# PROPERTIES OF DEUTERON ON THE LIGHT FRONT

**Satvir Kaur**

*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China*

**Jiatong Wu, Siqi Xu, Chandan Mondal, Xingbo Zhao, and James P. Vary**


*The XVIth Quark Confinement and the Hadron Spectrum Conference*

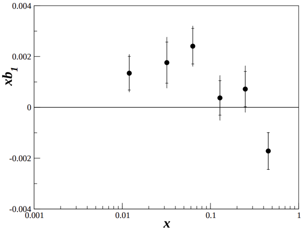
August 22, 2024

# Overview

# Motivation

- Spin-1 composite systems provide tensor structure functions:

  - Absent for spin-0 or 1/2 systems
  - Gluon Transversity
  - Proposals to study the structure of deuteron: JLab (approved), Fermilab (proposal in 2022), EICs ...
  - Largely unexplored field yet : can open a new field of spin physics




*PRL 95, 242001 (2005)*

PR12-13-011

**PR12-13-011**

The Deuteron Tensor Structure Function $b_1$

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

FERMILAB-PUB-22-381-V

**FERMILAB-PUB-22-381-V**

**The Transverse Structure of the Deuteron with Drell-Yan**

The SpinQuest Collaboration[a]

A Letter of Intent to Jefferson Lab PAC 42

Search for Exotic Gluonic States in the Nucleus

ELSEVIER

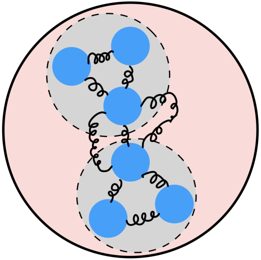
Progress in Particle and Nuclear Physics
Volume 119, July 2021, 103858

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

# Basis Light-Front Quantization (BLFQ)

- Non-perturbative approach based on the Hamiltonian formalism : $P^+P^- |\Psi\rangle = M^2 |\Psi\rangle$
  - To solve relativistic many-body bound state problems.
  - Successfully implemented to investigate the structures of various baryons and mesons.
  - Motivation : To extend the approach tc investigate light nuclei.



- $P^+$ : longitudinal momentum of the targeted nuclei
  $P^-$ : LF Hamiltonian

## Fock state expansion of the deuteron state

$$|\Psi\rangle_D = \psi_{6q} |qqq\ qqq\rangle + \psi_{6q+1g} |qqq\ qqq\ g\rangle + \psi_{6q+q\bar{q}} |qqq\ qqq\ q\bar{q}\rangle + \ \ldots$$

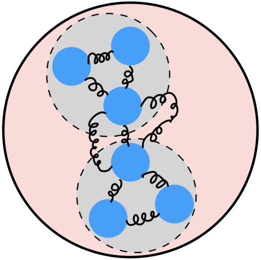
- $\psi_{\ldots}$ : LFWFs associated with the Fock components $|\ldots\rangle$.

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

# Basis Light-Front Quantization (BLFQ)

- Non-perturbative approach based on the Hamiltonian formalism :
  $P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$

  - To solve relativistic many-body bound state problems.
  - Successfully implemented to investigate the structures of various baryons and mesons.
  - Motivation : To extend the approach to investigate light nuclei.



- $P^+$ : longitudinal momentum of the targeted nuclei
  $P^-$ : LF Hamiltonian

## Fock state expansion of the deuteron state

$$|\Psi\rangle_D = \psi_{6q} |qqq\ qqq\rangle + \psi_{6q+1g} |qqq\ qqq\ g\rangle \Big\} + \psi_{6q+q\bar{q}} |qqq\ qqq\ q\bar{q}\rangle + \ \dots$$

- $\psi_{\dots}$ : LFWFs associated with the Fock components $|\dots\rangle$.

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

Basis function

2D-HO basis in transverse direction, $\Phi_{nm}(\mathbf{p}_\perp)$

Plane wave in longitudinal direction

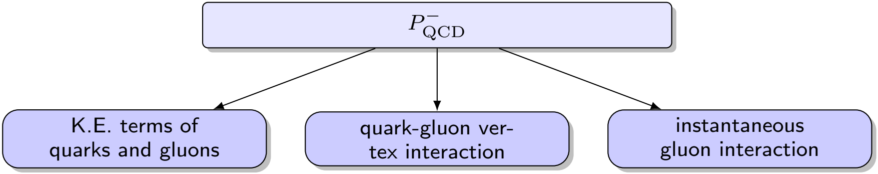
- Parton's basis state is identified by $|\alpha_i\rangle = |k_i, n_i, m_i, \lambda_i\rangle$
- Many-body basis states are identified as the direct product of the Fock-particle basis states $|\alpha\rangle = \otimes\, |\alpha_i\rangle$.

**Fock space truncation:**
$|\Psi\rangle_D = \psi_{6q}\, |qqq\ qqq\rangle + \psi_{6q1g}\, |qqq\ qqq\ g\rangle$

**Basis truncation:**
$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$
$\sum_i k_i = K$, $x = \frac{k_i}{K}$

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Téramond, PRC 81, 035205 (2010).

# LF Hamiltonian




Other interactions are absorbed into the involved parameters such as coupling constant and masses.

[1] BLFQ Collaboration, J. Lan et al., PLB 825 (2022) 136890

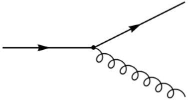
[2] BLFQ Collaboration, Z. Zhu et al., PLB 839 (2023) 137808

# Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\,\psi - \frac{1}{2}\int d^3x\, A_a^i (i\partial^\perp)^2 A_a^i$$

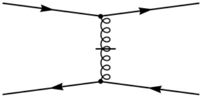
$$+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\,\psi$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c\psi \frac{1}{(i\partial^+)^2}(i\partial^+ A_a^\mu A_{\mu b})$$

$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a\psi$$



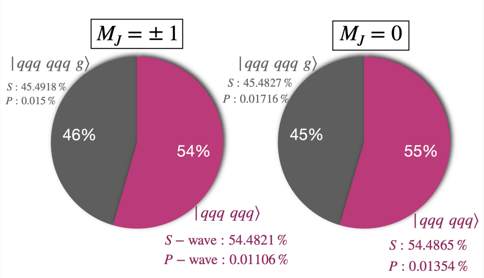
$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc} f^{ade}\, i\partial^+ A_b^\mu A_{\mu c}\frac{1}{(i\partial^+)^2}(i\partial^+ A_d^+ A_{\nu e})$$

$$+\frac{1}{4}g^2\int d^3x\, f^{abc} f^{ade}\, A_b^\mu A_c^\nu A_{\mu d} A_{\nu e}.$$

6

[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998)



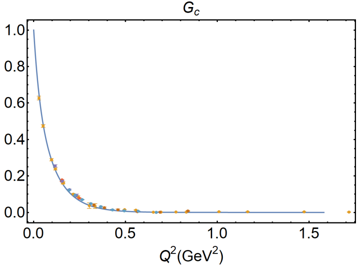

- Number of Color singlet states in $|qqq\ qqq\rangle$: 5
- Number of Color singlet states in $|qqq\ qqq\ g\rangle$: 16

| m_u | m_d | b | b_inst | g |
|---|---|---|---|---|
| 1.0 GeV | 0.95 GeV | 0.32 GeV | 5 GeV | 3.9 |

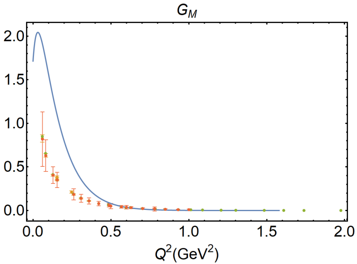
$$N_{max} = 8; K = 7$$

# EM Form Factors : Very preliminary results from BLFQ

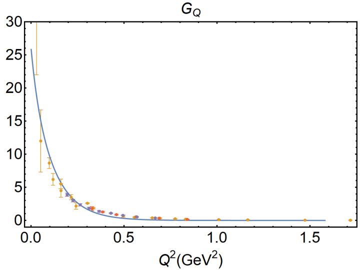
Charge FF



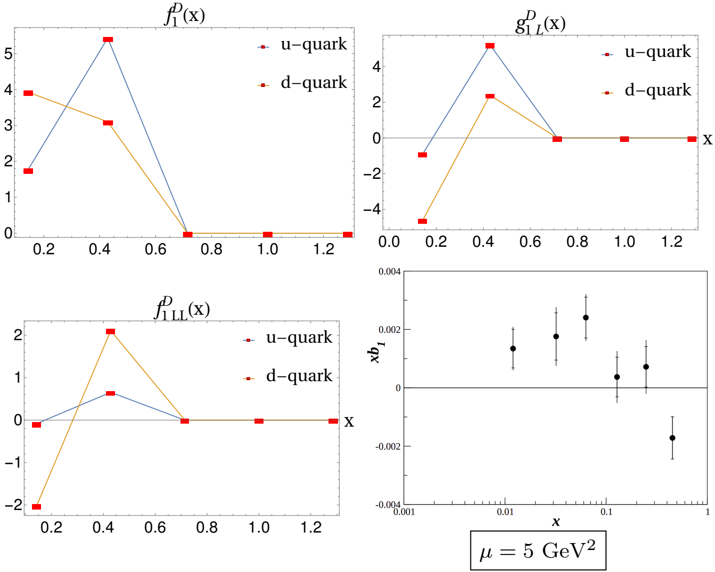

Magnetic FF




Quadrupole FF



# Structure Functions : Very preliminary results from BLFQ




- Capability to achieve the gluon transversity.

# Light-Front Schrödinger Wave Equation

- Light-front wave equation:

$$\left(\frac{m_p^2}{z}+\frac{m_n^2}{1-z}-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U_{\mathrm{eff}}\right)\chi(z)\,\phi(\zeta)=M_D^2\chi(z)\,\phi(\zeta)$$

- Light-front wavefunction:

$$\Psi(z,\zeta,\varphi)=\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}e^{iL\varphi}X(z)$$

- $\zeta=\sqrt{z(1-z)}\mathbf{b}_\perp$ , $X(z)=\sqrt{z(1-z)}\chi(z)$

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U_\perp(\zeta)\right)\phi(\zeta)=M_{\perp D}^2\phi(\zeta)$$

$$\left(\frac{m_p^2}{z}+\frac{m_n^2}{1-z}+U_\parallel(z)\right)\chi(z)=M_{\parallel D}^2\chi(z)$$

- Assumption : $U_{\mathrm{eff}}=U_\perp(\zeta)+U_\parallel(z)$ ; Masses : $M_D^2=M_{\perp D}^2+M_{\parallel D}^2$
- LFWF : $\Psi(z,\zeta)=\sqrt{z(1-z)}\chi(z)\,\phi(\zeta)$

[1] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015)

# Light-Front Holographic QCD : contains transverse dynamics

- Unique confining potential [1]: $U_\perp^{\rm LFH}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- Meson mass spectra:

$$M^2_{\perp D}(n_\perp, J, L) = 4\kappa^2\left(n_\perp + \frac{J+L}{2}\right) \quad ; \quad J = L + S$$

- Transverse part of the wave function:

$$\phi_{n_\perp L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n_\perp!}{(n_\perp+L)!}}\zeta^{1/2+L}\exp\left(\frac{-\kappa^2\zeta^2}{2}\right)L^L_{n_\perp}(\kappa^2\zeta^2)$$

- Transverse part of the deuteron LFWF in momentum space:

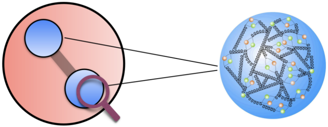
$$\Psi(z, k_\perp^2) = \frac{1}{\sqrt{z(1-z)}}\exp\left(-\frac{k_\perp^2}{2\kappa^2 z(1-z)}\right)$$

[1] S.J. Brodsky, G.F. de Téramond, H.G. Dosh, J. Erlich, Physics Reports 584, 1 (2015)

# The 't Hooft Equation : contains longitudinal dynamics

- Derived from the $(1+1)$-dim QCD Lagrangian in the large $N_c$ limit [1] :

$$\left(\frac{m_p^2}{z}+\frac{m_n^2}{1-z}\right)\chi(z)+\frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y\frac{\chi(z)-\chi(Z)}{(z-Z)^2}=M_{\parallel D}^2\chi(z)$$



- Extend to light nuclei

$$\mathrm{m}_n=m_p=0.8\ \mathrm{GeV};\ \kappa=0.13\pm0.0065\ \mathrm{GeV};\ g=0.5\pm0.025\ \mathrm{GeV}$$

$$M_D=\sqrt{M_{\perp D}^2+M_{\parallel D}^2}=1.80\pm0.09\ \mathrm{GeV}$$

[1] G. 't Hooft, Nucl. Phys. B 75 (1974) 461–470

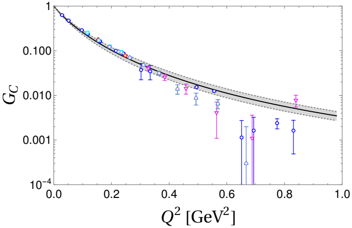
[2] Phys. Lett. B 823, 136754(2021); Phys. Rev. D 104, 074013 (2021)

[3] Phys. Lett. B 836, 137628 (2023)

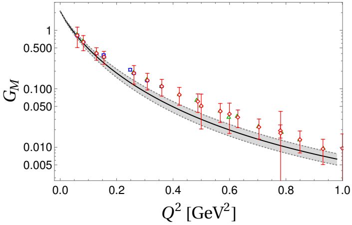
[4] Phys. Rev. D 109, 094017 (2024)

# Electromagnetic form factors: Preliminary

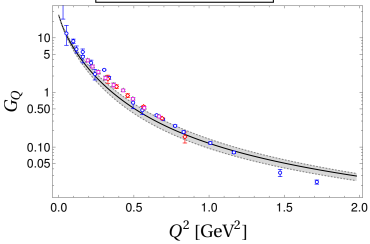
*Charge FF*




*Magnetic FF*




*Quadrupole FF*




$G_C(0) = 1$, $G_M(0) = M_D^2 \mathcal{Q}_D = 25.83$,

$G_Q(0) = \frac{M_D}{M_N} \mu_D = 1.714$

Charge radius:

$$\langle r_c^2 \rangle = -6 \lim_{Q^2 \to 0} \frac{d}{dQ^2} G_C(Q^2).$$

Deuteron charge radius: $2.09 \pm 0.10$ fm

# Structure Functions

- Structure functions: $x \sum_f e_f^2 \, \{\text{PDF}\}^D(x, Q^2)$
    - PDF of deuteron at the level of its valence quarks

$$\{\text{PDF}\}^D(x, Q^2) = \frac{1}{2} \sum_{\text{nucleon}} \int_x^1 \frac{\mathrm{d}y}{y} \mathcal{F}^{\text{nucleon}}(y) \otimes \{\text{PDF}\}^f \left(\frac{x}{y}, Q^2\right)$$

where $\mathcal{F}^{\text{nucleon}}$: nucleon longitudinal momentum distribution

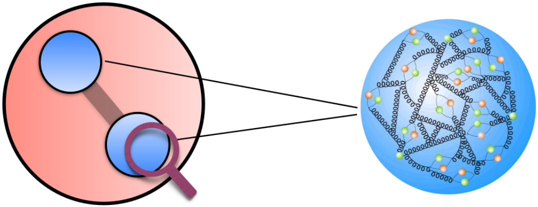
- Nucleon longitudinal momentum distribution functions:

$$\begin{aligned} f_1^N(z) &= \frac{1}{6}\left[P_\uparrow^1(z) + P_\uparrow^{-1}(z) + P_\uparrow^0(z)\right] \\ g_{1L}^N(z) &= \frac{1}{4}\left[P_\uparrow^1(z) - P_\downarrow^1(z)\right] \\ f_{1LL}^N(z) &= \frac{1}{4}\left[2P_\uparrow^0(z) - \left(P_\uparrow^1(z) + P_\uparrow^{-1}(z)\right)\right] \end{aligned}$$

where $P_\uparrow^\Lambda(z) = \int \mathrm{d}^2\mathbf{k}_\perp \sum_{\bar{h}} \left|\Psi_{\uparrow\bar{h}}^\Lambda(z, \mathbf{k}_\perp)\right|^2$ ;

$P_\downarrow^\Lambda(z) = \int \mathrm{d}^2\mathbf{k}_\perp \sum_{\bar{h}} \left|\Psi_{\downarrow\bar{h}}^\Lambda(z, \mathbf{k}_\perp)\right|^2$

- $\{\text{PDF}\}^f$ @ $Q^2 = 5$ GeV$^2$ are obtained from NNPDF global fits.



$$F_2 \rightarrow f_1^N \otimes \text{unpolarized NNPDF}$$

$$g_1 \rightarrow g_{1L}^N \otimes \text{helicity NNPDF}$$

$$b_1 \rightarrow f_{1LL}^N \otimes \text{unpolarized NNPDF}$$

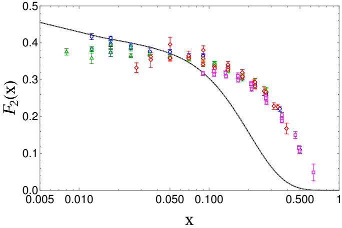
PHYSICAL REVIEW C VOLUME 44, NUMBER 3 SEPTEMBER 1991

**Convenient parametrization for deep inelastic structure functions of the deuteron**

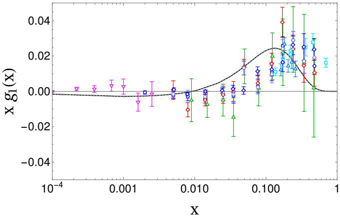
Hafsa Khan and Pervez Hoodbhoy
*Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan*
(Received 27 December 1990)

# Structure Functions: Preliminary

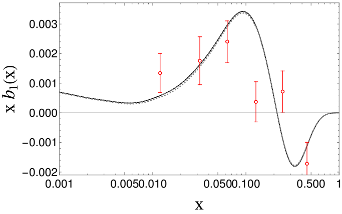
*unpolarized*



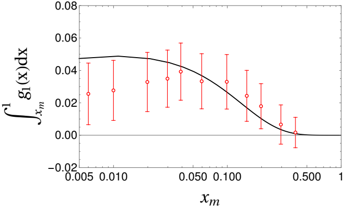

*helicity*




*tensor − polarized*




*Moments of* $g_1$




[1] A. Airapetian, et al. (HERMES Collaboration), PRL 95, 242001 (2005)

# Conclusion

- Deuteron, a lightest nuclei with spin-1, contains enriched information at the level of its partons.
- Showed some very preliminary results on deuteron structure functions from LF Hamiltonian approach.
  - Qualitatively consistent results.
  - able to achieve gluon transversity distribution.
  - able to study the color structure of deuteron.
- studied the deuteron structure functions using LF holographic QCD approach alongwith the 't Hooft Equation.
  - Reasonable consistency with the experimental data.

# LIGHT CONE 2024

## Hadron Physics in the EIC era

The Institute of Modern Physics,
Chinese Academy of Sciences,
Huizhou Campus, China.

November 25-29, 2024

### Physics Topics and Tools

- Physics of EIC and EicC
- Hadron spectroscopy and reactions
- Hadron/nuclear structure
- Spin physics
- Relativistic many-body physics
- QCD phase structure
- Light-front field theory
- AdS/CFT and holography
- Nonperturbative QFT methods
- Effective field theories
- Lattice field theories
- Quantum computing
- Present and future facilities

### International Advisory Committee

- Stanley J. Brodsky (SLAC)
- Ho-Meoyng Choi (Kyungpook National U.)
- Stanislaw D. Glazek (Warsaw U.)
- Chueng-Ryong Ji (NCSU)
- Dayashankar Kulshreshtha (Delhi U.)
- Gerald A. Miller (INT & U. Washington)
- Wally Melnitchouk (TJNAF)
- Barbara Pasquini (Pavia U.)
- Wayne Nicholas Polyzou (U. of Iowa)
- Nico G. Stefanis (Ruhr U.)
- Wojciech Broniowski (JKU & Cracow, INP)
- Tobias Frederico (ITA)
- John R. Hiller (Idaho U.)
- Vladimir Karmanov (Lebedev Inst.)
- Cédric Lorcé (Ecole Polytechnique)
- Anuradha Misra (Mumbai U.)
- Teresa Peña (IST & Lisboa U.)
- Giovanni Salmè (INFN Roma)
- James P. Vary (Iowa State U.)

### Local Organizing Committee

- Xingbo Zhao (IMP,chair)
- Jiangshan Lan (IMP,co-chair)
- Chandan Mondal (IMP)
- Satvir Kaur (IMP)
- Siqi Xu (IMP)
- Yair Mulian (IMP)
- Yuxiang Zhao (IMP)

Registration and abstract submission opens : 1st April, 2024
Abstract submission deadline : 31st August, 2024
Registration closes : 31st October, 2024

lightcone2024@impcas.ac.cn https://indico.impcas.ac.cn/event/55


THANK YOU
谢谢

# Deuteron wavefunction

Eigenvalue equation: $P^{-}_{QCD}P^{+}|\text{Deuteron}\rangle = M_D^2|\text{Deuteron}\rangle$ where $M_D \sim 1.84\ GeV$

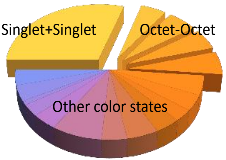
Hamiltonian in Light Front:

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

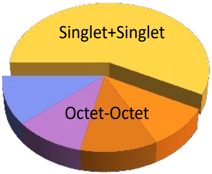
$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^{\mu}T^{a}\,\psi\,A^{a}_{\mu} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+}$$

Fock sector expansion: $|\text{Deuteron}\rangle = \psi_{6q}|qqq\ qqq\rangle + \psi_{6q1g}|qqq\ qqq\ g\rangle$

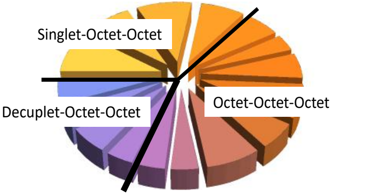
21 color state in total
(1 Singlet-Singlet + 4 Singlet-Octet-Octet+ 16 others)




5 color states
(1 Singlet-Singlet + 4 Octet-Octet)




16 color states
(4 Singlet-Octet-Octet + 8 Octet-Octet_Octet + 4 Decuplet-Octet-Octet)



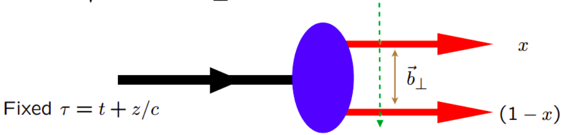
# Light-Front Holographic QCD : contains transverse dynamics

LF(3+1) ⟷ $AdS_5$ de Teramond, sjb

**Light-Front Holographic Dictionary**

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

***Light-Front Holography***: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

[1] S.J. Brodsky, arXiv-hep:1611.07194