Revealing the transverse force distributions in the nucleon from lattice QCD

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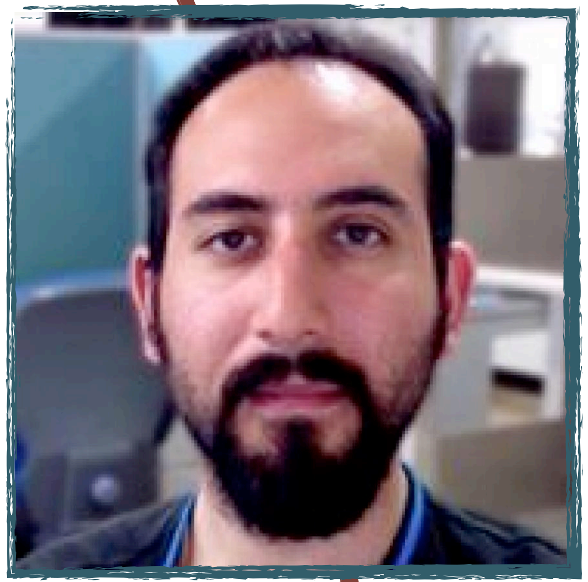
Joshua Crawford,
U.Adelaide PhD(2025)



K. Utku Can
U.Adelaide

QCDSF

Granada, Lattice 2017

Outline

Confinement:
static colour charges

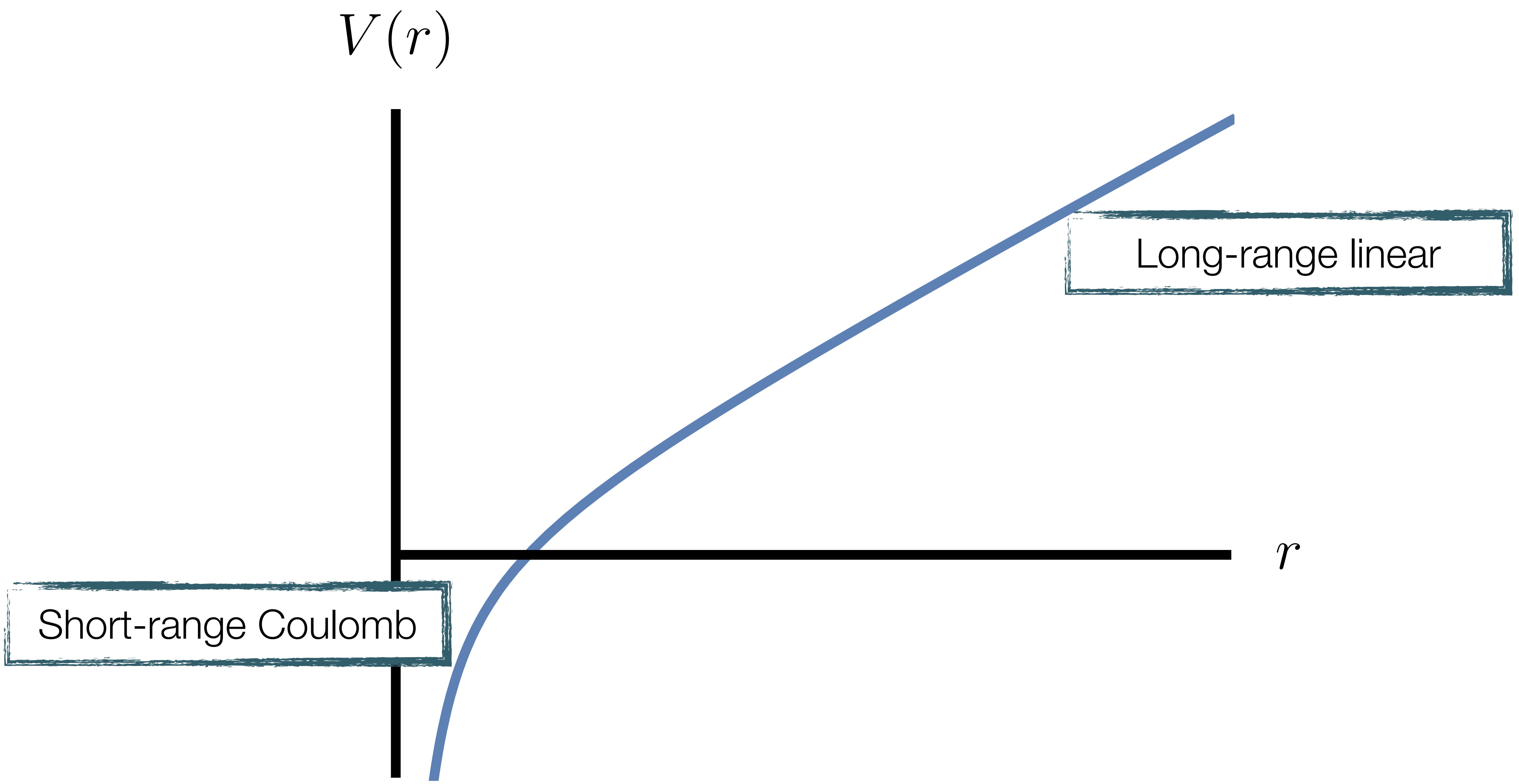
Generalised parton distributions:
mapping the position and
momentum of quarks

Forces in the nucleon

Mapping the force
distributions in lattice
QCD

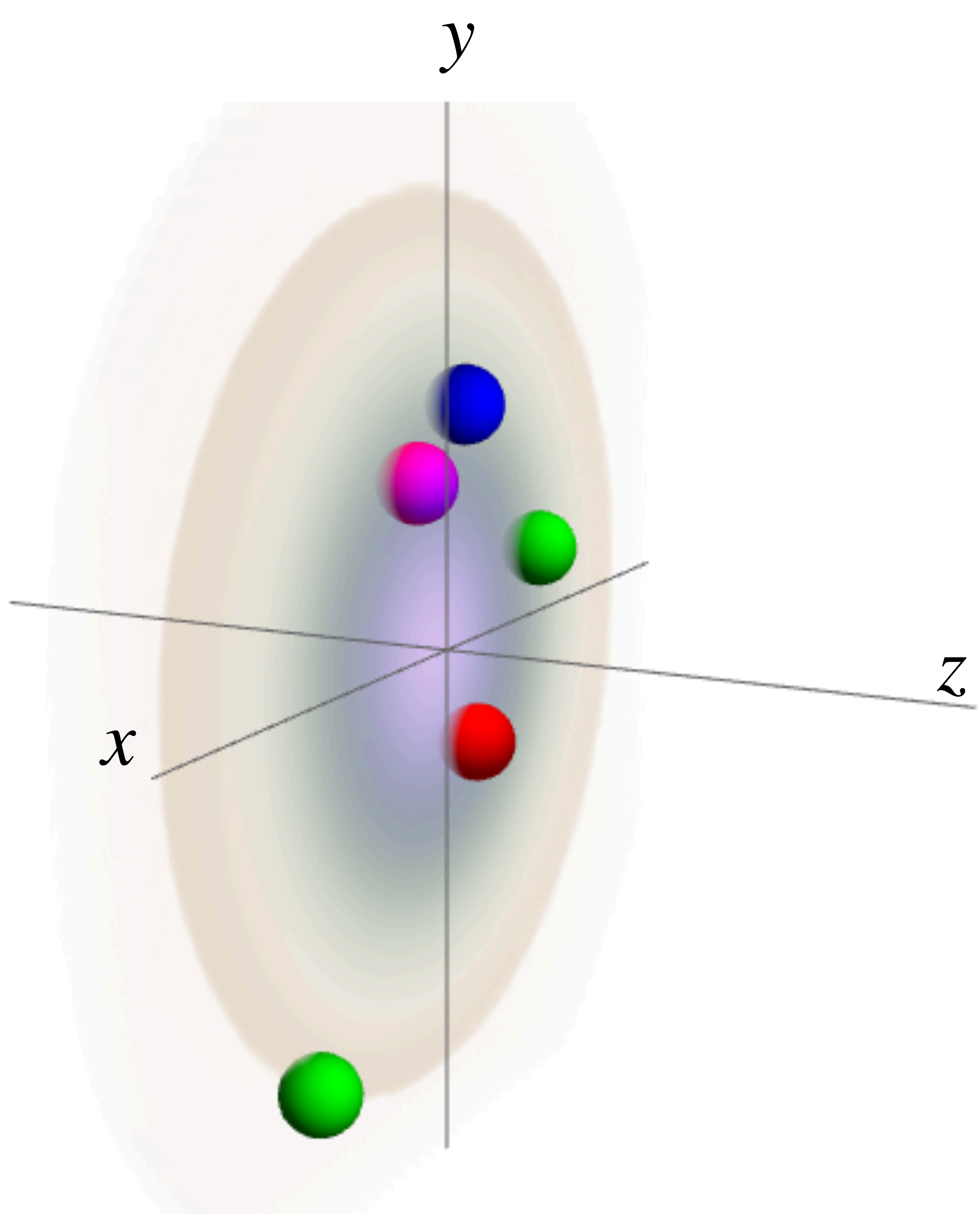
Confinement:
Dynamics of light
quarks in a nucleon

Confinement: static quark potential



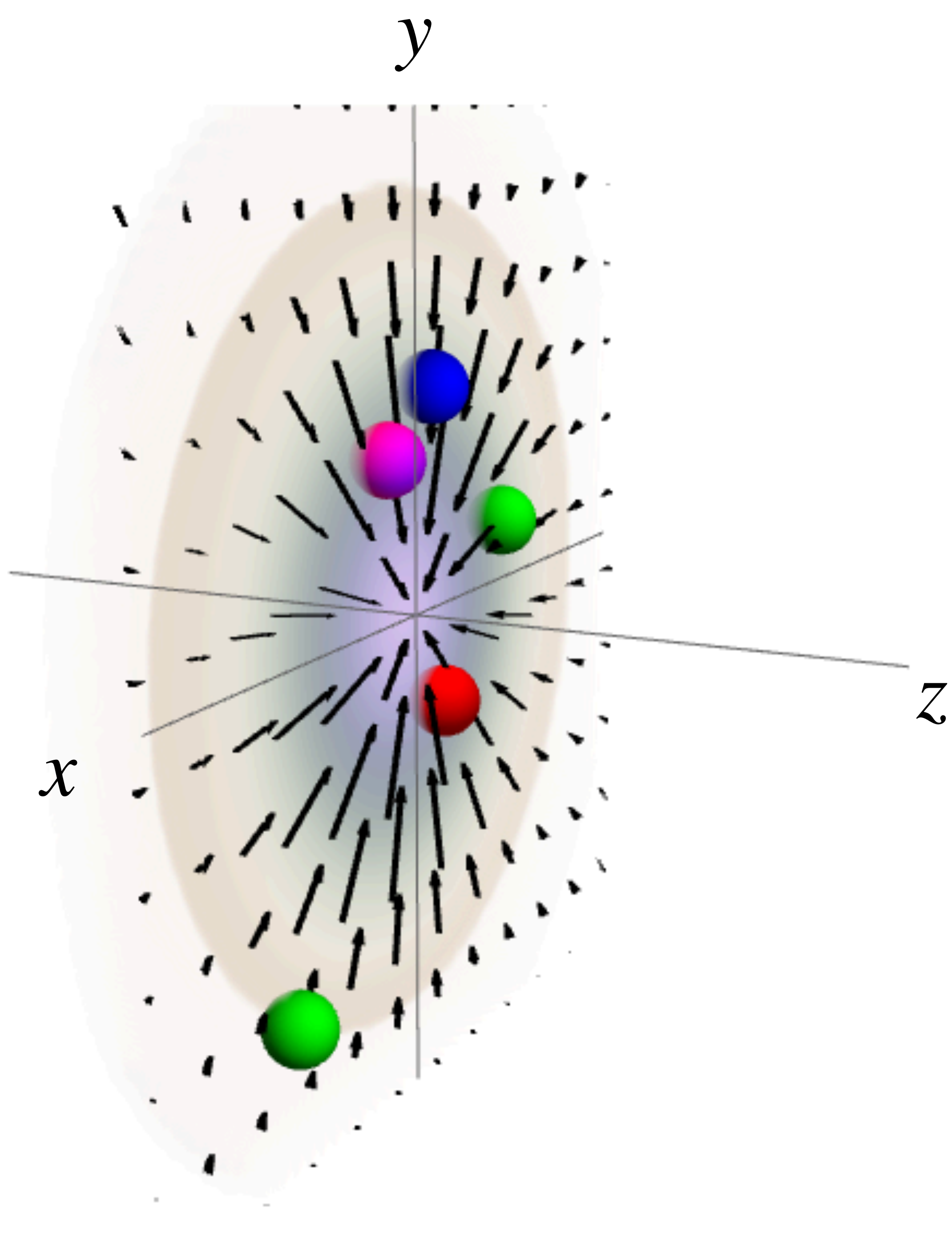
Charting the energy dependence between static quark-antiquark pair
(in pure Yang-Mills)

Imaging nucleon structure



Modern language to describe the (longitudinal) **momentum** and (transverse) **position** of quarks: Generalised parton distributions

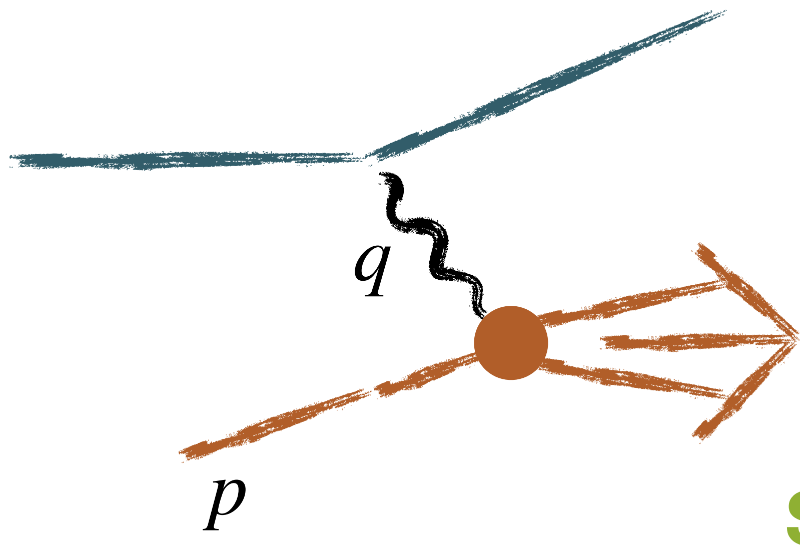
Imaging nucleon structure



Modern language to describe the (longitudinal) **momentum** and (transverse) **position** of quarks: Generalised parton distributions

Can we go further to describe the forces acting on these quarks?

Inelastic scattering



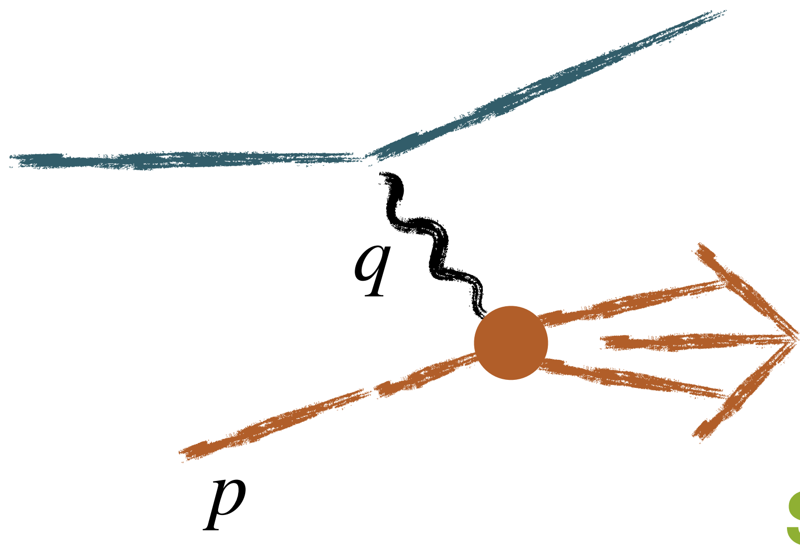
Hadron tensor

$$W_{\mu\nu} = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right)$$
$$+\frac{ig_1}{p\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma\right)\ ,$$

Scaling functions

g_2: No simple partonic interpretation!

Inelastic scattering



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$$+ \frac{ig_1}{p\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma\right) ,$$

Scaling functions

In the deep inelastic region, large Q^2, these functions map onto the parton distributions

g_2: No simple partonic interpretation!

Colour-Lorentz force

Burkardt, PRD(2013)

- While no simple parton interpretation, moment of the g_2 structure function can be expressed in terms of a local matrix element

$$\int dx\, x^2 \bar{g}_2(x) = \frac{d_2}{3} \equiv \frac{1}{6}\sum_q e_q^2 d_2^q$$

$$d_2^q = \frac{1}{2MP^+P^+S^x}\langle P,S\,|\,\overline{\psi_q}(0)\gamma^+ gG^{+y}(0)\psi_q(0)\,|\,P,S\rangle$$

Quark current density coupled to colour-Lorentz force

$$G^{+y} = \frac{1}{\sqrt{2}}\left(G^{0y} + G^{zy}\right) = -\frac{1}{\sqrt{2}}\left[\vec{E}_c + \vec{v}\times\vec{B}_c\right]^y = -\frac{1}{\sqrt{2}}F^y!$$

Transverse densities

- Electromagnetic current

$$\langle p', s' | \overline{\psi} \gamma^\mu \psi | p, s \rangle = \overline{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(t) \right] u(p, s)$$

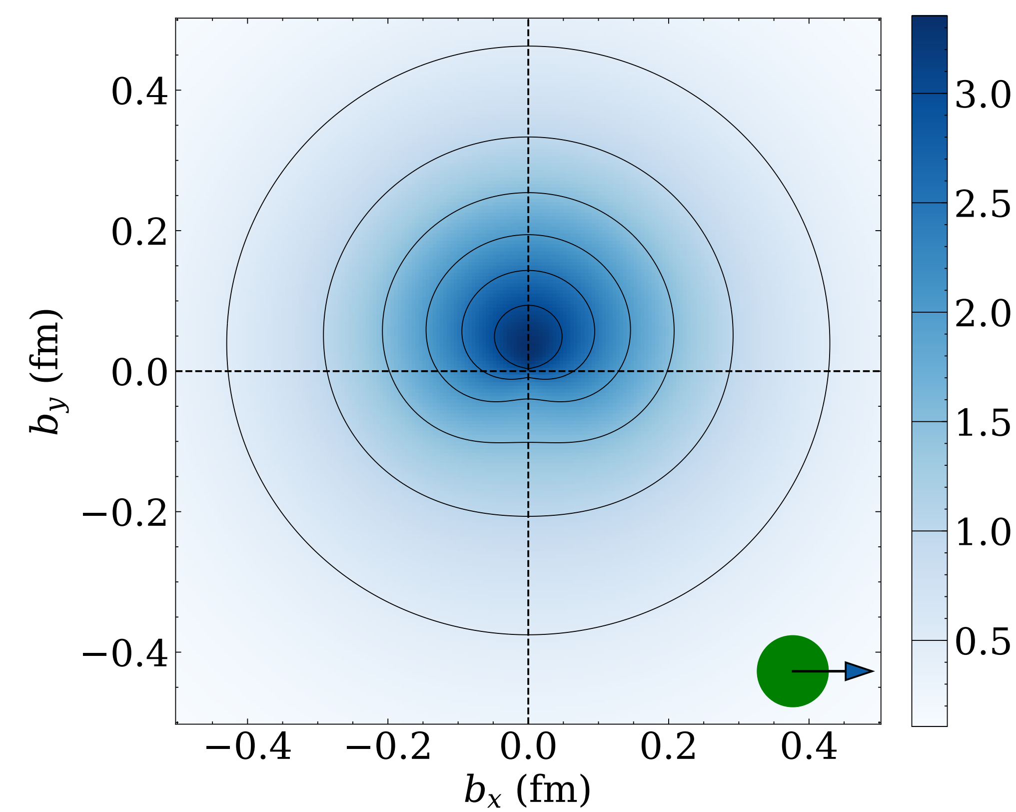
Dirac Pauli

2D Fourier transforms

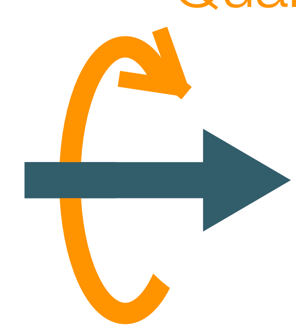
$$\tilde{F}_1(b^2),\ \tilde{F}_2(b^2)$$

Quark densities

$$q(\mathbf{b}) = \frac{1}{2} \left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N} \frac{d}{db^2} \tilde{F}_2(b^2) \right]$$



Quark orbital motion



Proton spin axis

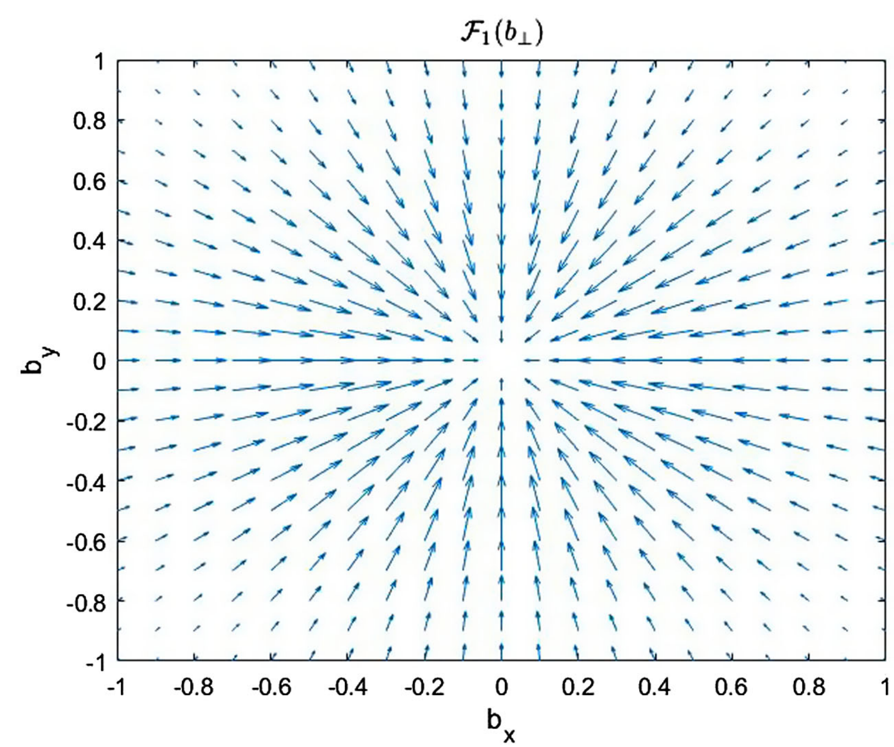
Twist-3 off-forward matrix elements

Aslan, Burkardt & Schlegel, PRD(2019)

$$\langle p', s' | \overline{\psi} \gamma^+ i g G^{+i} \psi | p, s \rangle = \overline{u}(p', s') \Big[P^+ \Delta^i \gamma^+ \Phi_1(t) + M P^+ i \sigma^{+i} \Phi_2(t) + \frac{1}{M} P^+ \Delta^i i \sigma^{+\Delta} \Phi_3(t) \Big] u(p, s),$$

where $P^\mu = (p' + p)^\mu / 2$, $\Delta^\mu = (p' - p)^\mu$, $t = -\Delta^2$ and $\sigma^{\mu\Delta} = \sigma^{\mu\nu} \Delta_\nu$.

Off-forward matrix elements → Form factors → 2D Fourier transform → Force distributions in the transverse plane

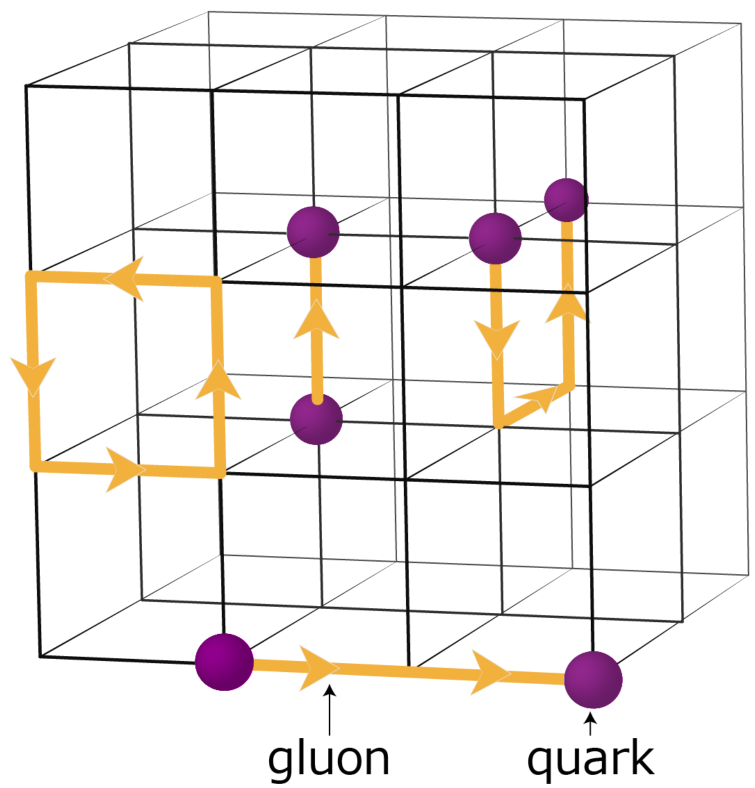


Resolving forces from lattice QCD

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time
- Approximate path integral by Monte-Carlo methods
  - Computationally intensive, large-scale supercomputing
- Controlled systematics
  - **Lattice spacing**, lattice volume, quark masses



Results shown today:

$m_\pi \sim 420\,\mathrm{MeV}$

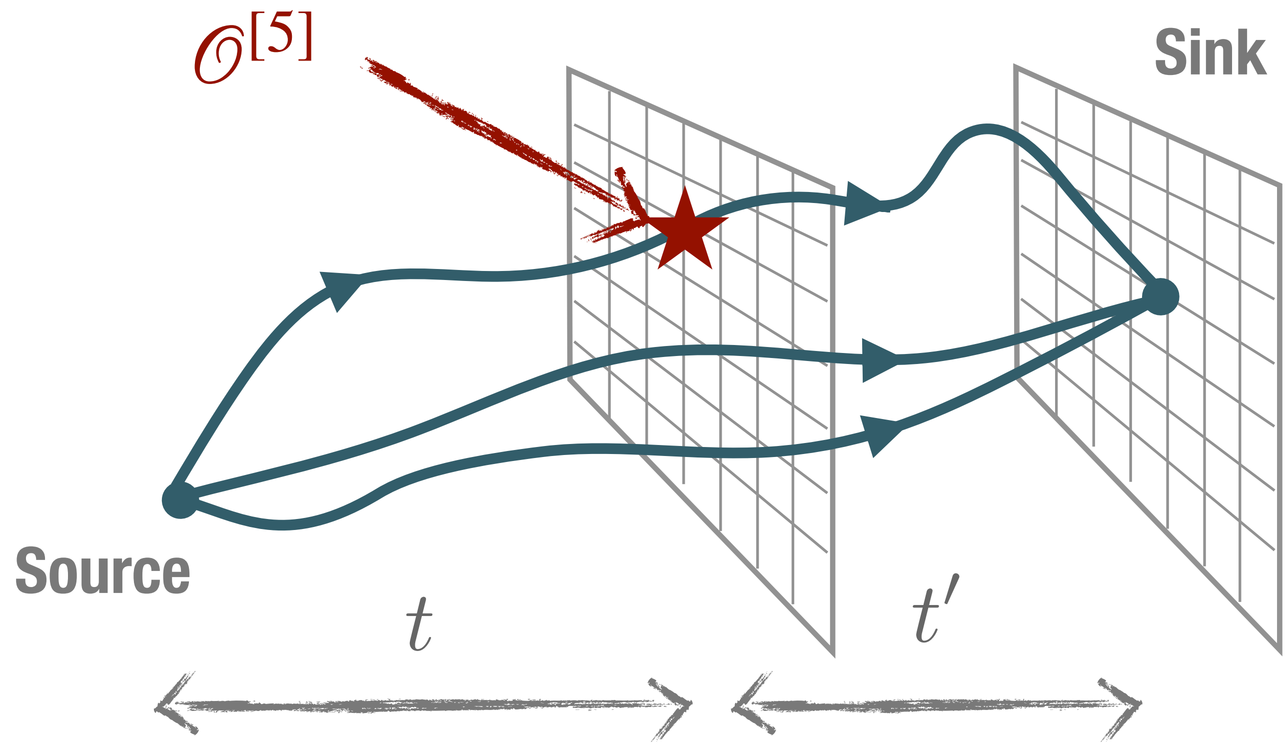
$L \sim 2.4 - 3.4\,\mathrm{fm}$

$a \sim 0.052 - 0.074\,\mathrm{fm}$

3-pt functions on the lattice

- Compute 3-point correlators on the lattice

Crawford, RDY et al. arxiv:2408.0362

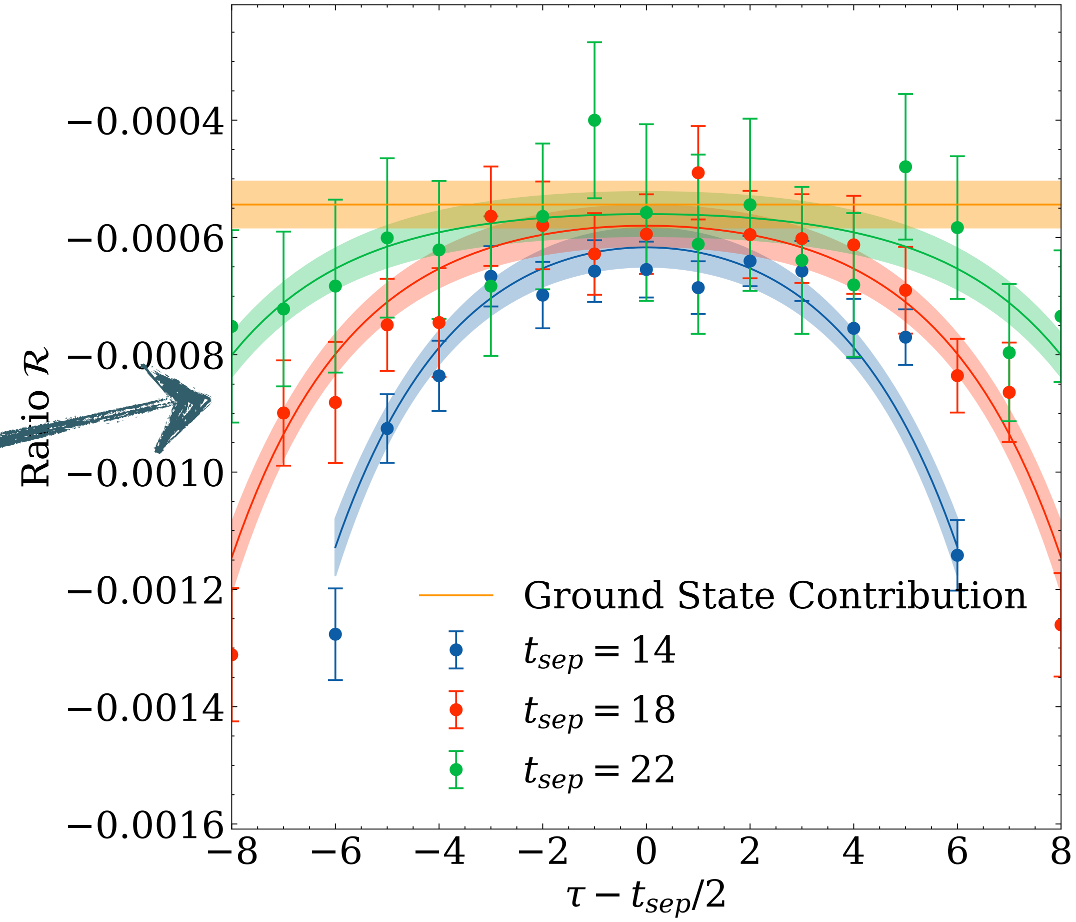


Large Euclidean time for ground-state dominance

$$\mathcal{O}^{[5]}_{[i\{j]4\}} = -\frac{g}{6}\overline{\psi}\left(\tilde{G}_{ij}\gamma_4 + \tilde{G}_{i4}\gamma_j\right)\psi - \text{traces}$$

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$

3 source–sink separations to establish ground-state dominance

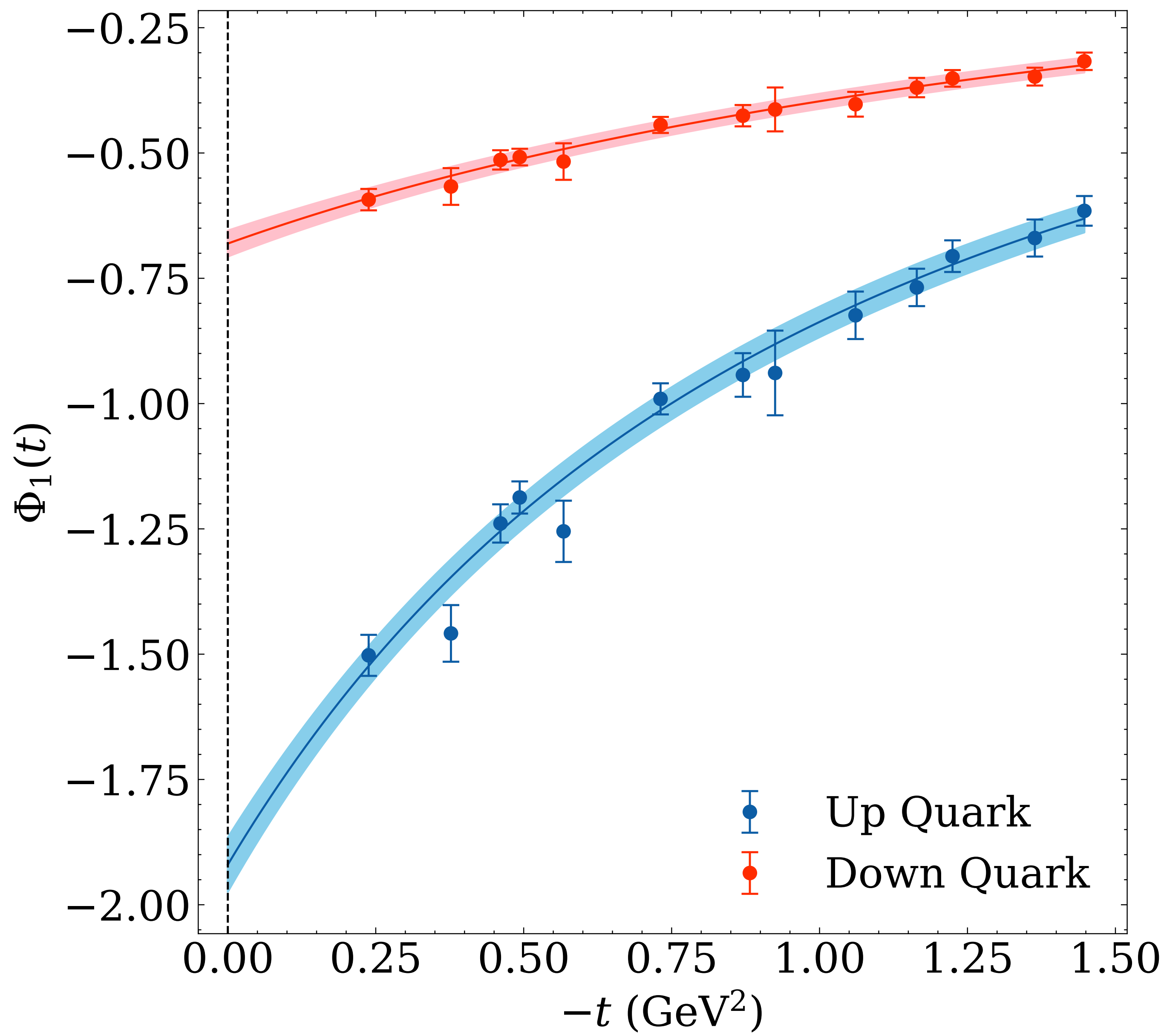


Φ_1 form factor

- Φ_1: isotropic force distribution
- Dipole fits to lattice results

Form factors are negative
⇒ attractive forces 😅

$\beta = 5.95$ ($a \sim 0.052$ fm)



$$\langle p', s' | \overline{\psi} \gamma^+ i g G^{+i} \psi | p, s \rangle = \overline{u}(p', s') \Big[P^+ \Delta^i \gamma^+ \Phi_1(t) + M P^+ i \sigma^{+i} \Phi_2(t) + \frac{1}{M} P^+ \Delta^i i \sigma^{+\Delta} \Phi_3(t) \Big] u(p, s),$$

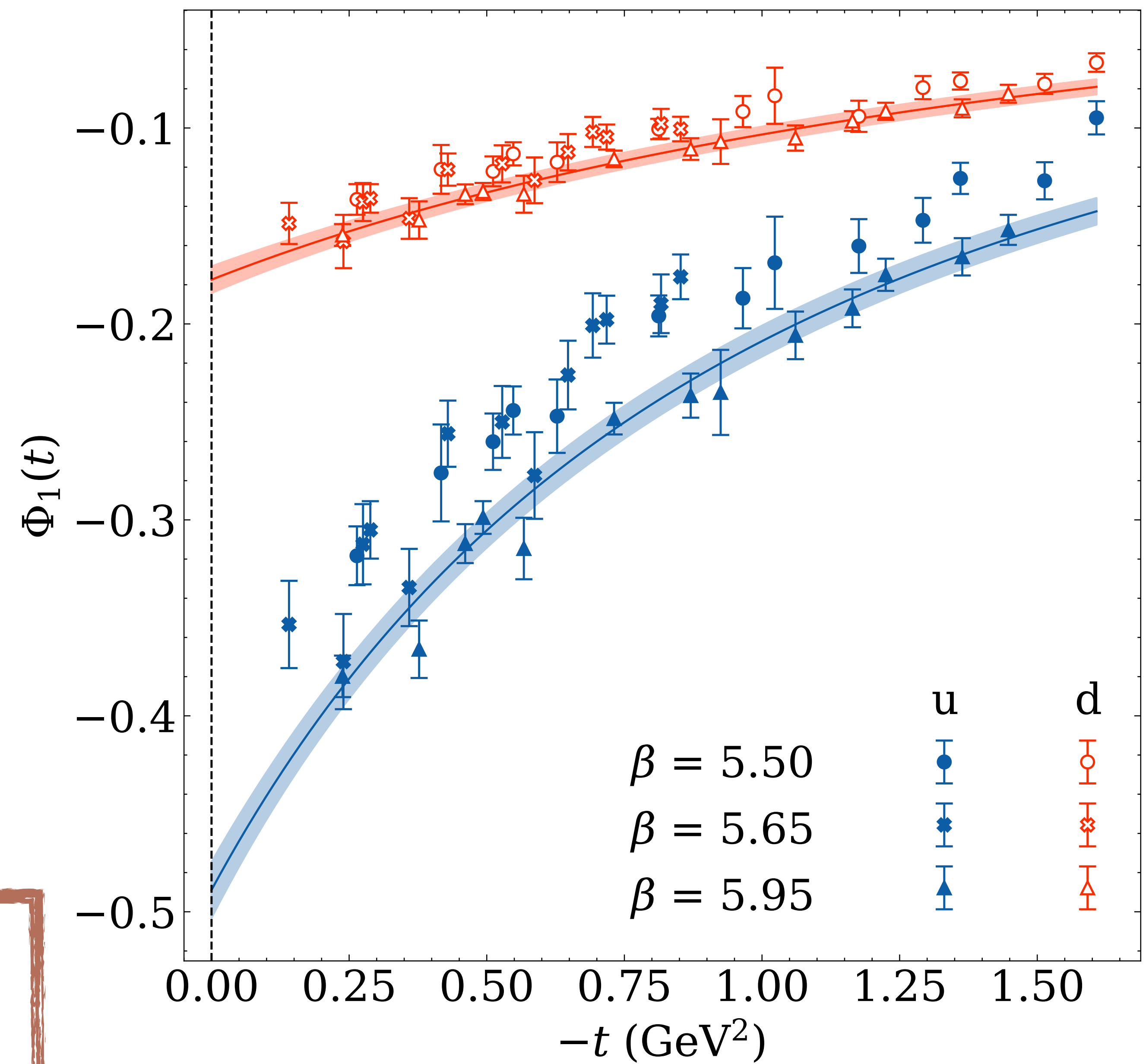
Estimating discretisation artefacts

- Extract form factors at 3 lattice spacings, $a \sim 0.74, 0.68, 0.52\,\mathrm{fm}$

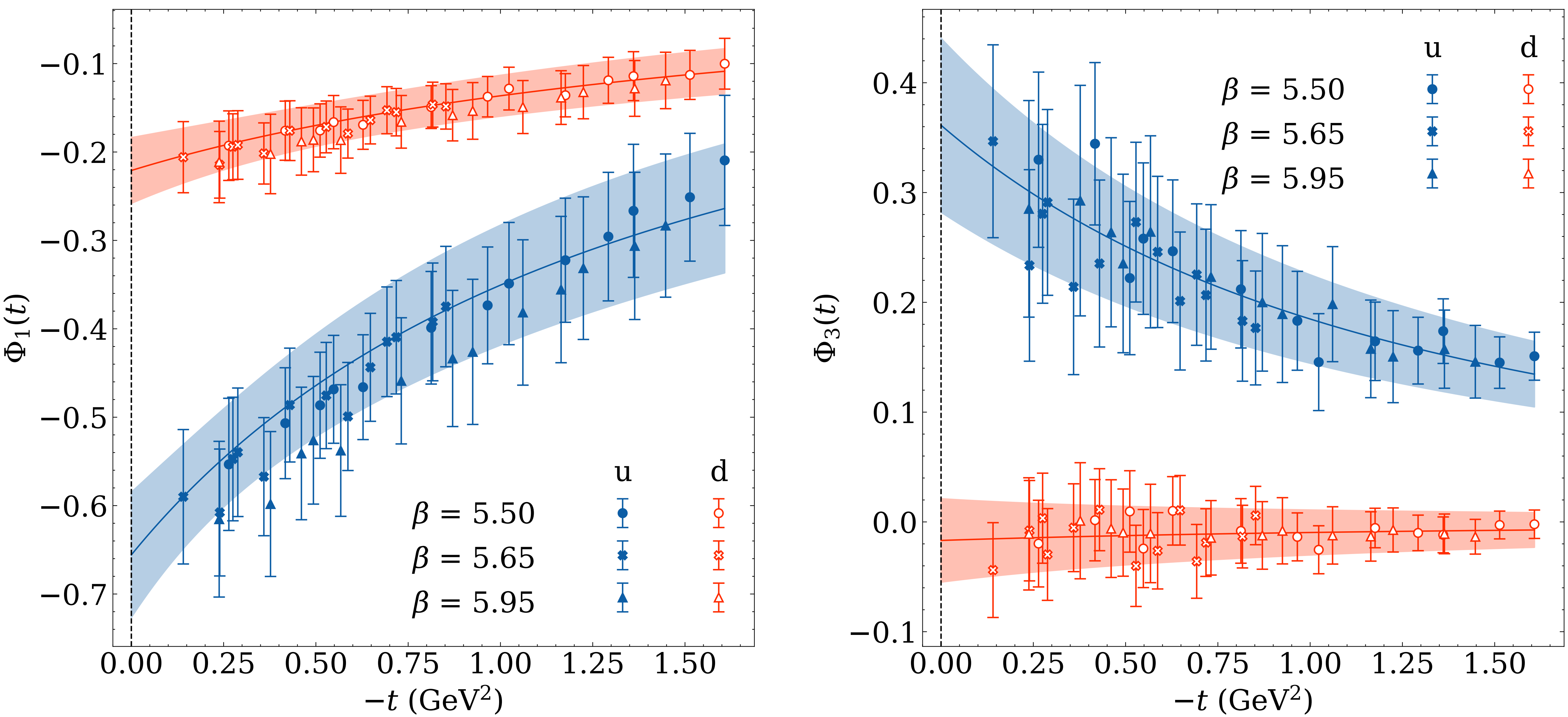
- Model a dependence in magnitude and slope

$$\Phi_i(t,a) = \frac{\Phi_i(0) + b_i a}{\left(1 + t\left(\frac{1}{\Lambda_i^2} + c_i a\right)\right)^2},$$

Some tension between different lattices; mostly in overall normalisation



Global analysis combining different a



Error bars here include estimate for $a \to 0$

Transverse force densities

- **2D Fourier transform** to impact parameter space

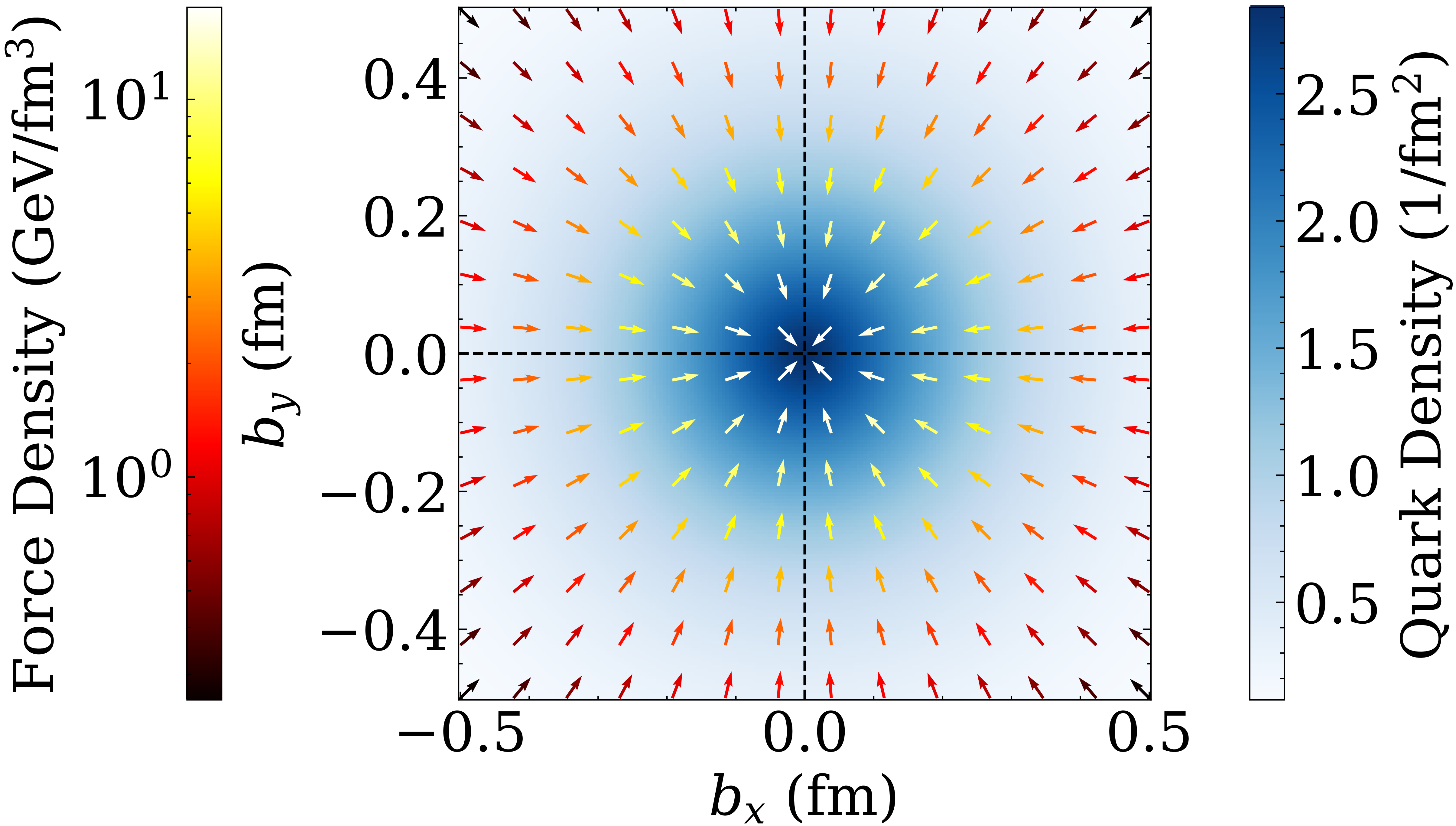
$$\mathcal{F}^i_{ss'}(\mathbf{b}) = -2\sqrt{2}P^+\frac{d}{db^2}\tilde{\Phi}_1(b^2) + \sqrt{2}m_N\epsilon^{ij}S^j\tilde{\Phi}_2(b^2) - \frac{2\sqrt{2}\epsilon^{jk}S^k}{m_N}\left[\delta^{ij}\frac{d}{db^2} + 2b^ib^j\frac{d^2}{(db^2)^2}\right]\tilde{\Phi}_3(b^2)$$

Isotropic

- Compare force density with quark (charge) density:

$$q(\mathbf{b}) = \frac{1}{2}\left[\tilde{F}_1(b^2) + \frac{b^j\epsilon^{ji}S^i}{m_N}\frac{d}{db^2}\tilde{F}_2(b^2)\right]$$

Isotropic

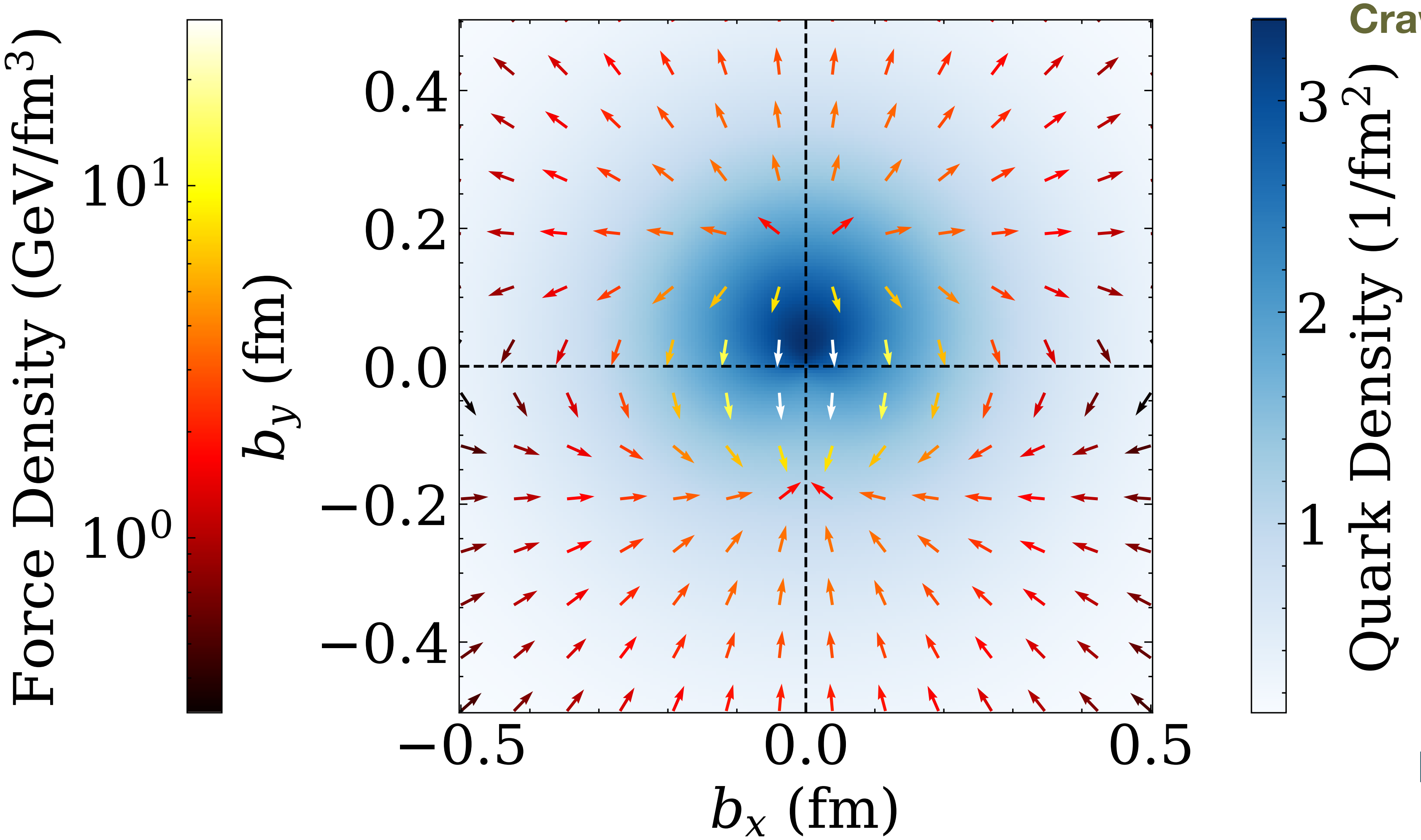


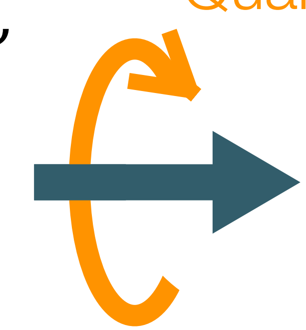
Force densities

Shown with corresponding quark densities

Transverse force densities (polarised)

Crawford, RDY et al. arxiv:2408.0362





Distorted quark distribution

$$q(\mathbf{b}) = \frac{1}{2}\left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N}\frac{d}{db^2}\tilde{F}_2(b^2)\right]$$

$$\mathcal{F}^i_{ss'}(\mathbf{b}) = -2\sqrt{2}P^+\frac{d}{db^2}\tilde{\Phi}_1(b^2) + \sqrt{2}m_N\epsilon^{ij}S^j\tilde{\Phi}_2(b^2) - \frac{2\sqrt{2}\epsilon^{jk}S^k}{m_N}\left[\delta^{ij}\frac{d}{db^2} + 2b^ib^j\frac{d^2}{(db^2)^2}\right]\tilde{\Phi}_3(b^2)$$

Non-trivial force structures

Local forces: *dividing out the quark densities*

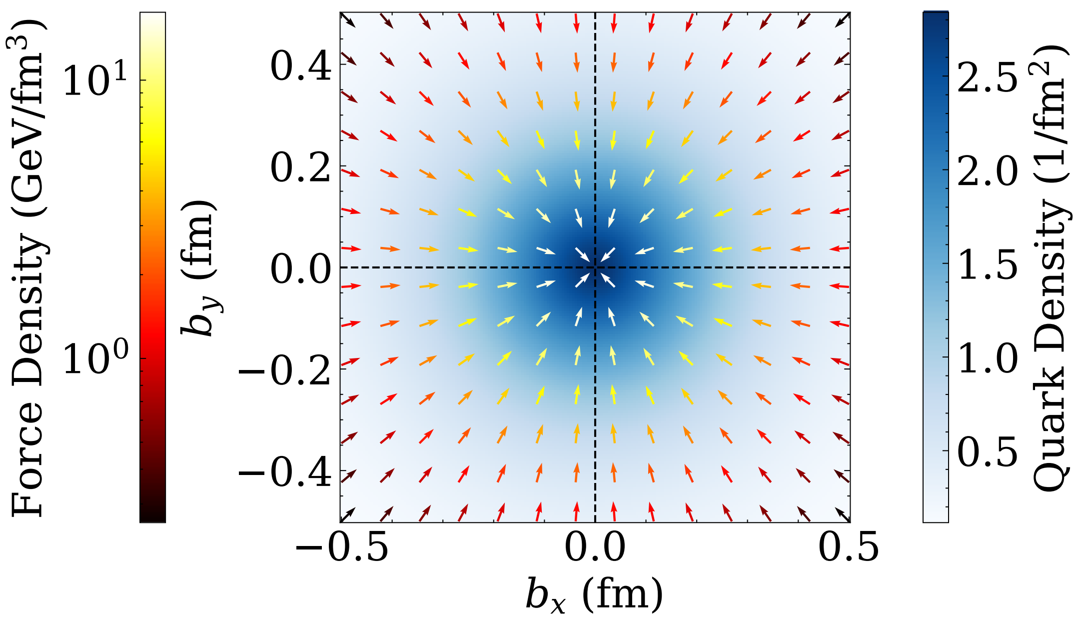
- Very tempting to describe local forces:

Force densities = “quark density” x “force”

$$\mathscr{F} \sim \langle \rho F \rangle$$

- Suggest modelling the local forces as

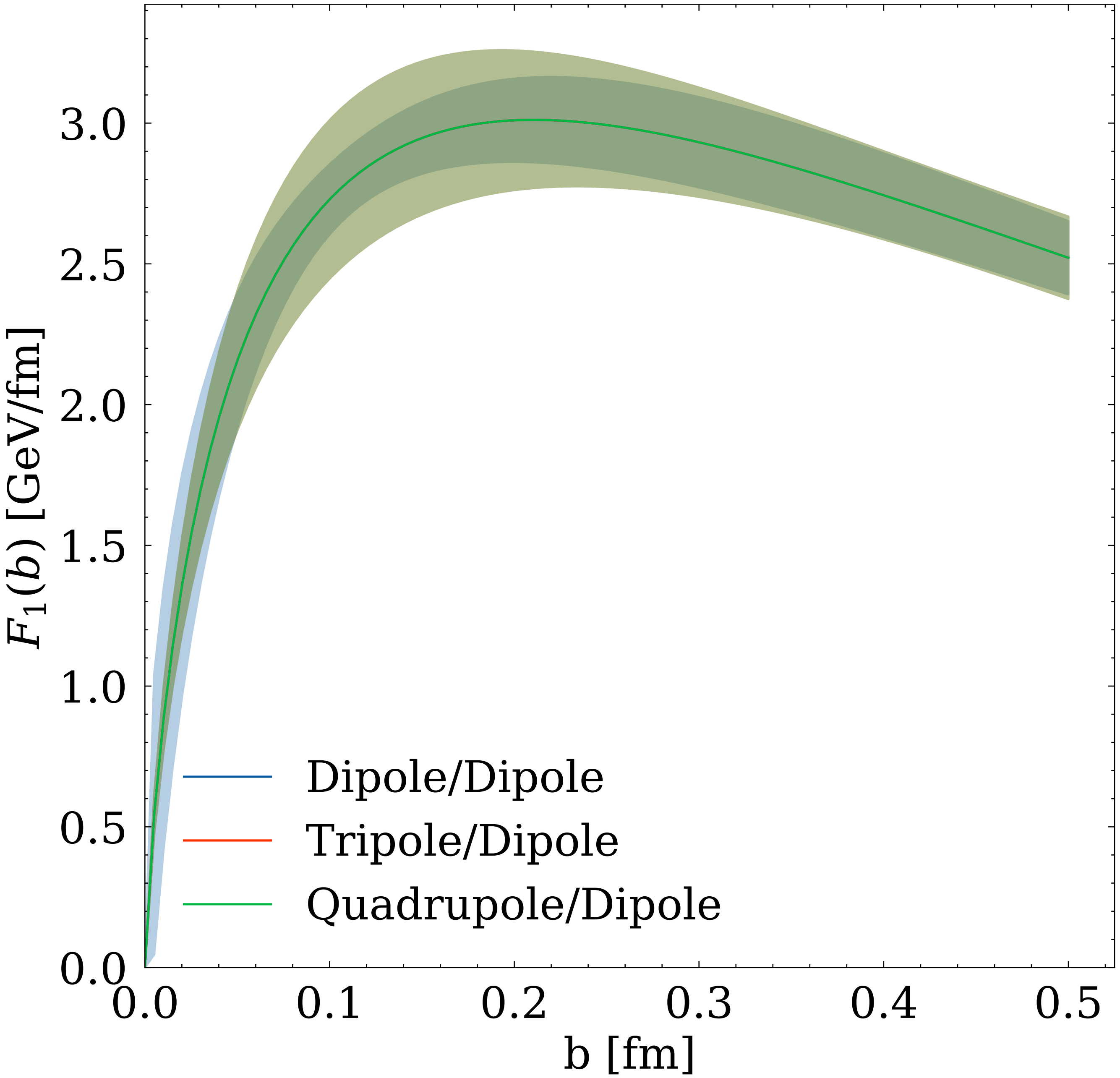
$$F \sim \frac{\langle \rho F \rangle}{\langle \rho \rangle}$$



Local forces (spin indep: Φ_1)

$$\sim \frac{\langle \overline{\psi}\gamma^+ F^{+b}\psi\rangle}{\langle \overline{\psi}\gamma^+\psi\rangle}$$

Strong forces at intermediate distances



Onset of something that mimics static quark potential?

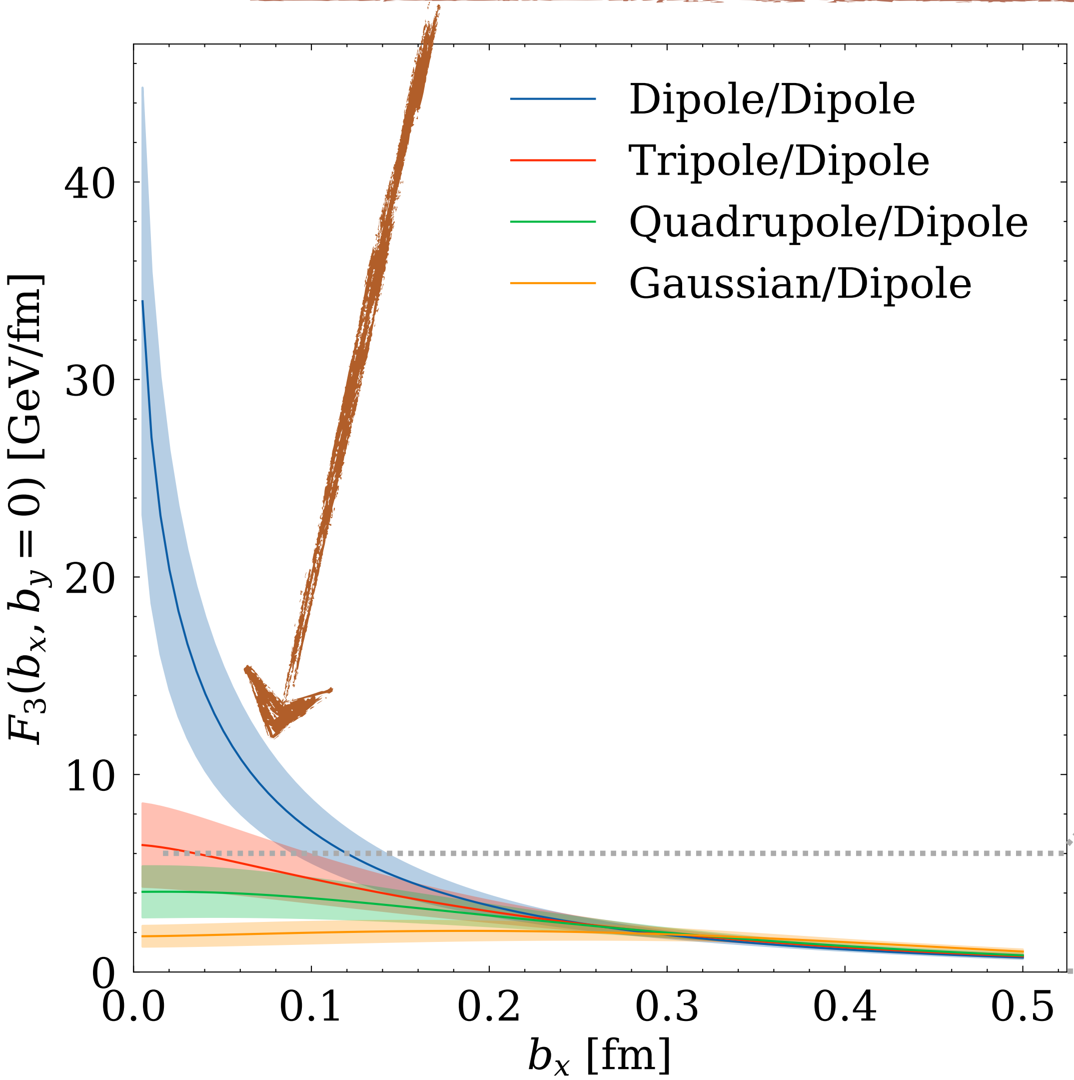
… for the future

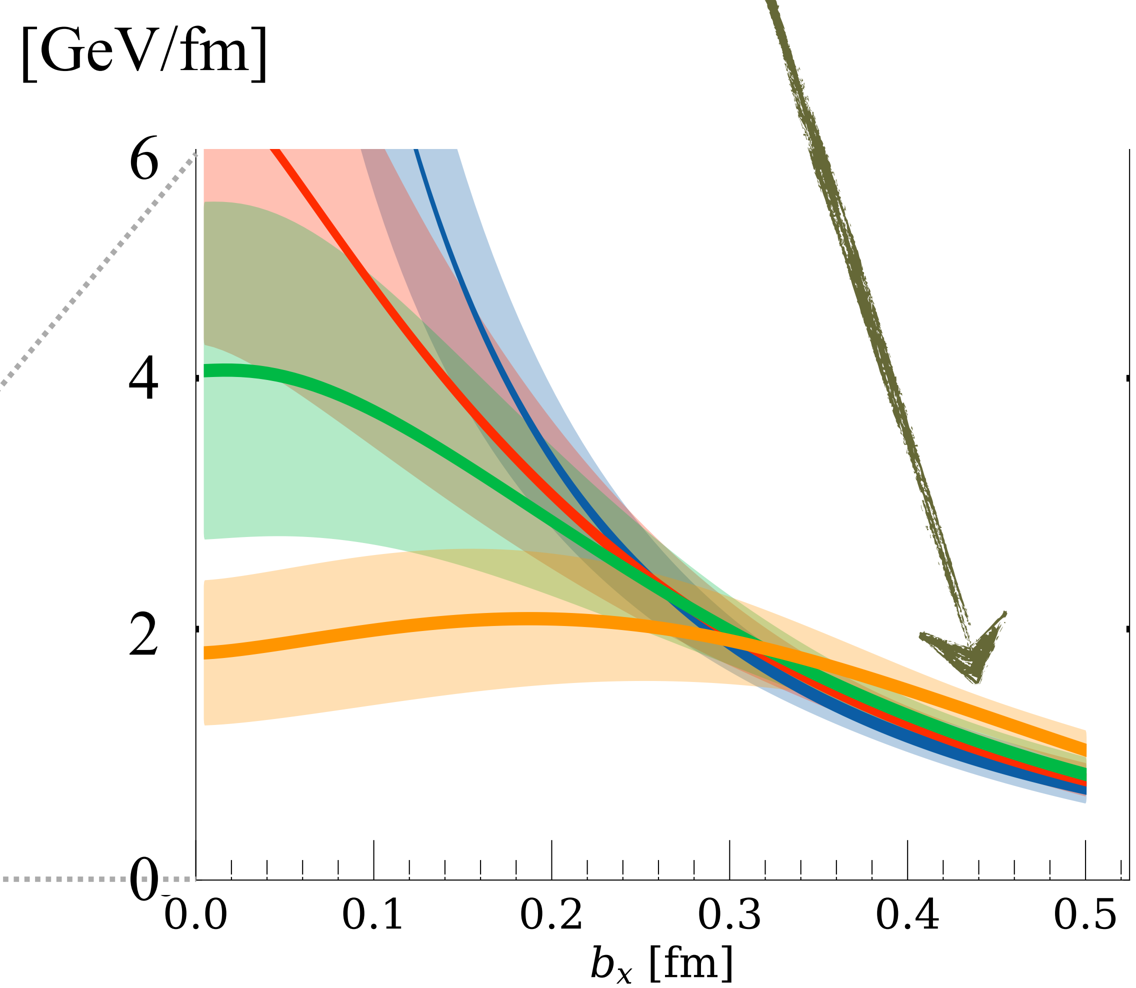
Local forces (spin dep: Φ_3)

$$\sim \frac{\langle \overline{\psi}\gamma^+ F^{+j}\psi \rangle}{\langle \overline{\psi}\gamma^+\psi \rangle}$$

Short distance behaviour quite sensitive to model: need good FFs at large Q^2

Asymptotic trend? … need better FFs at smaller Q^2



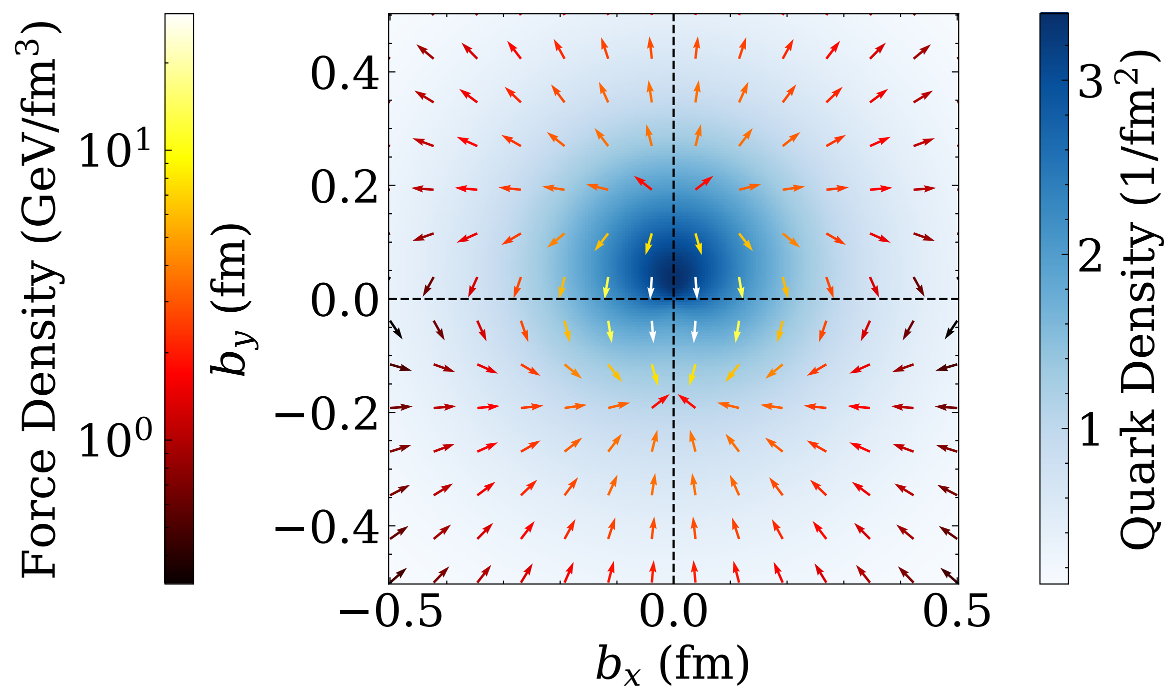


Sketch magnitude of force along b_x (fixed $b_y = 0$)

Transverse force densities: A new paradigm for describing the confinement dynamics of light quarks in hadrons

GPDs a very powerful language for describing the structure of hadrons

Twist-3 matrix elements offer new opportunities to probe interaction dynamics of partons



First force distributions from lattice QCD!

Future: lighter quark masses; control discretisation; improved renormalisation…

Crawford, RDY et al. arxiv:2408.0362

Full Lattice Details

- Use gauge ensembles generated by CSSM/QCDSF/UKQCD collaborations[4].
- Fermions described by stout-smeared non-perturbatively $\mathcal{O}(a)$ improved Wilson (SLiNC) action[5].
- Use tree-level Symanzik improved gluon action.
- All ensembles at SU(3) symmetric point.

N_f	β	$L^3 \times T$	a (fm)	m_π, m_K (MeV)	t_{sep}/a	N_{meas}
$2+1$	5.50	$32^3 \times 64$	0.074	465	11, 13, 15	3528
$2+1$	5.65	$48^3 \times 96$	0.068	412	11, 14, 17	1074
$2+1$	5.95	$48^3 \times 96$	0.052	418	14, 18, 22	1014

[4] Haar, T. R., Nakamura, Y., and Stüben, H. *EPJ Web Conf.* 2018. arXiv: `hep-lat/1711.03836`.

[5] Cundy, N. et al. *Phys. Rev. D.* 2009. arXiv: `hep-lat/0901.3302`.

Operator Mixing and Renormalisation

- Our operator mixes with lower dimensional operators, contaminating the signal.
- We incorporate this mixing when renormalising in the RI′-MOM scheme:

$$\mathcal{O}_R^{[5]}(\mu) = Z^{[5]}(a\mu)\left(\mathcal{O}^{[5]}(a) + \frac{1}{a}\frac{Z^{\sigma}(a\mu)}{Z^{[5]}(a\mu)}\mathcal{O}^{\sigma}(a)\right)$$

- Mixing coefficient determined both through LPT and non-perturbatively.
- Multiplicative renormalisation constant $Z^{[5]}(a\mu)$ computed using the procedure outlined by RQCD[9].
- Cannot match to $\overline{\text{MS}}$ at this time as perturbative calculations not available.

[9] Bürger, S. et al. *Phys. Rev. D*. 2022. arXiv: `hep-lat/2111.08306`.

Mixing Coefficient Calculation

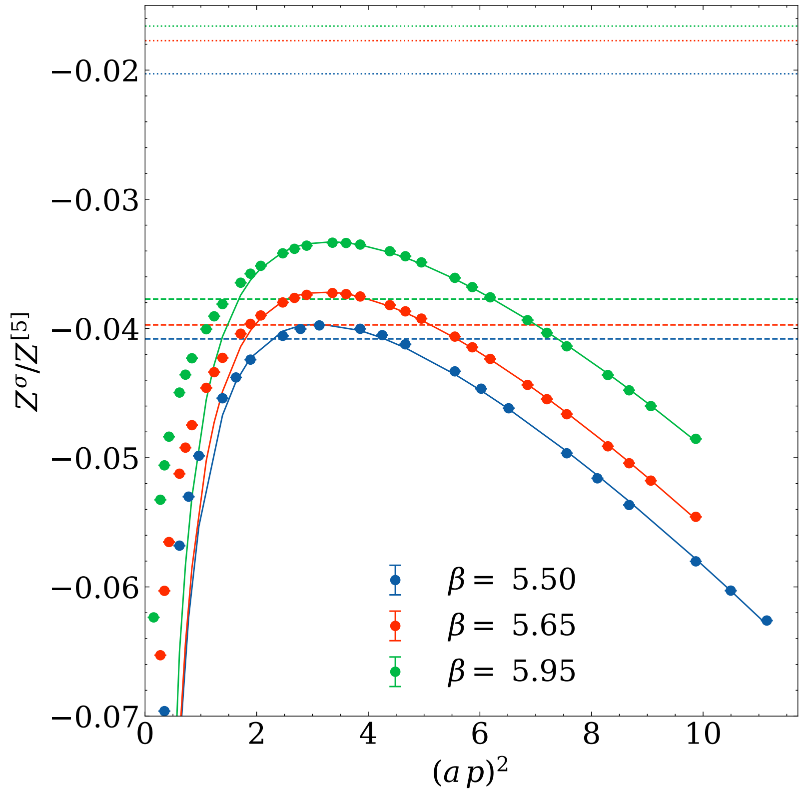


Figure: Non-perturbative calculation of the mixing coefficient $Z^\sigma/Z^{[5]}$.

- We compute the amputated 3-pt Greens function on the lattice and match it to the continuum tree-level result:

$$\mathrm{Tr}\left[\Gamma_R^{[5]}(p)\Gamma^{\sigma}_{tree}(p)^{-1}\right]_{p^2=\mu^2} = 0$$

- We fit the data using the form:

$$\frac{Z^\sigma}{Z^{[5]}} = \frac{A}{(ap)^2} + B + C(ap)^2 + D(ap)^4$$

- Extract the constant piece B.

RI′-MOM Procedure[10]

1. Compute $Z^{[5]}$ on each lattice by matching to tree-level results,

$$\frac{1}{12}\,\mathrm{Tr}\left[\Gamma_R^{[5]}(p)\Gamma_{tree}^{[5]}(p)^{-1}\right]_{p^2=\mu^2} = 1.$$

2. Choose a reference scale $\mu_0 = 2$ GeV and compute the ratio $Z^{[5]}(\mu)/Z^{[5]}(\mu_0)$ on all lattices.
3. Extrapolate this ratio to the continuum and define it as $R(\mu, \mu_0)$.
4. $Z^{[5]}(\mu')$ for each lattice, at some intermediate scale μ', is then calculated as

$$Z^{[5]}(\mu') = R(\mu', \mu_0) Z^{[5]}(\mu_0).$$

5. Evolve to some common scale μ through the one-loop formula,

$$Z^{[5]}(\mu) = \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)}\right)^{-B} Z^{[5]}(\mu'), \qquad B = \frac{1}{\frac{11}{3}N_c - \frac{2}{3}N_f}\left(3N_c - \frac{1}{6}\left(N_c - \frac{1}{N_c}\right)\right)$$

[10] Bürger, S. et al. *Phys. Rev. D*. 2022. arXiv: `hep-lat/2111.08306`.