Centre vortex geometry at finite temperature

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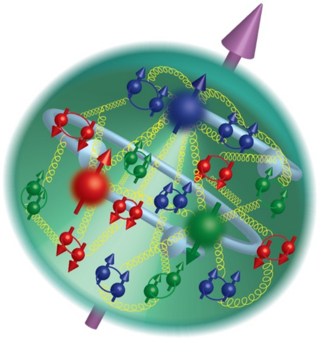
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

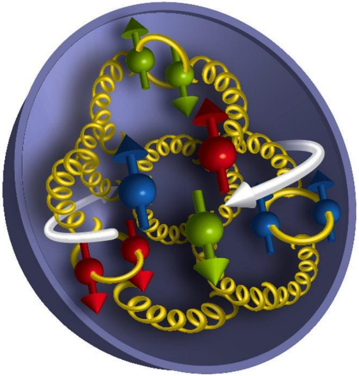
THE UNIVERSITY of ADELAIDE

Images of the proton



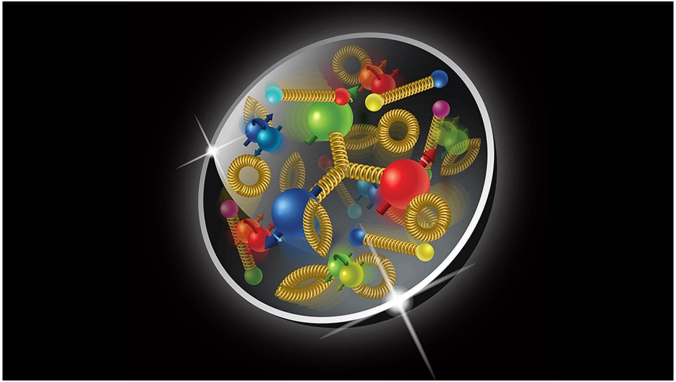
- Artistic rendering of proton structure revealing its intricate and dynamic system of quarks and gluons. (Image by Argonne National Laboratory.)

Images of the proton



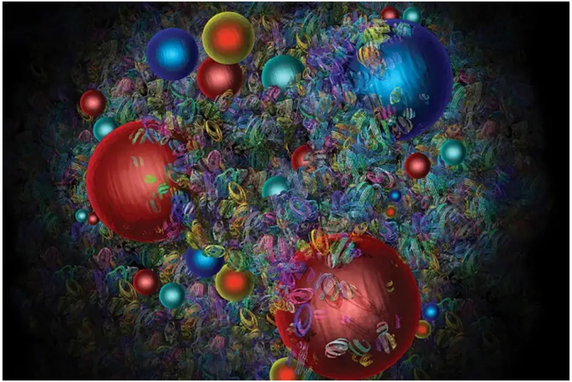
- The internal structure of a proton, with quarks, gluons, and quark spin shown. The nuclear force acts like a spring, . . . Brookhaven National Laboratory.

Images of the proton



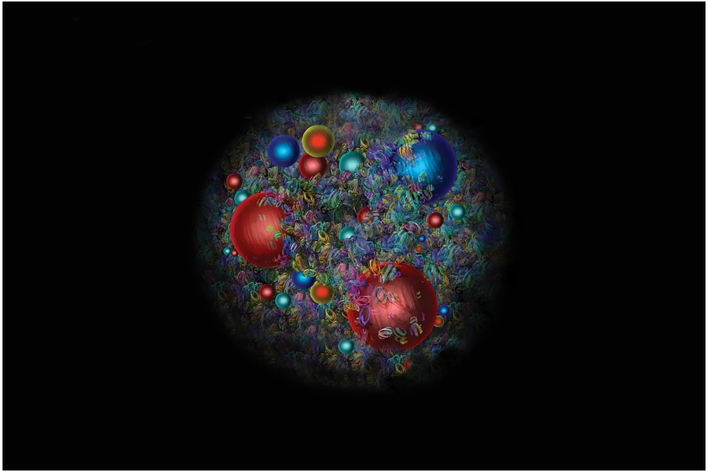
- Quark and gluon sea: in this illustration of the proton the large spheres represent the three valence quarks, the small spheres other quarks and the springs the gluons holding them together. (Courtesy: Brookhaven National Laboratory.)

Images of the proton

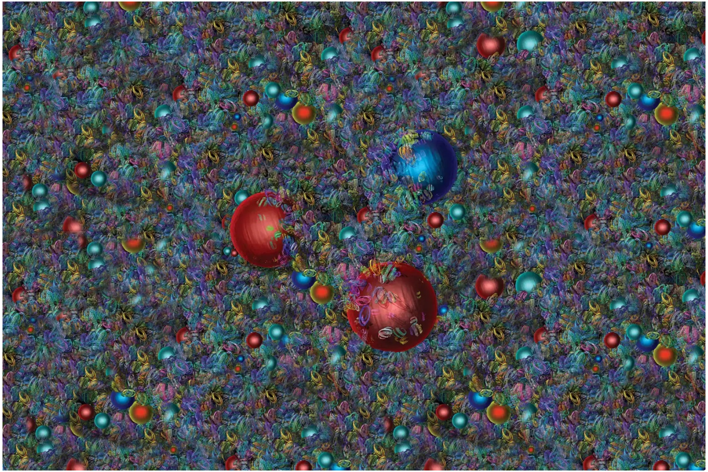


- **Glorious complexity** An artist's impression of the mayhem of quarks and gluons inside the proton. Credit: Daniel Dominguez (CERN).

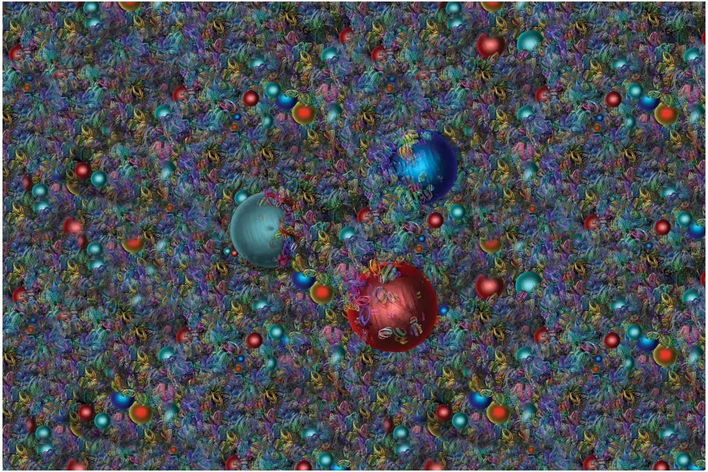
Images of the proton



The reality of a lattice QCD calculation



Valence quarks embedded in ground-state QCD-vacuum fields



Introduction

- What aspect of the nontrivial ground-state field structure confines quarks?
- Is there an essential aspect that captures the salient features of QCD?
  - Confinement.
  - Dynamical generation of mass via chiral symmetry breaking.

Centre-Vortices in the Ground-State QCD-Vacuum Fields

- Foundations
  - Spaghetti Vacuum, a condensate of vortices with finite thickness.
    - H. B. Nielsen and P. Olesen, Nucl. Phys. B **160** (1979), 380-396
  - Importance of the Z_N centre of the $SU(N)$ gauge group.
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- Renewed Excitement...
  - L. Del Debbio, M. Faber, J. Greensite,... Phys. Rev. D **55** (1997), 2298-2306
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  - L. Del Debbio, M. Faber, J. Giedt, J. Greensite,... Phys. Rev. D **58** (1998), 094501
  - R. Bertle, M. Faber, J. Greensite and S. Olejnik, JHEP **03** (1999), 019
  - M. Faber, J. Greensite, S. Olejnik and D. Yamada, JHEP **12** (1999), 012

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- Review: "The confinement problem in lattice gauge theory,"
  J. Greensite, Prog. Part. Nucl. Phys. **51** (2003), 1. 500+ citations

Centre-Vortices in the Ground-State QCD-Vacuum Fields

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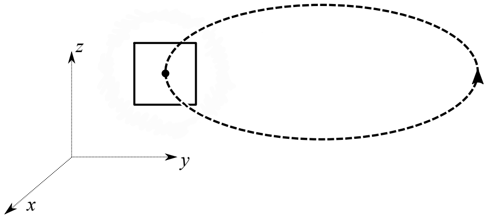
- How does centre-vortex geometry change at finite temperature?
- What happens to centre vortices in the process of deconfinement?

What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the ‘axis of rotation’ of the vortex.



A centre vortex (dashed line) intersecting a lattice plaquette (solid square).

Vortex Structure in the Colour Fields of the QCD Vacuum

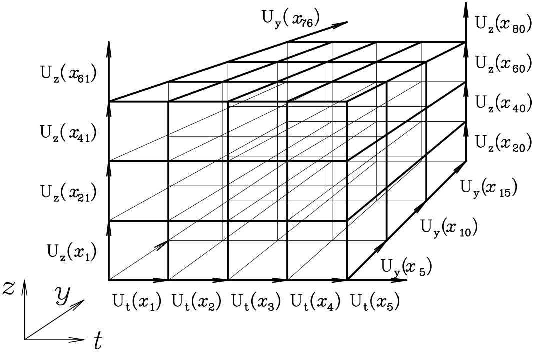
How do you find centre vortices?

Lattice Links

- On the lattice, the gluon-field is encoded in terms of the link variable

$$U_\mu(x) \simeq \exp\left(i\, a\, g\, A_\mu(x)\right) \ ,$$

a 3×3 complex special-unitary matrix.

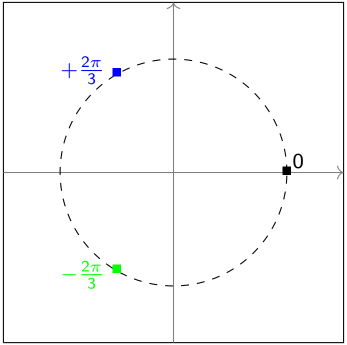


Centre Group of SU(3)

- Centre elements commute with every group element,

$$z = \exp\left(\frac{2\pi i}{3} m\right) I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3 .$$

- Each of the three centre phases corresponds to a centre element of SU(3).



1. Maximal Centre Gauge

- Gauge transformations bring the links close to an element of the group centre

$$z = \exp\left(\frac{2\pi i}{3}\, m\right) I \,, \;\; m \in \{-1,\, 0,\, +1\} \,.$$

- This is done by maximising the functional

$$R = \sum_x \sum_\mu \left| \, \mathrm{tr}[U_\mu(x)] \, \right|^2$$

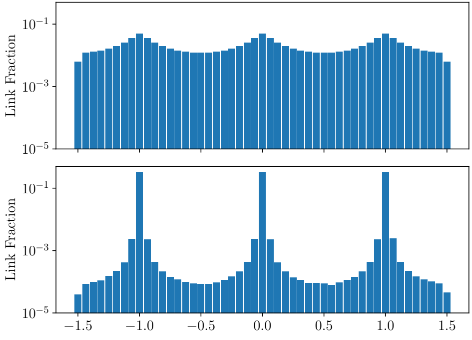
- This is called **Maximal Centre Gauge**

1. Maximal Centre Gauge

- Distribution of link phases.

$$\operatorname{tr} U_\mu^{\mathrm{MCG}}(x) = \underbrace{r_\mu(x)}_{\text{real}} \exp\underbrace{\left(\frac{2\pi i}{3}\,\phi_\mu(x)\right)}_{-\pi < \text{phase} \le \pi}, \quad -\frac{3}{2} < \phi_\mu(x) \le \frac{3}{2}.$$

- $\phi_\mu(x)$ before gauge fixing.
- $\phi_\mu(x)$ after gauge fixing.



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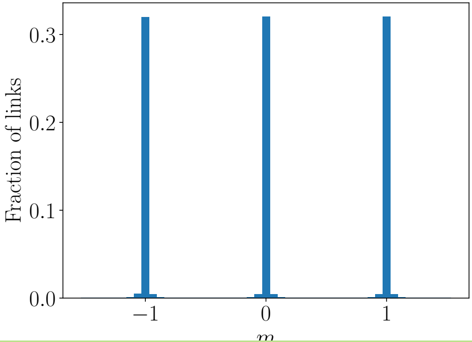
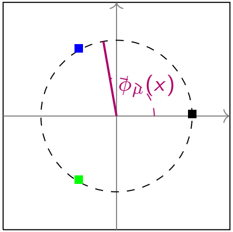


2. Centre Projection

- Project onto $Z(3)$

$$U_\mu^{\mathrm{MCG}}(x) \to Z_\mu(x) = \exp\left(\frac{2\pi i}{3}\, m_\mu(x)\right) I\,, \;\; m_\mu(x) \in \{-1,\, 0,\, +1\}\,.$$



- Eight degrees of freedom are replaced by one of the three cube-roots of 1.

2. Centre Projection

- This projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu(x)$
  - Vortex Only - $Z_\mu(x)$
  - Vortex Removed - $R_\mu(x) = Z_\mu^\dagger(x)\, U_\mu(x)$

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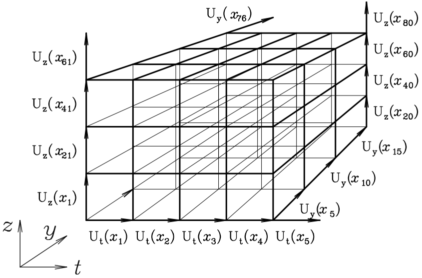
- Untouched Ensembles
  - $32^3 \times N_t$ Iwasaki pure gauge (PG), spacing $a = 0.100$ fm
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.1022$ fm, $m_\pi = 701$ MeV
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.0933$ fm, $m_\pi = 156$ MeV
    - S. Aoki, *et al.* (PACS-CS), Phys. Rev. D **79**, 034503.

3. Identifying Vortices

- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).



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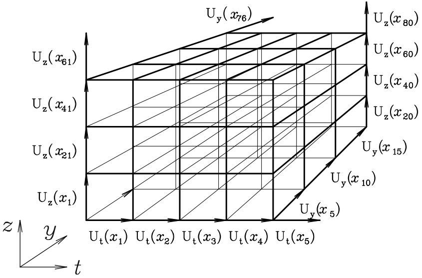


3. Identifying Vortices

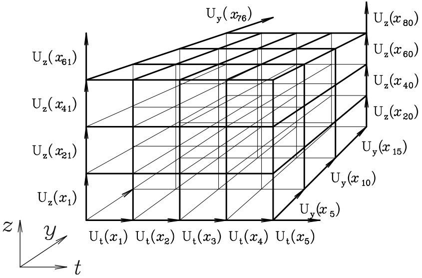
- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).

- Each plaquette takes a value from $Z(3)$.

- Non-trivial plaquettes with values

$$\exp\left(\frac{2\pi i}{3}\, m\right) \neq 1\,, \quad i.e.\ m \in \{-1, +1\}\,,$$

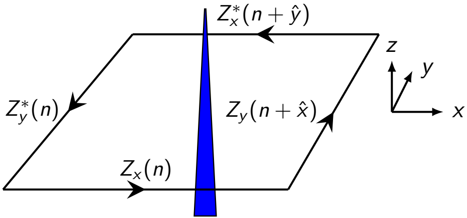
identify our thin vortices.



What do centre vortices look like?

Rendering Projected Vortices

- Vortex sheets are sliced to vortex lines in a 3D slice of the 4D lattice.
- Flow of centre charge $+1$ is indicated using a right-handed coordinate system.
- For example,
  - An $m = +1$ vortex in the x-y plane is plotted in the $+\hat{z}$ direction.



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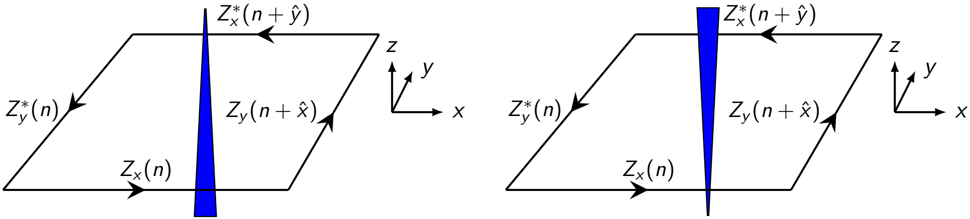
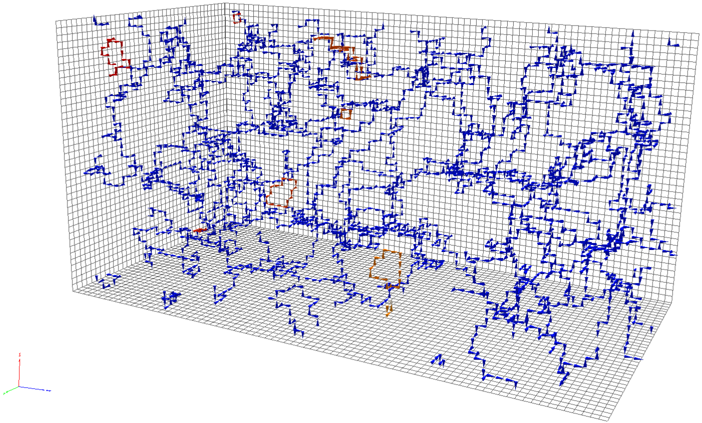
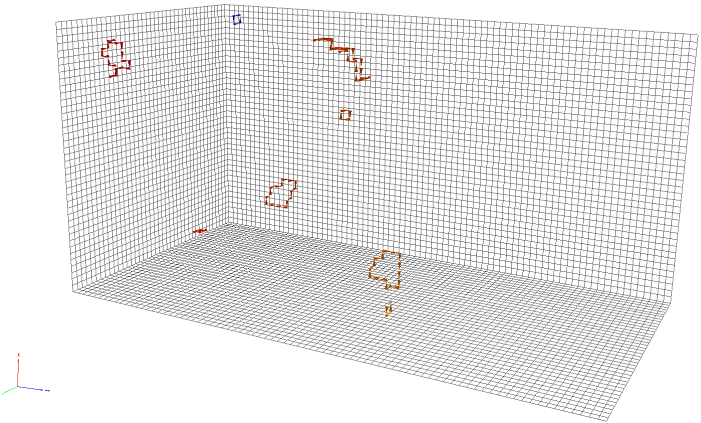
  - An $m = -1$ vortex in the x-y plane is plotted in the $-\hat{z}$ direction.



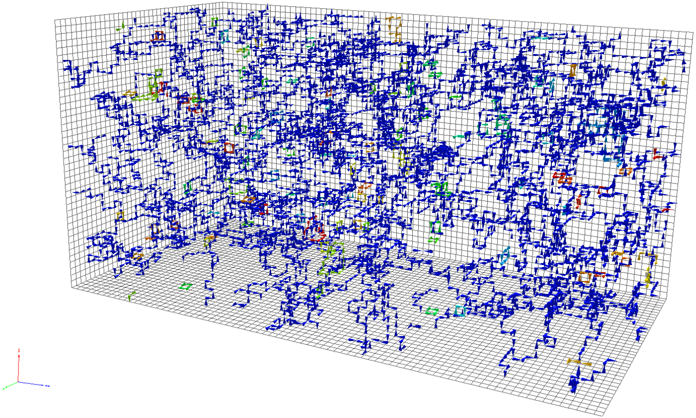
Vortices on a Pure-Gauge $32^3 \times 64$ Lattice



Secondary Loops on a Pure-Gauge $32^3 \times 64$ Lattice

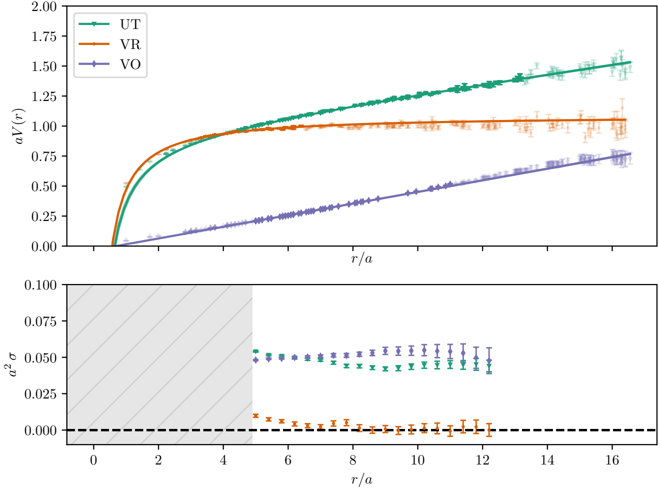


$2+1$ Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



Static Quark Potential

- Vortices capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.

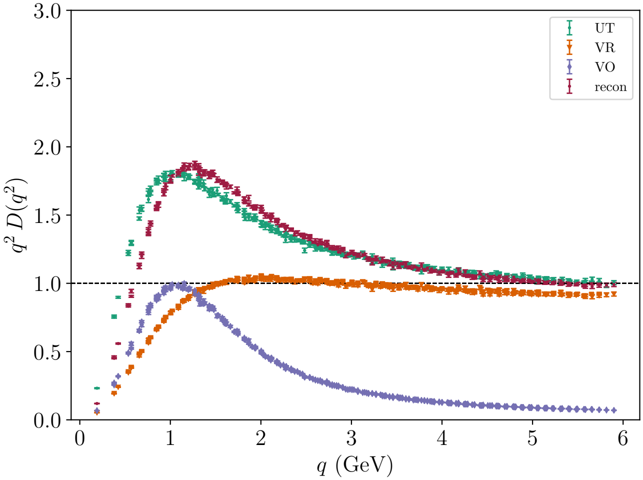


Landau-Gauge Gluon Propagator

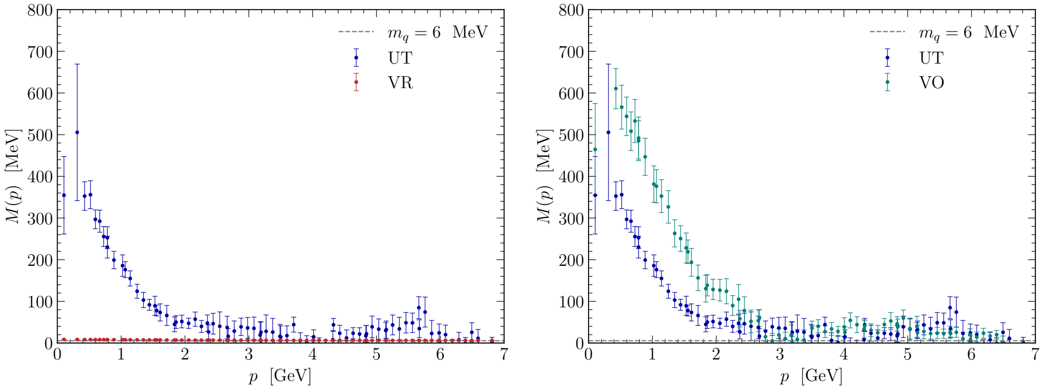
- Scalar gluon propagator

$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \to \frac{1}{q^2} \text{ at tree level}$$

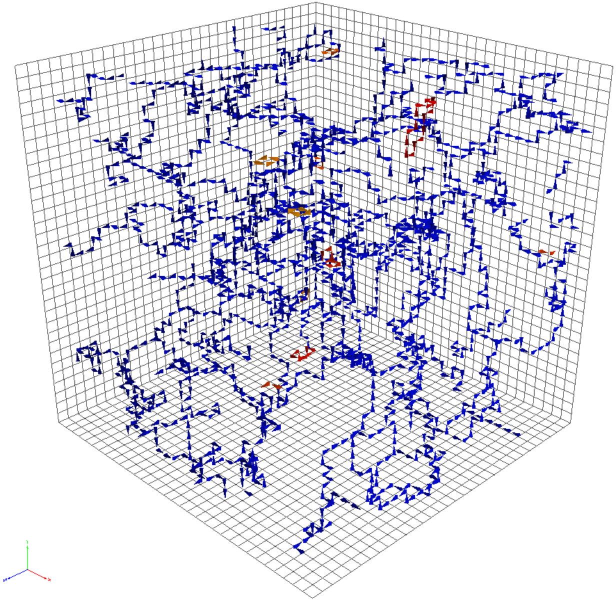
- Vortex Removal (VR) almost eliminates infrared enhancement.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstructed propagator.



Dynamical Mass Generation in the Quark Propagator



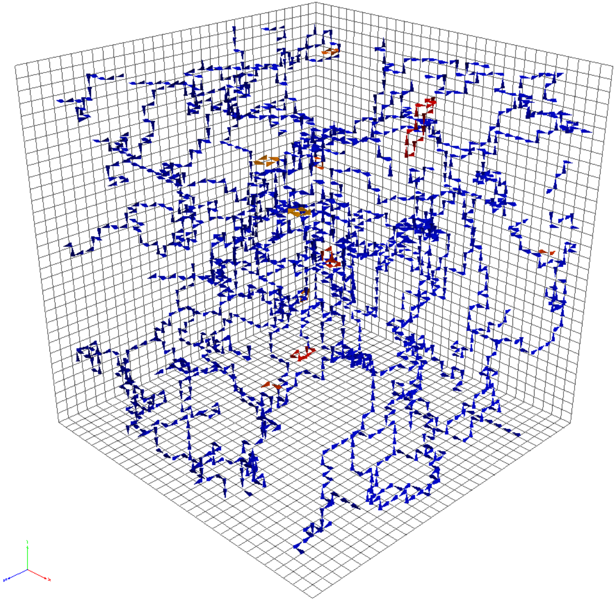
Centre Vortex Structure at Finite Temperature



- Temporal (fixed t) slice
- Viewing spatial plaquettes
  - x-y, x-z, y-z plaquettes
- $32^3 \times N_t$ lattices

N_t	T (MeV)	T/T_c
64	31	0.11
12	164	0.61
8	247	0.91
6	329	1.22
5	395	1.46
4	493	1.83

$T_c = 270$ MeV



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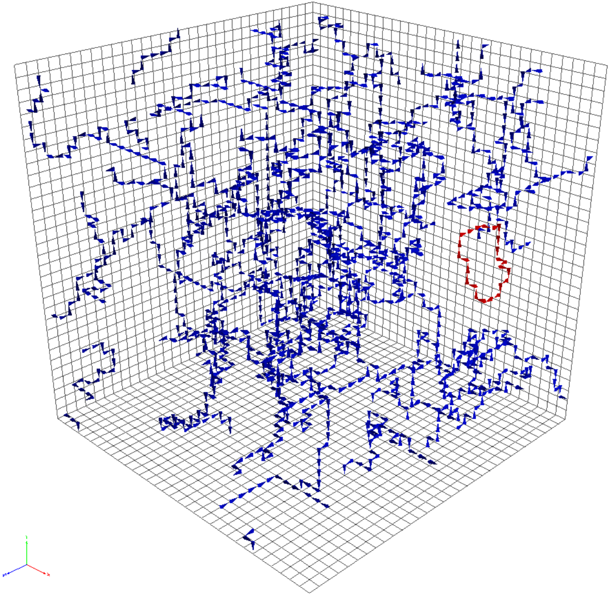

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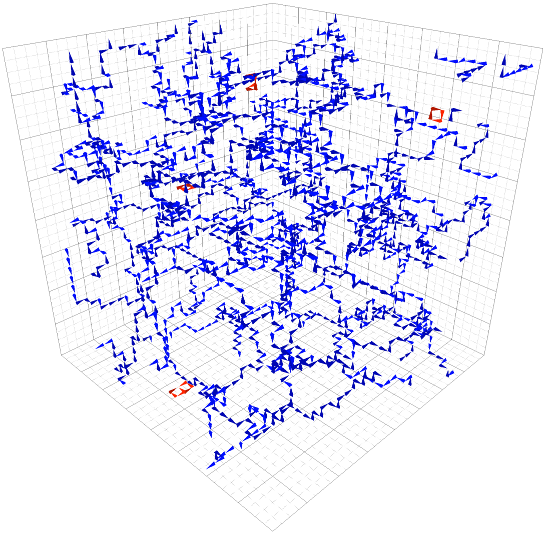

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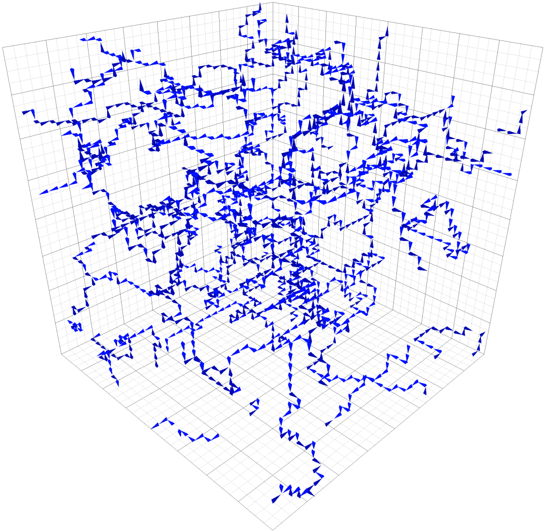

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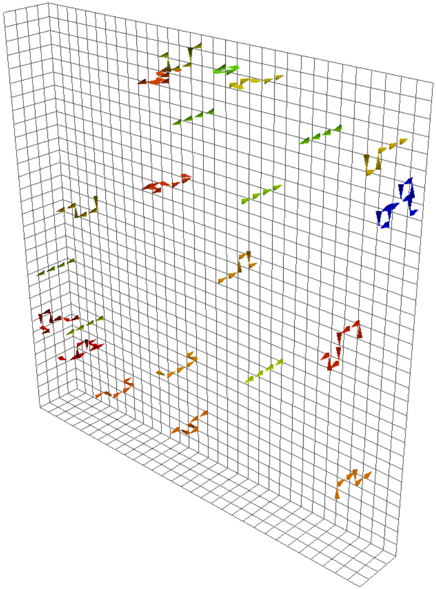

- Spatial (fixed x) slice
- Viewing mixed plaquettes
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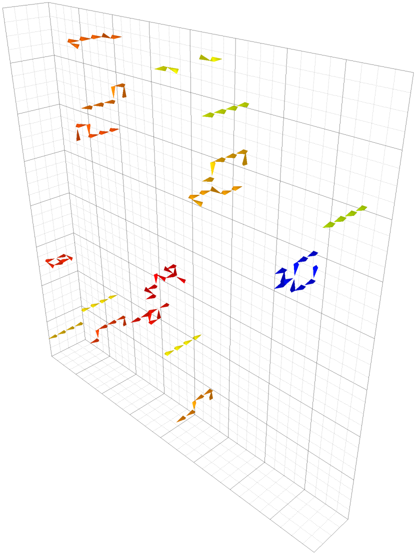

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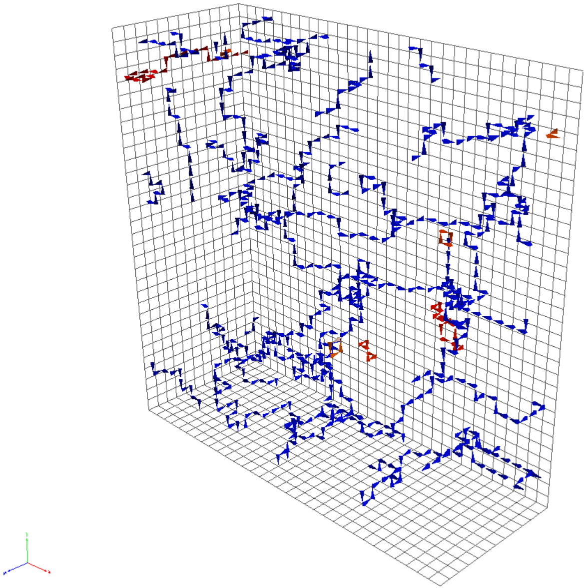

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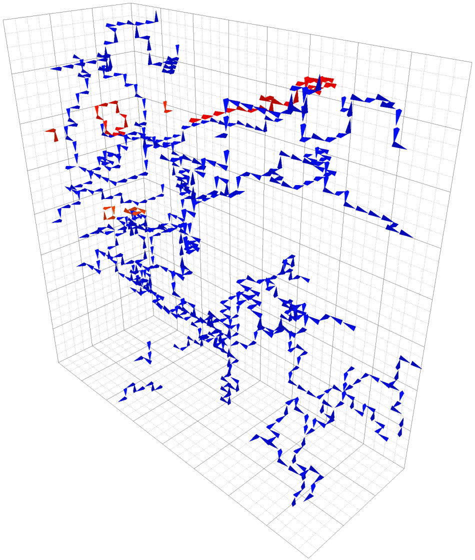

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Quantifying Centre Vortex Structure at Finite Temperature

Temporal Alignment of the Vortex Sheet

- Denote the centre charge, m, of a plaquette as

$$P_{\mu\nu}(\vec{x},t) = \exp\left(\frac{2\pi i}{3}\, m_{\mu\nu}(\vec{x},t)\right)\mathbb{I}\,, \qquad \text{with } m_{\mu\nu}(\vec{x},t) \in \{-1,0,1\}\,.$$

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- Identify cases where a vortex at one time remains in place at a later time.

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- Identify cases where a vortex at one time remains in place at a later time.
- Define the indicator function

$$\chi_{\mu\nu}(\mathbf{x},t;\tau) = \begin{cases} 1, & m_{\mu\nu}(\mathbf{x},t)\, m_{\mu\nu}(\mathbf{x},t+\tau) > 0 \\ 0, & \text{otherwise} \end{cases}\,,$$

Temporal Alignment of the Vortex Sheet

- Denote the centre charge, m, of a plaquette as

$$P_{\mu\nu}(\vec{x},t) = \exp\left(\frac{2\pi i}{3}\, m_{\mu\nu}(\vec{x},t)\right)\mathbb{I}\,, \qquad \text{with } m_{\mu\nu}(\vec{x},t) \in \{-1,0,1\}\,.$$

- Identify cases where a vortex at one time remains in place at a later time.
- Define the indicator function

$$\chi_{\mu\nu}(\mathbf{x},t;\tau) = \begin{cases} 1, & m_{\mu\nu}(\mathbf{x},t)\, m_{\mu\nu}(\mathbf{x},t+\tau) > 0 \\ 0, & \text{otherwise} \end{cases},$$

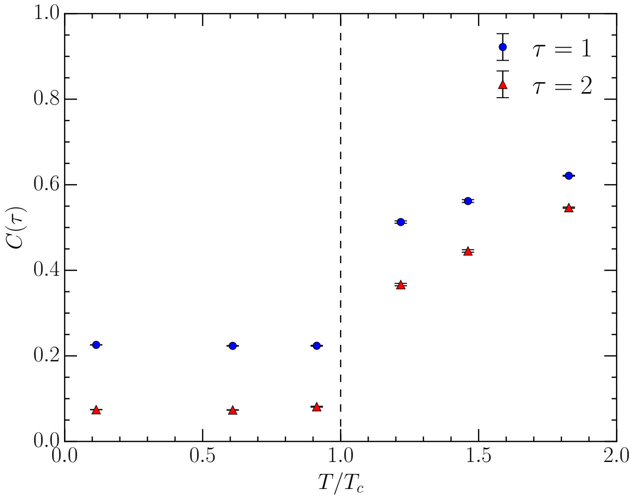
- Calculate the correlation measure

$$C(\tau) = \frac{1}{N_{\rm vor}\, N_t} \sum_{\substack{\mathbf{x},\, t,\\ i,\, j}} \chi_{ij}(\mathbf{x},t;\tau)\,,$$

where $N_{\rm vor}$, the average number of vortices per slice, ensures $C(\tau=0)=1$.

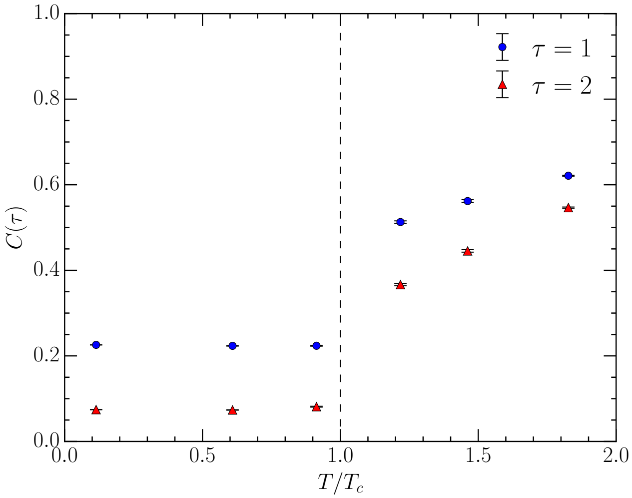
Temporal Alignment of the Vortex Sheet

- $C(\tau)$ changes significantly across the critical temperature, T_c.



Temporal Alignment of the Vortex Sheet

- $C(\tau)$ changes significantly across the critical temperature, T_c.
- Rising values indicate continued alignment with temperature.



Temporal Alignment of the Vortex Sheet

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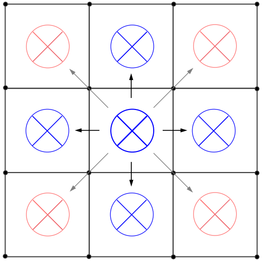
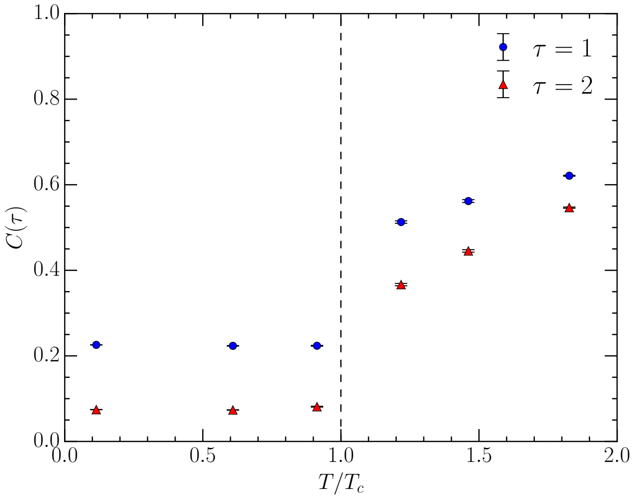
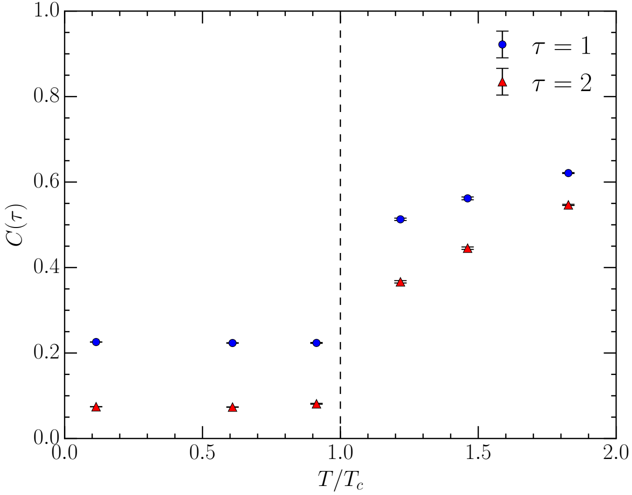
- $C(\tau = 1) \simeq 0.2$ below T_c.





Temporal Alignment of the Vortex Sheet

- Examined ergodicity via an ensemble of 100 hot starts.



Temporal Alignment of the Vortex Sheet

- Examined ergodicity via an ensemble of 100 hot starts.
- Examined lattice spacing dependence by
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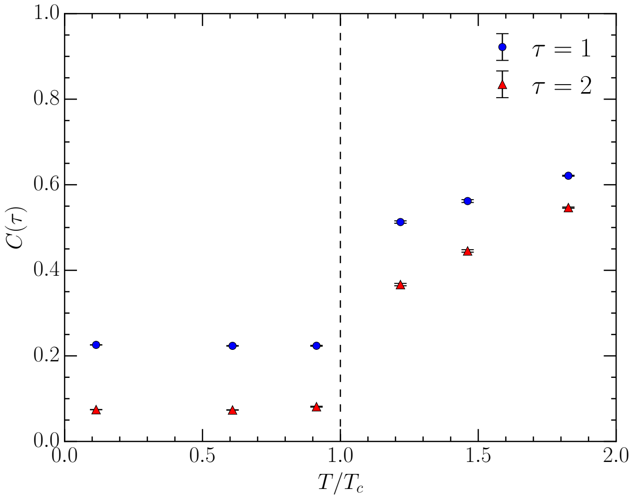
  - Doubling the lattice dimensions to maintain the physical volume.



Temporal Alignment of the Vortex Sheet

- Examined ergodicity via an ensemble of 100 hot starts.
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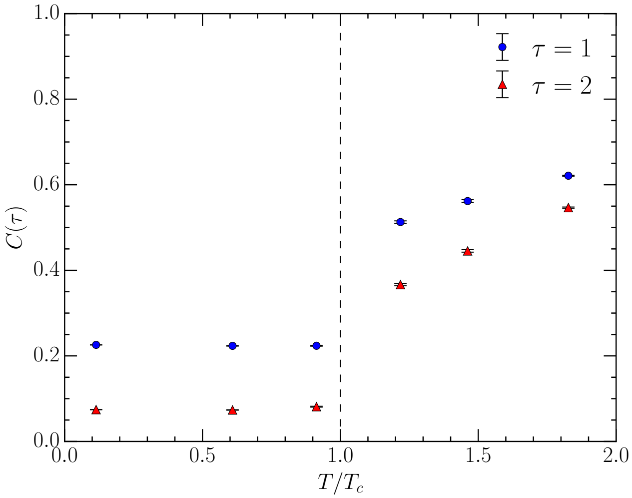
- Considered the simple Wilson gauge action as an alternative.



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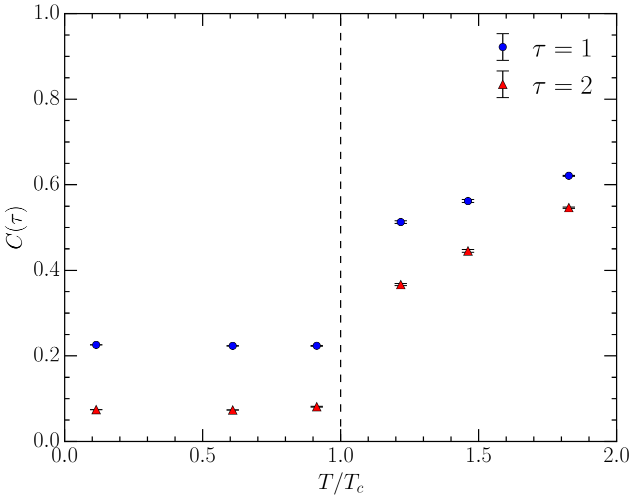
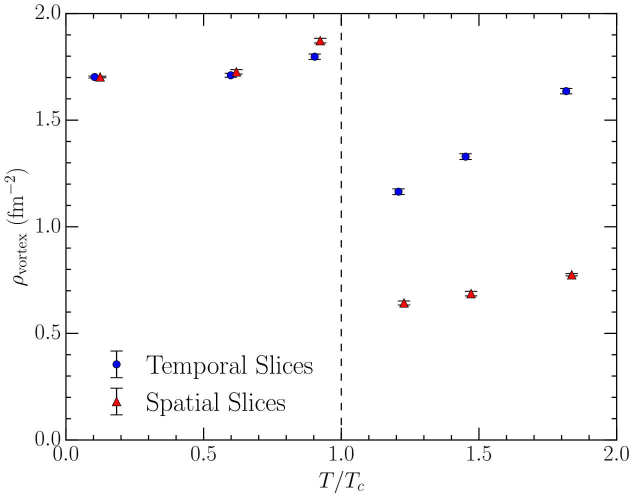
- Considered the simple Wilson gauge action as an alternative.
- The correlations persist.



Vortex Area Density

- The vortex density of a slice is

$$\rho_{\text{vortex}} = \frac{\text{Number of Vortices}}{\text{Number of Plaquettes}}\,\frac{1}{a^2}\,.$$



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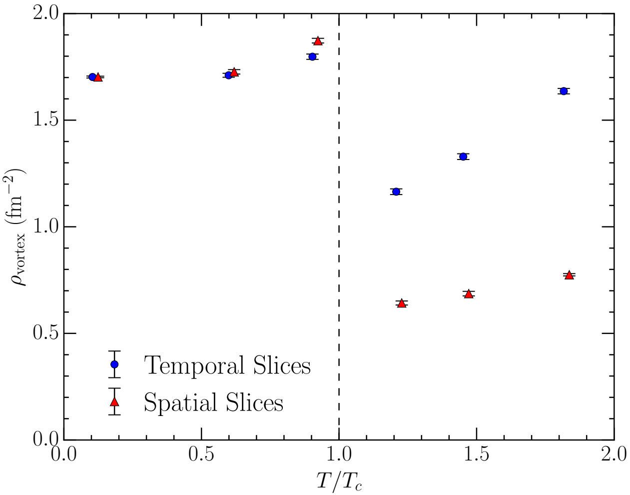

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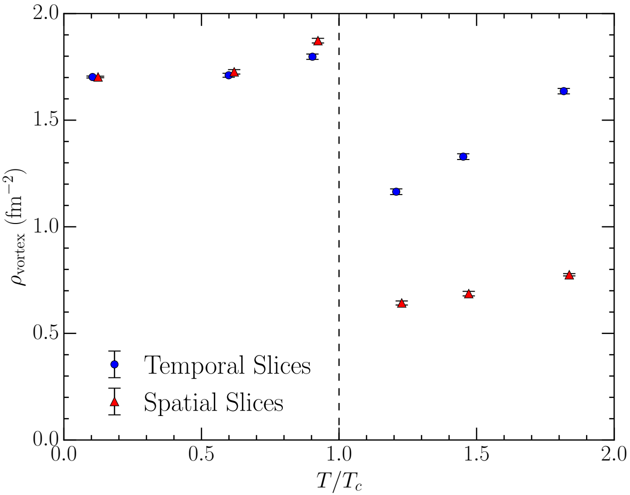
- The sharp spatial-slice decrease as T_c is crossed coincides with the absence of a percolating cluster.



Vortex Area Density for Spatial Slices

- Spatial Slices:
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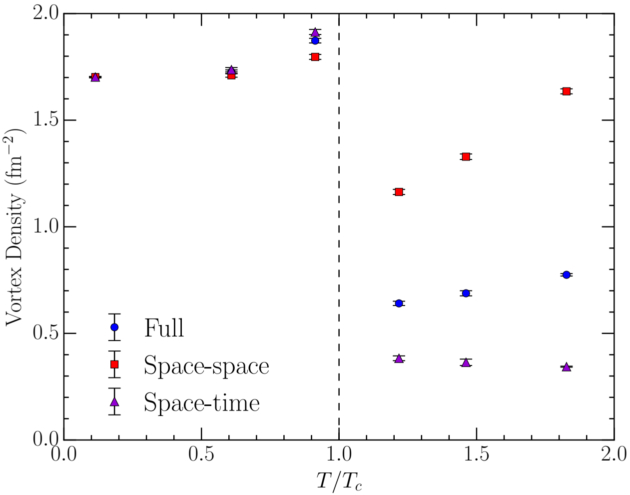

Vortex Area Density for Spatial Slices

- Spatial Slices:
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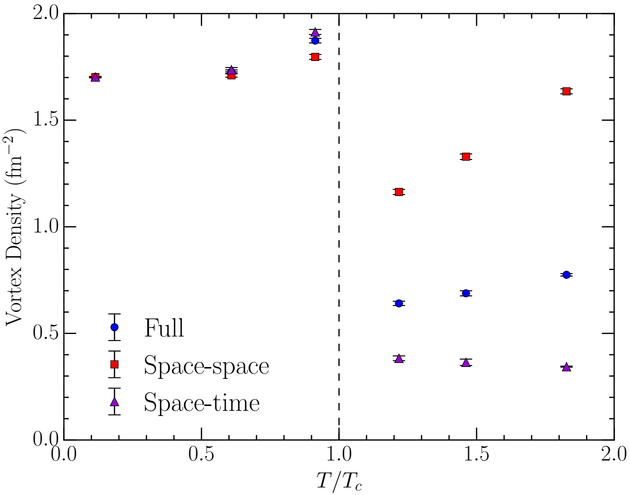

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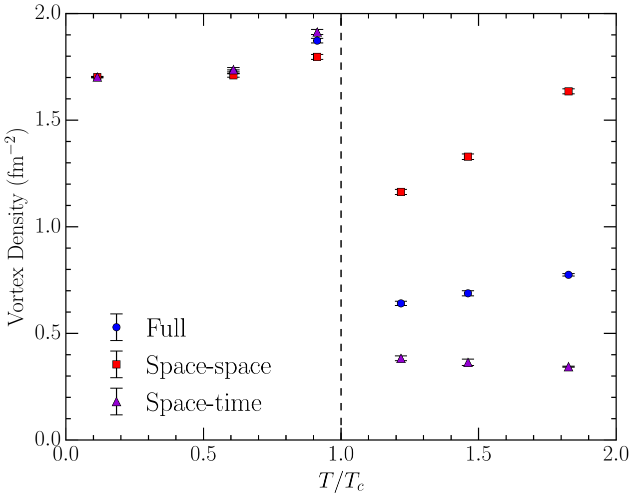

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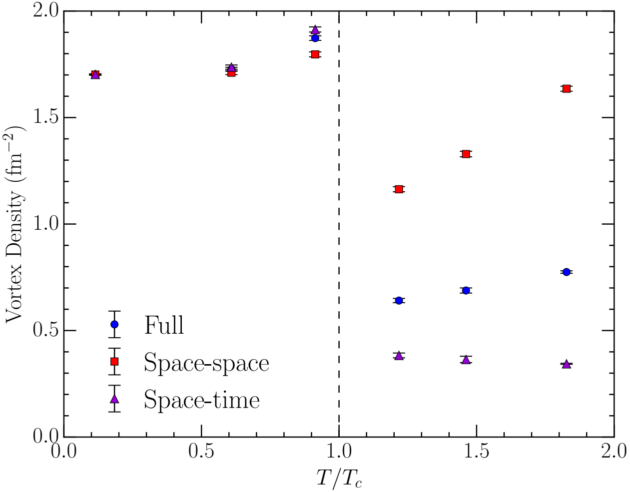

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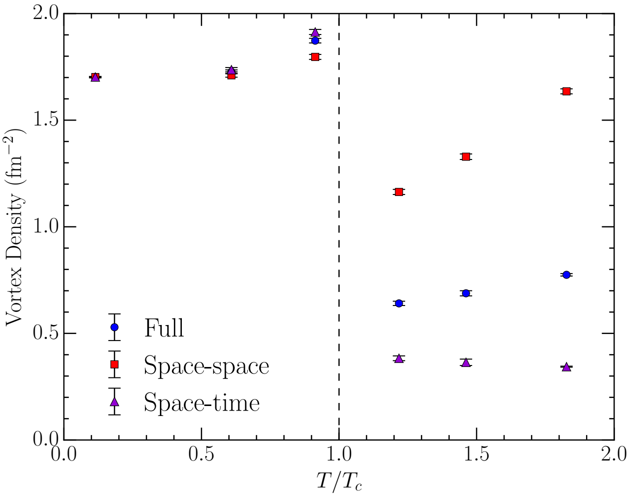
- Mild increase in the full density is due to rapid increase in the spatial plaquettes.



Vortex Area Density for Spatial Slices

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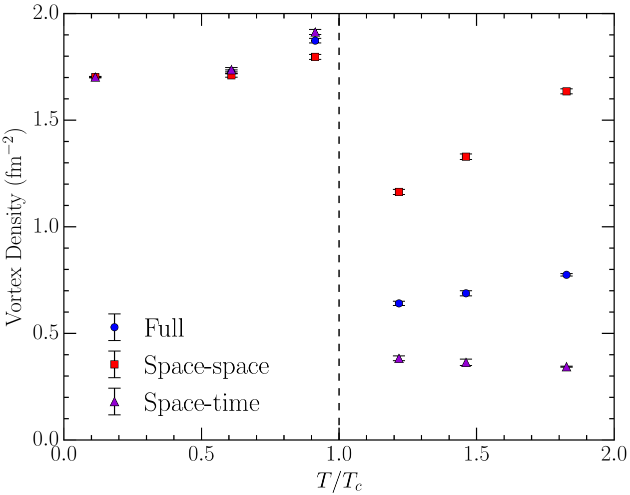
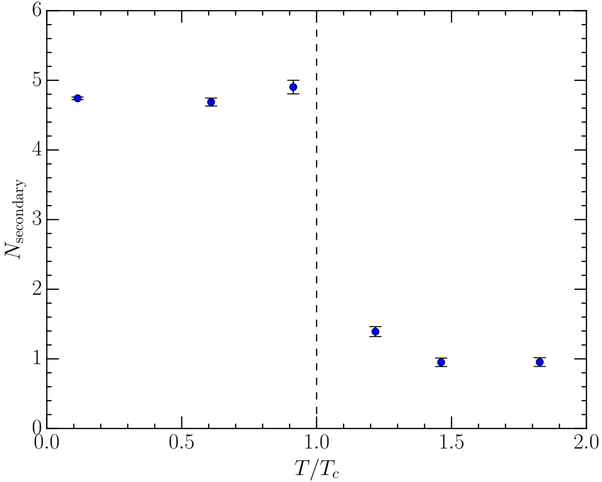
- Consistent with continued temporal alignment of the vortex sheet.
- Note increase in space-time vortex density just below T_c.



Non-percolating secondary vortex clusters in temporal slices

- Secondary clusters are smaller loops, disconnected from the main percolating cluster.



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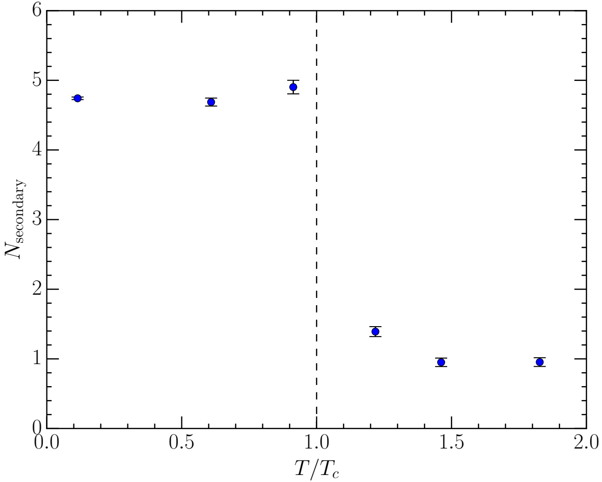

Non-percolating secondary vortex clusters in temporal slices

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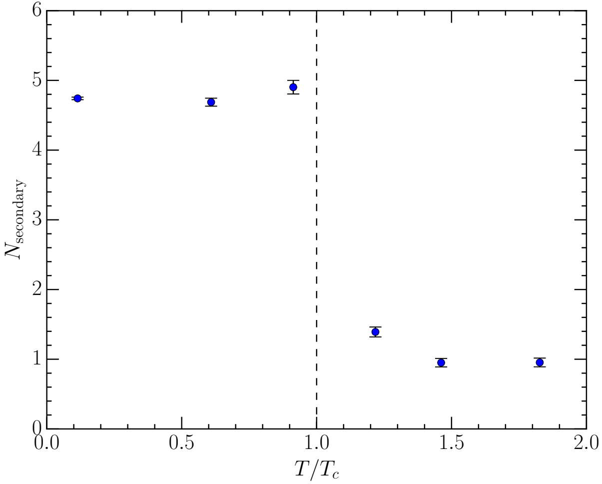
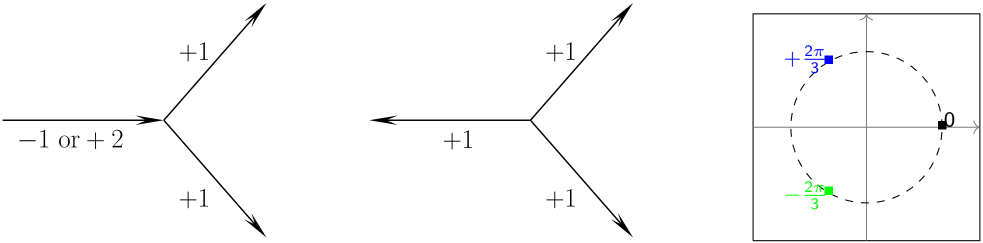
- $N_{\mathrm{secondary}}$ is the average number seen on a temporal slice.
- Pronounced drop:
  - A key aspect of vortex geometry characterising deconfinement.



Branching Point Geometry

Branching Points versus Monopoles

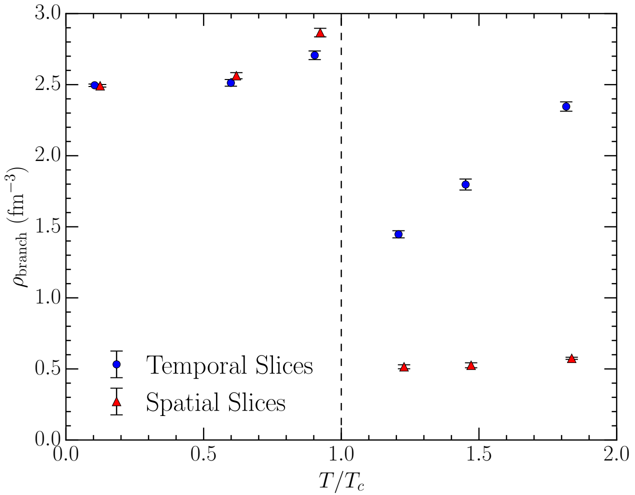


- Our convention illustrates the directed flow of charge $m = +1$.
- Arrows indicate the direction of flow for the labelled charge.
- However, a vortex monopole with charge $+1$ flowing out of the vertex (centre) is equivalent to
  a vortex branching point with centre charge $+2$ flowing into a vertex (left).

Branching-point geometry

- Branching-point volume density

$$\frac{\text{Number of branching points}}{\text{Number of elementary cubes}}\,\frac{1}{a^3}\,.$$



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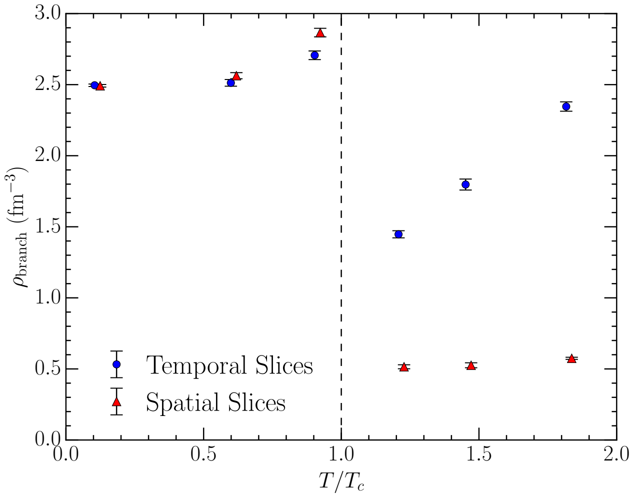

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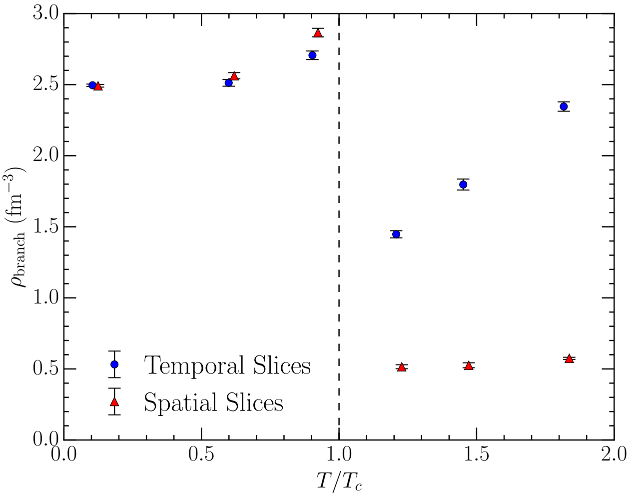

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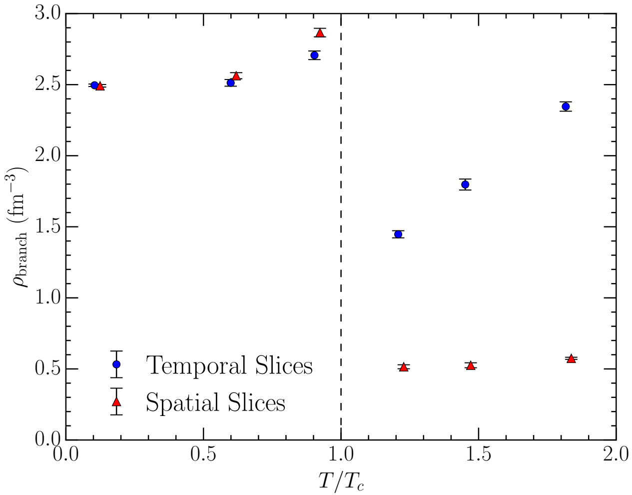
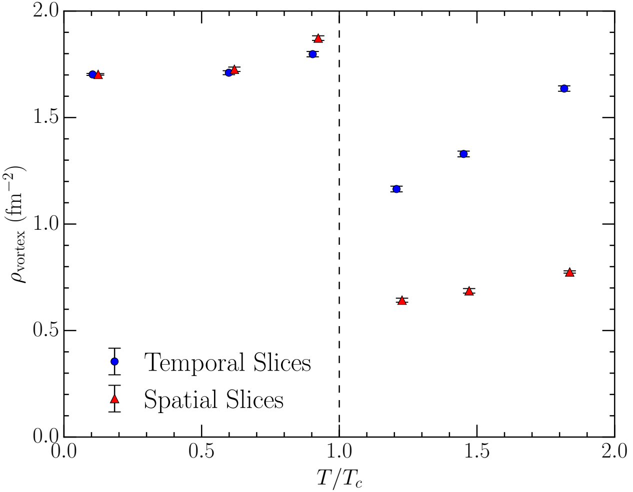
- There are two drivers behind the rapid increase in ρ_{branch} at large T.



Vortex Area Density

- Already observed an increase in the vortex density
  - Naturally allows for more branching chances.



Linear Branching Density

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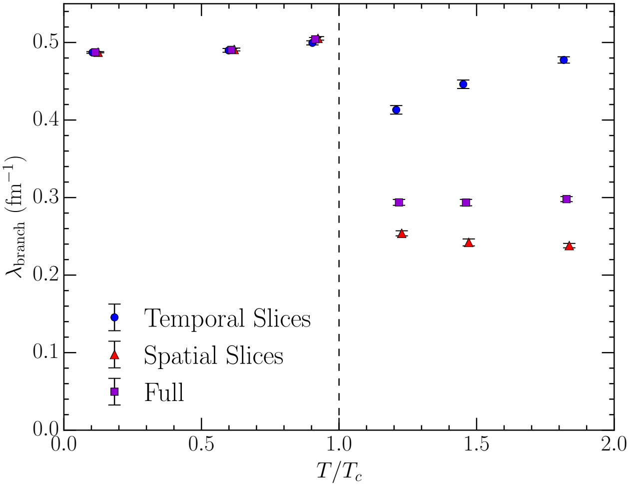
- Define a linear branching density

$$\sim \frac{\text{Number of branching points}}{\text{Number of vortices}} \,.$$

$$\lambda_{\text{branch}} = \frac{\rho_{\text{branch}}}{3\,\rho_{\text{vortex}}} \,.$$



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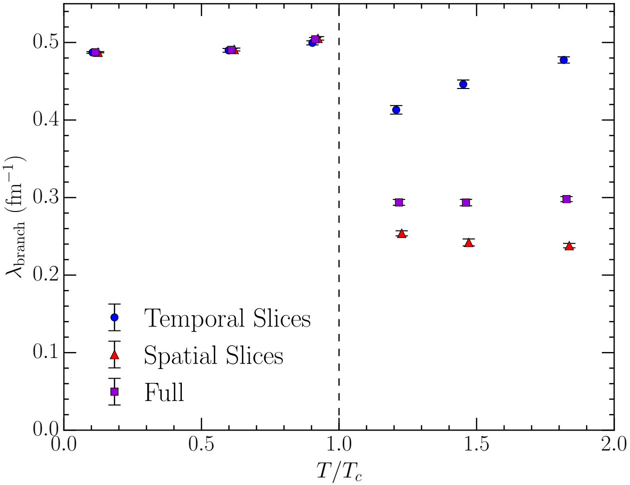

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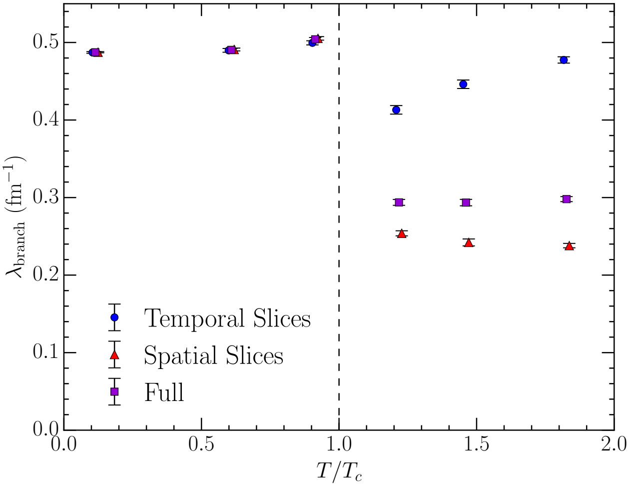
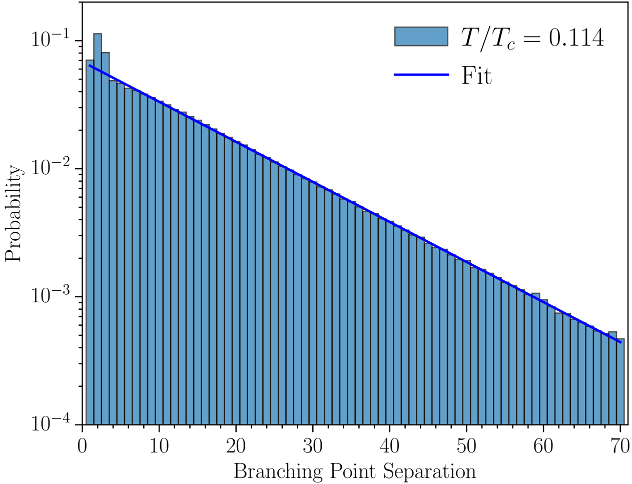
- Describes a rate of branching (branches/fm).
- Full reports an average over the 4 slice directions.



Histogram of Vortex Chain Lengths in Temporal (fixed t) Slices

- Histogram of vortex chain lengths in the percolating cluster.
- Chain length is the number of jets from one branching point to the next.
- The histograms are normalised to unit probability.



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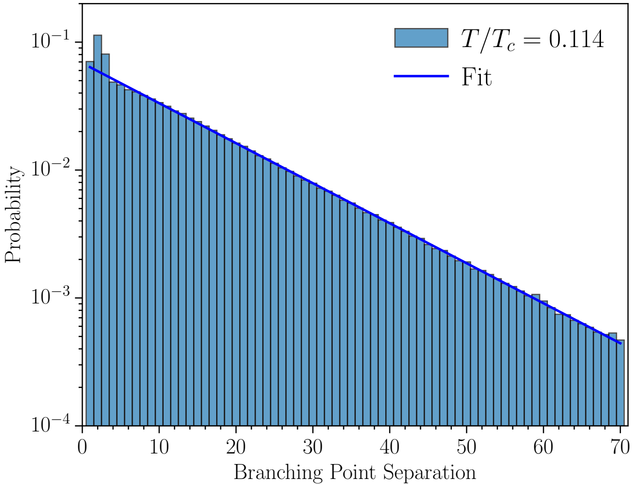
- Moderate size loops, $k > 5$, are exponentially distributed.
  - Fixed probability of branching .
  - Branching is independent of length.



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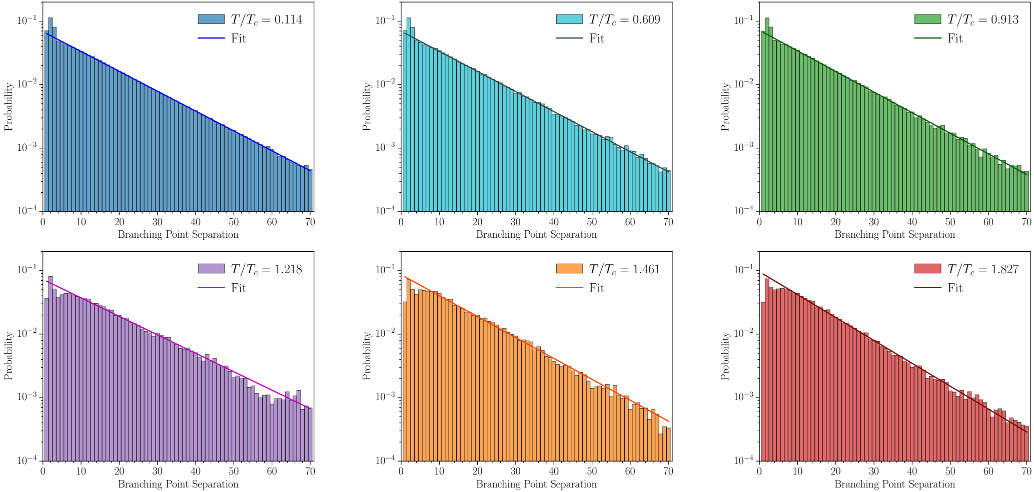
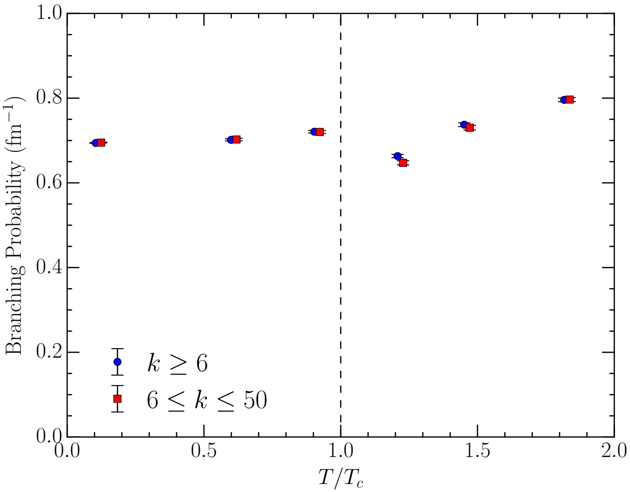


Branching Probability of a Vortex Path

- Long-range ($k \geq 6$) branching probability.
- Chain lengths $k \leq 70$ and $k \leq 50$ are considered.
- Note the mild drop across the phase transition.
- Probability grows with temperature to large values.



Selected References

- "Centre vortex geometry at finite temperature," J. A. Mickley, W. Kamleh and DBL, Accepted Phys. Rev. D (2024) [arXiv:2405.10670 [hep-lat]].
- "Centre vortex structure in the presence of dynamical fermions," J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **107** (2023) no.9, 094507 [arXiv:2302.05897 [hep-lat]].
- "Impact of dynamical fermions on the center vortex gluon propagator," J. Biddle, W. Kamleh and DBL, Phys. Rev. D **106** (2022) 014506 [arXiv:2206.02320 [hep-lat]].
- "Static quark potential from centre vortices in the presence of dynamical fermions," J. Biddle, W. Kamleh and DBL, Phys. Rev. D **106** (2022) 054505 [arXiv:2206.00844 [hep-lat]].
- "Dynamical fermions, centre vortices, and emergent phenomena," DBL, J. Biddle, W. Kamleh and A. Virgili, EPJ Web Conf. **274** (2022), 01002 [arXiv:2211.13421 [hep-lat]]. *Plenary presentation summary to appear in the proceedings of the XVth Quark Confinement and the Hadron Spectrum conference, 1st-6th August 2022, Stavanger, Norway.*

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  - Smaller secondary clusters are suppressed.
- Observe a rapid rise in the branching point density with high T..
  - The branching probability and branching rate increase with T.
  - The vortex density increases $\rightarrow$ more opportunities to branch.

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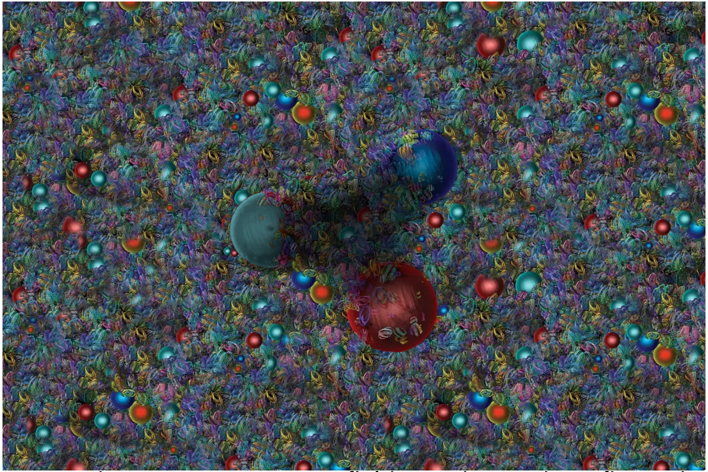
  - A commensurate condensation of branching points on temporal slices.
- An exchange that leaves the linear branching probability averaged over the four slice dimensions invariant.

Additional Information

Ironic incongruity



- Static valence quarks suppress vacuum fields as they induce flux-tube tunnels.

Restoration of Chiral Symmetry

- If vortices are responsible for $D\chi SB$, then their removal should restore chiral symmetry
$$\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{A}}$$
- Expect hadrons related by chiral transformations to become degenerate

$$\begin{array}{ccc} \pi & \xleftrightarrow{\mathrm{U}(1)_{\mathrm{A}}} & a_0 \\ \rho & \xleftrightarrow{\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}}} & a_1 \\ N & \xleftrightarrow{\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}}} & \Delta \end{array}$$

- At light quark masses, all symmetries are observed to be restored.
- A. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) 125002 [arXiv:1708.06789 [hep-lat]].

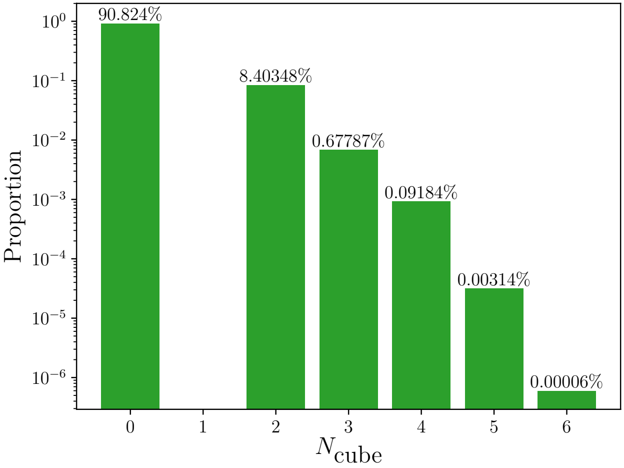
Visualising Centre Vortices

- Consider the number of vortices entering a 3D cube on the dual lattice.

$N_{\text{cube}}(\tilde{x})$	Interpretation
0	No vortices present.
1	Terminating vortex, forbidden by Bianchi*.
2	Vortex line flowing through the cube.
3	Simple three-way vortex monopole.
4	Vortex intersection.
5	Complex five-way monopole path.
6	Vortex intersections or double monopoles.

*Bianchi identity implies a continuous flow of centre vortex flux through a spatial cube.

Visualising Centre Vortices



Space-Time Oriented Vortices

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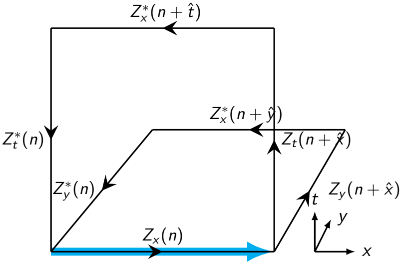
- The three jets associated with the spatial x-y, y-z and z-x plaquettes, are complemented by
  - Jets in the three forward time x-t, y-t and z-t plaquettes, and
  - Jets in the three backward time x-t, y-t and z-t plaquettes.
- See "Visualization of center vortex structure," to link vortices to topological charge.

  J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].

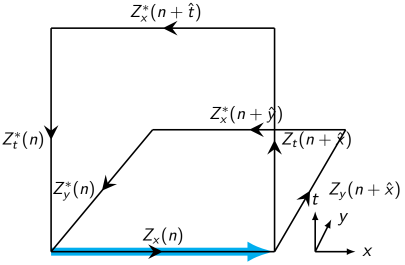
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex.



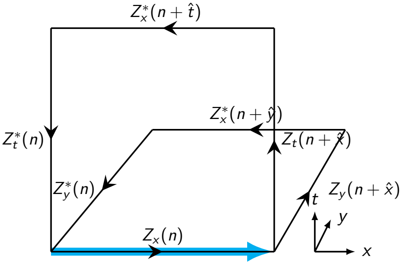
Rendering Space-Time Oriented Projected Vortices

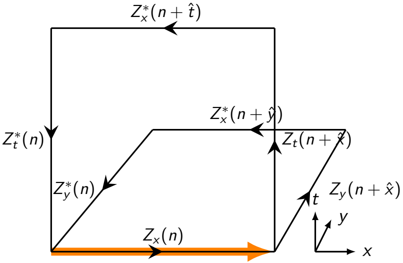
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.



Rendering Space-Time Oriented Projected Vortices

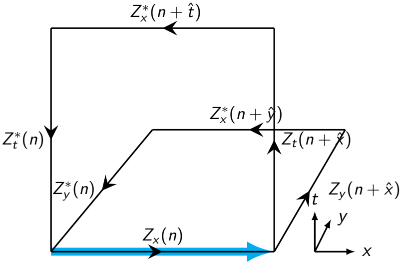
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.

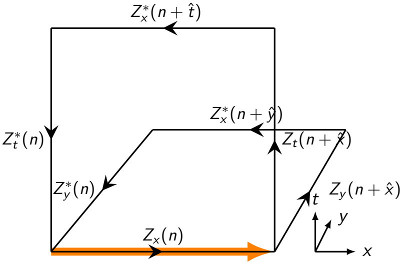




Rendering Space-Time Oriented Projected Vortices

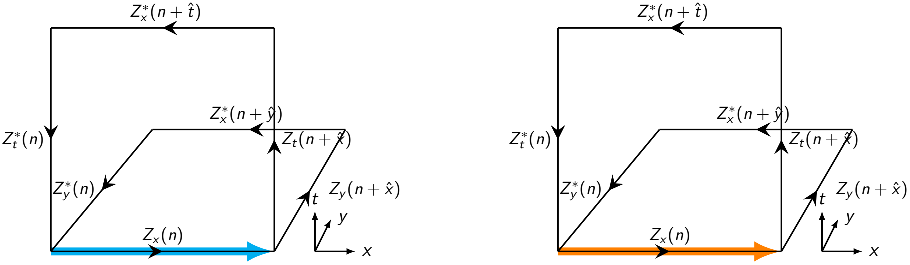
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.



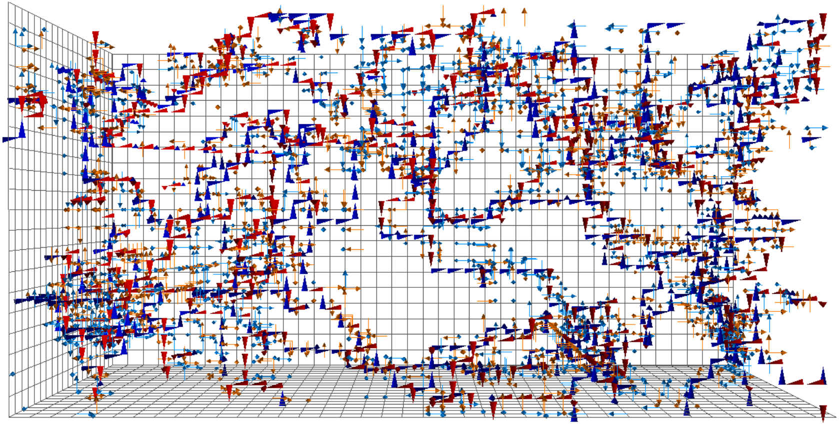


Rendering Space-Time Oriented Projected Vortices

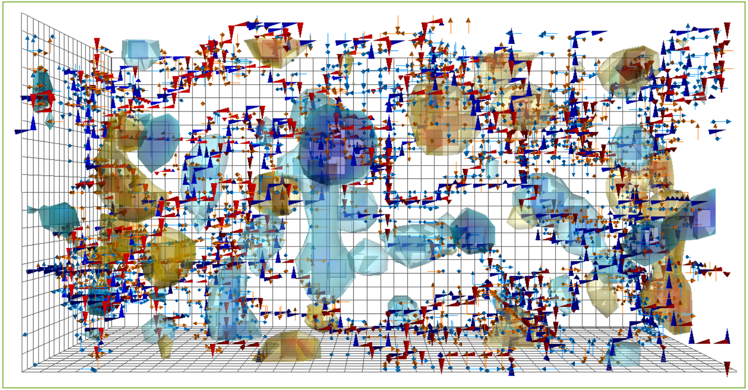
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.



Time slice $t = 1$



Animation of Centre Vortex Structure Google: YouTube CSSM Visualisations



Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition.
  `http://www.avs.com/solutions/express/`
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.
  `https://www.pdf3d.com/products/pdf3d-reportgen/`
- Imported into LaTeX via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).
  `ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/`