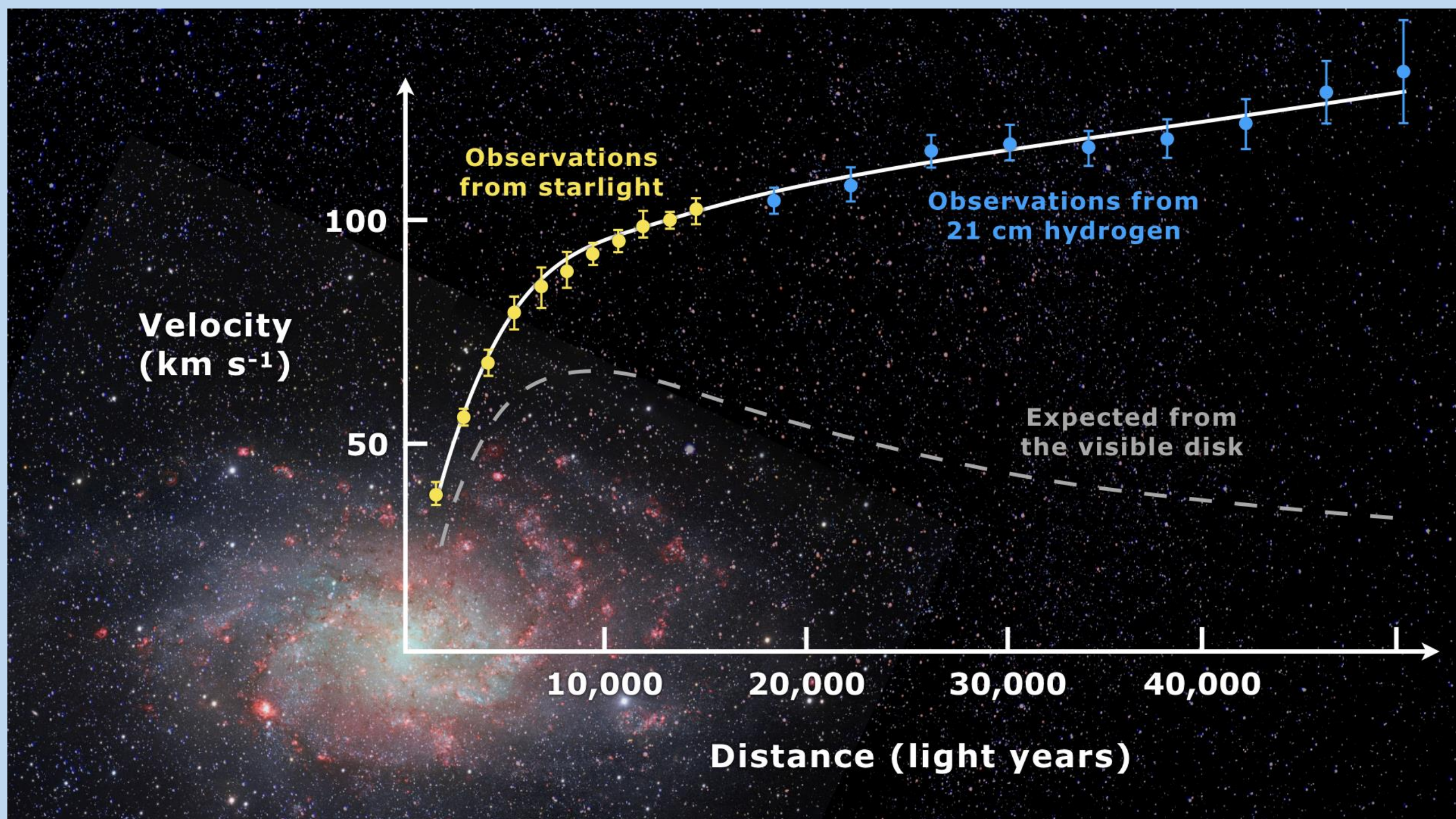


Image from De Leo [1], highlighting discrepancy in observed and expected galactic rotational curves.

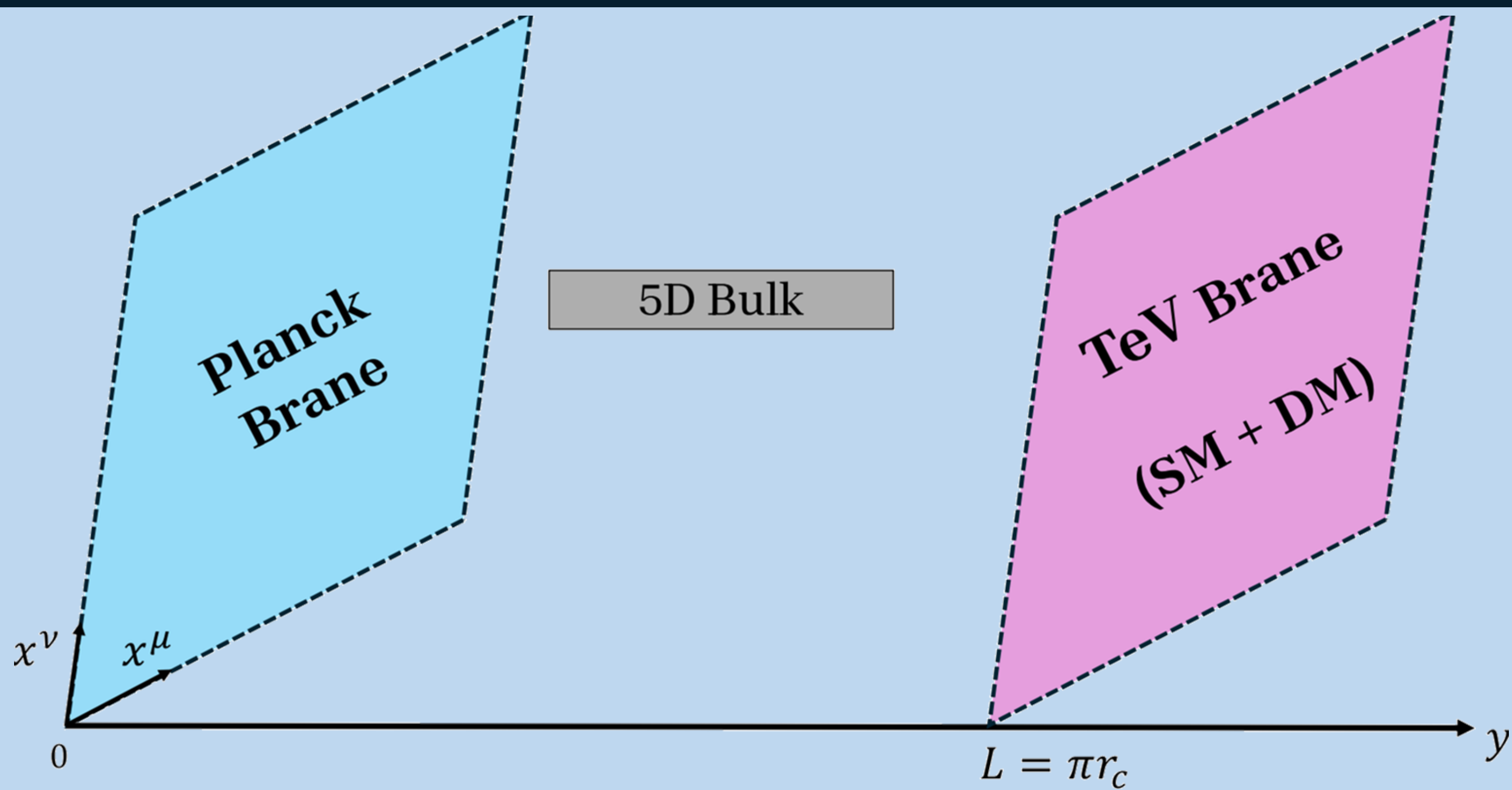
Compact Extra-Dimensions

- No Higgs mechanism for spin-2 fields - look to extra-dimensions for equivalent mechanism and hierarchy solution.
- 4D coordinate x , additional coordinate y , expansion parameter κ_5 .
- Weak field expansion $G_{AB} = \eta_{AB} + \kappa_5 \tilde{h}_{AB}$.
- Assume KK decomposition of wavefunctions - ψ_n is extra-dim profile of mode:

$$\hat{\phi}(x, y) \sim \sum_n \phi_n(x) \psi_n(y).$$

- The ψ_n satisfy Sturm-Liouville equation - generates **discrete mass spectrum without breaking coordinate invariance!**
- Particle coupling \propto overlap integral of profiles ψ_n .
- Need appropriate boundary conditions - Randall-Sundrum 1 (RS1) Model [2,3].

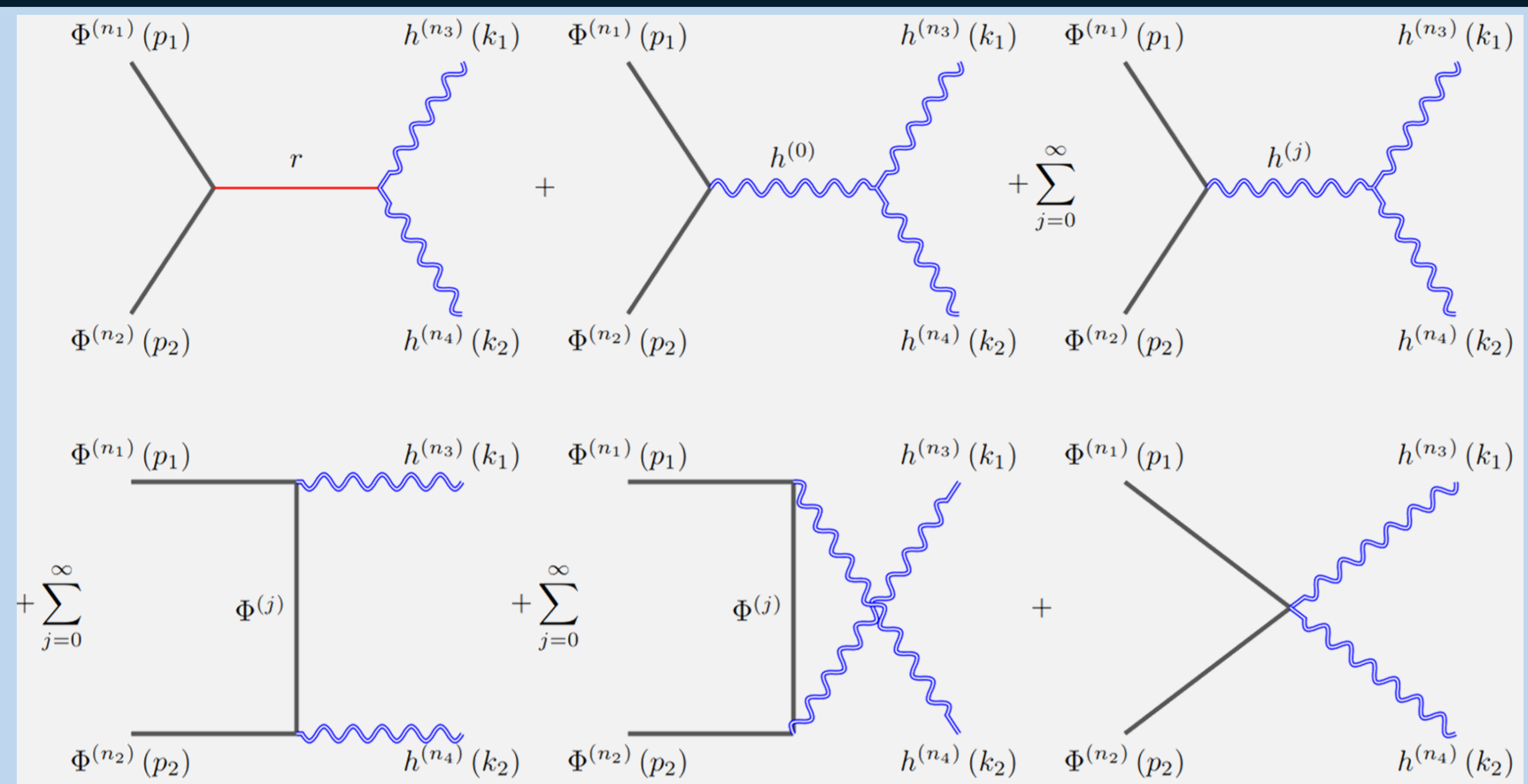
Randall-Sundrum 1 Model



- Two 4D spacetime slices (branes) at $y = 0$ (Planck) and $y = L = \pi r_c$ (TeV).
 - Warped geometries described by below metric, with warping k and fluctuations $\tilde{h}_{\mu\nu}$ (Spin-2) and \hat{r} (Spin-0 / Radion):
- $$G_{AB} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu} + \kappa_5 \tilde{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}, \quad \hat{u} = \frac{\kappa_5 \hat{r}}{2\sqrt{6}} e^{k(2|y|-L)}.$$
- A **natural solution** to electroweak hierarchy problem.
 - Planck Higgs VEV **exponentially warped down** compared to TeV Higgs VEV.
 - Effective 4D gravitational coupling **minimally altered**:

$$\langle \Phi \rangle_{\text{TeV}} = e^{-kL} \langle \Phi \rangle_{\text{Pl}} \quad \kappa_4^2 = \left(\frac{k}{1 - e^{-2kL}} \right) \kappa_5^2.$$

Matter Calculation



All annihilation processes of bulk and brane-localised DM candidates $\Phi = S, \psi, V$ (Scalar, Fermion and Vector) to external spin-2 modes (h).

- Testing for unitarity – expect no additional scale dependence in amplitudes.**
- Unitarity requires high energy amplitudes grow $\propto s\kappa_4^2 = E^2/\Lambda^2$, where Λ is the effective cutoff energy.
- In full extra-dimensional models, do **not** expect additional scales due to higher dimensional **diffeomorphism** – general coordinate invariance of General Relativity.
- Longitudinal modes of spin-2 fields naively generate the worst behaviour at E^2/m_n^2 per leg – we examine these modes in detail.

Results

Elastic Brane Localised (B-L) Matter Scattering ($n_3 = n_4 = n$)

B-L Scalar: $\mathcal{M} = -\frac{s\kappa_4^2(3 \cos 2\theta + 1)}{96} [k^{(n)}(\bar{y})]^2 + \mathcal{O}(s^0).$

B-L Fermion: $\mathcal{M} = \frac{s\kappa_4^2 \sin 2\theta}{32} [k^{(n)}(\bar{y})]^2 + \mathcal{O}(s^0).$

B-L Vector: $\mathcal{M} = \frac{s\kappa_4^2(3 \cos 2\theta + 1)}{96} [k^{(n)}(\bar{y})]^2.$

Elastic Bulk Matter Scattering ($n_1 = n_2 = m, n_3 = n_4 = n$)

Bulk Scalar: $\mathcal{M} = \frac{s\kappa_4^2(1 - \cos 2\theta)}{32} \left\langle k^{(n)} k^{(n)} f_S^{(m)} f_S^{(m)} \right\rangle_S + \mathcal{O}(s^0).$

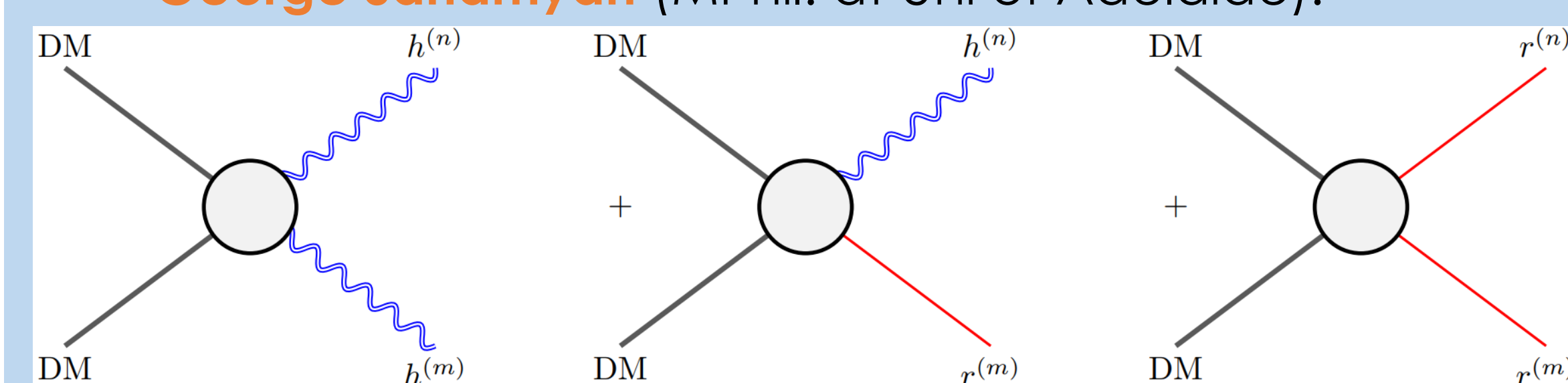
Bulk Fermion: $\mathcal{M} = \frac{s\kappa_4^2 \sin 2\theta}{32} \left\langle k^{(n)} k^{(n)} f_{\psi_{L/R}}^{(m)} f_{\psi_{L/R}}^{(m)} \right\rangle_\psi + \mathcal{O}(s^0).$

Bulk Vector: $\mathcal{M} = \frac{s\kappa_4^2(3 \cos 2\theta + 1)}{96} \left\langle k^{(n)} k^{(n)} f_V^{(m)} f_V^{(m)} \right\rangle_V + \mathcal{O}(s^0).$

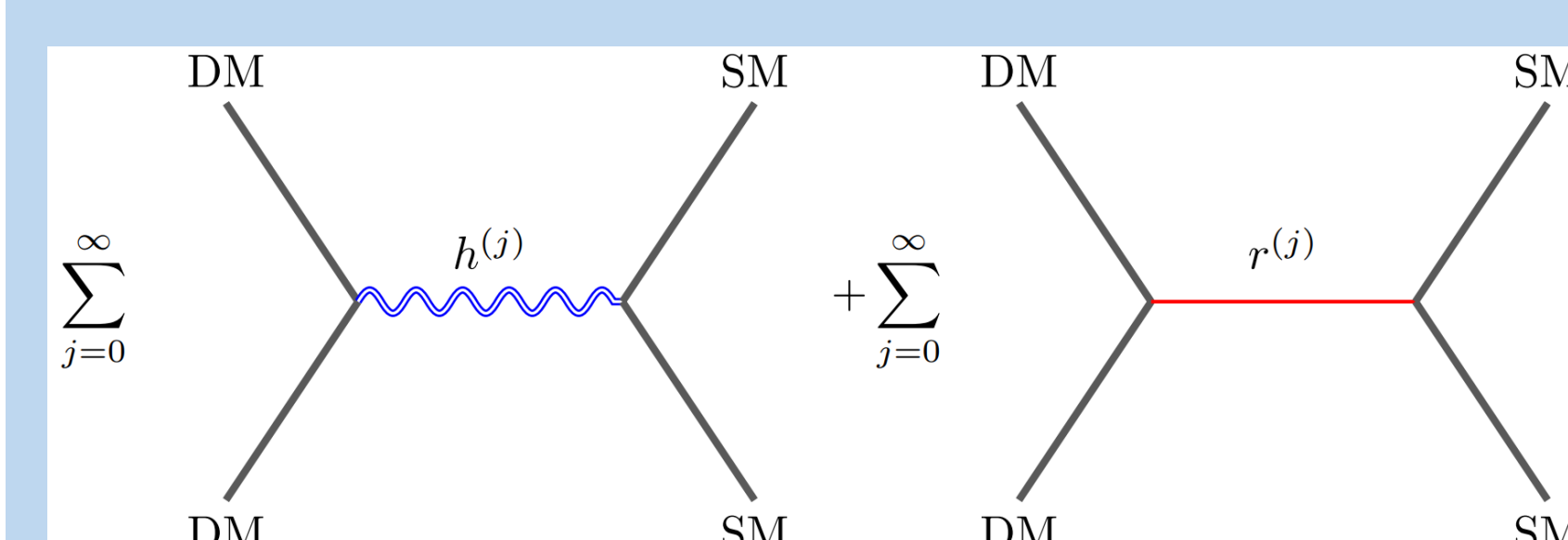
Amplitudes grow $\propto s\kappa_4^2$ and external spin-2 modes behave as scalar fields $k^{(n)}$.
→ Goldstone Equivalence Theorem verification (see Phys. Rev. D 109, 075016)!

Future Work

- Radion field introduces **long-range force** → **branes can collapse**.
- Introduce **bulk scalar** field to stabilise geometry.
 - Mixes with radion → creates massive KK tower (**Goldberger-Wise GW** [3]).
- Test unitarity for external Goldberger-Wise states included.
 - Scanning parameter space to find DM family which saturates the relic density – **Results to be released shortly** in collaboration with **George Sanamyan** (MPhil. at Uni of Adelaide).



New GW states included – update unitarity results.



KK portal to produce SM in stabilised geometry.

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References

- [1] De Leo, M 2018, *Rotation curve of spiral galaxy Messier 33 (Triangulum)*, viewed 18/08/24, [https://commons.wikimedia.org/wiki/File:Rotation_curve_of_spiral_galaxy_Messier_33_\(Triangulum\).png](https://commons.wikimedia.org/wiki/File:Rotation_curve_of_spiral_galaxy_Messier_33_(Triangulum).png).
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), arXiv:hep-ph/9905221.
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- [4] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. 83, 4922 (1999), arXiv:hep-ph/9907447.