Anisotropic pressure and novel first-order phase transition in SU(3) Yang-Mills theory on $\mathbb{T}^2 \times \mathbb{R}^2$

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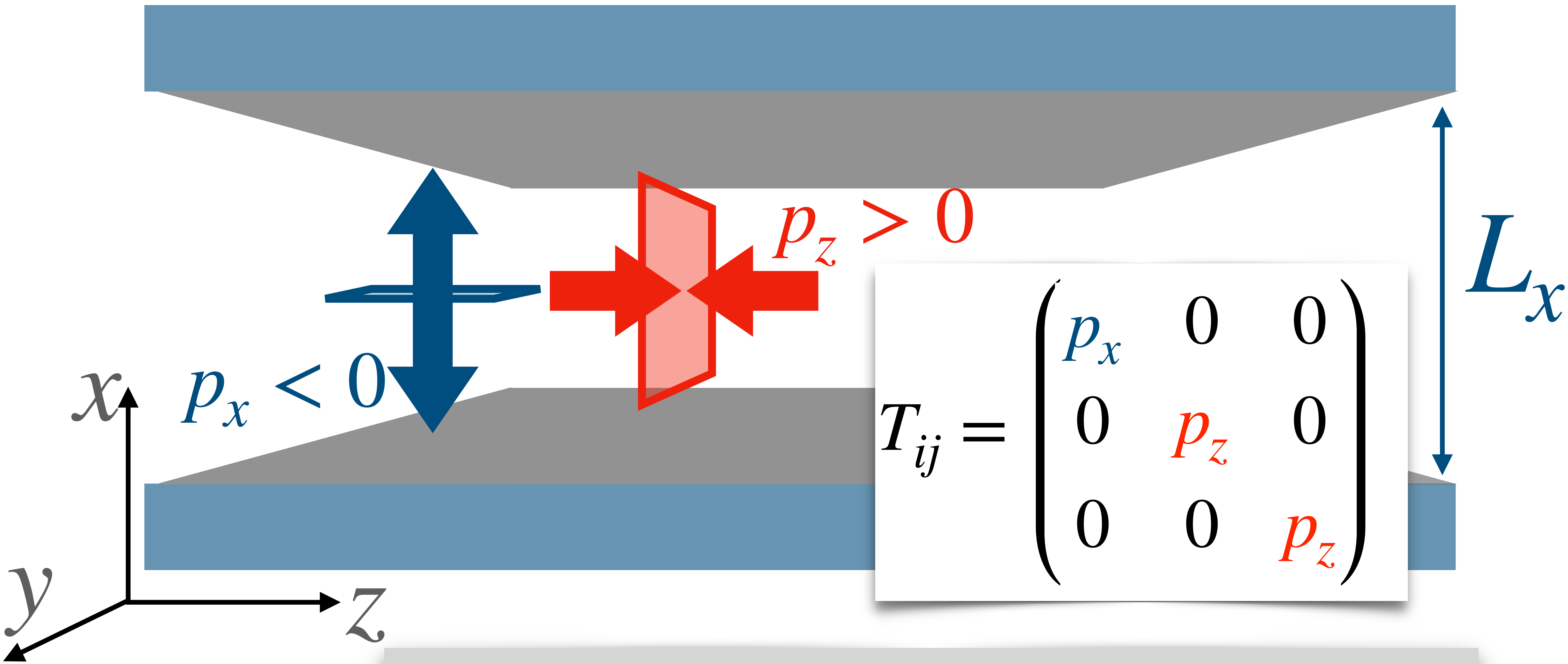
Anisotropic pressure

Casimir effect

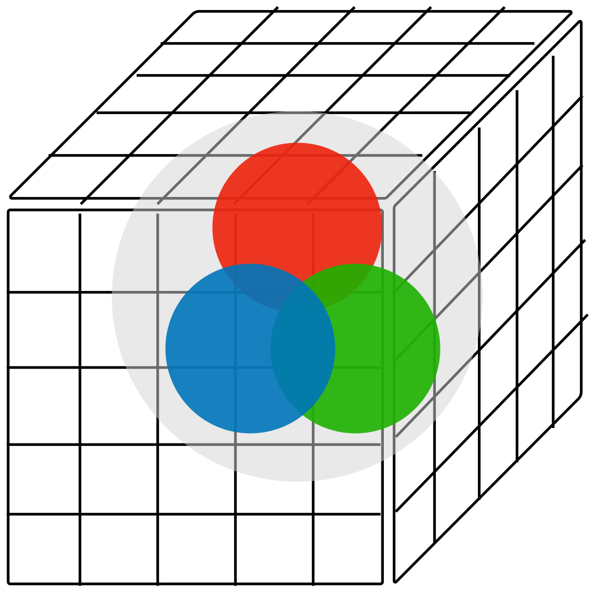


$$T_{ij} = \begin{pmatrix} p_x & 0 & 0 \\ 0 & p_z & 0 \\ 0 & 0 & p_z \end{pmatrix}$$

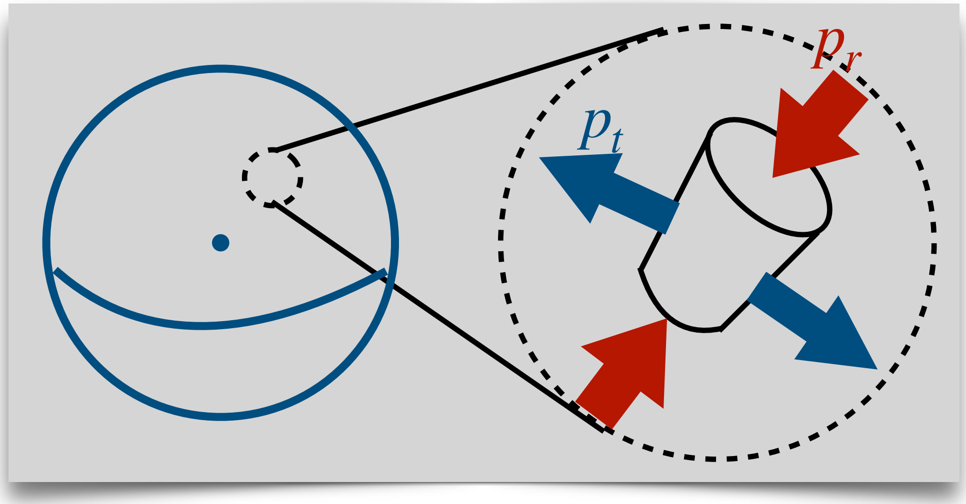
- Horizontal direction: Repulsive force
  Between conductors: Attractive force
- $p_x \neq p_z \rightarrow$ **Anisotropic** pressure

Anisotropic pressure system for QCD

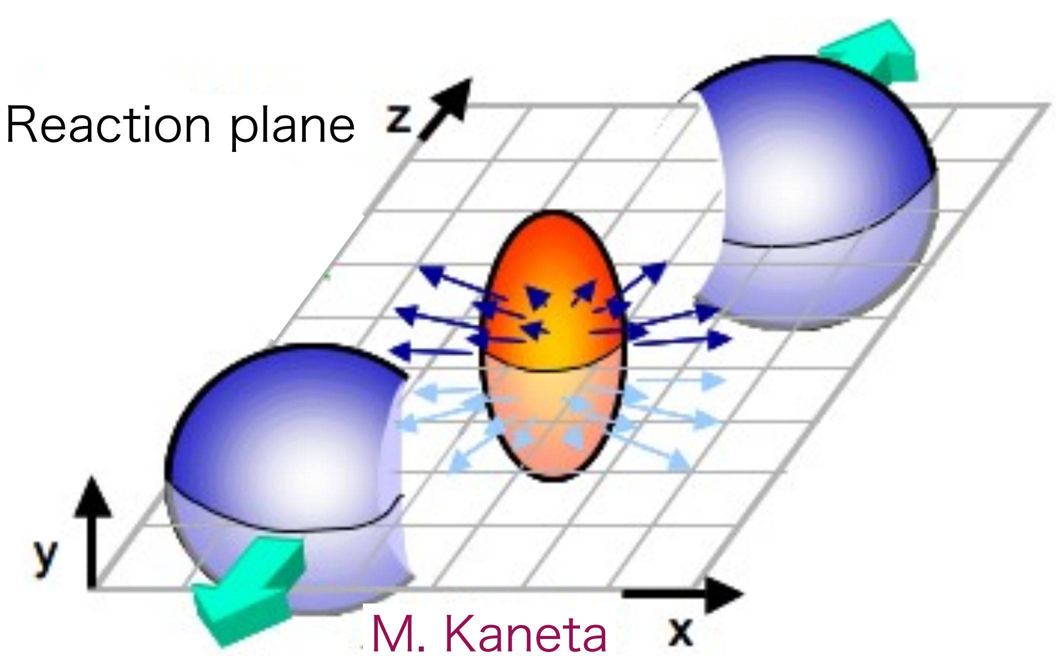
Lattice simulation



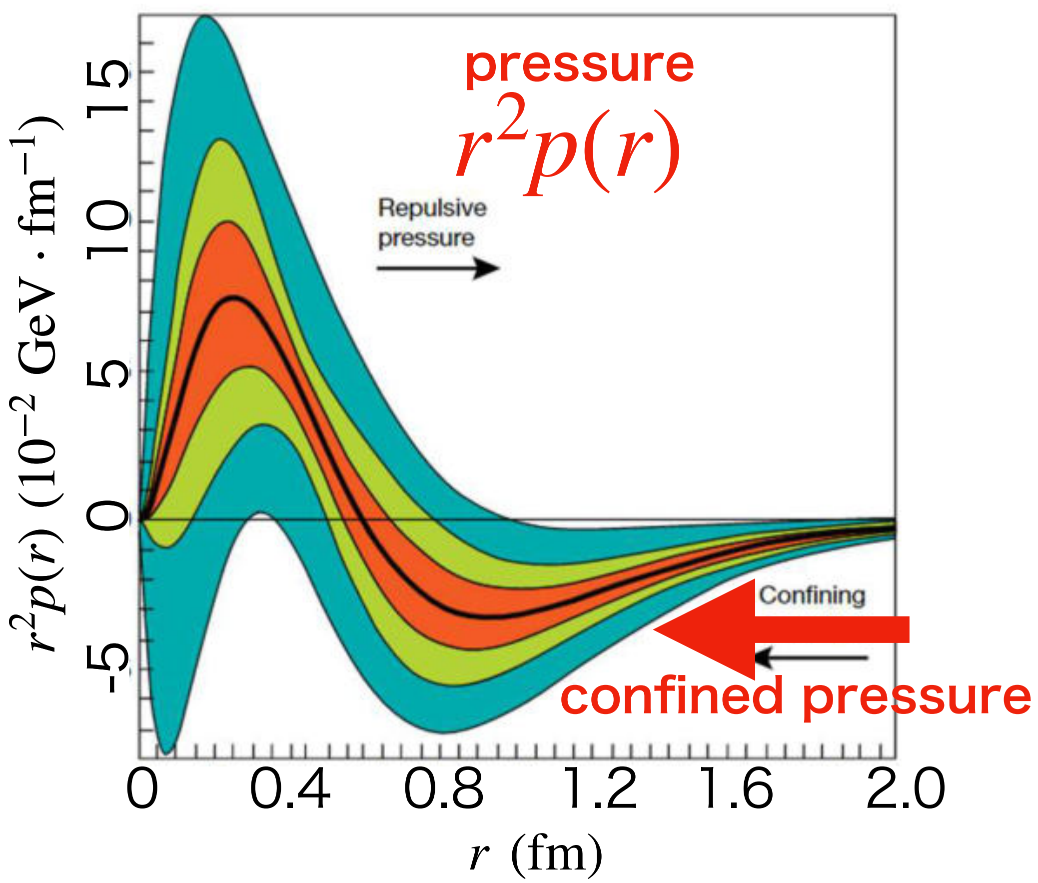
Hadron



Fire ball in heavy ion collision



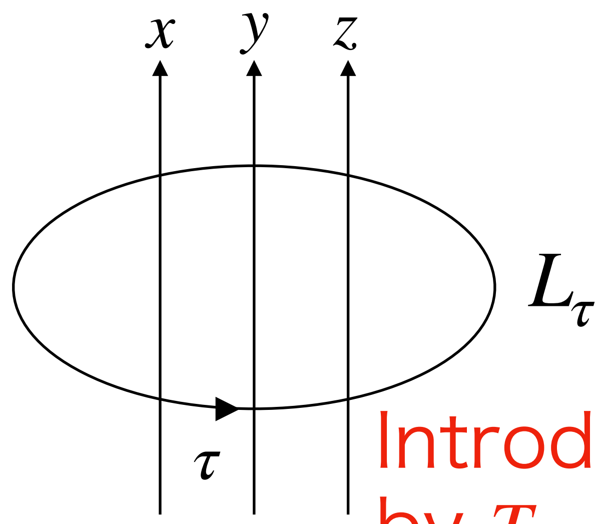
[Burkert, Elouadrhiri, Girod, Nature (2018)]



Force inside nucleon

Pure Yang-Mills theory on $\mathbb{T}^2 \times \mathbb{R}^2$

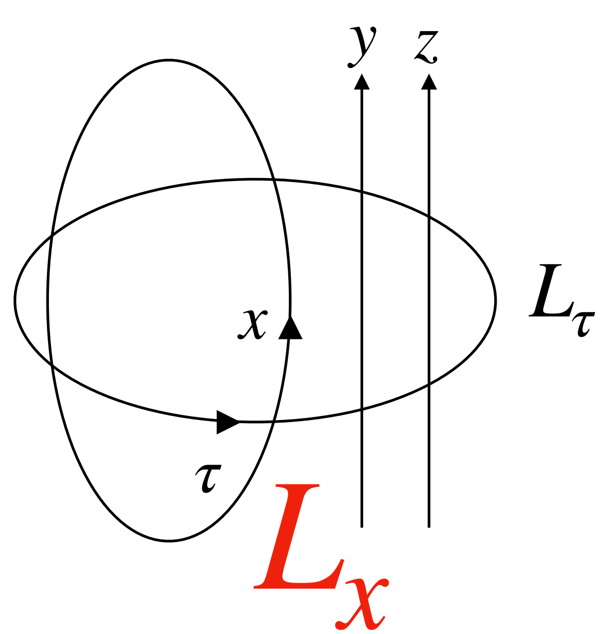
- Finite temperature ($\mathbb{S}^1 \times \mathbb{R}^3$) ~Periodic Boundary Condition (PBC)



Introduce temp. by $T = 1/L_\tau$

Isotropic pressure
$$p_x = p_y = p_z$$

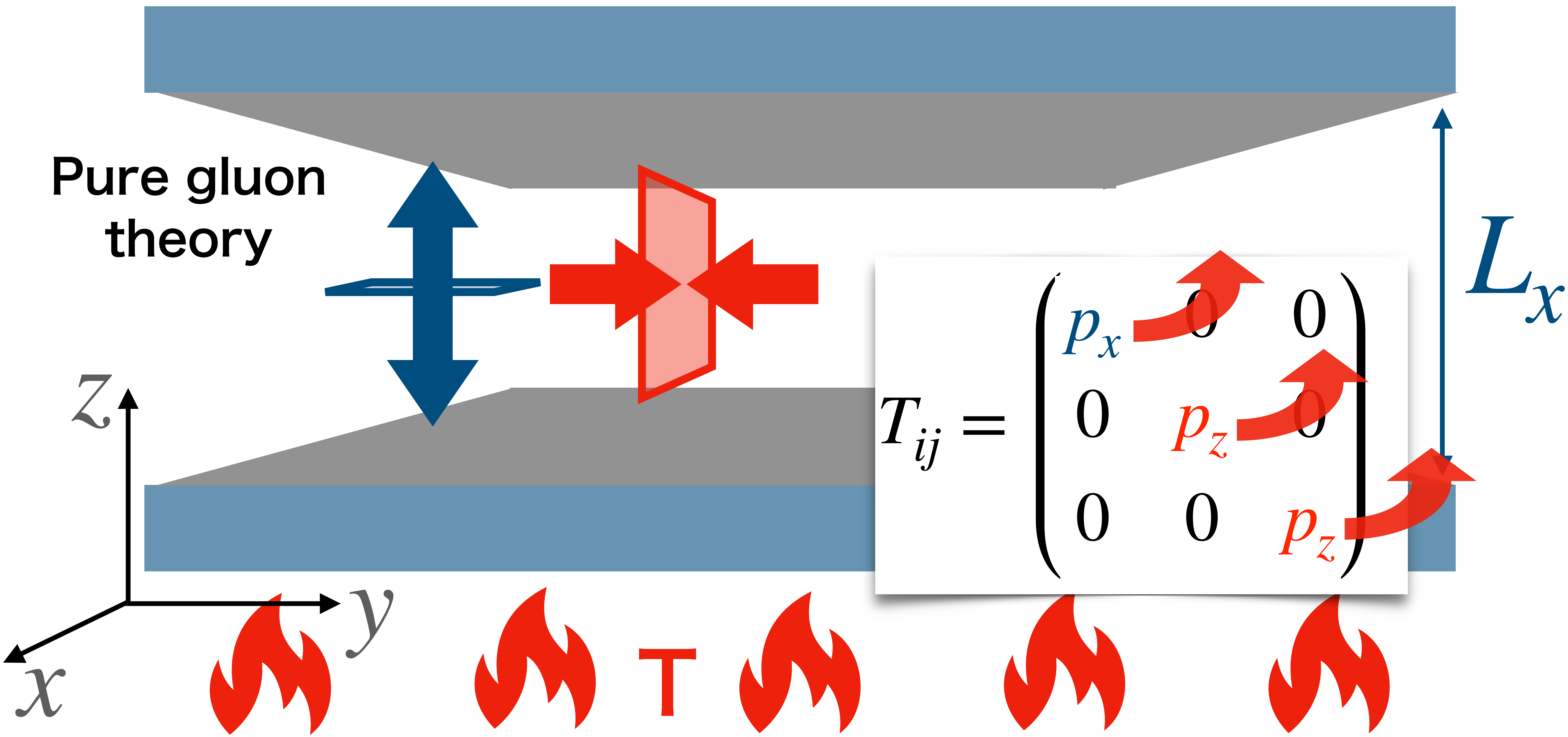
- The space-time in this talk ($\mathbb{T}^2 \times \mathbb{R}^2$) ~PBC along x direction



Anisotropic pressure
$$p_x \neq p_y = p_z$$

New QCD phase diagram

Thermal Casimir effect

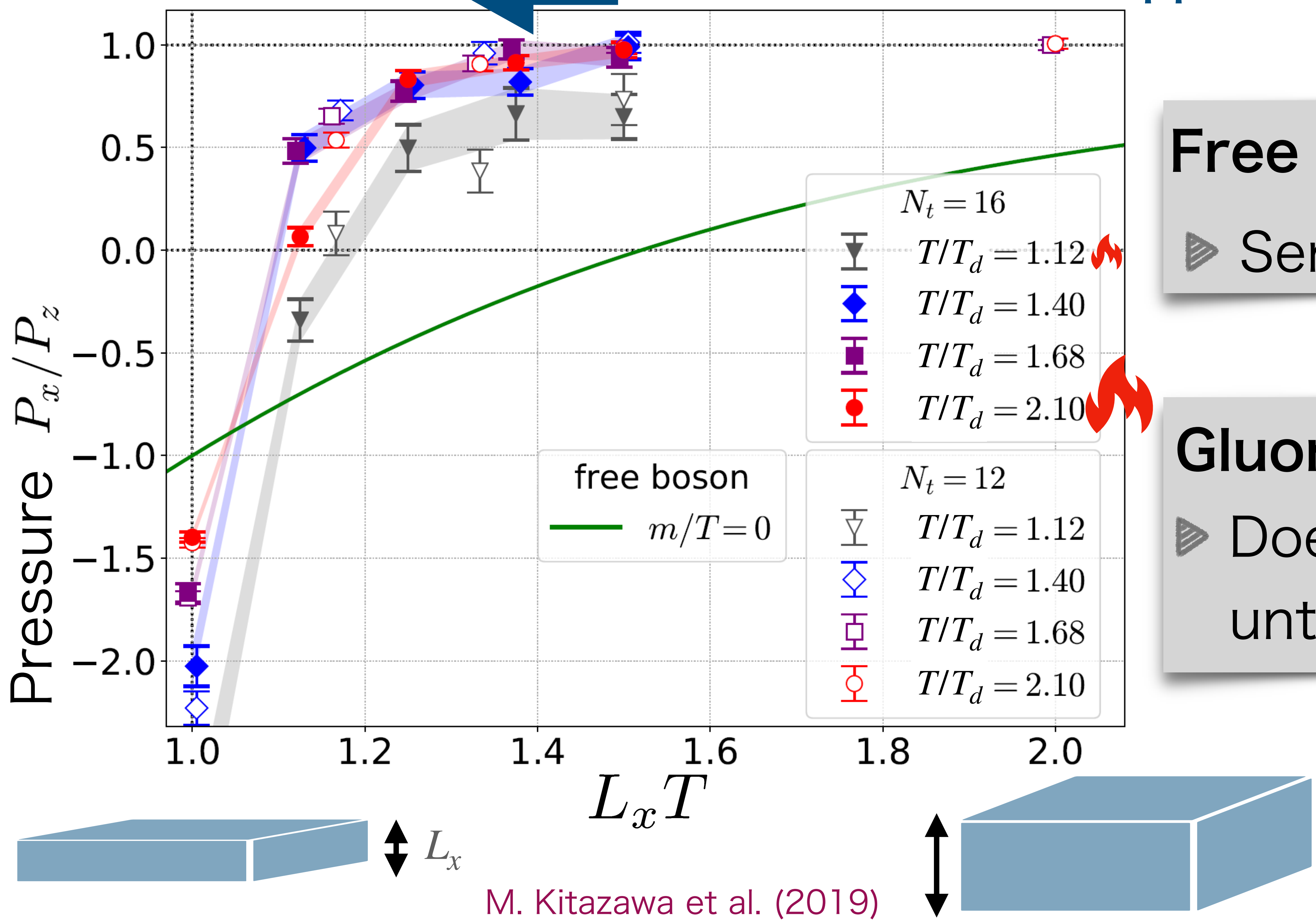


- Investigate L_x, T dependence of the thermodynamics in the pure YM theory

The thermodynamics on $\mathbb{T}^2 \times \mathbb{R}^2$ ~lattice results~

Lattice QCD simulation

Finite volume effect appears



M. Kitazawa et al. (2019)

Free massless boson

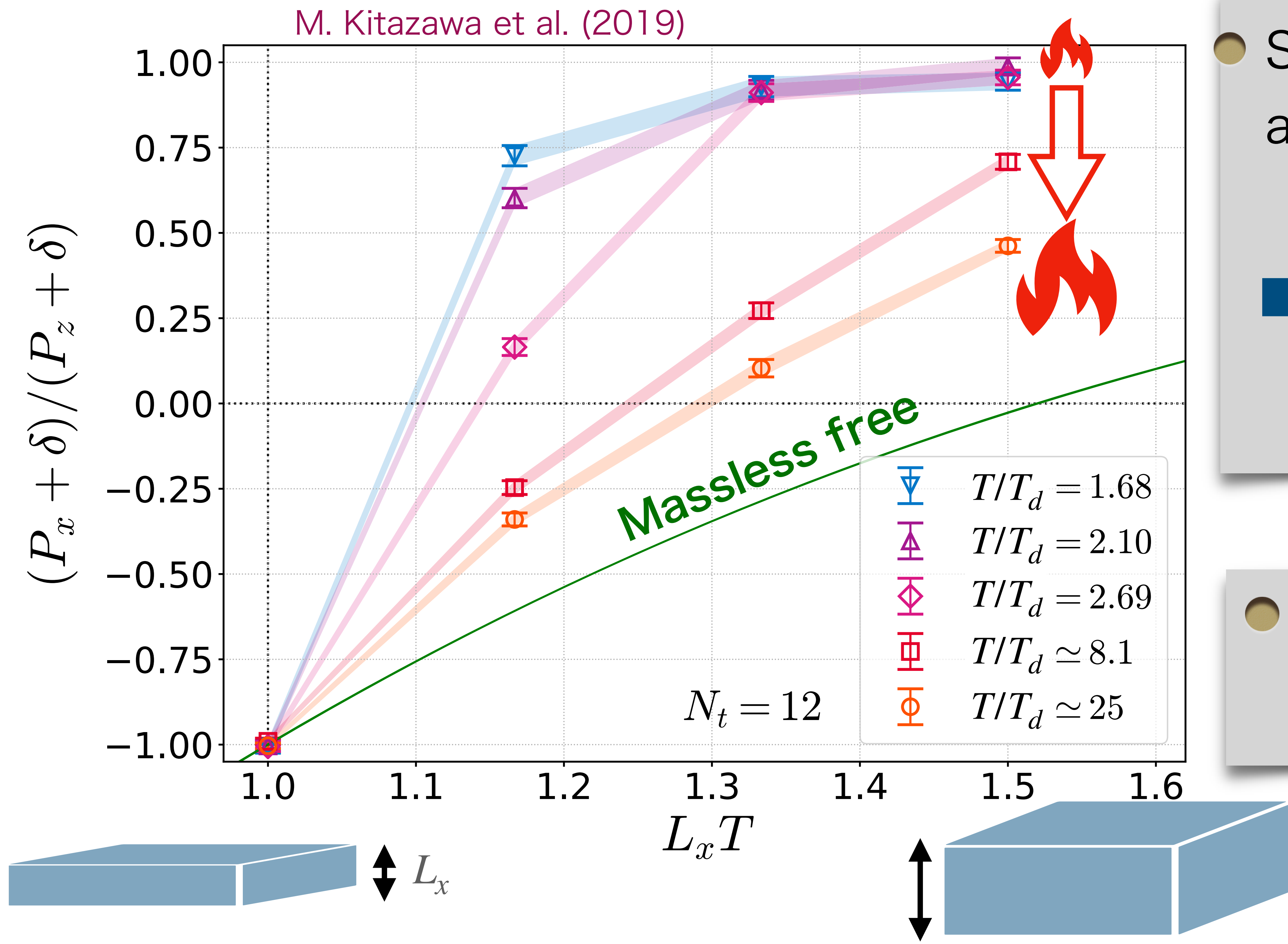
- Sensitive to BC

Gluon fields

- Does not feel BC until L_x is very smaller

Lattice analysis at high temperature

Lattice QCD simulation



- Small lattice spacing at high temp.

$$\rightarrow \frac{p_x + \Delta/4}{p_z + \Delta/4}$$

Δ : trace anomaly

- Change behavior at $T/T_d = 2.69$

Elucidate the physics **behind the results** by **a model**

Model construction

Polyakov loop effective model

Polyakov loop Ω_c

Oder parameter of deconfinement

(Symmetry : **Center symmetry**)

$$\Omega_c(x, \mathbf{x}_c^\perp) = \frac{1}{N}\mathrm{Tr}\left[\mathscr{P}\exp\left(i\int_0^{L_c} A_c(x_c, \mathbf{x}_c^\perp)dx_c\right)\right]$$

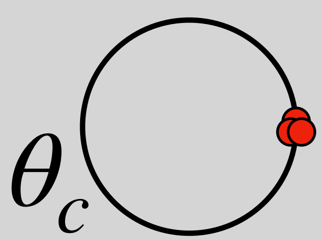
$$x_c = \tau,\ \mathbf{x}_c^\perp = (x, y, z) \text{ for } c = \tau$$

$$x_c = x,\ \mathbf{x}_c^\perp = (\tau, y, z) \text{ for } c = x$$

$$A_c = \frac{1}{L_c}\begin{pmatrix} (\theta_c)_1 & 0 & 0 \\ 0 & (\theta_c)_2 & 0 \\ 0 & 0 & (\theta_c)_3 \end{pmatrix}$$

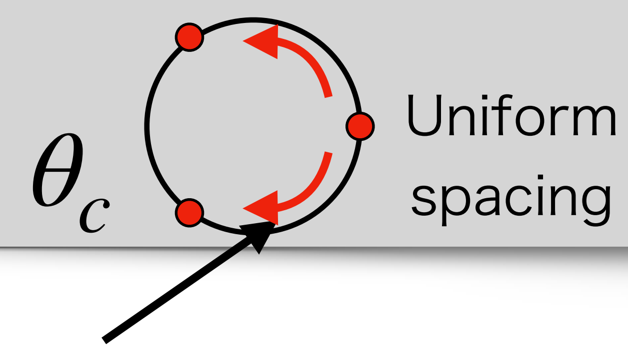
Polyakov loop eigenvalues (Mean field approximation)

Confined phase $\langle \Omega_c \rangle = 0$



$$\vec{\theta}_c = (\phi_c,\ 0,\ -\phi_c)$$

Deconfined phase $\langle \Omega_c \rangle \neq 0$

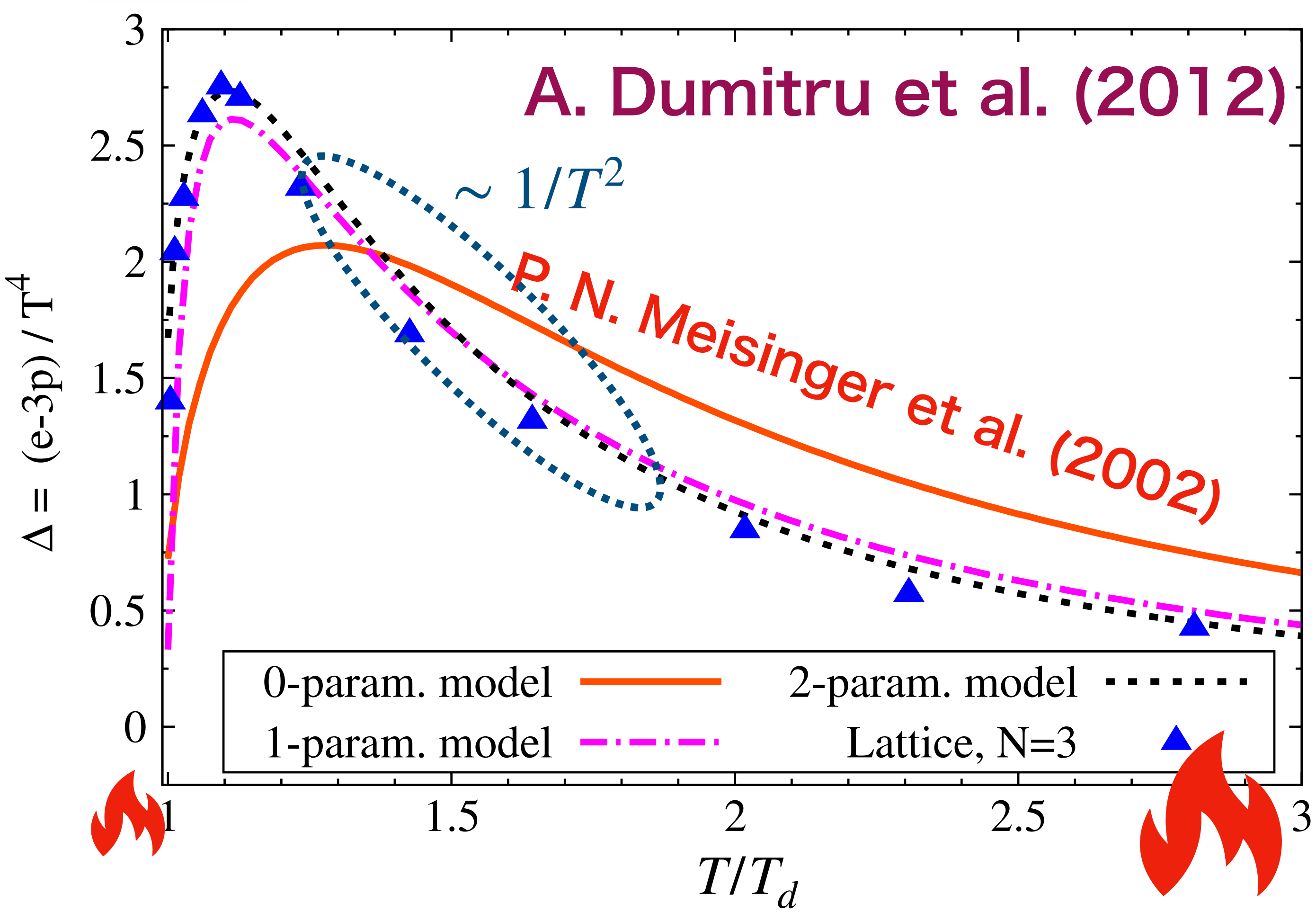


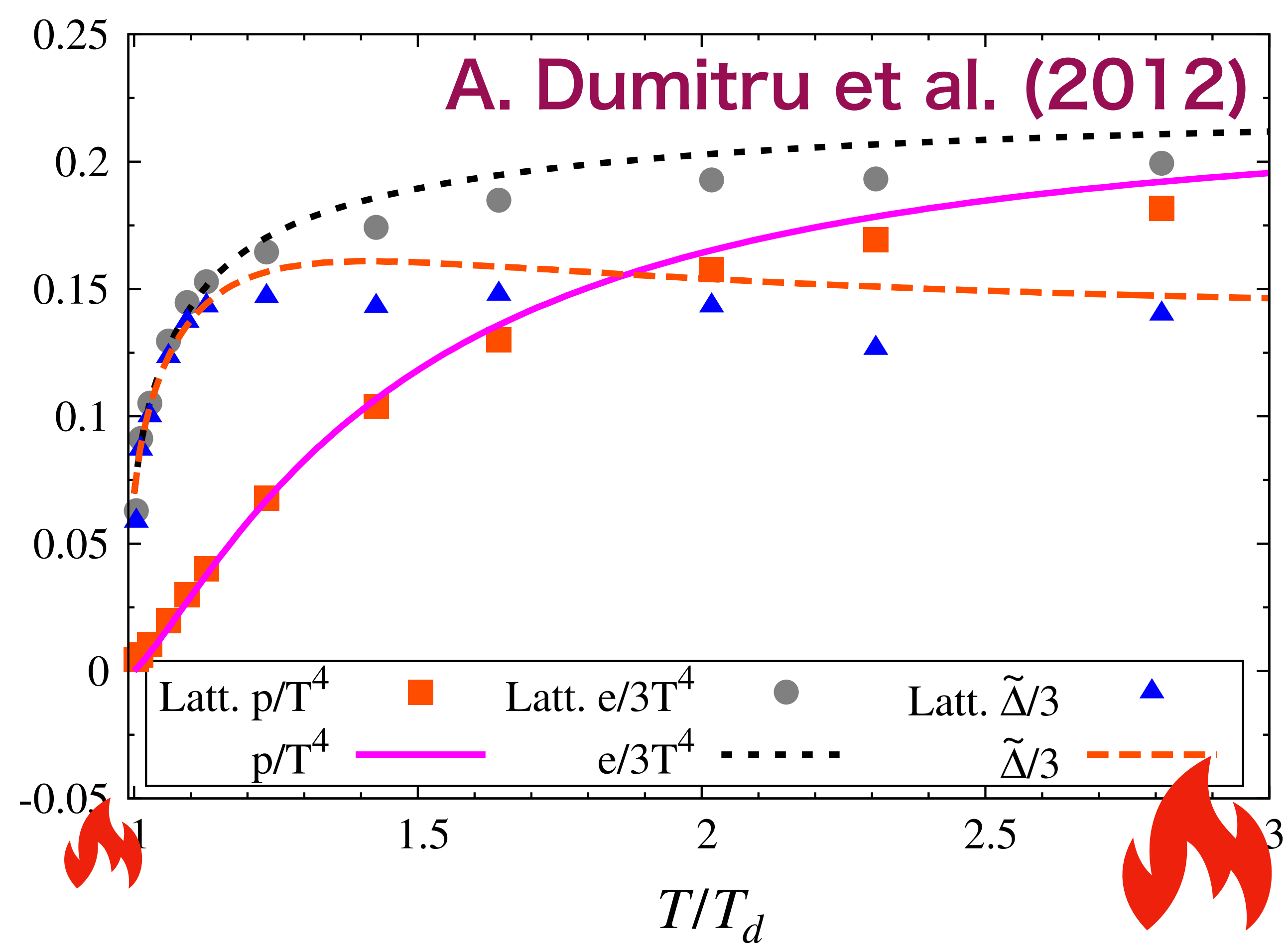
- Order parameter of confined/deconfined
- **Eigenvalues changes** → thermodynamics behavior

P. N. Meisinger et al. (2002)

Improvement to the Polyakov loop model

- Meisinger's model is simple and **qualitatively** reproduce the lattice data. P. N. Meisinger et al. (2002)
- Dumitru et al. extends this model with two parameters and Dumitru's model **quantitatively** reproduce the lattice data A. Dumitru et al. (2012)





Free energy from Polyakov loop

Free energy

P. N. Meisinger et al. (2002)
A. Dumitru et al. (2012)

$$f(L_c; P_c) = f_{\text{pert}}(L_c; \Omega_c) + f_{\text{pot}}(L_c; \Omega_c)$$

Perturbative term

Free energy of massless gluon with constant background fields A_τ

$$f_{\text{pert}}(L_\tau; \theta_\tau) = \sum_{n \in \mathbb{Z}} \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}_A \ln\left[\left(\frac{2\pi n}{\beta} - A_\tau\right)^2 + \vec{k}^2\right]$$

Potential term

→Provoke **phase transition**

Based on the Meisinger's model (with two parameters)

By using the Polyakov loop eigenvalues

→ Construct a **Model on T2xR2** based on the **Finite temperature model**

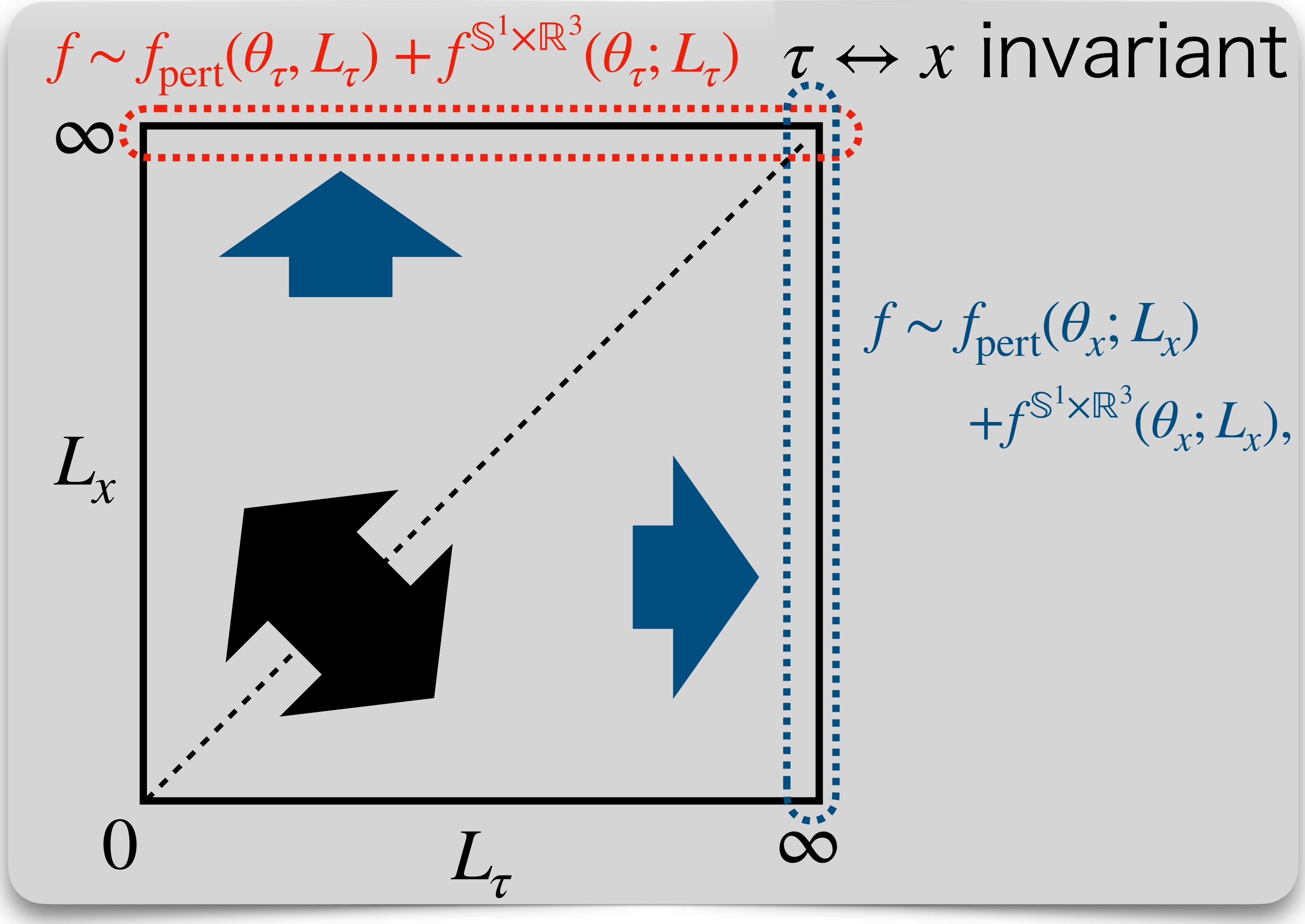
A. Dumitru et al. (2012)

Constraint on free energy

- Invariant under an **exchange** of τ, x

$$f(L_\tau, L_x; \theta_\tau, \theta_x) = f(L_x, L_\tau; \theta_x, \theta_\tau)$$

- Reduce to $\mathbb{S}^1 \times \mathbb{R}^3$ at **Limit** $L_x \to \infty$



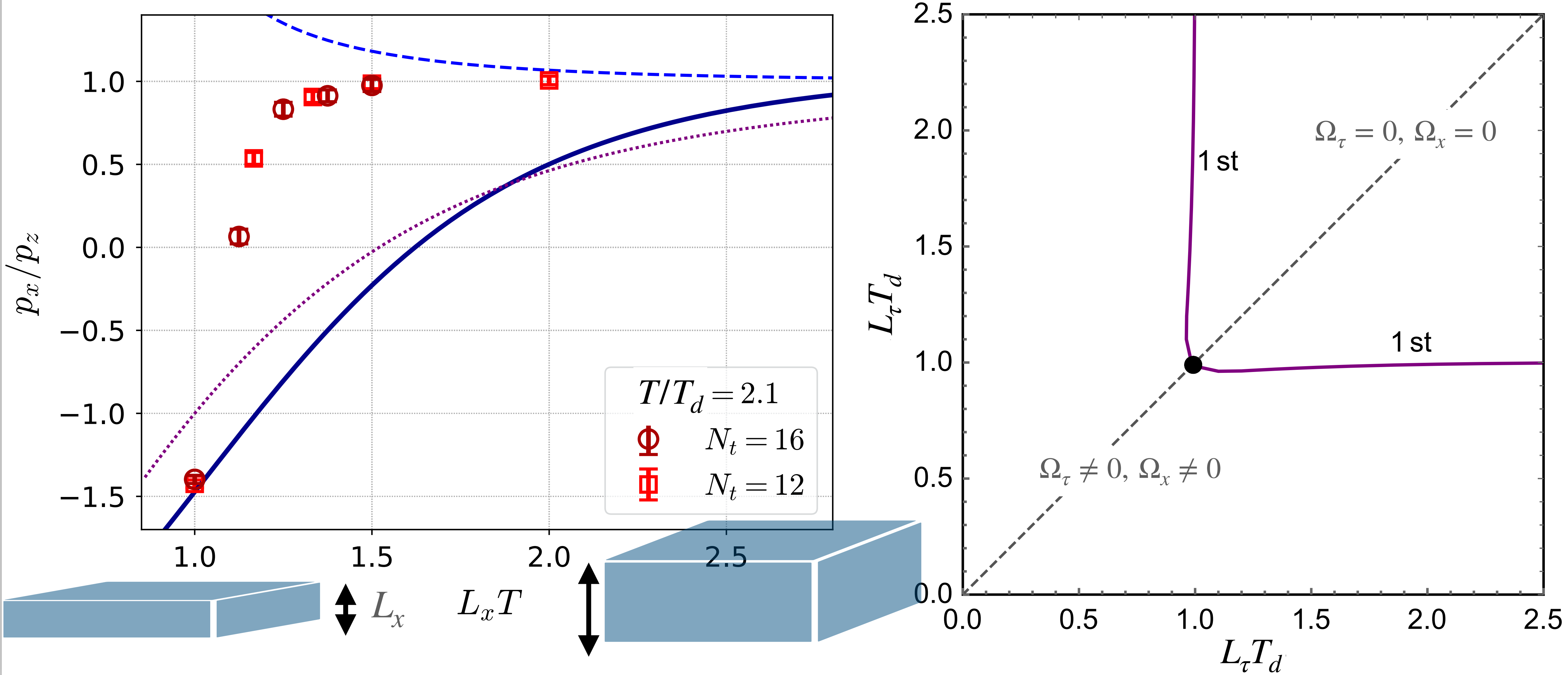
Separable extensions from potential term of **Finite temp. Polyakov loop model**

$$f_{\rm pot}^{\mathbb{T}^2\times\mathbb{R}^2}(L_\tau, L_x; \theta_\tau, \theta_x) = f_{\rm pot}^{\mathbb{S}^1\times\mathbb{R}^3}(L_\tau; \theta_\tau) + f_{\rm pot}^{\mathbb{S}^1\times\mathbb{R}^3}(L_x; \theta_x)$$

Separable ansatz

$$f_{\mathrm{pot}}^{\mathbb{T}^2\times\mathbb{R}^2}(L_\tau, L_x; \theta_\tau, \theta_x) = f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(L_\tau; \theta_\tau) + f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(L_x; \theta_x)$$

Suenaga-Kitazawa. (2023)



Does not capture the Lattice results as well

Cross term

Free energy on $\mathbb{T}^2 \times \mathbb{R}^2$

New term (non-separable)

$$f = f_{\text{pert}}(L_\tau, L_x; \theta_\tau, \theta_x) + f_{\text{pot}}(L_\tau, L_x; \theta_\tau, \theta_x) + f_{\text{cross}}$$

Separable potential term

Cross term

- Possible eigenvalues depend. (invariant under Z_3 trans.) ~$\mathcal{O}(\Omega^3)$

$\mathrm{Tr}(\Omega_c)\mathrm{Tr}(\Omega_c^\dagger)$, Uniqe to SU(3) $\mathrm{Tr}(\Omega_c^2)\mathrm{Tr}(\Omega_c) + \mathrm{Tr}(\Omega_c^{\dagger 2})\mathrm{Tr}(\Omega_c^\dagger)$

$[\mathrm{Tr}(\Omega_c^3) + \mathrm{Tr}(\Omega_c^{\dagger 3})]$, $\mathrm{Tr}(\Omega_c)^3 + \mathrm{Tr}(\Omega_c^\dagger)^3$

Only Two Independent term

$$f_{\text{cross}} = g(L_\tau, L_x)\Big[c_4 \mathrm{Tr}(\Omega_\tau)^2 \mathrm{Tr}(\Omega_x)^2 + c_5\left(\mathrm{Tr}(\Omega_\tau)^2\mathrm{Tr}(\Omega_x^3) + \mathrm{Tr}(\Omega_\tau^3)\mathrm{Tr}(\Omega_x)^2\right) + c_6 \mathrm{Tr}(\Omega_\tau^3)\mathrm{Tr}(\Omega_x^3)\Big]$$

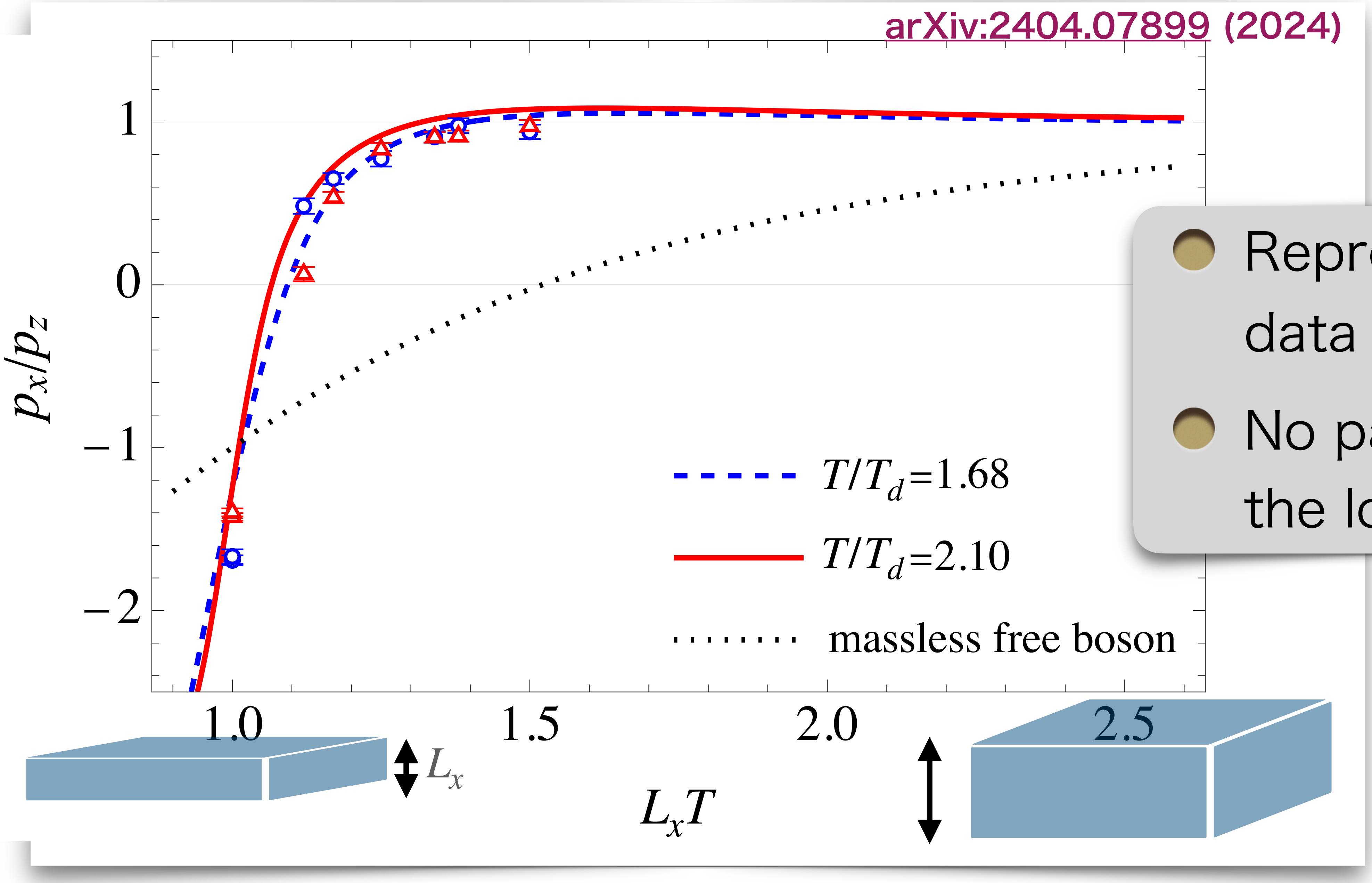
$(\Omega_x = \Omega_\tau)$

$\tau \leftrightarrow x$ **invariant**

Results

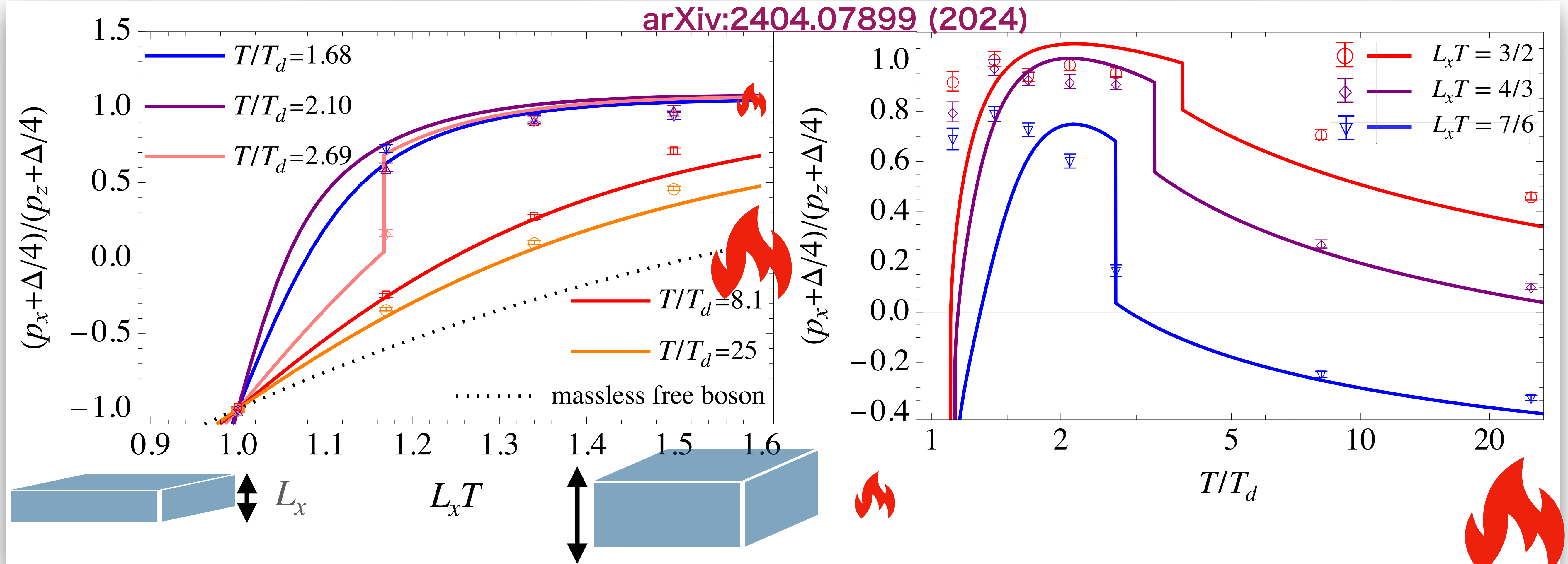
Results~Thermodynamics~

$$(c_4, c_5, c_6) = (0.11,\ 0.06,\ -0.03)$$



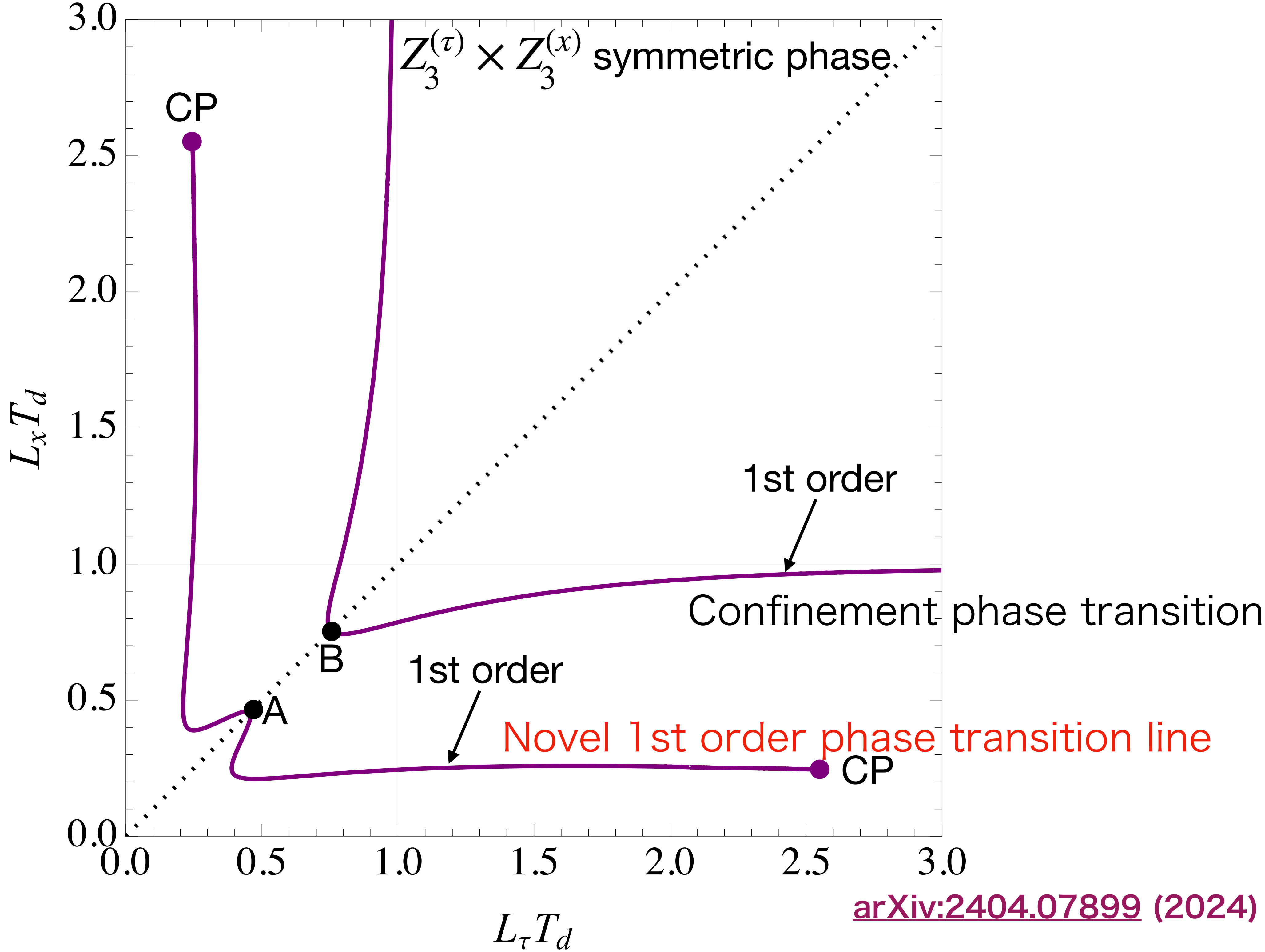
- Reproduce the lattice data at $T/T_d > 1.5$
- No parameter explained the low temp.

Thermodynamics at high temperature



- Reproduce the lattice results at high temp.
- Suggest the new 1st order phase transition
- Effective model at $T/T_d > 1.5$

Phase diagram

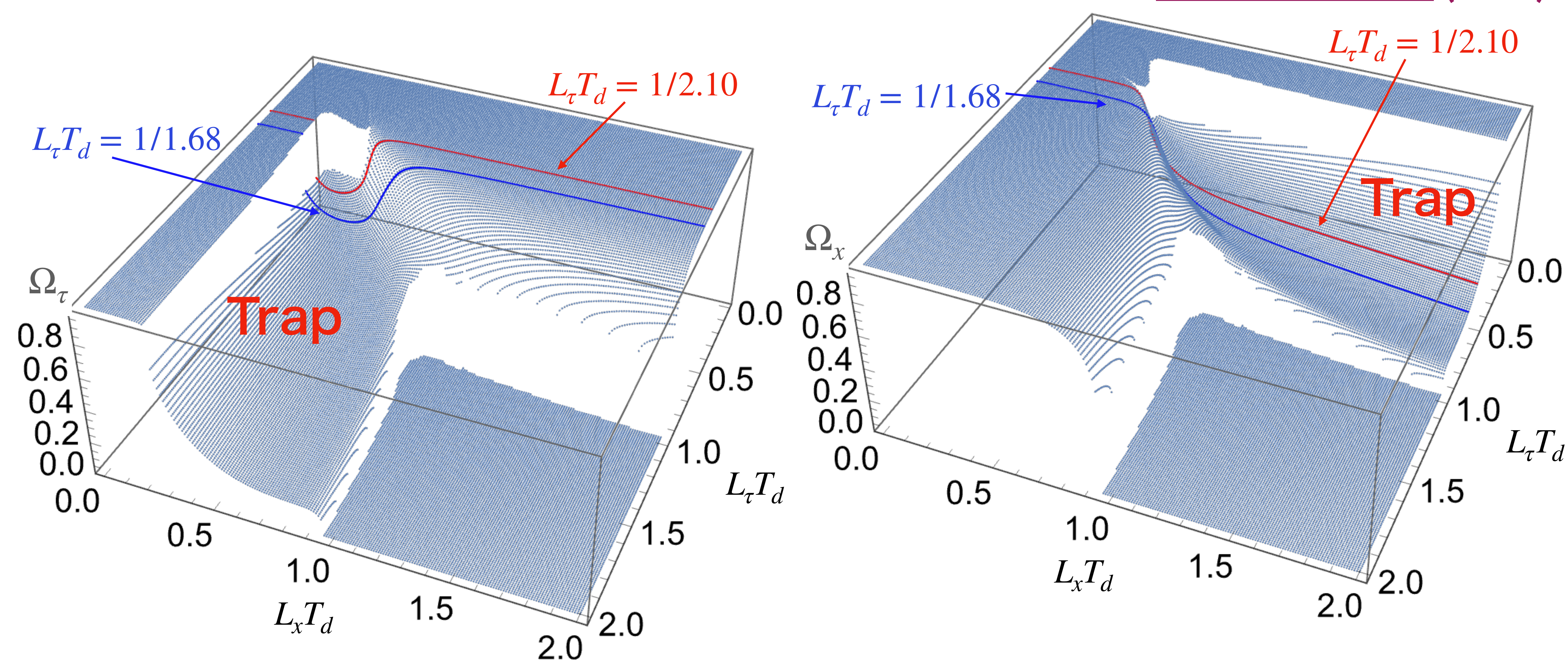


arXiv:2404.07899 (2024)

What is the mechanism to reproduce Lattice results ?

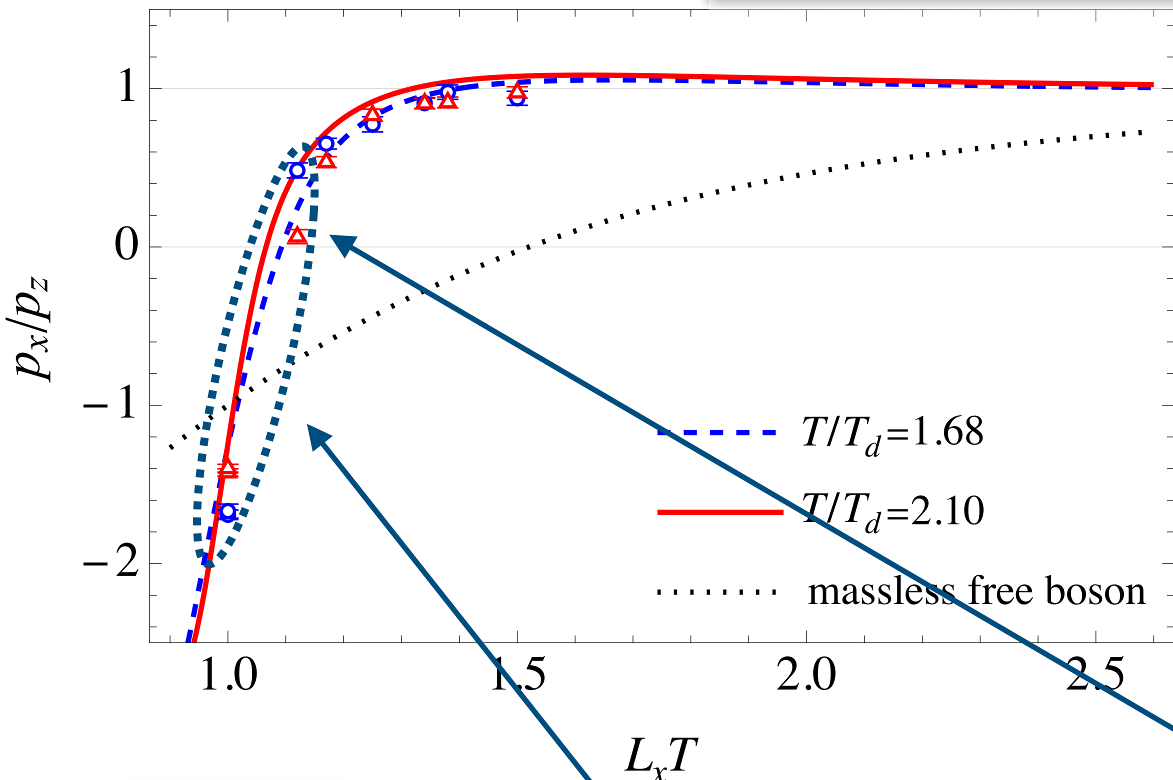
Polyakov loops

arXiv:2404.07899 (2024)

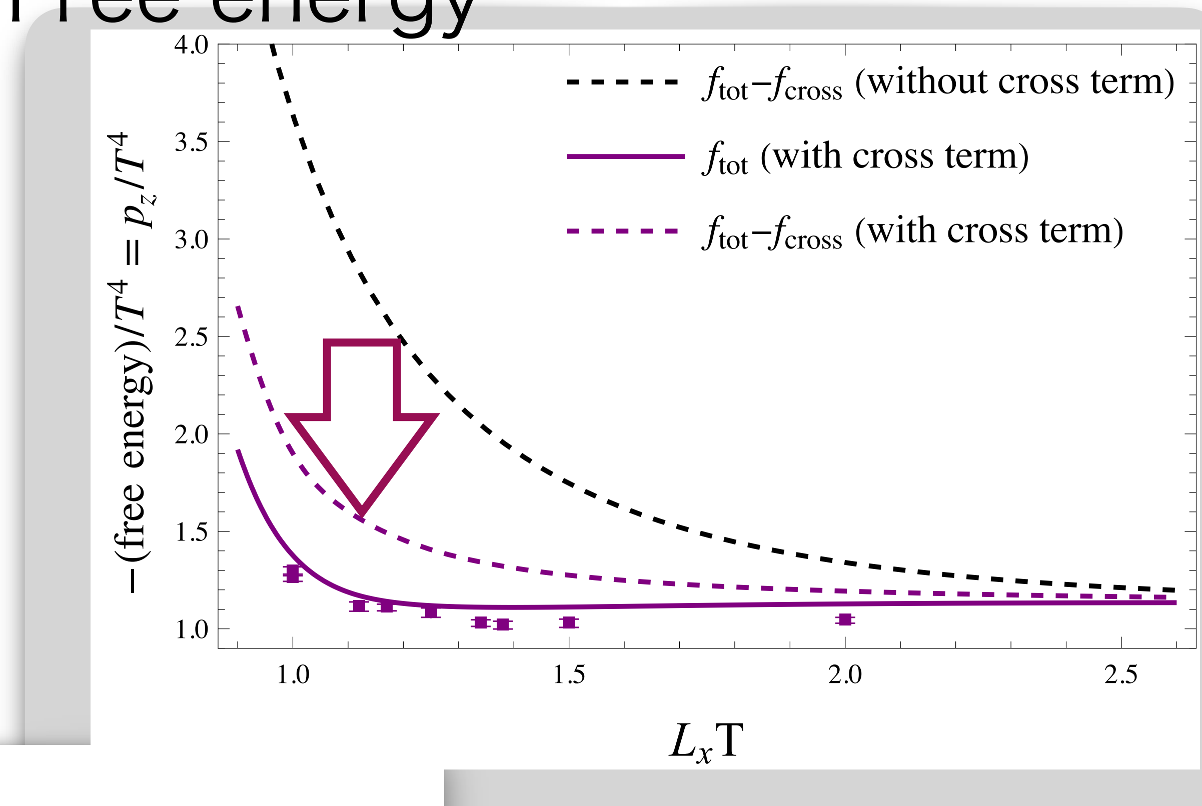


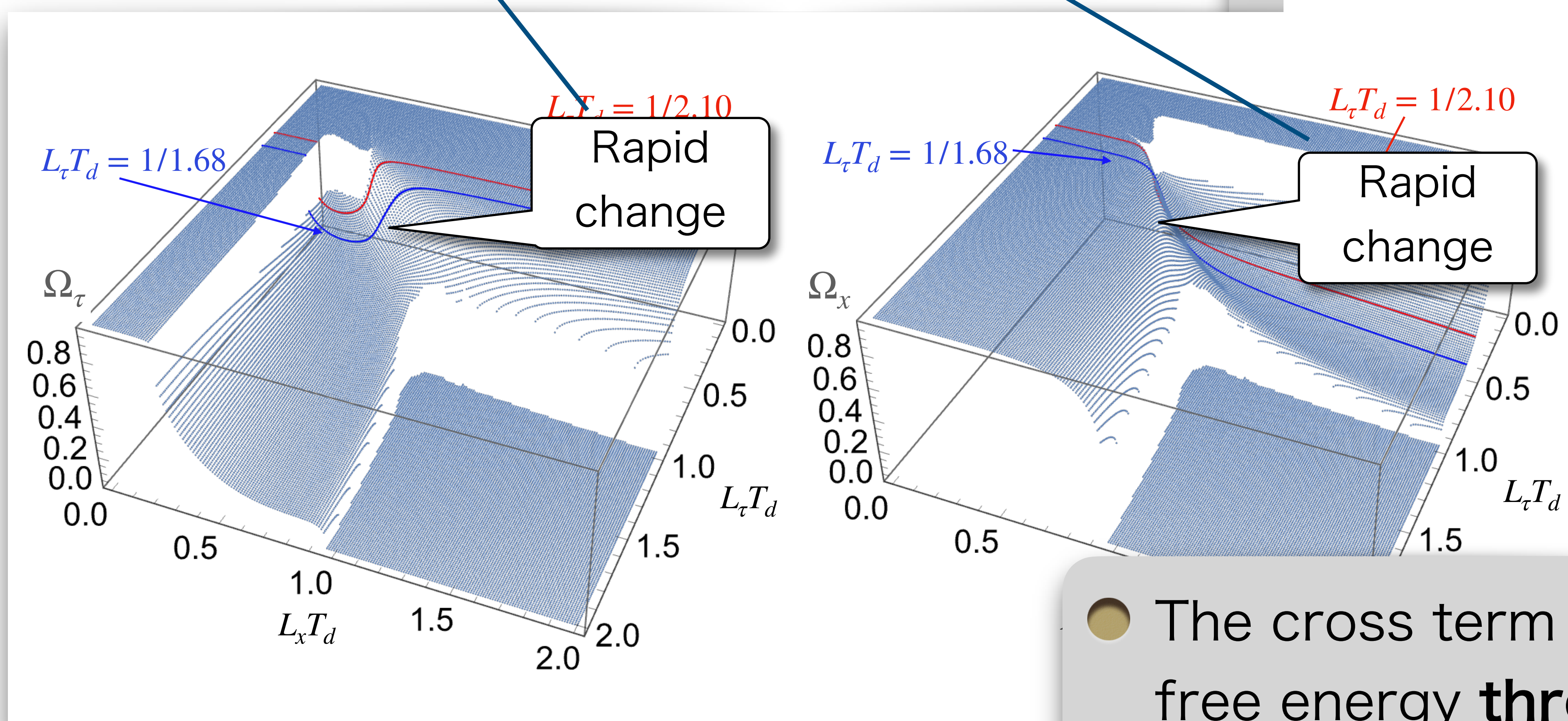
Polyakov loops geometry explains Lattice thermodynamics

Polyakov loop → p_x/p_z



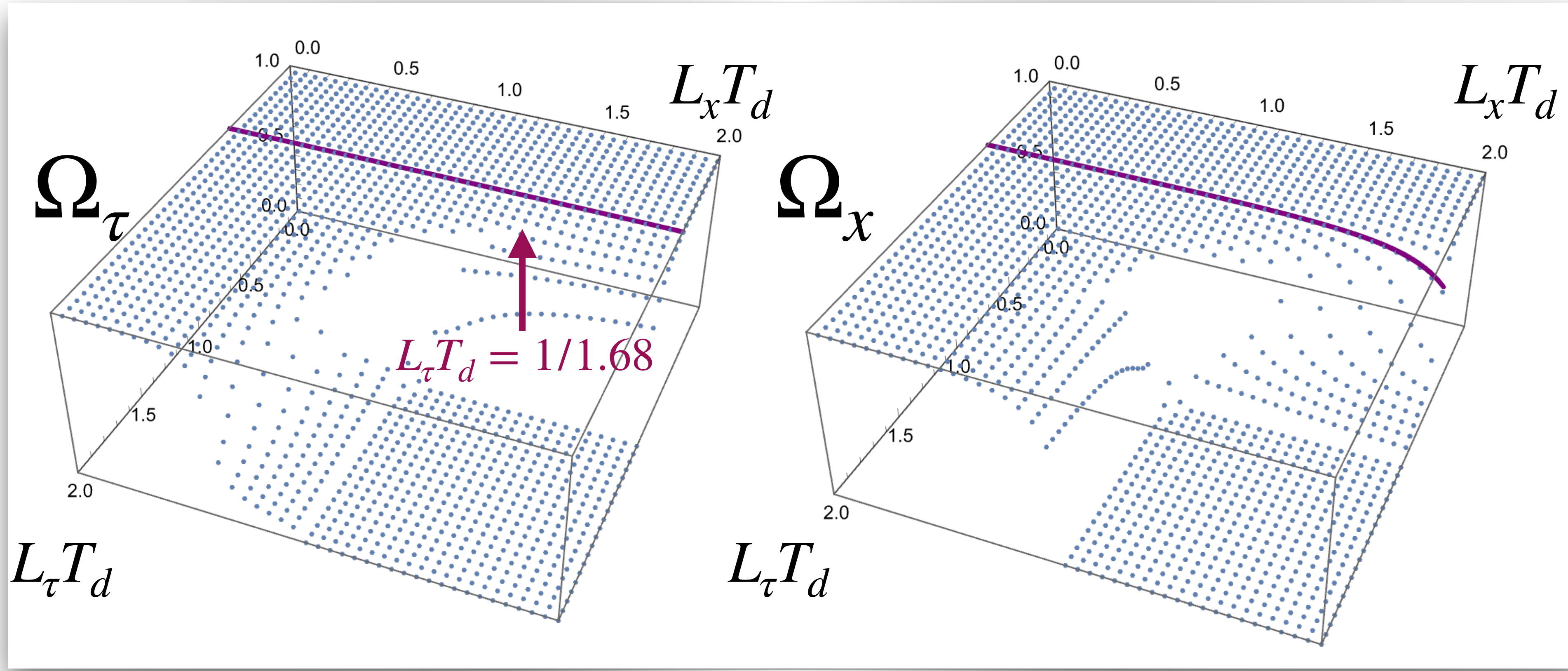
Free energy

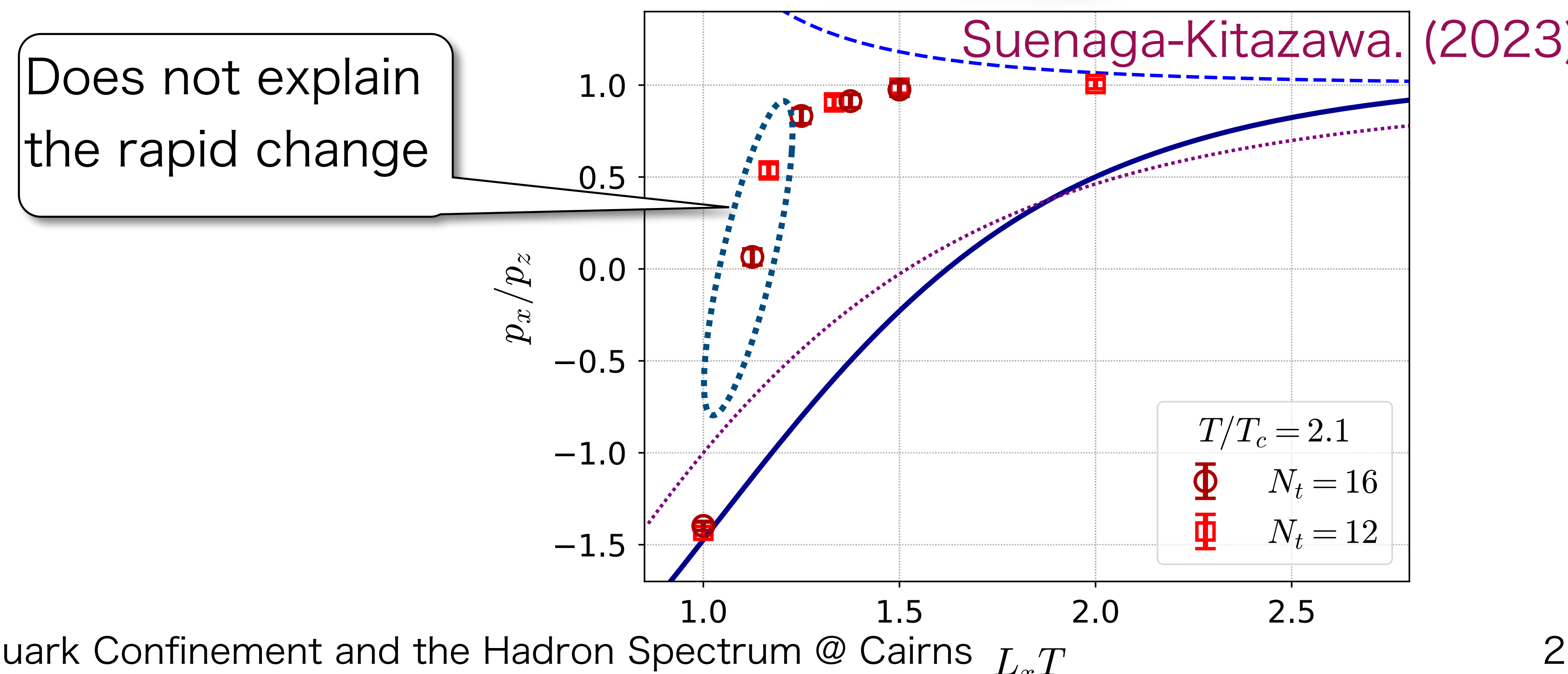




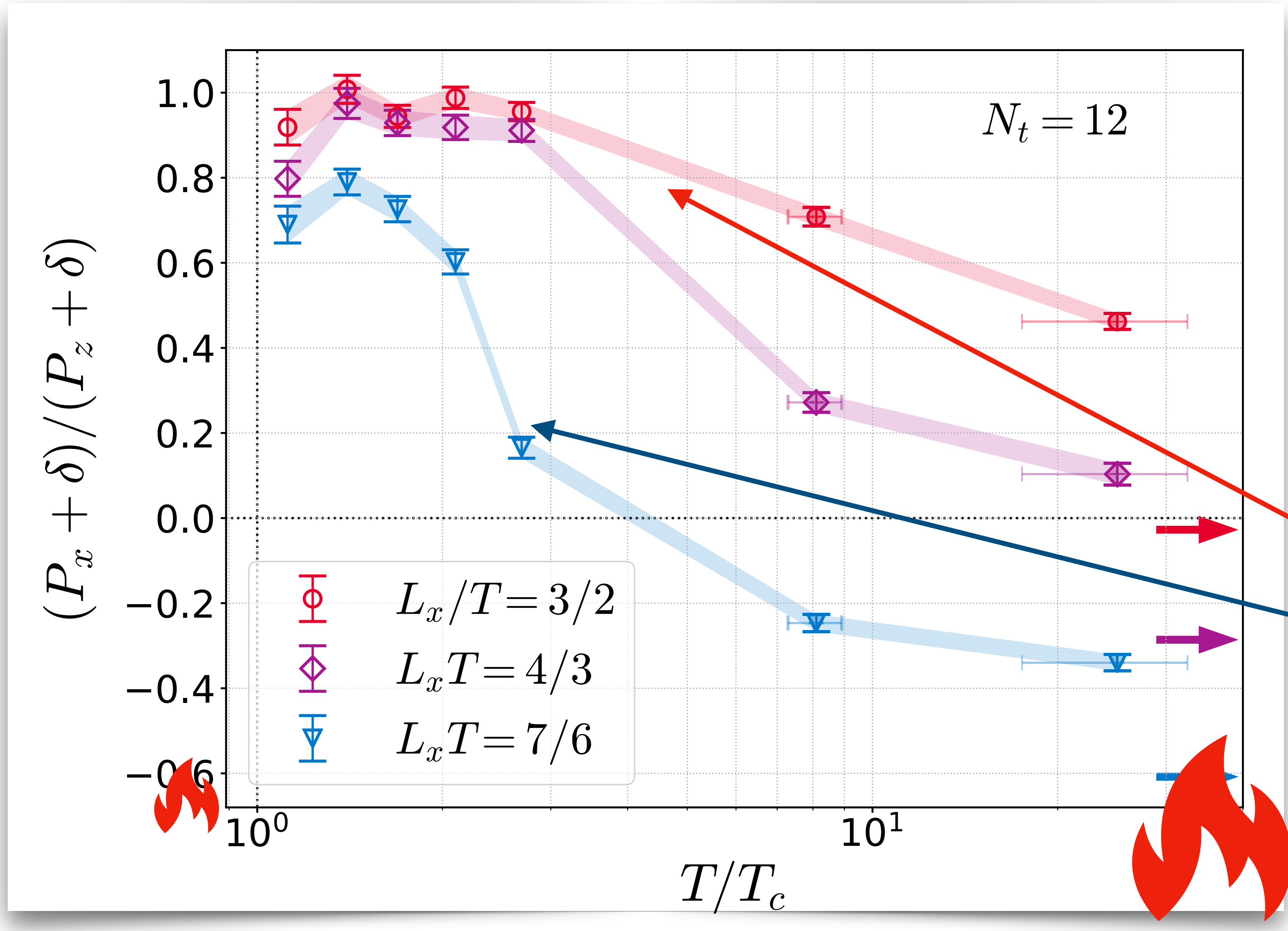
- The cross term **indirectly** pushes down free energy **through Polyakov loops**

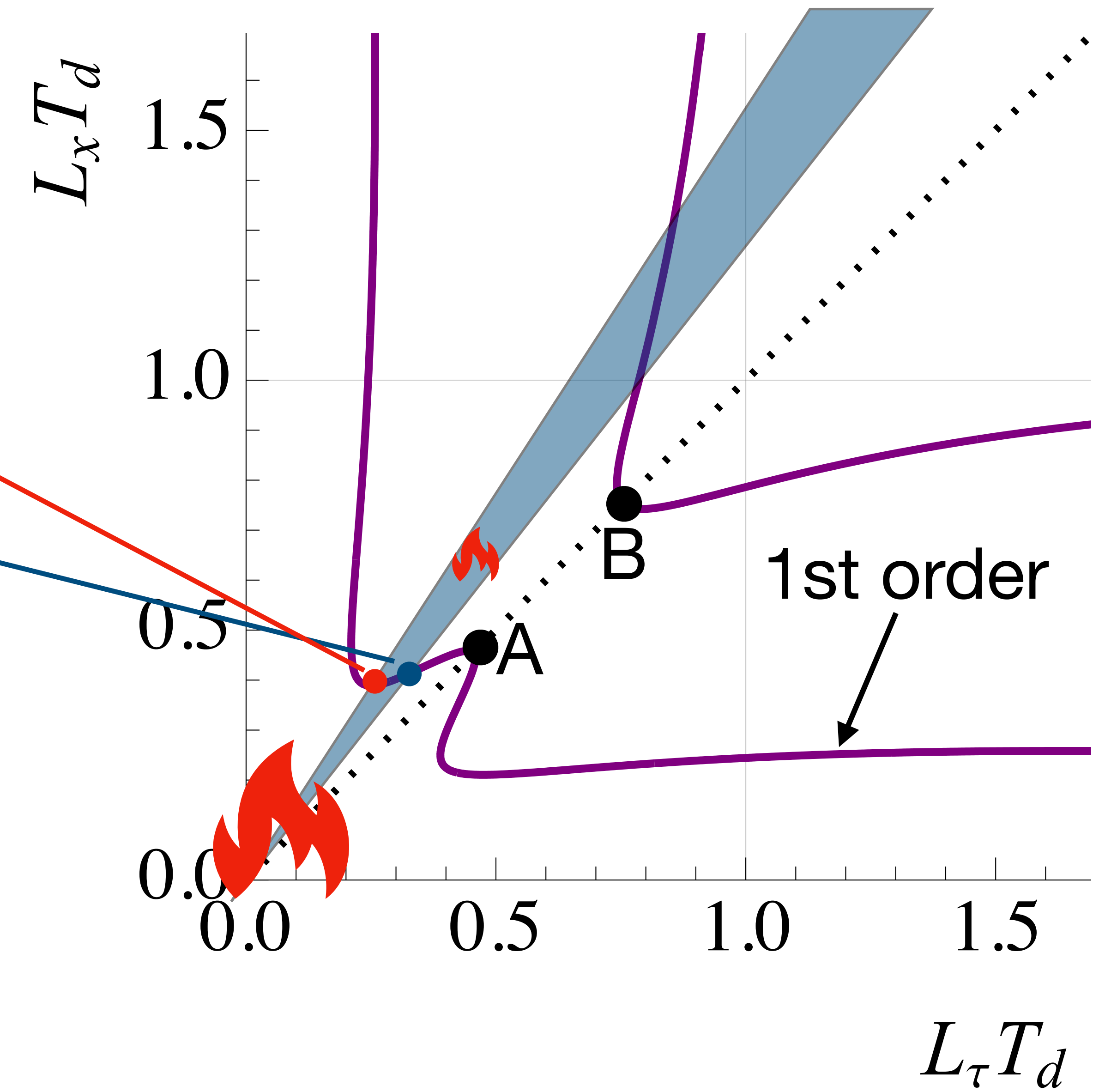
Polyakov loop of separable pot





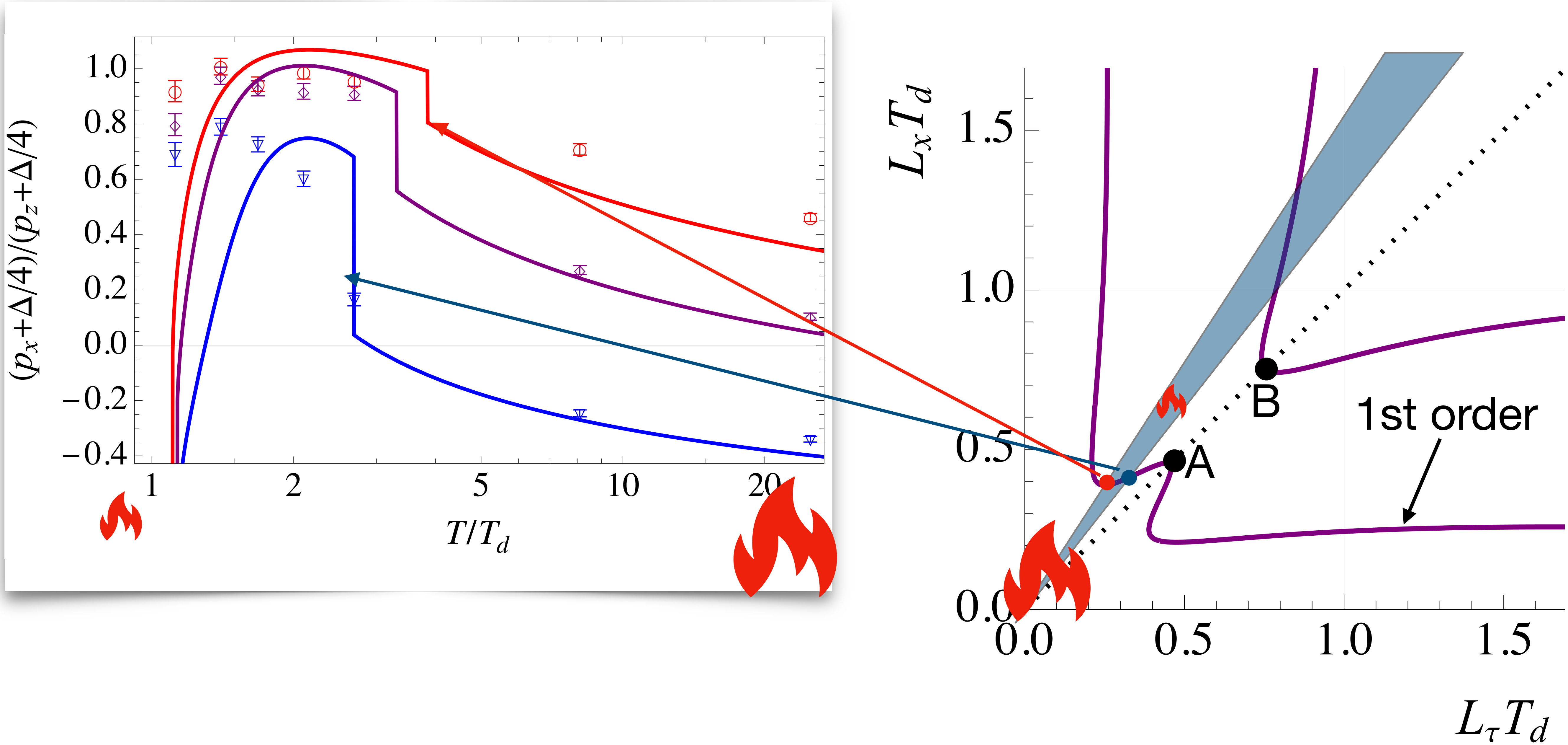
Polyakov loop → Thermodynamics at high temp.





- The temperature dependence of the pressure ratio has a **rapidly decreasing region**.

Polyakov loop → Thermodynamics at high temp.



- The **Rapid change** can be understood from the **jump** of the Polyakov loops.

Summary

- Discuss the **Anisotropic pressure in YM** on $\mathbb{T}^2 \times \mathbb{R}^2$
- Lattice results show the **unique behavior**
- Construct **Polyakov loop effective model** on $\mathbb{T}^2 \times \mathbb{R}^2$
- **Cross term** lead to change the behavior of Polyakov loops
- This change explain the **lattice results** and predict the **new 1st order phase transition**.

Thank you for your attention

Back up

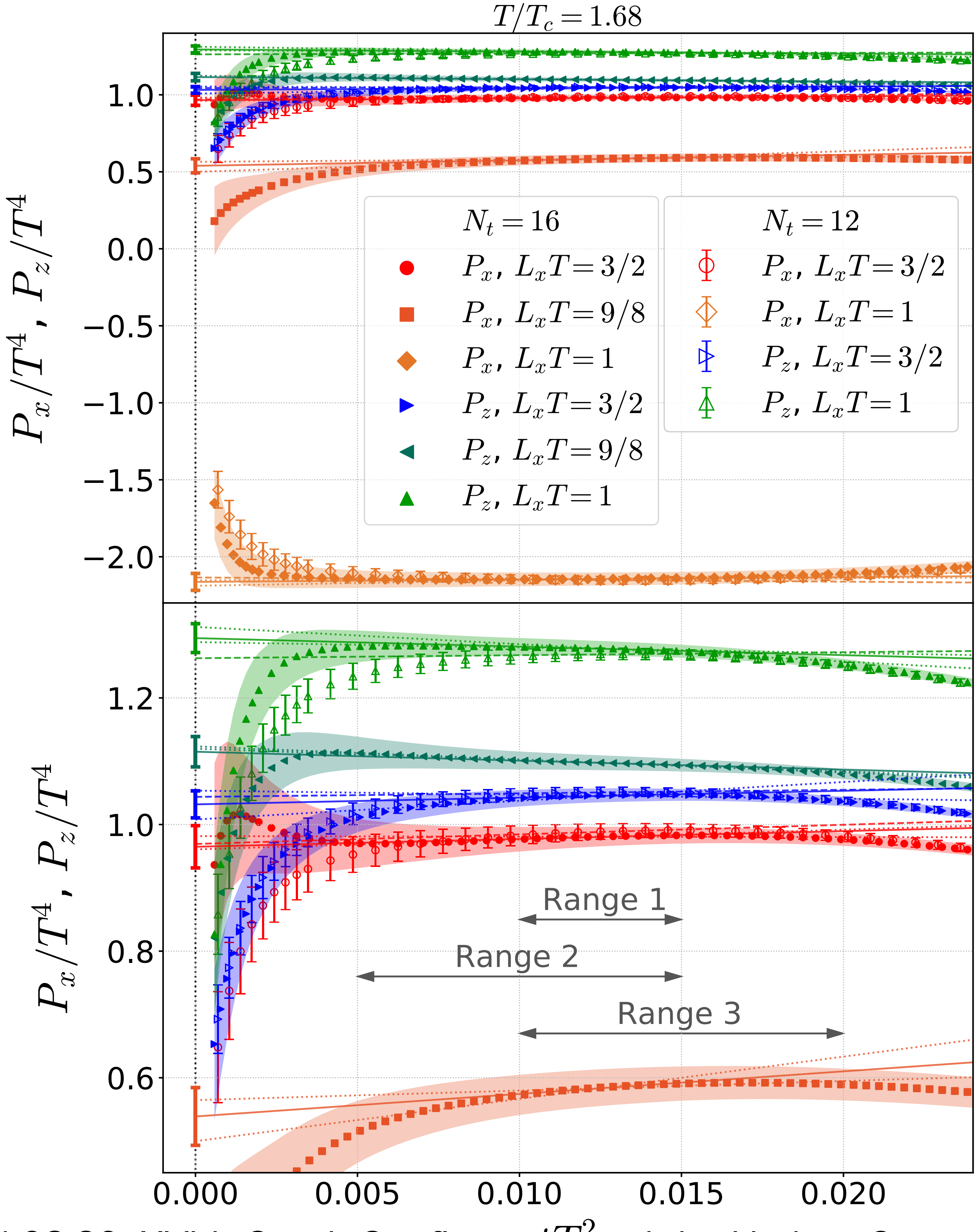
Back up

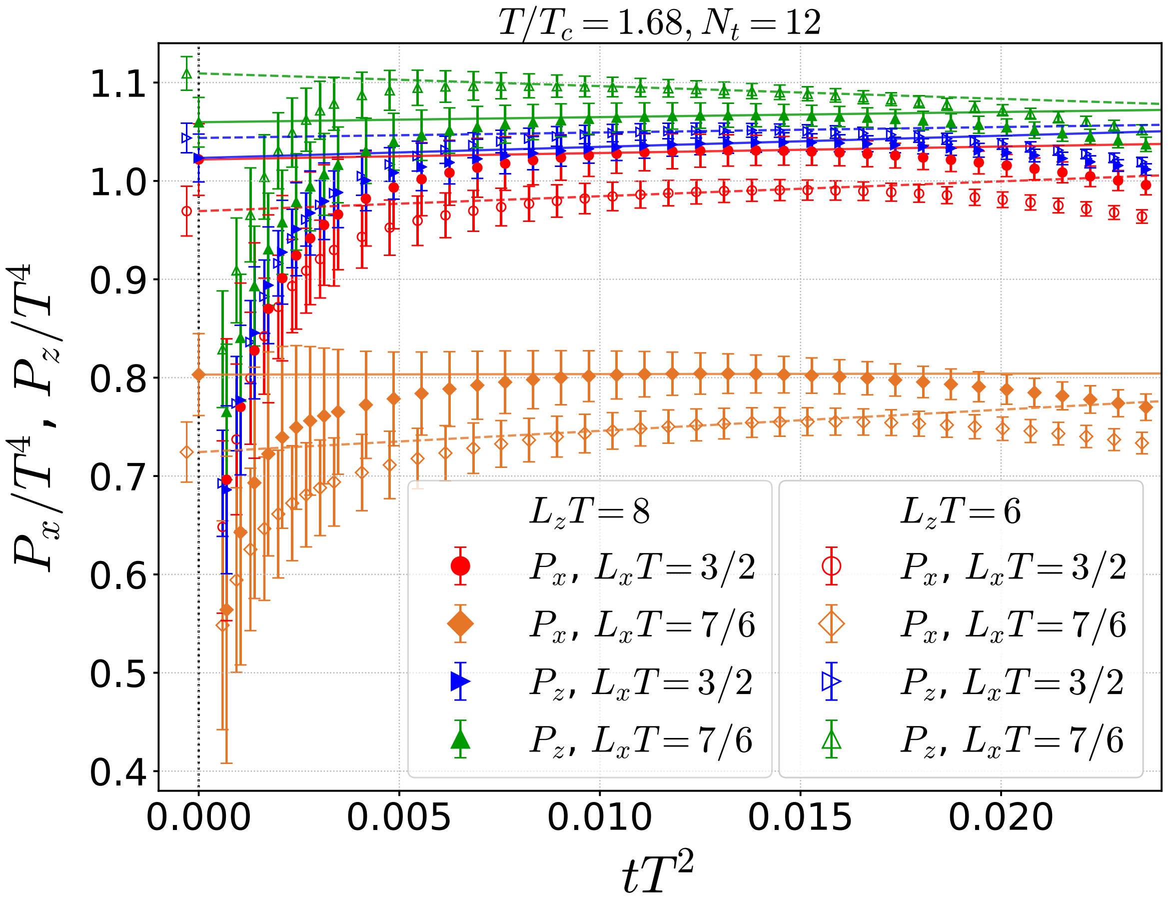
TABLE I. Simulation parameters $\beta = 6/g_0^2$ and lattice volume $N_x \times N_z^2 \times N_\tau$ for each temperature T. The vacuum subtraction is performed on lattices with $N_{\rm vac}^4$.

T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	⋯
≃8.1	8.0	72	12	12, 14, 16, 18	⋯
≃25	9.0	72	12	12, 14, 16, 18	⋯

M. Kitazawa et al. (2019)

Back up

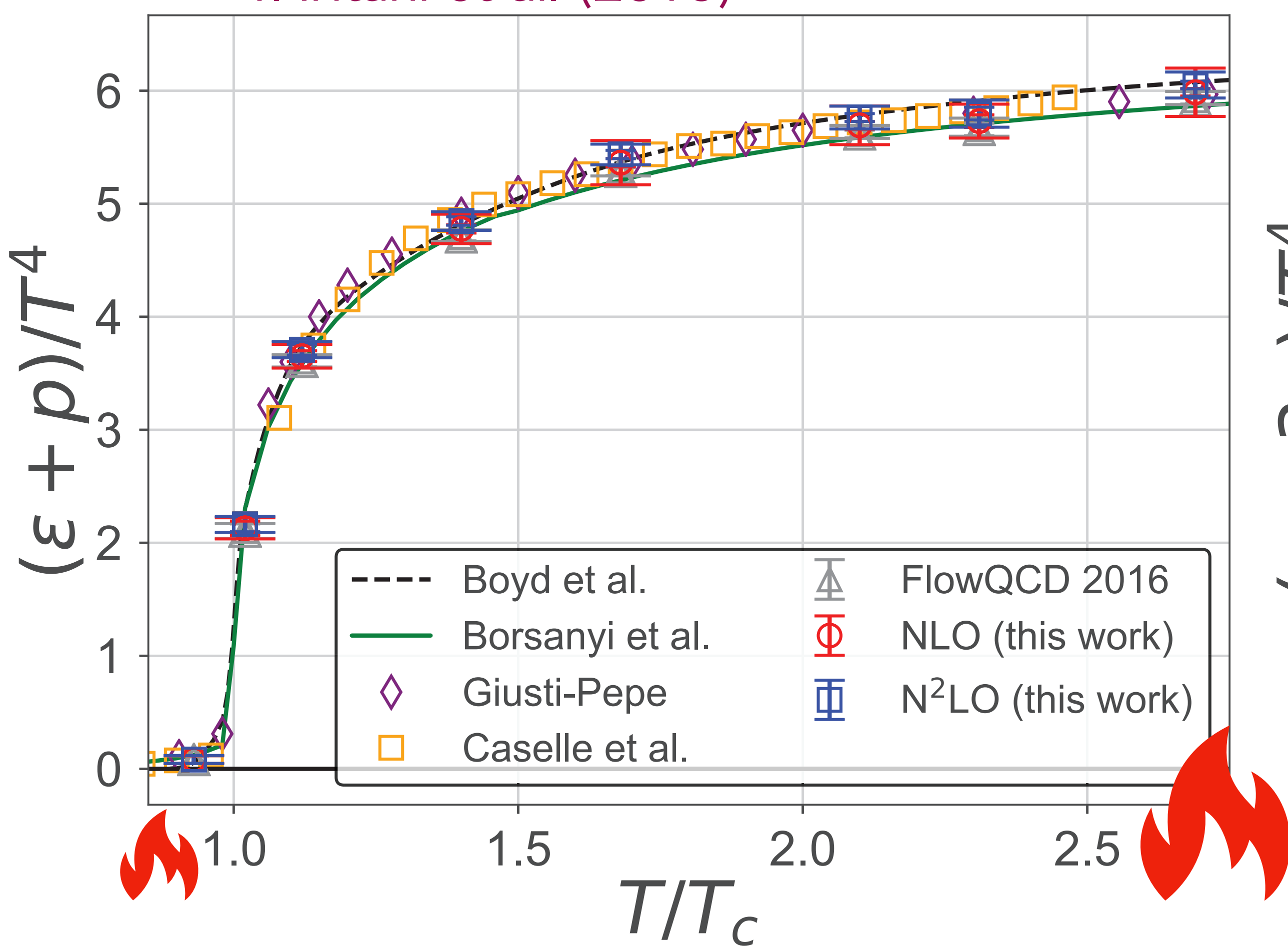


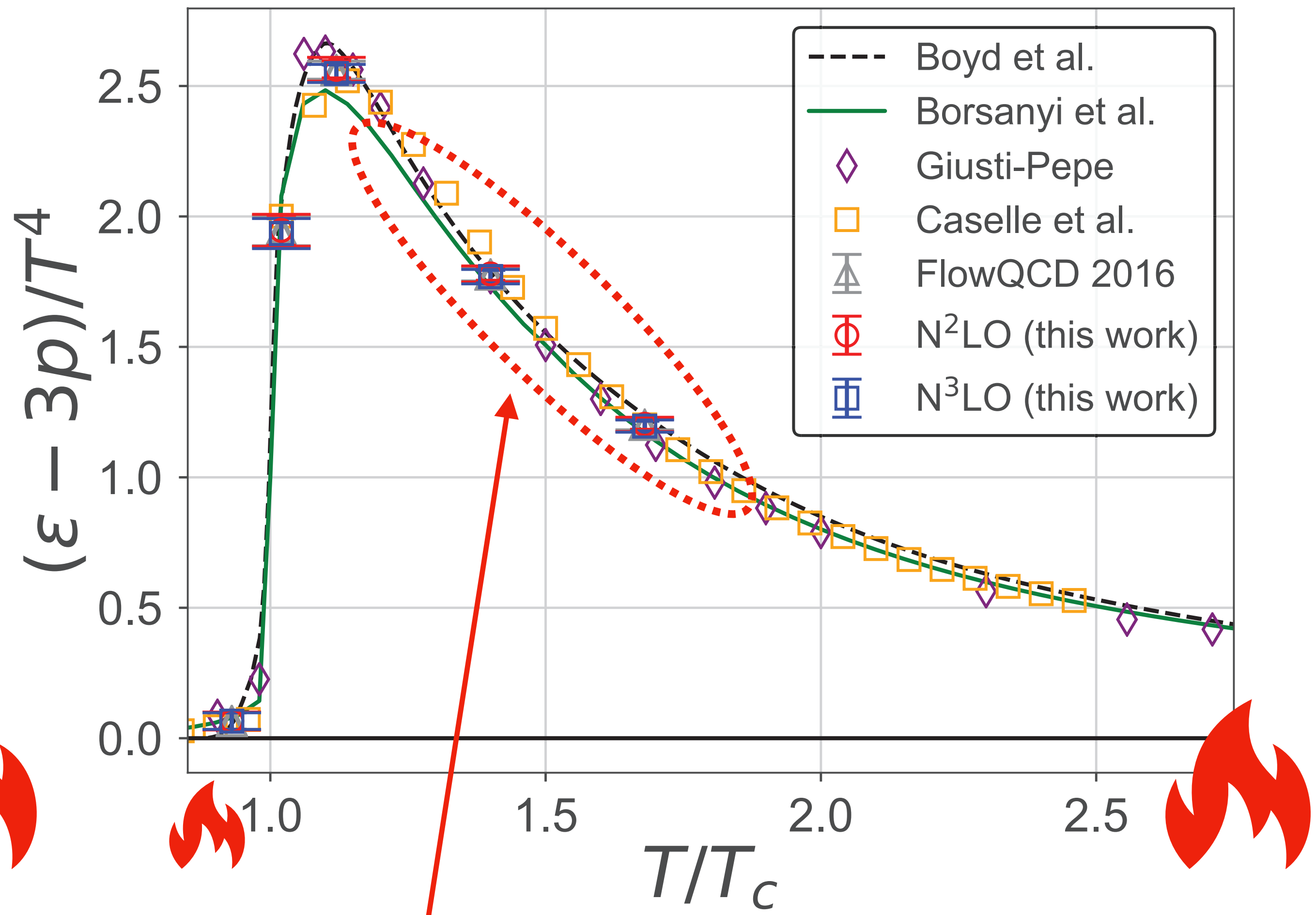


M. Kitazawa et al. (2019)

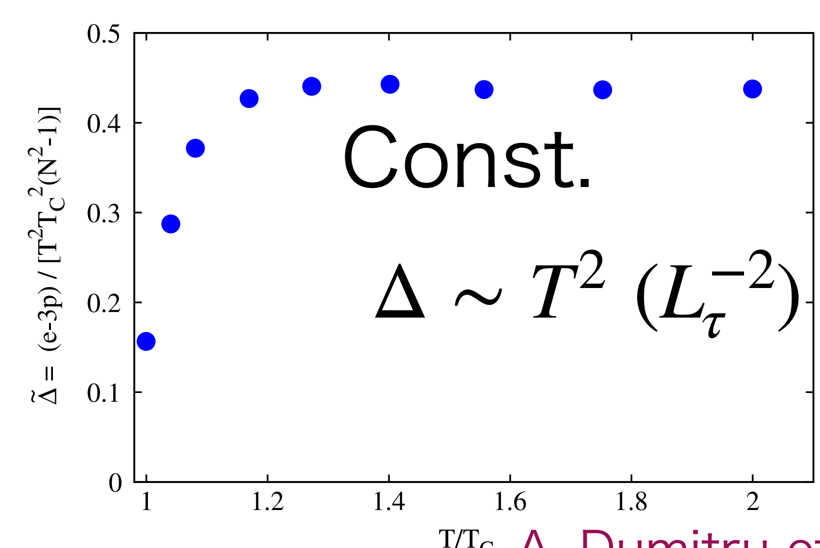
Modelが持つべき特徴（$\mathbb{S}^1\times\mathbb{R}^3$）

T. Iritani et al. (2019)





- 1st order phase transition
- $T/T_c : 1.2 \sim 2.0 \longrightarrow \Delta \sim T^2$
- $T/T_c > 2.0 \longrightarrow \Delta \sim T^4$



A. Dumitru et al. (2012)

Potential term (non-pert.)

Free energy

P. N. Meisinger et al. (2002)

$$f(L_\tau;\theta_\tau) = \sum_{n\in\mathbb{Z}} \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}_A \ln\left[\left(\frac{2\pi n}{\beta} - A_\tau\right)^2 + \vec{k}^2 + m_g^2\right] = f_{\mathrm{pert}} + \boxed{m_g^2 F(L_\tau,\theta_\tau)} + \mathcal{O}(m_g^4)$$

$$f_{\mathrm{pot}} \sim L_\tau^{-2}$$

Without parameter

A. Dumitru et al. (2012) ⟶ **Two parameter model**

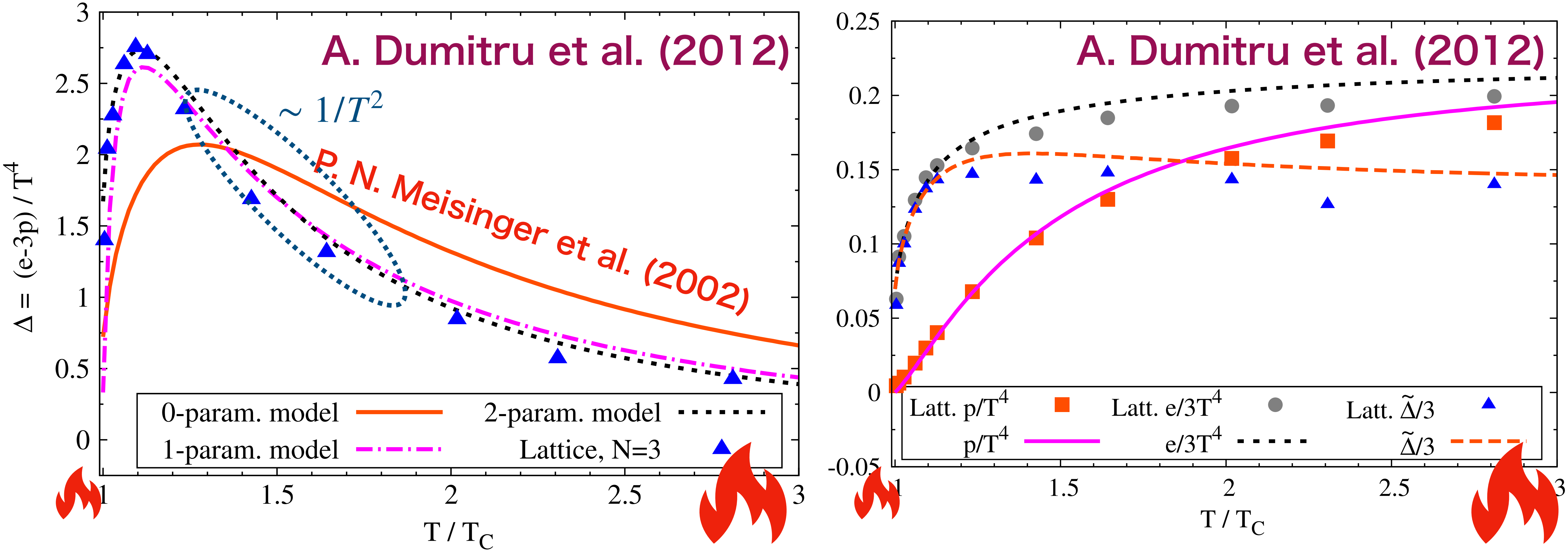
$$f_{\mathrm{pot}} \longrightarrow c_1 F(L_\tau,\theta_\tau) + c_2 F'(L_\tau,\theta_\tau)$$

With parameters

- **"Two parameter model"** on $\mathbb{S}^1 \times \mathbb{R}^3$

Result

- **"Two parameter model" on $\mathbb{S}^1 \times \mathbb{R}^3$**



Well-explain the thermodynamic of lattice near T_c

Extend two parameter model on $\mathbb{T}^2 \times \mathbb{R}^2$

Model ~ Two Polyakov loop~

- Polyakov loops along two compactified directions

$$P_\tau = \frac{1}{N_c}\mathrm{Tr}\left[\mathscr{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right] \qquad P_x = \frac{1}{N_c}\mathrm{Tr}\left[\mathscr{P}\exp\left(i\int_0^{L_x} A_x dx\right)\right]$$

- Assumption : Diagonalized background gauge fields

$$A_y, A_z = 0$$

$$A_\tau = \frac{1}{L_\tau}\begin{pmatrix} (\theta_\tau)_1 & 0 & 0 \\ 0 & (\theta_\tau)_2 & 0 \\ 0 & 0 & (\theta_\tau)_3 \end{pmatrix} \qquad A_x = \frac{1}{L_x}\begin{pmatrix} (\theta_x)_1 & 0 & 0 \\ 0 & (\theta_x)_2 & 0 \\ 0 & 0 & (\theta_x)_3 \end{pmatrix}$$

$$f = f_{\mathrm{pert}}^{\mathbb{T}^2\times\mathbb{R}^2}(L_\tau, L_x; \theta_\tau, \theta_x) + f_{\mathrm{pot}}^{\mathbb{T}^2\times\mathbb{R}^2}(L_\tau, L_x; \theta_\tau, \theta_x)$$

Cross term

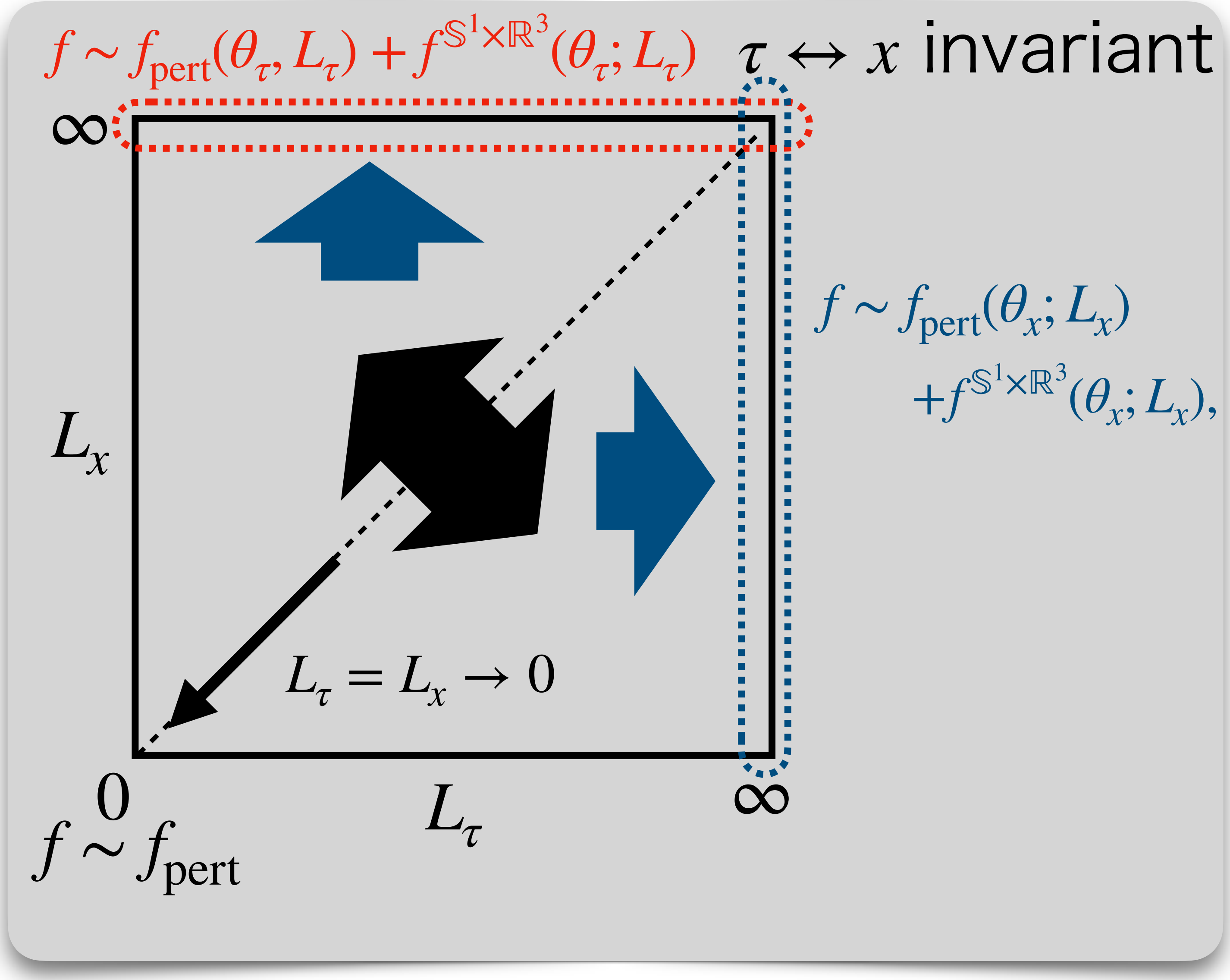
$$f_{\text{cross}} = g(L_\tau, L_x)\Big[c_4 \text{Tr}(P_\tau)^2\text{Tr}(P_x)^2 + c_5\left(\text{Tr}(P_\tau)^2\text{Tr}(P_x^3) + \text{Tr}(P_\tau^3)\text{Tr}(P_x)^2\right) + c_6\text{Tr}(P_\tau^3)\text{Tr}(P_x^3)\Big]$$

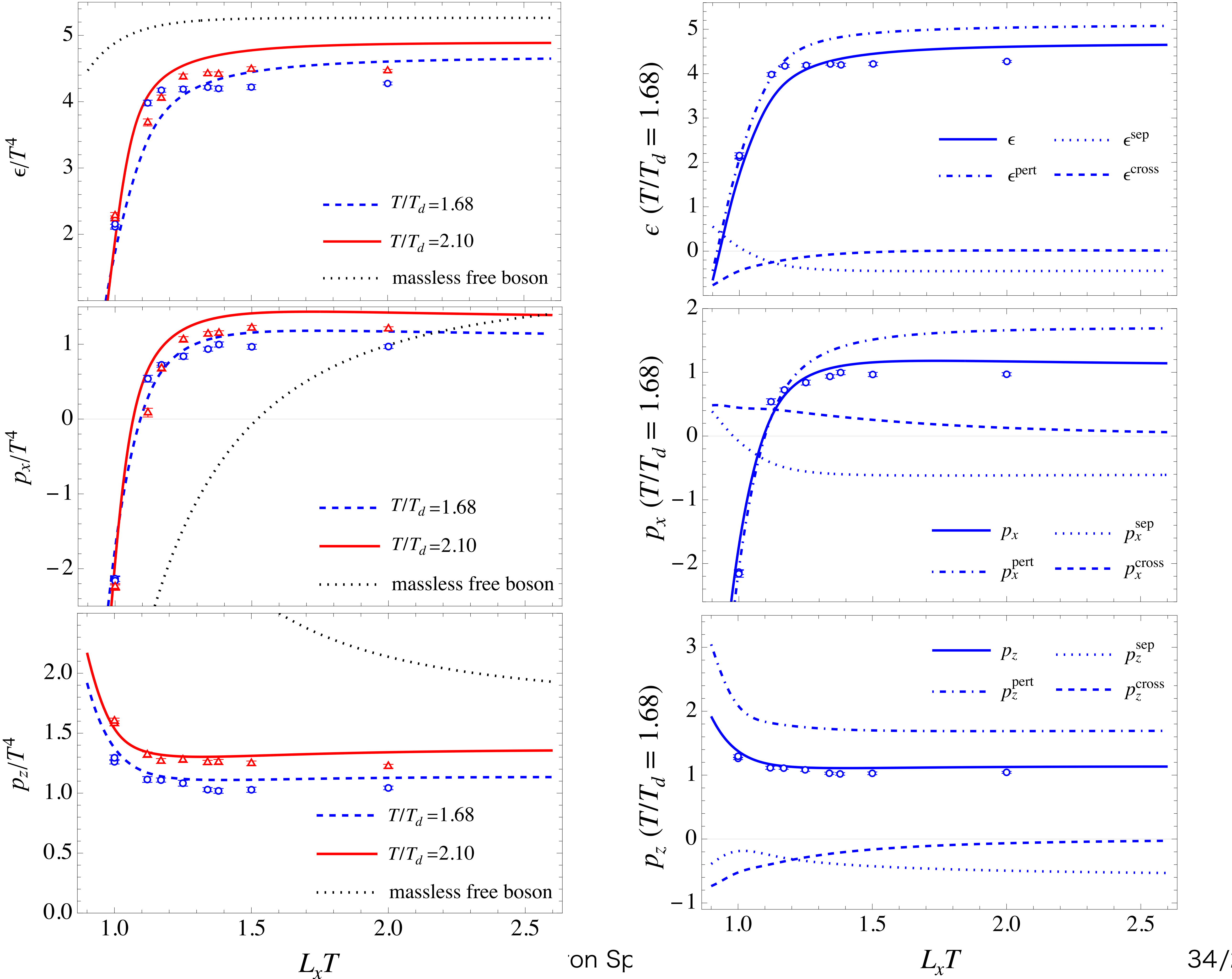
$$T_d^{-2n+4}(L_\tau^2 + L_x^2)^{-n}$$

- Restriction of n

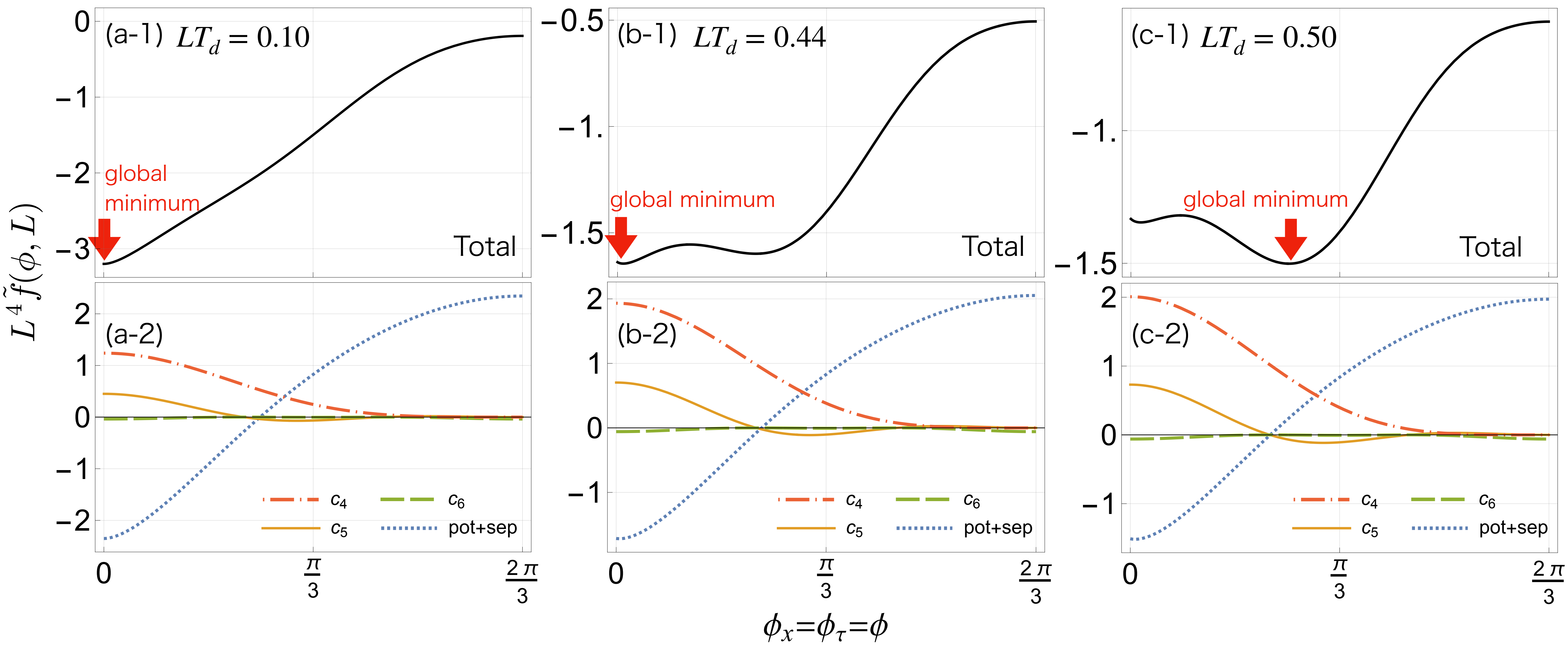
→ $(i)\ 1.5 < n < 2$

- $f_{\text{pert}} \sim \mathcal{O}(L_c^{-4})$
- $f_{\text{pot}}^{\mathbb{S}^1\times\mathbb{R}^3} \sim \mathcal{O}(L_c^{-2})$
- $\lim_{L_c\to\infty} f_{\text{pert}} = \mathcal{O}(L_c^{-3})$

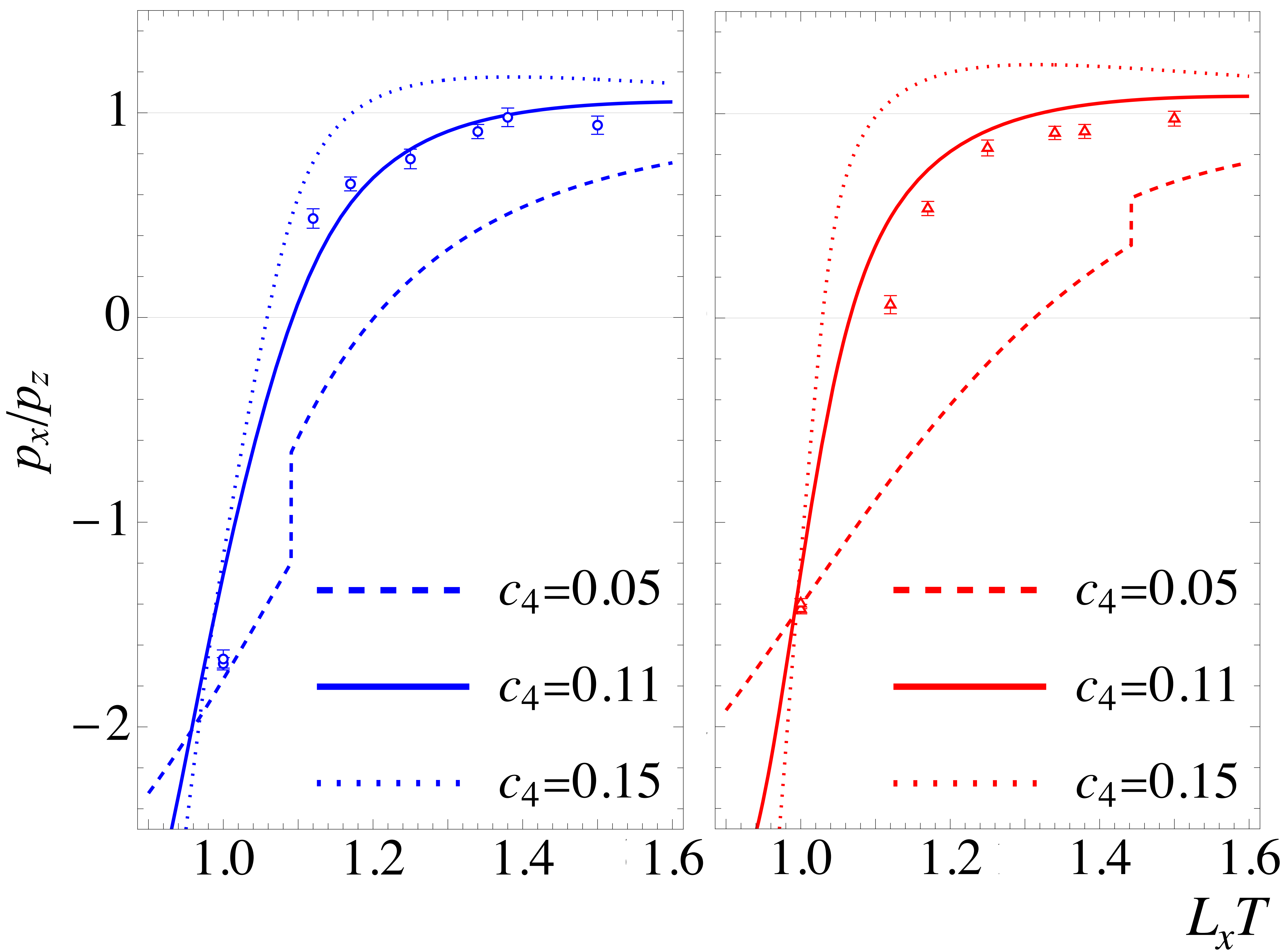




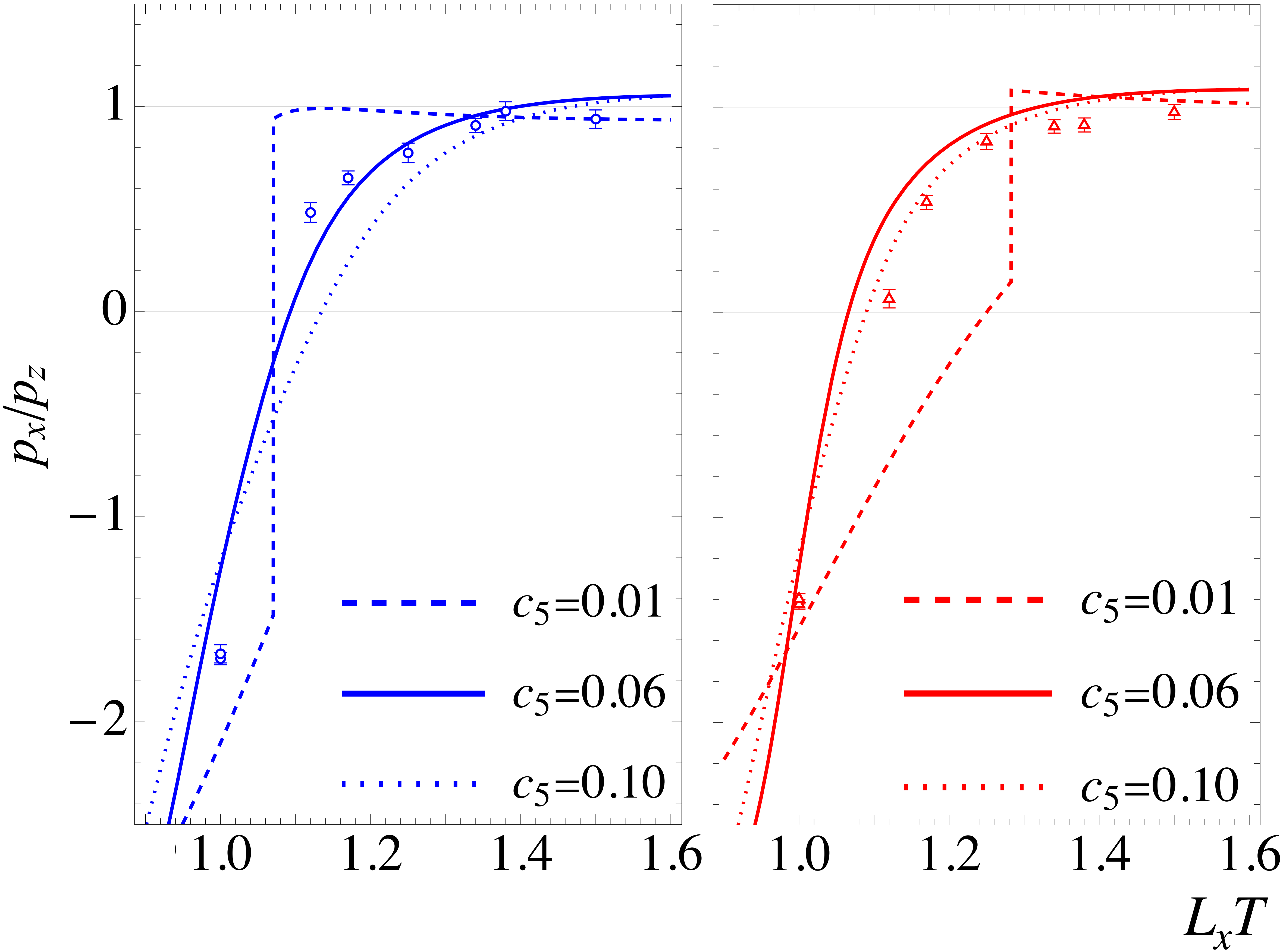
ϵ/T^4
T/T_d =1.68
T/T_d =2.10
massless free boson
p_x/T^4
T/T_d =1.68
T/T_d =2.10
massless free boson
p_z/T^4
T/T_d =1.68
T/T_d =2.10
massless free boson
L_xT
ϵ (T/T_d = 1.68)
ϵ
ϵ^sep
ϵ^pert
ϵ^cross
p_x (T/T_d = 1.68)
p_x
p_x^sep
p_x^pert
p_x^cross
p_z (T/T_d = 1.68)
p_z
p_z^sep
p_z^pert
p_z^cross
L_xT



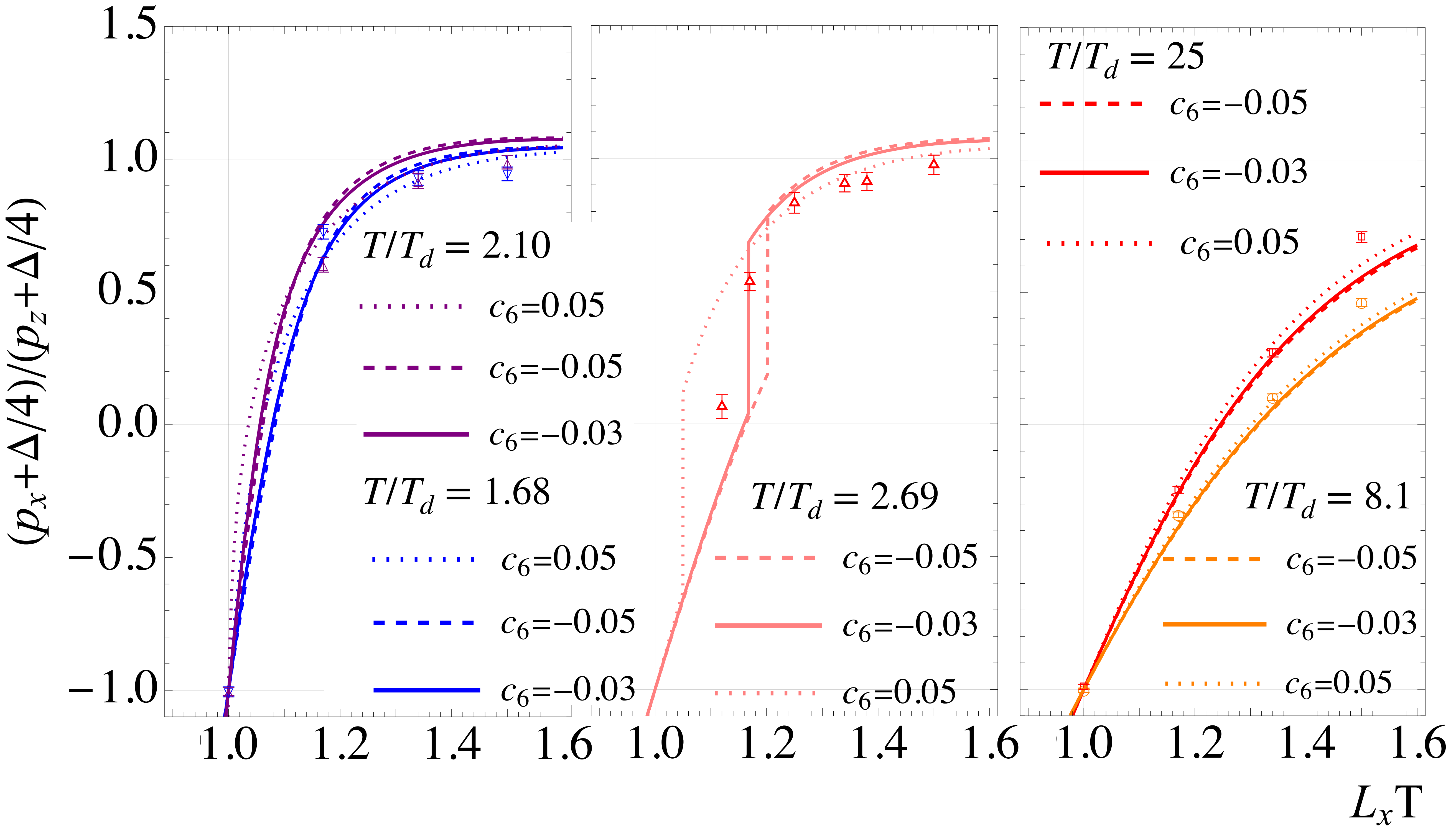
(a-1) $LT_d = 0.10$
global minimum
Total
(b-1) $LT_d = 0.44$
global minimum
Total
(c-1) $LT_d = 0.50$
global minimum
Total
(a-2)
(b-2)
(c-2)
c_4
c_5
c_6
pot+sep
$L^4\tilde{f}(\phi, L)$
$\phi_x=\phi_\tau=\phi$



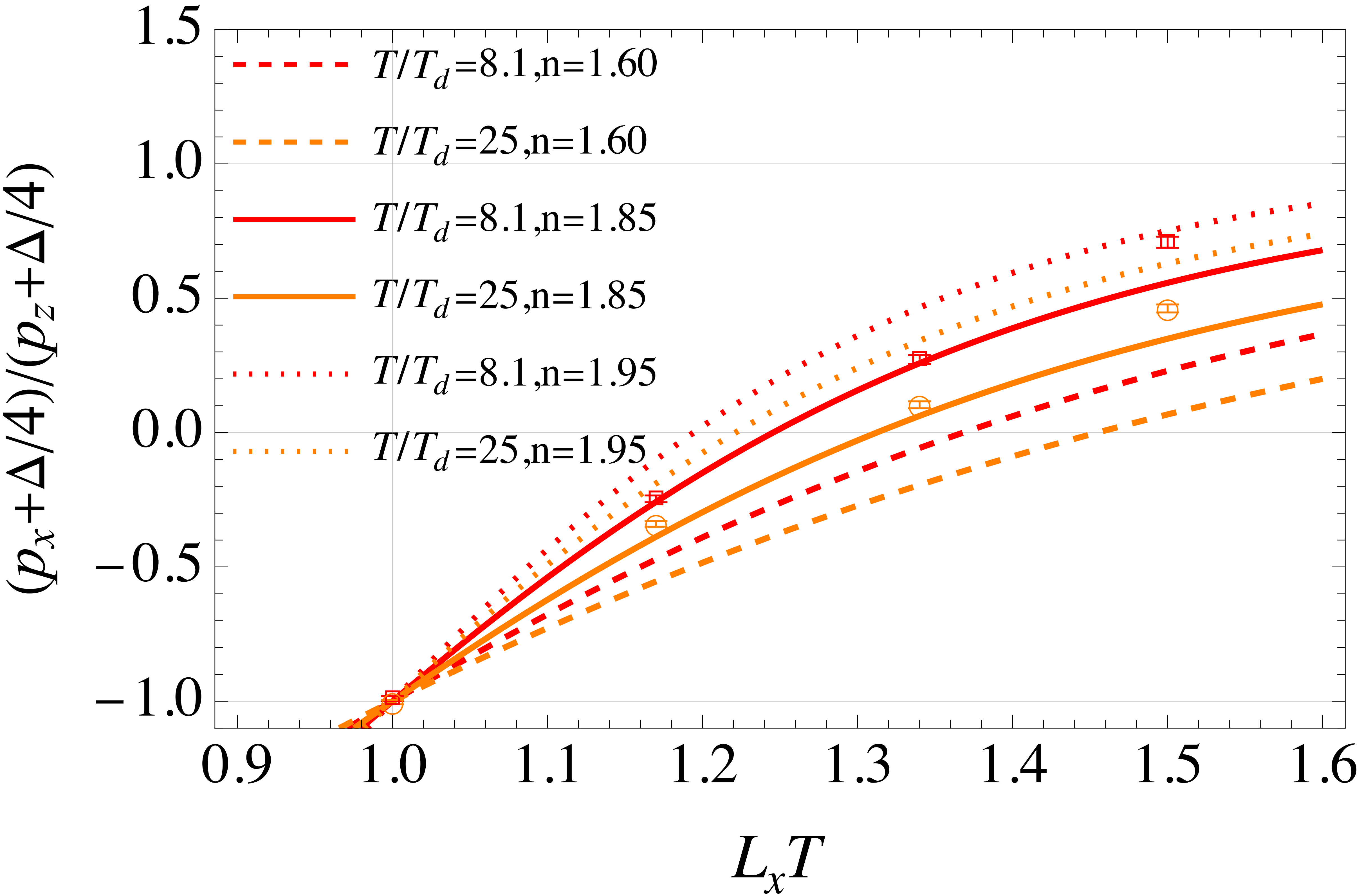
1
0
−1
−2
p_x/p_z
c_4=0.05
c_4=0.11
c_4=0.15
1.0
1.2
1.4
1.6
c_4=0.05
c_4=0.11
c_4=0.15
1.0
1.2
1.4
1.6
L_xT



1
0
−1
−2
p_x/p_z
1.0
1.2
1.4
1.6
c_5=0.01
c_5=0.06
c_5=0.10
L_xT



1.5
1.0
0.5
0.0
−0.5
−1.0
$(p_x+\Delta/4)/(p_z+\Delta/4)$
$T/T_d = 2.10$
c_6=0.05
c_6=−0.05
c_6=−0.03
$T/T_d = 1.68$
c_6=0.05
c_6=−0.05
c_6=−0.03
$T/T_d = 2.69$
c_6=−0.05
c_6=−0.03
c_6=0.05
$T/T_d = 25$
c_6=−0.05
c_6=−0.03
c_6=0.05
$T/T_d = 8.1$
c_6=−0.05
c_6=−0.03
c_6=0.05
1.0
1.2
1.4
1.6
L_xT



1.5
1.0
0.5
0.0
−0.5
−1.0
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
$(p_x+\Delta/4)/(p_z+\Delta/4)$
L_xT
T/T_d=8.1,n=1.60
T/T_d=25,n=1.60
T/T_d=8.1,n=1.85
T/T_d=25,n=1.85
T/T_d=8.1,n=1.95
T/T_d=25,n=1.95

“Two parameter model” on $\mathbb{S}^1 \times \mathbb{R}^3$ A. Dumitru et al. (2012)

Free energy

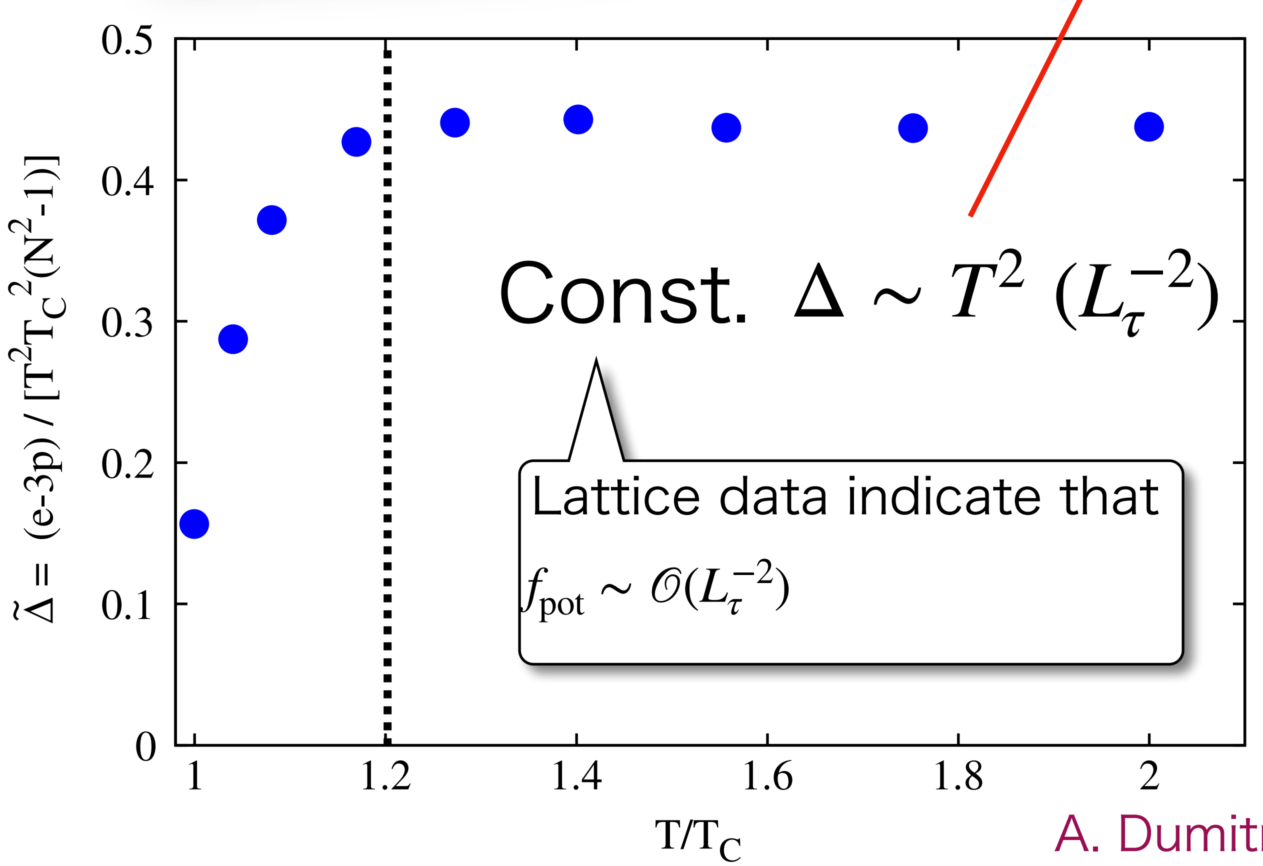
①T_cがdeconfined temperatureになる
②$T = T_c$でpressureがゼロ

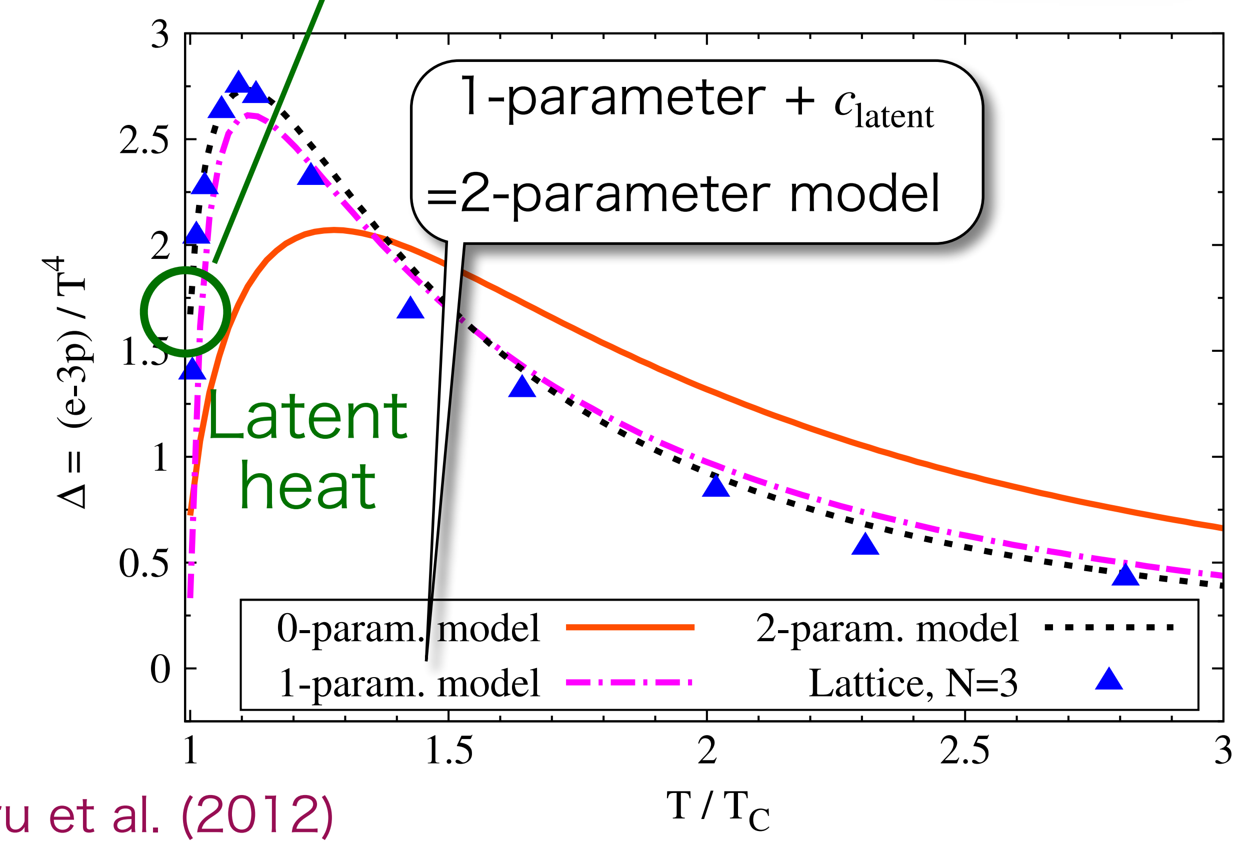
Four possible term
→**independent parameters are only two**

$$B_2(x) = x(1 - |x|)$$
$$B_4(x) = x^2(1 - |x|)^2$$

$$f_{\mathrm{tot}}(\vec{\theta}_\tau; L_\tau) = f_{\mathrm{pert}}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) + f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) + 8c_{\mathrm{latent}}\frac{\pi^2 T_c^4}{45}$$

$$f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) = -\frac{4\pi^2}{6}\frac{T_c^2}{L_\tau^2}\left(\frac{1}{5}c_1 B_2\left(\frac{\phi_\tau}{2\pi}\right) + c_2 B_4\left(\frac{\phi_\tau}{2\pi}\right) - \frac{2}{15}c_3\right)$$





A. Dumitru et al. (2012)

Model ～One loop free energy～

- **Usisng the background field methodを用いる**
- **Regularization** + the high-temperature expansion

One loop perturbative free energy

$$f_{\mathrm{pert}}(\vec{\theta}_\tau,\vec{\theta}_x;L_\tau,L_x) = -\frac{8\pi^2}{45L_\tau^4} + \frac{8\phi_\tau^2(\phi_\tau-\pi)^2+\phi_\tau^2(\phi_\tau^2-2\pi)^2}{6\pi^2L_\tau^4} - \frac{8\pi^2}{45L_x^4} + \frac{8\phi_x^2(\phi_x-\pi)^2+\phi_x^2(\phi_x^2-2\pi)^2}{6\pi^2L_x^4} - \frac{8}{\pi^2}\sum_{l_\tau,l_x=1}^{\infty}\frac{1+2\cos(\phi_\tau l_\tau)\cos(\phi_x l_x)+\cos(2\phi_x l_\tau)\cos(2\phi_x l_x)}{((l_\tau L_\tau)^2+(l_x L_x)^2)^4}$$

$$\lim_{L_c\to\infty} f_{\mathrm{pert}} = \mathcal{O}(L_c^{-3})$$

Total free energy

Dominant at deconfined phase

$$f_{\mathrm{tot}} = f_{\mathrm{pert}}(\vec{\theta}_\tau,\vec{\theta}_x;L_\tau,L_x) + f_{\mathrm{pot}}(\vec{\theta}_\tau,\vec{\theta}_x,;L_\tau,L_x)$$

Dominant at confined phase

これは適宜指摘した方が良い？
少なくとも位置が悪い

Model ～Potential term～

- The general properties of the free energy

1. YM theory on $\underline{\mathbb{T}^2 \times \mathbb{R}^2}$ is **invariant** under $\tau \leftrightarrow x$

$$f_{\rm tot}(\vec{\theta}_\tau, \vec{\theta}_x; L_\tau, L_x) = f_{\rm tot}(\vec{\theta}_x, \vec{\theta}_\tau; L_x, L_\tau)$$

2. At $L_\tau \to \infty$ $(\tau \to 0)$, the system is **irrelevant** for the **BC**.

$P_\tau = 0$ $\quad$ ($\tau \leftrightarrow x$ symmetry: P_x at $L_x \to \infty$)

3. For $L_x \to \infty$, $\mathbb{T}^2 \times \mathbb{R}^2 \to \mathbb{S}^1 \times \mathbb{R}^3$

$$f_{\rm pot}^{\mathbb{T}^2\times\mathbb{R}^2} \to f_{\rm pot}^{\mathbb{S}^1\times\mathbb{R}^3}$$

4. For $L_\tau = L_x \to 0$ $(T \to \infty)$, $f_{\rm pert}$ is **dominant** contribution

$$f_{\rm pert} \sim \mathcal{O}(L_\tau^{-4}, L_x^{-4})$$

Model ～One loop free energy～

- We use **the background field method**
- **Regularization** + the high-temperature expansion

One loop perturbative free energy

間違いあり

$$f_{\mathrm{pert}}(\vec{\theta}_\tau, \vec{\theta}_x; L_\tau, L_x) = -\frac{8\pi^2}{45L_\tau^4} + \frac{2\pi^2}{3L_\tau^4}B_4\left(\frac{\phi_\tau}{2\pi}\right) - \frac{8\pi^2}{45L_x^4} + \frac{2\pi^2}{3L_x^4}B_4\left(\frac{\phi_x}{2\pi}\right)$$
$$-\frac{8}{\pi^2}\sum_{l_\tau, l_x=1}^{\infty}\frac{1+2\cos(\phi_\tau l_\tau)\cos(\phi_x l_x)+\cos(2\phi_x l_\tau)\cos(2\phi_x l_x)}{((l_\tau L_\tau)^2+(l_x L_x)^2)^4}$$

$$B_4\left(\frac{\phi_\tau}{2\pi}\right) = \frac{8\phi_\tau^2(\phi_\tau-\pi)^2+\phi_\tau^2(\phi_\tau^2-2\pi)^2}{4\pi^4}$$

Total free energy

Dominant at deconfined phase

$$f_{\mathrm{tot}} = f_{\mathrm{pert}}(\vec{\theta}_\tau, \vec{\theta}_x; L_\tau, L_x) + f_{\mathrm{pot}}(\vec{\theta}_\tau, \vec{\theta}_x, ; L_\tau, L_x)$$

Dominant at confined phase

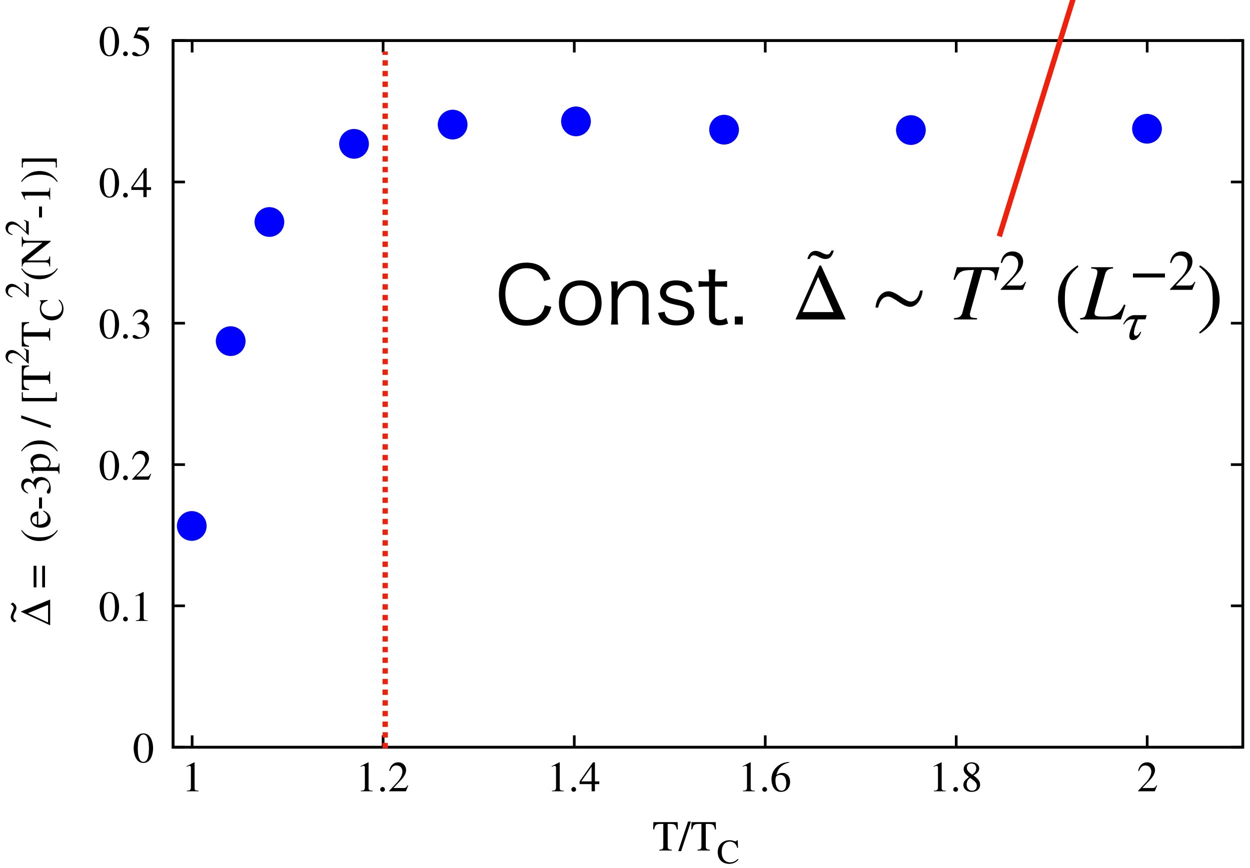
Model ～Potential term～

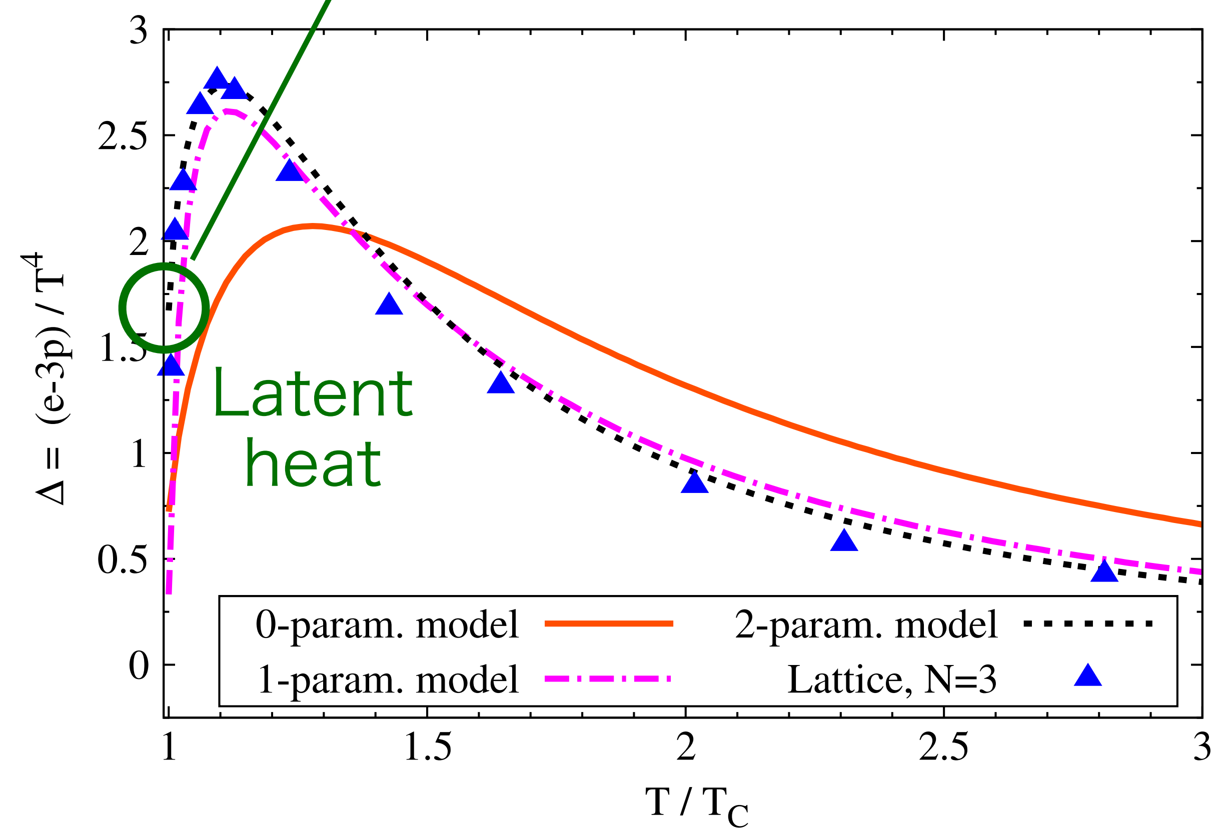
- **"Two parameter model" on $\mathbb{S}^1 \times \mathbb{R}^3$**

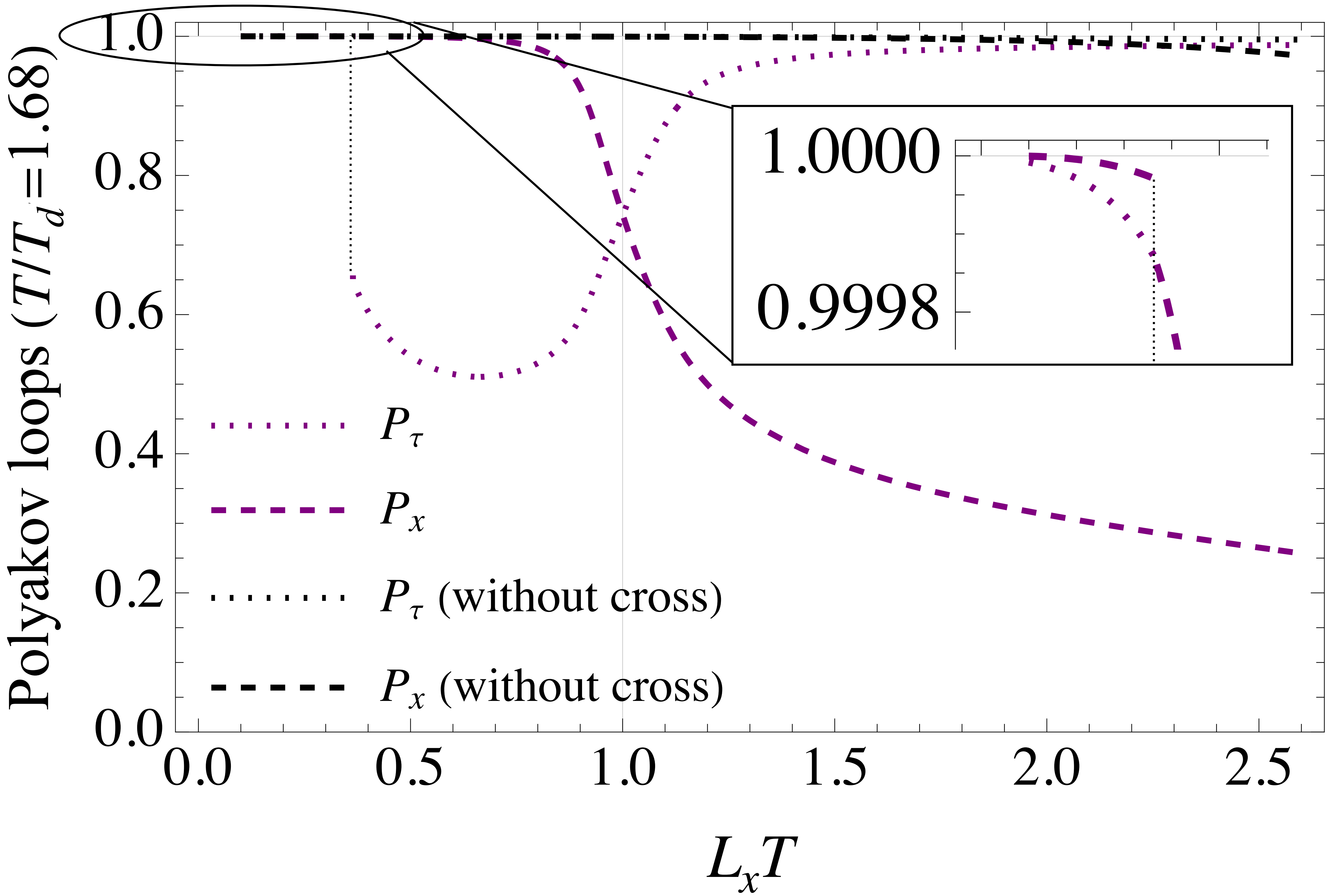
Free energy

$$f_{\rm tot}(\vec{\theta}_\tau; L_\tau) = f_{\rm pert}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) + f_{\rm pot}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) + 8c_{\rm latent}\frac{\pi^2 T_d^4}{45}$$

$$f_{\rm pot}^{\mathbb{S}^1\times\mathbb{R}^3}(\vec{\theta}_\tau, L_\tau) = -\frac{4\pi^2}{6}\frac{T_d^2}{L_\tau^2}\left(\frac{1}{5}c_1 B_2\left(\frac{\phi_\tau}{2\pi}\right) + c_2 B_4\left(\frac{\phi_\tau}{2\pi}\right) - \frac{2}{15}c_3\right)$$







Polyakov loops (T/T_d=1.68)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
L_xT
1.0000
0.9998
P_τ
P_x
P_τ (without cross)
P_x (without cross)

$$\frac{\partial}{\partial \beta}\sum_l \ln\left[(\omega_l - \varphi/\beta)^2 + E^2\right] = n(E,\varphi),$$

$$n(E,\varphi) = E\Big(1 + \frac{1}{e^{\beta E + i\varphi} - 1} + \frac{1}{e^{\beta E - i\varphi} - 1}\Big),$$

$$p_x^{\mathrm{pert}} = -\left.\frac{\partial(L_x f_{\mathrm{pert}})}{\partial L_x}\right|_{\Omega_\tau,\Omega_x}$$

$$= \frac{2}{L_\tau}\sum_{j,k=1}^{3}\Big(1 - \frac{\delta_{jk}}{3}\Big)\sum_{\ell_\tau}\int \frac{d^2 p_L}{(2\pi)^2} n(\mathcal{E}, (\Delta\theta_x)_{jk})$$

(5

$$\mathcal{E}^2 = (\omega_\tau - (\Delta\theta_\tau)_{jk}/L_\tau)^2 + \mathbf{p}_L^2.$$

(6

$$p_z^{\mathrm{pert}} = -\frac{\partial}{\partial L_x}(L_x f_{\mathrm{pert}})$$

$$= \frac{2}{L_\tau}\sum_{j,k=1}^{3}\Big(1 - \frac{\delta_{jk}}{3}\Big)\sum_{\ell_\tau}\int \frac{d^2 p_L}{(2\pi)^2}$$

$$\times \ln(1 - e^{-\beta\mathcal{E} + i(\Delta\theta_x)_{jk}})(1 - e^{\beta\mathcal{E} - i(\Delta\theta_x)_{jk}}).$$