Spatial Imaging of Proton from a Light-front Hamiltonian Approach

Xingbo Zhao

With

Yiping Liu, Chandan Mondal, Siqi Xu,
Ziqi Zhang, Yang Li, James Vary

(BLFQ collaboration)

The XVIth Quark Confinement and the Hadron Spectrum Conference
Cairns, Australia, 08/22/2024

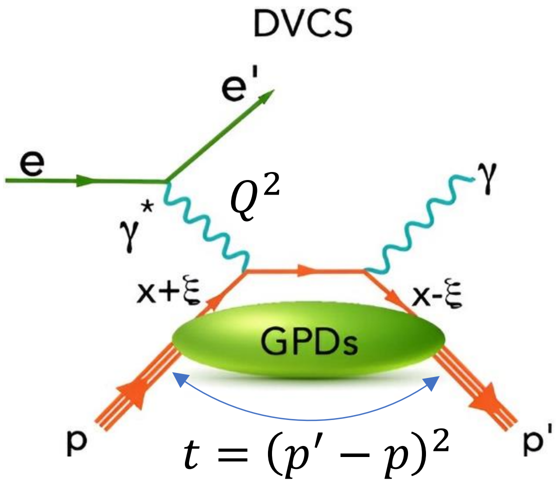
Outline

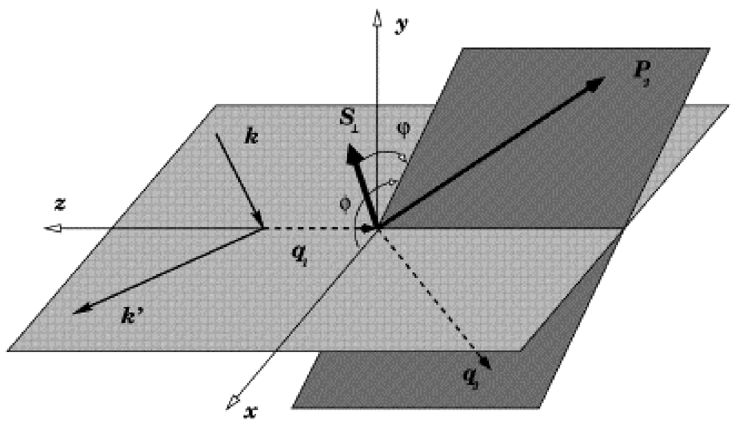
- Spatial Imaging on the light-front through GPDs
- Basis Light-front Quantization
- Numerical results for proton
  - Form factors and PDFs
  - GPDs at zero-skewness
  - Skewed GPDs
- Conclusions

3D Structure in Coordinate Space

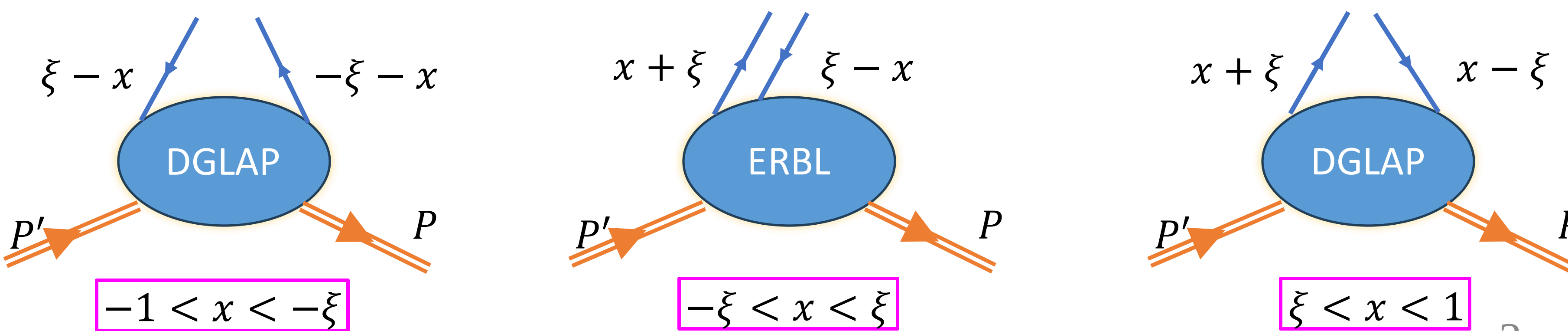
- Deeply Virtual Compton Scattering (DVCS) [X. Ji, Phys. Rev. D 55, 7114 (1997)]

$$e(p) + P(P) \to e'(p') + P'(P') + \gamma$$





- Generalized Parton Distribution Functions (GPDs)



Definition of GPDs

- GPDs are defined through the following bilocal operator on the light front

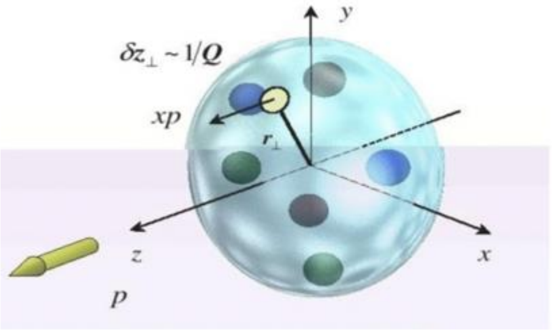
$$F^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\Delta^2) = \int \frac{dz^-}{4\pi} e^{ip\cdot x} \left\langle P',\Lambda' \middle| \bar{\psi}\left(-\frac{z}{2}\right) \mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right) \Gamma \psi\left(\frac{z}{2}\right) \middle| P,\Lambda \right\rangle \Big|_{z^+=z^\perp=0}$$

[Diehl, 2003]

- GPDs are parameterized by taking different Γ matrices

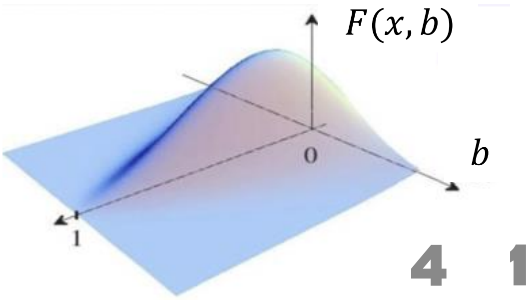
$$F^{[\gamma^+]}_{\Lambda'\Lambda} = \frac{1}{2P^+}\bar{u}\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\Delta}}{2M}E(x,\xi,t)\right]u$$

$$F^{[\gamma^+\gamma_5]}_{\Lambda'\Lambda} = \frac{1}{2P^+}\bar{u}\left[\gamma^+\gamma_5 \widetilde{H}(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\tilde{E}(x,\xi,t)\right]u$$



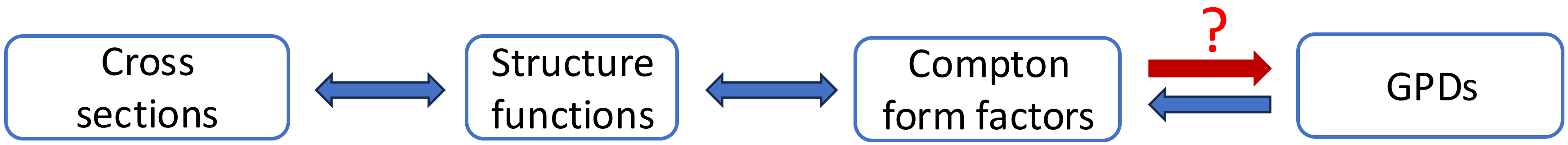
- Fourie transform of GPDs at $\xi = 0$ with respect to t produces spatial imaging

$$F(x,b) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F(x,\xi=0,t=-\Delta_\perp^2)$$



GPDs and Compton Form Factors

- Cross sections to Compton form factors



- GPDs to Compton form factors (CFFs)

CFFs ← → GPDs

$$\mathcal{F}(\xi,-t) = \int_{-1}^{1} dx \left(\frac{1}{x-\xi-i\epsilon} - \frac{1}{x+\xi-i\epsilon} \right) F(x,\xi,-t)$$

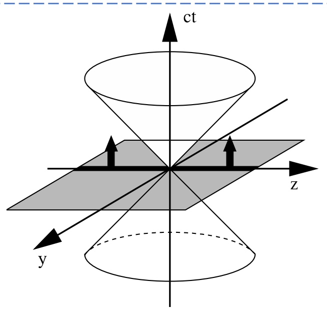
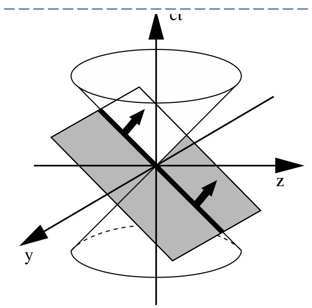
$$\tilde{\mathcal{F}}(\xi,-t) = \int_{-1}^{1} dx \left(\frac{1}{x-\xi-i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right) \tilde{F}(x,\xi,-t)$$

- Compton form factors are integrations of GPDs over x
- Challenging to extract GPDs from CFFs
- Exploring GPDs from theory is interesting

See J. Qiu's talk on 08/19

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Main advantages:

- Simple vacuum
- GPDs are defined on the light front
- Frame-independent wave function

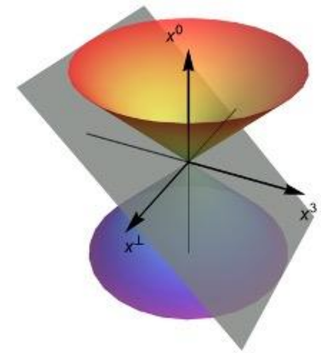
$$F^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\Delta^2) = \int \frac{dz^-}{4\pi} e^{ip\cdot x} \langle P',\Lambda' \rvert \bar{\psi}\left(-\tfrac{z}{2}\right) \mathcal{W}\left(-\tfrac{z}{2},\tfrac{z}{2}\right) \Gamma \psi\left(\tfrac{z}{2}\right) \lvert P,\Lambda \rangle \Big|_{z^+=z^\perp=0}$$

Basis Light-Front Quantization

- Hamiltonian eigenvalue equation

[Dirac, 1949]
[Vary, et.al, Phys.Rev.C '10]

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$



P^-: Light-Front Hamiltonian
$|\beta\rangle$: Eigenstates (wave function)
P^-_β: Eigenvalues (mass)

- Basis setup
  - Fock sector expansion: $|P,\Lambda\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + \cdots\cdots$
  - Single particle basis in $|qqq\rangle$ $\quad |\alpha\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$

2D harmonic oscillator	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \lvert m_i\rvert + 1) \le N_{\max}$	$\sum_i k_i^+ = K_{\max}$	$\Lambda = \sum_i (\lambda_i + m_i)$

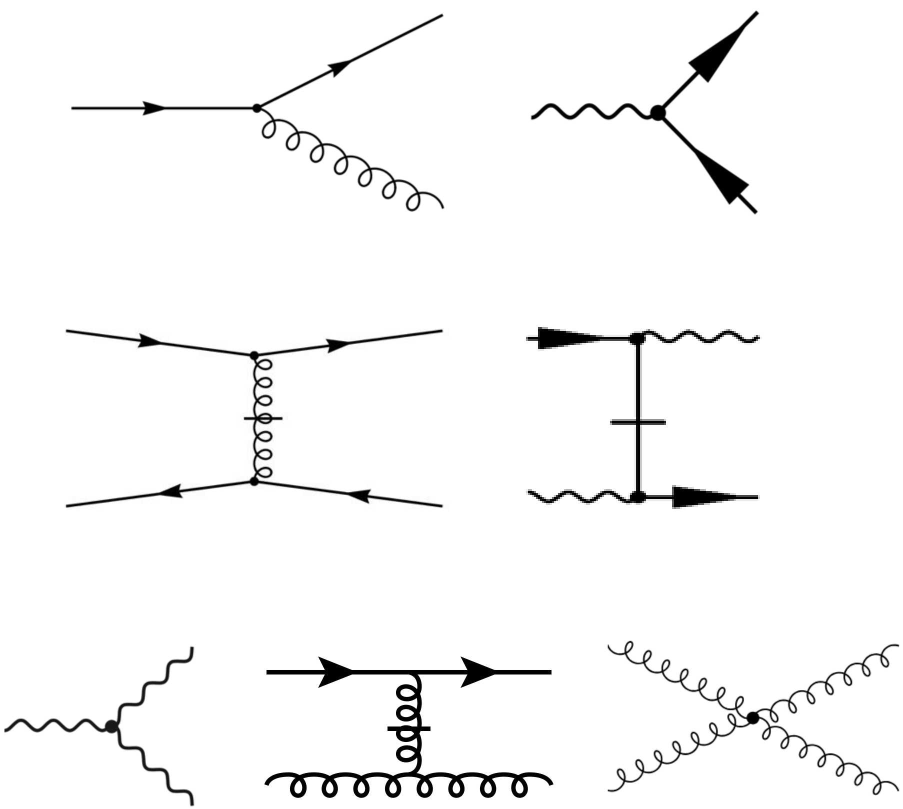
- Advantages:
  - Rotational symmetry in transverse plane
  - Exact factorization between center-of-mass motion and intrinsic motion
  - Harmonic oscillator basis supplies adequate infrared behavior

Light-Front Hamiltonian

- QCD light-front Hamiltonian can be derived from QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \xrightarrow{A^+ = 0}$$

$$\begin{aligned} P^-_{QCD} = & \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A \\ & + g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\ & + \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\ & - \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\ & + \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\ & + \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\ & - \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c} \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\ & + \tfrac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}. \end{aligned}$$



ψ: quark field operator
A^a_μ: gluon field operator

Publications on Nucleon GPDs

$$|\text{Proton}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle + \cdots$$

- $|\boldsymbol{qqq}\rangle$**:**

  Proton GPD [Xu et al., Phys.Rev.D104,094036 (2021)]

  Proton angular momentum [Liu et al., Phys.Rev.D105,094018 (2022)]

  Proton twist-3 GPDs [Zhang et al., Phys.Rev.D109,034031 (2024)]

  Proton chiral odd GPDs [Kaur et al., Phys. Rev. D 109, 014015 (2024)]

- $|\boldsymbol{qqq}\rangle + |\boldsymbol{qqqg}\rangle$**:**

  Proton spin structure [Xu et al., Phys.Rev.D,108 9, 094002 (2023)]

  Gluon GPDs [Lin et al., Phys.Lett.B,847 138305 (2023)]

- $|\boldsymbol{qqq}\rangle + |\boldsymbol{qqqg}\rangle + |\boldsymbol{qqqq\bar{q}}\rangle$

  Proton structure with sea quarks [arxiv:2408.xxxxx]

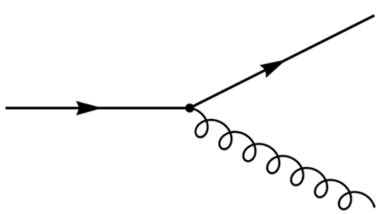
Light-Front Hamiltonian

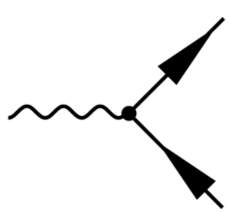
$$|\text{Proton}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle + \Psi_{31}|qqq\,u\bar{u}\rangle + \Psi_{32}|qqq\,d\bar{d}\rangle + \Psi_{33}|qqq\,s\bar{s}\rangle$$

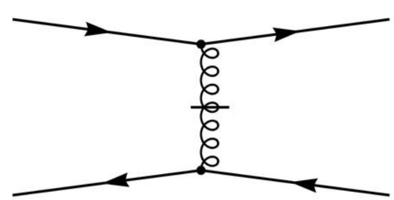
$$P^- = H_{K.E.} + H_{Interact}$$

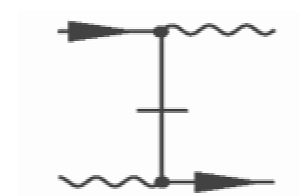
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}A_\nu \gamma^\nu \psi$$









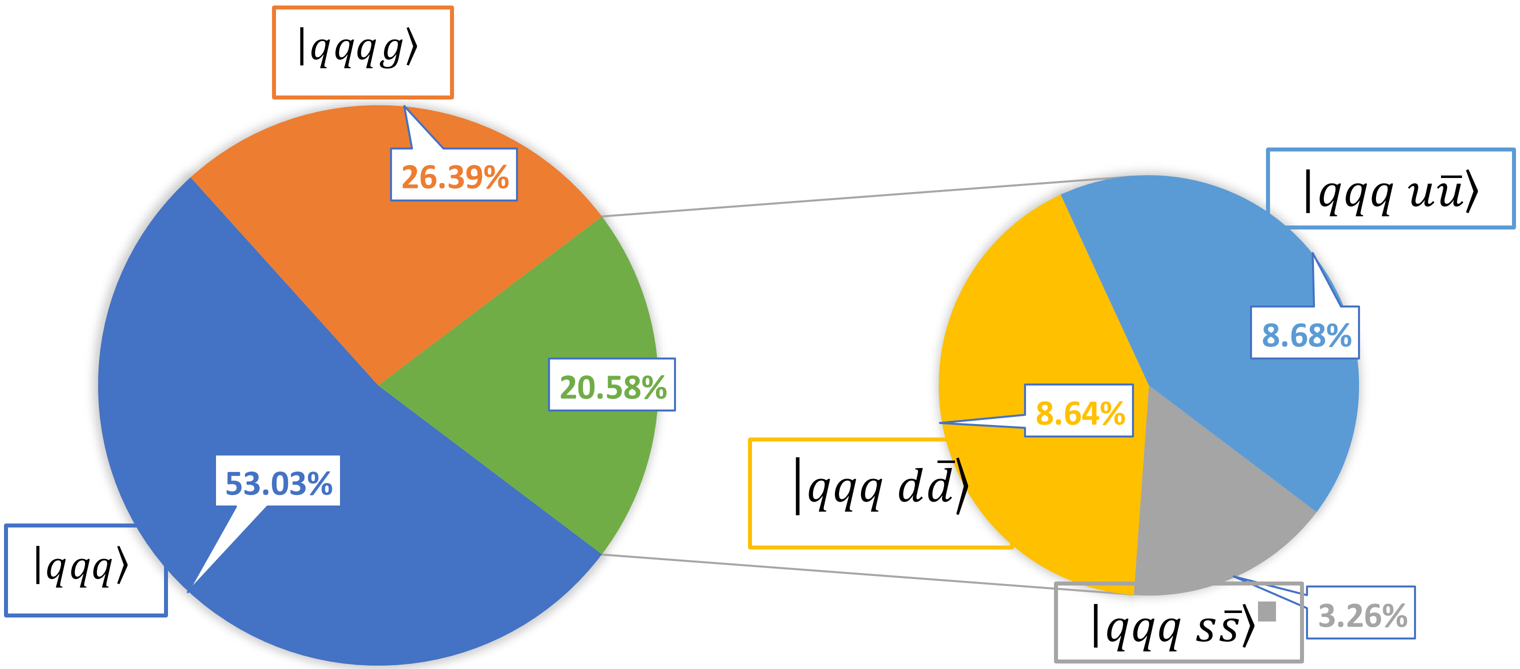
Fock Sector Decomposition

$$|\text{Proton}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 16$

m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

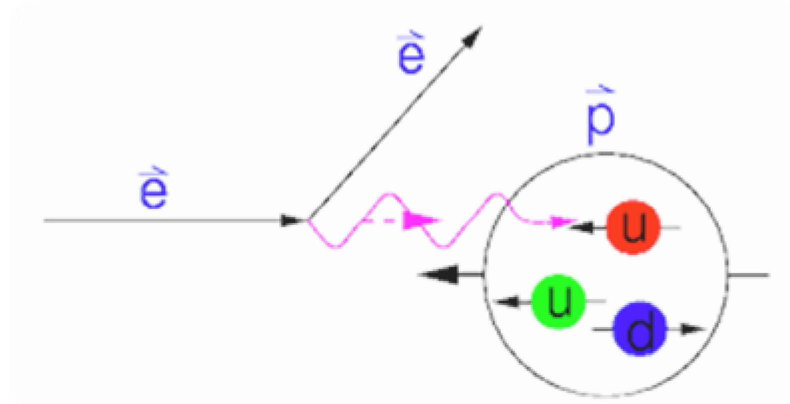
In five quark Fock sectors, current quark masses are used



Electromagnetic Form Factor

- Elastic scattering of proton

$$e(p) + h(P) \rightarrow e(p') + h(P')$$



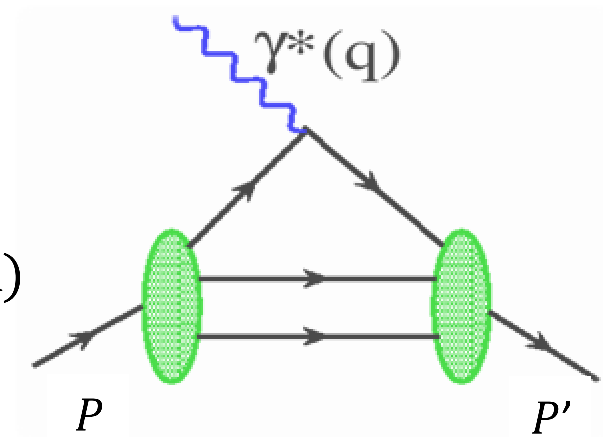
[R. Hofstadter, Nobel Prize 1961]

- Elastic electron scattering established the extended nature of the proton

$$\langle P', \Lambda' |\, J^{\mu}(0)\, | P, \Lambda \rangle =$$

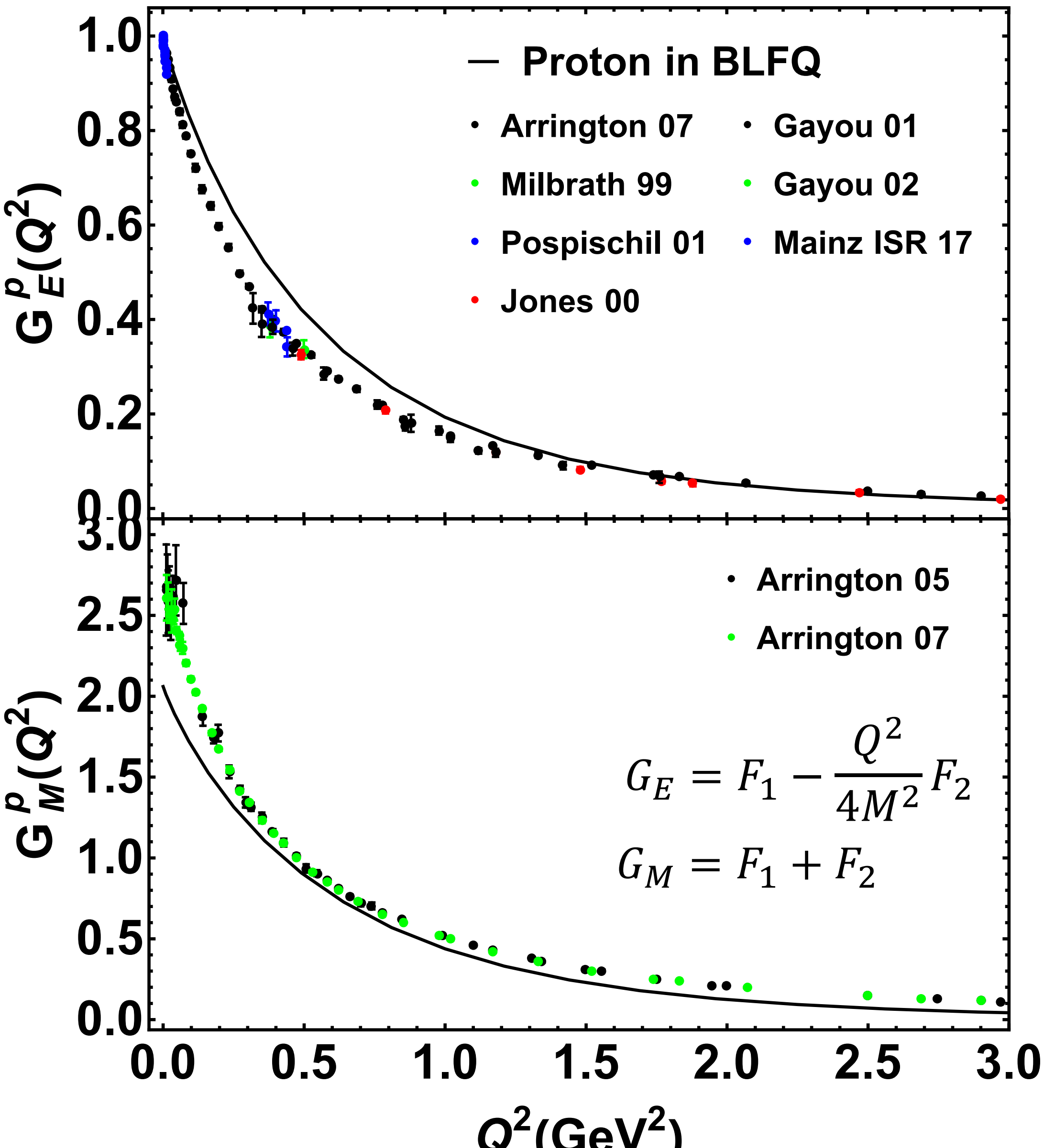
$$\bar{u}(\mathrm{P}', \Lambda') \left[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} F_2(q^2) \right] u(P, \Lambda)$$

Dirac Form Factor　　Pauli Form Factor



- Fourier transformation of these form factors provides spatial distributions of charge and magnetization

Electromagnetic Form Factors



$$G_E = F_1 - \frac{Q^2}{4M^2} F_2$$

$$G_M = F_1 + F_2$$

Electromagnetic Radii

Charge radius

$$\sqrt{\langle r_E^2 \rangle} = 0.72 \pm 0.05 \text{ fm}$$

$$\sqrt{\langle r_E^2 \rangle_{\text{exp}}} = 0.840^{+0.003}_{-0.002} \text{ fm}$$

Magnetic radius

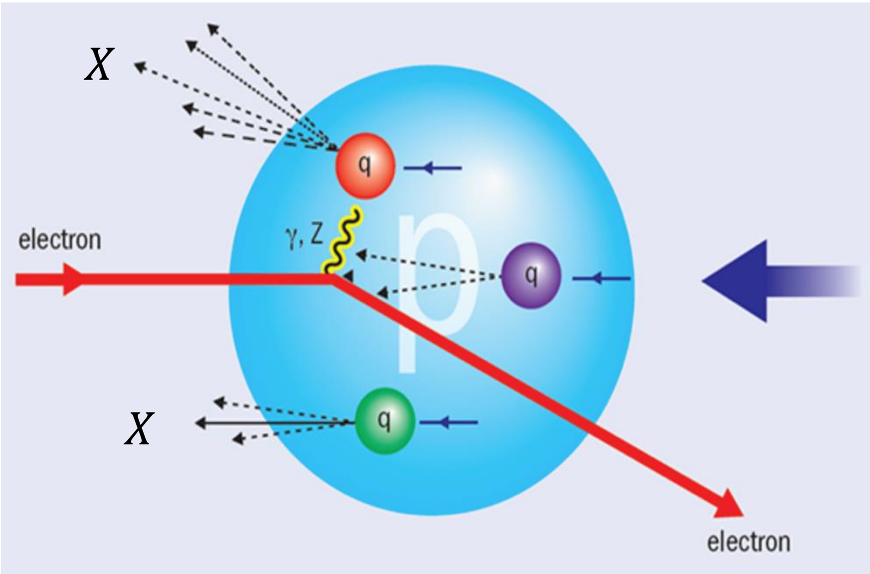
$$\sqrt{\langle r_M^2 \rangle} = 0.73 \pm 0.02 \text{ fm}$$

$$\sqrt{\langle r_M^2 \rangle_{\text{exp}}} = 0.849^{+0.003}_{-0.003} \text{ fm}$$

[arXiv: 2408.11298]

Parton Distribution Functions

- Deep Inelastic Scattering (SLAC 1968)



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1\ \mathrm{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \mathrm{fm}$$

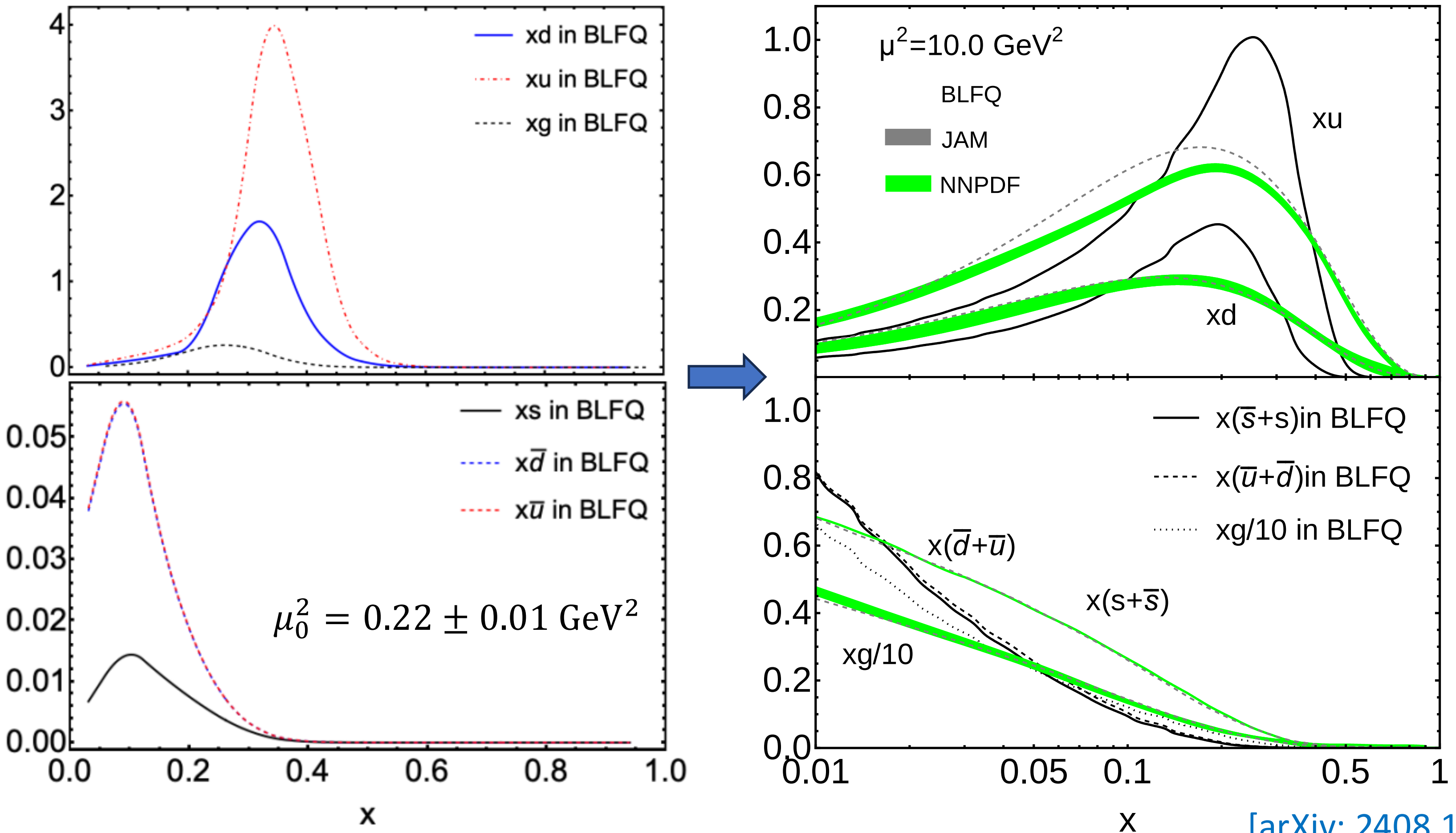
Discovery of spin ½ quarks and partonic structure

- Parton distribution functions (PDFs) are extracted from DIS processes

$$\Phi^{[\Gamma]}(x, Q^2) = \int \frac{dz^-}{8\pi}\, e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(z) \Gamma \psi(0) | P, \Lambda \rangle$$

- Encode longitudinal momentum distribution and polarization of the constituents

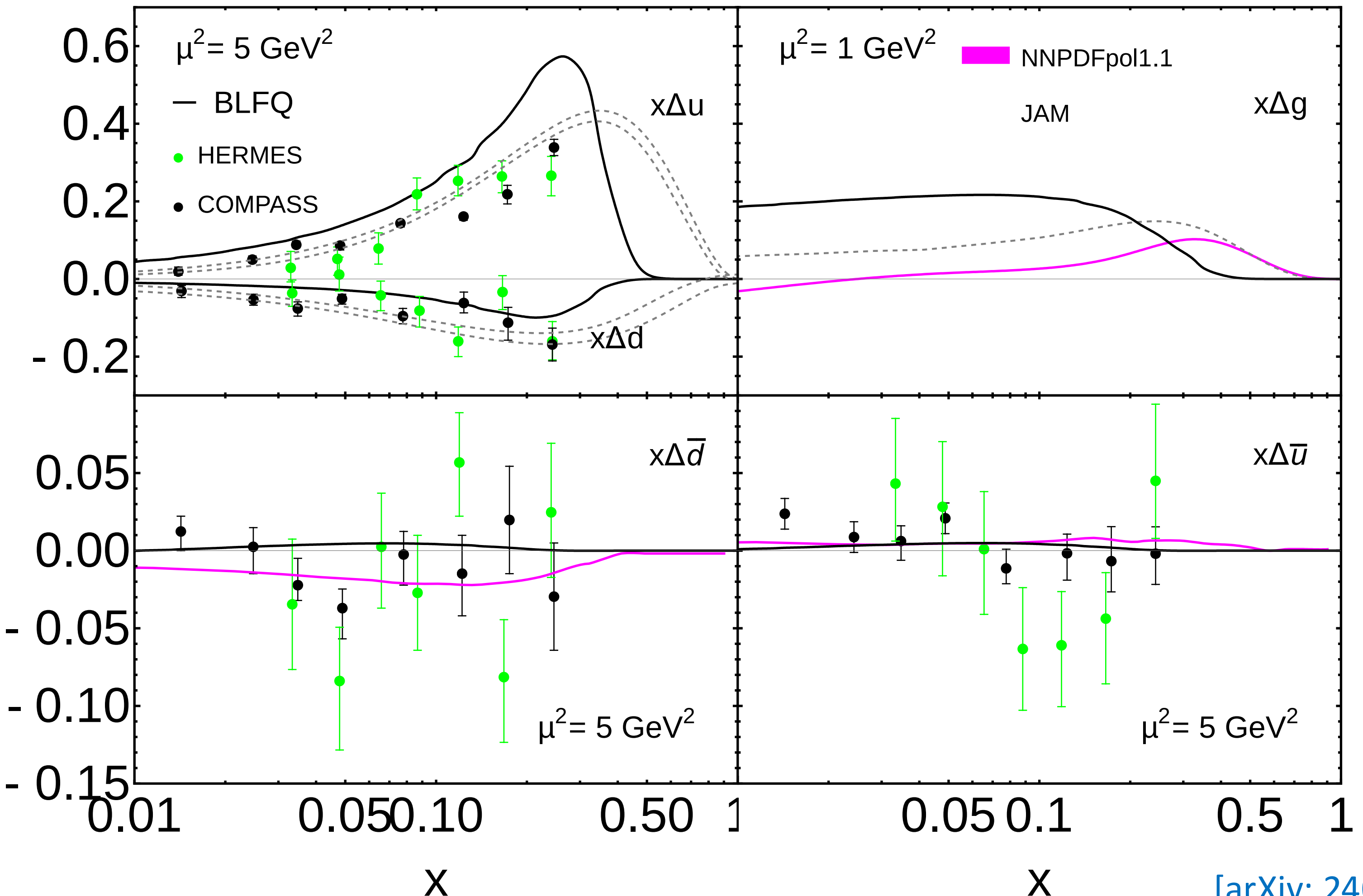
Unpolarized PDF



[arXiv: 2408.11298]

- Fitting the initial scale by matching the $\langle x \rangle$ moment at $10\ \mathrm{GeV}^2$
- Narrower peak than global fits

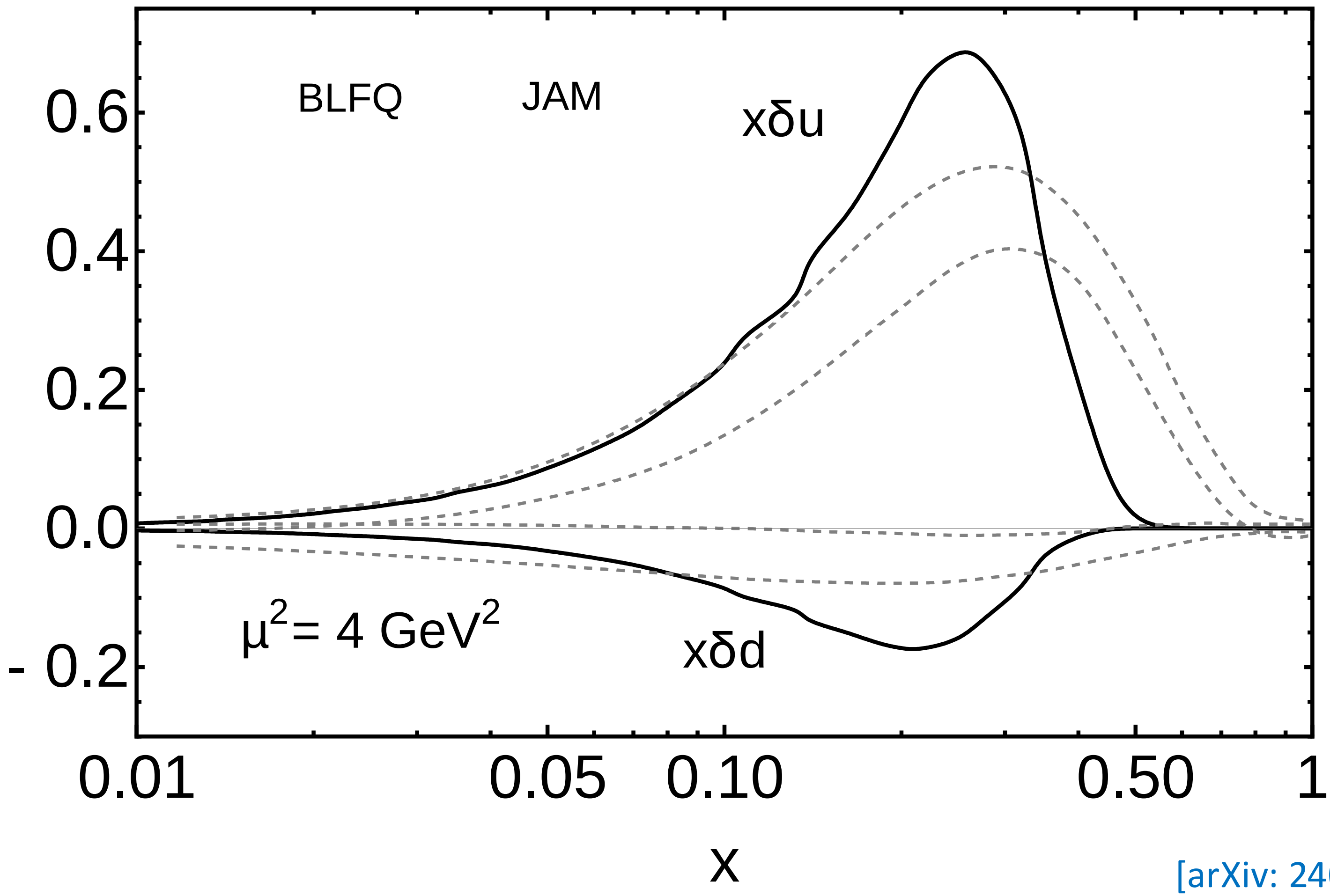
Helicity PDF



[arXiv: 2408.11298]

- Small-x region reasonably agrees with global fit/exp. data
- Narrower peak than global fits

Transversity PDF

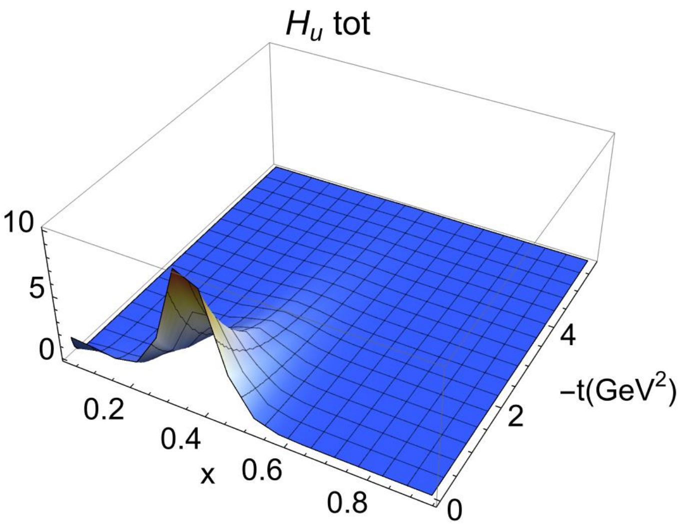


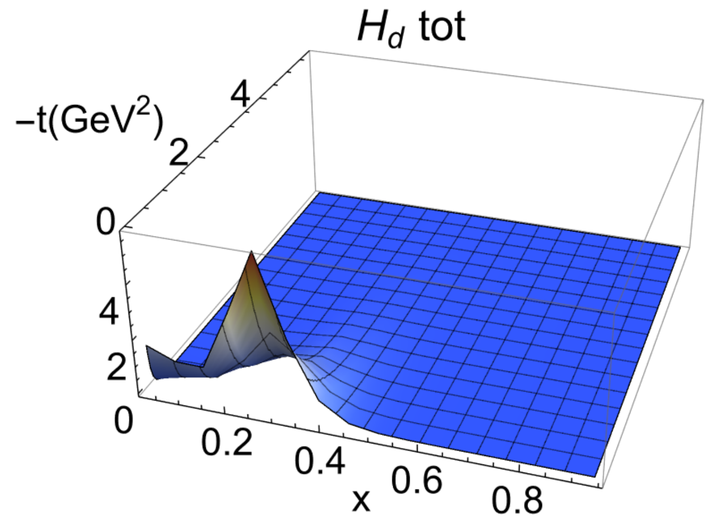
[arXiv: 2408.11298]

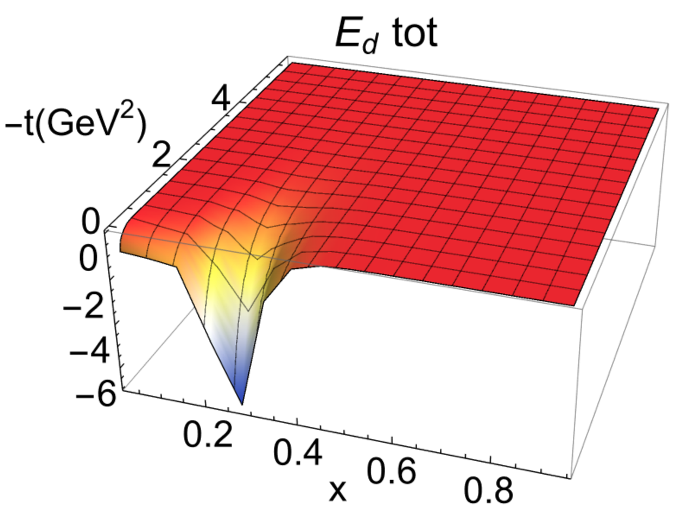
- Narrower peak than global fits
- Tensor charges: $\delta u = 0.81 \pm 0.08$, $\delta d = -0.22 \pm 0.01$

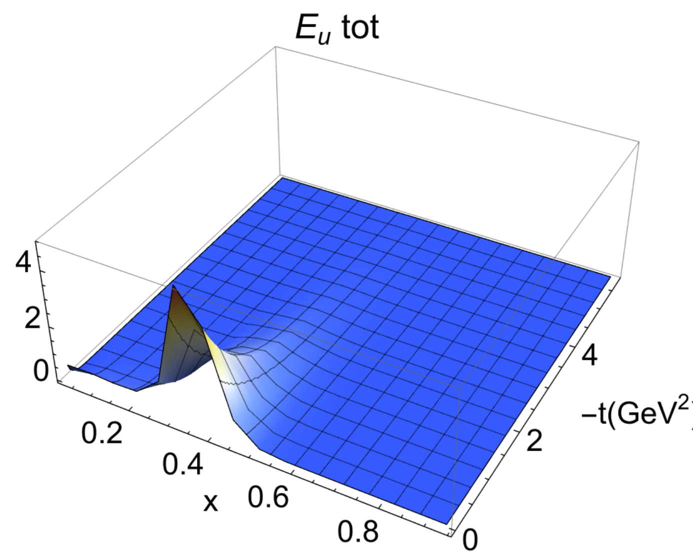
[PRL 132, 091901 (2024)] [PRD 98, 091501 (2018)]

u and d Quark GPDs at $\xi = 0$



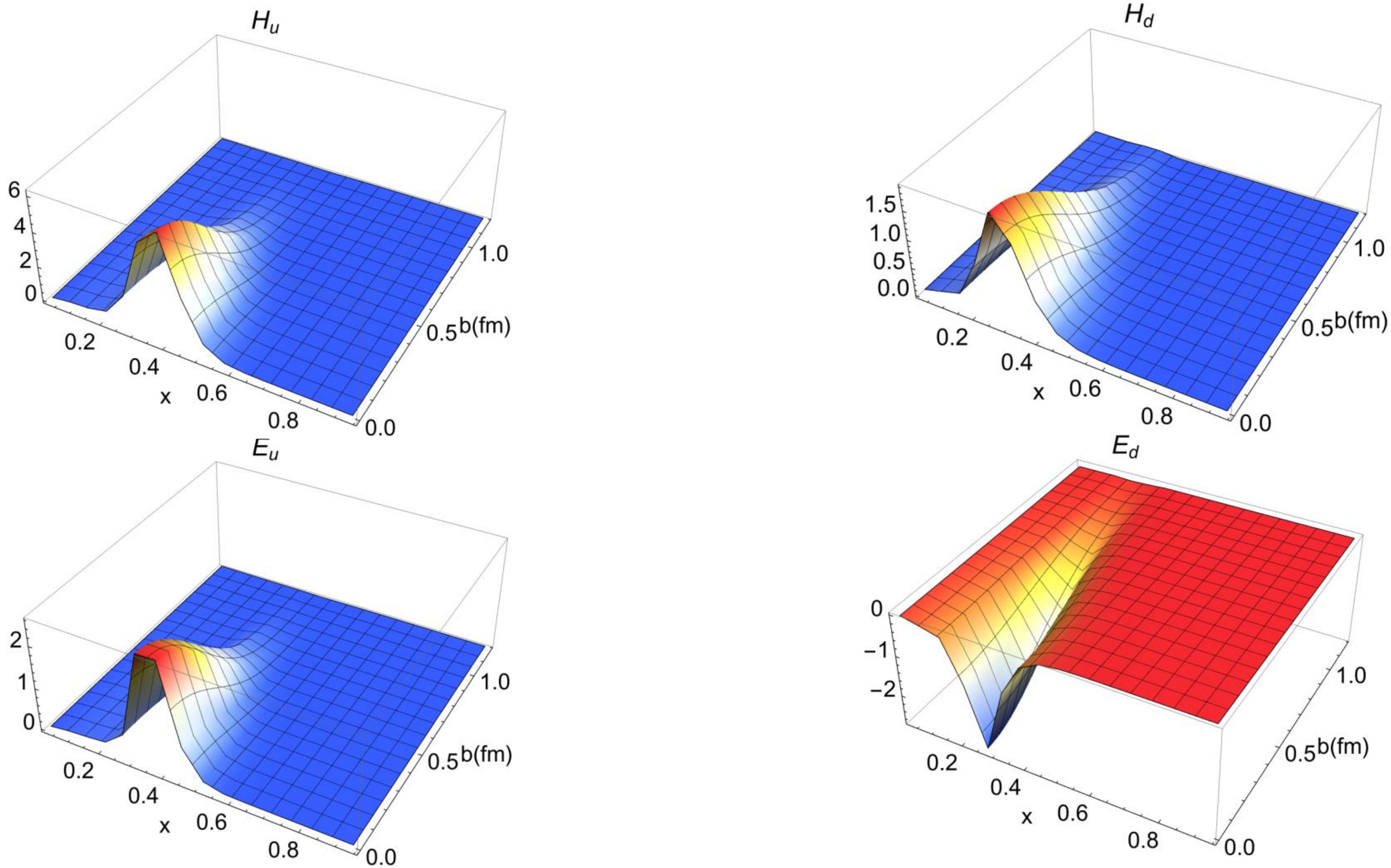






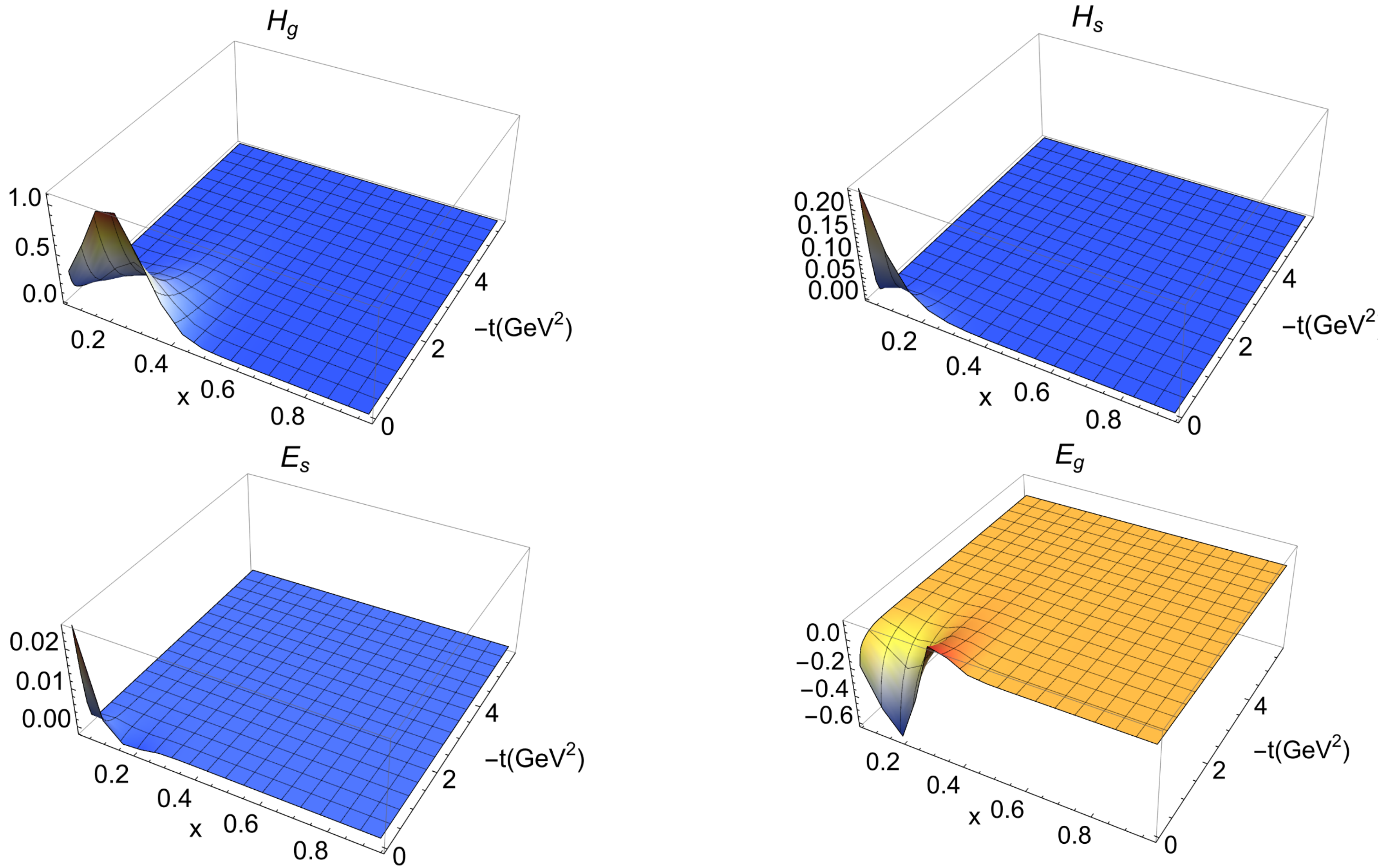
- Contributions from all Fock sectors
- Achieved qualitative features compared to various models

GPDs in Impact Parameter Space



- Concentrate near the center $b = 0$
- Qualitative features agree with other approaches

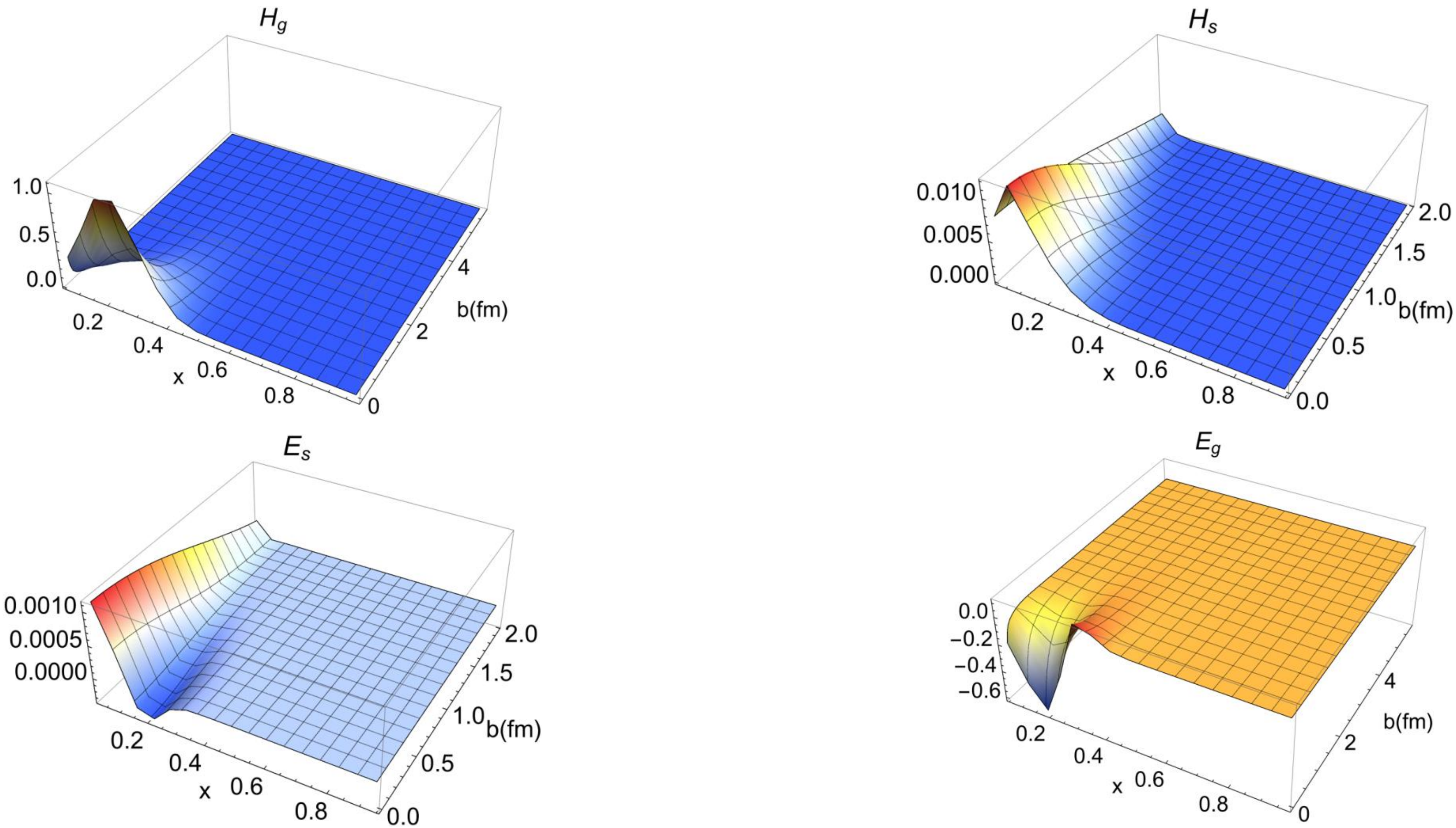
Gluon and s Quark GPDs at $\xi = 0$



- Dominate at small x region
- E_g oscillates over x

[PRD 105, 054509 (2022)]

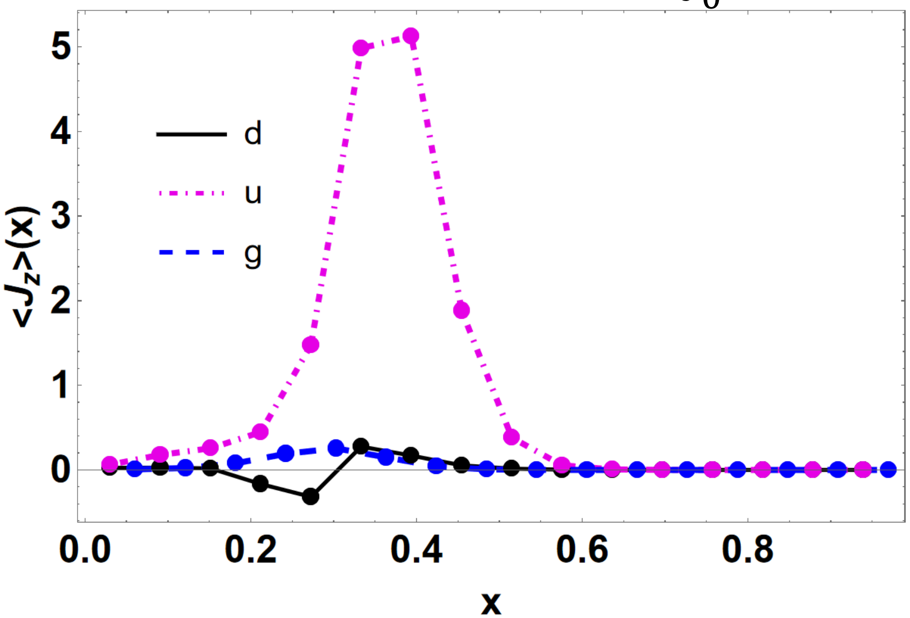
GPDs in Impact Parameter Space

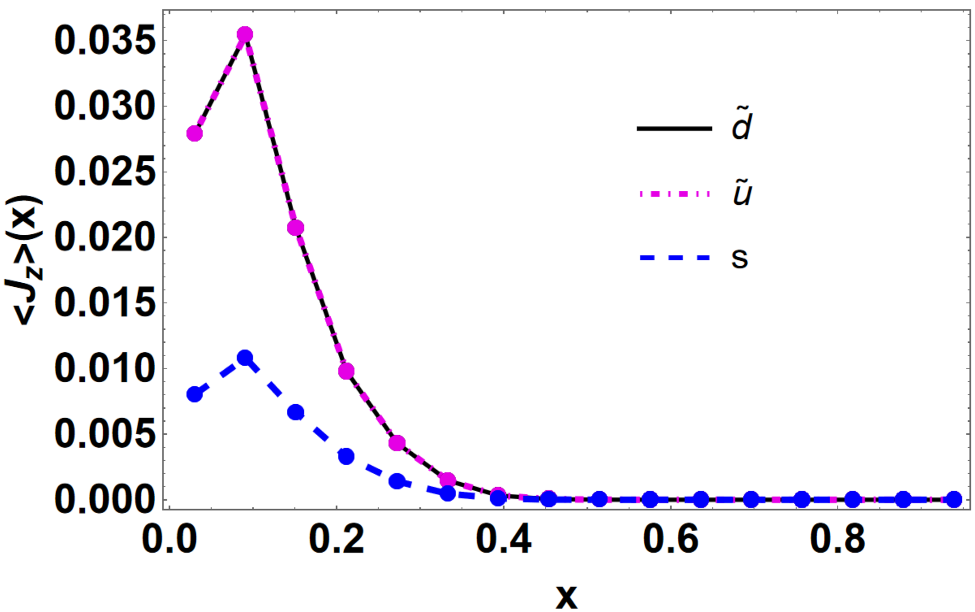


- Concentrate near the center $b = 0$ and small x

Angular Momentum Distribution

$$J_{q,g} = \frac{1}{2}\int_0^1 x[H_{q,g}(x,0,0) + E_{q,g}(x,0,0)]dx$$



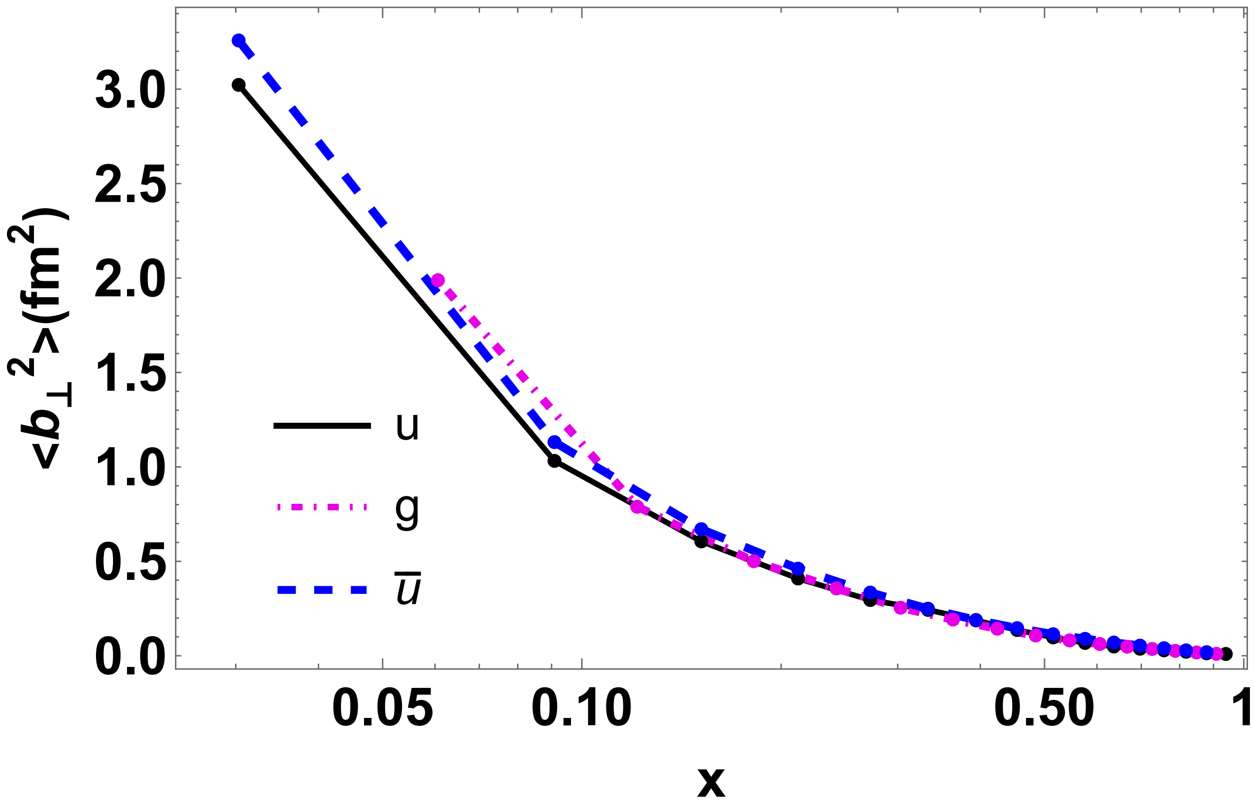


parton	d	u	dbar	ubar	s/sbar	g
percentage	0.63%	93.02%	0.63%	0.63%	0.19%	4.71%

- u is dominant, gluon contributes about 5%, d is negative
- $\bar{u}$ quark is almost the same as $\bar{d}$
- d=$\bar{d}$ is a coincidence

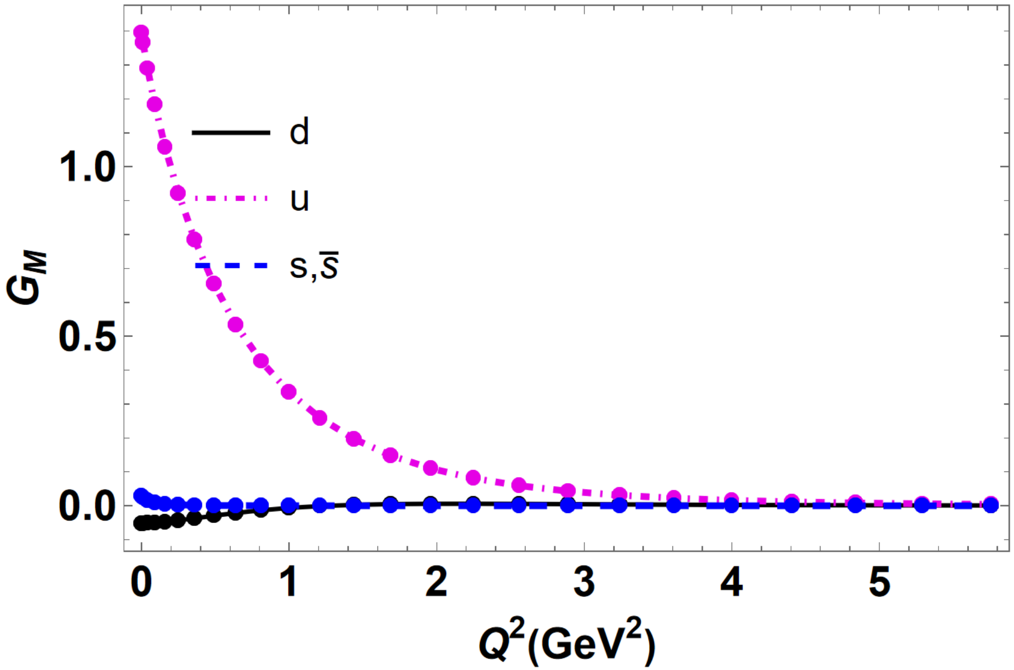
Squared Radius

$$\langle b_\perp^2 \rangle^{q/g}(x) = \frac{\int \mathrm{d}^2 b_\perp (b_\perp)^2 \mathcal{H}(x,0,b_\perp)}{\int \mathrm{d}^2 b_\perp \mathcal{H}(x,0,b_\perp)}$$



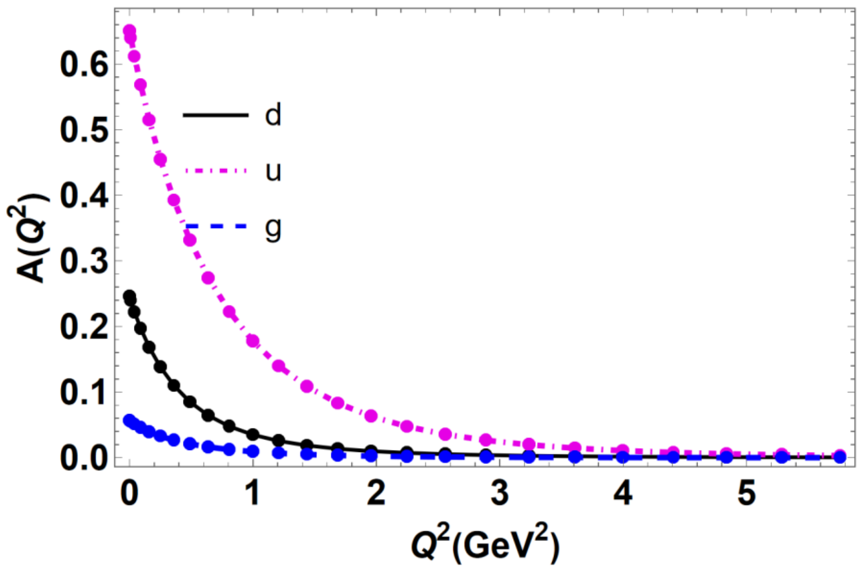
- Gluon radius > sea quark radius > valence quark radius
- As $x \to 1$, nucleon behaves like point particle

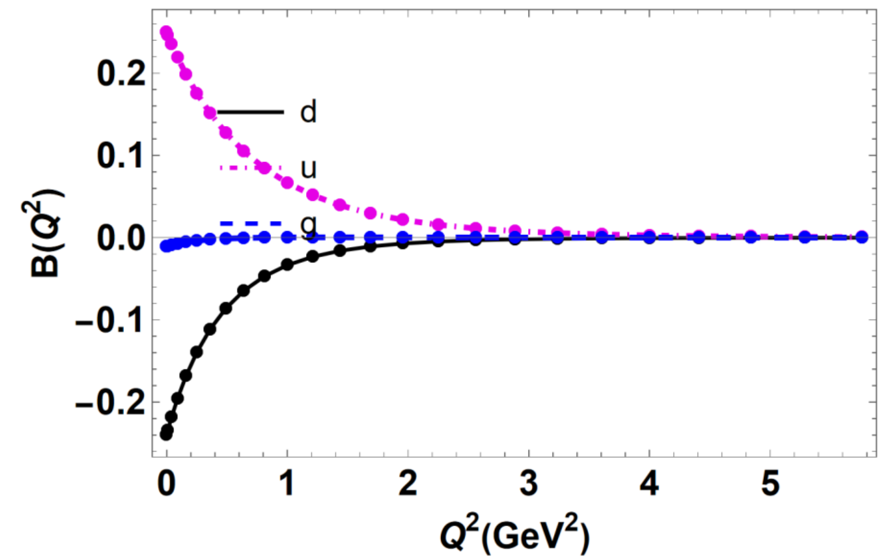
Magnetic Form Factor for Different Quarks

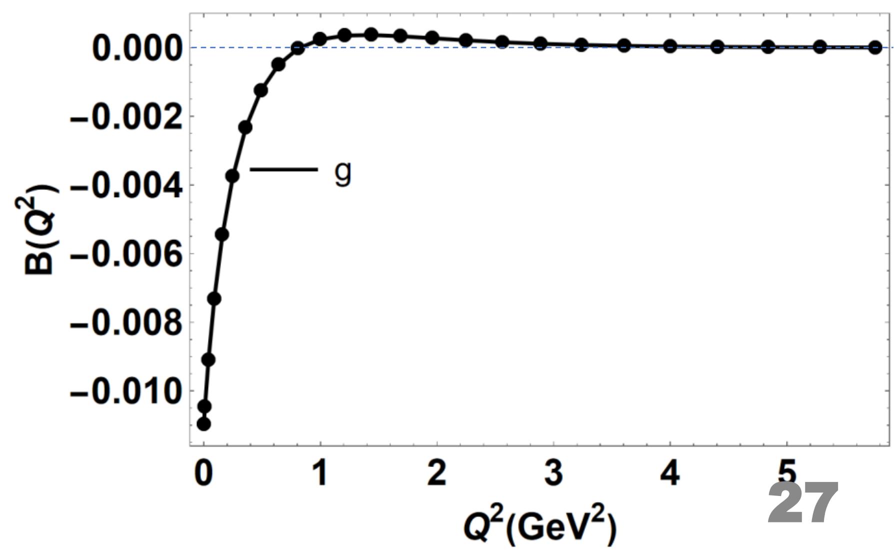


- $G_M^d(0) = -0.053$
- $G_M^u(0) = 1.396$
- $G_M^s(0) = -0.028$

Gravitational Form Factors



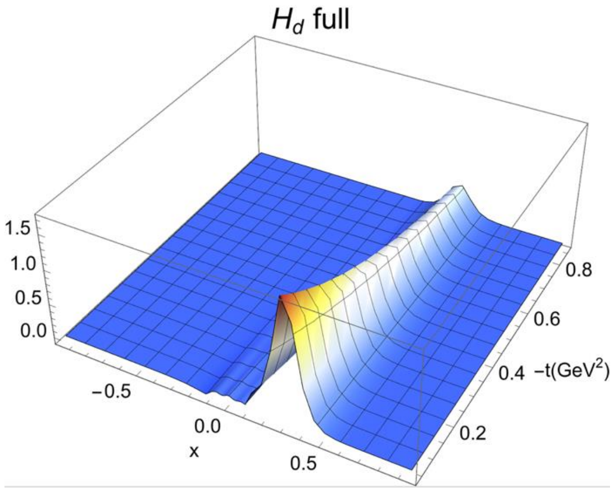


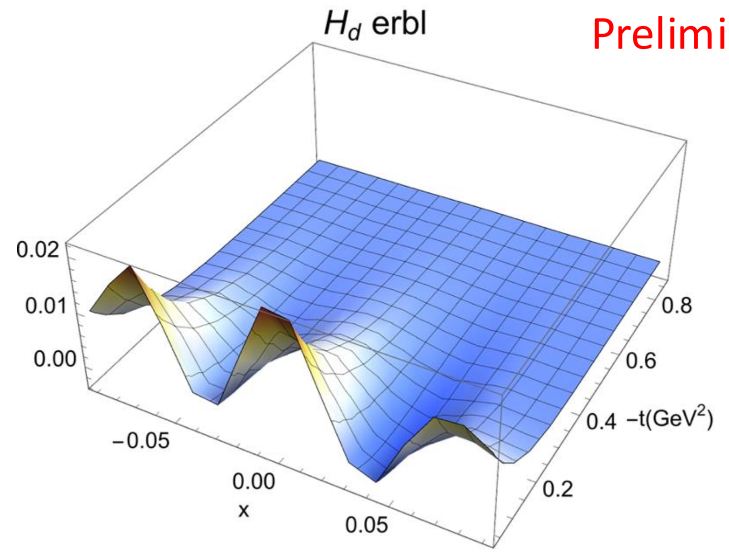


- Ji's sum rule is satisfied
- Qualitative agreement with lattice data

[PRD 105, 054509 (2022)]

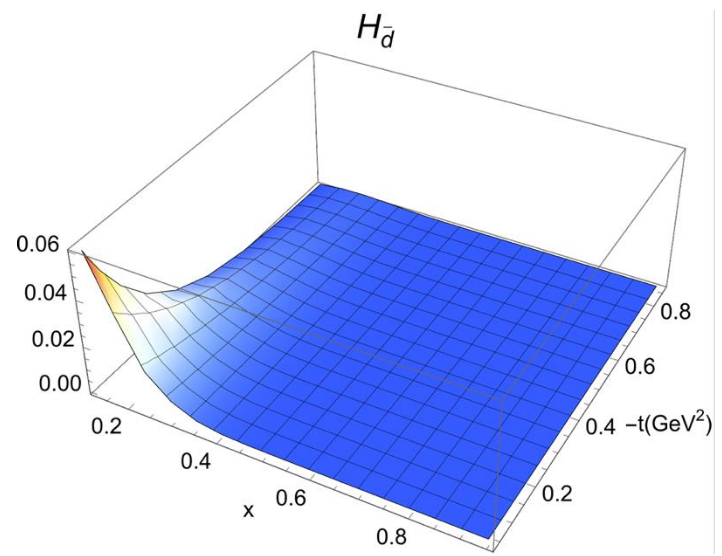
GPDs at $\xi = 0.1$





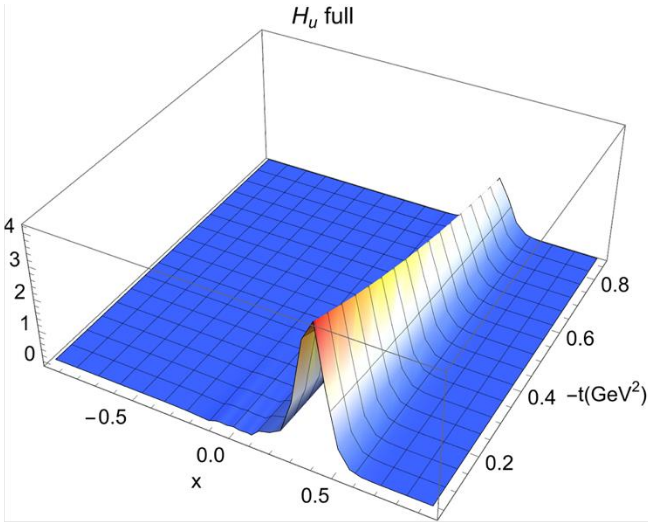
Preliminary

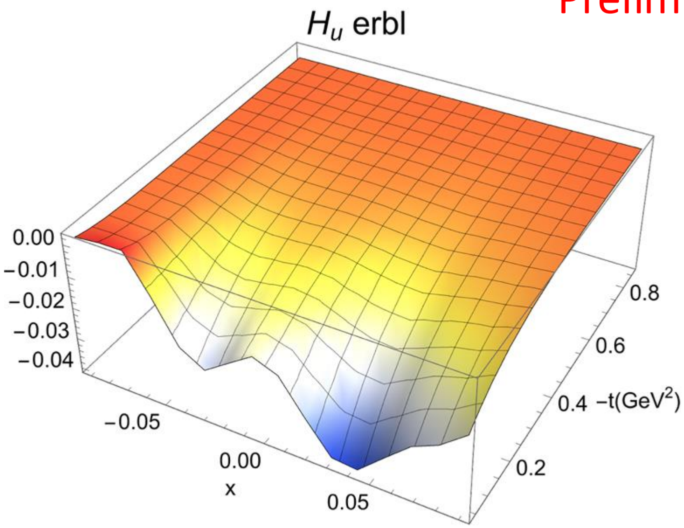
- **We use symmetric frame**
- d quark GPD H from $-1 < x < 1$
- At $\xi = 0.1$, DGLAP region dominates
- Discontinuity at $x = \pm\xi$

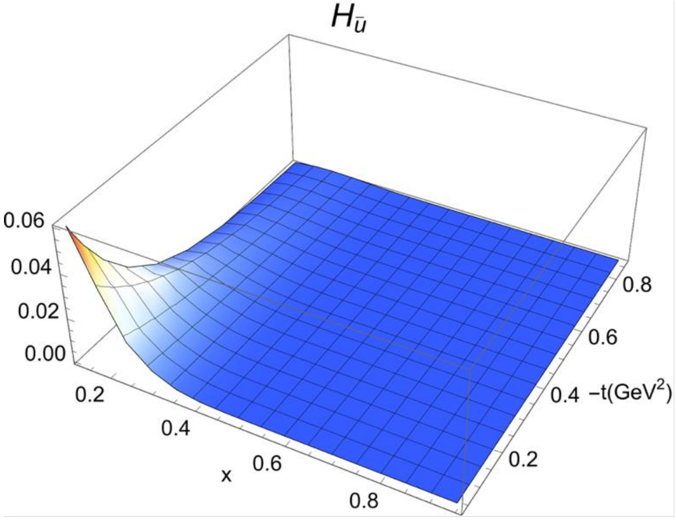


GPDs at $\xi = 0.1$

Preliminary



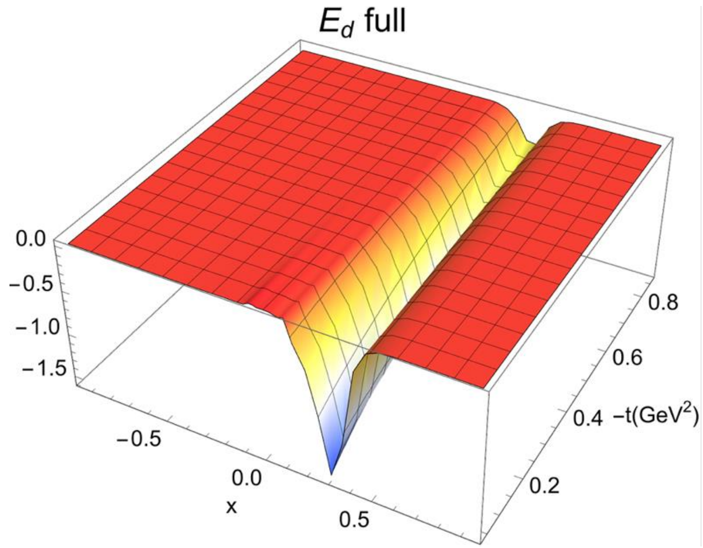


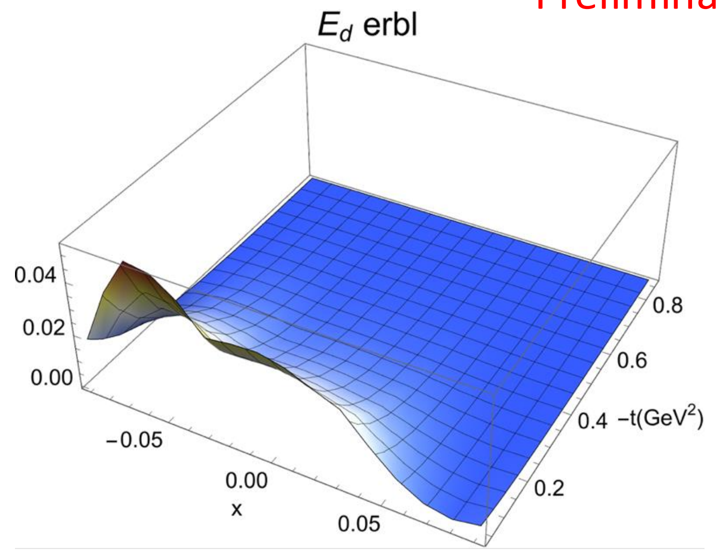


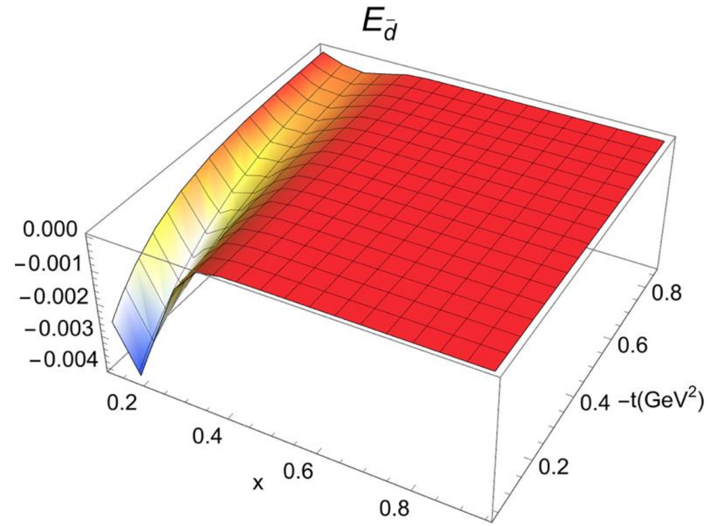
- u quark GPD H from $-1 < x < 1$
- At $\xi = 0.1$, DGLAP region dominates
- Discontinuity at $x = +\xi$, not sure at $x = -\xi$

GPDs at $\xi = 0.1$

Preliminary

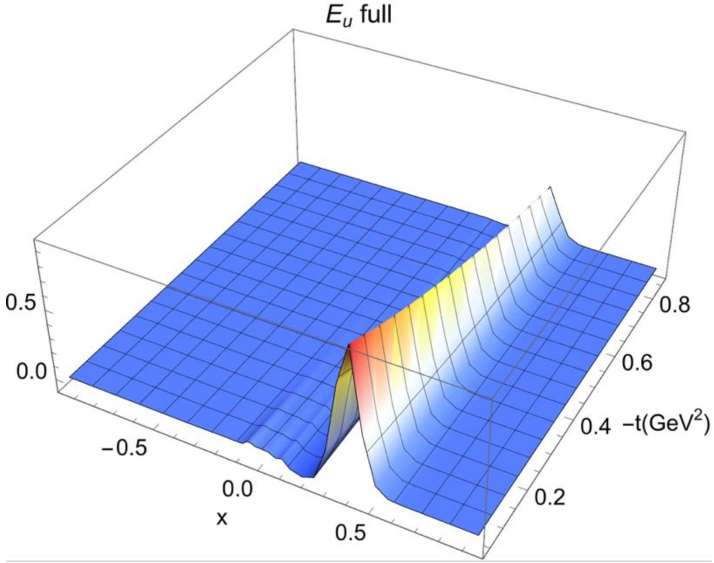


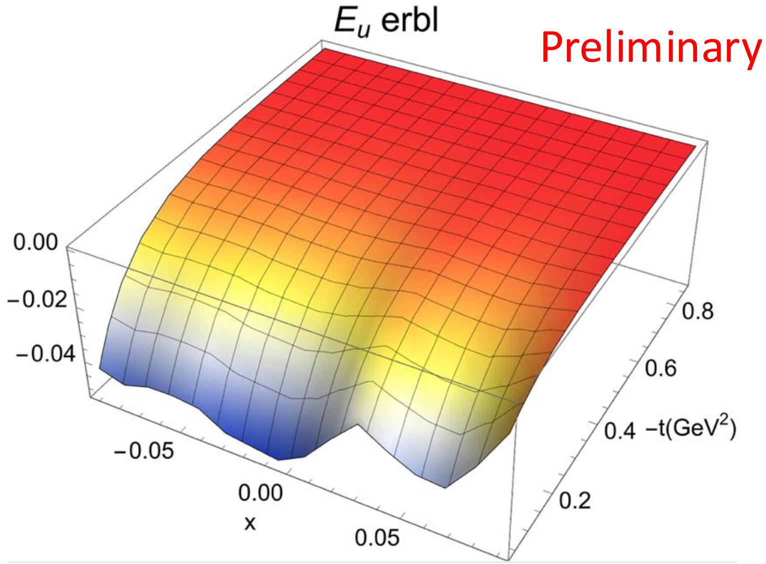


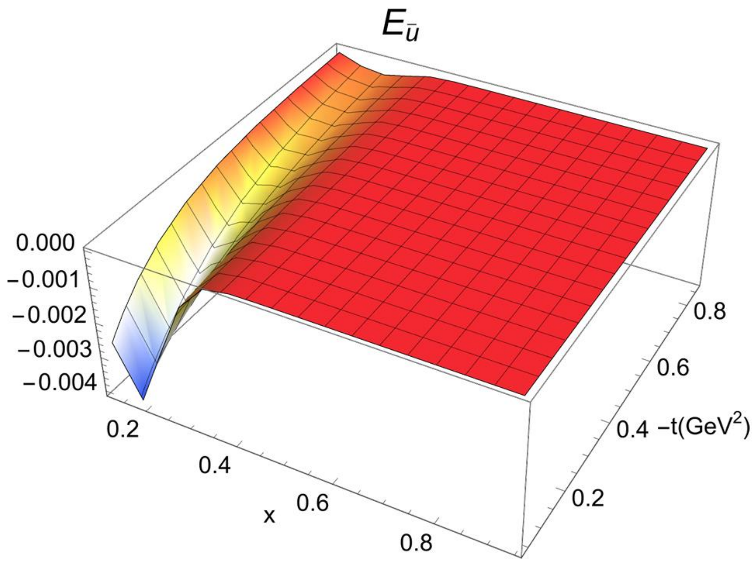


- d quark GPD E from $-1 < x < 1$
- At $\xi = 0.1$, DGLAP region dominates
- Discontinuity at $x = \pm\xi$

GPDs at $\xi = 0.1$

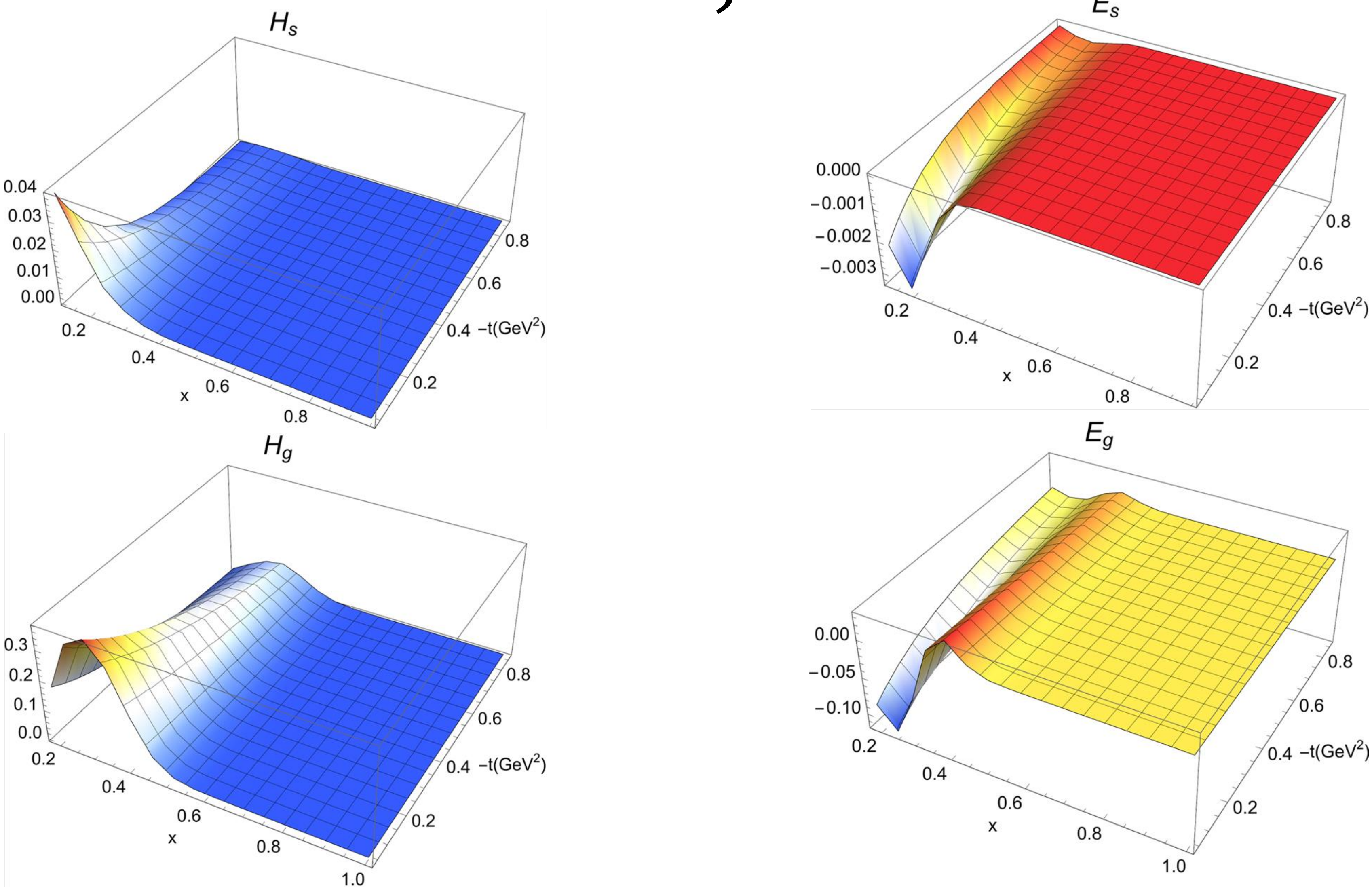






- u quark GPD E from $-1 < x < 1$
- GPD E for u quark is smaller than d quark
- Discontinuity at $x = -\xi$, not sure at $x = \xi$

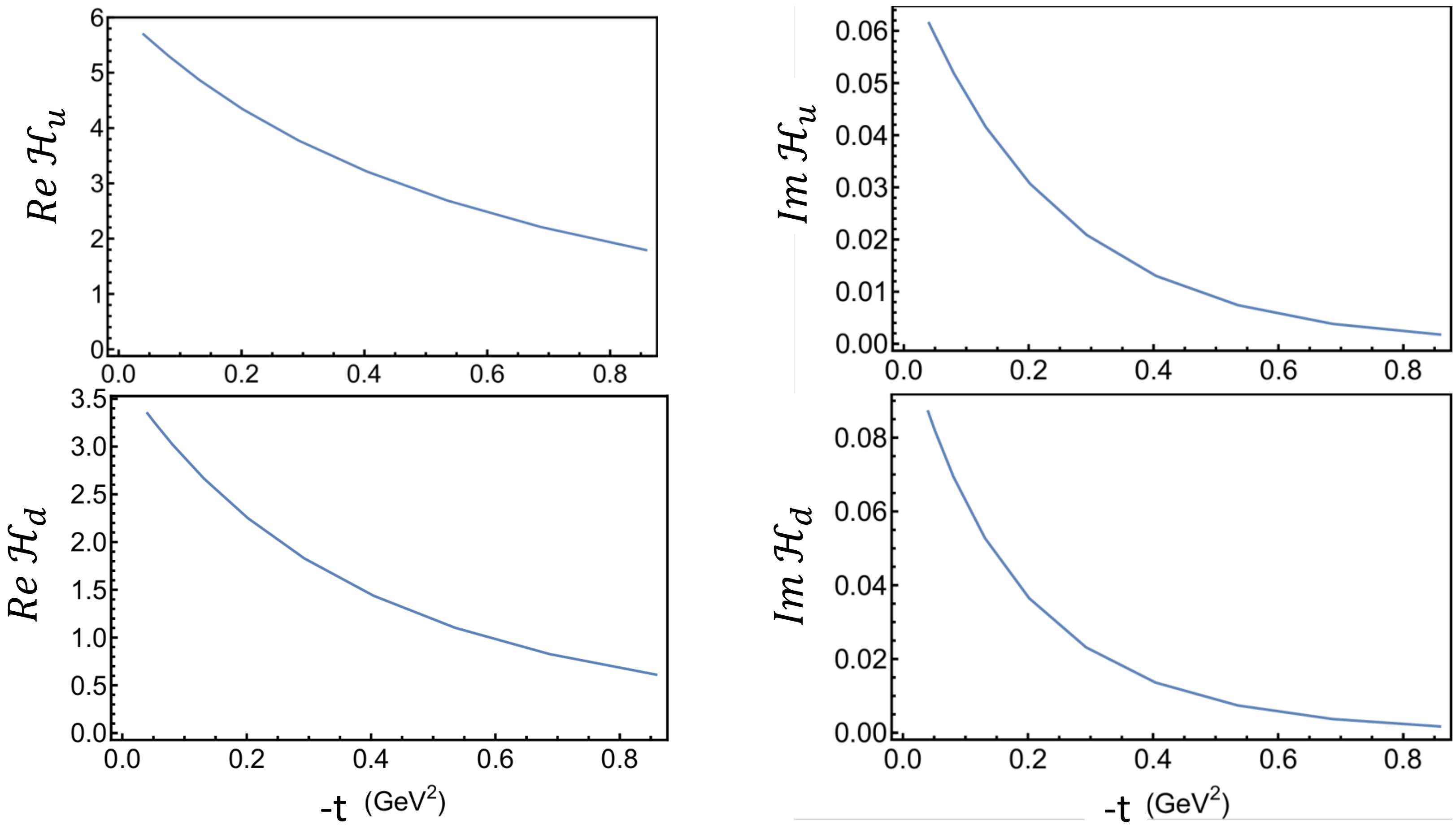
GPDs at $\xi = 0.1$



- Gluon and s quark GPDs in DGLAP region at $\xi = 0.1$
- For non-zero skewness, E_s changes sign

Compton Form Factors at $\xi = 0.1$

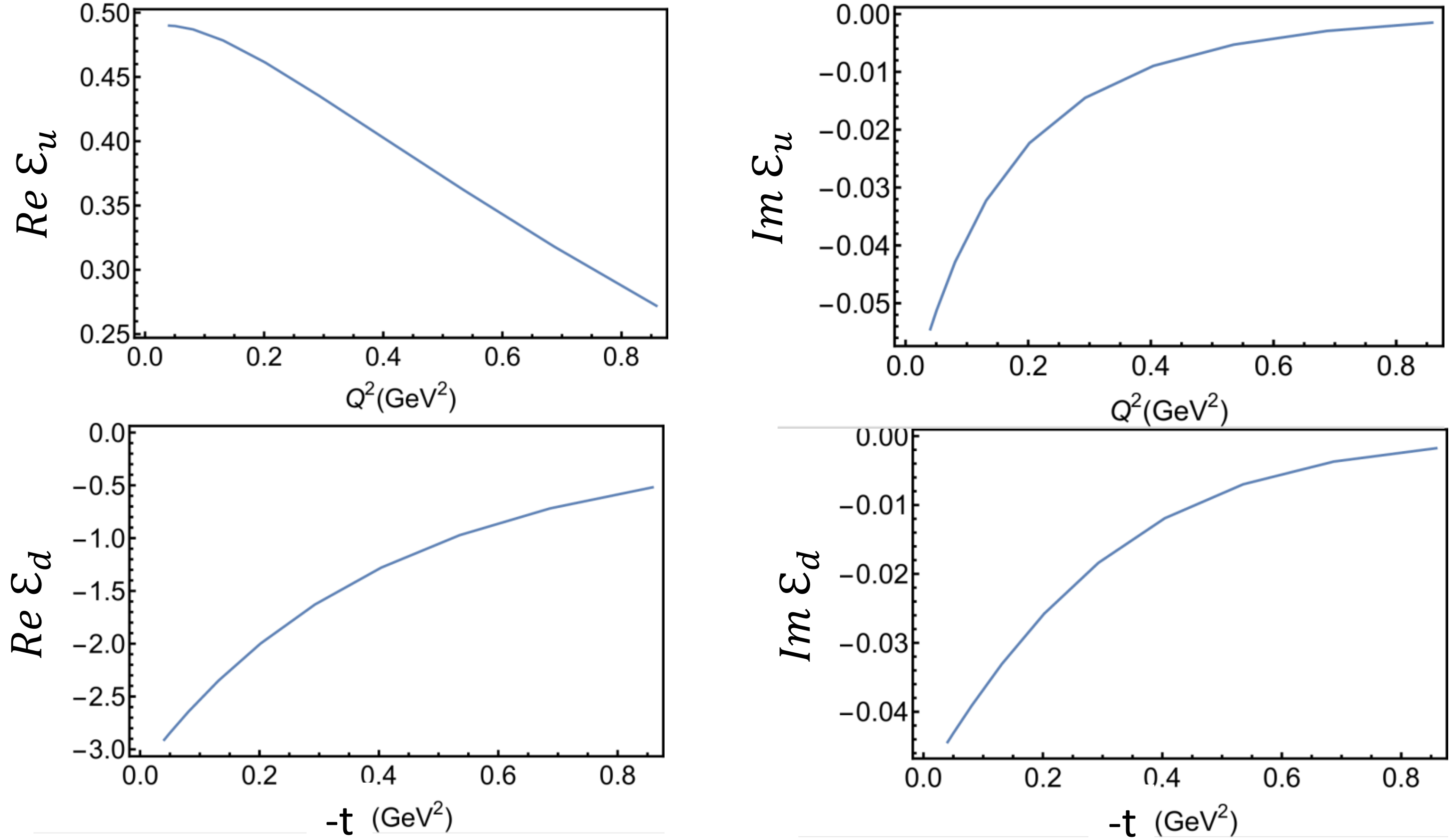
$$\mathcal{F}(\xi, -t) = \int_{-1}^{1} dx \left(\frac{1}{x - \xi - i\epsilon} - \frac{1}{x + \xi - i\epsilon} \right) F(x, \xi, -t)$$



- Real part of $\mathcal{H}_d$ falls faster than $\mathcal{H}_u$
- Imaginary part of $\mathcal{H}$ falls faster than real part

Compton Form Factors at $\xi = 0.1$

$$\mathcal{F}(\xi, -t) = \int_{-1}^{1} dx \left(\frac{1}{x - \xi - i\epsilon} - \frac{1}{x + \xi - i\epsilon} \right) F(x, \xi, -t)$$



- Real part of $\mathcal{E}_d$ follows the trend of electromagnetic form factors
- Imaginary part of u falls faster than d

Conclusions

- GPDs provide spatial imaging of proton on the light front
- BLFQ: a nonperturbative framework based on light-front QCD Hamiltonian
- Based on $|qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle$ sectors we investigate proton 3D structure
- Achieved qualitative features compared to other approaches
- Explored the GPDs of valence and sea quarks and gluon
- Explored skewness-dependent GPDs including both DGLAP and ERBL regions, and investigated the Compton form factors

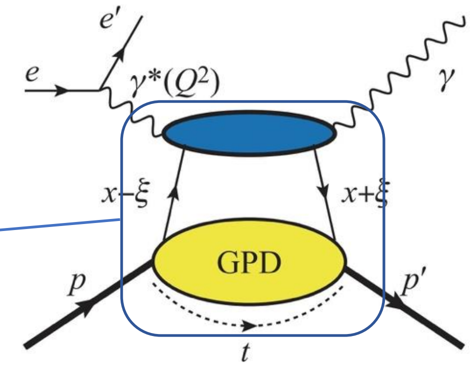
Thank you!


IMP
LIGHT CONE 2024
Hadron Physics in the EIC era
The Institute of Modern Physics, Chinese Academy of Sciences, Huizhou Campus, China.
November 25-29, 2024
Physics Topics and Tools
» Physics of EIC and EicC
» Hadron spectroscopy and reactions
» Hadron/nuclear structure
» Spin physics
» Relativistic many-body physics
» QCD phase structure
» Light-front field theory
» AdS/CFT and holography
» Nonperturbative QFT methods
» Effective field theories
» Lattice field theories
» Quantum computing
» Present and future facilities
International Advisory Committee
» Stanley J. Brodsky (SLAC)
» Ho-Meoyng Choi (Kyungpook National U.)
» Stanislaw D. Glazek (Warsaw U.)
» Chueng-Ryong Ji (NCSU)
» Dayashankar Kulshreshtha (Delhi U.)
» Gerald A. Miller (INT & U. Washington)
» Wally Melnitchouk (TJNAF)
» Barbara Pasquini (Pavia U.)
» Wayne Nicholas Polyzou (U. of Iowa)
» Nico G. Stefanis (Ruhr U.)
» Wojciech Broniowski (JKU & Cracow, INP)
» Tobias Frederico (ITA)
» John R. Hiller (Idaho U.)
» Vladimir Karmanov (Lebedev Inst.)
» Cédric Lorcé (Ecole Polytechnique)
» Anuradha Misra (Mumbai U.)
» Teresa Peña (IST & Lisboa U.)
» Giovanni Salmè (INFN Roma)
» James P. Vary (Iowa State U.)
Local Organizing Committee
» Xingbo Zhao (IMP,chair)
» Jiangshan Lan (IMP,co-chair)
» Yang Li (USTC)
» Chandan Mondal (IMP)
» Satvir Kaur (IMP)
» Siqi Xu (IMP)
» Yair Mulian (IMP)
» Yuxiang Zhao (IMP)
SCNT
Registration and abstract submission opens : 1st April, 2024
Abstract submission deadline : 31st August, 2024
Registration closes : 31st October, 2024

GPDs and Compton Form Factors

$$\mathcal{T}_{\text{DVCS}} = \frac{1}{Q^2}\bar{u}(k',h')\gamma^\rho u(k,h) \sum_\Lambda \varepsilon^*_\rho(q,\Lambda)\varepsilon_\nu(q,\Lambda) T^{\mu\nu} \varepsilon^*_\mu(q',\Lambda')\,,$$

$$\underset{(L)}{T^{\mu\nu}} \equiv i \int \mathrm{d}^4x e^{i(q+q')z/2} \left\langle P',S' \middle| \mathrm{T}\left\{ J^\mu\left(\frac{z}{2}\right) J^\nu\left(-\frac{z}{2}\right)\right\} \middle| P,S \right\rangle$$



$$T^{\mu\nu}_{(L)} = T^{\mu\nu}_{(2)} + T^{\mu\nu}_{(3)} + T^{\mu\nu}_{(4)} \qquad T^{\mu\nu}_{(NL)} = T^{\mu\nu}_{(3)} + T^{\mu\nu}_{(4)} + T^{\mu\nu}_{(5)} \qquad \cdots\cdots\cdots$$

$$T^{\mu\nu}_{(2)} = \int_{-1}^{1} \mathrm{d}x \sum_q \left(\mathscr{T}^{\mu\nu}_{(2)} C^{q[-]}_{(0)}(x,\xi) n^\rho W^{[\gamma_\rho]}(x,\xi,t) + \widetilde{\mathscr{T}}^{\mu\nu}_{(2)} C^{q[+]}_{(0)}(x,\xi) n^\rho W^{[\gamma_\rho\gamma^5]}(x,\xi,t) \right)$$

$$T^{\mu\nu}_{(3)} = \int_{-1}^{1} \mathrm{d}x \sum_q \left(\mathscr{T}^{\mu\nu,\rho}_{(3)} C^{q[-]}_{(0)}(x,\xi) W^{[\gamma^\perp_\rho]}(x,\xi,t) + \widetilde{\mathscr{T}}^{\mu\nu,\rho}_{(3)} C^{q[+]}_{(0)}(x,\xi) W^{[\gamma^\perp_\rho\gamma^5]}(x,\xi,t) \right)$$

$$\mathscr{T}^{\mu\nu}_{(2)} = -\frac{1}{2}\left[g^{\mu\nu}_\perp - \frac{1}{p\cdot\bar{q}}\left(p^\mu q'^\nu_\perp + q^\mu_\perp p^\nu\right)\right]\,, \qquad \mathscr{T}^{\mu\nu,\rho}_{(3)} = \frac{1}{2p\cdot\bar{q}}\left[q'^\mu g^{\nu\rho}_\perp + g^{\mu\rho}_\perp\left(q^\nu + 4\xi p^\nu\right)\right]\,,$$

$$\widetilde{\mathscr{T}}^{\mu\nu}_{(2)} = \frac{i}{2}\left[\epsilon^{\mu\nu}_\perp - \frac{1}{p\cdot\bar{q}}\left(-p^\mu \epsilon^{\nu\rho}_\perp q'^\perp_\rho + \epsilon^{\nu\rho}_\perp q^\perp_\rho p^\nu\right)\right] \quad \widetilde{\mathscr{T}}^{\mu\nu,\rho}_{(3)} = \frac{i}{2p\cdot\bar{q}}\left[\epsilon^{\mu\nu\rho\sigma}\bar{q}_\sigma + \xi\left(p^\mu\epsilon^{\rho\nu}_\perp + p^\nu\epsilon^{\rho\mu}_\perp\right)\right]\,.$$

Cross-section ⟷ Compton form factors

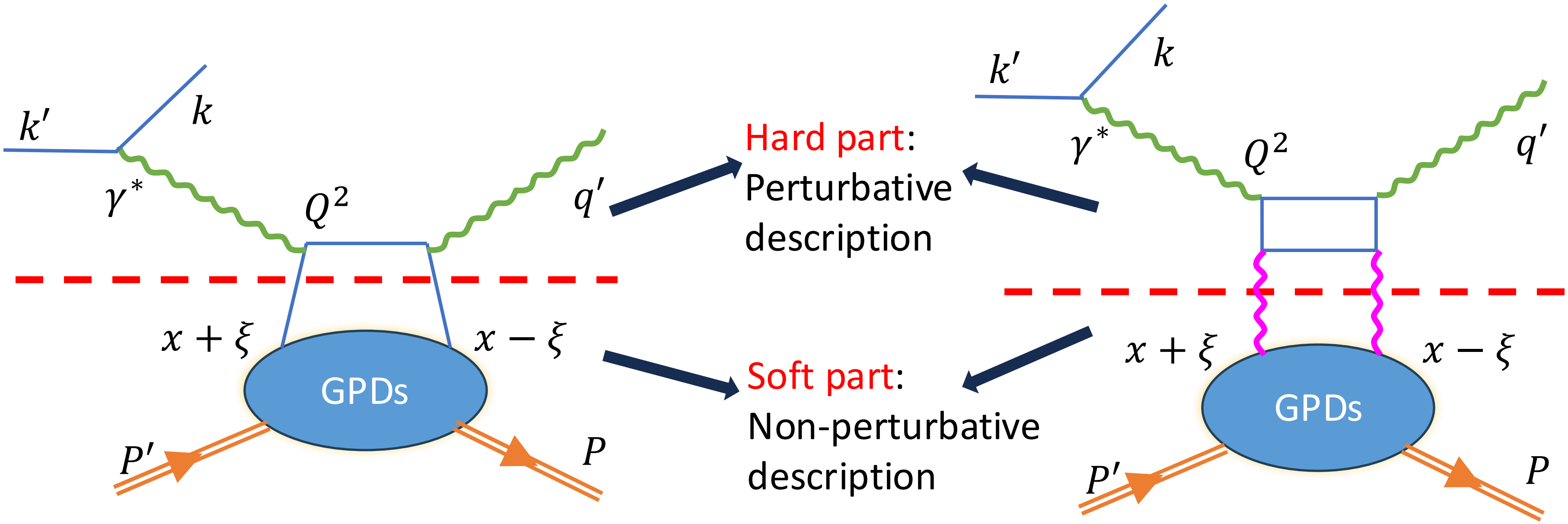
GPDs and Compton Form Factors

$$
\begin{aligned}
\frac{d^5\sigma_{\mathrm{DVCS}}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \Gamma|T_{\mathrm{DVCS}}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T}+\epsilon F_{UU,L}+\epsilon\cos 2\phi F_{UU}^{\cos 2\phi}+\sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}+(2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} \\
&\quad+(2\Lambda)\Big[\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}+\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}+(2h)\Big(\sqrt{1-\epsilon^2}F_{LL}+2\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}\Big)\Big] \\
&\quad+(2\Lambda_T)\Big[\sin(\phi-\phi_S)(F_{UT,T}^{\sin(\phi-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi-\phi_S)})+\epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)}+\epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad+\sqrt{2\epsilon(1+\epsilon)}(\sin\phi_S F_{UT}^{\sin\phi_S}+\sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)})\Big] \\
&\quad+(2h)(2\Lambda_T)\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)}+\sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\quad+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}.
\end{aligned}
$$

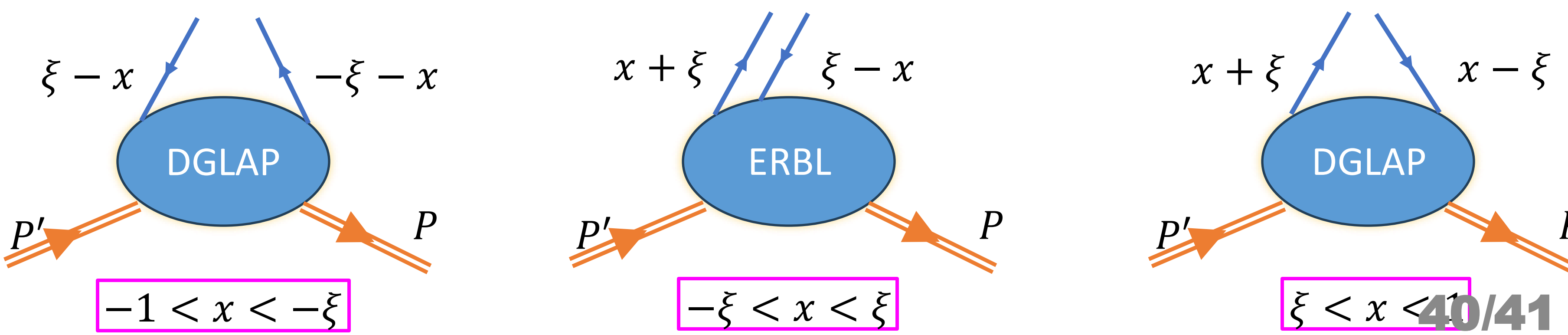
Cross-section ⟷ Structure functions

Deeply Virtual Compton Scattering

- The deeply virtual Compton scattering describes the process: $e + p \rightarrow e + p + \gamma$



- Soft part ⟹ Generalized parton distribution functions (GPDs)



Definition of GPDs

- **GPDs are defined through the following bilocal operator:** [Stephan.M, 2009]

$$F_{\Lambda'\Lambda}^{[\Gamma]}(x,\xi,\Delta^2) = \int \frac{dz^-}{4\pi} e^{ip\cdot x} \left\langle P',\Lambda' \middle| \bar{\psi}\left(-\frac{z}{2}\right) \mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right) \Gamma \psi\left(\frac{z}{2}\right) \middle| P,\Lambda \right\rangle \Bigg|_{z^+=z^\perp=0}$$

- **GPDs are parameterized by taking different Γ matrices:**

$$F_{\Lambda'\Lambda}^{[\gamma^+]} = \frac{1}{2P^+} \bar{u} \left[\gamma^+ H + \frac{i\sigma^{+\Delta}}{2M} E \right] u,$$

$$F_{\Lambda'\Lambda}^{[\gamma^+\gamma_5]} = \frac{1}{2P^+} \bar{u} \left[\gamma^+\gamma_5 \widetilde{H} + \frac{\Delta^+\gamma_5}{2M} \widetilde{E} \right] u,$$

$$F_{\Lambda'\Lambda}^{[i\sigma^{j+}\gamma_5]} = -\frac{i\varepsilon_T^{ij}}{2P^+} \bar{u} \left[i\sigma^{+i} H_T + \frac{\gamma^+\Delta_T^i - \Delta^+\gamma^i}{2M} E_T + \frac{P^+\Delta_T^i - \Delta^+ P_T^i}{M^2} \widetilde{H}_T + \frac{\gamma^+ P_T^i - P^+\gamma^i}{M} \widetilde{E}_T \right] u$$

- **We use symmetric frame**

Light-Front Hamiltonian (qqq)

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

[S. Xu et al, PRD 104 094036(2021)]

$$H_{trans} \sim \kappa_T^4 r^2$$

[S. J. Brodsky, G. de Teramond arXiv: 1203.4025]

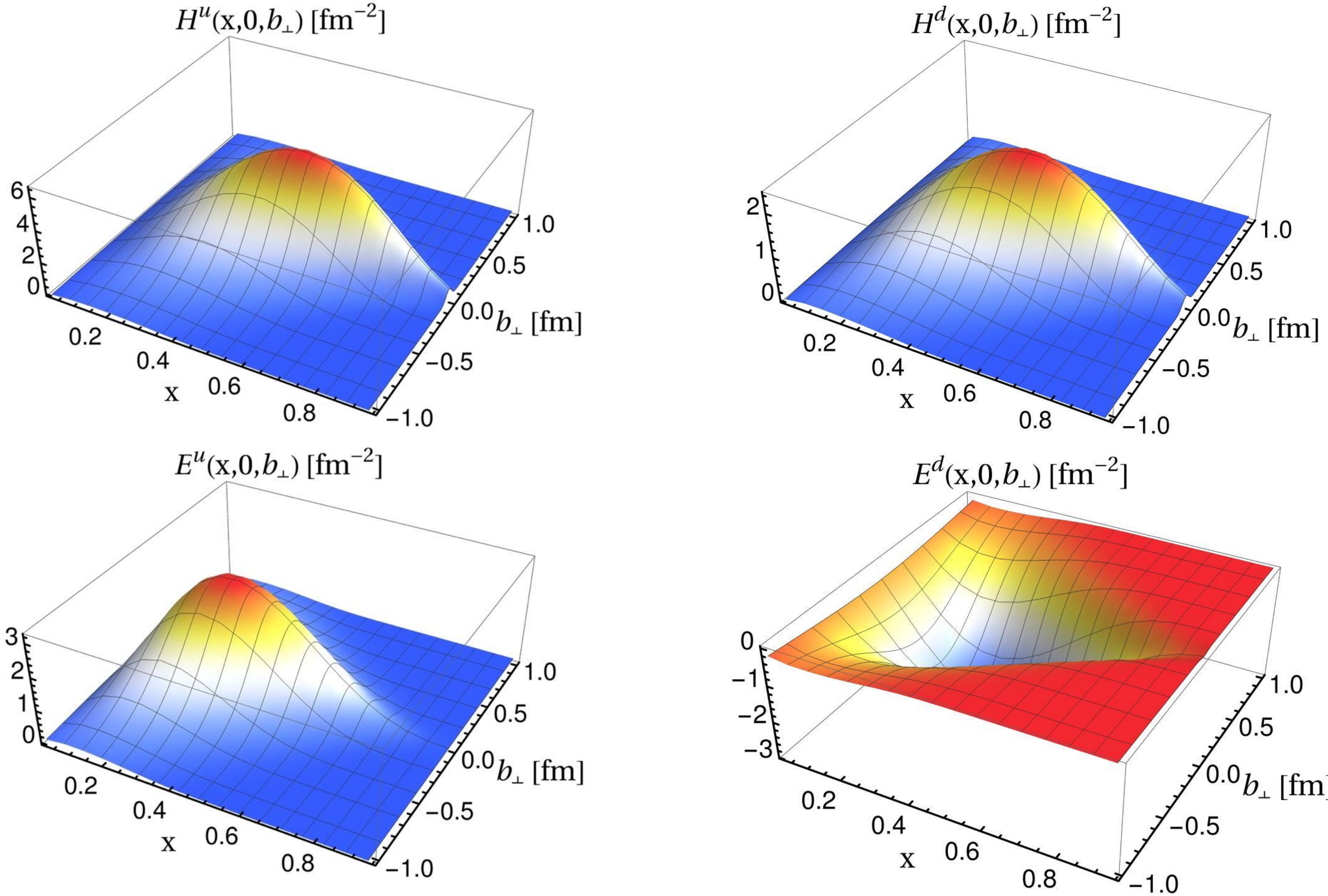
$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$

[Y. Li, X. Zhao , P Maris , J. P. Vary, PLB 758(2016)]

$$H_{OGE} = \frac{1}{2} \sum_{a \neq b} \frac{C_F 4\pi\alpha_s}{Q_{ab}^2} \bar{u}(k'_a, s'_a)\gamma^\mu u(k_a, s_a)\bar{u}(k'_b, s'_b)\gamma^\nu u(k_b, s_b) g_{\mu\nu}$$

[S. Xu et al, PRD 104 094036(2021)]

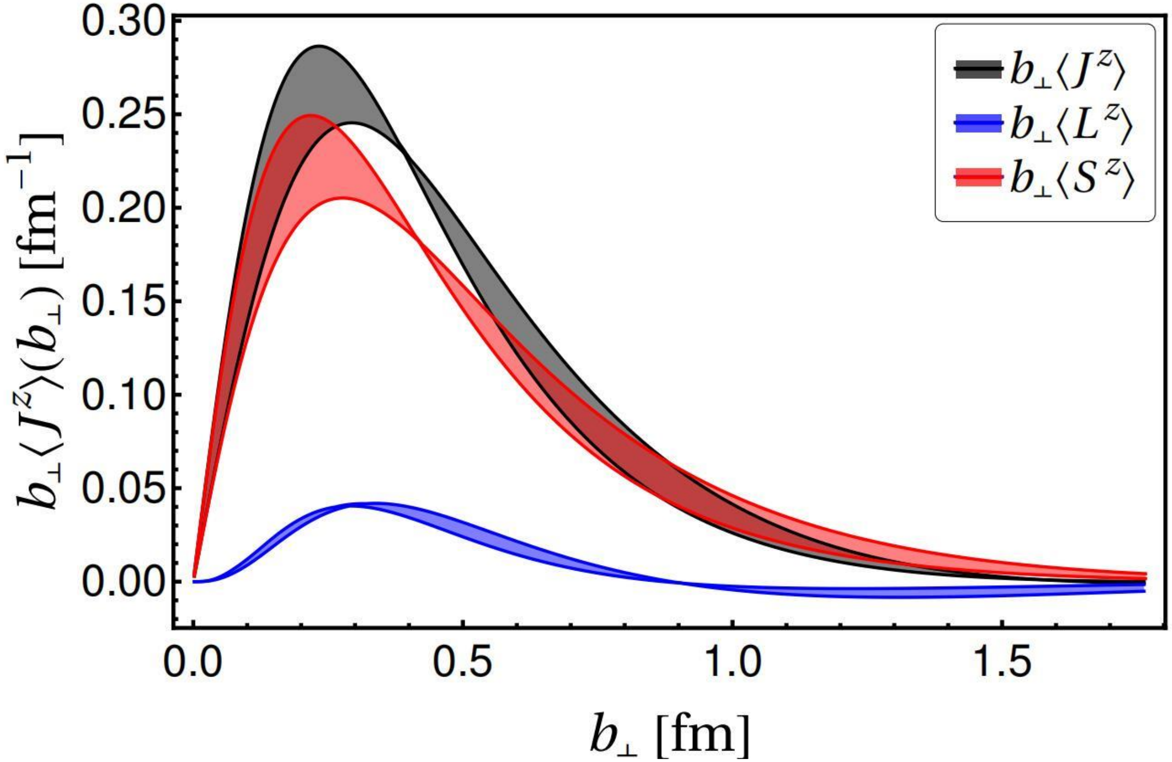
Result without Dynamical Gluon



- 0 skewness GPDs in transverse coordinate space

Result without Dynamical Gluon

Angular momentum decomposite: $\frac{1}{2} = S_{Ji}^{q} + L_{Ji}^{q} + J_{Ji}^{g}$

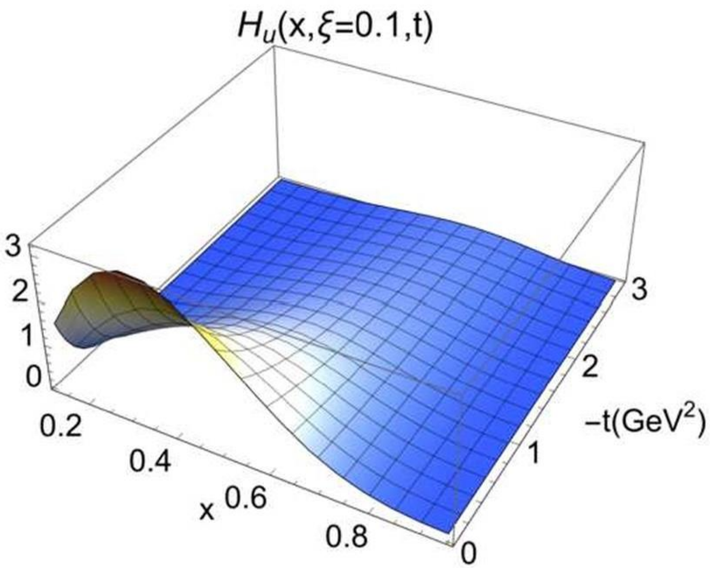


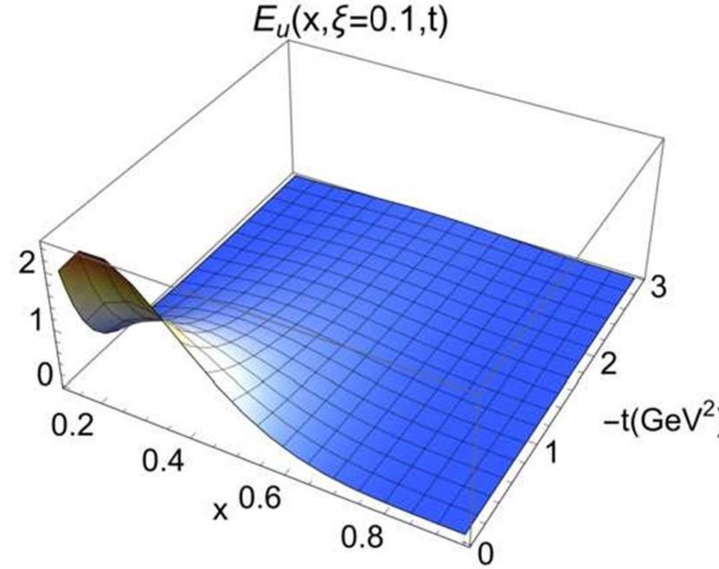
Proton angular momentum distribution in coordinate space

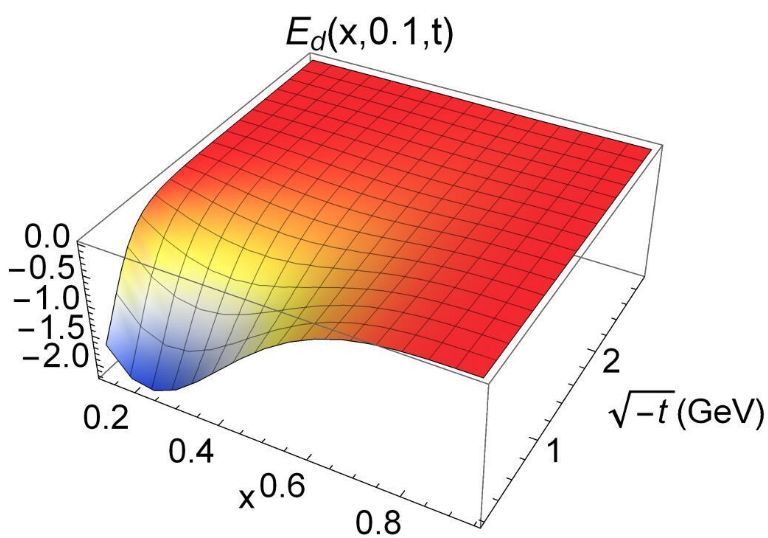
- 91% quark spin, 9% orbital angular momentum
- Angular momentum distribution concentrates in 1fm
- Orbital angular momentum contributes positively inside 0.8fm, and negatively outside 0.8fm.

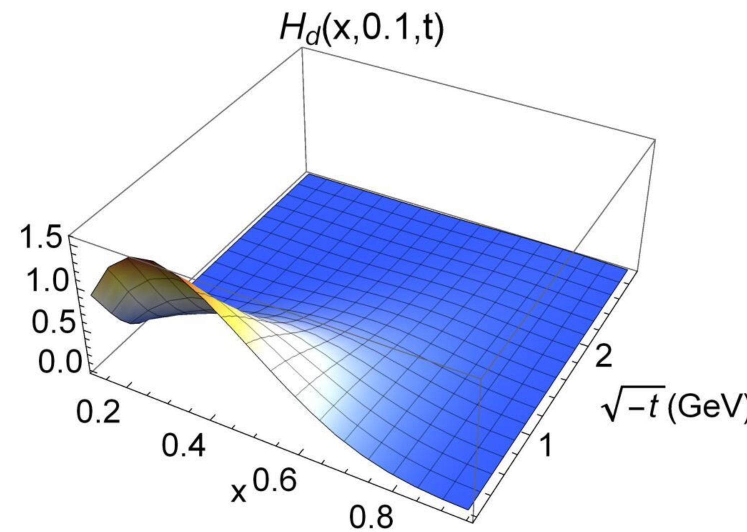
Gluon contributes 0 under qqq Fock sector

Result without Dynamical Gluon





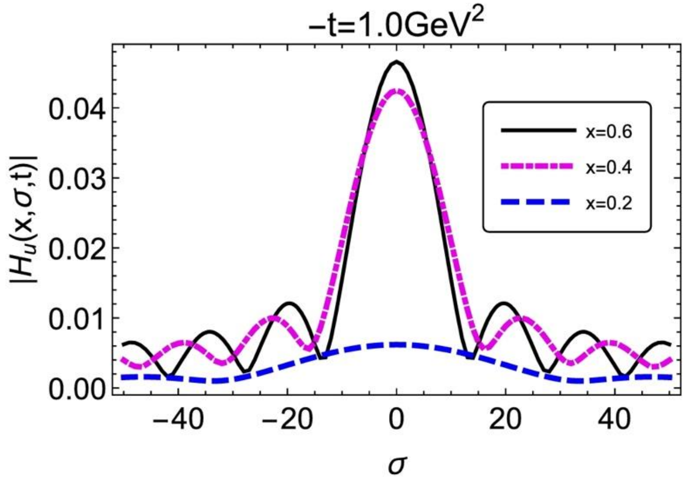


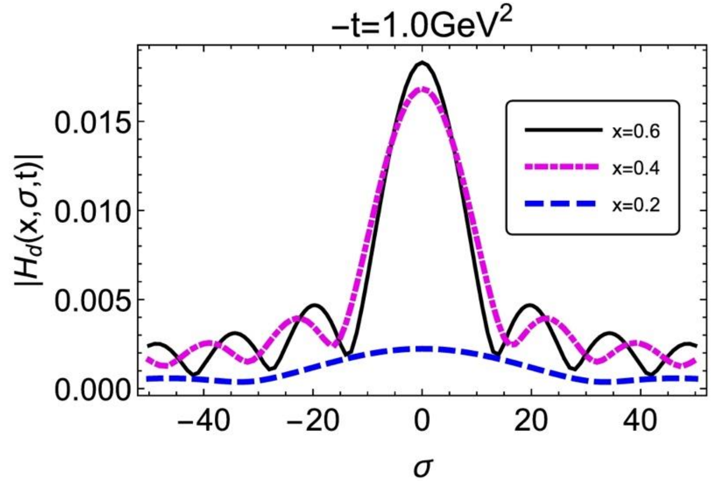


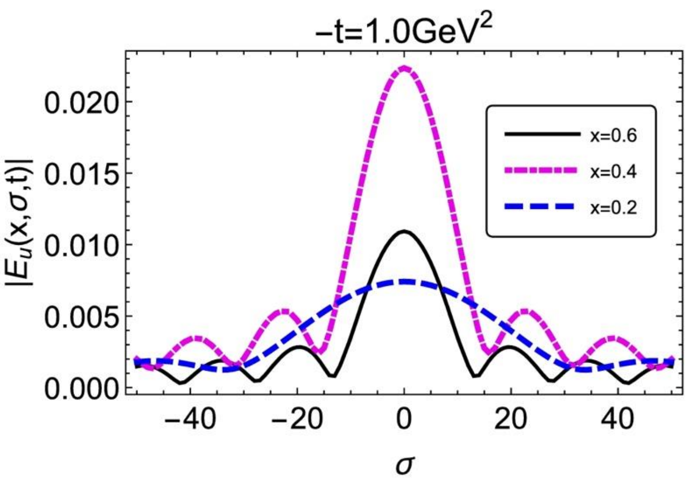
- nonzero skewness GPDs

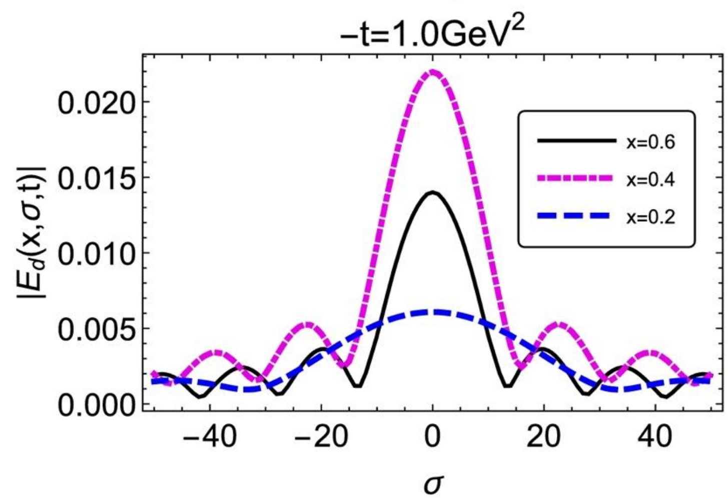
Result without Dynamical Gluon

$$f(x,\sigma,t) = \int_0^{\xi_f} \frac{d\xi}{2\pi} e^{i\xi P^+ b^-/2} F(x,\xi,t) = \int_0^{\xi_f} \frac{d\xi}{2\pi} e^{i\xi\sigma} F(x,\xi,t),$$









- GPDs in longitudinal coordinate space
- We can get differaction pattern as expected

Light-Front Hamiltonian (qqq, qqqg)

$$P^{-} = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_{i} \frac{p_i^2 + m_q^2}{p_i^+}$$ [S. Xu et al, PRD 104 094036(2021)]

$$H_{trans} \sim \kappa_T^4 r^2$$ [S. J. Brodsky, G. de Teramond arXiv: 1203.4025]

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$ [Y. Li, X. Zhao , P Maris , J. P. Vary, PLB 758(2016)]

$$H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j) \quad (|qqq\rangle)$$

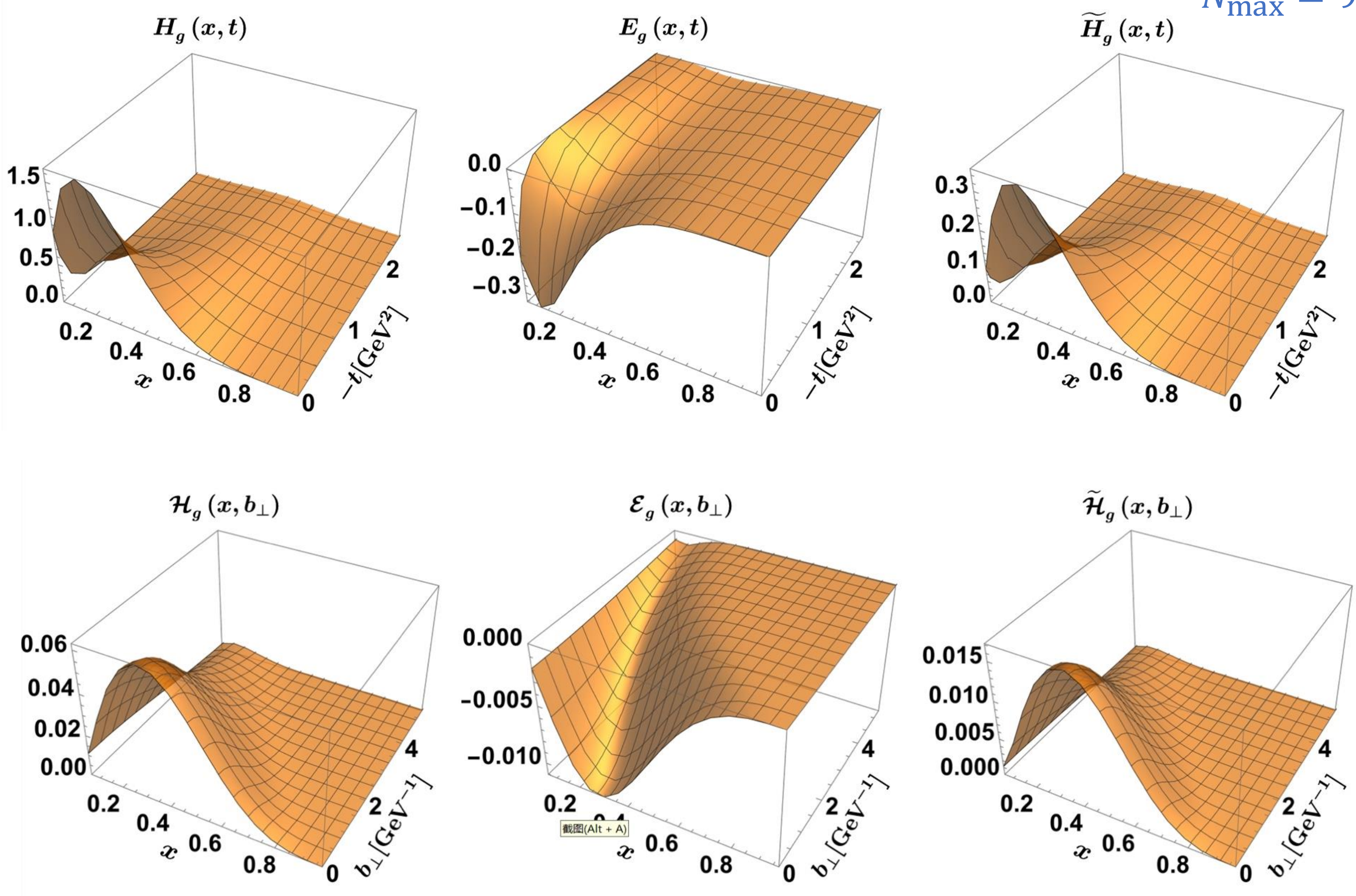
$$H_{Interact} = H_{Vertex} + H_{inst}$$

$$= g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+ \frac{1}{(i\partial^+)^2} j^+$$ $(|qqq\rangle + |qqqg\rangle)$

Result with Dynamical Gluon

$\xi = 0$

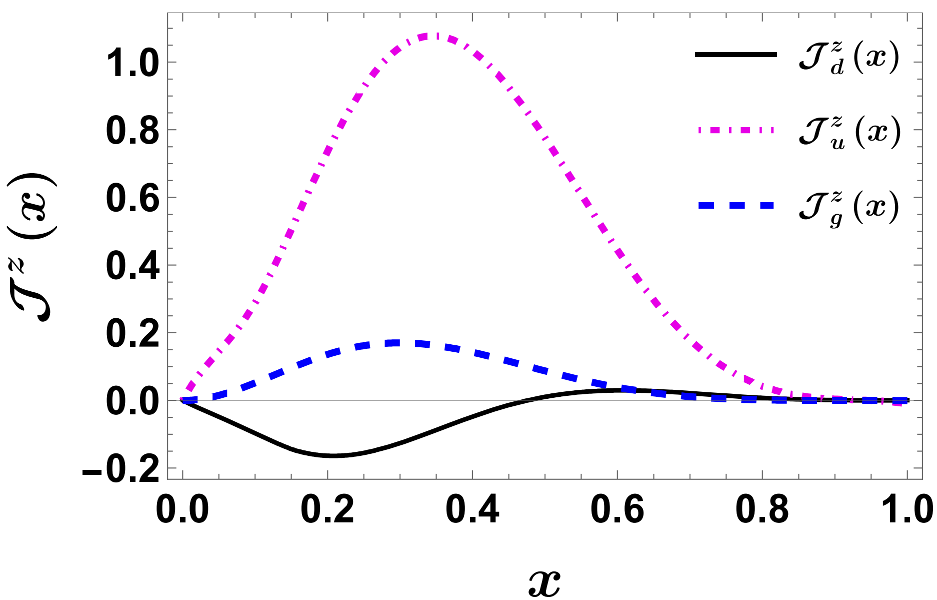
$N_{\max} = 9 \;\; K_{\max} = 16$



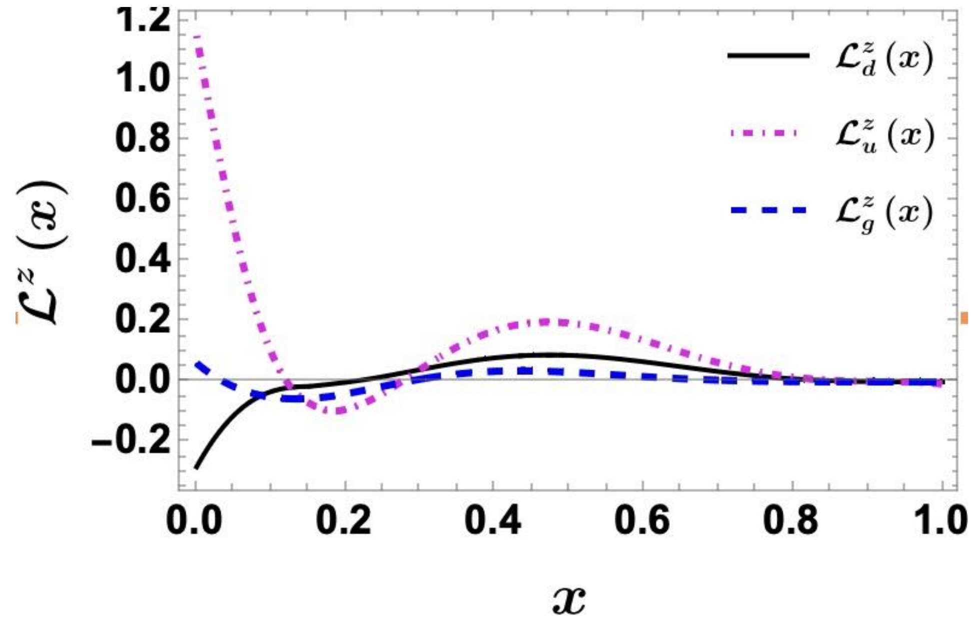
Result with Dynamical Gluon

$$\mathcal{J}^z(x) = \frac{1}{2}x\,[H(x,0,0) + E(x,0,0)]$$

-- B. Lin et. al., Phys. Lett. B 847 (2023) 138305



Total angular momentum for different parton



Orbital angular momentum for different parton

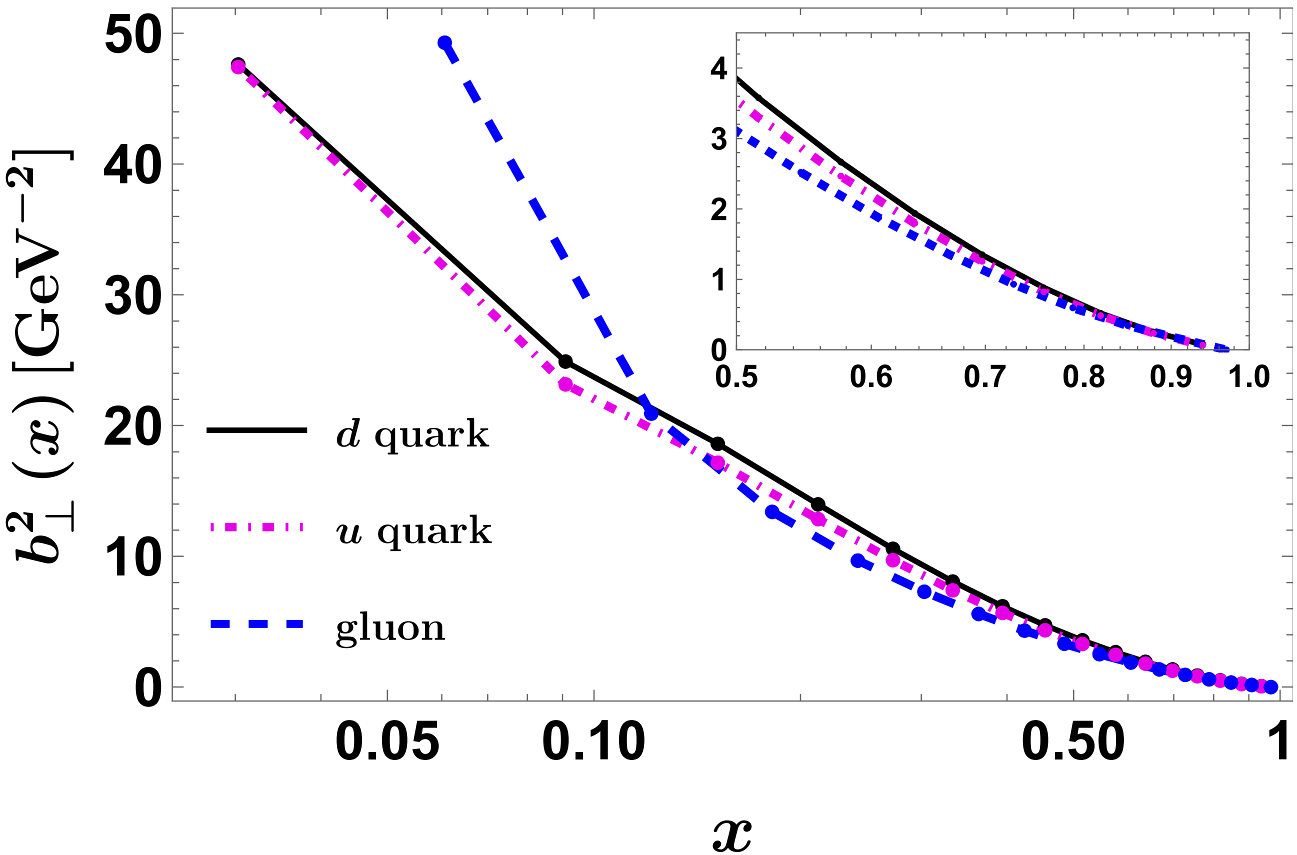
J_d	J_u	J_g	L_d	L_u	L_g
-7.7%	94.5%	13.2%	2.9%	22.0%	-12.6%

- 12.3% orbital angular momentum, 87.7% quark and gluon spin

Result with Dynamical Gluon

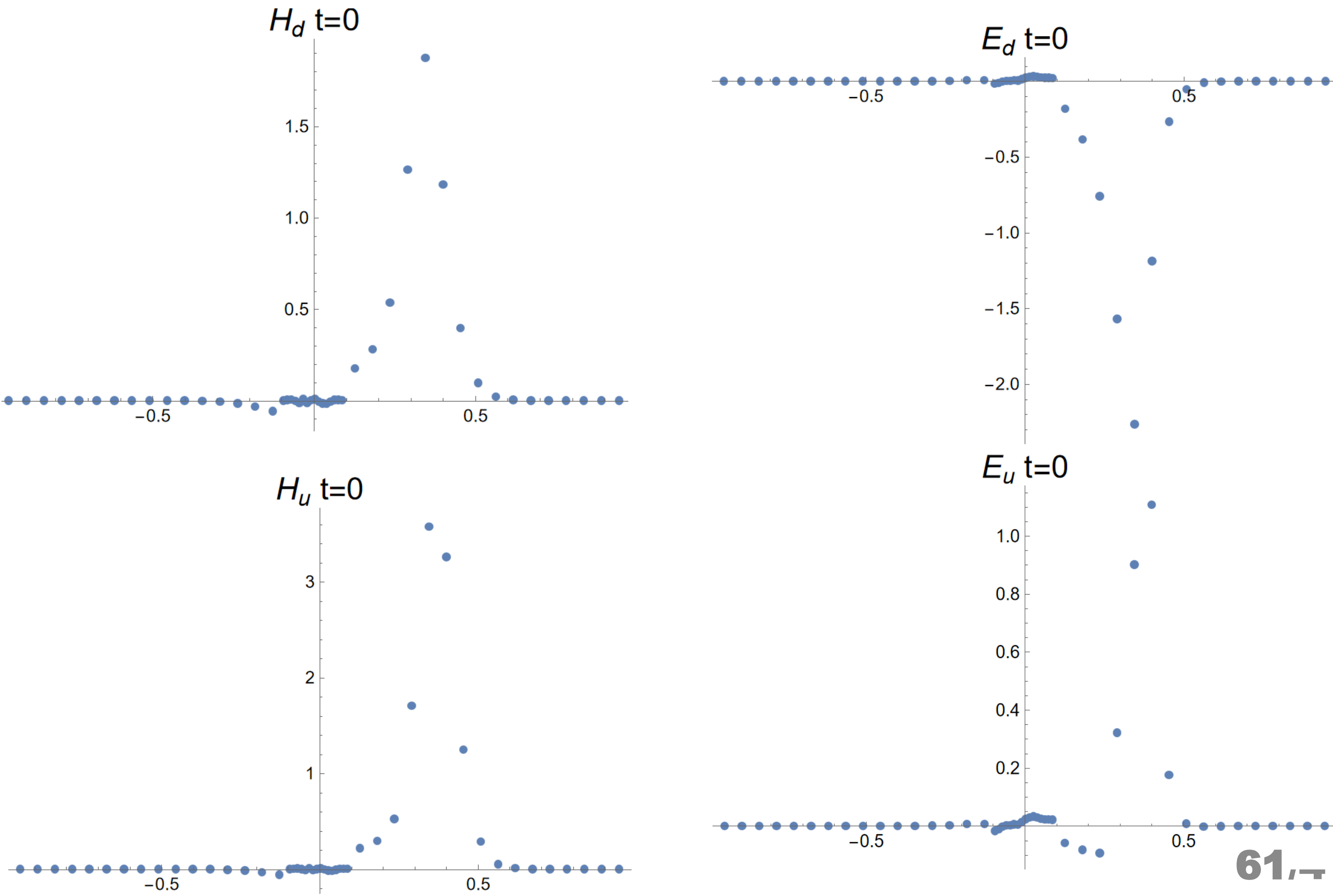
$$\langle b_{\perp}^2 \rangle^{q/g}(x) = \frac{\int d^2 b_{\perp} (b_{\perp})^2 \mathcal{H}(x, 0, b_{\perp})}{\int d^2 b_{\perp} \mathcal{H}(x, 0, b_{\perp})}$$

-- [1] R. Dupre et. al., Phys. Rev. D 95 (1) (2017) 011501



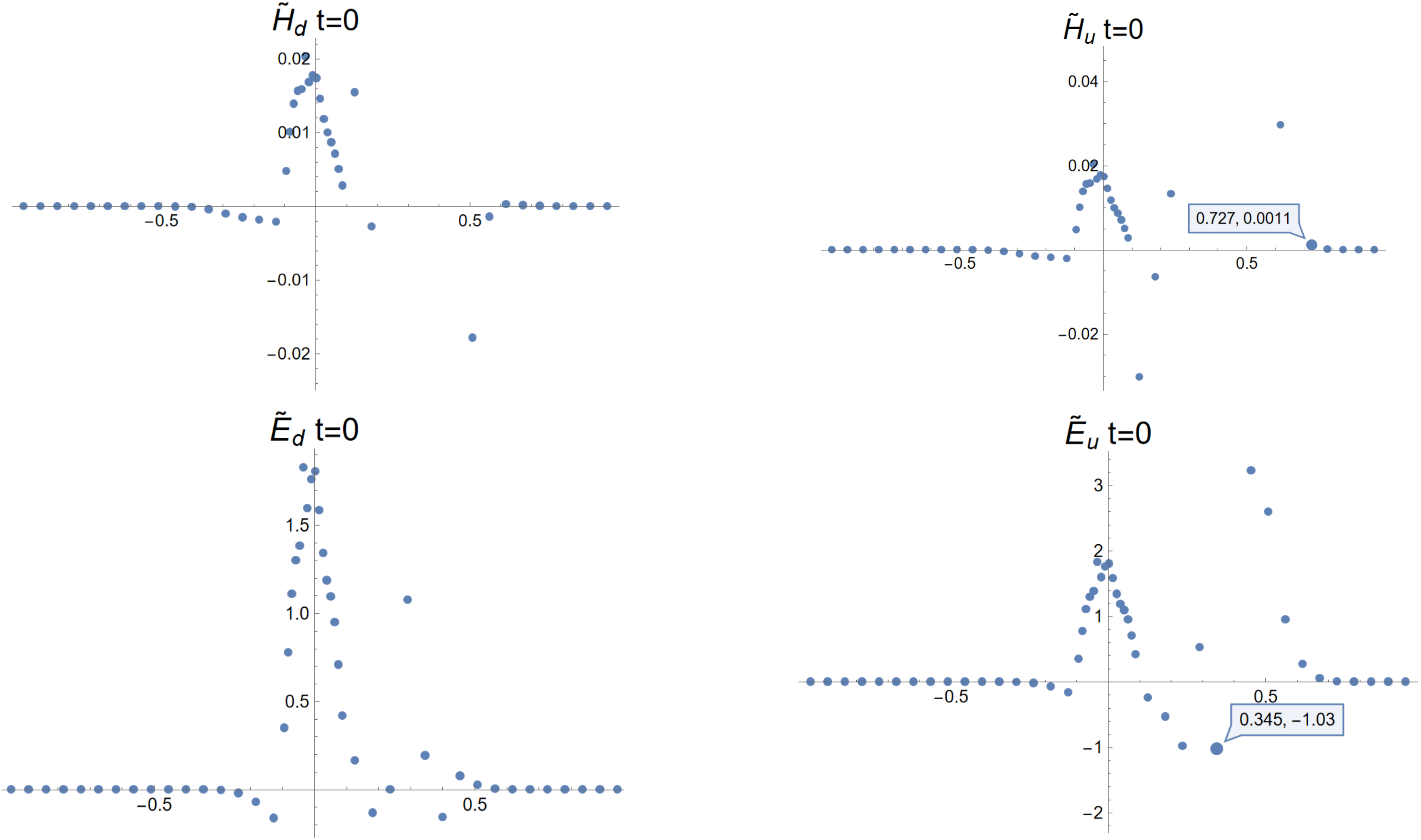
- At x>0.1, quark radius>gluon radius
- At 0.05<x<0.1, gluon radius > quark radius
- As $x \to 1$, nucleon behaves like point particle

2D GPDs at t=0



H_d t=0
1.5
1.0
0.5
−0.5
0.5
E_d t=0
−0.5
0.5
−0.5
−1.0
−1.5
−2.0
H_u t=0
3
2
1
−0.5
0.5
E_u t=0
1.0
0.8
0.6
0.4
0.2
−0.5
0.5

2D GPDs at t=0


$\tilde{H}_d$ t=0
0.02
0.01
−0.01
−0.02
−0.5
0.5
$\tilde{H}_u$ t=0
0.04
0.02
−0.02
−0.5
0.5
0.727, 0.0011
$\tilde{E}_d$ t=0
1.5
1.0
0.5
−0.5
0.5
$\tilde{E}_u$ t=0
3
2
1
−1
−2
−0.5
0.5
0.345, −1.03

Twist-3 generalized parton distribution

The twist-3 GPDs are parameterized by taking different Dirac gamma matrices:

[Stephan.M 2009]

$$F_{\Lambda'\Lambda}^{[1]} = \frac{M}{2(P^+)^2}\bar{u}\left[\gamma^+ H_2 + \frac{i\sigma^{+\Delta}}{2M}E_2\right]u,$$

$$F_{\Lambda'\Lambda}^{[\gamma_5]} = \frac{M}{2(P^+)^2}\bar{u}\left[\gamma^+\gamma_5 \widetilde{H}_2 + \frac{P^+\gamma_5}{M}\tilde{E}_2\right]u,$$

$$F_{\Lambda'\Lambda}^{[\gamma^j]} = \frac{M}{2(P^+)^2}\bar{u}\left[i\sigma^{+j}H_{2T} + \frac{\gamma^+\Delta_T^j - \Delta^+\gamma^j}{2M}E_{2T} + \frac{P^+\Delta_T^j - \Delta^+P_T^j}{M^2}\widetilde{H}_{2T} + \frac{\gamma^+P_T^j - P^+\gamma^j}{M}\tilde{E}_{2T}\right]u,$$

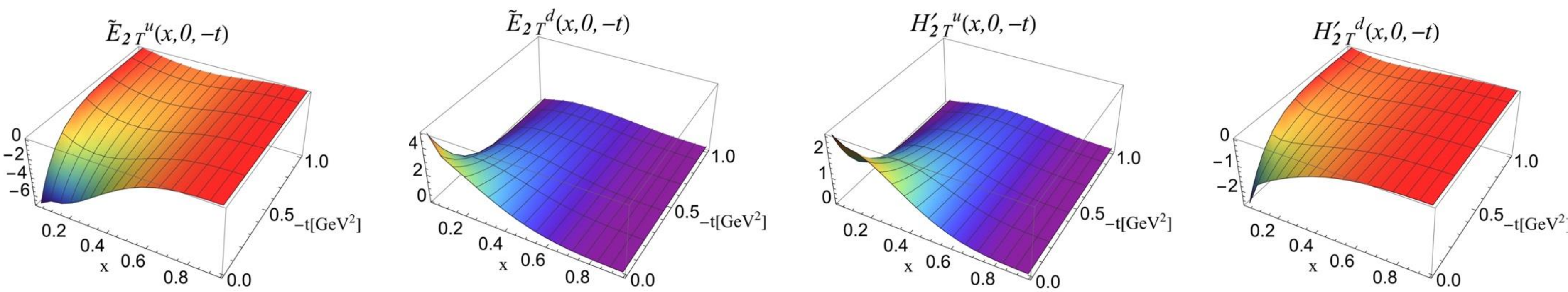
$$F_{\Lambda'\Lambda}^{[\gamma^j\gamma_5]} = \frac{-i\varepsilon_T^{ij}M}{2(P^+)^2}\bar{u}\left[i\sigma^{+i}H'_{2T} + \frac{\gamma^+\Delta_T^i - \Delta^+\gamma^i}{2M}E'_{2T} + \frac{P^+\Delta_T^j - \Delta^+P_T^j}{M^2}\widetilde{H}'_{2T} + \frac{\gamma^+P_T^j - P^+\gamma^j}{M}\tilde{E}'_{2T}\right]u,$$

$$F_{\Lambda'\Lambda}^{[i\sigma^{ij}\gamma_5]} = \frac{-i\varepsilon_T^{ij}M}{2(P^+)^2}\bar{u}\left[\gamma^+ H'_2 + \frac{i\sigma^{+\Delta}}{2M}E'_2\right]u$$

$$F_{\Lambda'\Lambda}^{[i\sigma^{+-}\gamma_5]} = \frac{M}{2(P^+)^2}\bar{u}\left[\gamma^+\gamma_5\widetilde{H}'_2 + \frac{P^+\gamma_5}{M}\tilde{E}'_2\right]u$$

Twist-3 generalized parton distribution

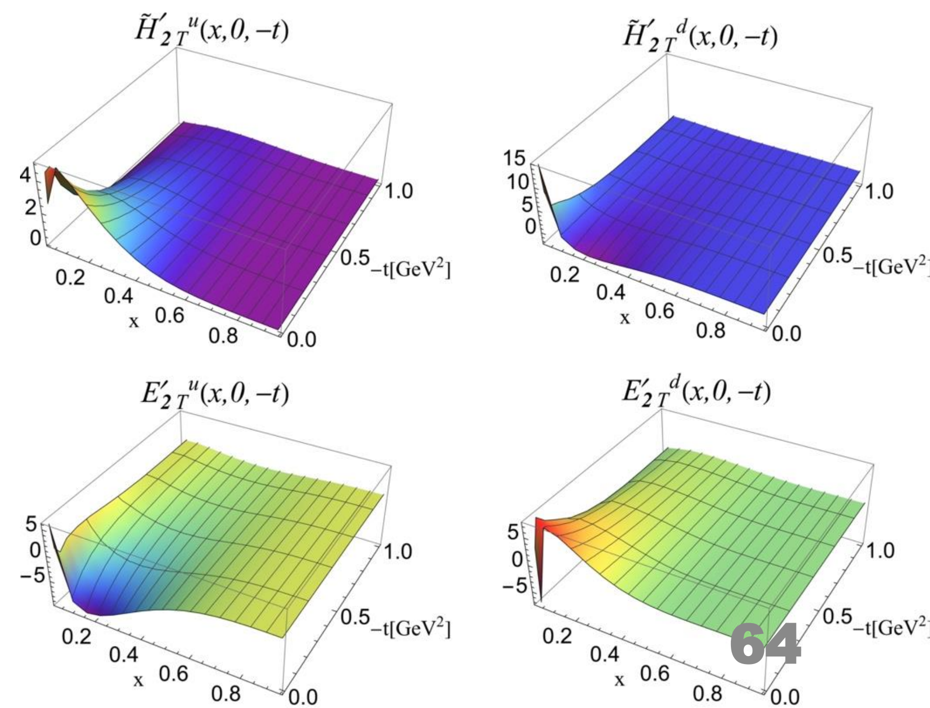
We calculated these twist-3 GPDs based on the LFWFs obtained by applying BLFQ framework, and results are given:



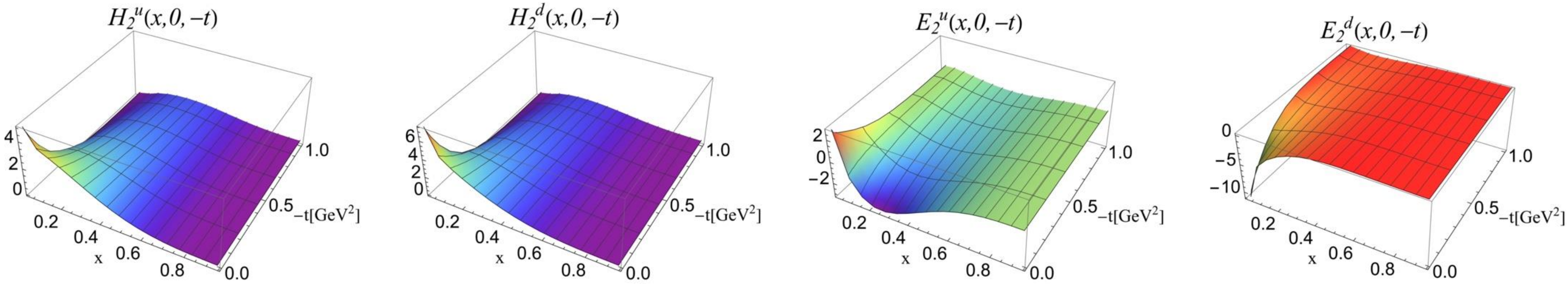
$\Gamma = \gamma^i$ ⟷ $H_{2T}, E_{2T}, \widetilde{H}_{2T}, \widetilde{E}_{2T}$

$\Gamma = \gamma^i\gamma_5$ ⟷ $H'_{2T}, E'_{2T}, \widetilde{H}'_{2T}, \widetilde{E}'_{2T}$

Results at qqq Fock state



Twist-3 generalized parton distribution



$H_2, E_2 \longleftrightarrow \Gamma = 1$

$\widetilde{H}_2, \tilde{E}_2 \longleftrightarrow \Gamma = \gamma_5$

$H'_2, E'_2 \longleftrightarrow \Gamma = i\sigma^{ij}\gamma_5$

$\widetilde{H}'_2, \tilde{E}'_2 \longleftrightarrow \Gamma = i\sigma^{+-}\gamma_5$