Invited talk at “XVIth Quark confinement and the Hadron Spectrum”, Cairns, 21 Aug 2024

QCD data-driven holographic modeling

Koji Hashimoto (Kyoto U.)

“AI for an inverse problem: Physical model solving quantum gravity” Workshop at ICML 2024
w/ K.Matsuo, M. Murata, G.Ogiwara (Saitama Tech), D.Takeda (Kyoto)

“Deriving dilaton potential in improved holographic QCD from chiral condensate” 2209.04638

“Deriving dilaton potential in improved holographic QCD from meson spectrum” 2108.08091
w/ K.Ohashi (Keio), T.Sumimoto (Osaka u)

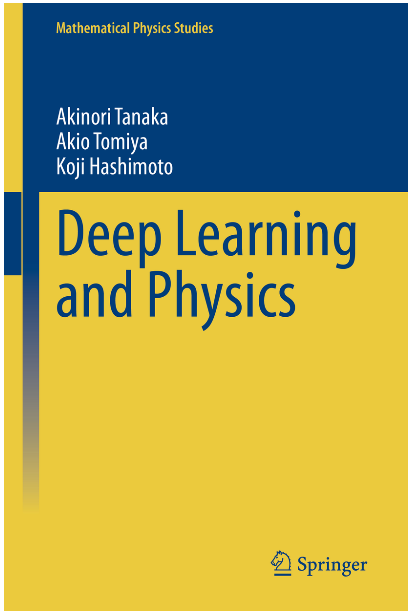
“Neural ODE and Holographic QCD” 2006.00712
w/ H.Y.Hu, Y.Z.You (UCSD)

“Deep Learning and AdS/QCD” 2005.02636
w/ T. Akutagawa, T. Sumimoto (Osaka u)

“Deep Boltzmann Machine and AdS/CFT” 1903.04951

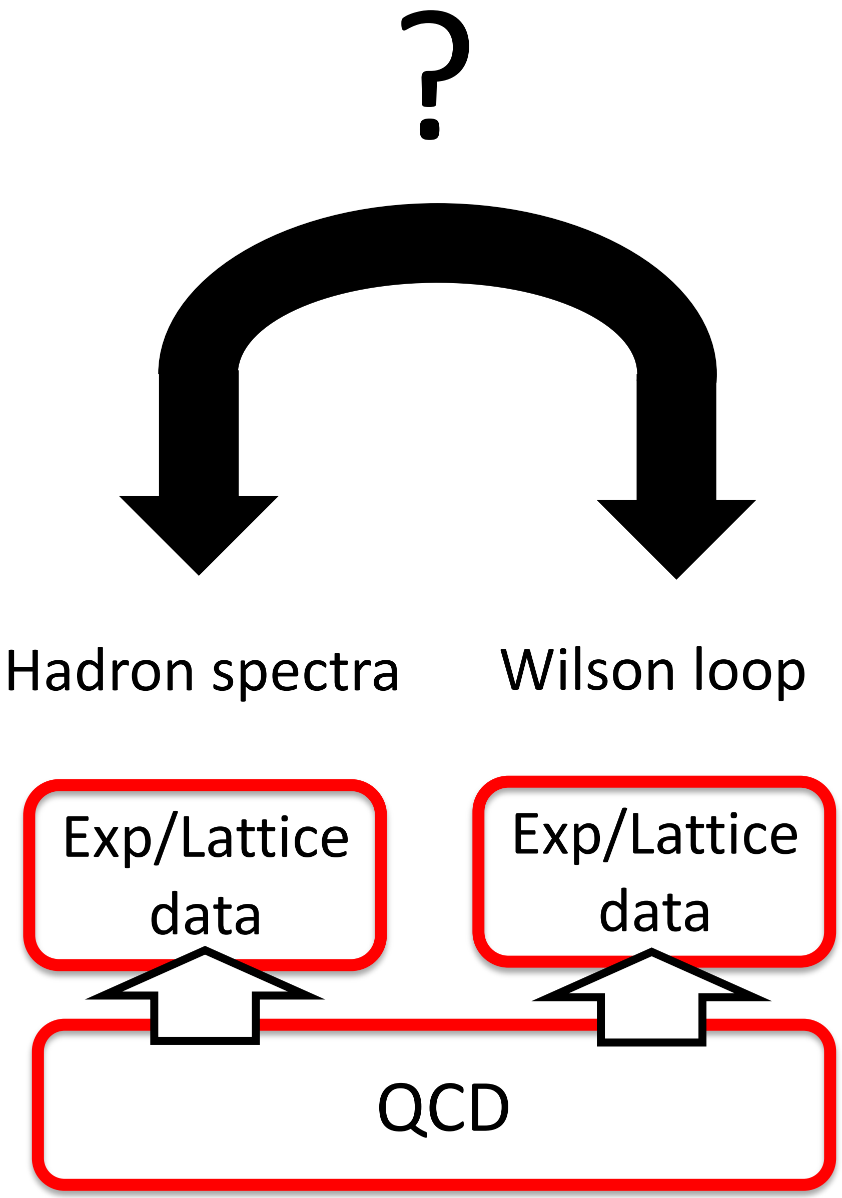
“Deep Learning and Holographic QCD” 1809.10536
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)

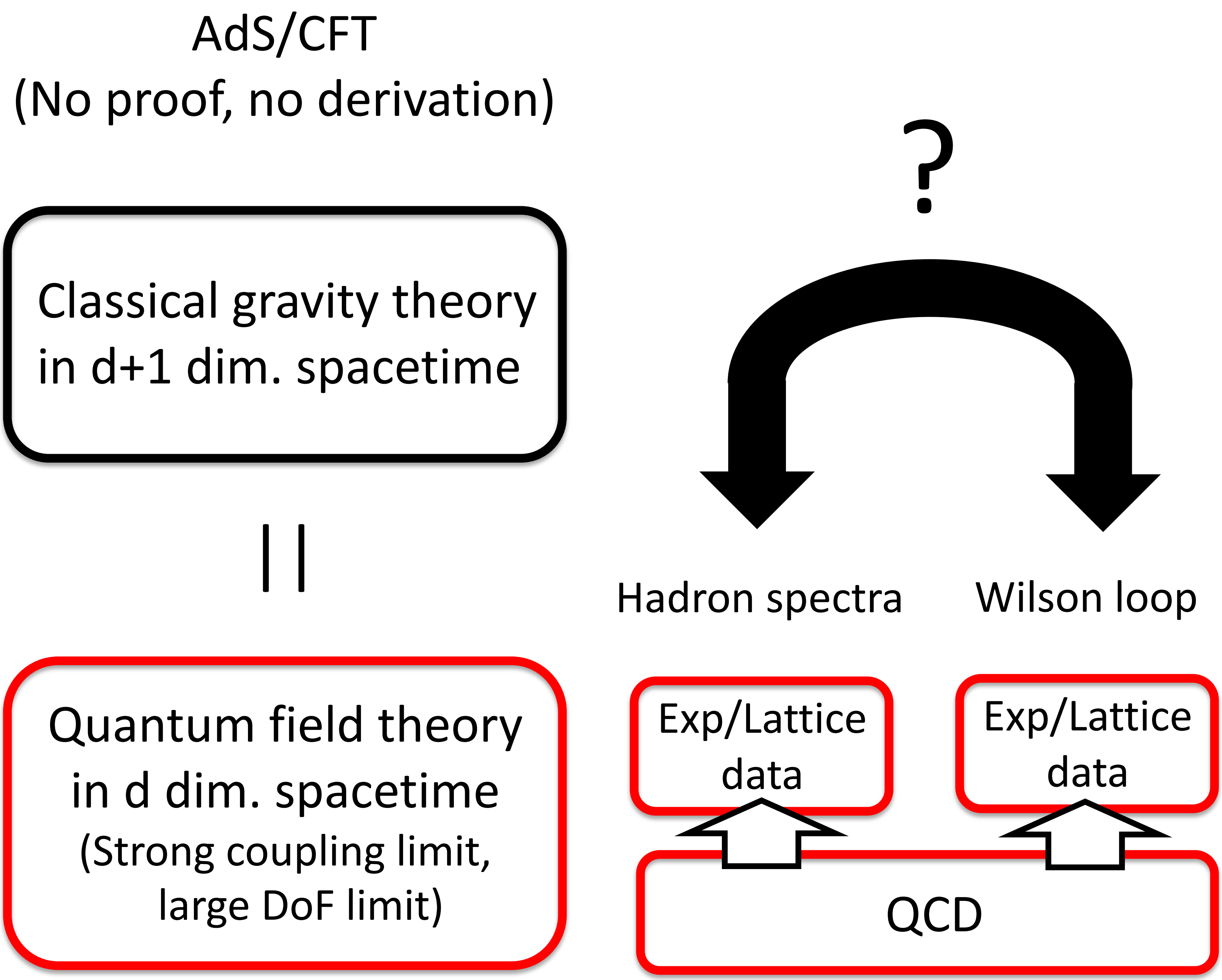
“Deep Learning and AdS/CFT” 1802.08313
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)



Bulk reconstruction by deep learning

?
Quark confinement ⇔ Hadron spectrum


?
Hadron spectra
Wilson loop
Exp/Lattice data
Exp/Lattice data
QCD


AdS/CFT
(No proof, no derivation)
Classical gravity theory in d+1 dim. spacetime
||
Quantum field theory in d dim. spacetime
(Strong coupling limit, large DoF limit)
?
Hadron spectra
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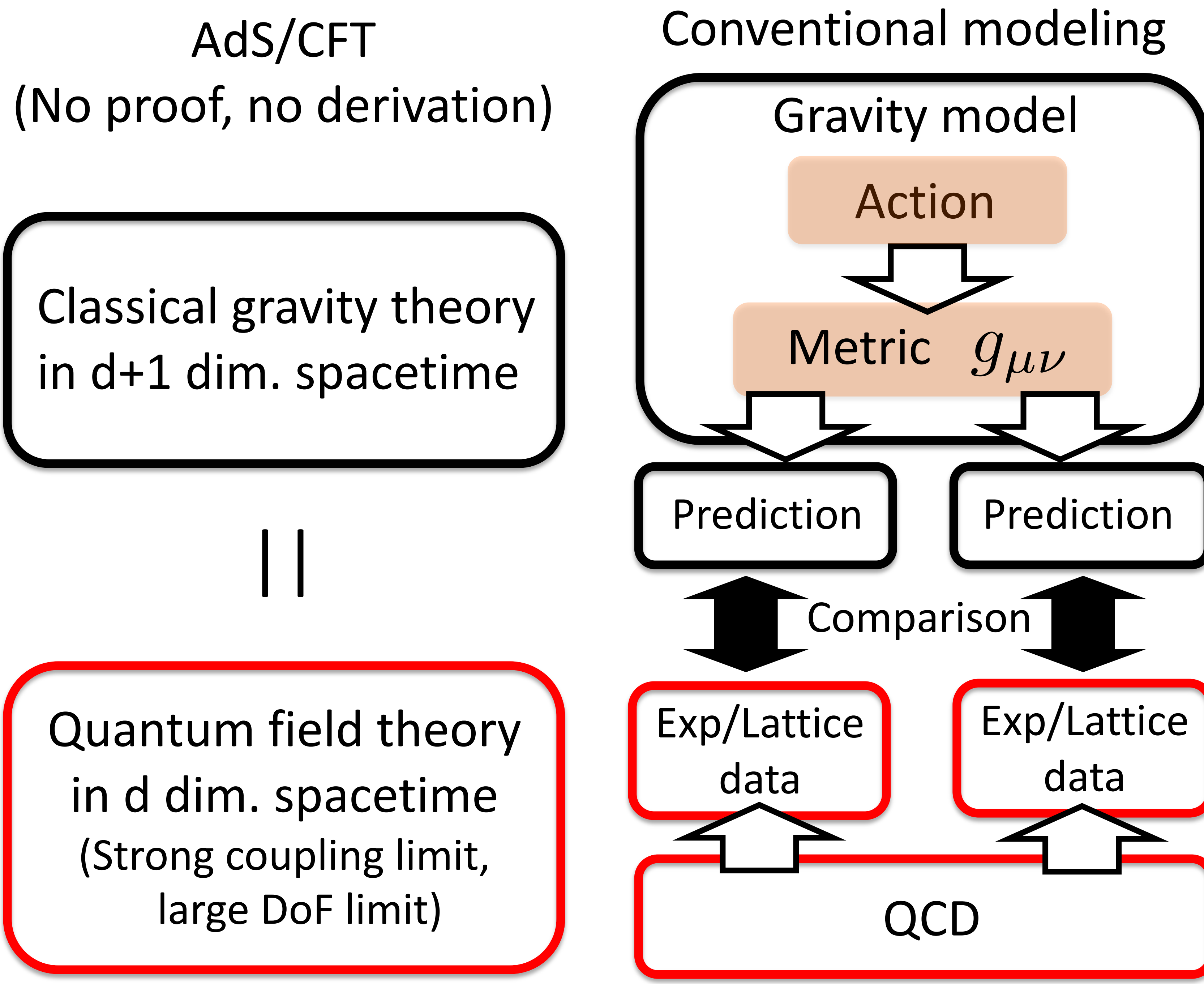
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Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
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Prediction
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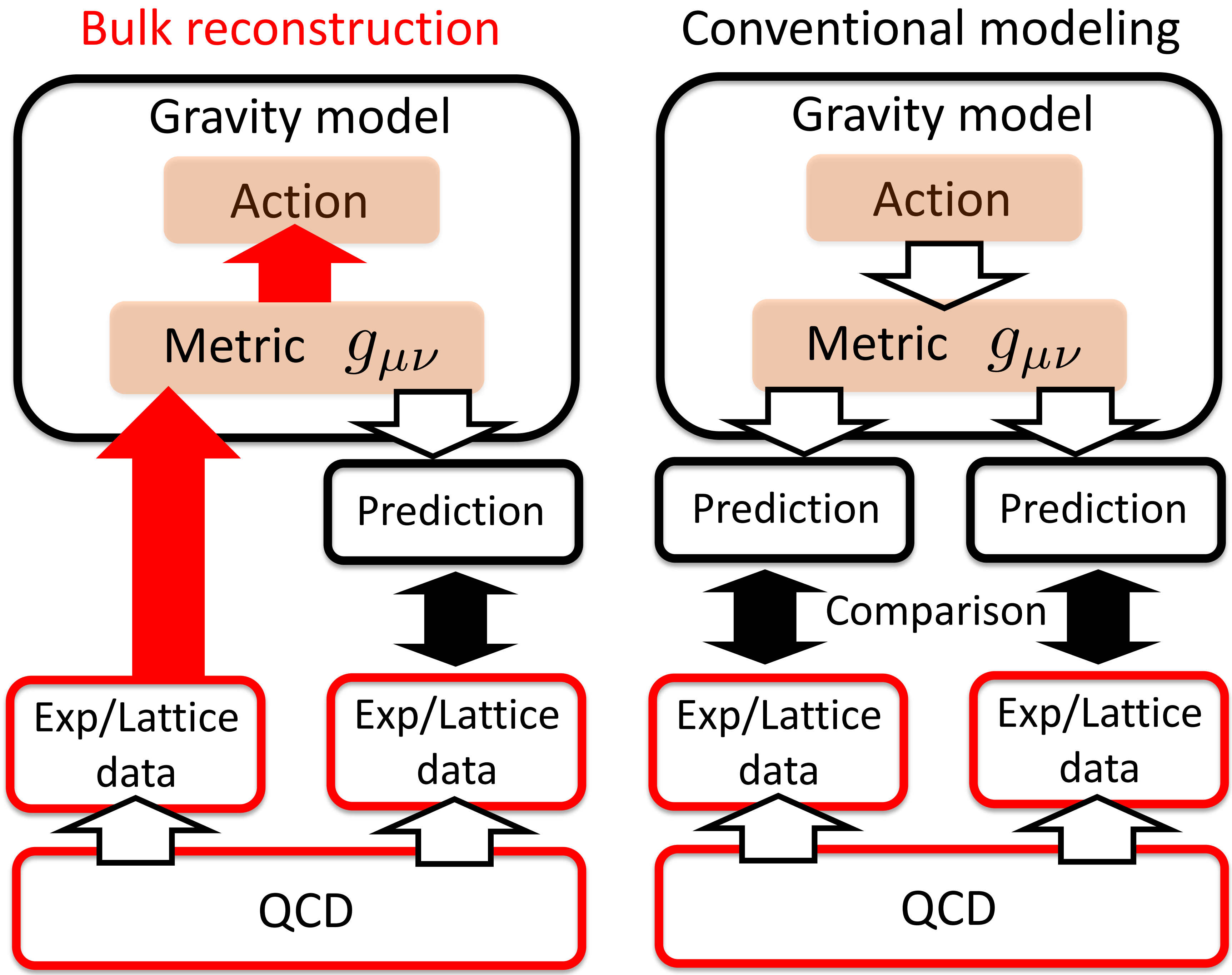

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Comparison of solvers

Reconstruction method	No use of Einstein eq	Lattice input
Holographic renormalization [deHaro Solodukhin Skenderis 00]		✓
Entanglement, Complexity [Hammersley 07] [Bilson 08]… [KH Watanabe 21]	✓	
Correlators [Hammersley 06] [Hubeny Liu Rangamani 06]	✓	
AdS/DL [KH Tanaka Tomiya Sugishita 18]	✓	✓
Wilson loop [KH 20]	✓	✓

Cf. Matti Järvinen’s talk on V-QCD

Emergent spacetime as a neural network

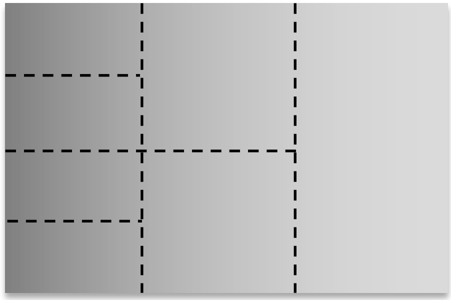
Quantum
gravity
in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

||

Quantum
mechanics
in d-dim.

Anti de Sitter
spacetime



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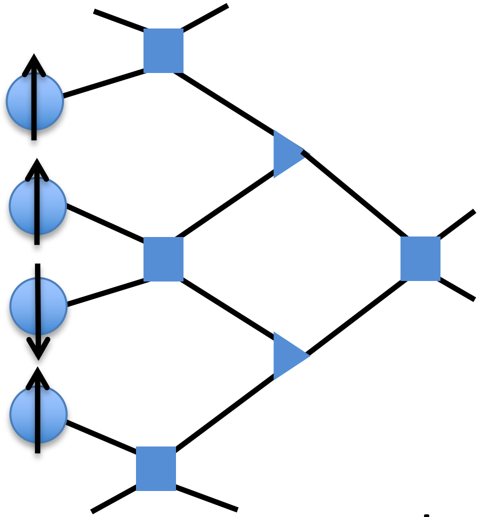
Swingle `10



Tensor network

Emergent spacetime as a neural network

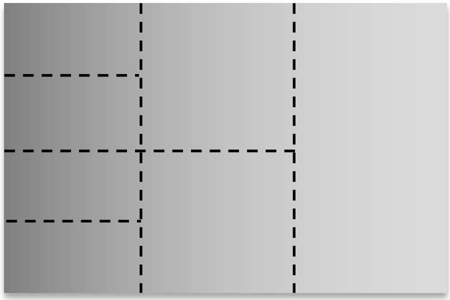
Quantum gravity in (d+1)-dim.

'tHooft `93
Susskind `94
Maldacena `97

||

Quantum mechanics in d-dim.

Anti de Sitter spacetime



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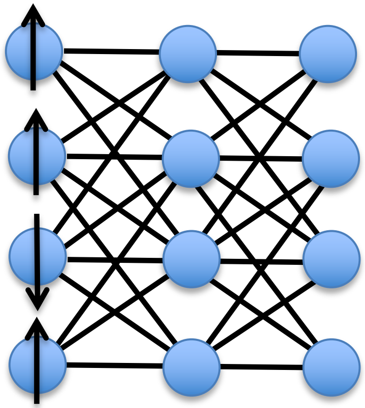
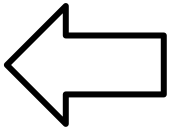
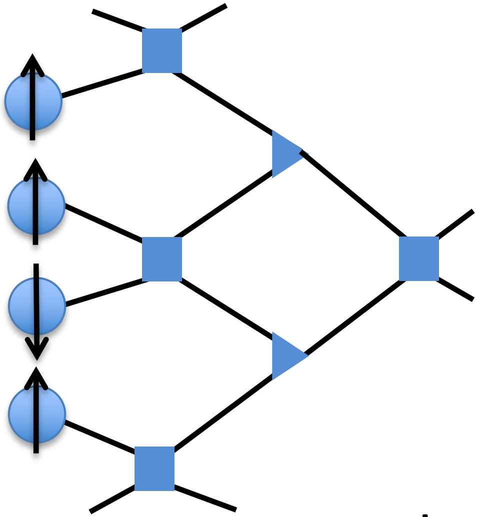
Swingle `10



Neural network



Carleo, Troyer `17



Tensor network

Emergent spacetime as a neural network

Quantum gravity in (d+1)-dim.

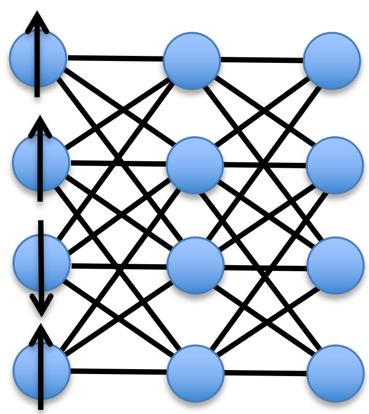
'tHooft `93
Susskind `94
Maldacena `97

||

Quantum mechanics in d-dim.

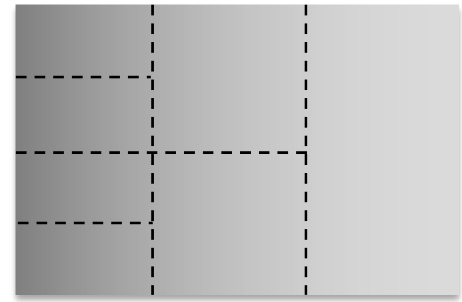
General spacetime

|| ?

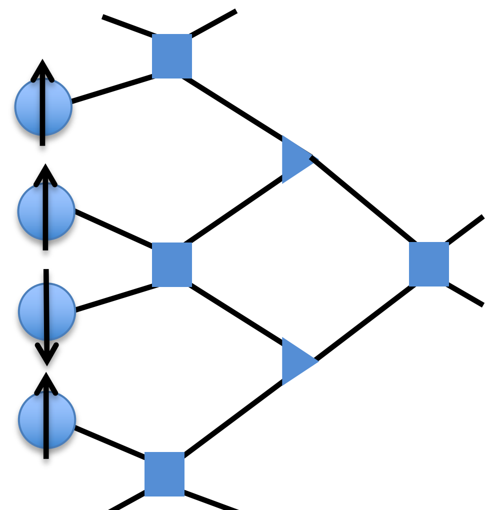


Neural network

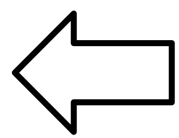
Anti de Sitter spacetime



|| Swingle `10



Tensor network



Carleo, Troyer `17

Bulk reconstruction by deep learning

② Space emergent from data

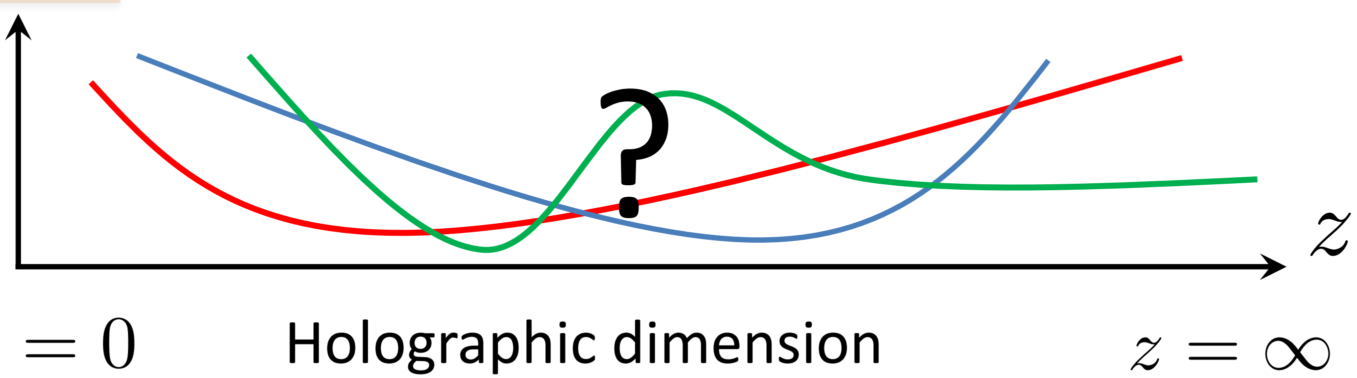
holographic QCD model for meson spectra

[Karch, Kaz, Son, Stephanov `06]

Vector meson spectra are eigenvalues $\omega^2 = m_n^2$ of normalizable solutions of the differential eq:

$$\frac{\partial}{\partial z}\left(e^{-B(z)}\frac{\partial}{\partial z}v_n(z)\right) + \omega^2 e^{-B(z)} v_n(z) = 0$$

$B(z)$: metric in the emergent dimension



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$$\frac{\partial}{\partial z}\left(e^{-B(z)}\frac{\partial}{\partial z}v_n(z)\right) + \omega^2 e^{-B(z)}v_n(z) = 0$$

Model : Classical 5-d gauge theory in unknown dilaton gravity b.g.

$$S = \int d^4x dz\, e^{-\Phi}\sqrt{-g}\,(F_{MN})^2$$

Dilaton $\Phi(z)$, metric $\quad ds^2 = e^{2A(z)}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

AdS boundary ($z \sim 0$) : $B(z) \equiv \Phi(z) - A(z) \sim \log z$

Solve EoM for the gauge field $A_\mu(z, x^\mu) = v_n(z)\rho_\mu(x^\mu)$

② Space emergent from data

Bring the bulk EoM to neural network

2005.02636

Bulk EoM $$\frac{\partial}{\partial z}\left(e^{-B(z)}\frac{\partial}{\partial z}v_n(z)\right)+m_n^2 e^{-B(z)}v_n(z)=0$$

⇩

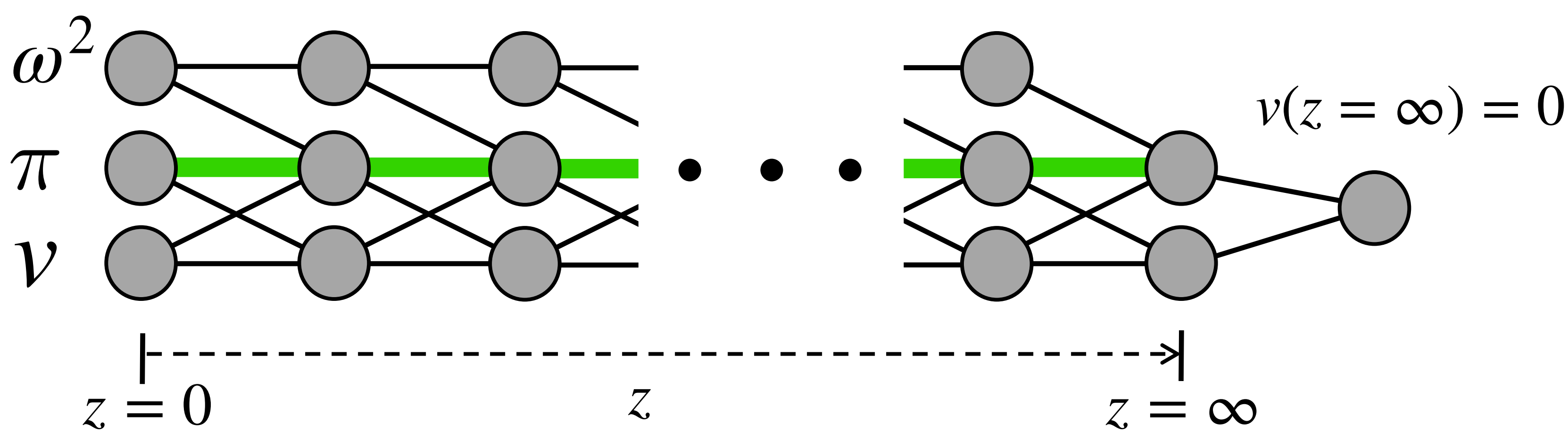
Discretization

Hamilton form

$$\begin{cases} v_n(z+\Delta z) = v_n(z) + \Delta z\, \pi_n(z) \\ \pi_n(z+\Delta z) = \pi_n(z) + \Delta z \left(B'(z)\pi_n(z) - \omega^2 v_n(z)\right) \end{cases}$$

⇩

Neural-Network representation



② Space emergent from data

Training with QCD data: hadron spectra

2005.02636

Data : PDG data for rho meson mass

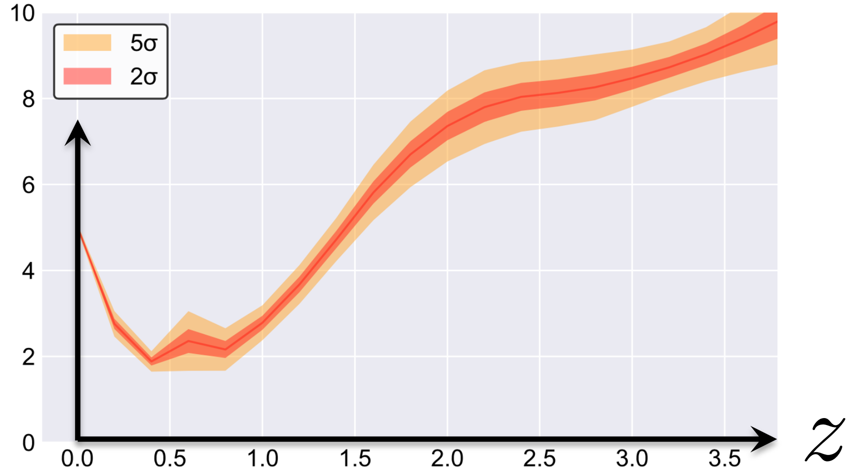
$$m_\rho^{(1)} = 0.77 \text{ GeV}, \; m_\rho^{(2)} = 1.45 \text{ GeV}$$

Deep learning

Result :

Emergent metric

$$B'(z) = \Phi'(z) - A'(z)$$



Bulk reconstruction by deep learning

1. Why and how? — 6 pages

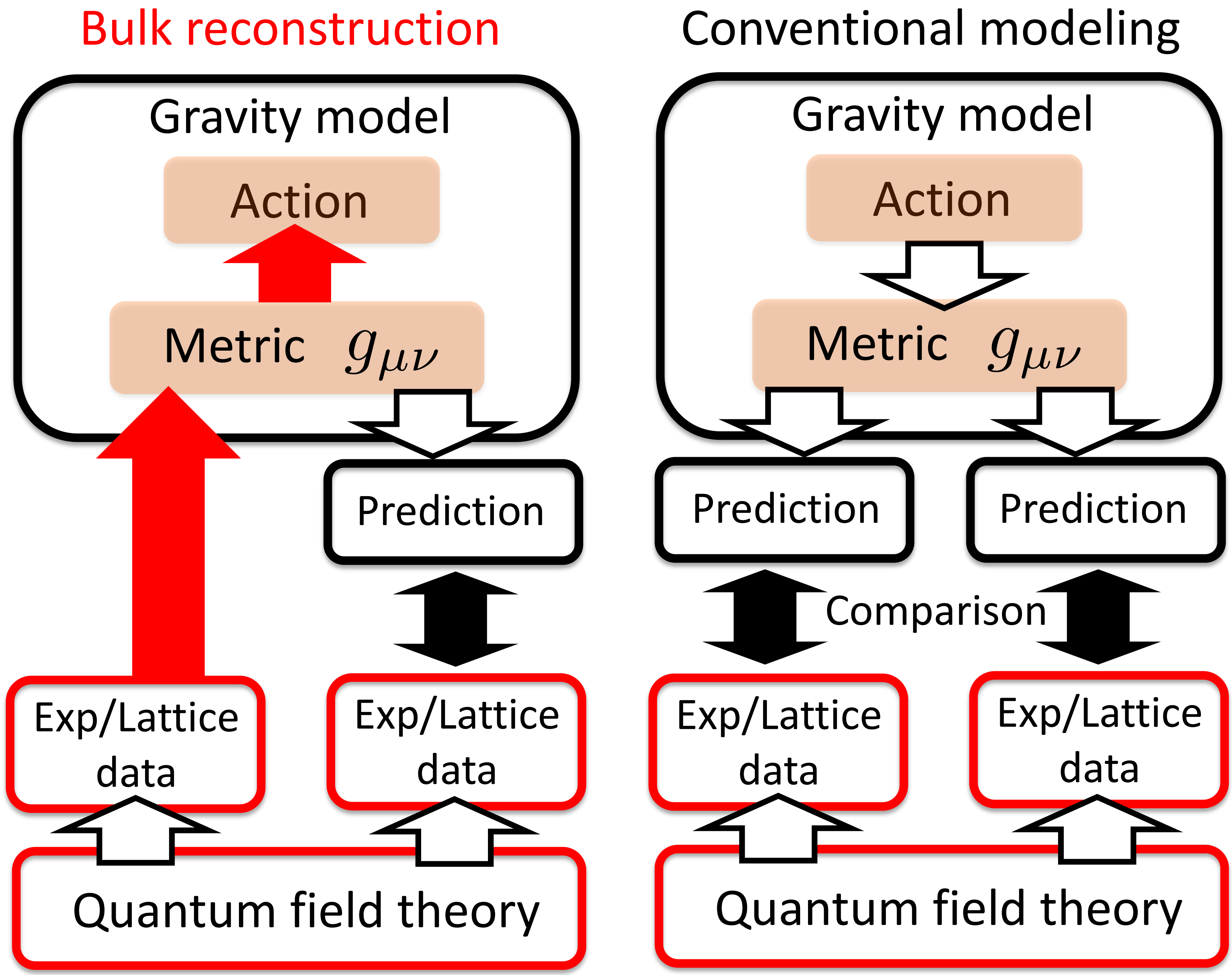
   1809.10536, 1903.04951

2. Space emergent from data — 4 pages

   2005.02636 (1802.08313, 1809.10536, 2006.00712, 2409.?????)

3. Gravity reconstructed — 7 pages

   2108.08091, 2209.04638



Bulk reconstruction
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
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Quantum field theory
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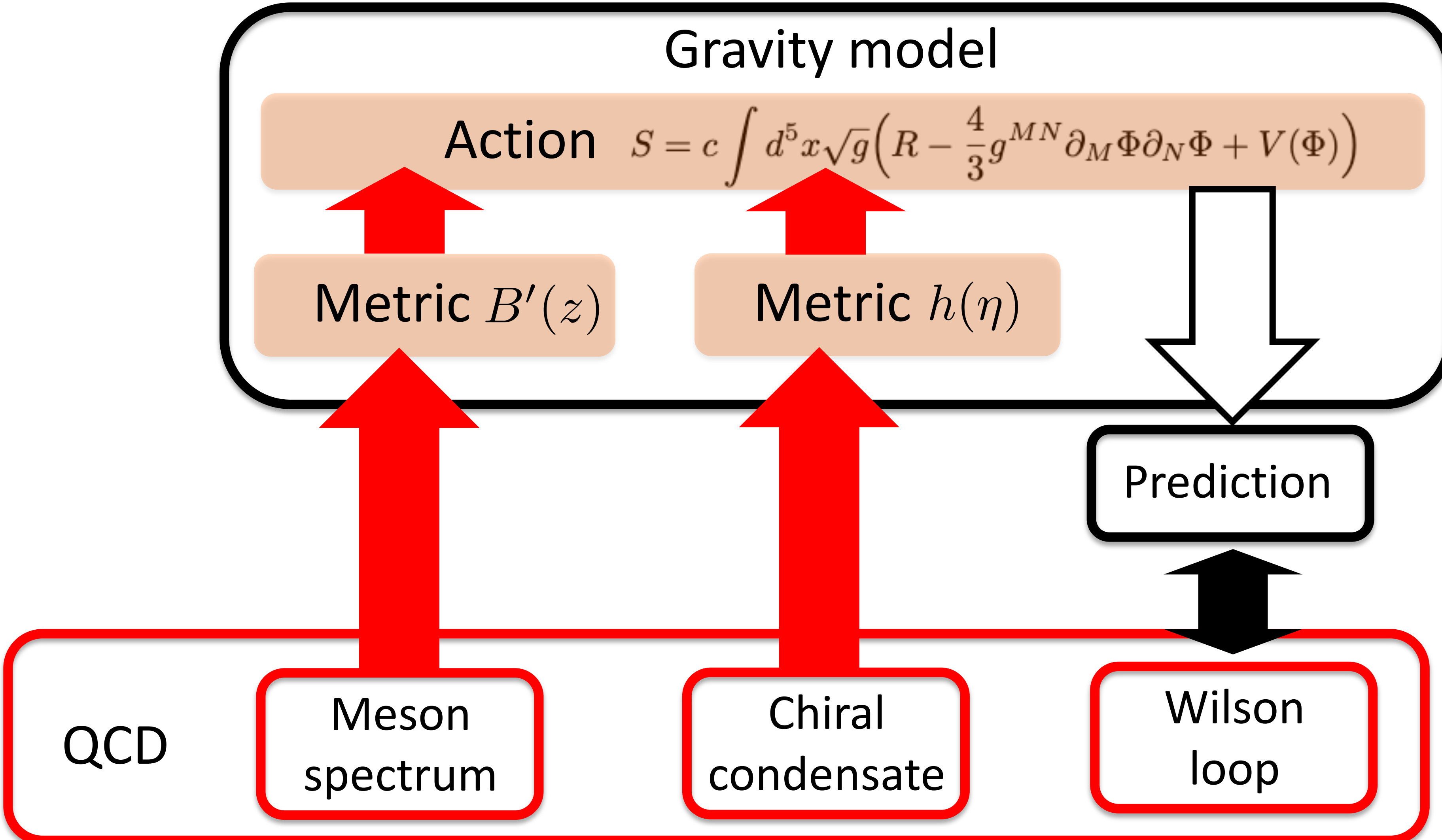
Exp/Lattice data
Exp/Lattice data
Quantum field theory

③ Gravity reconstructed

Two independent information of metric



Two independent information of metric

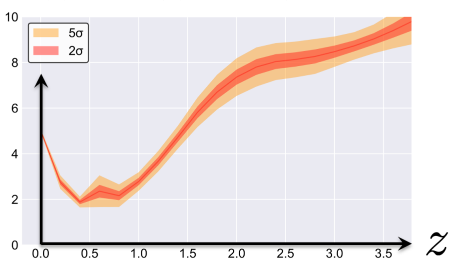
Gravity model

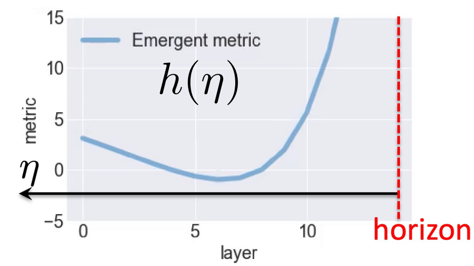
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Metric $B'(z)$

Metric $h(\eta)$

$B'(z) = \Phi'(z) - A'(z)$





Meson spectrum

Chiral condensate

③ Gravity reconstructed

Deriving the dilaton potential (T=0)

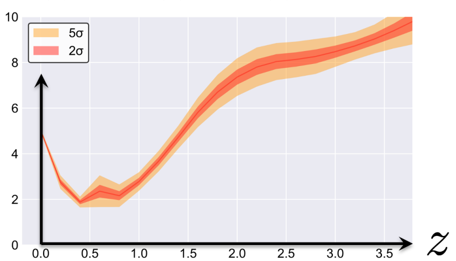
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

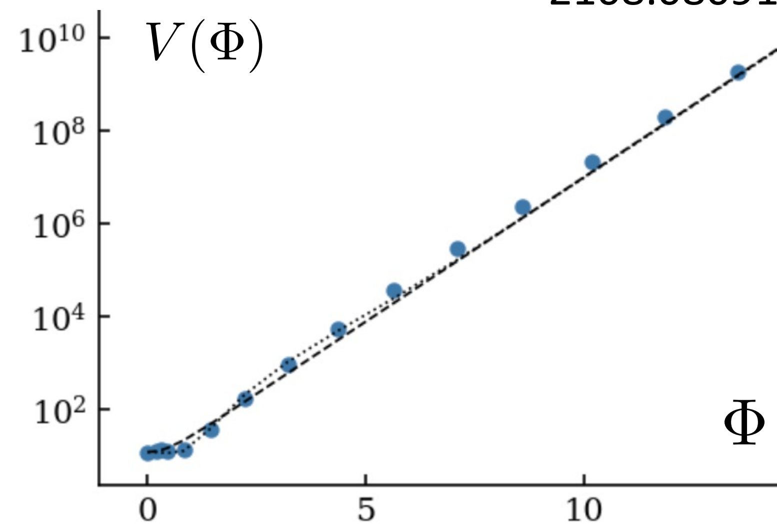
Metric $B'(z)$

Meson spectrum

$B'(z) = \Phi'(z) - A'(z)$



2108.08091



$V = 12\cosh(1.433\Phi)$

$V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$

Cf. [Gubser, Nellore, 0804.0434]

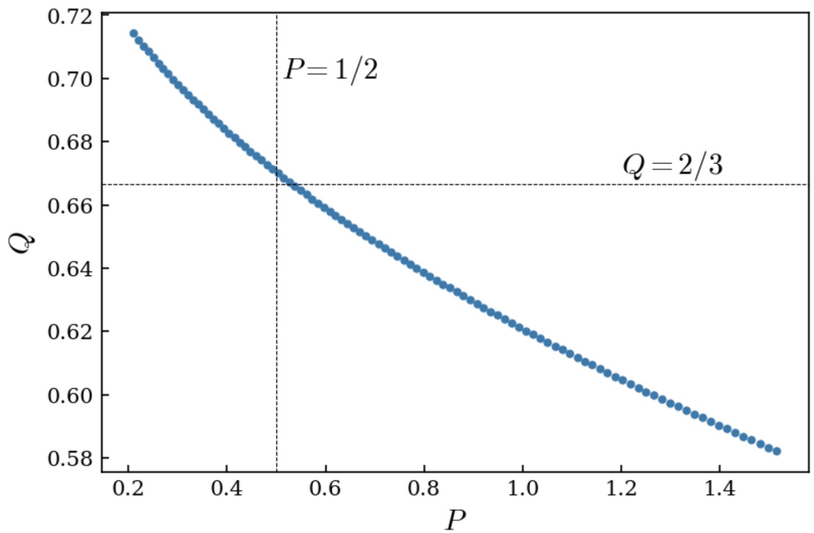
③ Gravity reconstructed

It's a nice dilaton potential !

Gravity model

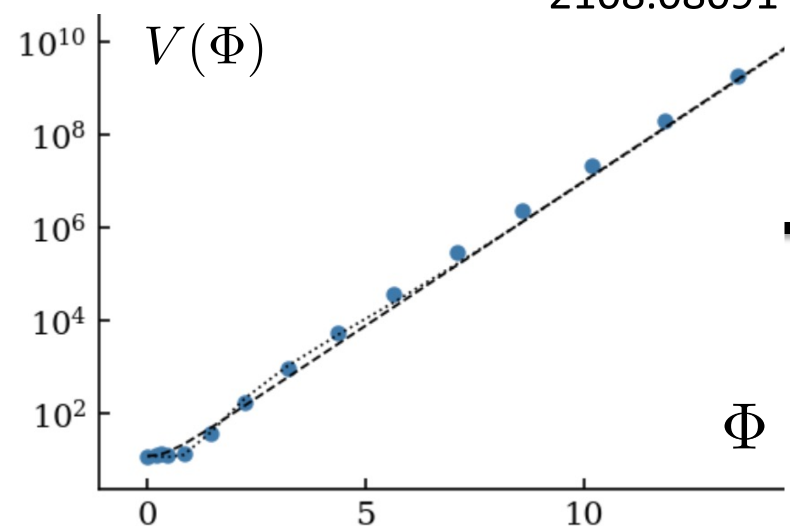
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

2108.08091



Fit the asymptotic part by $V(\Phi)\sim e^{2Q\Phi}\Phi^P$ for different values of dilaton initial cond.

Cf. [Gursoy, Kiritsis, 0707.1324]
[Gursoy, Kiritsis, Nitti, 0707.1349]



$V = 12\cosh(1.433\Phi)$ (dashed)

$V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$ (dotted)

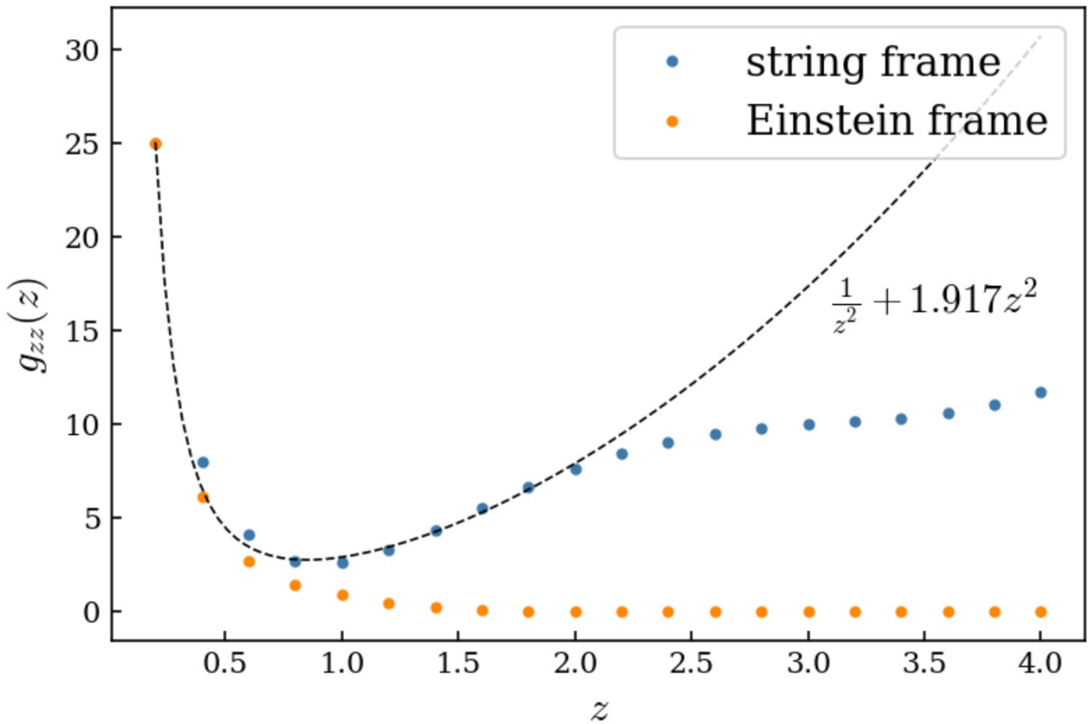
Cf. [Gubser, Nellore, 0804.0434]

③ Gravity reconstructed

String frame metric has a bottom

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$



Prediction

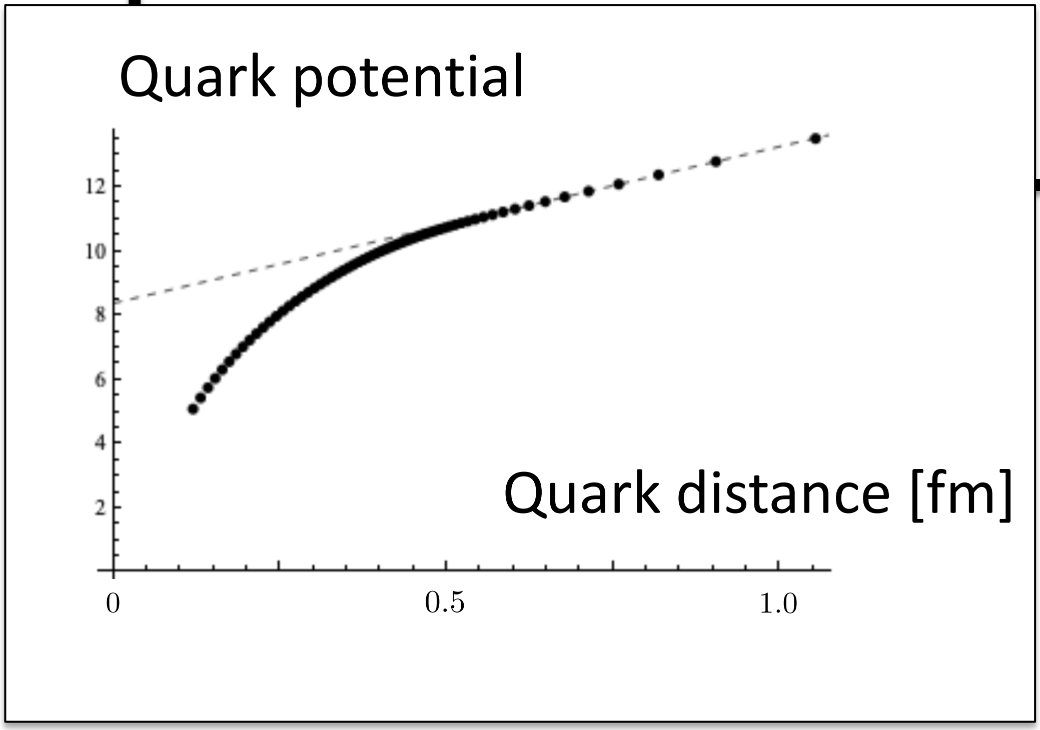
Wilson loop

③ Gravity reconstructed

Prediction of string breaking (T=0)

Gravity model

Action $S = c \int d^5x \sqrt{g} \left(R - \frac{4}{3} g^{MN} \partial_M \Phi \partial_N \Phi + V(\Phi) \right)$



Prediction

Wilson loop

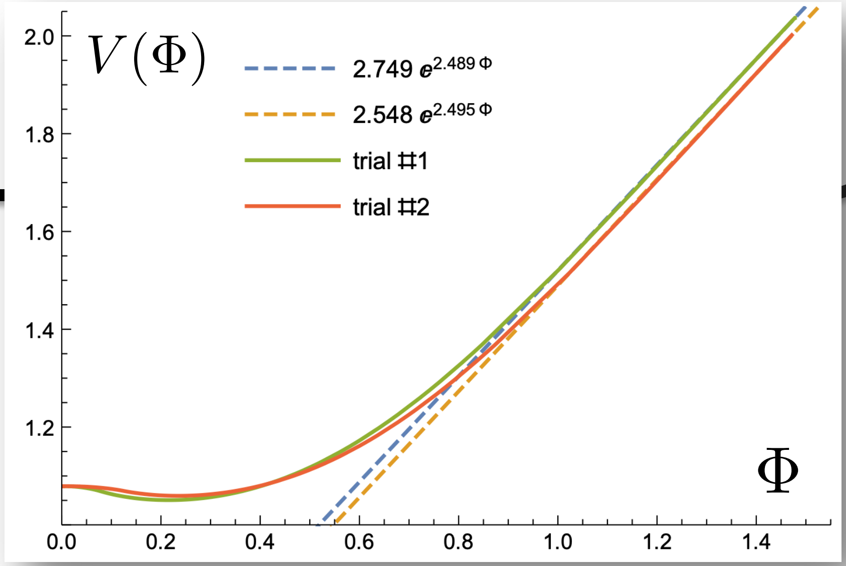
Deriving the dilaton potential (finite T)

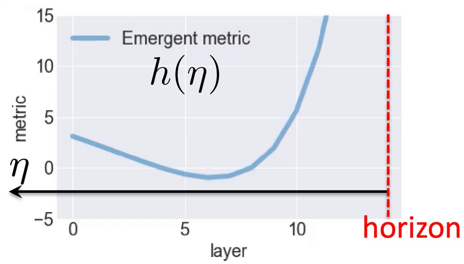
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

2209.04638

Metric $h(\eta)$





Chiral condensate

③ Gravity reconstructed

Prediction of string breaking (finite T)

Gravity model

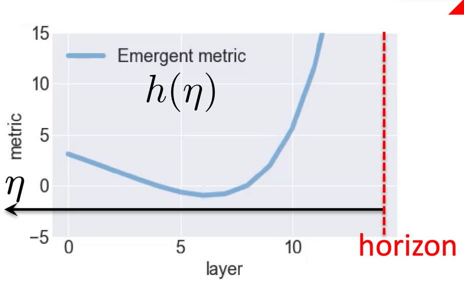
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

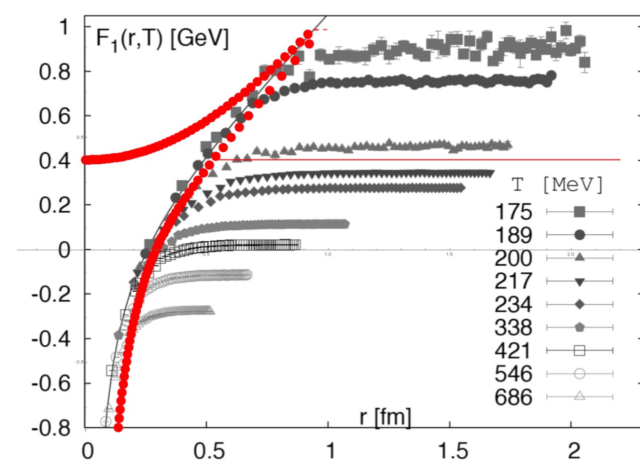
Metric $h(\eta)$

Prediction

Chiral condensate

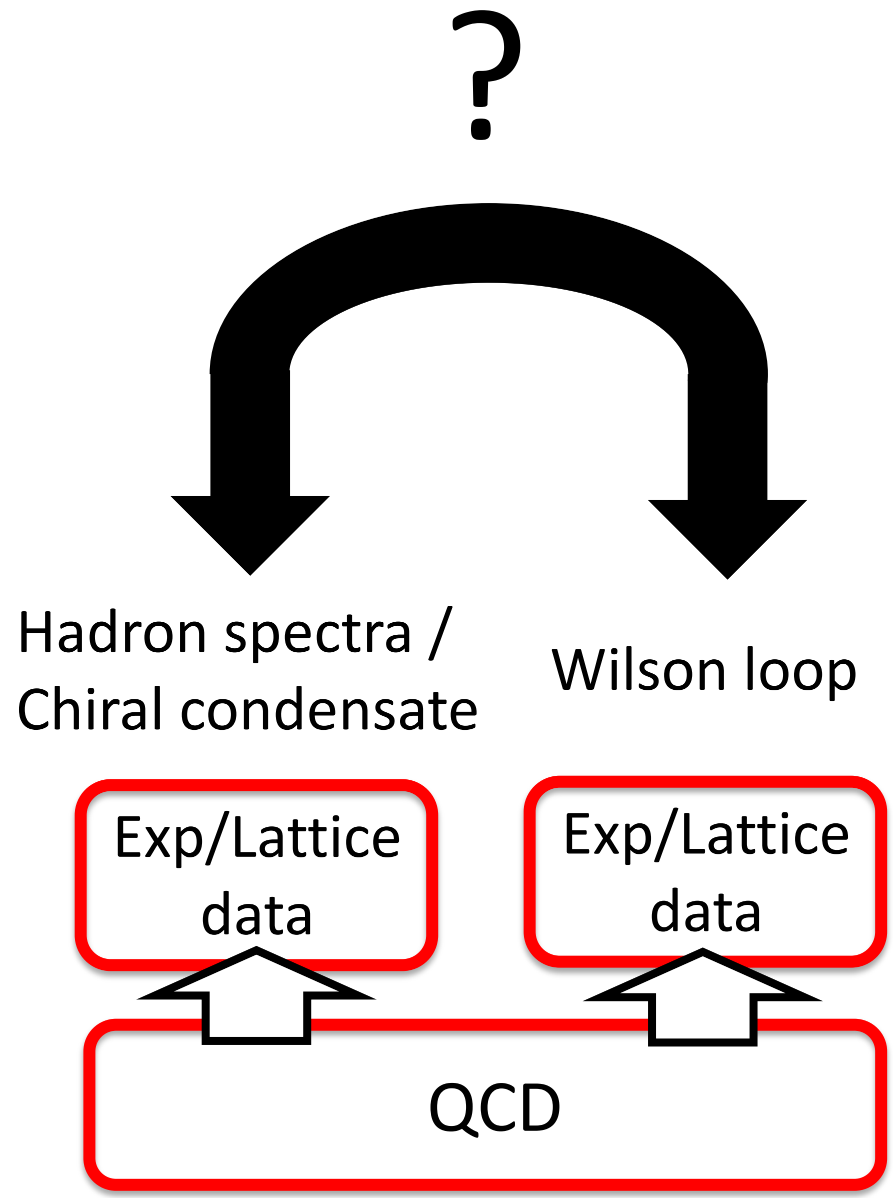
Wilson loop





Lattice data (grey) : Petreczky, J.Phys.G37(2010)094009

Bulk reconstruction by deep learning


?
Hadron spectra /
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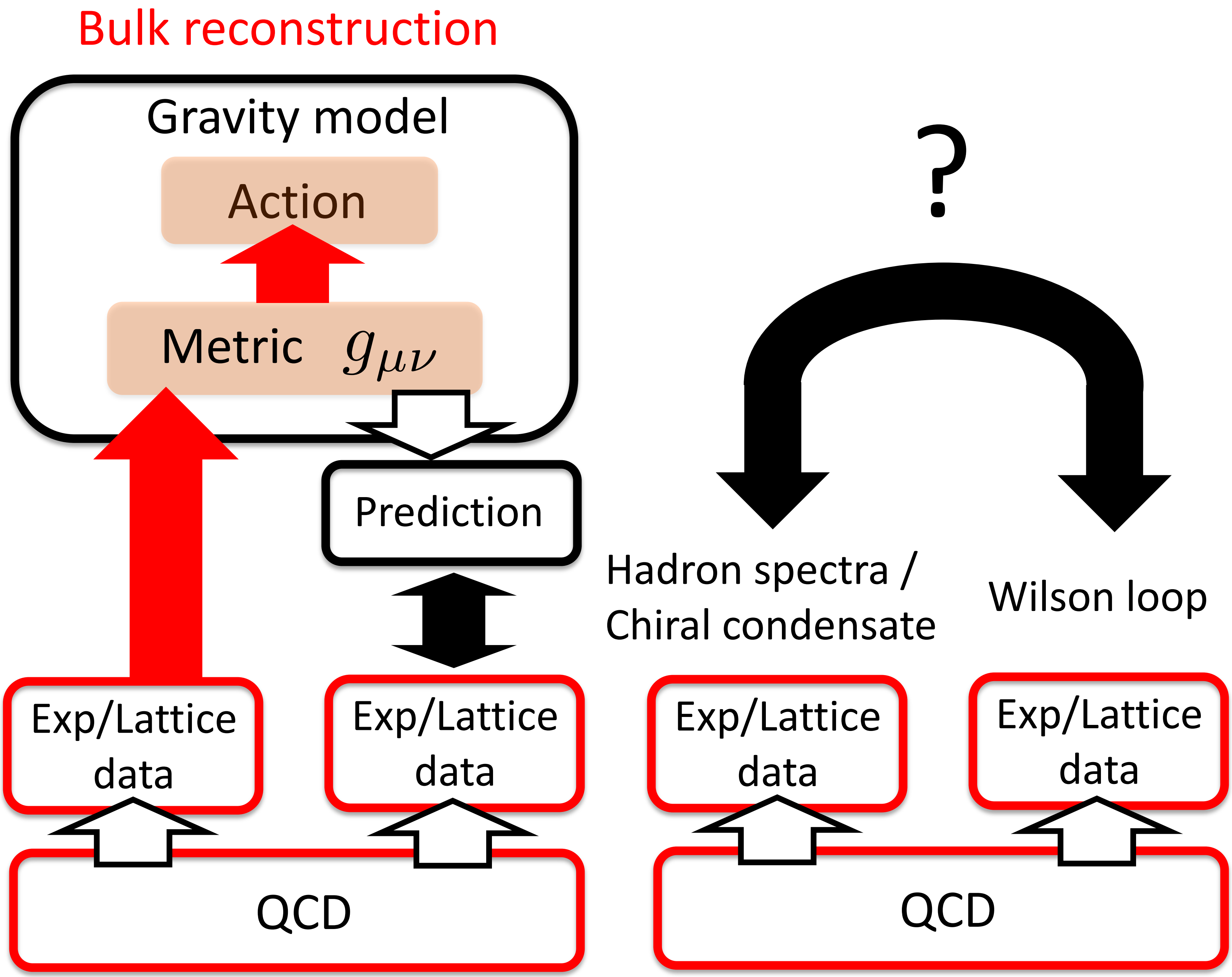

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Exp/Lattice data
QCD

Summary :

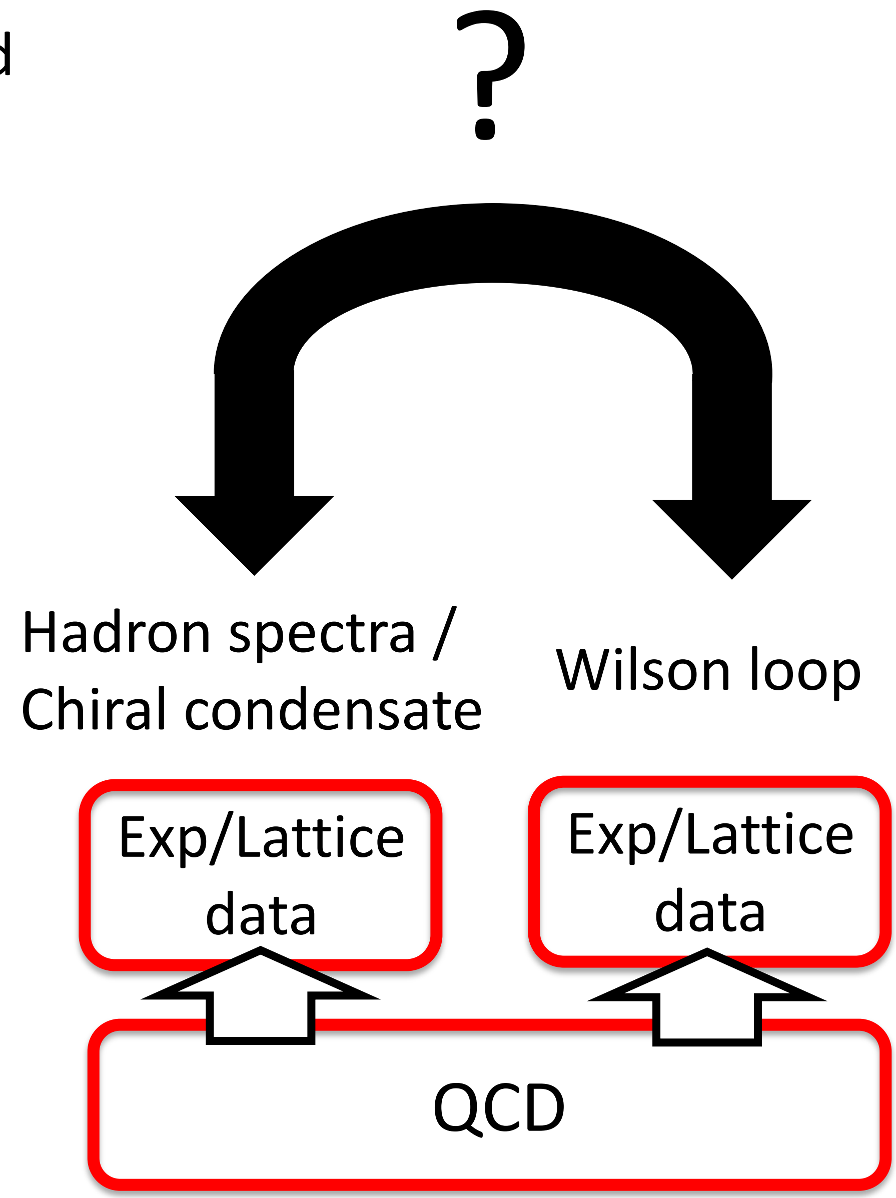
Deep learning enables us to find a gravity model dual to QCD, in a data-driven way

Physics :

Chiral sym. breaking has the same origin as confinement!

Challenge :

Can we pin down the unique gravity model consistent with various QCD data?

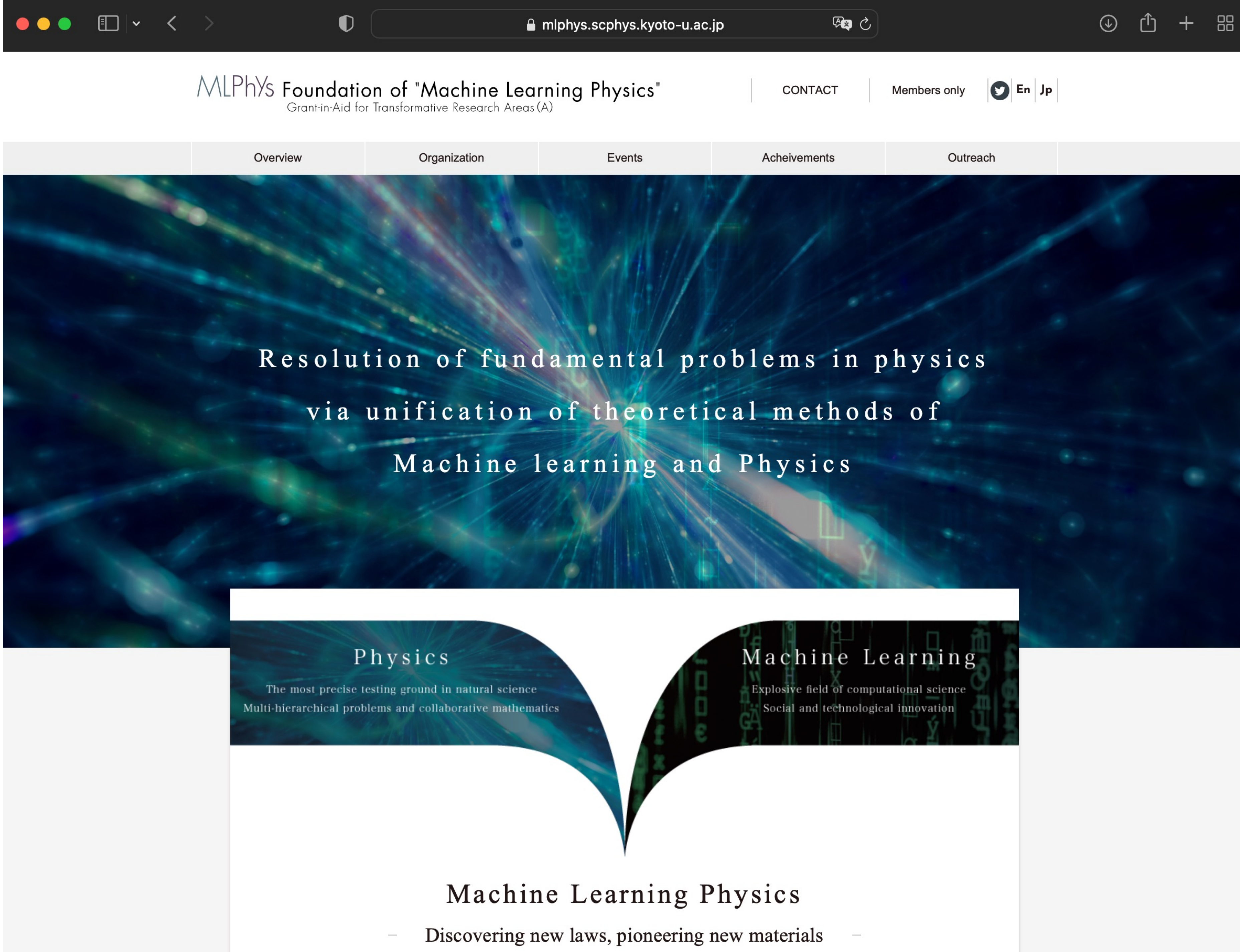


MLPhys 学術変革領域研究(A) 学習物理学の創成
Foundation of "Machine Learning Physics"

2024 **Dec.10-12** Yukawa Institute, Kyoto University

Overview

The String Data International Conference Series is the first meeting in the world to combine data science and string theory. It is evolving not just as a unique field of string theory with machine learning, but with broader topics connecting theoretical physics and AI. The research


mlphys.scphys.kyoto-u.ac.jp
MLPhYs Foundation of "Machine Learning Physics"
Grant-in-Aid for Transformative Research Areas (A)
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Organization
Events
Acheivements
Outreach
Resolution of fundamental problems in physics
via unification of theoretical methods of
Machine learning and Physics
Physics
The most precise testing ground in natural science
Multi-hierarchical problems and collaborative mathematics
Machine Learning
Explosive field of computational science
Social and technological innovation
Machine Learning Physics
Discovering new laws, pioneering new materials

Ⓐ Bulk from chiral condensate

Simplest holographic model

Classical scalar field theory in unknown 5-dim. spacetime

1802.08313
1809.10536

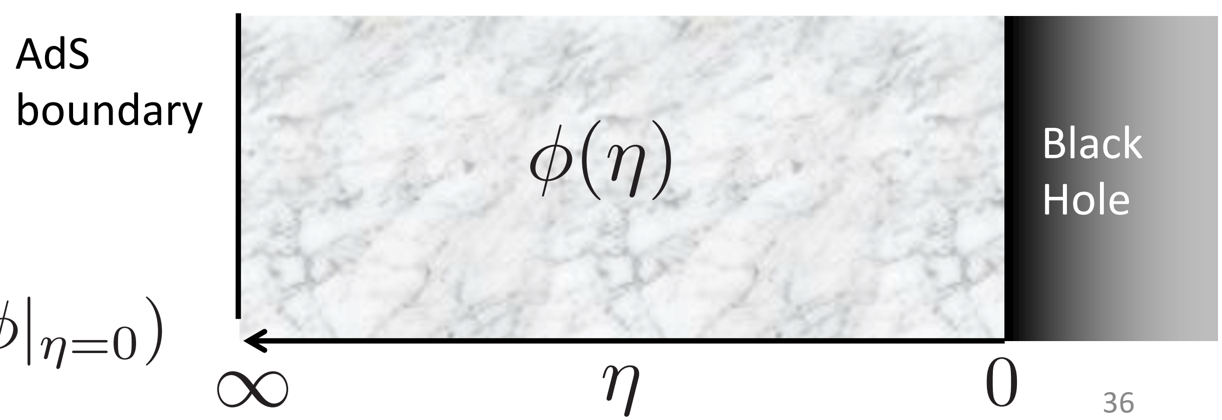
$$S = \int d\eta d^4x \sqrt{\det g}\,\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$\begin{cases} ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \\ V[\phi] = -\dfrac{3}{L^2}\phi^2 + \dfrac{\lambda}{4}\phi^4 \end{cases}$$

Data: $(m_q, \langle \bar{q}q \rangle)$

⇕

$(\phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=0})$



Ⓐ Bulk from chiral condensate

Relation to QCD data

Boundary condition for the metric components

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2, \; g \sim \text{const.}$

Solve eq. of motion to get response $\langle\bar{\psi}\psi\rangle_{m_q}$. [Klebanov, Witten `98]

- AdS boundary ($\eta \sim \infty$) : $\phi = m_q e^{-\eta} + \langle\bar{\psi}\psi\rangle e^{-3\eta}$
- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \,\big|_{\eta=0} = 0$

A. Bulk from chiral condensate

Equation of motion as a feedforward NN

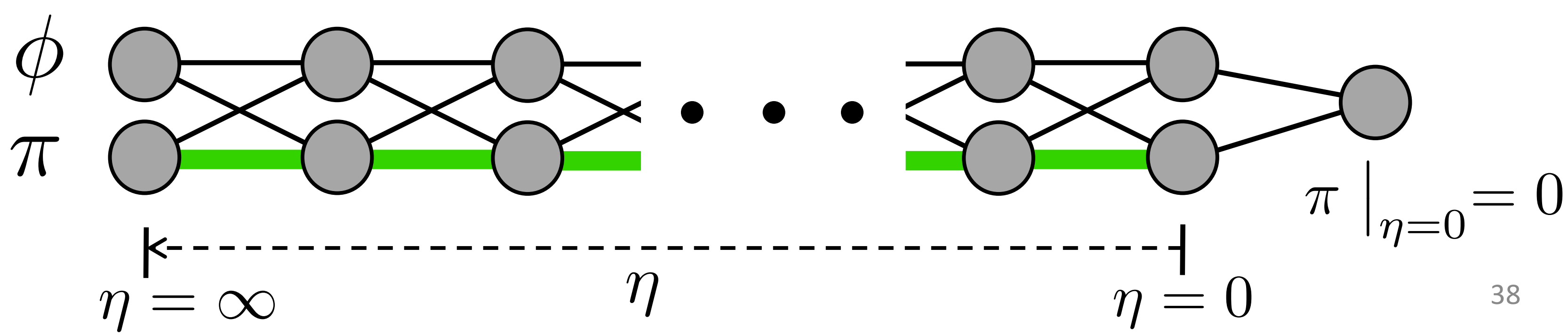
Eq. of motion $\partial_\eta^2 \phi + h(\eta)\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$

metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

Discretization
Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \frac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

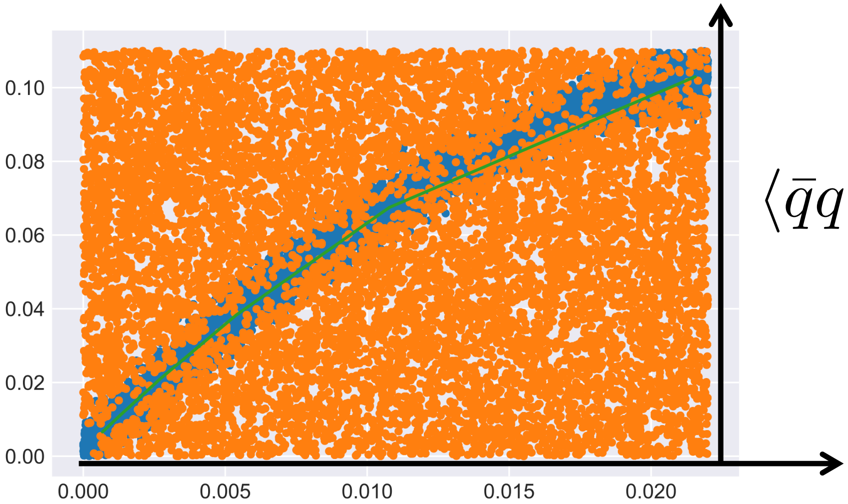
Feedfoward neural network for classification



Ⓐ Bulk from chiral condensate

Training with QCD data : quark condensate

Lattice QCD data at T=207[MeV]



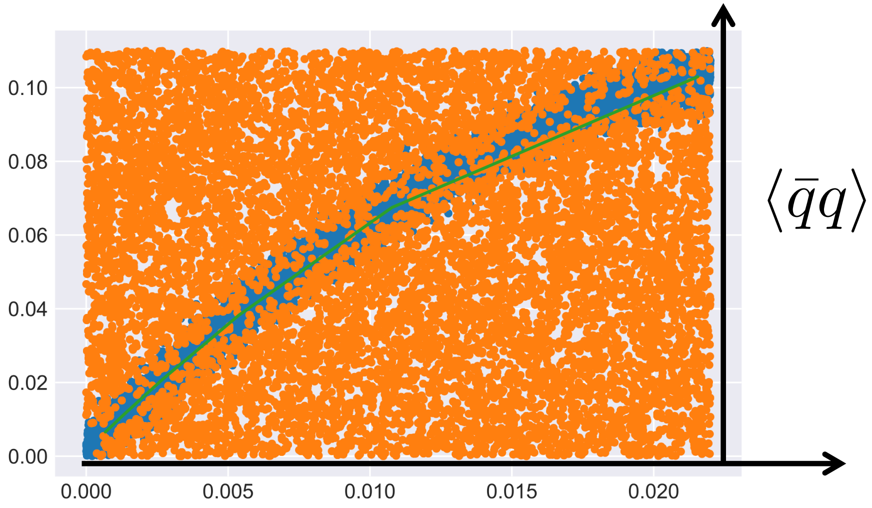
$\langle \bar{q}q \rangle$: Quark condensate

m_q : Quark mass

Ⓐ Bulk from chiral condensate

Training with QCD data : quark condensate

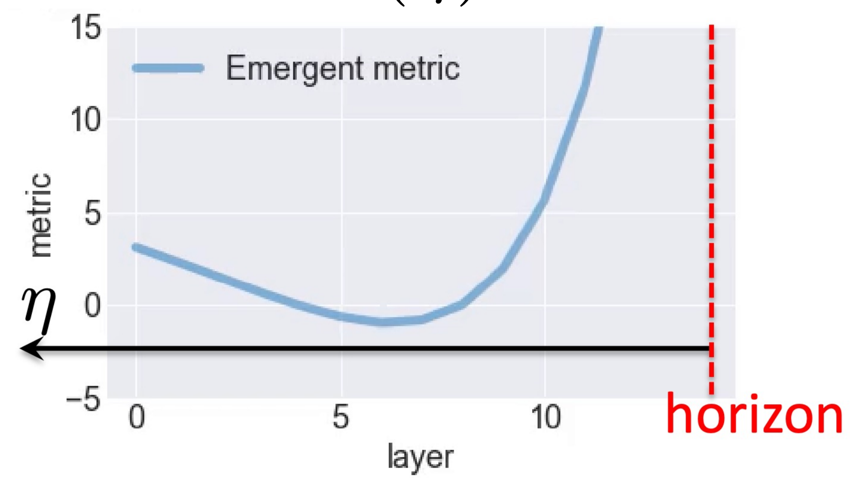
Lattice QCD data at T=207[MeV]



m_q : Quark mass

Bulk reconstructed!

$h(\eta)$



Trained values of potential :

1/L = 237(3)[MeV] ,

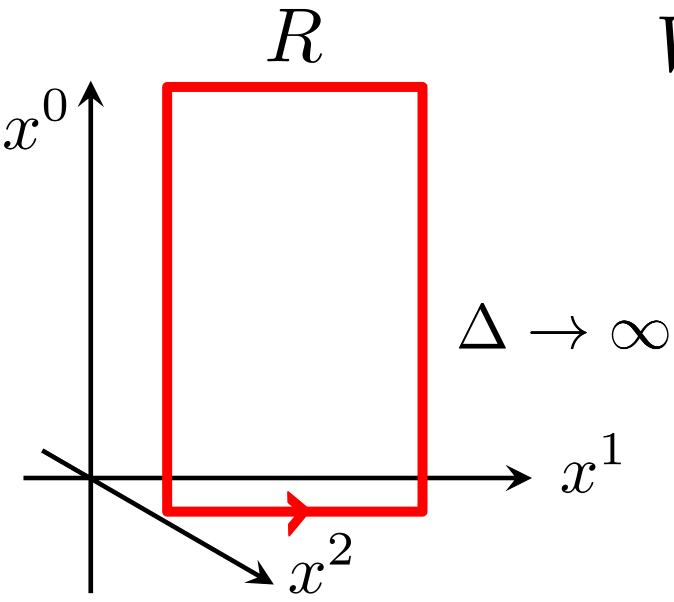
λ/L = 0.0127(6)

Ⓐ Review: bulk reconstruction

AdS/CFT is made by surfaces!

Surface dim.	AdS_{d+1}	CFT_d
1 + 1	Worldsheet	Wilson loop
(d-1) + 1	Worldvolume	Flavor partition fn.
(d-1) + 0	RT surface	Entanglement
d + 0	Volume	Complexity
d + 0 (w/ boundary)	Island	Page curve

Wilson loop



$$W \equiv \mathrm{Ptr}\exp\left[i\int_{\square} A_\mu(x)dx^\mu\right]$$

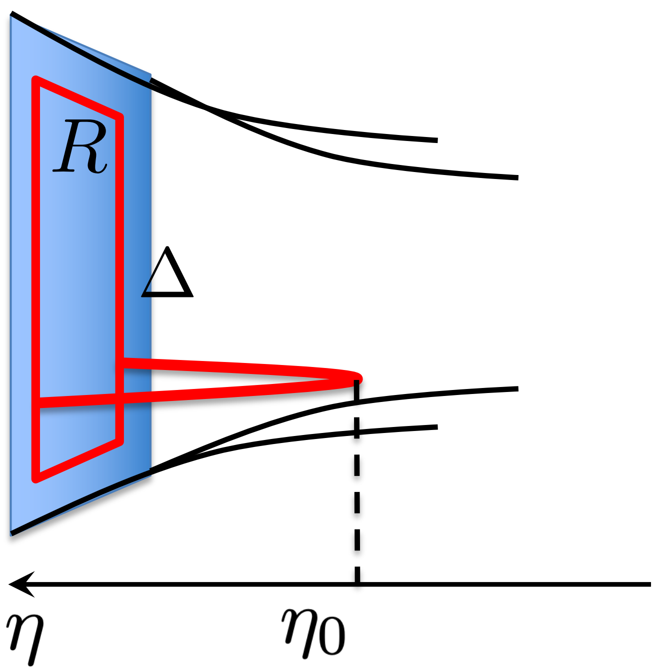
In confining phase,

$$\langle W\rangle \sim \exp[-E(R)\Delta]$$

Quark potential

Ⓐ Review: bulk reconstruction

AdS/CFT calculations of Wilson loop



Bulk metric in string frame

$$ds^2 = f(\eta)(-dt^2 + d\vec{x}^2) + d\eta^2$$

⇩ [Maldacena 98] [Rey Yee 98]

Nambu-Goto string solution

$$E(\eta_0) = \frac{1}{\pi\alpha'}\int_{\eta_0}^{\infty} d\eta\, \sqrt{f(\eta)}\sqrt{\frac{f(\eta)g(\eta)}{f(\eta)g(\eta) - f(\eta_0)g(\eta_0)}}$$

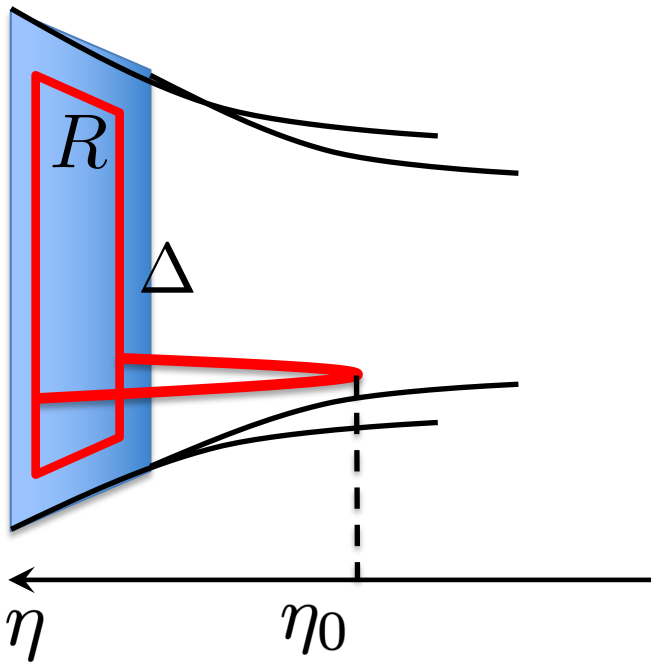
$$R(\eta_0) = 2\int_{\eta_0}^{\infty} d\eta\, \frac{1}{\sqrt{g(\eta)}}\sqrt{\frac{f(\eta_0)g(\eta_0)}{f(\eta)g(\eta) - f(\eta_0)g(\eta_0)}}$$

⇩

Quark potential $E(R)$

(A) Review: bulk reconstruction

AdS/CFT calculations of Wilson loop



Bulk metric in string frame

$$ds^2 = f(\eta)(-dt^2 + d\vec{x}^2) + d\eta^2$$

cena 98] [Rey Yee 98]

Nambu-Goto str

$$E(\eta_0) = \frac{1}{\pi\alpha'}\int_{\eta_0}^{\infty} d\eta \sqrt{}$$

$$R(\eta_0) = 2\int_{\eta_0}^{\infty} d\eta \frac{}{\sqrt{}}$$

Bulk Recon -struction

Quark potential $E(R)$

A. Review: bulk reconstruction

Wilson loop bulk reconstruction formula

[KH 20]

$$E(R) \Rightarrow ds^2 = f(\eta)(-dt^2 + d\vec{x}^2) + d\eta^2$$

Given a quark potential $E(R)$, solve

$$f_0 = 2\pi\alpha' \frac{dE(R)}{dR}$$

to get R as a function of f_0. Then substitute it to the following differential equation

$$\frac{d\eta(f)}{df} = \frac{1}{\pi}\sqrt{f}\frac{d}{df}\int_\infty^f df_0 \frac{R(f_0)}{\sqrt{f_0^2 - f^2}} \, .$$

Integrate this to find $\eta = \eta(f)$. Finally, invert it to find a bulk metric $f(\eta)$.