Study of the sub-leading twist GTMD E_{21} for proton in the light-front quark-diquark model

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Outline

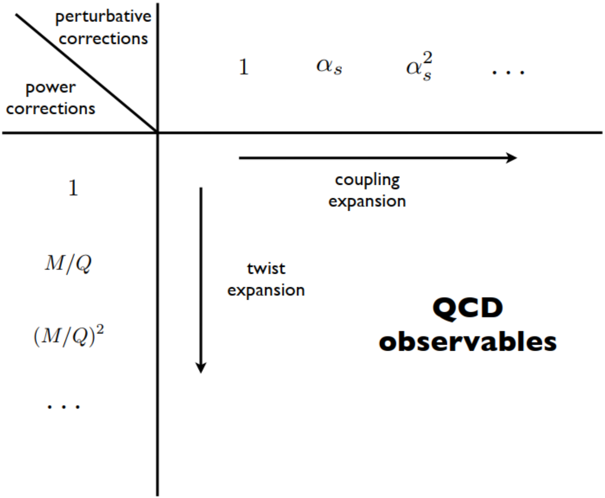
Internal Structure of the Hadrons

- Quantum Chromodynamics (QCD) provides a fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.

Quantum chromodynamics (QCD): Theory of Strong Interactions

- Many fundamental questions have not been resolved. The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.
- α_s is small at high energies and QCD can be used perturbatively.
- α_s becomes large at low energies and one has to use other methods such as effective Lagrangian models to describe physics.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc..

Twist?



- https://inspirehep.net/literature/1493030

Outline

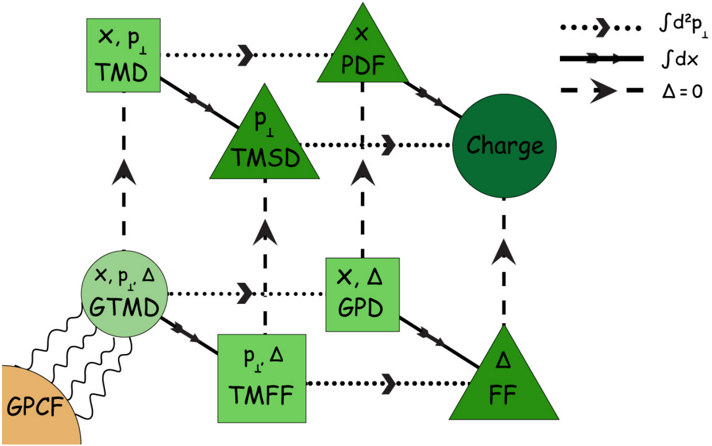


Figure 1: The relationship between generalized parton correlation functions (GPCFs) and various distribution functions is illustrated.

- S. Sharma and H. Dahiya, Eur. Phys. J. A 59, 235 (2023)

Outline

Light-Front Quark-Diquark Model

- In this model the proton is described as an aggregate of an active quark and a diquark spectator of definite mass.

- The proton has spin-flavor $SU(4)$ structure and it has been expressed as a made up of isoscalar-scalar diquark singlet $|u\,S^0\rangle$, isoscalar-vector diquark $|u\,A^0\rangle$ and isovector-vector diquark $|d\,A^1\rangle$ states as

$$|P;\pm\rangle = C_S|u\,S^0\rangle^{\pm} + C_V|u\,A^0\rangle^{\pm} + C_{VV}|d\,A^1\rangle^{\pm}.$$

Here, the scalar and vector diquark has been denoted by S and A respectively. Their isospin has been represented by the superscripts on them.

- T. Maji and D. Chakrabarti, Phys. Rev. D 94, 094020 (2016)

Light-Front Quark-Diquark Model

- For the scalar $|\nu\, S\rangle^{\pm}$ and vector diquark $|\nu\, A\rangle^{\pm}$ case, the expansion of the Fock-state in two particles for $J^z = \pm 1/2$ can be specified as

$$|\nu\, S\rangle^{\pm} = \sum_{\lambda^q} \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \psi^{\pm(\nu)}_{\lambda^q}(x,\mathbf{p}_\perp) \Big| \lambda^q, \lambda^s; xP^+, \mathbf{p}_\perp \Big\rangle,$$

$\lambda^S = 0$ (singlet)

$$|\nu\, A\rangle^{\pm} = \sum_{\lambda^q}\sum_{\lambda^A} \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \psi^{\pm(\nu)}_{\lambda^q\lambda^A}(x,\mathbf{p}_\perp) \Big| \lambda^q, \lambda^A; xP^+, \mathbf{p}_\perp \Big\rangle.$$

$\lambda^A = \pm 1, 0$ (triplet)

Light-Front Quark-Diquark Model

	λ^q	λ^{Sp}	LFWFs for $J^z = +1/2$	LFWFs for $J^z = -1/2$
S	$+1/2$	0	$\psi_{+}^{+(\nu)} = N_S\ \varphi_1^{(\nu)}$	$\psi_{+}^{-(\nu)} = N_S\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-}^{+(\nu)} = -N_S\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-}^{-(\nu)} = N_S\ \varphi_1^{(\nu)}$
A	$+1/2$	$+1$	$\psi_{+\,+}^{+(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{+\,+}^{-(\nu)} = 0$
	$-1/2$	$+1$	$\psi_{-\,+}^{+(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$	$\psi_{-\,+}^{-(\nu)} = 0$
	$+1/2$	0	$\psi_{+\,0}^{+(\nu)} = -N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$	$\psi_{+\,0}^{-(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-\,0}^{+(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-\,0}^{-(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$
	$+1/2$	-1	$\psi_{+\,-}^{+(\nu)} = 0$	$\psi_{+\,-}^{-(\nu)} = -N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$
	$-1/2$	-1	$\psi_{-\,-}^{+(\nu)} = 0$	$\psi_{-\,-}^{-(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$

Light-Front Quark-Diquark Model

- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction and the parameters a_i^ν, b_i^ν and δ^ν are established as

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

Input Parameters

- The parameters a_i^ν and b_i^ν have been fitted at model scale $\mu_0 = 0.8$ GeV using the Dirac and Pauli data of form factors.

ν	a_1^ν	b_1^ν	a_2^ν	b_2^ν	δ^ν
u	0.280	0.1716	0.84	0.2284	1.0
d	0.5850	0.7000	0.9434	0.64	1.0

Table 1: Values of model parameters corresponding to up and down quarks.

ν	N_S	N_0^ν	N_1^ν
u	2.0191	3.2050	0.9895
d	2.0191	5.9423	1.1616

Table 2: Values of normalization constants N_i^2 corresponding to both up and down quarks.

Input Parameters

- The AdS/QCD scale parameter (κ) is chosen to be 0.4 GeV.
- Constituent quark mass (m) and the proton mass (M) are taken to be 0.055 GeV and 0.938 GeV sequentially.
- The coefficients C_i of scalar and vector diquarks are given as

$$
\begin{aligned}
C_S^2 &= 1.3872, \\
C_V^2 &= 0.6128, \\
C_{VV}^2 &= 1.
\end{aligned}
$$

Outline

- Internal Structure of the Hadrons
- Distribution Functions
- Light-Front Quark-Diquark Model
- GTMD Correlator & Parameterization
- Result & Discussion
- Summary

GTMD Correlator

- The unintegrated quark-quark GTMD correlator $W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S)$ for Double Drell Yan process in the Drell-Yan-West frame ($\Delta^+ = 0$) is given by

$$W^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2}\int \frac{dz^-}{(2\pi)}\frac{d^2 z_T}{(2\pi)^2} e^{ip.z}\langle P^f; \Lambda^{N_f}|\bar{\psi}(-z/2)\Gamma\mathcal{W}_{[-z/2,z/2]}\psi(z/2)|P^i; \Lambda^{N_i}\rangle\Big|_{z^+=0}.$$

- $|P^i; \Lambda^{N_i}\rangle$ and $|P^f; \Lambda^{N_f}\rangle$ are the initial and final states of the proton having momentum P^i and P^f with helicities Λ^{N_i} and Λ^{N_f}, respectively.
- We follow light-cone gauge $A^+ = 0$ and the convention $z^\pm = (z^0 \pm z^3)$ of the symmetric frame is used, where specific kinematics can be provided by

$$P \equiv \left(P^+, \frac{M^2 + \vec{\Delta}_\perp^2/4}{P^+}, 0_\perp\right),$$
$$\Delta \equiv \left(0, 0, \vec{\Delta}_\perp\right).$$

- The movement of Wilson line $\mathcal{W}_{[0,z]}$ is through the path $[0, 0, 0_\perp] \to [0, 1, 0_\perp] \to [0, 1, z_\perp] \to [0, z^-, z_\perp]$ & its value is chosen to be 1.

GTMD Parameterization for proton at twist-3

$$W^{[1]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P^f,\Lambda^{N_F})\Big[E_{2,1} + \frac{i\sigma^{i+}p^i_\perp}{P^+}\,E_{2,2} + \frac{i\sigma^{i+}\Delta^i_\perp}{P^+}\,E_{2,3} + \frac{i\sigma^{ij}p^i_\perp\Delta^j_\perp}{M^2}\,E_{2,4}\Big]u(P^i,\Lambda^{N_i}),$$

$$W^{[\gamma^5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P^f,\Lambda^{N_F})\Big[-\frac{i\varepsilon^{ij}_T p^i_\perp\Delta^j_\perp}{M^2}\,E_{2,5} + \frac{i\sigma^{i+}\gamma_5 p^i_\perp}{P^+}\,E_{2,6} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}\,E_{2,7} + i\sigma^{+-}\gamma_5\,E_{2,8}\Big]u(P^i,\Lambda^{N_i}),$$

$$W^{[\gamma^j]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P^f,\Lambda^{N_F})\Big[\frac{p^j_\perp}{M}\,F_{2,1} + \frac{\Delta^j_\perp}{M}\,F_{2,2} + \frac{M\,i\sigma^{j+}}{k^+}\,F_{2,3} + \frac{p^j_T\,i\sigma^{k+}p^k_\perp}{M\,P^+}\,F_{2,4} + \frac{\Delta_\perp{}^j\,i\sigma^{k+}p^k_T}{M\,P^+}\,F_{2,5} + \frac{\Delta^j_\perp\,i\sigma^{k+}\Delta^k_\perp}{M\,P^+}\,F_{2,6} + \frac{p^i_\perp\,i\sigma^{ij}}{M}\,F_{2,7} + \frac{\Delta^i_\perp\,i\sigma^{ij}}{M}\,F_{2,8}\Big]u(P^i,\Lambda^{N_i}),$$

GTMD Parameterization for proton at twist-3

$$W^{[\gamma^j\gamma^5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\bar{u}(P^f,\Lambda^{N_F})\Big[-\frac{i\varepsilon_T^{ij}p_\perp^i}{M}G_{2,1} - \frac{i\varepsilon_T^{ij}\Delta_\perp^i}{M}G_{2,2} + \frac{M\, i\sigma^{j+}\gamma_5}{P^+}G_{2,3} + \frac{p_\perp^j\, i\sigma^{k+}\gamma_5 p_\perp^k}{M\,P^+}G_{2,4} + \frac{\Delta_\perp{}^j\, i\sigma^{k+}\gamma_5 p_\perp^k}{M\,P^+}G_{2,5} + \frac{\perp^j\, i\sigma^{k+}\gamma_5\Delta_\perp^k}{M\,P^+}G_{2,6} + \frac{p_\perp^j\, i\sigma^{+-}\gamma_5}{M}G_{2,7} + \frac{\Delta_\perp^j\, i\sigma^{+-}\gamma_5}{M}G_{2,8}\Big]u(P^i,\Lambda^{N_i}),$$

$$W^{[i\sigma^{ij}\gamma^5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = -\frac{i\varepsilon_T^{ij}}{2P^+}\bar{u}(P^f,\Lambda^{N_F})\Big[H_{2,1} + \frac{i\sigma^{k+}p_\perp^k}{P^+}H_{2,2} + \frac{i\sigma^{k+}\Delta_\perp^k}{P^+}H_{2,3} + \frac{i\sigma^{kl}p^k\Delta_\perp^l}{M^2}H_{2,4}\Big]u(P^i,\Lambda^{N_i}),$$

$$W^{[i\sigma^{+-}\gamma^5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\bar{u}(P^f,\Lambda^{N_F})\Big[-\frac{i\varepsilon_T^{ij}p_\perp^i\Delta_\perp^j}{M^2}H_{2,5} + \frac{i\sigma^{i+}\gamma_5 p_\perp^i}{P^+}H_{2,6} + \frac{i\sigma^{i+}\gamma_5\Delta_\perp^i}{P^+}H_{2,7} + i\sigma^{+-}\gamma_5 H_{2,8}\Big]u(P^i,\Lambda^{N_i}).$$

- S. Meissner, A. Metz and M. Schlegel, JHEP 08, 056 (2009)

Outline

Explicit Results

- For proton, the twist-3 GTMD $E^{\nu}_{21}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for scalar and vector di-quark is given as

$$E^{\nu(S)}_{2,1} = \frac{C_S^2 N_s^2}{16\pi^3}\left[m\left(\frac{T^{\nu}_{11}}{xM} + \left(\mathbf{p}_\perp^2 - (1-x)^2\frac{\mathbf{\Delta}_\perp^2}{4}\right)\frac{T^{\nu}_{22}}{x^3M^3}\right) + (1-x)\mathbf{\Delta}_\perp^2\frac{T^{\nu}_{12}}{2x^2M^2}\right],$$

$$E^{\nu(A)}_{2,1} = \frac{C_A^2}{16\pi^3}\left(\frac{1}{3}|N_0^{\nu}|^2 + \frac{2}{3}|N_1^{\nu}|^2\right)\left[m\left(\frac{T^{\nu}_{11}}{xM} + \left(\mathbf{p}_\perp^2 - (1-x)^2\frac{\mathbf{\Delta}_\perp^2}{4}\right)\frac{T^{\nu}_{22}}{x^3M^3}\right)\right.$$
$$\left. + (1-x)\mathbf{\Delta}_\perp^2\frac{T^{\nu}_{12}}{2x^2M^2}\right].$$

x and $p_\perp$ Dependence

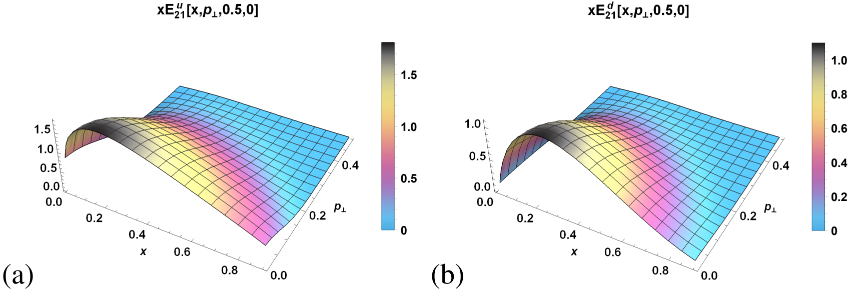


Figure 2: The GTMD $xE^{\nu}_{21}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{p}_\perp$ at $\mathbf{\Delta}_\perp = \mathbf{0}$ (i.e., at TMD limit). The left and right column correspond to u and d quarks sequentially.

x and $\Delta_\perp$ Dependence

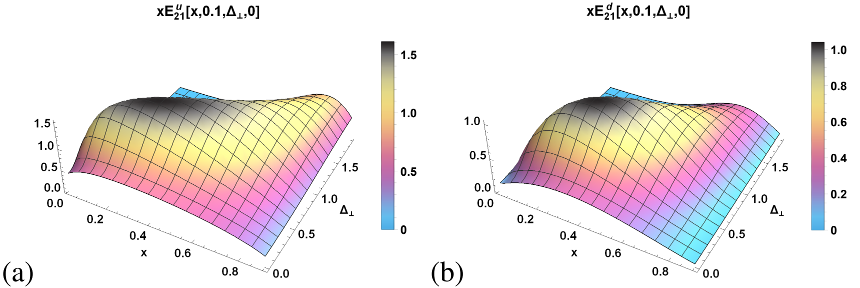


Figure 3: The GTMD $xE^{\nu}_{21}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{\Delta}_\perp$ at $\mathbf{p}_\perp = 0.1$ and $\theta = 0$. The left and right column correspond to u and d quarks sequentially.

Transverse Momentum Form Factors

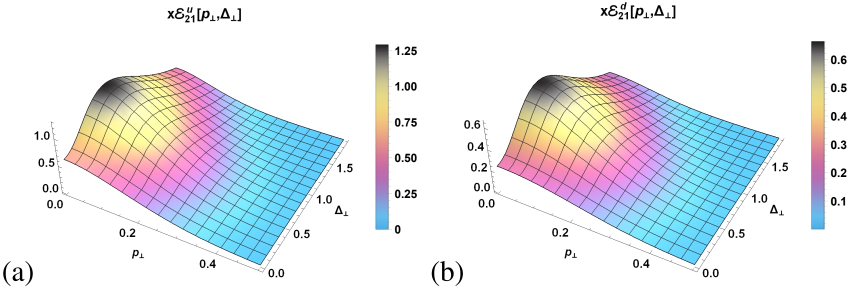
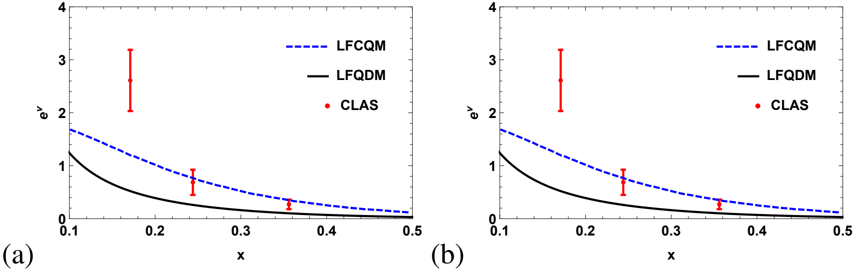


Figure 4: The TMFF $xE^{\nu}_{21}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to $\mathbf{p}_\perp$ and $\mathbf{\Delta}_\perp$ at $x = 0.3$ and $\theta = 0$. The left and right column correspond to u and d quarks sequentially.

Comparison with Phenomenology



LFQDM - $1GeV^2$ S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)

LFCQM - $1GeV^2$ *S. Rodini and B. Pasquini, Nuovo Cim. C 42, 112 (2019)*

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Outline

Summary

- The GTMD $xE_{21}\nu(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ shows quark flavor symmetry.
- The GTMD $xE^{\nu}_{21}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ remains positive over the entire domain for both u and d quarks.
- Although GTMDs are primarily connected to the double Drell-Yan process, they provide valuable information linking GPDs, TMDs, and PDFs, which are explored in Deeply Virtual Compton Scattering, Semi-Inclusive Deep Inelastic Scattering, and Deep Inelastic Scattering experiments.

Thank you!

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