Isospin violation effects in the pion–nucleon sigma-term

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MH, Ruiz de Elvira, Kubis, Meißner PLB 843 (2023) 138001, PRL 115 (2015) 092301

Gupta, Park, MH, Mereghetti, Yoon, Bhattacharya PRL 127 (2021) 24

Scalar currents and searches for physics beyond the SM

- No scalar currents in Standard Model, but many BSM scenarios involve **scalar quark-level operators**
- Standard example from dark matter: Majorana WIMP χ

$$\mathcal{L} = \sum_q C_q^{SS} \bar{\chi}\chi \, m_q \bar{q} q$$

  but also $\mu \to e$ conversion in nuclei, EDMs, nuclear matter, ...

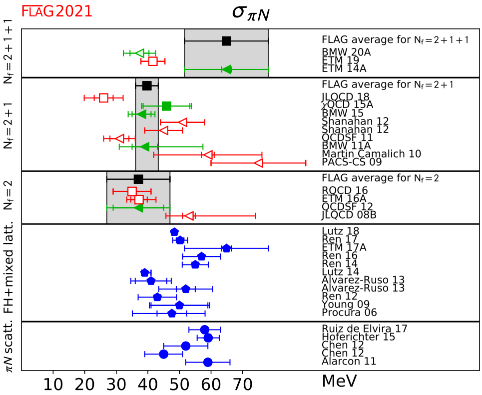
- Need nucleon matrix elements

$$\langle N | m_q \bar{q} q | N \rangle = f_q^N m_N$$

  for cross section off nuclei ("direct detection")

- **Pion–nucleon σ-term $\sigma_{\pi N}$**
  - Isoscalar–scalar coupling: $\sigma_{\pi N} = m_N (f_u^N + f_d^N)$
  - Related to pion–nucleon scattering
  - **Rare opportunity to check lattice BSM nucleon matrix elements against experiment**

Status of the σ term around 2021



- Phenomenology: $\sigma_{\pi N} \approx 60\,\text{MeV}$, lattice: $\sigma_{\pi N} \approx 40\,\text{MeV}$
- Cross check failed, isospin violation a possible explanation due to $\sigma_{\pi N} \propto M_\pi^2$?
  - Isospin breaking corrections to Cheng–Dashen low-energy theorem
  - Definition of isospin limit

Extracting $\sigma_{\pi N}$ from πN scattering: low-energy theorem

- No scalar probe, but still relation to experiment! How?

  $\hookrightarrow$ **low-energy theorem**

- Goes back to Cheng, Dashen; Brown, Pardee, Peccei 1971

- Relates $\sigma_{\pi N}$ to πN scattering amplitude, but at **unphysical kinematics**

  $\hookrightarrow$ analytic continuation to the Cheng–Dashen point

- **No chiral logs** at one-loop order! Bernard, Kaiser, Meißner 1996

- **Protected by *SU*(2)**

  $\hookrightarrow$ expected correction: $\sigma_{\pi N} M_\pi^2 / m_N^2 \sim 1\,\text{MeV}$

How to define the isospin limit?

- Typical ChPT convention for **isospin limit**

  $\hookrightarrow$ **charged particle masses** M_{π^+}, M_{K^+}, m_p, ...
- Why? Most data available for charged particles
- Examples:
  - $\pi\pi$: $\pi^+\pi^-$ atoms, $\pi N \to \pi\pi N$ data
  - πN: $\pi^- p$, $\pi^- d$ atoms, $\pi N \to \pi N$ data

  $\hookrightarrow$ natural to use charged-particle masses to minimize corrections
- Standard example: **$\pi\pi$ scattering lengths** Colangelo, Gasser, Leutwyler 2001

$$a_0^0 = 0.220(5) \qquad a_0^2 = -0.0444(10)$$

- IB corrections important when comparing to $K \to 3\pi$ and $K_{\ell 4}$ data due to neutral-pion thresholds
- a_0^I vanish in the chiral limit

  $\hookrightarrow$ choice of isospin conventions matters

Isospin violation in $\sigma_{\pi N}$

- **Cheng–Dashen theorem**
  - Correction to scalar form factor (Δ_σ^p) and πN amplitude (Δ_D^p) MH et al. 2015

$$\Delta_\sigma^p = \sigma_p(2M_\pi^2) - \sigma_p = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{128\pi F_\pi^2}\Big(-7 + 2\sqrt{2}\log\big(1+\sqrt{2}\big)\Big) \qquad \Delta_\pi = M_{\pi^\pm}^2 - M_{\pi^0}^2$$

$$\Delta_D^p = F_\pi^2\Big\{D_p(0, 2M_\pi^2) - d_{00}^p - 2M_\pi^2 d_{01}^p\Big\} = \frac{23g_A^2 M_\pi^3}{384\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}\Big(3 + 4\sqrt{2}\log\big(1+\sqrt{2}\big)\Big)$$

  ↪ both defined with respect to the charged pion mass

  - Results in an upwards shift of $\frac{81 g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2} \simeq 3.4\,\text{MeV}$, "enhanced" by 4π

- **Isospin limit definition**
  - Lattice QCD prefers $\bar{\sigma}_{\pi N}$ defined at mass of the neutral pion

  ↪ $\Delta_\pi / M_\pi^2 \simeq 6\%$ correction!

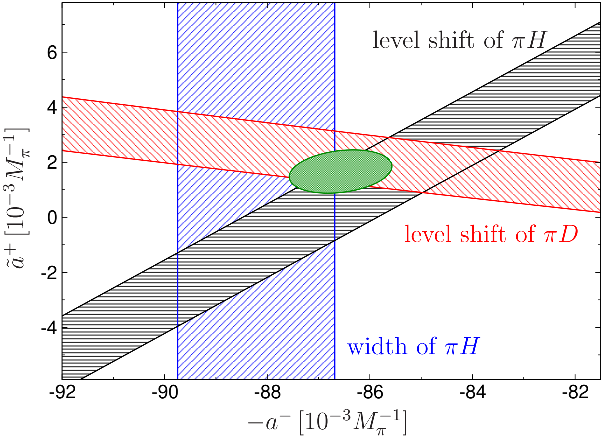
  - More careful analysis in ChPT MH et al. 2023

$$\Delta\sigma_{\pi N} \equiv \sigma_{\pi N} - \bar{\sigma}_{\pi N} = 3.1(5)\,\text{MeV}$$

  ↪ not negligible at current level of precision

- Both effects happen to almost cancel each other

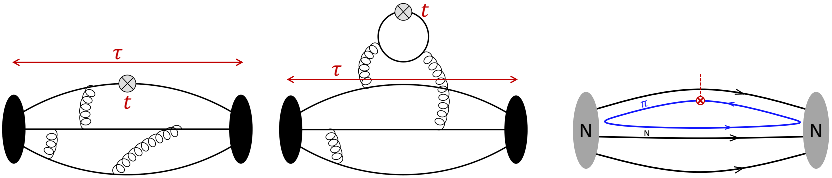
Latest update from pionic atoms



- Final PSI measurement of the width of pionic hydrogen Hirtl et al. 2021
- $\sigma_{\pi N}$ mainly sensitive to isoscalar scattering length a^+

  $\hookrightarrow$ $\sigma_{\pi N} = 59.0(3.5)$ MeV largely unchanged

The σ-term on the lattice



1. **Feynman–Hellmann method**
   - Derivative of the nucleon mass:

$$\langle N|m_q\bar{q}q|N\rangle = m_q\frac{\partial m_N}{\partial m_q}$$

   - Need very precise data of m_N near the physical point
2. **Direct method**
   - Calculate the three-point function
   - Noisier signal, control over **excited-state contamination** (ESC)

Two different fit strategies

$$C^{2\text{pt}}(\tau;\mathbf{k}) = \sum_{i=0}^{3} |\mathcal{A}_i(\mathbf{k})|^2 e^{-M_i\tau}$$

$$C_{\mathcal{S}}^{3\text{pt}}(\tau;t) = \sum_{i,j=0}^{2} \mathcal{A}_i\mathcal{A}_j^* \langle i|\mathcal{S}|j\rangle e^{-M_i t - M_j(\tau-t)}$$

1. "**Standard fit**" ($\{4, 3^*\}$)
   - Excited states determined from combined fit to $C^{2\text{pt}}$ to $C_{\mathcal{S}}^{3\text{pt}}$
   - Two excited states M_1, M_2 in $C_{\mathcal{S}}^{3\text{pt}}$, $\langle 2|\mathcal{S}|2\rangle$ not resolved
   - Flat priors on M_i
2. "**Excited-state fit**" ($\{4^{N\pi}, 3^*\}$)
   - Narrow-width prior for M_1 centered about the noninteracting energy of the lowest positive parity states $N(\mathbf{1})\pi(-\mathbf{1})$ or $N(\mathbf{0})\pi(\mathbf{0})\pi(\mathbf{0})$

$\hookrightarrow$ **for small pion masses**, the two strategies yield very different results, but lattice data not precise enough to decide see plenary talk by Rajan Gupta, Fr., 15:30

Role of excited states in ChPT

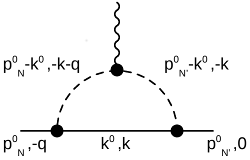
- Can estimate ESC in ChPT Tiburzi 2015, Bär 2015, ...
- Strategy
  - $N\pi$ and $\pi\pi$ cuts in chiral loops correspond to $N(\mathbf{k})\pi(-\mathbf{k})$ or $N(\mathbf{0})\pi(\mathbf{k})\pi(-\mathbf{k})$ excited states on the lattice
  - Wick-rotate Minkowski loop integrals to Euclidean space
  - Collect residues
- Amounts to a chiral expansion of the ratio $\mathcal{R}_\mathcal{S}(\tau,t) = C^{\text{3pt}}_\mathcal{S}(\tau;t)/C^{\text{2pt}}(\tau)$

$$\mathcal{R}_\mathcal{S}(\tau,t) = \mathcal{R}^{(0)}_\mathcal{S}(\tau,t) + \mathcal{R}^{(1)}_\mathcal{S}(\tau,t) + \mathcal{R}^{(2)}_\mathcal{S}(\tau,t)$$

with $\mathcal{R}^{(0)}_\mathcal{S} = \hat{m}g^{(0)}_\mathcal{S}$ and $\delta\sigma_{\pi N}(\tau,t) = \mathcal{R}_\mathcal{S}(\tau,t) - \lim_{t,\tau\to\infty}\mathcal{R}_\mathcal{S}(\tau,t)$

- Convergence set by $Q \in \{M_\pi, t^{-1}, t_B^{-1}, \tau^{-1}\}$, $t_B = \tau - t$

Leading loop contribution



- **Leading loop** gives

$$\mathcal{R}_{\mathcal{S}}^{(1)}(\tau,t)=\frac{3g_A^2M_\pi^2}{8F_\pi^2L^3}\sum_{\mathbf{k}}\frac{\mathbf{k}^2}{E_\pi^4}\bigg[1-e^{-E_{N\pi}t}-e^{-E_{N\pi}t_B}+\frac{1}{2}e^{-E_{N\pi}\tau}+\frac{1}{4}e^{-2E_\pi t}+\frac{1}{4}e^{-2E_\pi t_B}\bigg]$$

where

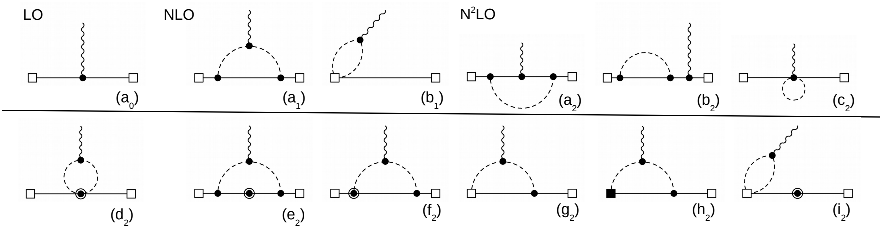
$$E_\pi=\sqrt{\mathbf{k}^2+M_\pi^2},\quad \widetilde{E}_N=\sqrt{m_N^2+\mathbf{k}^2}-m_N,\quad E_{N\pi}=E_\pi+\tilde{E}_N,\quad \mathbf{k}=\frac{2\pi\mathbf{n}}{L}$$

- First term reproduces **continuum result**

$$\frac{3g_A^2M_\pi^2}{8F_\pi^2}\frac{1}{L^3}\sum_{\mathbf{k}}\frac{\mathbf{k}^2}{E_\pi^4}=-\frac{9g_A^2M_\pi^3}{64\pi F_\pi^2}+\Delta_L\sigma_{\pi N}$$

- Excites-state sums converge at scales $\{t^{-1},t_B^{-1},\tau^{-1}\}$

Full N^{2}LO analysis

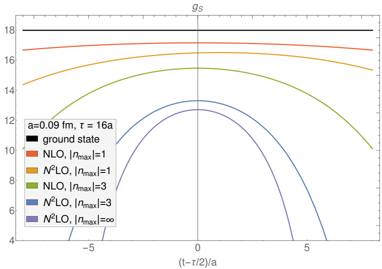


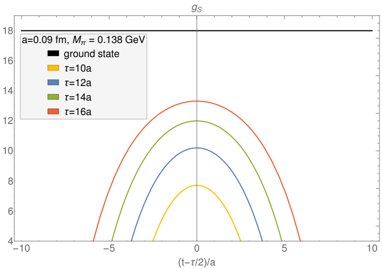
$$\mathcal{R}_{\mathcal{S}}^{(1)}(\tau,t) = \ldots - \frac{3M_\pi^2}{32F_\pi^2}\frac{1}{L^3}\sum_{\mathbf{k}}\frac{1}{E_\pi^2}\left(e^{-2E_\pi t} + e^{-2E_\pi t_B}\right) \qquad \mathcal{R}_{\mathcal{S}}^{(2)}(\tau,t) = \mathcal{R}^{(2)}_{\text{recoil}} + \mathcal{R}^{(2)}_{c_i}$$

$$\mathcal{R}^{(2)}_{\text{recoil}} = -\frac{9g_A^2M_\pi^2}{32m_NF_\pi^2L^3}\sum_{\mathbf{k}}\frac{(\mathbf{k}^2)^2}{E_\pi^5}\left[1 - \frac{2}{3}e^{-E_\pi t} - \frac{2}{3}e^{-E_\pi t_B} + \frac{2}{3}e^{-E_\pi \tau} - \frac{1}{6}e^{-2E_\pi t} - \frac{1}{6}e^{-2E_\pi t_B}\right]$$
$$+ \frac{3g_A^2M_\pi^2}{32m_NF_\pi^2L^3}\sum_{\mathbf{k}}\frac{1}{E_\pi}\left[1 - \frac{1}{2}e^{-2E_\pi t} - \frac{1}{2}e^{-2E_\pi t_B} + \frac{2\mathbf{k}^2}{E_\pi^2}\left(1 - e^{-E_\pi t} - e^{-E_\pi t_B} + e^{-E_\pi \tau}\right)\right]$$

$$\mathcal{R}^{(2)}_{c_i} = -\frac{3M_\pi^2}{4F_\pi^2}\frac{1}{L^3}\sum_{\mathbf{k}}\frac{1}{E_\pi^3}\left((c_2 + 2c_3)E_\pi^2 + (2c_1 - c_3)M_\pi^2\right)\left[1 - \frac{1}{2}e^{-2E_\pi t} - \frac{1}{2}e^{-2E_\pi t_B}\right]$$
$$+ \frac{3M_\pi^2}{F_\pi^2}\frac{1}{L^3}\sum_{\mathbf{k}}\frac{1}{E_\pi}c_1$$

Full N^{2}LO analysis

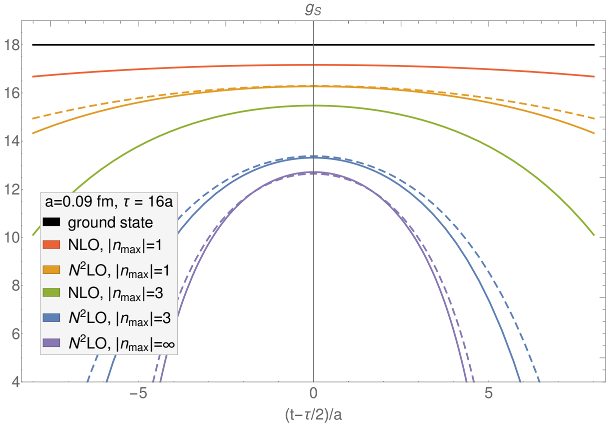




		NLO		N^{2}LO		$\delta\sigma_{\pi N}(16,8)$	NLO		N^{2}LO		$\delta\sigma_{\pi N}(12,6)$
$\lvert\mathbf{n}_{max}\rvert$	$m_\mathbf{n}$	loop	source	c_i	recoil	total	loop	source	c_i	recoil	total
0	1	0	−0.4	−1.0	−0.0	−1.4	0	−0.5	−1.2	−0.1	−1.7
1	6	−2.2	−0.6	−2.9	−0.2	−5.9	−2.7	−0.9	−4.3	−0.2	−8.0
$\sqrt{2}$	12	−4.4	−0.8	−4.6	−0.3	−10.0	−5.6	−1.2	−7.4	−0.3	−14.5
$\sqrt{3}$	8	−5.2	−0.8	−5.2	−0.3	−11.6	−6.7	−1.3	−8.7	−0.4	−17.1
2	6	−5.6	−0.9	−5.5	−0.3	−12.2	−7.3	−1.3	−9.4	−0.4	−18.4
$\sqrt{5}$	24	−6.5	−0.9	−6.2	−0.3	−14.0	−8.9	−1.4	−11.1	−0.5	−21.9
$\sqrt{6}$	24	−7.2	−0.9	−6.7	−0.4	−15.2	−10.1	−1.5	−12.4	−0.5	−24.5
$\sqrt{8}$	12	−7.4	−0.9	−6.8	−0.4	−15.5	−10.4	−1.5	−12.8	−0.5	−25.2
3	30	−7.7	−0.9	−7.0	−0.4	−16.0	−11.1	−1.6	−13.5	−0.5	−26.6
∞		−9.2	−0.9	−7.6	−0.4	−18.0	−14.2	−1.5	−16.3	−0.6	−32.6

- Functional form matches near $t \sim \tau/2$
- Applicability of ChPT?
- Largest $\{t,\tau\}$ suggest 10 MeV each from NLO and N^{2}LO loops

Impact of the Δ(1232)



- Dashed lines N^2LO + leading Δ(1232) effect

  ↪ chiral convergence looks very stable

Lattice calculation by Mainz group

Variation	$\sigma_{\pi N}$ [MeV]
$M_\pi < 220$ MeV	42.04(1.27)
$M_\pi < 285$ MeV	41.89(67)
no cut in M_π	41.67(44)
$M_\pi < 220$ MeV+ $\mathcal{O}(a)$	41.58(6.58)
$M_\pi < 285$ MeV+ $\mathcal{O}(a)$	39.31(3.15)
no cut in M_π+ $\mathcal{O}(a)$	37.55(1.82)
$M_\pi < 220$ MeV+ $\mathcal{O}(e^{-mL})$	42.45(1.33)
$M_\pi < 285$ MeV+ $\mathcal{O}(e^{-mL})$	42.43(79)
no cut in M_π+ $\mathcal{O}(e^{-mL})$	42.87(59)
$M_\pi < 220$ MeV+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	42.69(6.68)
$M_\pi < 285$ MeV+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	39.38(3.35)
no cut in M_π+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	39.34(2.08)
$M_\pi < 220$ MeV	46.81(1.14)
$M_\pi < 285$ MeV	43.71(62)
no cut in M_π	41.04(39)
$M_\pi < 220$ MeV+ $\mathcal{O}(a)$	51.38(5.87)
$M_\pi < 285$ MeV+ $\mathcal{O}(a)$	45.77(2.73)
no cut in M_π+ $\mathcal{O}(a)$	40.38(1.65)
$M_\pi < 220$ MeV+ $\mathcal{O}(e^{-mL})$	47.21(1.20)
$M_\pi < 285$ MeV+ $\mathcal{O}(e^{-mL})$	44.44(76)
no cut in M_π+ $\mathcal{O}(e^{-mL})$	42.79(56)
$M_\pi < 220$ MeV+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	52.26(5.93)
$M_\pi < 285$ MeV+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	47.13(2.90)
no cut in M_π+ $\mathcal{O}(a)$+ $\mathcal{O}(e^{-mL})$	43.83(1.87)

PRL 131 (2023) 26 [supplemental material]

- Two strategies for ESC
  - upper: “window”
  - lower: “two-state” (closest to “excited-state fit” above)
- Final result $\bar{\sigma}_{\pi N} = 43.7(3.6)$ MeV as average
- For “two-state” fit: systematic increase of (5–10) MeV when restricting results to low pion masses
- Most reliable result arguably $\bar{\sigma}_{\pi N} = 52.3(5.9)$ MeV
- Compares well with $\bar{\sigma}_{\pi N} = 55.9(3.5)$ MeV from phenomenology
- However: lattice data cannot distinguish yet among ESC strategies

Conclusions

- **Isospin violation in πN sigma-term**
  - Cheng–Dashen theorem: leads to increase of $\sigma_{\pi N}$
  - Definition of isospin limit reduces tension between lattice and phenomenology by $\simeq 3$ MeV
- **Excited-state contamination**
  - ChPT predicts large ESC, stable with explicit Δ
  - Most relevant for low pion masses
  - Potentially removes remaining tension with phenomenology
  - Lattice data cannot distinguish yet between analysis strategies