Universität Bielefeld

CRC-TR 211

QCHSC-2024

Probing the In- and Out-of-Equilibrium Chiral Magnetic Effects with Lattice QCD

XVIth Quark Confinement and the Hadron Spectrum 2024
Cairns, Australia

Dean Valois

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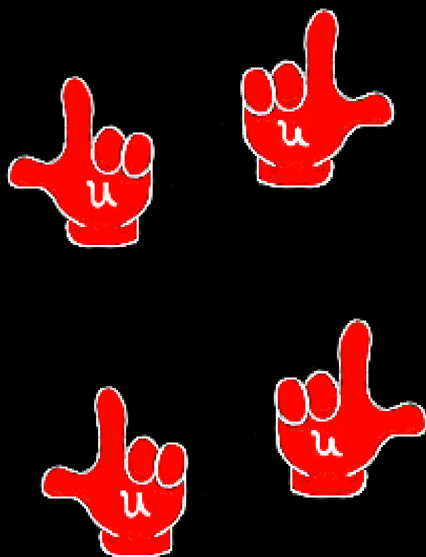
In collaboration with

Bastian Brandt, Gergely Endrődi
Eduardo Garnacho, Gergely Markó

WHAT IS THE CHIRAL MAGNETIC EFFECT ?



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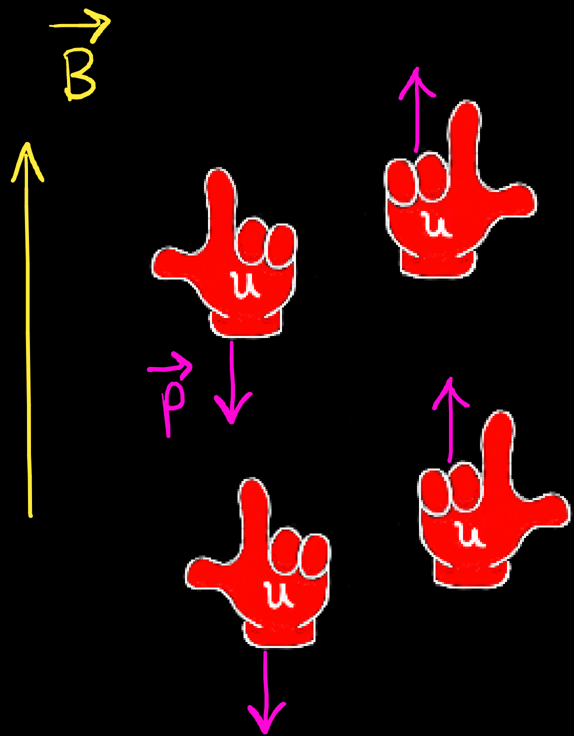

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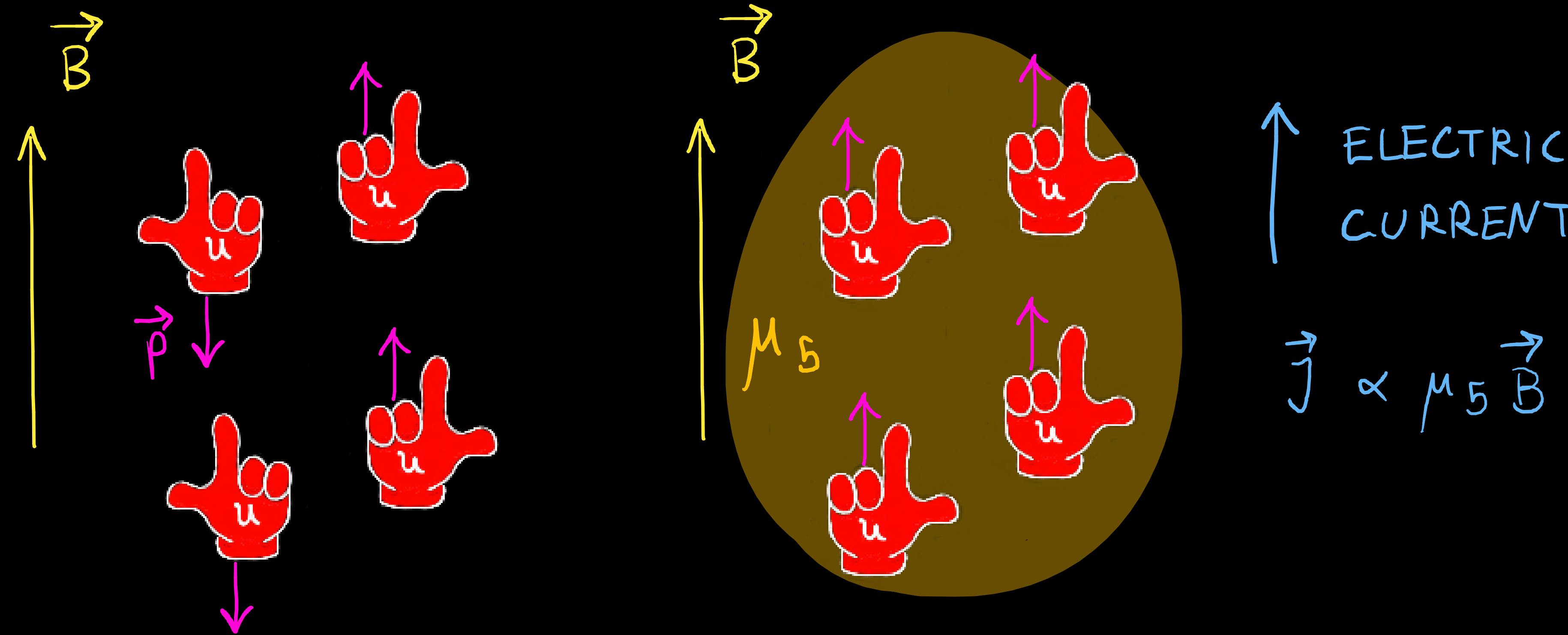

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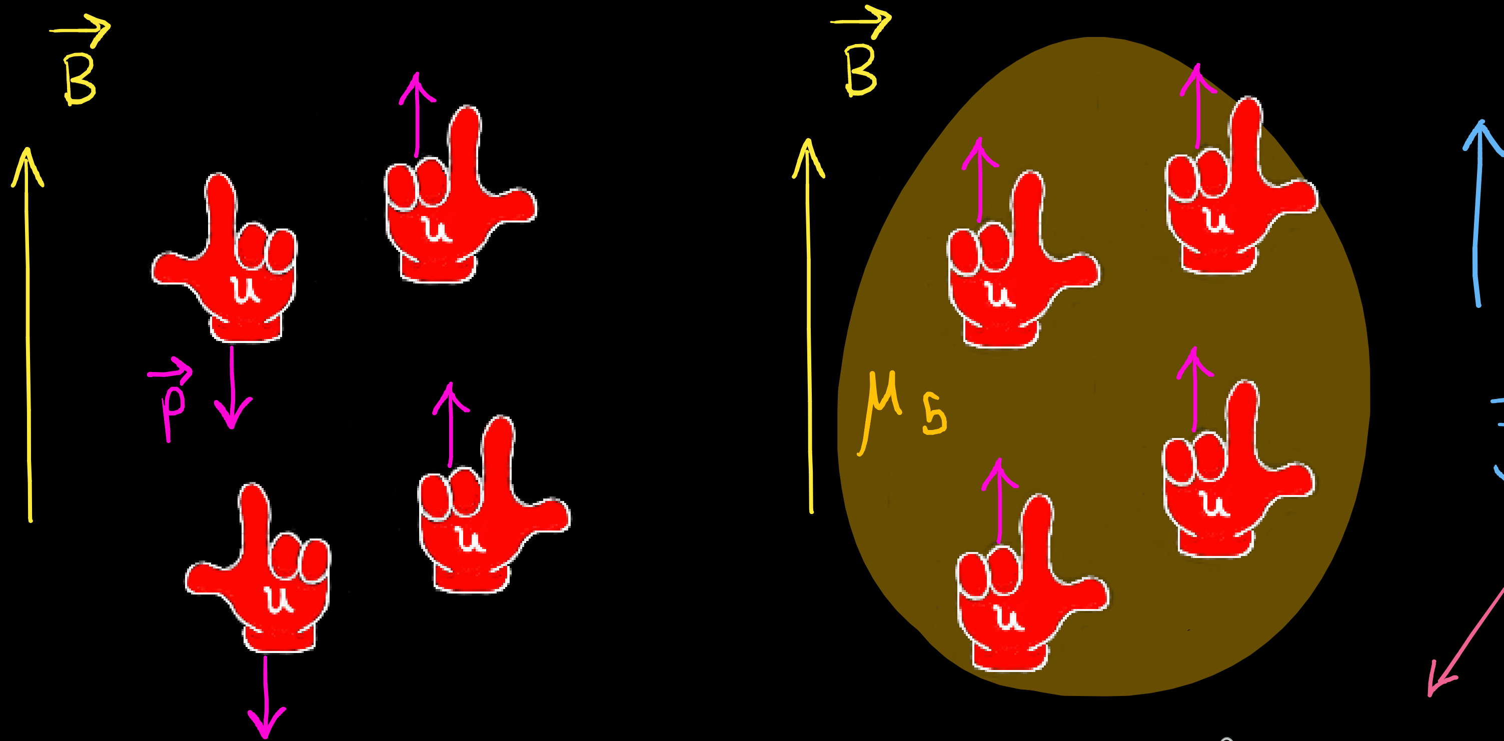

ELECTRIC CURRENT

$$\vec{J} \propto \mu_5 \vec{B}$$

THEORETICAL PREDICTIONS

K. Fukushima et al. 2008

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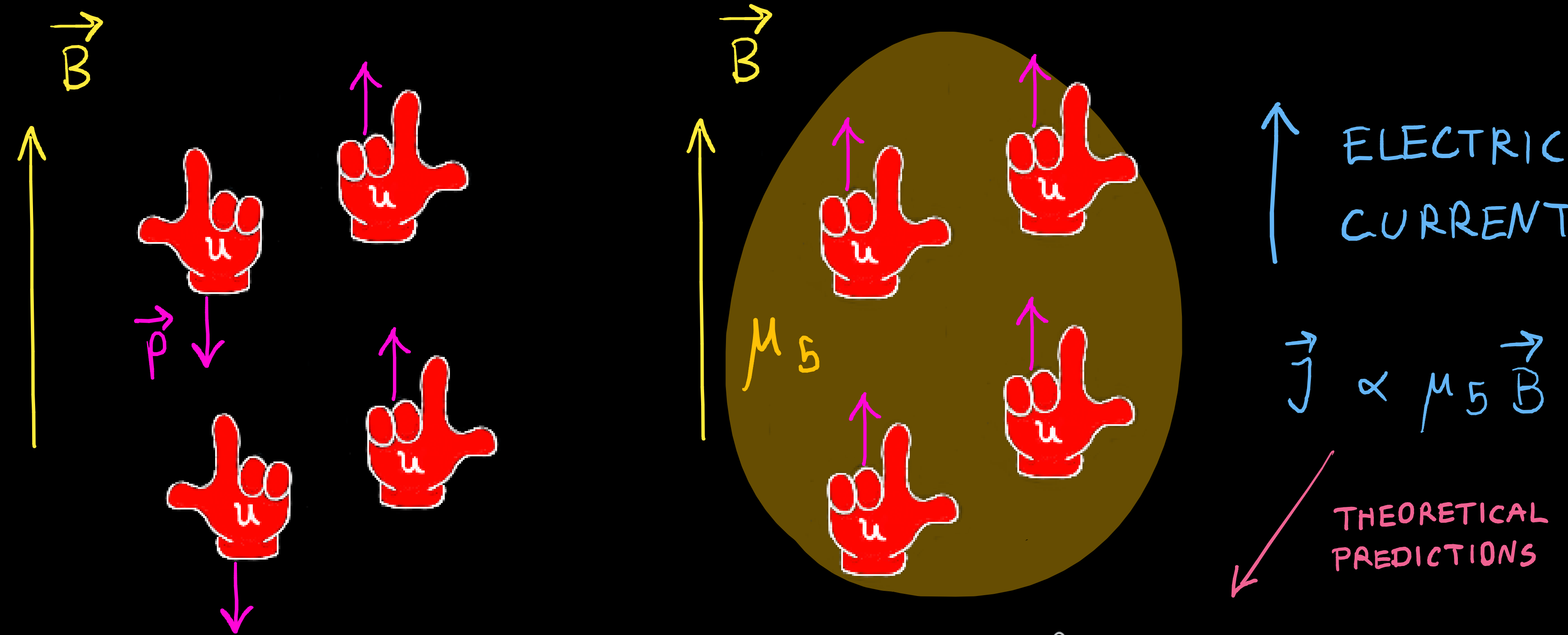

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IN CONDENSED MATTER:

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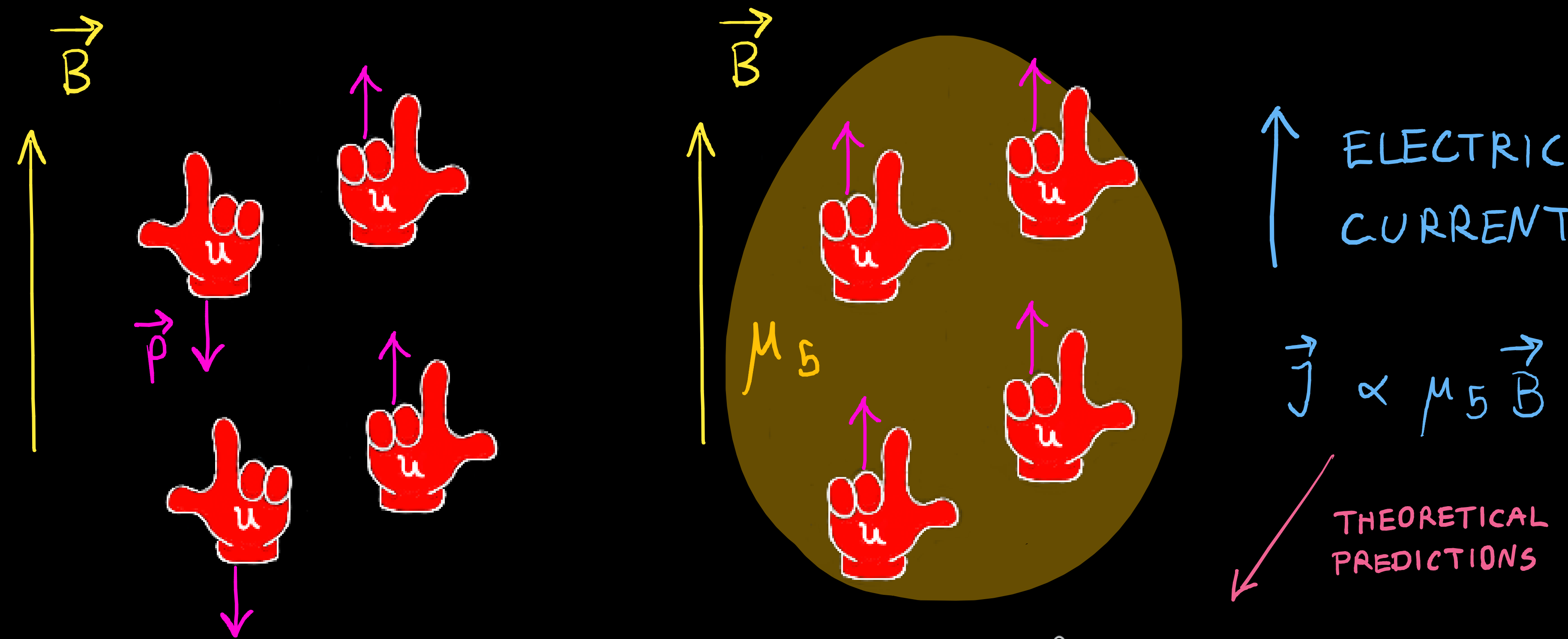

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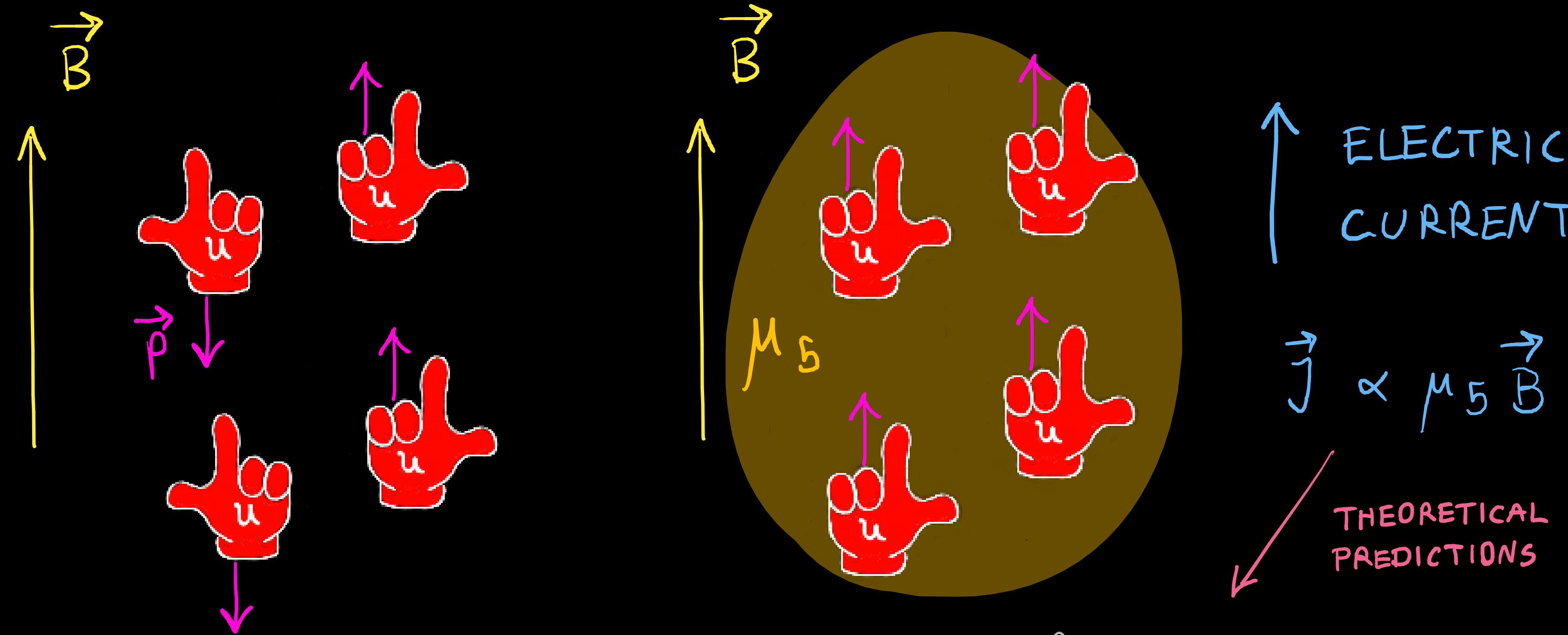

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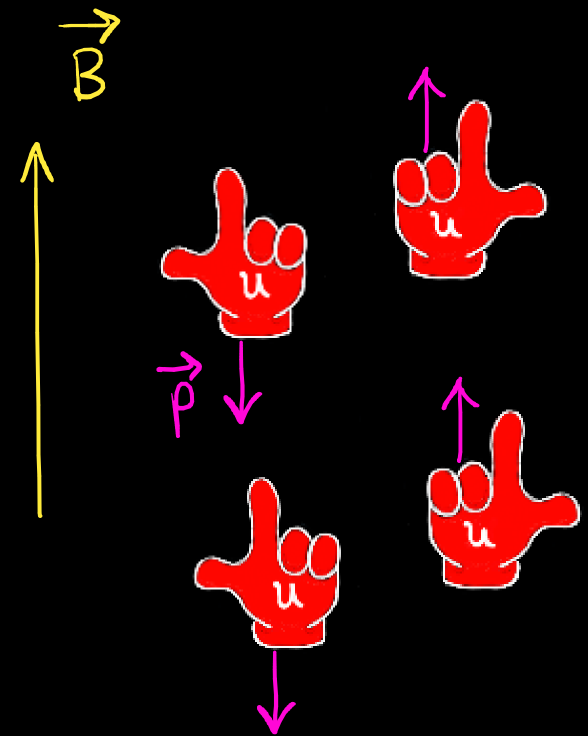

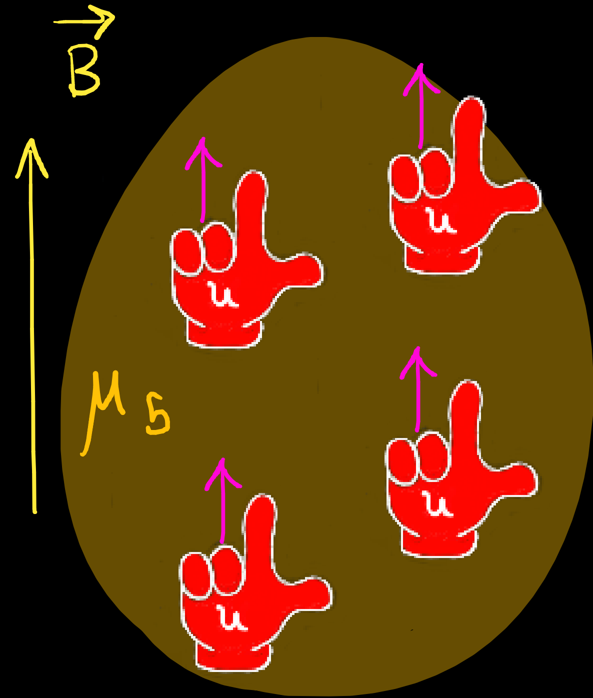

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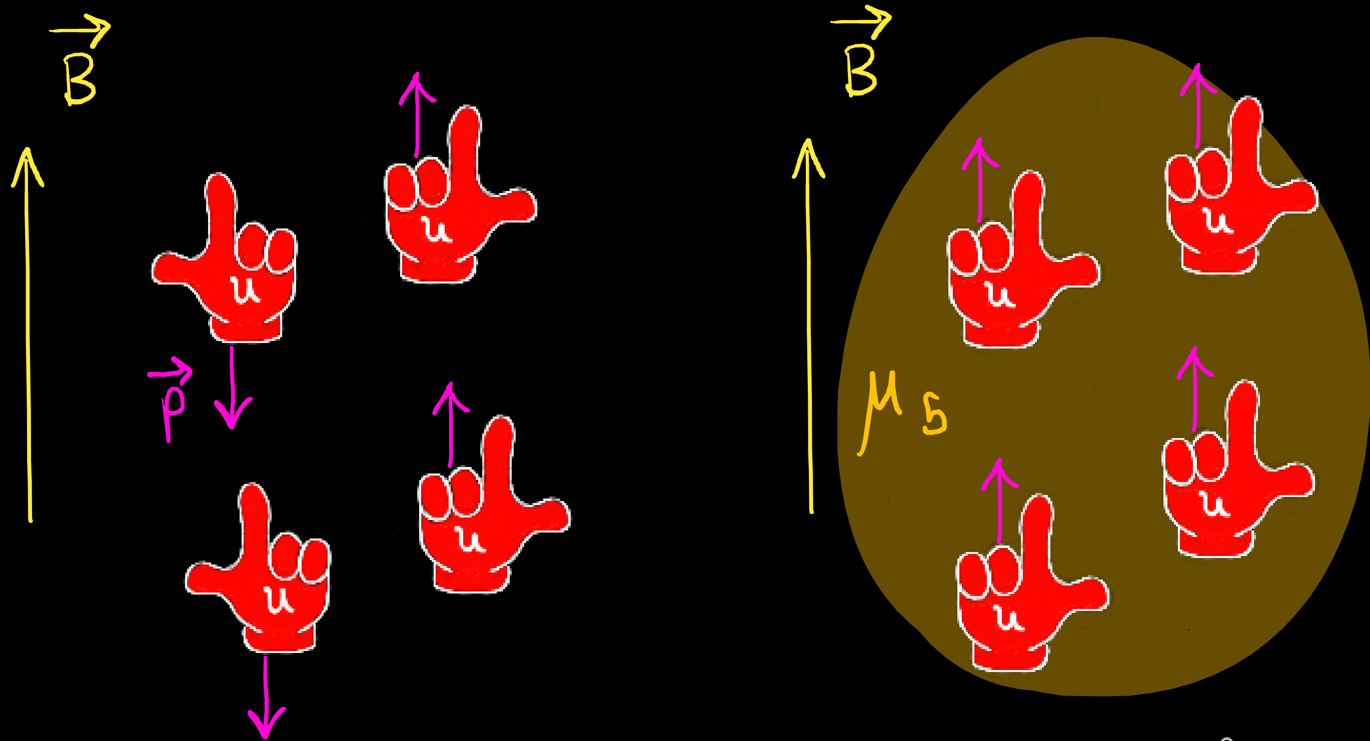

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EVIDENCE FOR CME IN HIC

📎 B. I. Abelev et al. (STAR Collaboration) 2009

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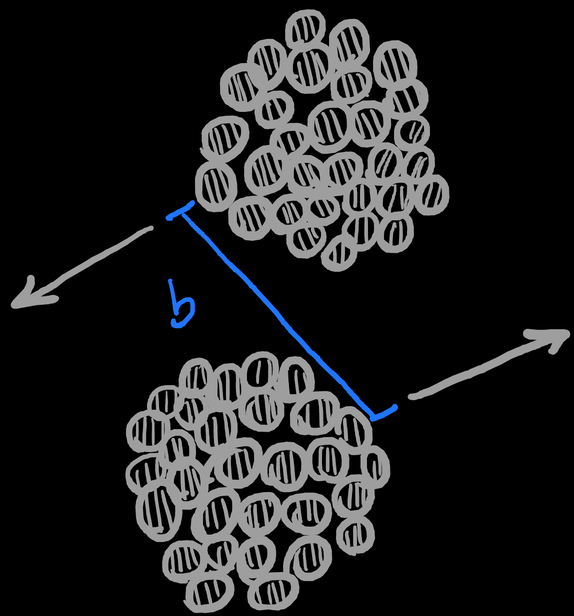
HEAVY-ION COLLISIONS (HIC)



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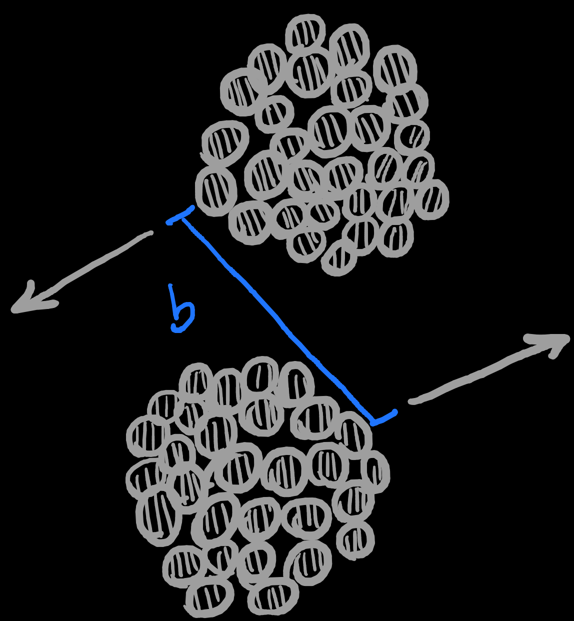
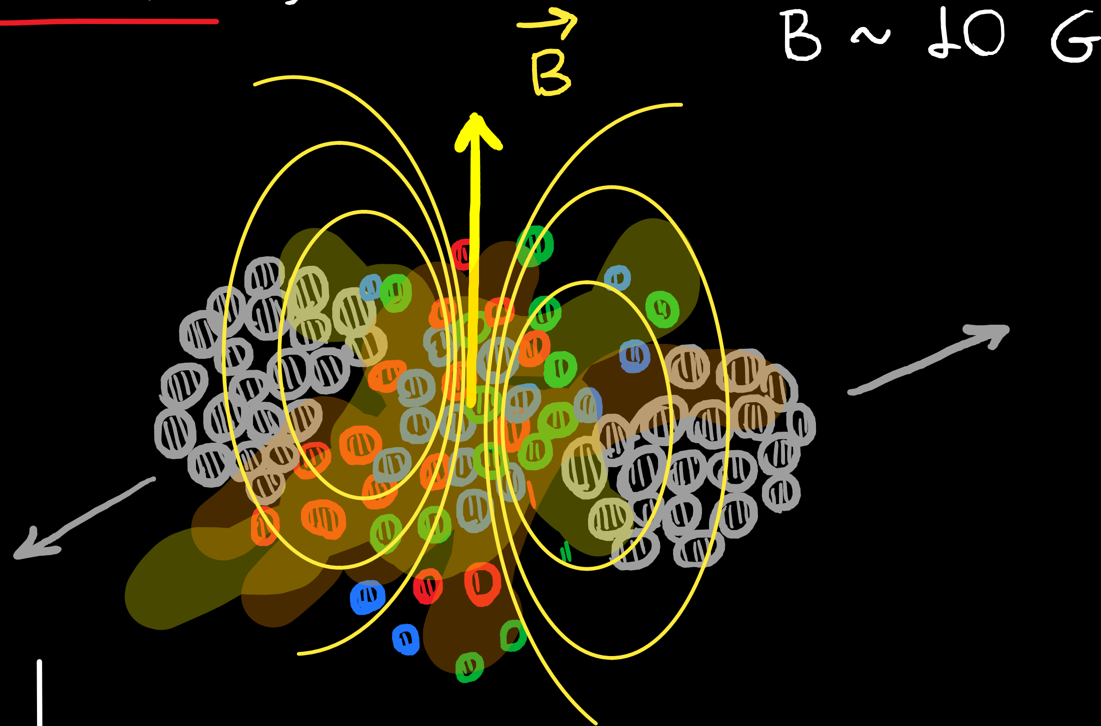
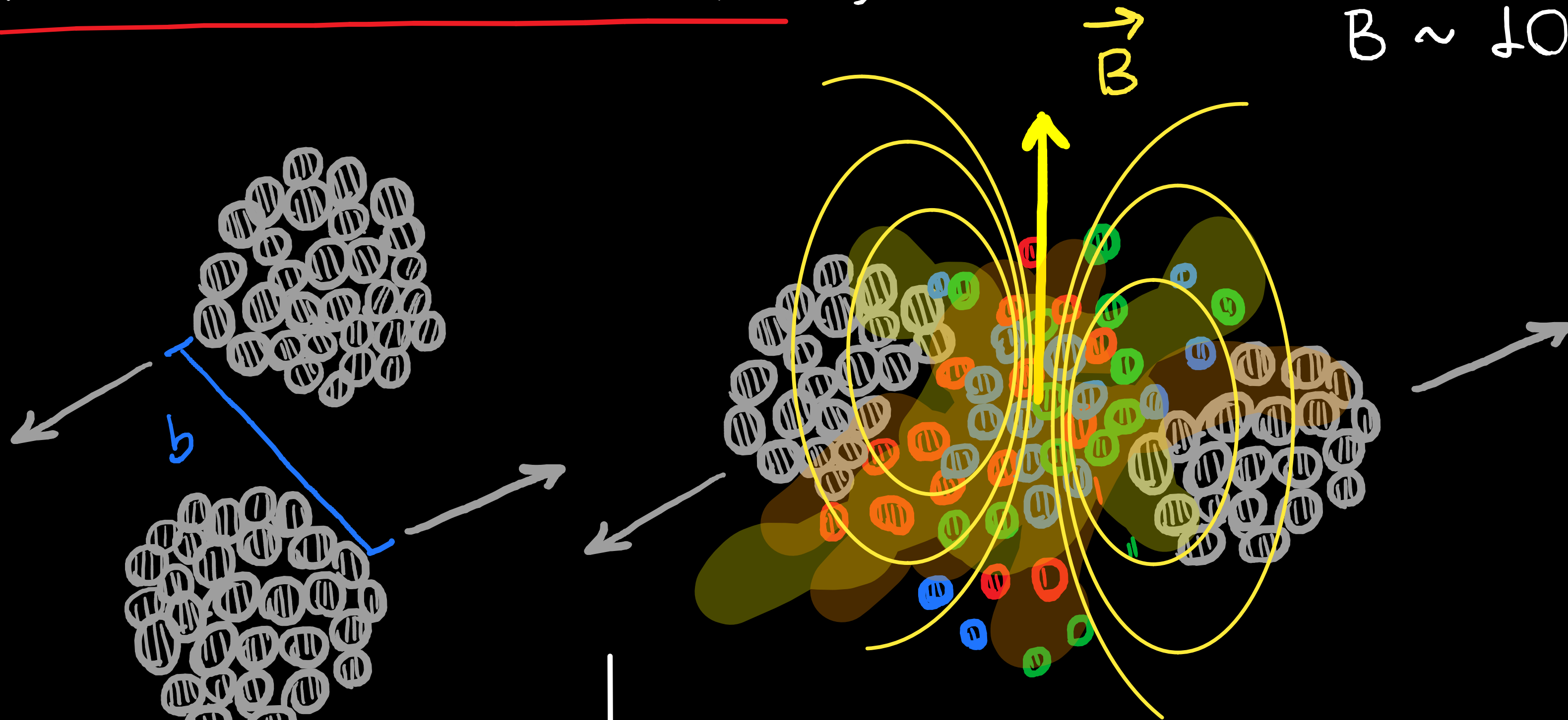




- RECENT RESULTS BY STAR
  - 📎 M. Abdulhamid et al. 2024

HEAVY-ION COLLISIONS (HIC)

$B \sim 10^{18}$ G



- RECENT RESULTS BY STAR
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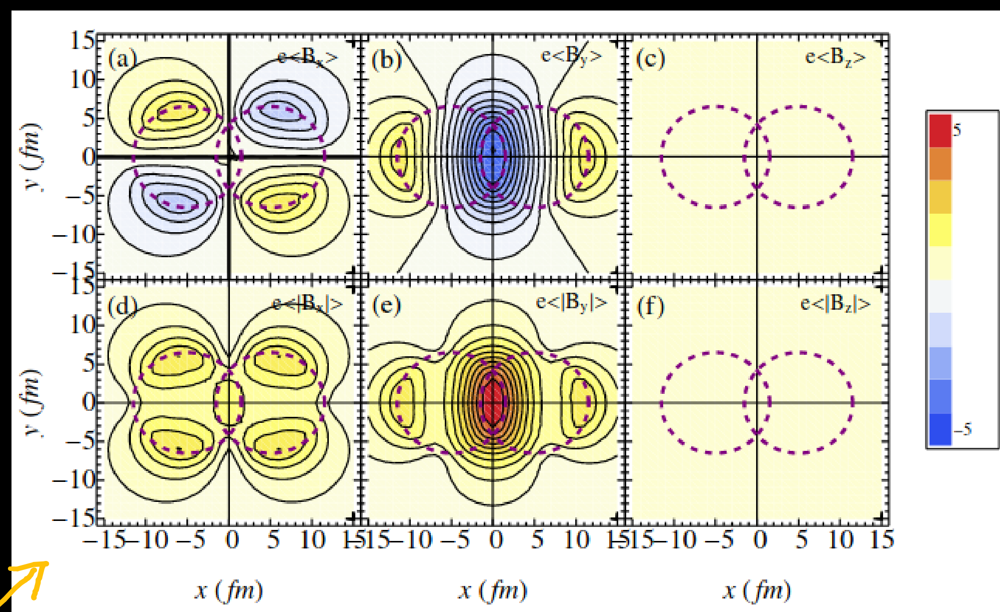
- MAGNETIC FIELDS IN HIC HAVE HIGHLY NON-TRIVIAL GEOMETRIES



📎 W. T. Deng & X. G. Huang 2012

OUTLINE

- BRIEF OVERVIEW OF CME
- PART I - EQUILIBRIUM CME WITH NON-UNIFORM $\vec{B}$
- PART II - OUT-OF-EQUILIBRIUM CME WITH UNIFORM $\vec{B}$
- SUMMARY & CONCLUSIONS

CME IN EQUILIBRIUM

$$j_3 = C_{CME}\, \mu_5\, B$$

CME IN EQUILIBRIUM

CME CONDUCTIVITY

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VECTOR CURRENT

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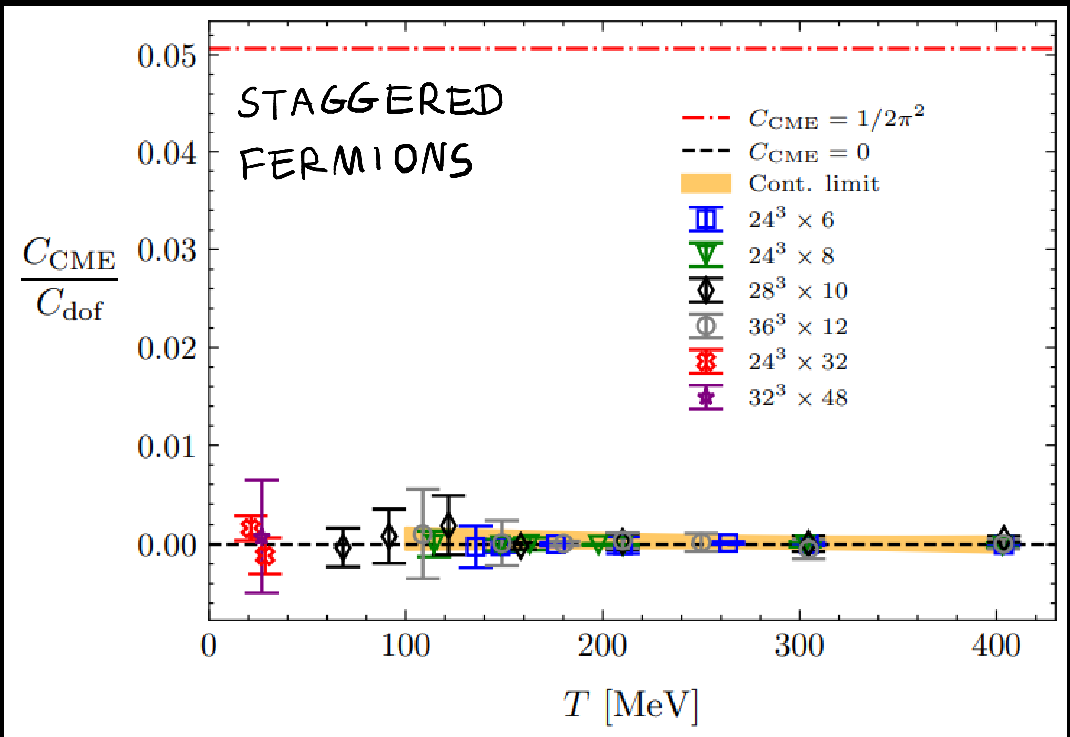

B. Brandt, G. Endrődi, E. Garnacho-Velasco, G. Markó 2024

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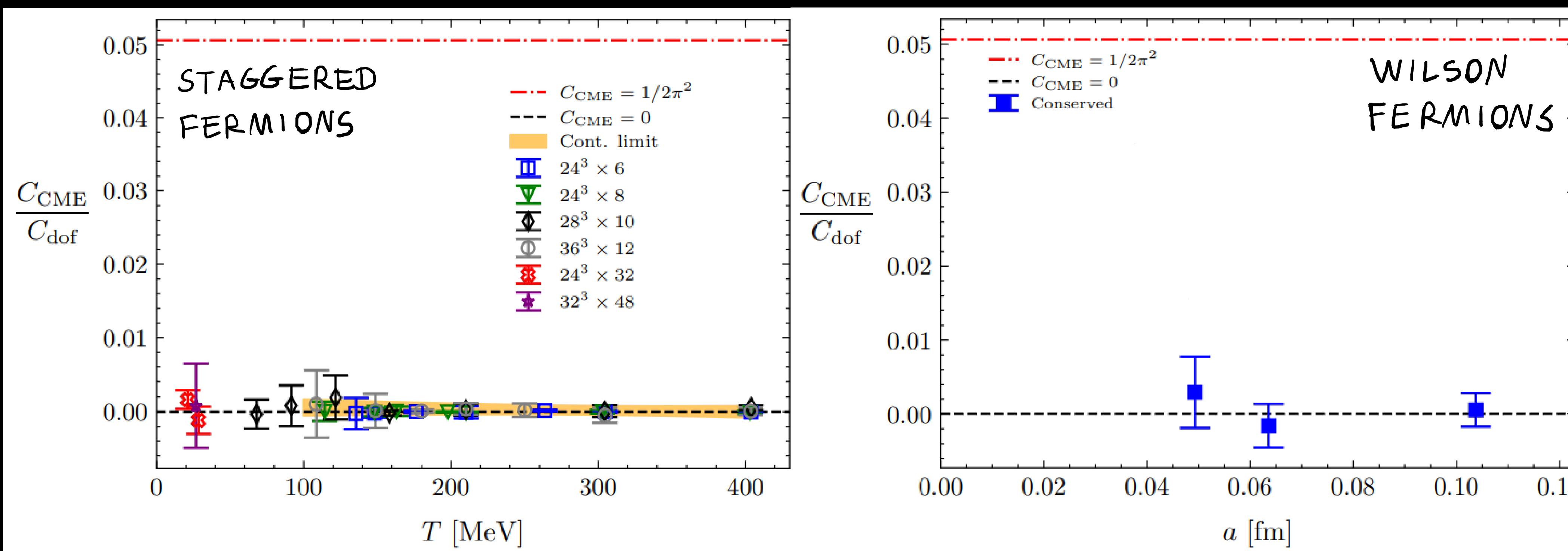

📎 B. Brandt, G. Endrődi, E. Garnacho-Velasco, G. Markó 2024

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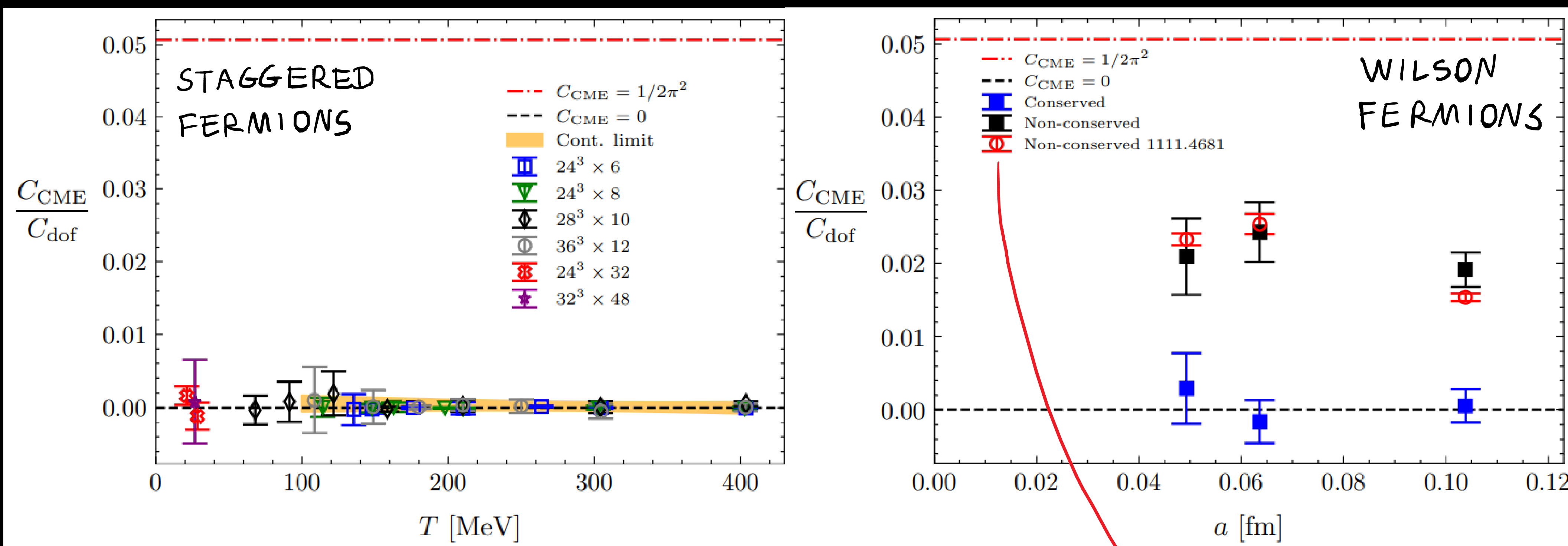

📎 B. Brandt, G. Endrődi, E. Garnacho-Velasco, G. Markó 2024

📎 A. Yamamoto 2011

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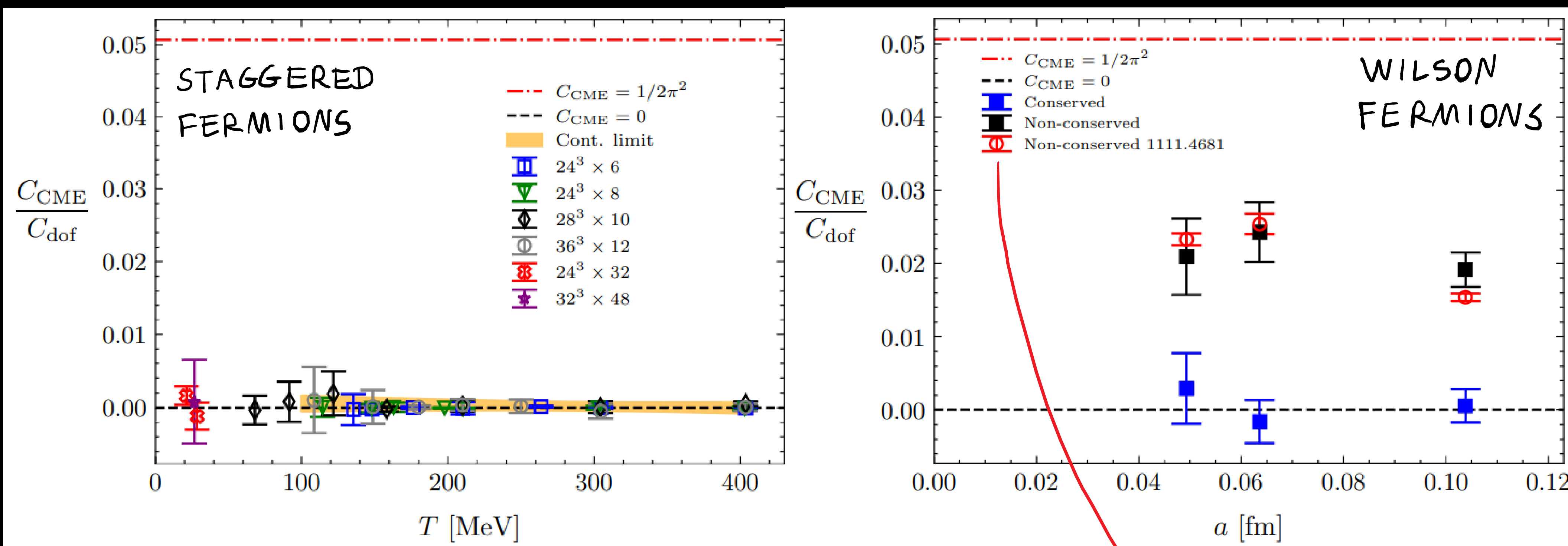

B. Brandt, G. Endrődi, E. Garnacho-Velasco, G. Markó 2024

A. Yamamoto 2011

CONCLUSION: FOR THE CORRECT CME, $\dot{j}_3$ HAS TO BE CONSERVED ON THE LATTICE!

CME & BLOCH'S THEOREM

- IN QUANTUM MECHANICS:

CME & BLOCH'S THEOREM

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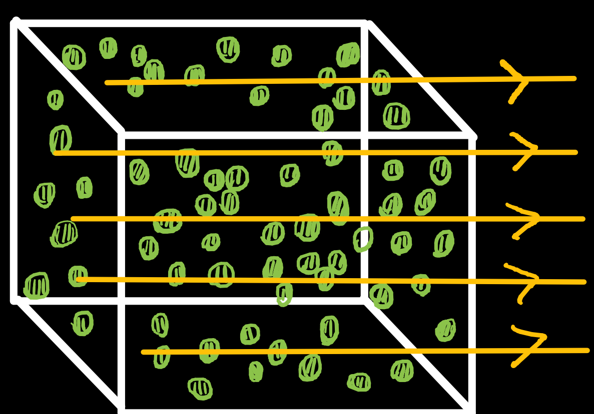
TOTAL CONSERVED CURRENT VANISHES ON THE GROUND STATE IN EQUILIBRIUM



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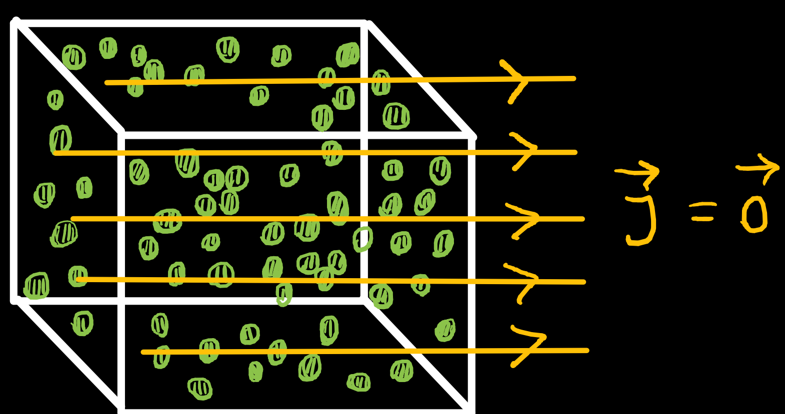

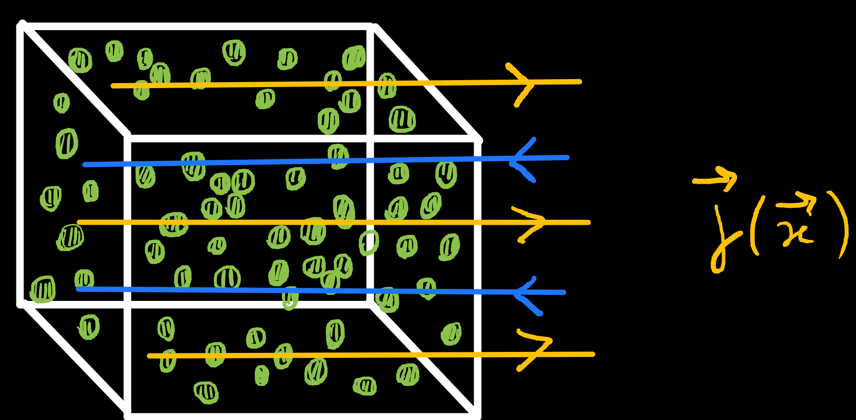

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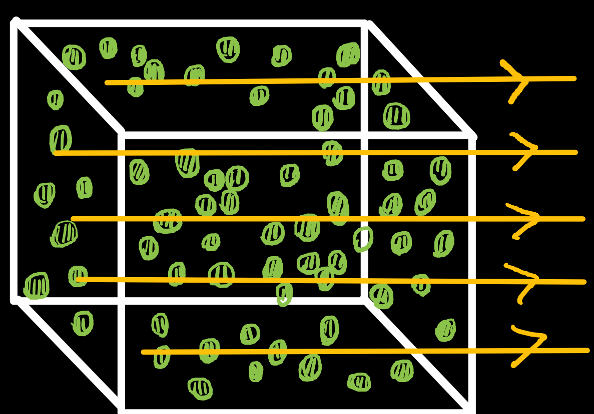

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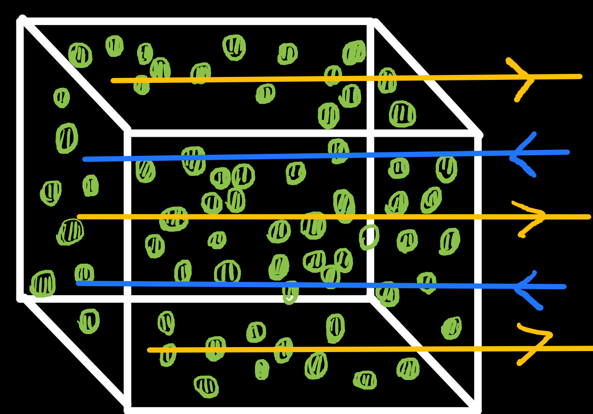

$\int_{\vec{x}} \vec{j}(\vec{x}) = \vec{J} = \vec{0}$

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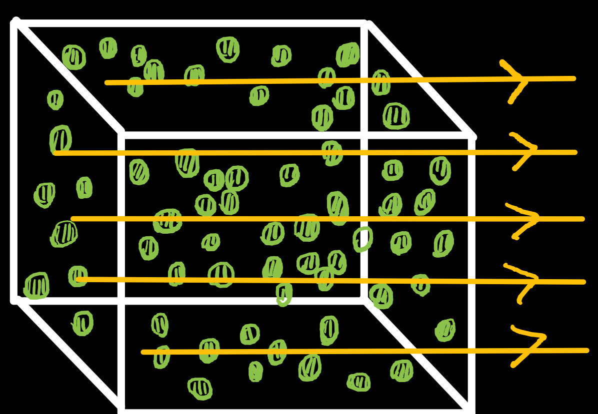

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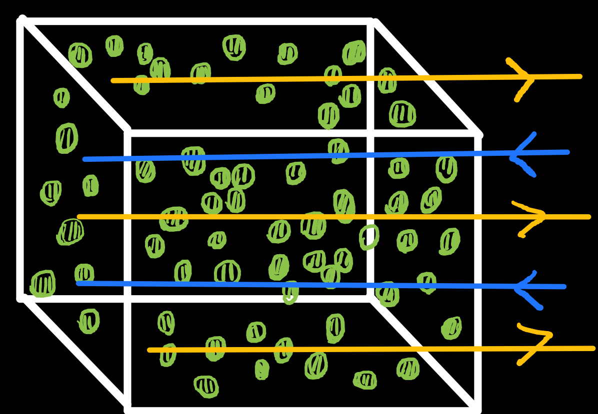

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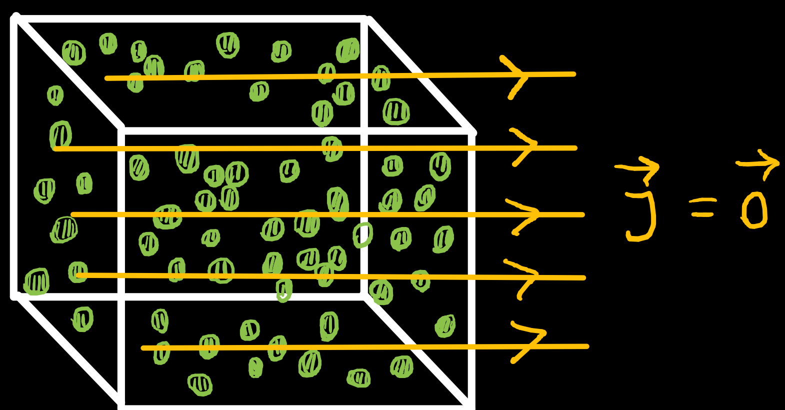

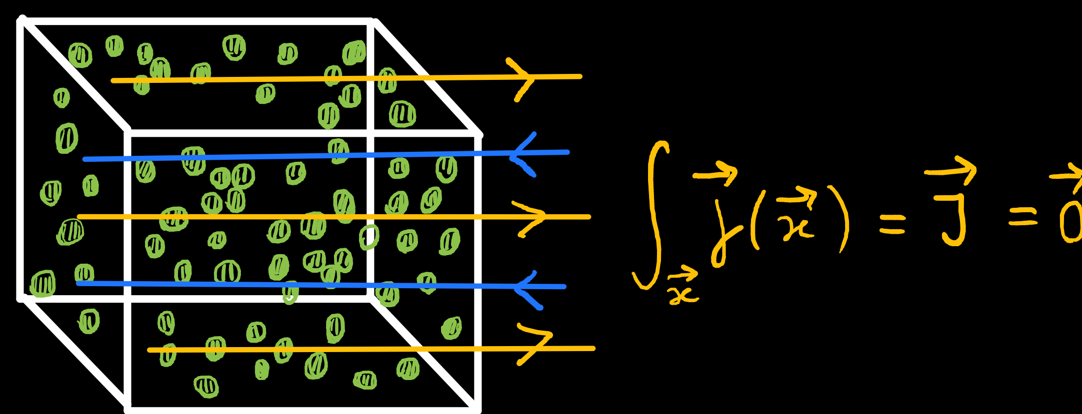

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WHAT GENERATES A NON-TRIVIAL LOCAL $j(\vec{x})$?

PART I –

EQUILIBRIUM CME WITH NON-UNIFORM $\vec{B}$

NON-UNIFORM $\vec{B}$ MODEL

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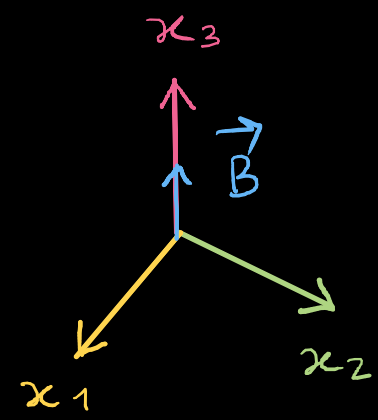
$$\vec{B}(x_1) = \frac{B}{\cosh\left(\frac{x_1}{\varepsilon}\right)^2} \hat{x}_3$$



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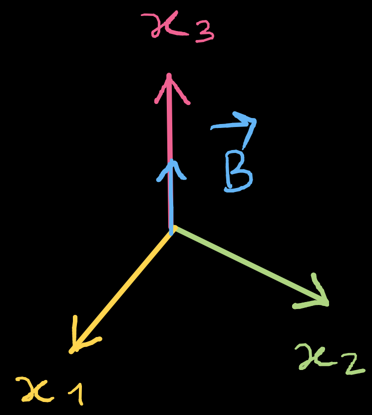

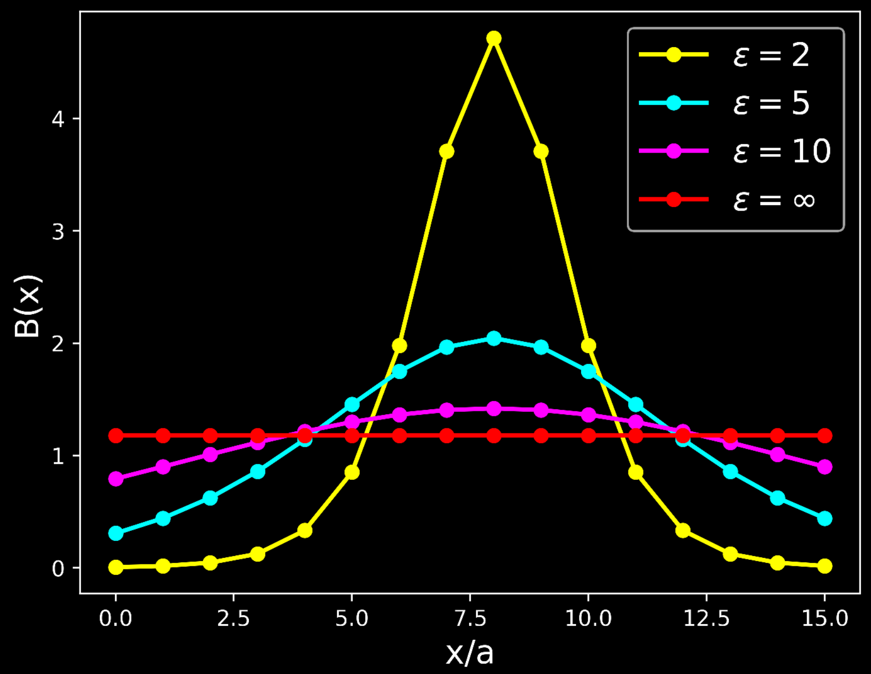

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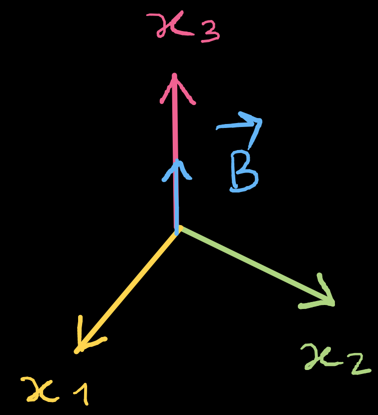

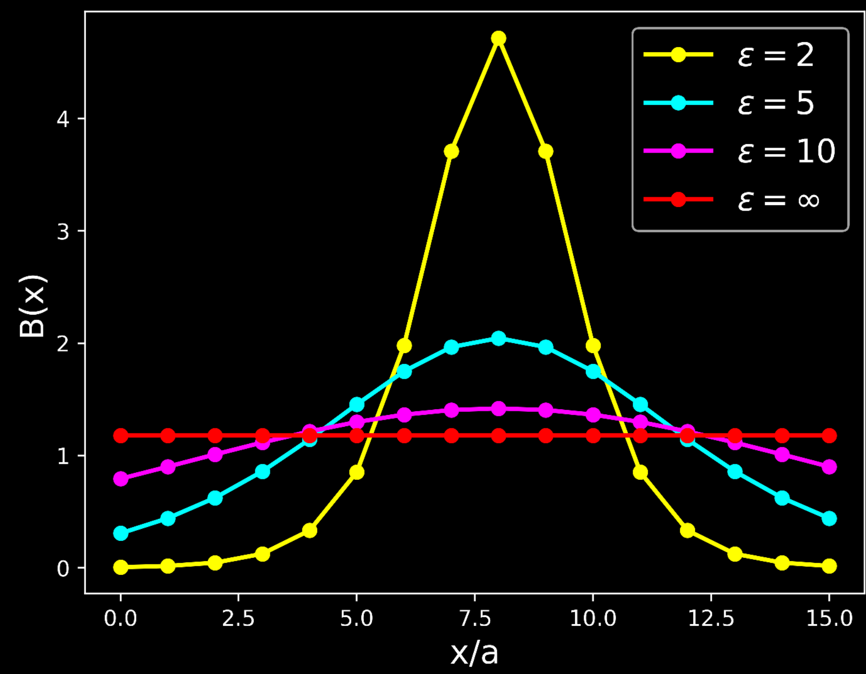

- ANALYTICALLY TREATABLE
  - 📎 Gaoqing Cao Phys. Rev. D 97, 054021 (2018)
- RESEMBLES $\vec{B}$ IN HIC
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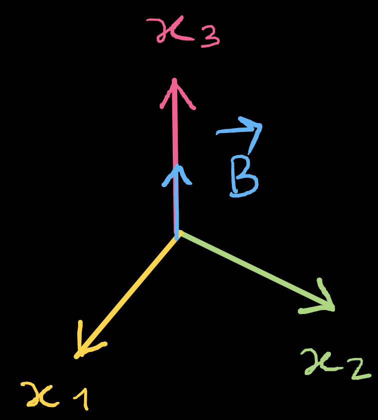

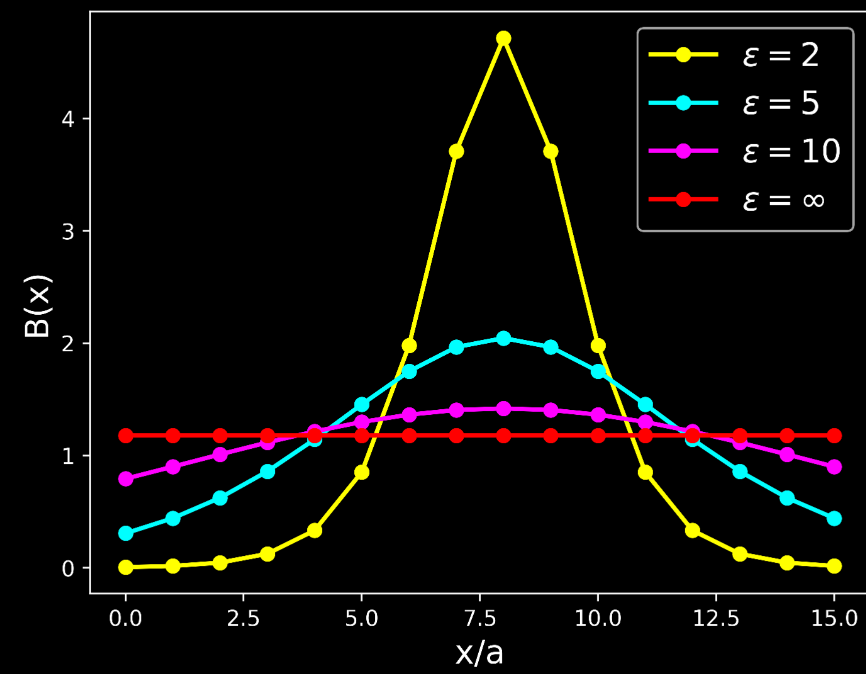

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QUANTIZATION ON A FINITE VOLUME :

$$eB = \frac{3\pi N_b}{\varepsilon L_y \tanh\left(\frac{L_x}{2\varepsilon}\right)}$$

$N_b \in \mathbb{Z}$

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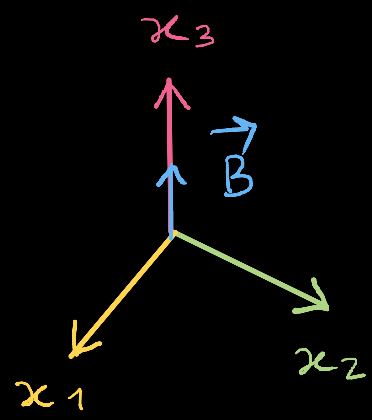

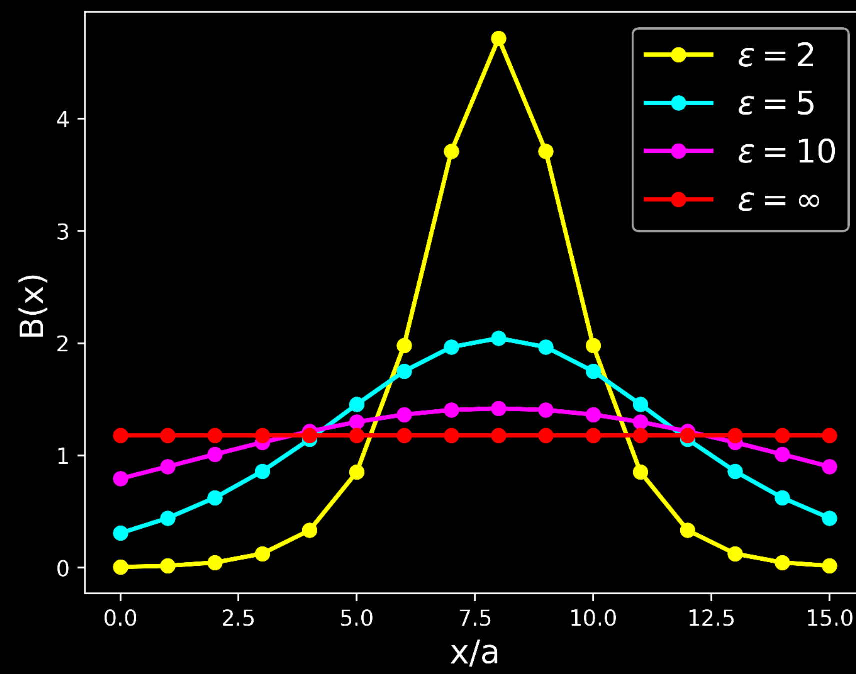

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- IMPACT ON PHASE DIAGRAM : 📎 B. Brandt et al. 2023
- USED AS A TOOL TO COMPUTE EoS : 📎 B. Brandt, G. Endrődi, G. Markó, A. D. M. Valois 2024

CME WITH FREE FERMIONS

$$j_3 = C_{CME}\, \mu_5\, B$$

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- OUR LATTICE OBSERVABLE: $G(x_1) \equiv \frac{1}{L^2} \frac{\partial \langle j_3(x_1) \rangle}{\partial \mu_5}\Big|_{\mu_5=0}$

CME WITH FREE FERMIONS

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CME WITH FREE FERMIONS

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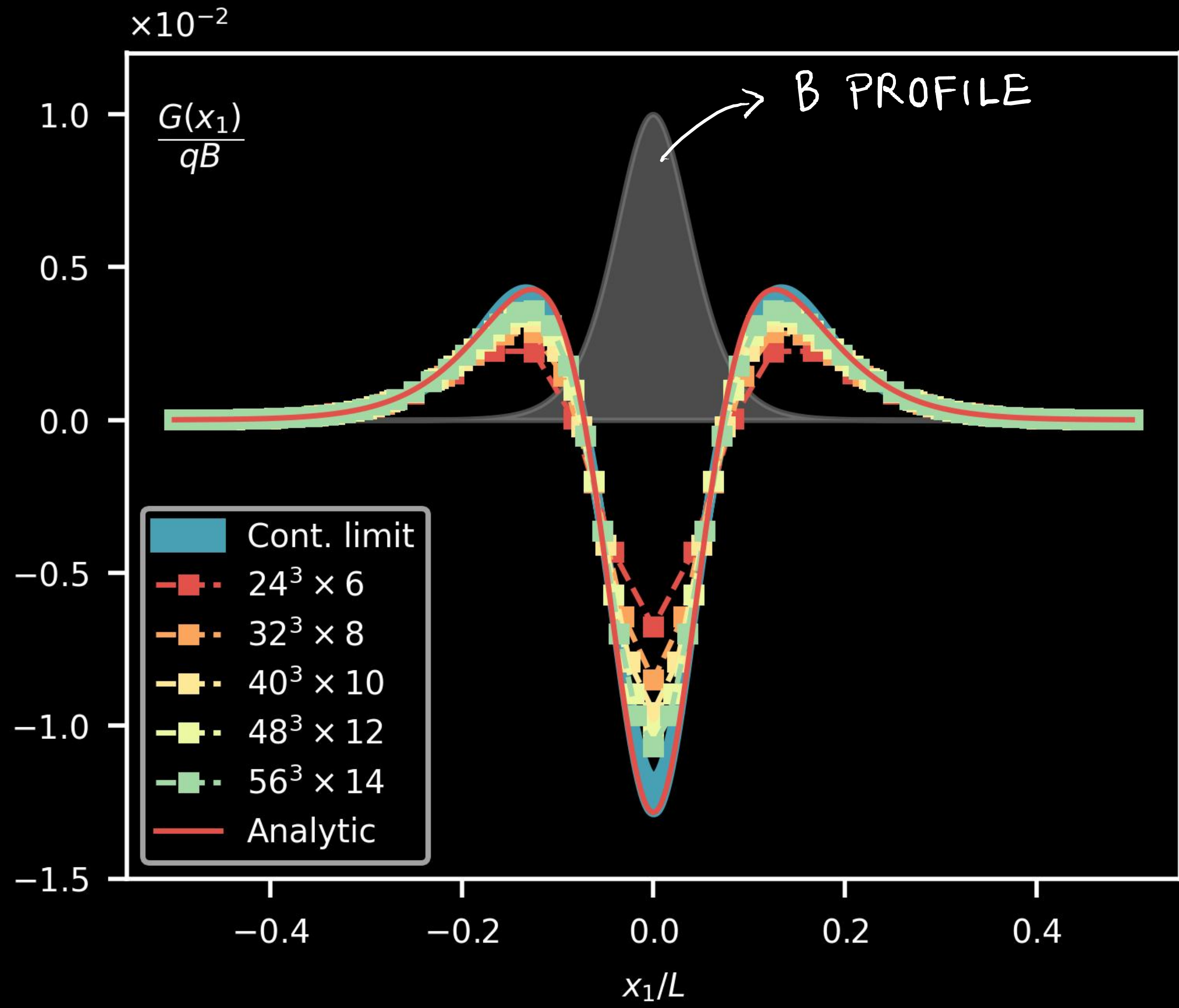

CME IN QCD

CME in QCD

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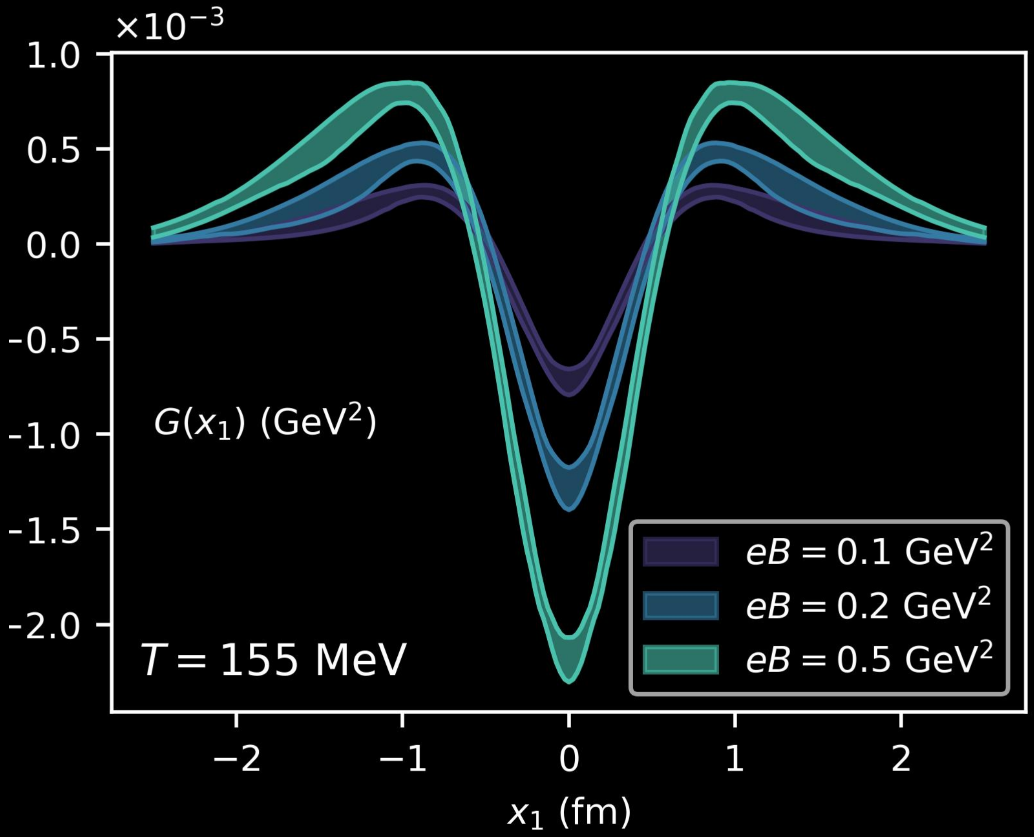

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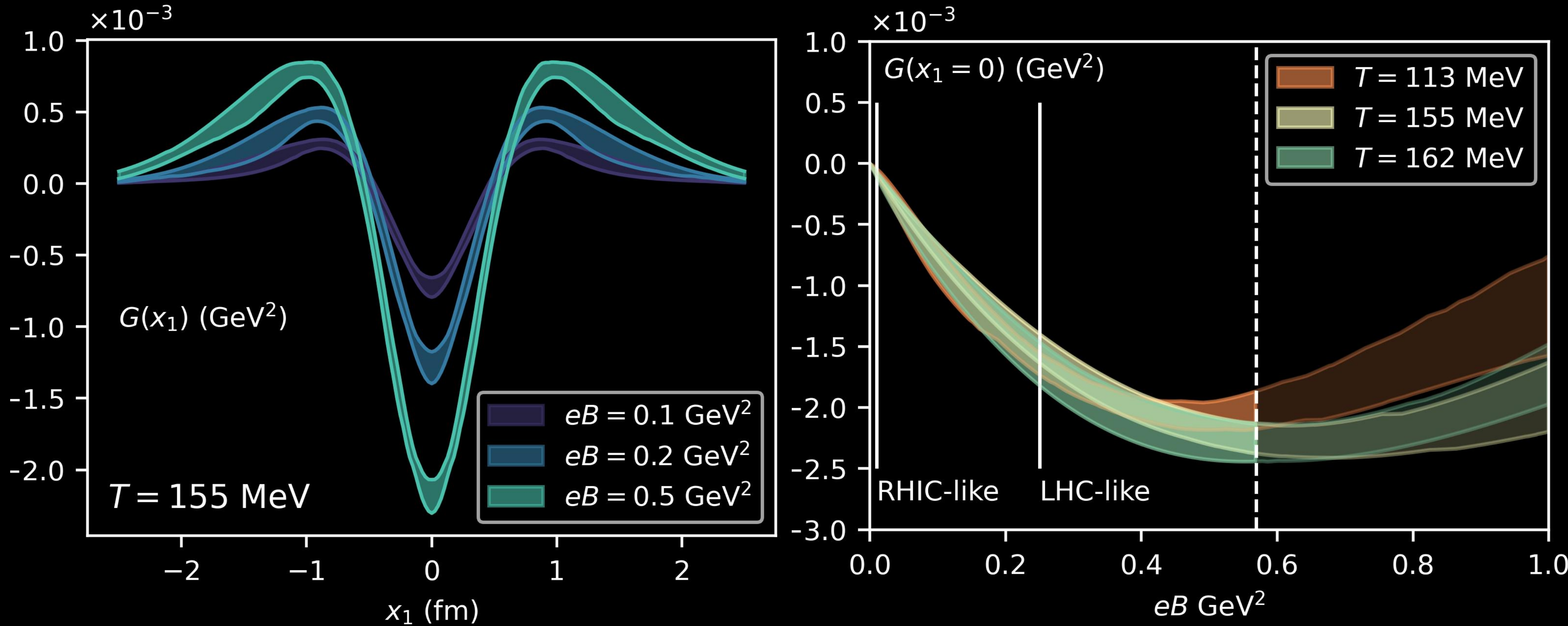

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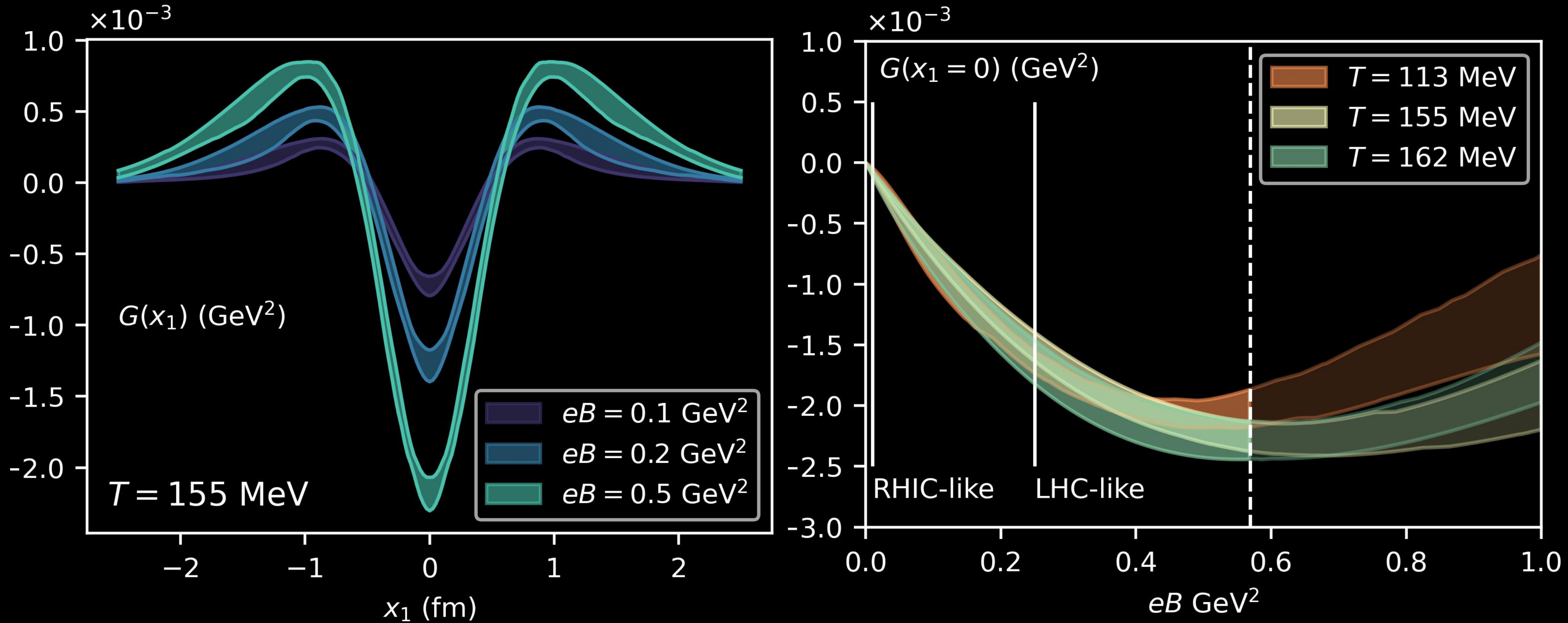

- J_3 FLOWS ALONG $x_3 \parallel B$

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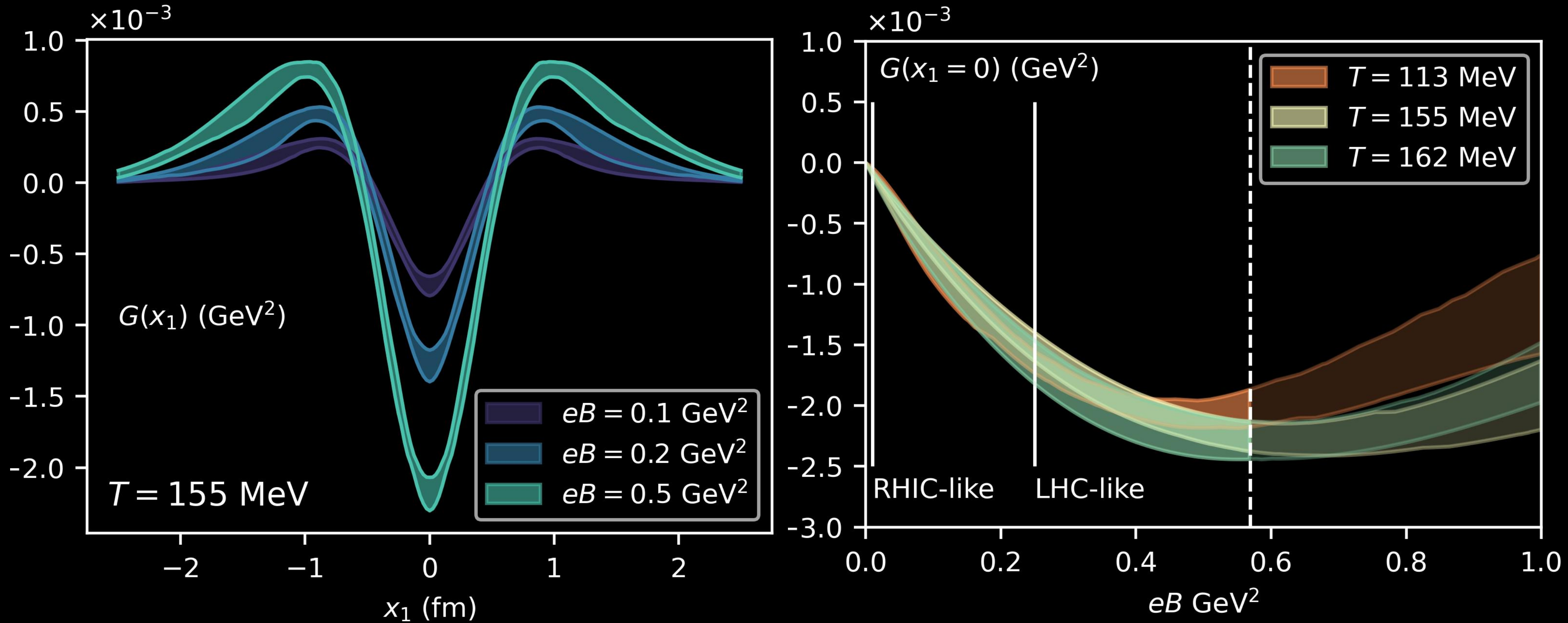
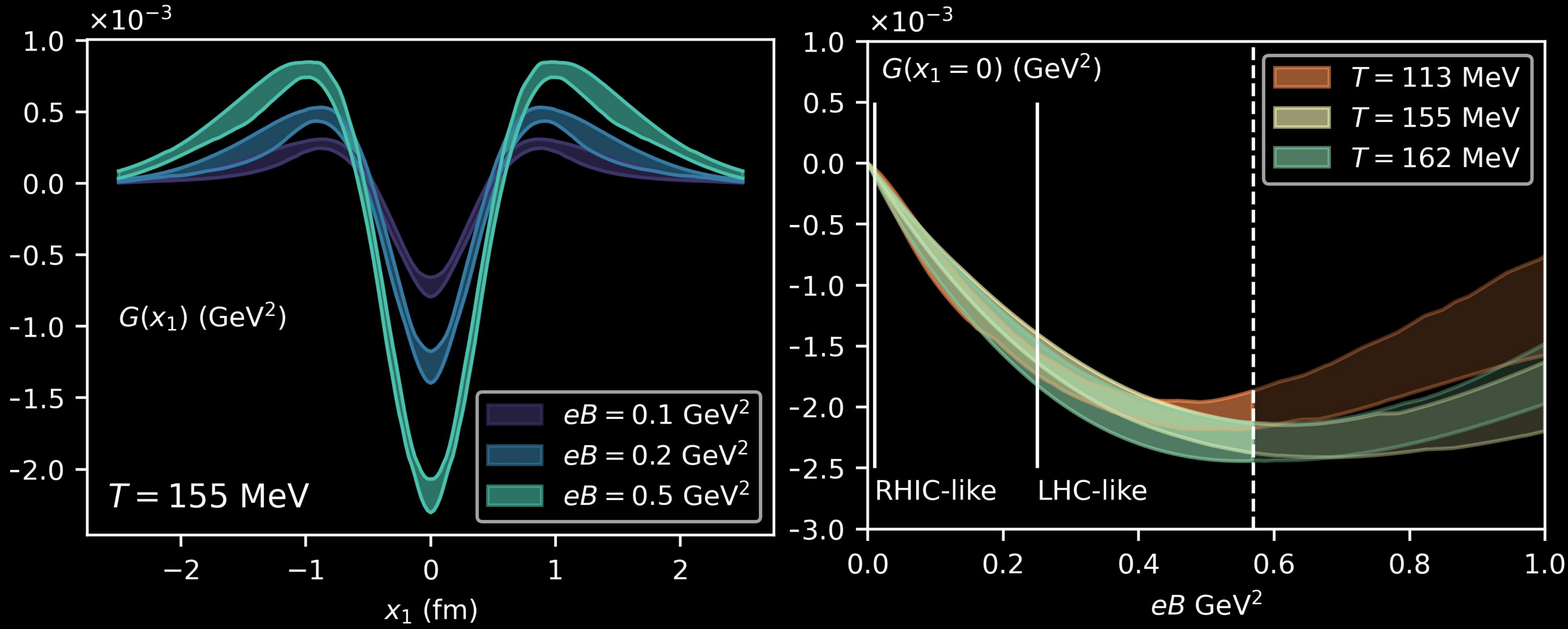


- J_3 FLOWS ALONG $x_3 \parallel B$
- J_3 IS INHOMOGENEOUS ALONG x_1

CME IN QCD

B. Brandt, G. Endrődi, E. Garnacho-Velasco, G. Markó, A. D. M. Valois (in prep.)



- J_3 FLOWS ALONG $x_3 \parallel B$
- J_3 IS INHOMOGENEOUS ALONG x_1
- $\int dx_1 \, G(x_1) = 0$

CME IN QCD

$$H(x_1, x_1') = \frac{T}{L^2} \frac{\delta \langle j_3(x_1) \rangle}{\delta \mu_5(x_1')} \bigg|_{\mu_5 = 0}$$

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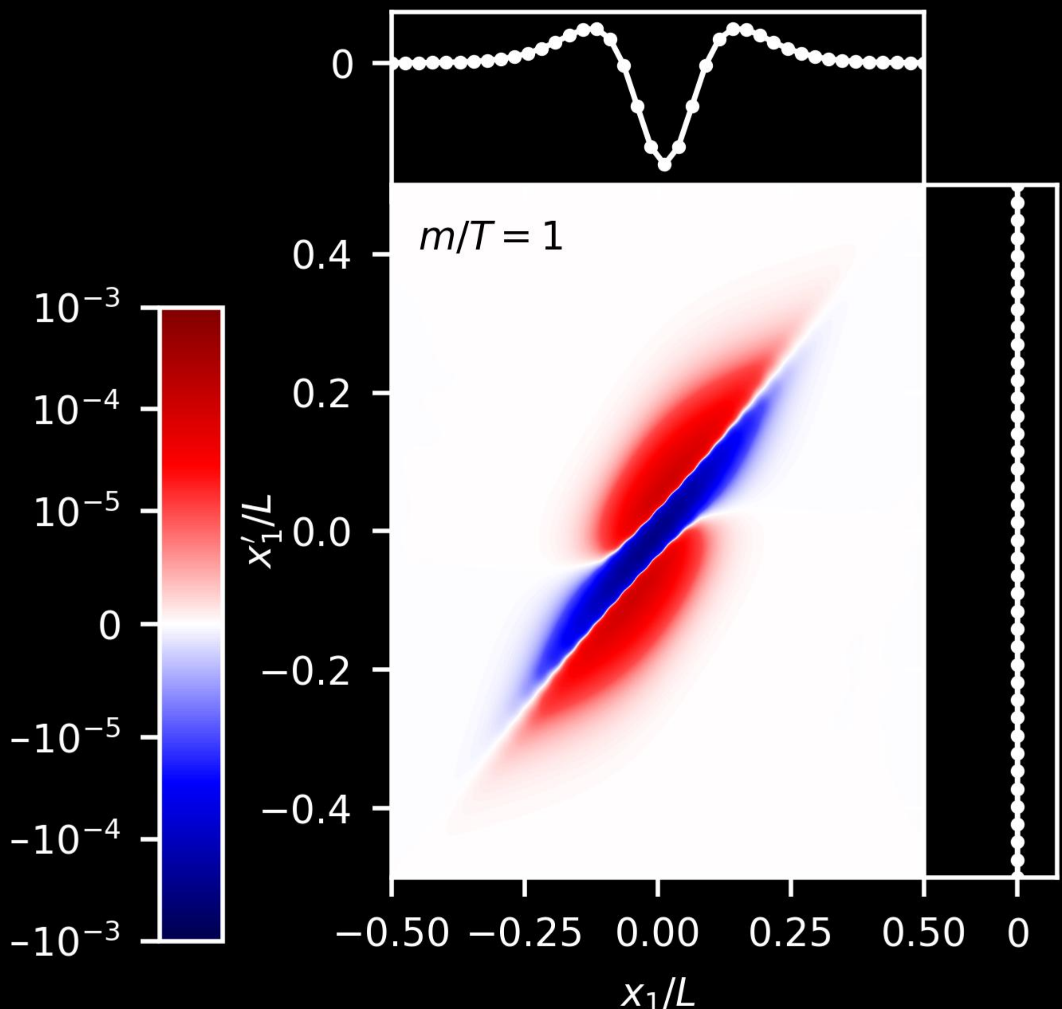

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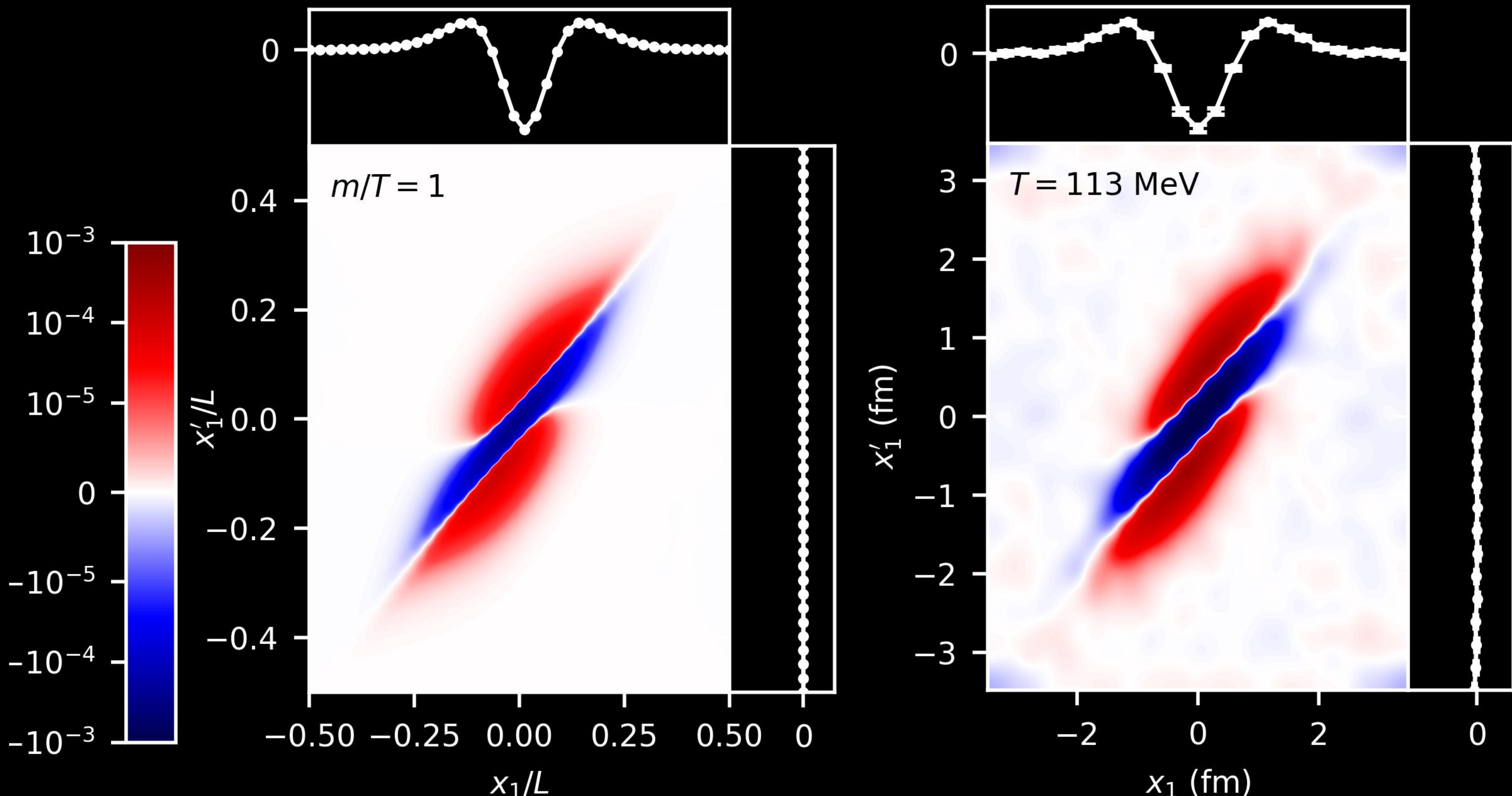

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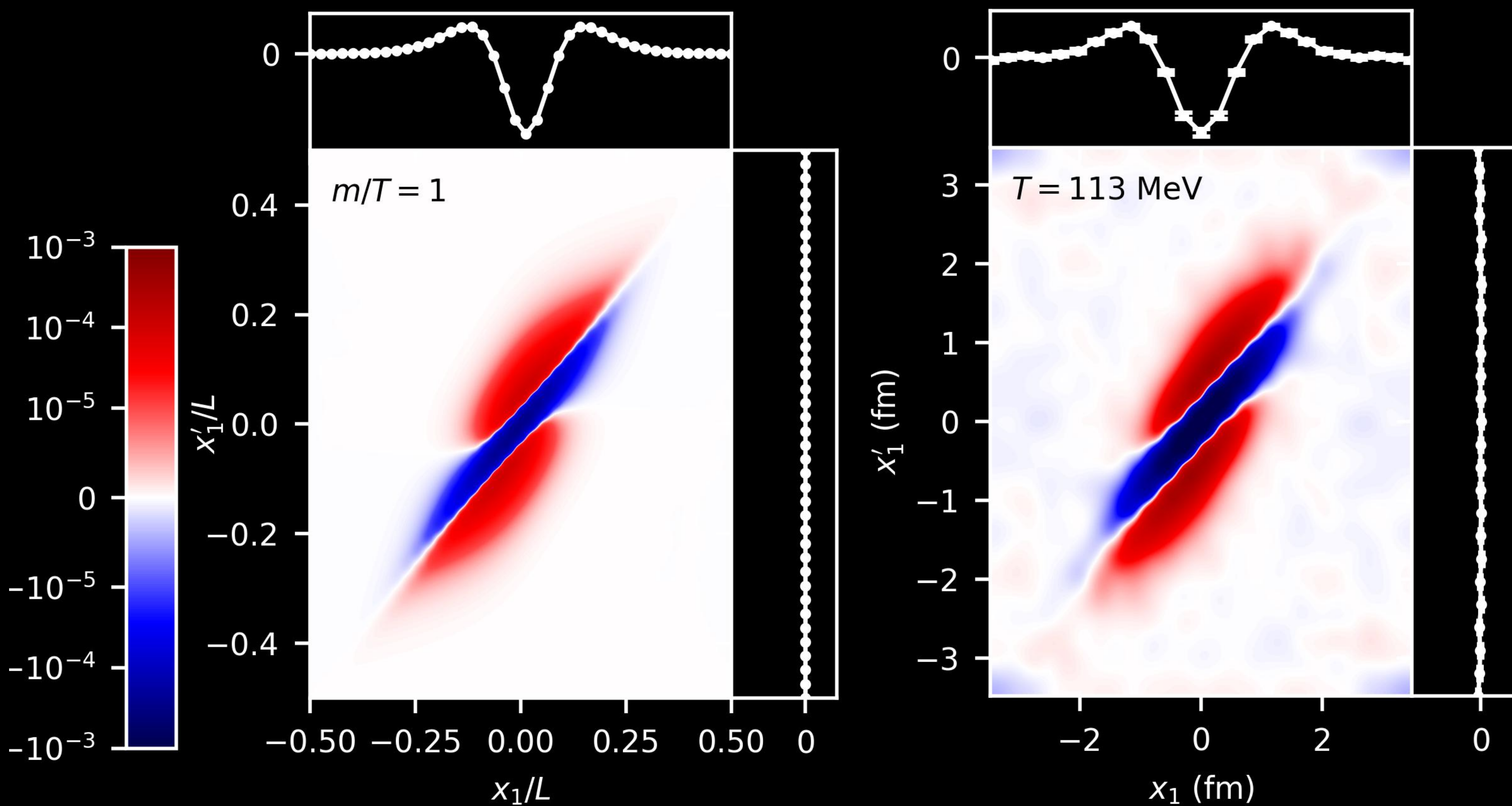

IF $\mu_5(x_1')$ IS KNOWN → CONVOLUTE $\mu_5(x_1')$ WITH $H(x_1, x_1')$

PART II –

OUT-OF-EQUILIBRIUM CME WITH UNIFORM B

LINEAR RESPONSE THEORY

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$$C_{CME}^{OUT} \sim \frac{1}{eB} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

KUBO FORMULA FOR CME

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E.G.: BACKUS-GILBERT BAYESIAN MEM GAUSSIAN

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P. V. Buividovich 2024

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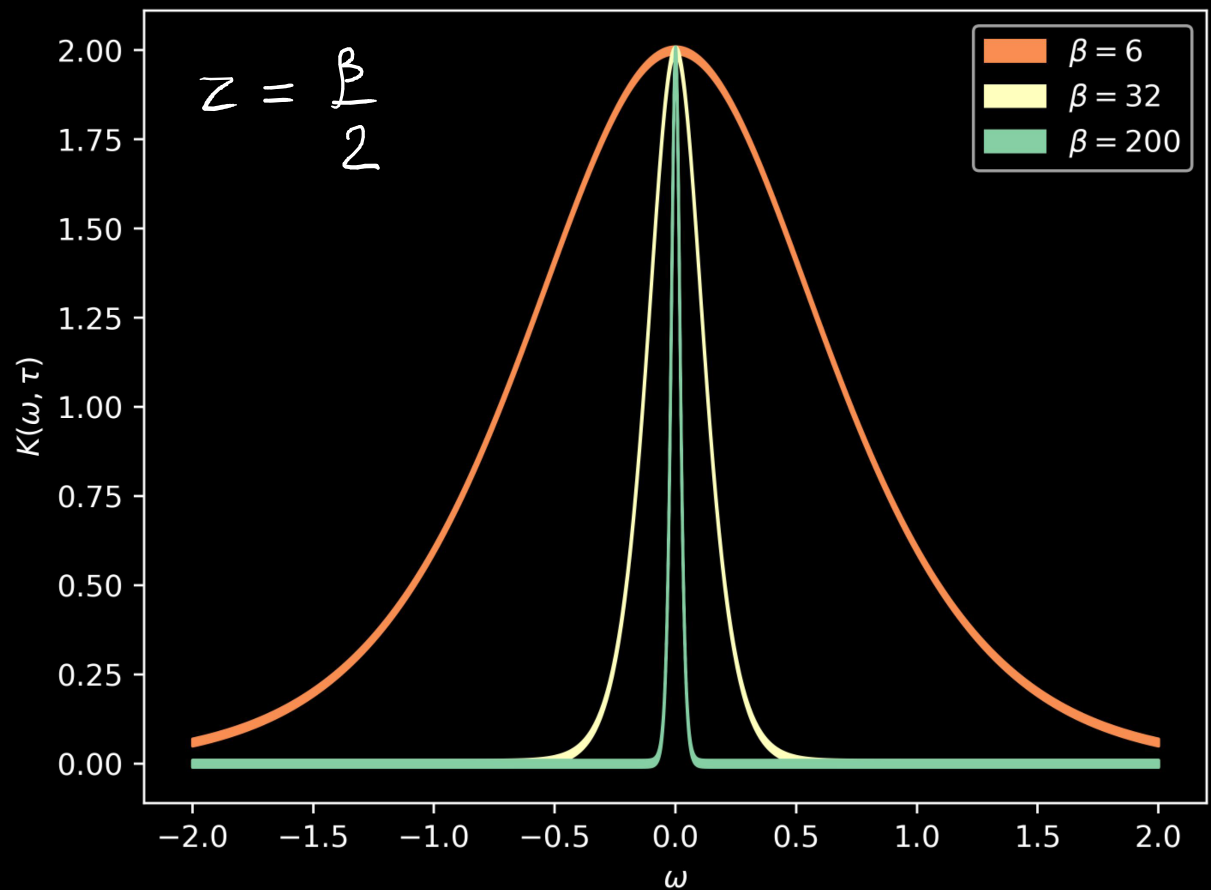

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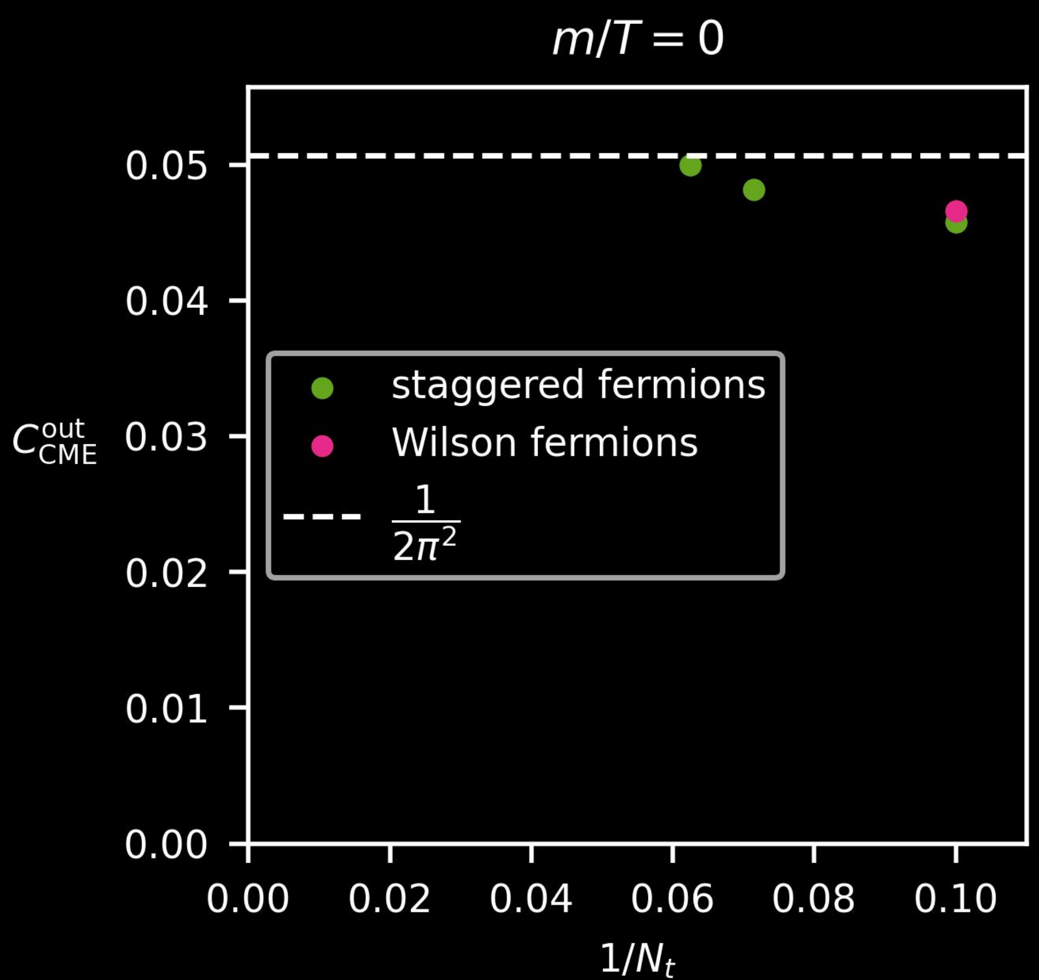

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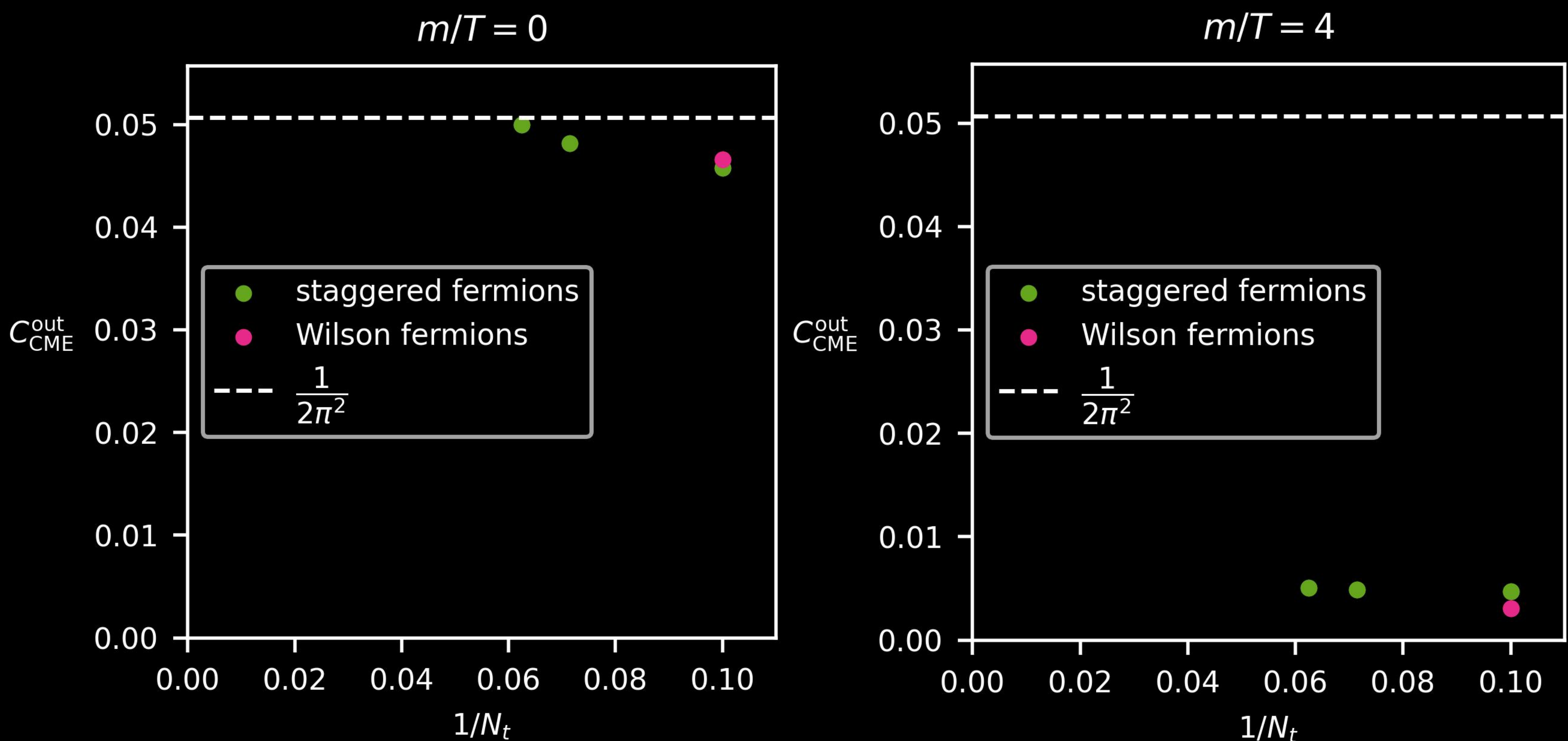

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OUT-OF-EQUI.

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