Crystalline Phases in QCD

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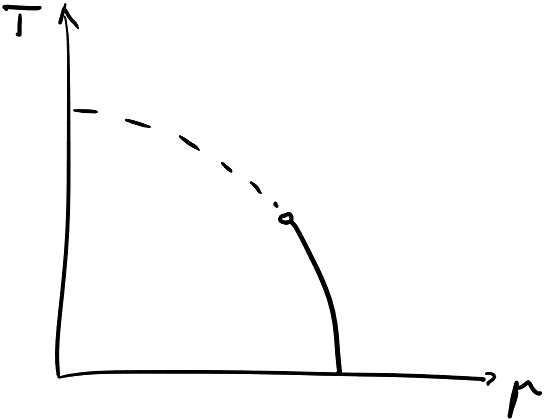
in collaboration with J. Bernhardt, M. Buballa & C.S. Fischer

Based on [Phys.Rev.D 108 (2023), arXiv:2406.00205]

Inhomogeneous Phases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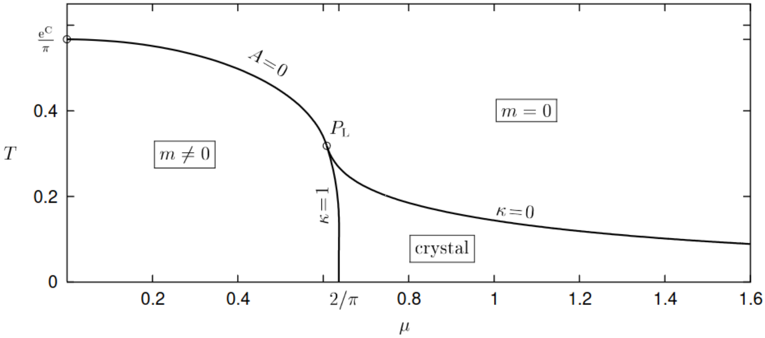
- In other words:
  Breaking of Chiral Symmetry and Translational Symmetry

Gross-Neveu Model

- The GN model in 1+1 dimensions

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\bar{\psi}\psi\bar{\psi}\psi$$

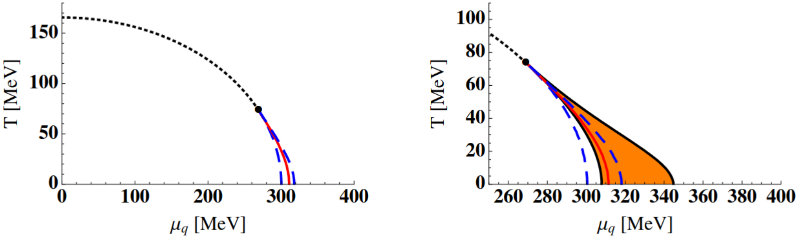


[M. Theis | J.Phys.A 39 (2006)]

Nambu-Jona–Lasinio Model

- The NJL model is similar. In 3+1 dimensions we have a scalar and a pseudoscalar contact vertex

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right)$$

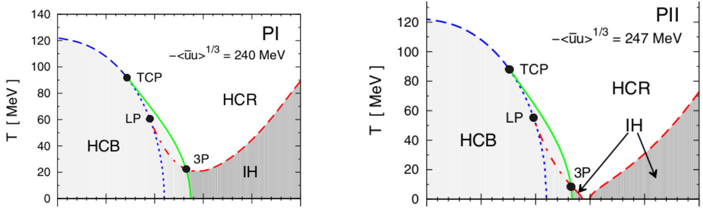


[D. Nickel | Phys.Rev.D 80 (2009)]

On Non-Locality

- Non-local NJL models or, non-local chiral quark models

$$\mathcal{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + \left(G(z)\bar{\psi}(x+z/2)\psi(x-z/2)\right)^2$$



[J. P. Carlomagno, et al | Phys. Rev. D 92, 056007 (2015)]

Ok... But Why Care?

- Could affect the EoS of neutron stars
  [S. Carignano, E. J. Ferrer, V. de la Incera, and L. Paulucci **PRD** (2015)]
  [M. Buballa and S. Carignano **EPJA** (2016)]
- If unstable, could have large life-times
  [J.P. Carlomagno and G. Krein **PRD** (2018)]
- Will leave signatures in HIC
  [R. Pisarsky and F. Rennecke **PRL** (2021)]

Moat Regimes in QCD and their Signatures in Heavy-Ion Collisions Fabian Rennecke

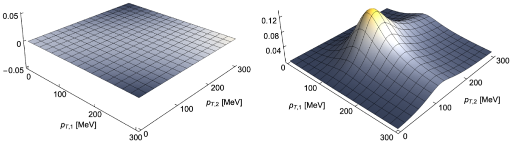


Figure 3: Connected two-particle correlation normalized with the spectra, $\Delta n_{12} = \langle (\frac{d^3N}{d\mathbf{p}^3})^2 \rangle_c / \langle \frac{d^3N}{d\mathbf{p}^3} \rangle^2$, as a function of the transverse momenta of the two particles. *Left:* Normal phase. *Right:* Moat regime.

Example: Mean-Field NJL

NJL

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left\{(\bar{\psi}\psi)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right\}$$

NJL

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$$\phi_S(x) = \langle\bar{\psi}(x)\psi(x)\rangle, \quad \phi_P(x) = \left\langle\bar{\psi}(x)i\gamma_5\tau^3\psi(x)\right\rangle$$

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⇓ Mean Field Free Energy ⇓

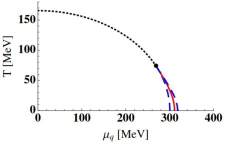
$$\Omega_{\mathrm{MF}}[\phi] = -\frac{T}{V}\,\mathrm{Tr}\log\left(\frac{S_0^{-1} + G\big(\phi_S(\mathsf{x}) + \phi_P(\mathsf{x})\big)}{T}\right) + G\frac{1}{V}\int d^3x\left(\phi_S^2(\mathsf{x}) + \phi_P^2(\mathsf{x})\right)$$

- Take the homogeneous case

$$\Omega_{\mathrm{MF}}(\bar{\phi}) = -\frac{T}{V}\,\mathrm{Tr}\log\left(\frac{S_0^{-1} + G(\bar{\phi}_S + \bar{\phi}_P)}{T}\right) + G\left(\bar{\phi}_S^2 + \bar{\phi}_P^2\right)$$

- Find the stationary points

$$\frac{\partial \Omega}{\partial \bar{\phi}} = 0 \qquad \Rightarrow$$



Stability Analysis

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$$\phi_S(x) = \bar{\phi}_S + \delta\phi_S(x)$$

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$$\Omega_{\mathrm{MF}} = \sum_{n=0}^{\infty} \Omega^{(n)}, \qquad \Omega^{(n)} \propto \mathcal{O}(\delta\phi_S^n)$$

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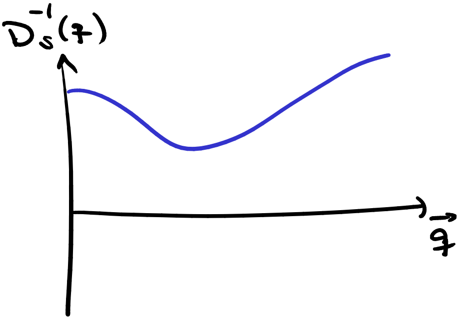
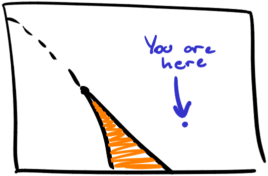
$\Downarrow$ Leading Order $\Downarrow$

$$\Omega^{(2)} = \frac{2G^2}{V} \int_{\vec{q}} |\delta\phi_S(\vec{q})|^2 D_S^{-1}(\vec{q})$$

Stability Analysis





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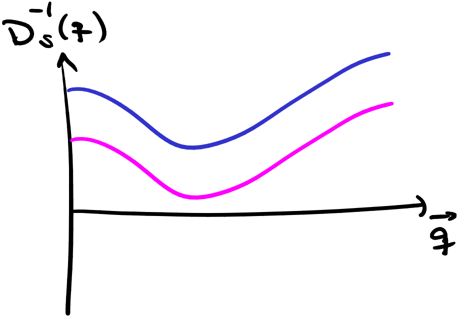

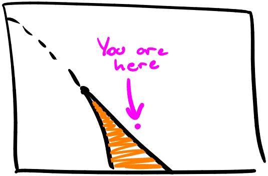

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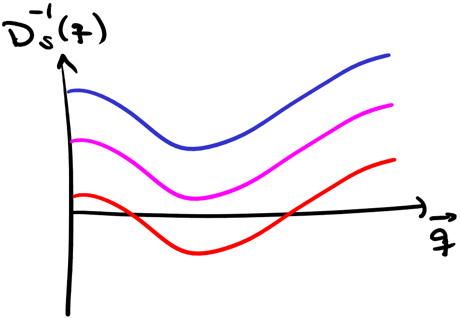
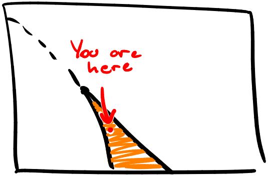




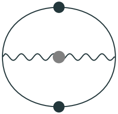
Towards QCD: 2PI Stability Analysis

The Analysis

The Analysis

- Start from a 2PI effective action

$$\Omega[S] = \mathrm{Tr}\,\log\left[S\right] + \mathrm{Tr}\left[\mathbf{1} - S_0^{-1}S\right] + \frac{1}{2}$$



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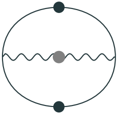

- Expand around homogeneous *propagator*

$$S(k_1, k_2) = \bar{S}(k_1)\delta(k_1 - k_2) + \delta S(k_1, k_2)$$

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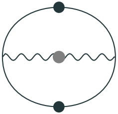

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- Calculate leading order term

$$\Omega^{(2)}[\delta S] = \mathrm{Tr}\left[\overline{\frac{\delta^2\Omega}{\delta S \delta S}}\delta S \delta S\right] \quad \neq \sum\!\!\!\!\!\!\int |\delta S|^2 \times f(k_1, k_2)$$

Dyson-Schwinger Equations

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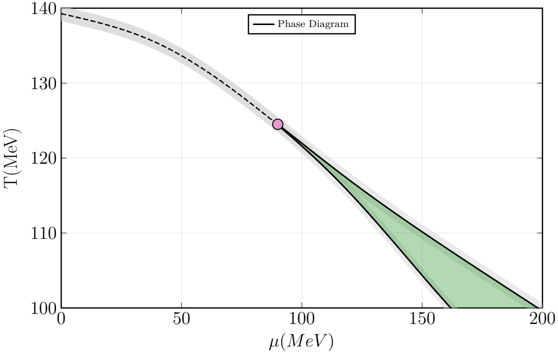
- This is the the quark's Dyson-Schwinger Equation!

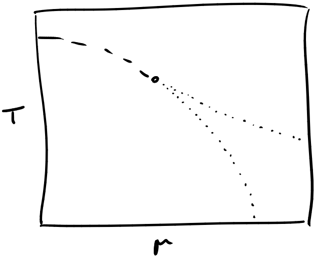
- For instance... Take the Qin-Chang model

$$= \left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] \times \lambda e^{q^2/\omega^2}$$



The Analysis

- Non trivial parts of the analysis
  - Must use a test-function δS
  - We employ a variational method. Pick a test-function with some free parameters and optimise parameters.
  - If the parameter space of our test-function is too small, we will get **wrong** results. Good to have a test!



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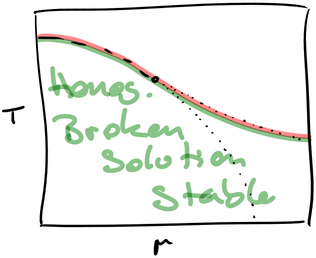

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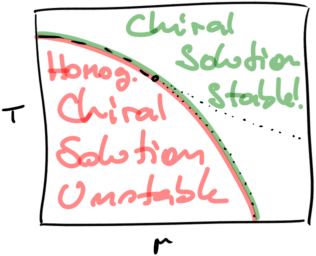
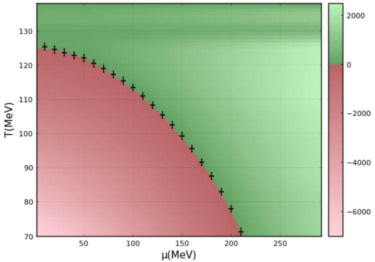
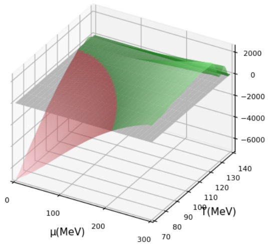


The Spinodal Test

- Let's probe *homogeneous* chiral symmetry breaking

$$S(k) = S(k)_{\text{chiral}} + \delta S(k)_{\text{breaks chiral symmetry}}$$





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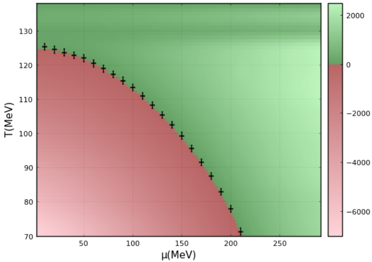
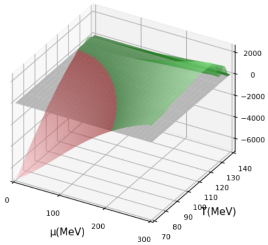


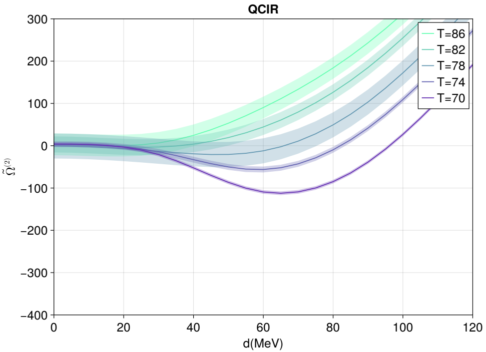


Now we can break translationa symmetry as well

$$\delta S(k) \to \delta S(k_1, k_2)$$

Towards the Full Analysis

- Away from the homogeneous case (where $d = k_1 - k_2 = 0$), we don't have a test.
- Therefore, what we can be *sure* of is, the stability at the edge of the homogeneous Wigner-Weyl "phase boundary"



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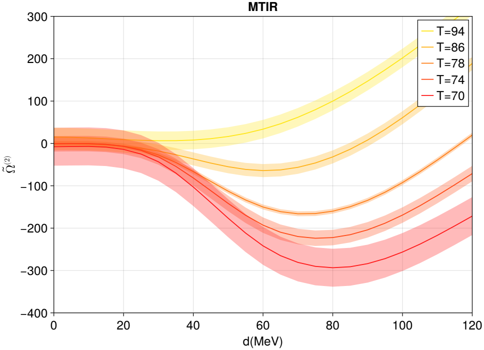
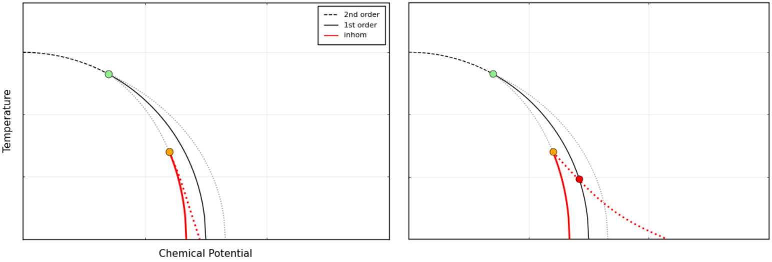


Conclusions



- The instability at the chiral phase stability boundary exists!
- It certainly persists for larger μ if T is lower
- Therefore, the two possible outcomes are, either it crosses the 1st order phase transition, and it is stable, or it doesn't
- This is a *very* conservative reading of our results!


Mundus Minimarum Particularum
CERN navem capere pisces
RHIC navem capere pisces
Immensus et terribilis ignotus Oceanus
Punctum Criticum
Magna Gluon Pulmenti
Nostra Materia Mundi
Hadrons Palus
Incertus Terra
Neutron Siderum
Temperatus
Potentia chemica

backup

Quantum Fluctuations

- The Landau-Peierls instability states that 1-dimensional condensates in 2^+ spacial dimensions are unstable. Due to transverse **phonon** fluctuations.

$$\text{Crystal} \rightarrow \text{Liquid Crystal}$$

- The Pisarski-Tsvelik-Valgushev (PTV) instability states that transverse bosonic fluctuations by Nambu-Goldstone bosons from **flavour** symmetry breaking disorder these phases.

$$\text{Crystal} \rightarrow \text{Quantum Pion Liquid}$$

- See Refs by Pisarsky, Renneke, Hidaka, de la Incera, Ferrer, etc

Ansatz in NJL

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- Chiral Density Wave:

$$\phi_S(\vec{x}) = -\frac{\Delta}{2G_S}\cos(\vec{q}\cdot\vec{x}), \qquad \phi_P(\vec{x}) = -\frac{\Delta}{2G_P}\sin(\vec{q}\cdot\vec{x})$$

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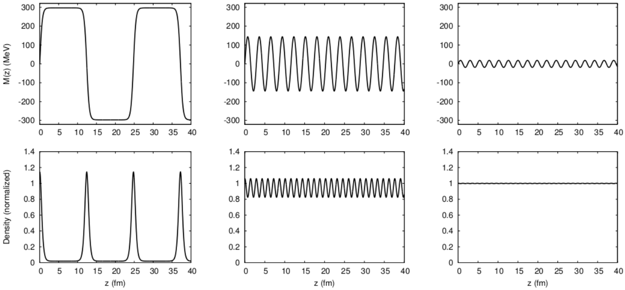

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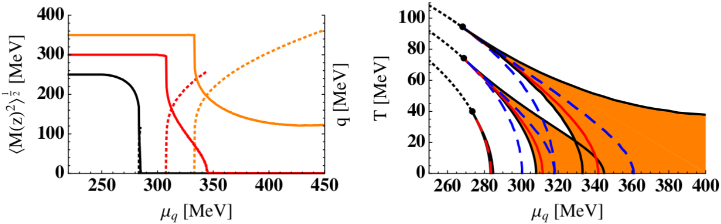

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