广州大学

GUANGZHOU UNIVERSITY

QCHSC 2024

NNLO QCD Corrections to Mesons EM Form Factors

arXiv:2312.17228
arXiv:2407.21120

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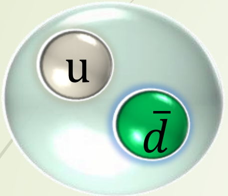
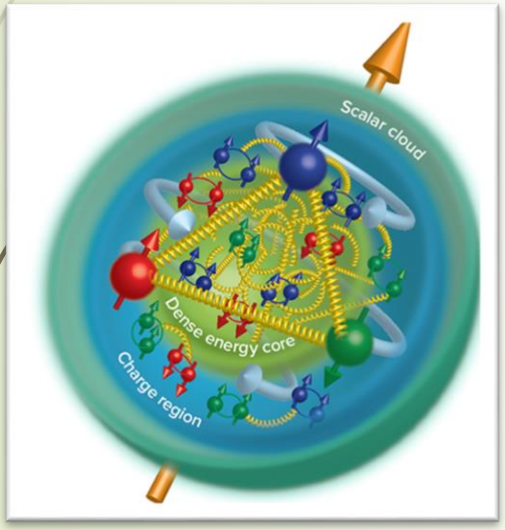
In collaboration with：Wen Chen, Feng Feng and Yu Jia

2024/8/20

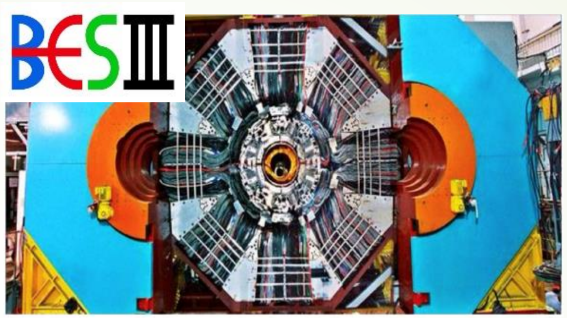
Motivation

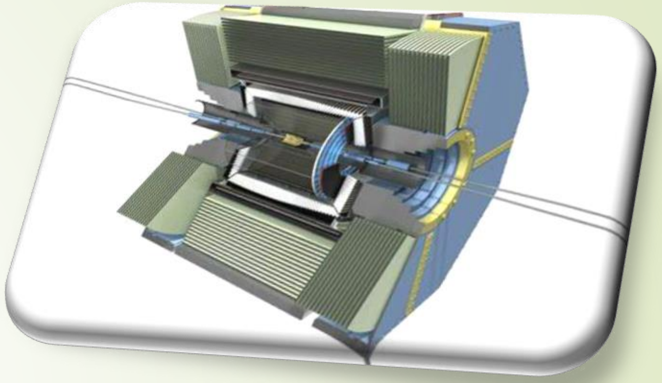
Unveiling the inner structure of hadron

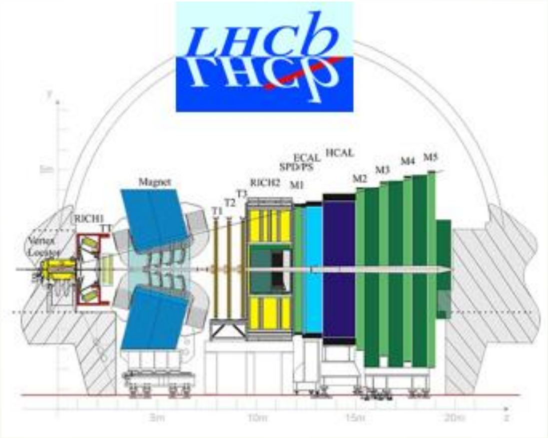


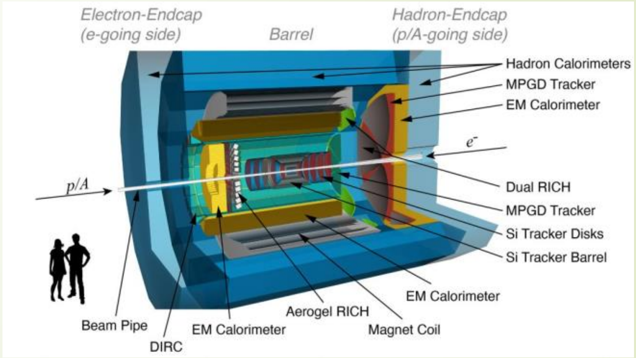


QCD









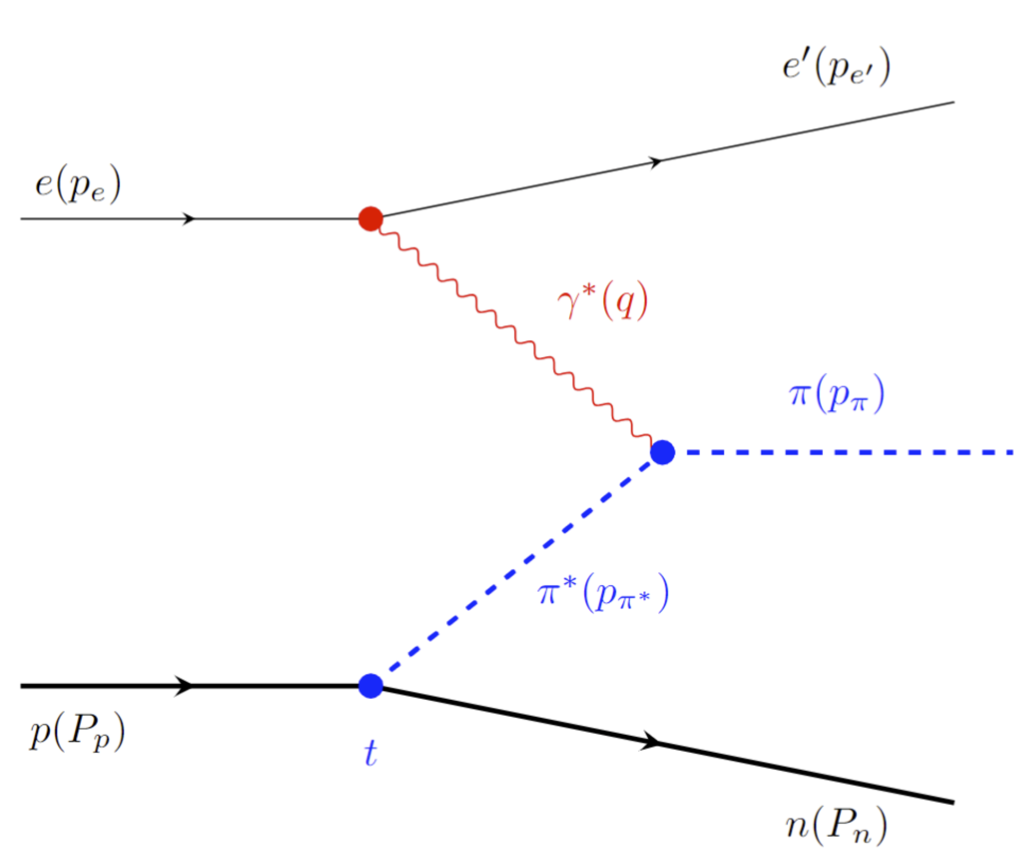
EM FFs: Probing the internal structure of hadrons

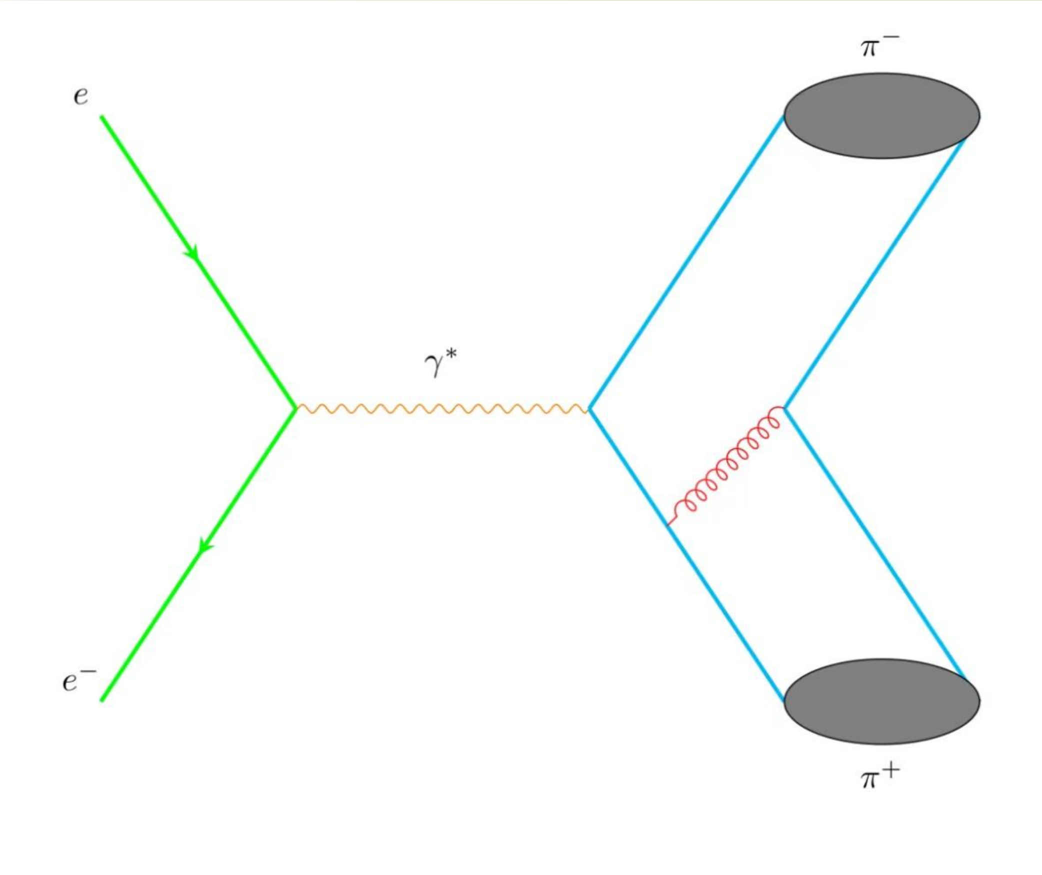
Charge Pion Form Factors:

$$\langle \pi^+(P') | J^\mu_{\text{em}} | \pi^+(P) \rangle = F_\pi(Q^2)(P^\mu + P'^\mu),$$

$$J^\mu_{\text{em}} = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f ,$$
$$Q^2 \equiv -(P' - P)^2.$$

At small Q^2： Chiral Perturbative Theory, Lattice QCD.
Relate to pion charge radius





Space Like

Sullivan Process

At small t is sensitive to pion form factor

EIC： Q^2 up to 30 GeV^2
arXiv:2208.14575

Time Like

BESIII
BarBar
Belle II
STCF

Probing pion Form Factors

In framework of Collinear Factorization, Large Q^2

(Lepage, Brodsky, Efremov, Radyushkin, Duncan, Mueller)

$$F_\pi(Q^2) = \iint dx\, dy\, \Phi_\pi^*(x, \mu_F) T(x, y, \frac{\mu_R^2}{Q^2}, \frac{\mu_F^2}{Q^2}) \Phi_\pi(y, \mu_F)$$

Leading-Twist Pion LCDA:

$$\Phi_\pi(x, \mu_F) = \int \frac{dz^-}{2\pi i} e^{iz^- xP^+} \langle 0 \,|\bar{d}(0)\gamma^+\gamma_5 \;\times\; \mathcal{W}(0, z^-) u(z^-)|\, \pi^+(P)\rangle$$

ERBL evolution equations

$$\frac{d\Phi_\pi(x, \mu_F)}{d \ln \mu_F^2} = \int_0^1 dy V(x, y)\, \Phi_\pi(y, \mu_F).$$

Hard Kernel

$$T(x, y, \mu_R^2/Q^2, \mu_F^2/Q^2)$$

Pertubative Expansion

$$T = \frac{16 C_F \pi \alpha_s}{Q^2} \left\{ T^{(0)} + \frac{\alpha_s}{\pi} T^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 T^{(2)} + \cdots \right\}$$

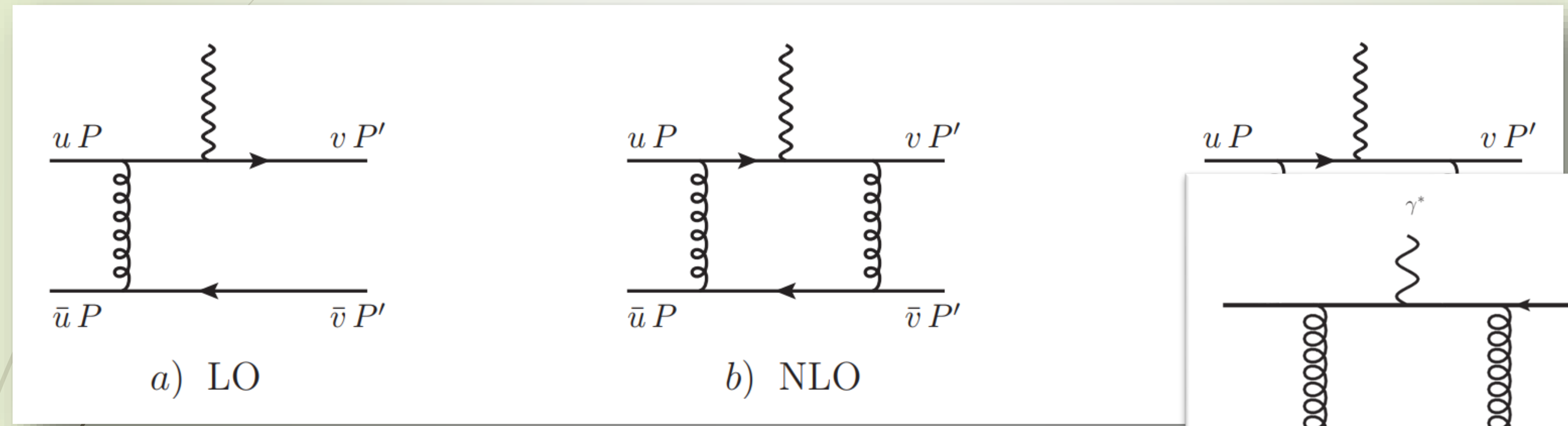
LO: 1977-1980 Lepage, Brodsky, Efremov, Radyushkin, Duncun, Muller…

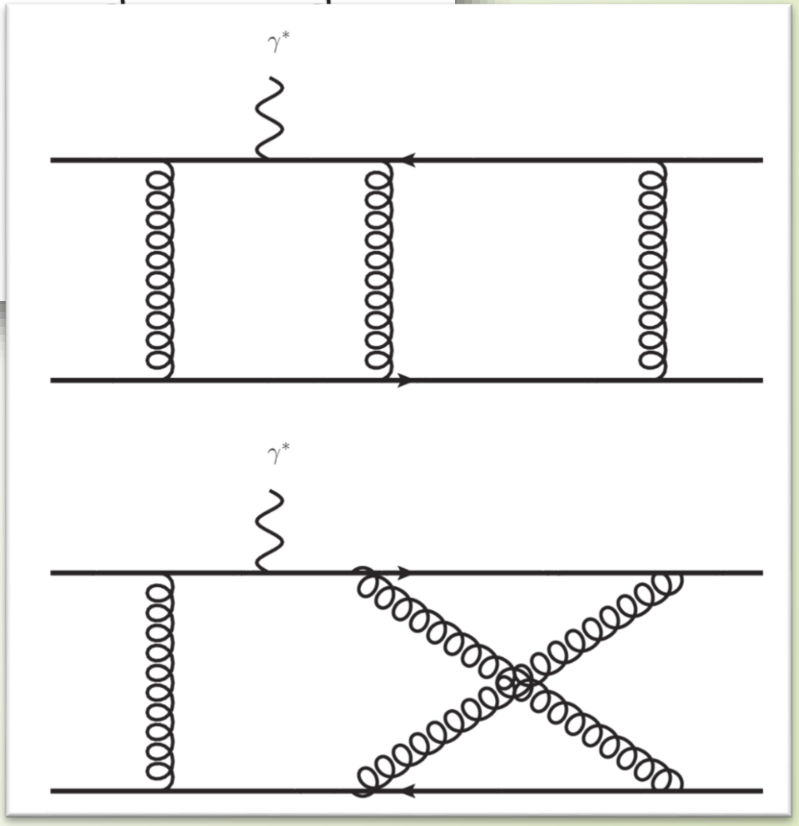
NLO: Field, Gupta, Otta, Chang(1981), Dittes, Radyushkin(1981), Sarmadi (1984),
Braaten,Tse(1987), Melic, Nizic and Passek (1998).

The goal of this work is to achieve NNLO calculations.

Partonic Process

$$\gamma^* + u(uP)\bar{d}(\bar{u}P) \rightarrow u(vP')\bar{d}(\bar{v}P')$$





Feynman Diagrams(about 1600)

Calculation Methods

- Diagrams and amplitudes : HepLib, FeynArts
- Partial Fraction: Apart
- Integrals Reduction: FIRE **(Integration-By-Parts)**
- Master Integrals Calculation: Differential Equations
- Validations of analytic results for MIs: AMFLOW, AmpRed

Feynman Integrals

Differential Equations (DE)

$$\frac{d}{dx}\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} = \begin{pmatrix} A_{11} & \dots & A_{1,n} \\ \vdots & & \vdots \\ A_{n1} & \dots & A_{n,n} \end{pmatrix}\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$$

Differential equations method: New technique for massive Feynman diagrams calculation

A.V. Kotikov (BITP, Kiev) (Jun, 1990)

Published in: *Phys.Lett.B* 254 (1991) 158-164

Differential equation method: The Calculation of N point Feynman diagrams

A.V. Kotikov (BITP, Kiev) (1991)

Published in: *Phys.Lett.B* 267 (1991) 123-127, *Phys.Lett.B* 295 (1992) 409-409 (erratum)

Multiloop integrals in dimensional regularization made simple

Johannes M. Henn (Princeton, Inst. Advanced Study) (Apr 5, 2013)

Published in: *Phys.Rev.Lett.* 110 (2013) 251601 • e-Print: 1304.1806 [hep-th]

Choosing canonical basis (Basis with uniform transcendentality)

For random integral basis g, we have:

$$\partial_x \vec{g}(x;\epsilon) = B(x,\epsilon)\,\vec{g}(x;\epsilon)$$

We can choose new basis f:

$$\vec{f} = T\vec{g},$$

The new DEs is simple and elegant

$$d\,\vec{f}(x,\epsilon) = \epsilon\,\left(d\,\tilde{A}\right)\,\vec{f}(x;\epsilon)$$

$$\tilde{A} = \left[\sum_k A_k \log \alpha_k(x)\right] .$$

Goncharov Polylogarithms

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,,$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!}\ln^n x\,.$$

$$G_{a_1,\ldots,a_m}(x)G_{b_1,\ldots,b_n}(x) = \sum_{c\in a\amalg b} G_{c_1,c_2,\ldots,c_{m+n}}(x)\,.$$

A. B. Goncharov, *Multiple polylogarithms, cyclotomy and modular complexes*, Math. Res. Lett. **5**, (1998) 497–516, [arXiv:1105.2076].

Numerical evaluation of multiple polylogarithms

Jens Vollinga (Mainz U., Inst. Phys.), Stefan Weinzierl (Mainz U., Inst. Phys.) (Oct, 2004)

Published in: *Comput.Phys.Commun.* 167 (2005) 177 • e-Print: hep-ph/0410259 [hep-ph]

GINAC

LO kernel

$$T^{(0)}(x,y) = \frac{e_u}{\bar{x}\bar{y}}(1-\epsilon) - \left[\begin{array}{c} e_u \to e_d \\ \bar{x} \to x,\, \bar{y} \to y \end{array}\right],$$

Perturbative expansion

$$F(u,v) = F^{(0)}(u,v) + \frac{\alpha_s}{\pi} F^{(1)}(u,v) + \left(\frac{\alpha_s}{\pi}\right)^2 F^{(2)}(u,v) + \cdots$$

Renormalized "pion" LCDA can be expressed as :

$$\Phi(x|u) = \Phi^{(0)}(x|u) + \frac{\alpha_s}{\pi} \Phi^{(1)}(x|u) + \left(\frac{\alpha_s}{\pi}\right)^2 \Phi^{(2)}(x|u) + \cdots$$

Renormalized LCDA

$$\Phi(x|u) = \int dy Z(x,y)\Phi_{\text{bare}}(y|u) = Z(x,u),$$

$$Z(x,y) = \delta(x-y) + \sum_{k=1}^{\infty} \frac{1}{\epsilon^k} Z_k(x,y),$$

$$V(x,y) = -\alpha_s \frac{\partial Z_1}{\partial \alpha_s}.$$

Related with EBRL kernel

$$\alpha_s \frac{\partial Z_2}{\partial \alpha_s} = \alpha_s \frac{\partial Z_1}{\partial \alpha_s} \otimes Z_1 + \beta(\alpha_s) \frac{\partial Z_1}{\partial \alpha_s},$$

arXiv:hep-ph/0512208

Matching

$$F(u,v) = \Phi(x|u) \otimes T(x,y) \otimes \Phi(y|v)$$

NLO

$$\begin{aligned} Q^2 F^{(1)}(u,v) \;&=\; T^{(1)}(u,v) + \Phi^{(1)}(x|u) \underset{x}{\otimes} T^{(0)}(x,v) \\ &\quad + \Phi^{(1)}(y|v) \underset{y}{\otimes} T^{(0)}(u,y), \end{aligned}$$

NNLO

$$\begin{aligned} Q^2 F^{(2)}(u,v) \;&=\; T^{(2)}(u,v) + \Phi^{(2)}(x|u) \underset{x}{\otimes} T^{(0)}(x,v) \\ &\quad + \Phi^{(2)}(y|v) \underset{y}{\otimes} T^{(0)}(u,y) \\ &\quad + \Phi^{(1)}(x|u) \underset{x}{\otimes} T^{(1)}(x,v) \\ &\quad + \Phi^{(1)}(y|v) \underset{y}{\otimes} T^{(1)}(u,y) \\ &\quad + \Phi^{(1)}(x|u) \underset{x}{\otimes} T^{(0)}(x,y) \underset{y}{\otimes} \Phi^{(1)}(y|v) \end{aligned}$$

LO and NLO Matching

$$F_0(u,v) = T_0(u,v)$$

$$F_1(u,v) = T_1(u,v) + \frac{1}{\epsilon} V_0(x,u) \otimes T_0(x,v) + \frac{1}{\epsilon} T_0(u,y) \otimes V_0(y,v)$$

NNLO Matching

$$F_2(u,v) = T_2(u,v)$$
$$+\frac{1}{\epsilon}\left[Z_1^{(1)}(x,u)\otimes T_1(x,v) + T_1(u,y)\otimes Z_1^{(1)}(y,v)\right] + \frac{1}{\epsilon^2} Z_1^{(1)}(x,u)\otimes T_0(x,y)\otimes Z_1^{(1)}(y,v)$$
$$+\frac{1}{\epsilon}\left[Z_2^{(1)}(x,u)\otimes T_0(x,v) + T_0(u,y)\otimes Z_2^{(1)}(y,v)\right] + \frac{1}{\epsilon^2}\left[Z_2^{(2)}(x,u)\otimes T_0(x,v) + T_0(u,y)\otimes Z_2^{(2)}(y,v)\right]$$

The convolutions above are calculated analytically.

All Infra-divergences are cancel.

We obtain analytic results for finite T_2

Asympotic Expressions of Kernel $T^{(1)}$

$$\lim_{\substack{x\to 0\\ y\to 0}} T^{(1)}(x,y,\mu) = -\frac{e_d}{36xy}\left[12\ln^2(xy) - 18\ln(xy) - \pi^2 + 30 - 3(8\ln(xy)-3)L_\mu\right],$$

$$\lim_{\substack{x\to 0\\ y\to 1}} T^{(1)}(x,y) = -\frac{e_d}{12x}\left[4\ln^2 x - \ln x\ln\bar{y} - 7\ln x - \ln\bar{y} + 15 - (8\ln x - 3)L_\mu\right]$$
$$+ \left[x\to\bar{y}, \bar{y}\to x, e_d\to -e_u\right].$$

NLO results agree with previous calculations

$T^{(2)}$ –asymptotic behavior

$$\lim_{\substack{x\to 0\\ y\to 0}} T^{(2)}(x,y,\mu) = -\frac{e_d}{18xy}\Big[\ln^4(xy) - \frac{15}{2}\ln^3(xy) - (\frac{5}{3}\pi^2 - \frac{367}{8})\ln^2(xy) - \frac{81}{4}\ln x\ln y$$
$$+(73\zeta_3 + \frac{137}{48}\pi^2 - \frac{1169}{8})\ln(xy) + \frac{83}{60}\pi^4 - \frac{219}{2}\zeta_3 - \frac{269}{24}\pi^2 + \frac{3177}{16}$$
$$+\frac{1}{8}(8\ln(xy) - 3)(4\ln(xy) - 15)L_\mu^2$$
$$-(4\ln^3(xy) - 21\ln^2(xy) - \frac{1}{3}(10\pi^2 - \frac{537}{2})\ln(xy) + 4\zeta_3 + \frac{25}{4}\pi^2 - 81)L_\mu\Big].$$

$T^{(2)}$ –asymptotic behavior

$$
\begin{aligned}
\lim_{\substack{x\to 0\\ y\to 1}} T^{(2)}(x,y,\mu) = & -\frac{e_d}{18x}\Big[\ln^4 x - \frac{1}{2}\ln^3 x\ln\bar{y} - \frac{5}{32}\ln^2 x\ln^2\bar{y} - \frac{1}{6}\ln x\ln^3\bar{y} \\
& -8\ln^3 x + 3\ln^2 x\ln\bar{y} + \frac{19}{8}\ln x\ln^2\bar{y} - \frac{1}{6}\ln^3\bar{y} \\
& -\frac{1}{48}\left(\left(89\pi^2-2181\right)\ln^2 x + 2\left(225-14\pi^2\right)\ln x\ln\bar{y} - 165\ln^2\bar{y}\right) \\
& -\frac{1}{8}\left(\left(-395\zeta_3-47\pi^2+1106\right)\ln x + \left(29\zeta_3+\pi^2+132\right)\ln\bar{y}\right) \\
& +\frac{1}{80}\left(32\pi^4-7670\zeta_3-430\pi^2+11505\right) + \\
& +(4\ln^2 x - \frac{33}{2}\ln x - \frac{2}{3}\pi^2 + \frac{13}{8})L_\mu^2 - (4\ln^3 x - \ln^2 x\ln\bar{y} \\
& -\frac{1}{2}\ln^2\bar{y}\ln x - 22\ln^2 x + \frac{15}{4}\ln x\ln\bar{y} - \frac{1}{2}\ln^2\bar{y} + (\frac{357}{4} - 4\pi^2)\ln x \\
& +\frac{19}{4}\ln\bar{y} - 7\zeta_3 + \frac{14}{3}\pi^2 - \frac{147}{4})L_\mu\Big] \\
& +[x\to\bar{y}, \bar{y}\to x, e_d\to -e_u].
\end{aligned}
$$

Can the endpoint logarithms be resummation?

Phenomenological Exploration

Leading-twist pion LCDA

$$\Phi_\pi(x,\mu_F) = \frac{f_\pi}{2\sqrt{2N_c}} {\sum_{n=0}}' a_n(\mu_F)\psi_n(x),$$

$$\psi_n(x) = 6x\bar{x}C_n^{3/2}(2x-1)$$

a_n **can be calculated by Lattice QCD**

The pion EM form factor

$$Q^2 F_\pi(Q^2) = \frac{(e_u - e_d) f_\pi^2}{24} \times \sum_{k=0} \left(\frac{\alpha_s}{\pi}\right)^{k+1} {\sum_{m,n}}' a_n(\mu_F) a_m(\mu_F) \mathcal{T}_{mn}^{(k)},$$

Two-fold Convolution

$$\mathcal{T}_{mn}^{(k)} = \frac{1}{(e_u - e_d)} \psi_m(x) \underset{x}{\otimes} T^{(k)}\left(x, y, \frac{\mu_R^2}{Q^2}, \frac{\mu_F^2}{Q^2}\right) \underset{y}{\otimes} \psi_n(y).$$

The convolutions can be evaluated analytically

$$\mathcal{T}_{mn}^{(0)} = 9,$$
$$\mathcal{T}_{00}^{(1)} = \frac{1}{4}(81L_\mu + 237),$$

$$L_\mu \equiv \ln(\mu^2/Q^2).$$

$$\mathcal{T}_{00}^{(2)} = \frac{729L_\mu^2}{16} - (8\zeta_3 + \frac{35\pi^2}{6} - \frac{2961}{8})L_\mu + 205\zeta_5 - \frac{3\pi^4}{20} - \frac{651\zeta_3}{2} - \frac{275\pi^2}{24} + 821.$$

About 10^5 terms appear in the calculation of convolutions

Phenomenological exploration

Inputs: Gegenbauer Moments

LaMET-Ji
arXiv: 1305.1539

LPC

$$a_2(2\ \mathrm{GeV}) = 0.258 \pm 0.087,$$
$$a_4(2\ \mathrm{GeV}) = 0.122 \pm 0.056,$$
$$a_6(2\ \mathrm{GeV}) = 0.068 \pm 0.038.$$

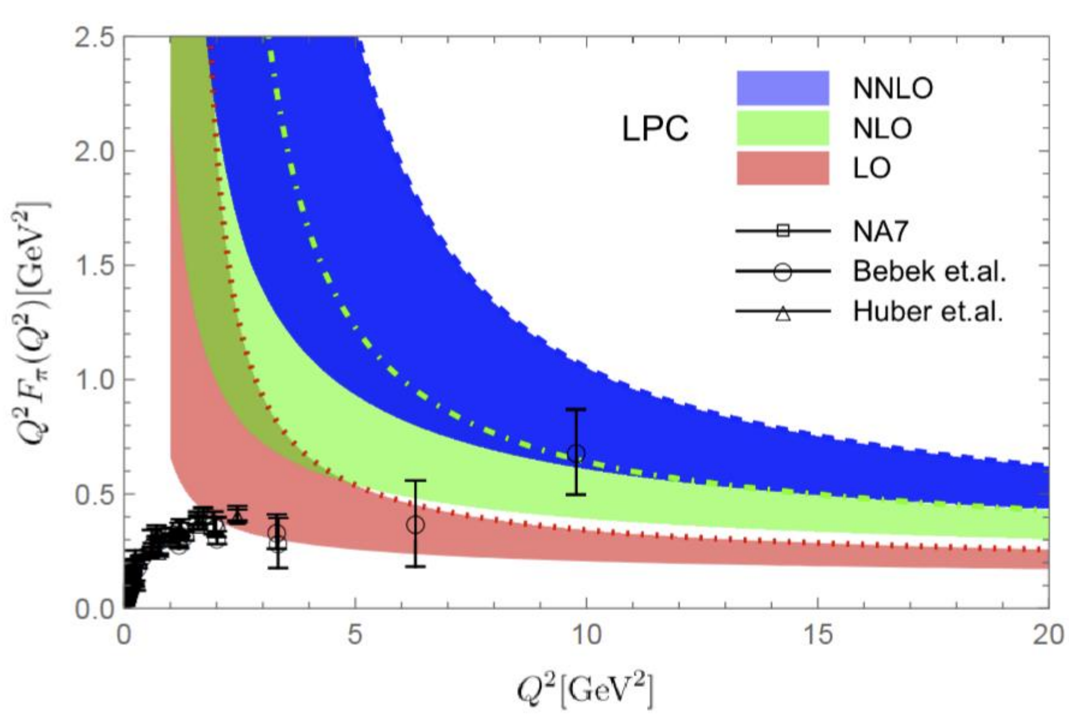
2201.09173

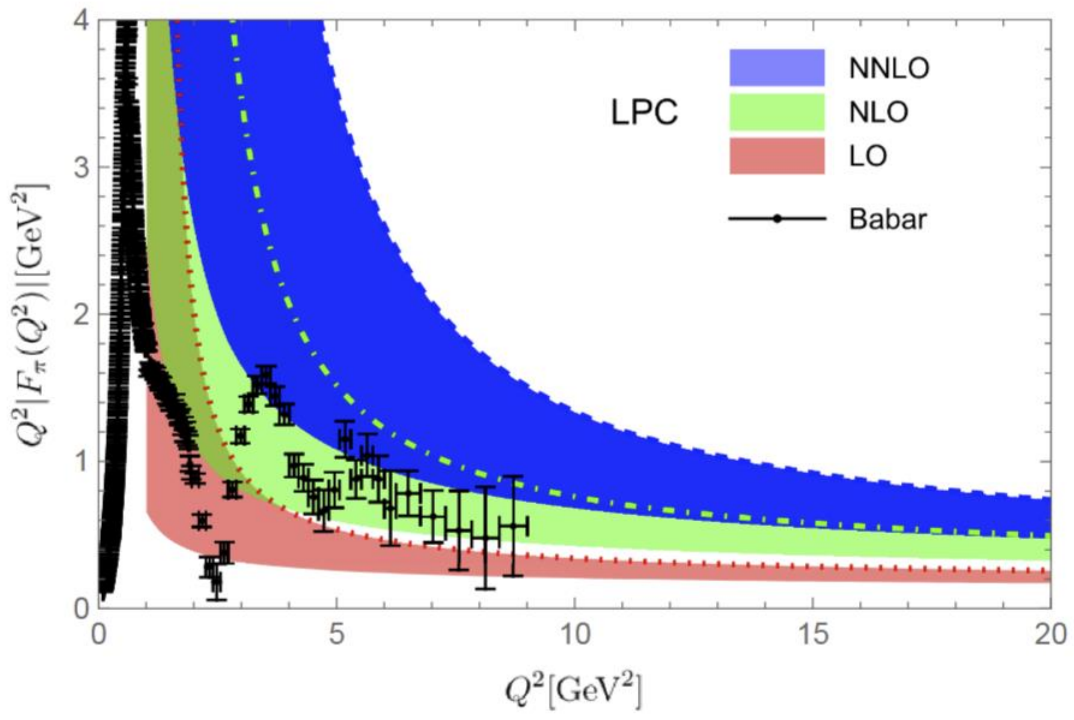
RQCD

$$a_2(2\ \mathrm{GeV}) = 0.116^{+0.019}_{-0.020},$$

1909.08038

Phenomenological exploration
Results with LPC inputs

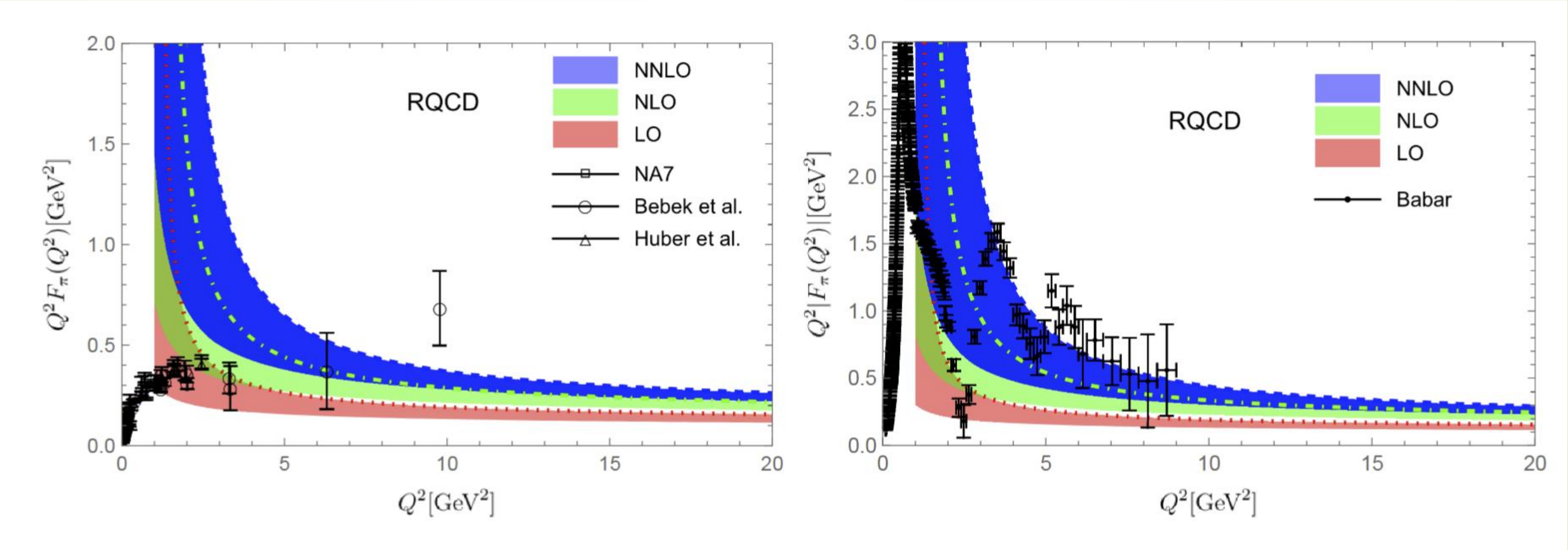




Space Like FF

Time Like FF

Results with RQCD inputs



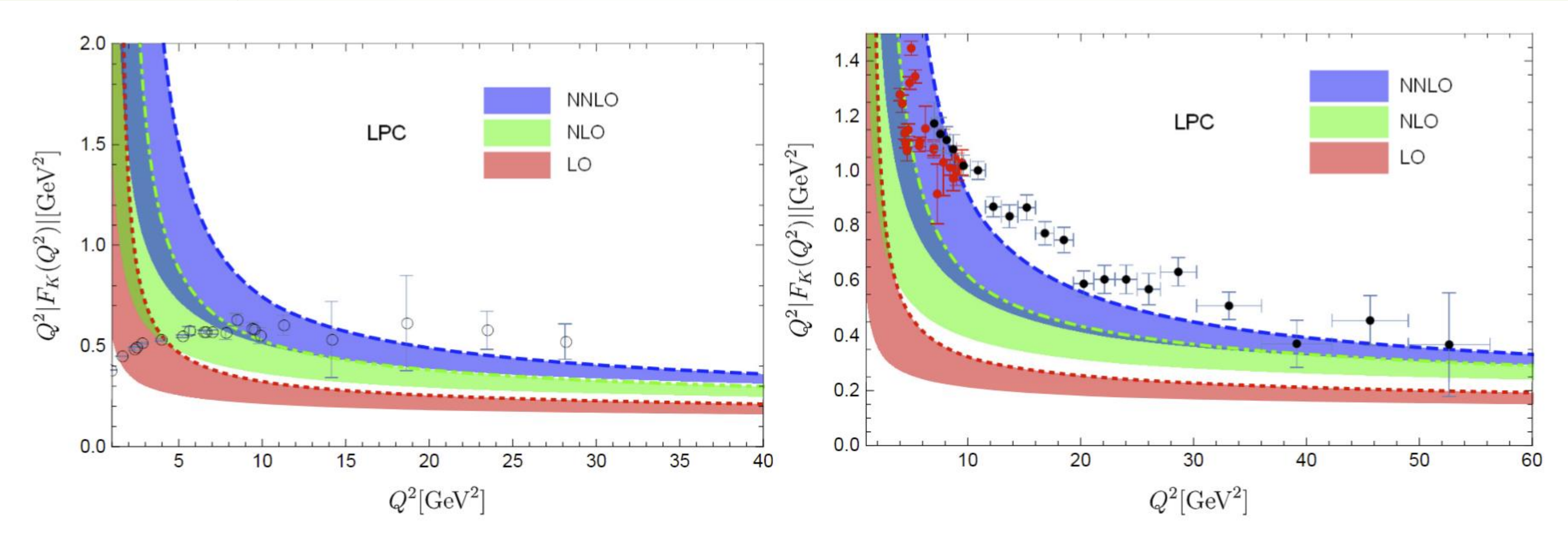
Space Like FF **Time Like FF**

Extend to Kaon(charge and neutral) Form Factors

$$Q^2 F_K(Q^2) = \frac{2C_F \pi^2 f_K^2}{3} \sum_{k=0} \left(\frac{\alpha_s}{\pi}\right)^{k+1} \times \sum_{m,n} (e_u - (-1)^{m+n} e_s) \mathcal{T}_{mn}^{(k)}\, a_m(\mu_F) a_n(\mu_F),$$

arXiv:2407.21120

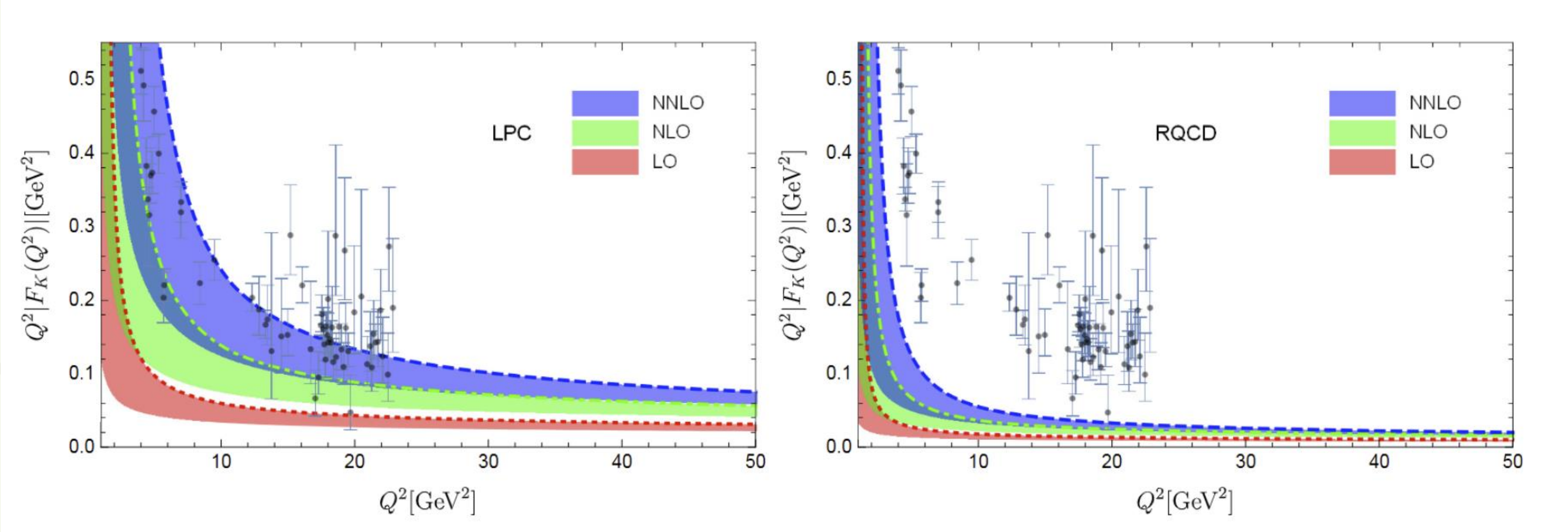
Charge Kaon FFs



Space like FFs, data taken form Ding *et.al.* arXiv:2404.04412

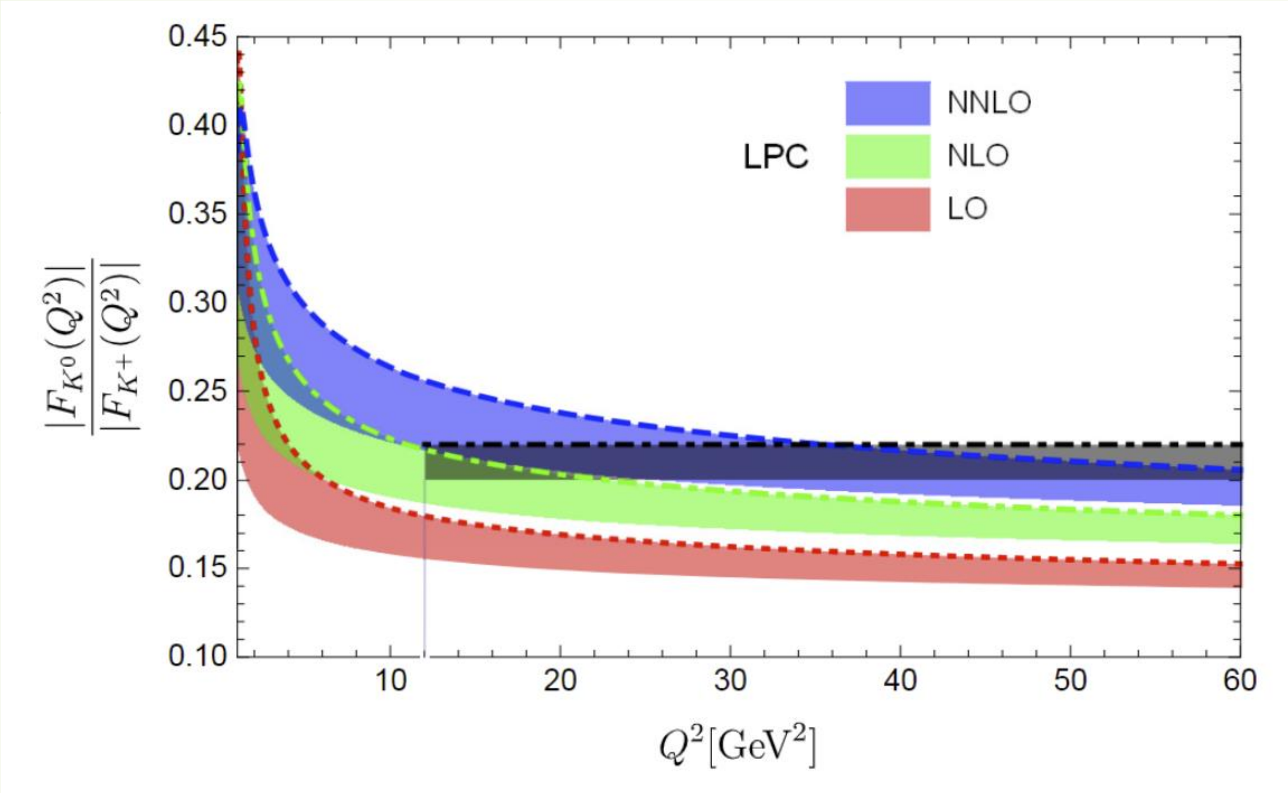
Time like FFs, data take from BESIII and BarBar

Neutral Kaon FFs



Time like FFs, data take from BESIII

SU(3) Flavor Breaking Effects



$$\frac{F^{\mathrm{BESIII}}_{K_S K_L}(12\mathrm{GeV}^2 < Q^2 < 25\mathrm{GeV}^2)}{F^{\mathrm{BESIII}}_{K^+K^-}(12\mathrm{GeV}^2 < Q^2 < 25\mathrm{GeV}^2)} = 0.21 \pm 0.01$$

Data take from BESIII

Conclusion

- The EM form factors for pion/kaon have been calculated up to NNLO QCD corrections.
- We verify the validity of the collinear factorization up to NNLO for this observable.
- The matching kernel have been obtained analytically.
- The NNLO corrections turn out to be positive and significant.
- The NNLO results can provides strong constraint on the Gegenbauer moments.

Thanks !