Understanding the nature of the $\Delta(1600)$

Liam Hockley, **Curtis Abell**, Derek Leinweber, Anthony Thomas

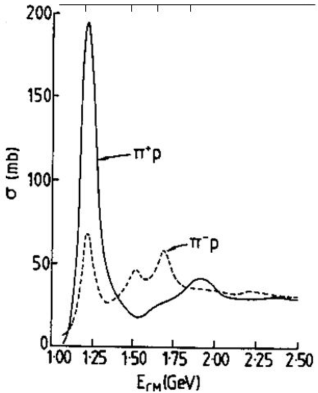
THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Baryon Resonances

- Manifest as "peaks" in the (πN) scattering cross-section
- Central position of the peak given as the mass, width inversely proportion to lifetime
- At first glance, correspond with a phase shift of 90^o
- Generally, describe resonances as poles in the scattering matrix

$$E_{\text{pole}} = M - \frac{\Gamma}{2} i$$

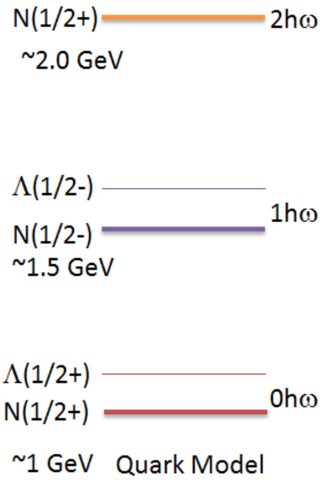


Baryon Resonances

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	****	Σ^0	$1/2^+$	****
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1580)$	$3/2^-$	*
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	***	$\Sigma(1620)$	$1/2^-$	*
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1660)$	$1/2^+$	***
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1670)$	$3/2^-$	****
$N(1700)$	$3/2^-$	***	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1750)$	$1/2^-$	***
$N(1710)$	$1/2^+$	****	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1775)$	$5/2^-$	****
$N(1720)$	$3/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1780)$	$3/2^+$	*
$N(1860)$	$5/2^+$	**	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1880)$	$1/2^+$	**
$N(1875)$	$3/2^-$	***	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1900)$	$1/2^-$	**
$N(1880)$	$1/2^+$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1910)$	$3/2^-$	***
$N(1895)$	$1/2^-$	****	$\Delta(2200)$	$7/2^-$	***	$\Sigma(1915)$	$5/2^+$	****
$N(1900)$	$3/2^+$	****	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1940)$	$3/2^+$	*
$N(1990)$	$7/2^+$	**	$\Delta(2350)$	$5/2^-$	*	$\Sigma(2010)$	$3/2^-$	*
$N(2000)$	$5/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(2030)$	$7/2^+$	****
$N(2040)$	$3/2^+$	*	$\Delta(2400)$	$9/2^-$	**	$\Sigma(2070)$	$5/2^+$	*
$N(2060)$	$5/2^-$	***	$\Delta(2420)$	$11/2^+$	****	$\Sigma(2080)$	$3/2^+$	*
$N(2100)$	$1/2^+$	***	$\Delta(2750)$	$13/2^-$	**	$\Sigma(2100)$	$7/2^-$	*
$N(2120)$	$3/2^-$	***	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2160)$	$1/2^-$	*

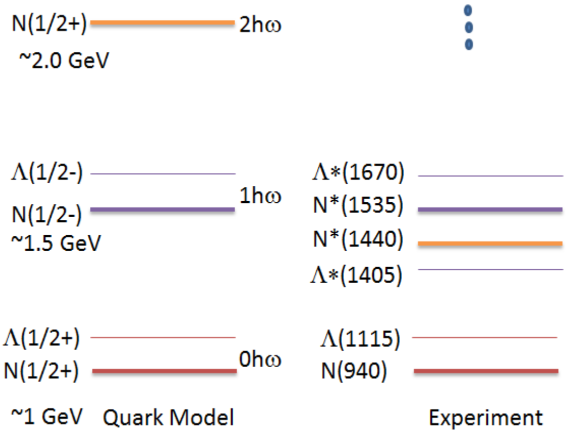
Baryon Resonances

- Quark model does a great job describing many of these states
- However, expect the spectrum to alternate in the signs of their internal parities as one climbs in energy



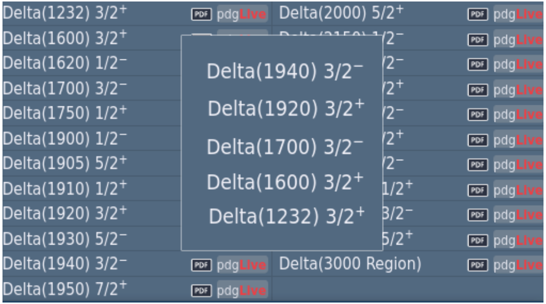
Baryon Resonances

- First positive parity nucleon excitation, $N^*(1440)$ lies below the first negative parity excitations
- Smaller than expected mass separation for $N^*(1535)$ and $N^*(1650)$
- $\Lambda^*(1405)$ sits lower than expected, two-pole structure



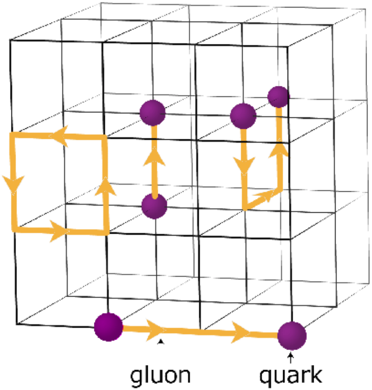
$\Delta\,\frac{3}{2}$ Spectrum

- Similar issues arise in the Δ spectrum
- Consider positive parity spectrum:
  - **** resonances: $\Delta(1232)$, $\Delta(1600)$
  - *** resonance: $\Delta(1920)$
- Infinite volume: search for poles in P_{33} scattering data
- Finite volume: lattice QCD



Lattice QCD

- Discretise space-time into a 4D lattice with periodic boundary conditions, of volume $L^3 \times L_{\text{time}}$, with finite spacing a
- Working at larger-than-physical pion mass reduces computational requirements
- Rotation to Euclidean space prevents asymptotic states in scattering theory
- Require formalism to bridge scattering and lattice QCD



[1]

[1] 10.1051/epjconf/202024509008

Hamiltonian Effective Field Theory

- Use experimental scattering data to constrain a Hamiltonian, and connect to finite-volume lattice QCD
- Construct two bare basis states: $|\Delta_1\rangle$ with mass m_{Δ_1}, $|\Delta_2\rangle$ with mass m_{Δ_2}
- Three scattering states:
  - p-wave $|\pi N(k)\rangle$ with energy $\omega_{\pi N}(k) = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_N^2}$
  - p-wave $|\pi \Delta_p(k)\rangle$ with energy $\omega_{\pi \Delta}(k) = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_\Delta^2}$
  - f-wave $|\pi \Delta_f(k)\rangle$ with energy $\omega_{\pi \Delta}(k) = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_\Delta^2}$
- Regularise p-wave interactions with dipole form factor, with regulator parameter Λ:

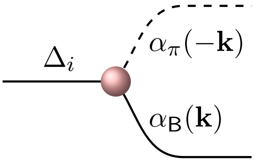
$$u(k) = \frac{1}{\left(1 + (k/\Lambda)^2\right)^2}$$

- Tripole regularises f-wave interactions

Hamiltonian Effective Field Theory

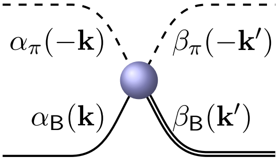
- Parametrise interaction between bare states $|\Delta_i\rangle$ and scattering states $|\alpha(k)\rangle$ with potential $G_\alpha^{\Delta_i}(k)$

$$G_\alpha^{\Delta_i}(k) = \frac{g_\alpha^{\Delta_i}}{2\pi}\left(\frac{k}{f_\pi}\right)^l \sqrt{\omega_{\alpha_\pi}(k)}\, u(k)$$



- Parametrise interaction between scattering states $|\alpha(k)\rangle$ and $|\beta(k')\rangle$ with potential $V_{\alpha\beta}(k,k')$

$$V_{\alpha\beta}(k,k') = \frac{v_{\alpha\beta}}{4\pi^2}\left(\frac{k}{f_\pi}\right)^l \left(\frac{k'}{f_\pi}\right)^{l'} \frac{\tilde{u}(k)\,\tilde{u}(k')}{\omega_\pi(k)\,\omega_\pi(k')}$$



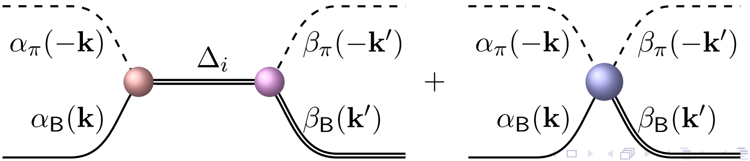
Infinite-Volume Scattering

- Formulate the coupled-channel scattering equations to solve for scattering observables

$$T_{\alpha\beta}(k,k';E) = \tilde{V}_{\alpha\beta}(k,k';E) + \sum_{\gamma} \int dq\, q^2 \frac{\tilde{V}_{\alpha\gamma}(k,q;E)\, T_{\gamma\beta}(q,k';E)}{E - \omega_\gamma(q) + i\varepsilon} \quad (1)$$

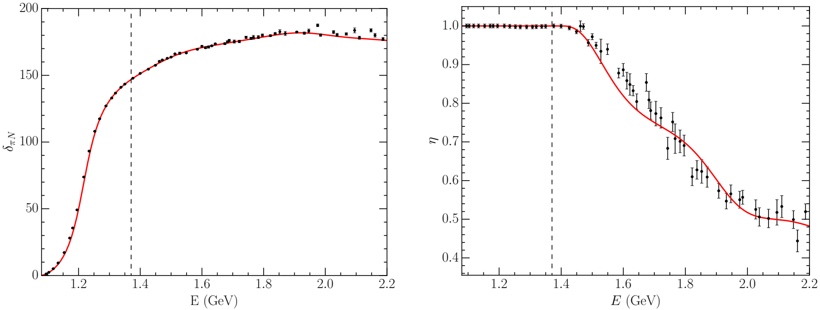
- α, β and γ run over scattering states: $|\pi N\rangle$, $|\pi\Delta_p\rangle$, $|\pi\Delta_f\rangle$

$$\tilde{V}_{\alpha\beta}(k,k';E) = \sum_{i=1}^{2} \frac{G_\alpha^{\Delta_i}(k)\, G_\beta^{\Delta_i}(k')}{E - m_{\Delta_i}} + V_{\alpha\beta}(k,k') \quad (2)$$



Fitting to Scattering Data

- From T-matrix, compare phase shifts and inelasticities with experimental data.
- Vary bare masses, coupling strengths, regulator strengths to describe this data
- Find $m_{\Delta_1} = 1.389$ GeV, $m_{\Delta_2} = 2.318$ GeV



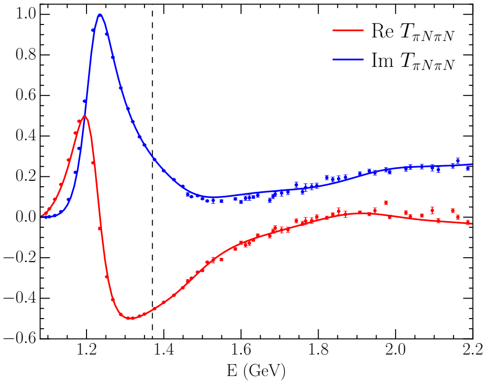
T-Matrix Poles

- PDG pole positions:

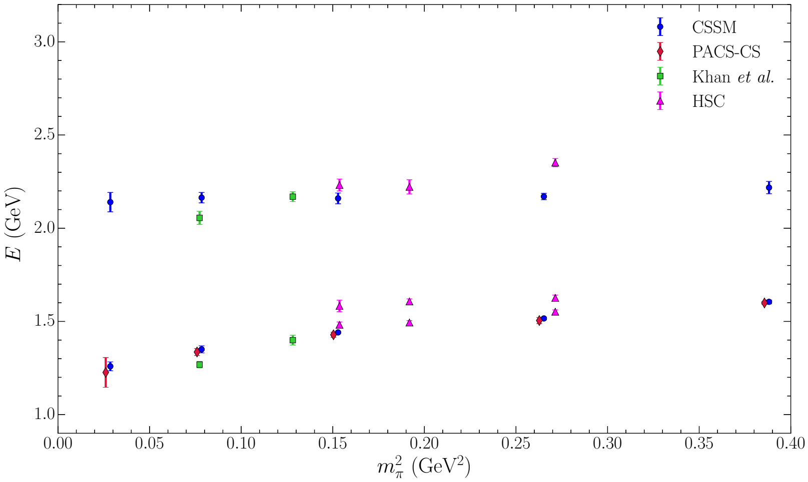
$$E_{\Delta(1232)} = (1.210 \pm 0.001) - (0.050 \pm 0.001)i\,,$$
$$E_{\Delta(1600)} = (1.52^{+0.07}_{-0.05}) - (0.14^{+0.02}_{-0.07})i\,.$$
$$E_{\Delta(1920)} = (1.90 \pm 0.05) - (0.15 \pm 0.10)i$$

- HEFT pole positions:

$$E_1 = 1.211 - 0.049i\,,$$
$$E_2 = 1.46 - 0.16i\,.$$
$$E_3 = 1.93 - 0.20i\,.$$



$3/2^+$ Spectrum in Lattice QCD



Finite-Volume HEFT

- Formulate HEFT in a finite-volume L^3 to compare with lattice QCD data
- Momentum is discretised

$$k_n = \frac{2\pi n}{L}\,, \quad n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,, \quad n_i \in \mathbb{Z}^+$$

- Finite-range regulator $u(k) = \left(1 + (k/\Lambda)^2\right)^{-2}$ removes ultraviolet contributions
- Maximum momentum in the Hamiltonian matrix, k_{max}, set for $u^2(k_{\text{max}}) \sim 10^{-4}$
- Increase hadron masses with m_π^2, fit bare mass slopes to lattice QCD data

$$m_{\Delta_i}(m_\pi^2) = m_{\Delta_i}\big|_{\text{phys}} + \alpha_{\Delta_i}\left(m_\pi^2 - m_\pi^2\big|_{\text{phys}}\right)$$

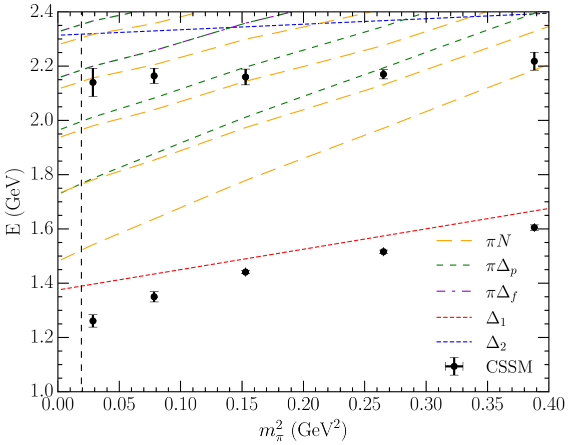
Finite-Volume Free Hamiltonian

- Non-interacting basis states form the free Hamiltonian

$$H_0 = \begin{pmatrix} m_{\Delta_1} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & m_{\Delta_2} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \omega_{\pi N}(k_1) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \omega_{\pi \Delta_p}(k_1) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \omega_{\pi \Delta_f}(k_1) & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{\pi N}(k_2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \omega_{\pi \Delta_f}(k_n) \end{pmatrix}$$

- $\omega_{\pi B}(k) = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_B^2}$

$L \sim 3$ fm: Non-interacting energy levels



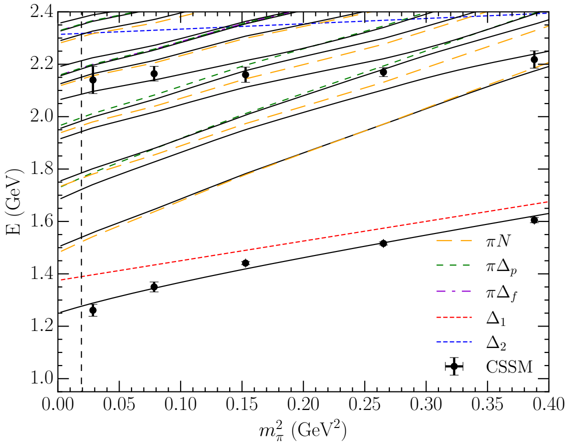
Finite-Volume Interaction Hamiltonian

$$
\begin{array}{c}
\qquad\qquad |\Delta_1\rangle \qquad\quad |\Delta_2\rangle \qquad\quad |\pi N(k_1)\rangle \qquad\quad |\pi\Delta_p(k_1)\rangle \qquad\quad |\pi\Delta_f(k_1)\rangle \qquad\quad |\pi N(k_2)\rangle
\end{array}
$$

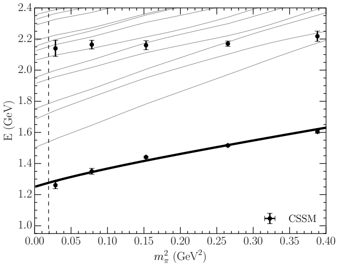
$$
H_I = \begin{pmatrix}
0 & 0 & \bar{G}^{\Delta_1}_{\pi N}(k_1) & \bar{G}^{\Delta_1}_{\pi\Delta_p}(k_1) & \bar{G}^{\Delta_1}_{\pi\Delta_f}(k_1) & \bar{G}^{\Delta_1}_{\pi N}(k_2) & \cdots \\
0 & 0 & \bar{G}^{\Delta_2}_{\pi N}(k_1) & \bar{G}^{\Delta_2}_{\pi\Delta_p}(k_1) & \bar{G}^{\Delta_2}_{\pi\Delta_f}(k_1) & \bar{G}^{\Delta_2}_{\pi N}(k_2) & \cdots \\
\bar{G}^{\Delta_1}_{\pi N}(k_1) & \bar{G}^{\Delta_2}_{\pi N}(k_1) & \bar{V}_{\pi N\pi N}(k_1,k_1) & \bar{V}_{\pi N\pi\Delta_p}(k_1,k_1) & \bar{V}_{\pi N\pi\Delta_f}(k_1,k_1) & \bar{V}_{\pi N\pi N}(k_1,k_2) & \cdots \\
\bar{G}^{\Delta_1}_{\pi\Delta_p}(k_1) & \bar{G}^{\Delta_1}_{\pi\Delta_p}(k_1) & \bar{V}_{\pi\Delta_p\pi N}(k_1,k_1) & \bar{V}_{\pi\Delta_p\pi\Delta_p}(k_1,k_1) & \bar{V}_{\pi\Delta_p\pi\Delta_f}(k_1,k_1) & \bar{V}_{\pi\Delta_p\pi N}(k_1,k_2) & \cdots \\
\bar{G}^{\Delta_1}_{\pi\Delta_f}(k_1) & \bar{G}^{\Delta_1}_{\pi\Delta_f}(k_1) & \bar{V}_{\pi\Delta_f\pi N}(k_1,k_1) & \bar{V}_{\pi\Delta_f\pi\Delta_p}(k_1,k_1) & \bar{V}_{\pi\Delta_f\pi\Delta_f}(k_1,k_1) & \bar{V}_{\pi\Delta_f\pi N}(k_1,k_2) & \cdots \\
\bar{G}^{\Delta_1}_{\pi N}(k_2) & \bar{G}^{\Delta_2}_{\pi N}(k_2) & \bar{V}_{\pi N\pi N}(k_2,k_1) & \bar{V}_{\pi\Delta_p\pi N}(k_2,k_1) & \bar{V}_{\pi\Delta_f\pi N}(k_2,k_1) & \bar{V}_{\pi N\pi N}(k_2,k_2) & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

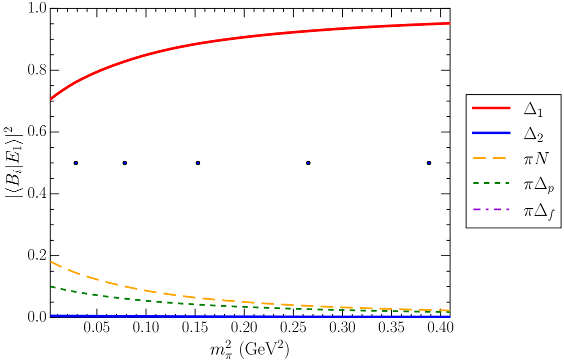
- Solve the eigenvalue equation for $H = H_0 + H_I$

$L \sim 3$ fm: Interacting Energy Levels

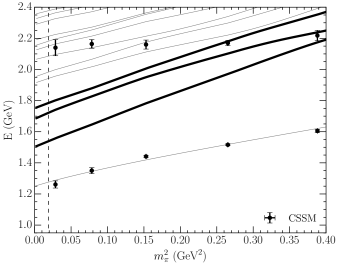


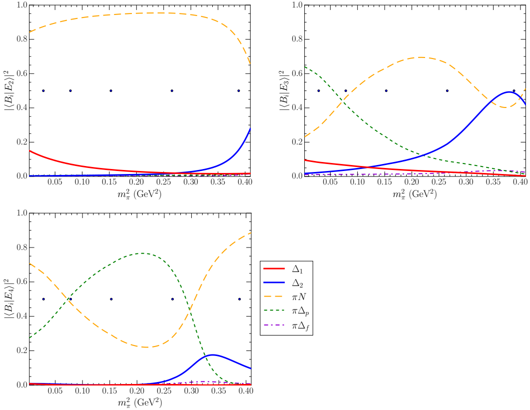
$L \sim 3$ fm Eigenvectors: E_1



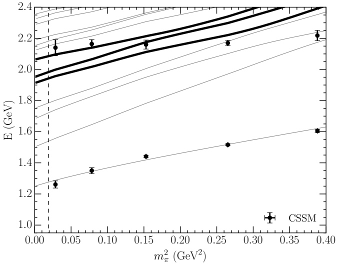


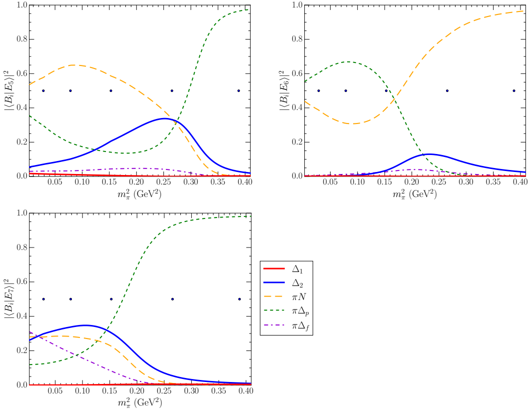
$L \sim 3$ fm Eigenvectors: E_2, E_3, E_4



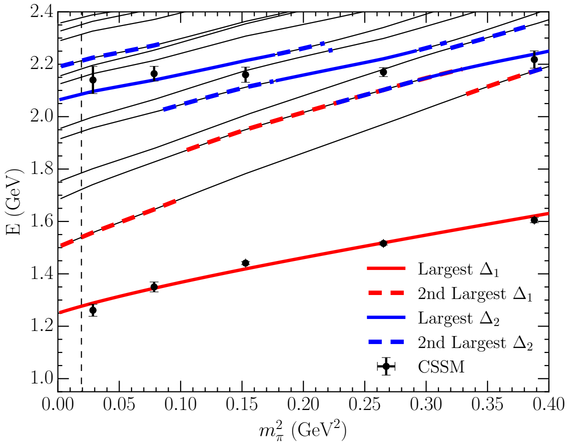


$L \sim 3$ fm Eigenvectors: E_5, E_6, E_7, E_8





$L \sim 3$ fm: Bare State-Dominated Energy Levels



Conclusion

- Identity of resonances resolved by bridging experiment and lattice QCD
- Able to fit to experimental results using two bare-basis states in HEFT, describing scattering data and PDG poles
- Large second bare state hints at dynamically generated $\Delta(1600)$
- Constructing a finite-volume Hamiltonian, able to describe 3 fm lattice QCD data
- Eigenvector structure further supports dynamically generated $\Delta(1600)$
- Interpretation of $\Delta(1600)$ as dynamically generated through rescattering is consistent with both experiment and lattice QCD
- Insight gained into structure of the $\Delta(1920)$