# Lattice extraction of TMD soft function and CS kernel using the Wilson line with the auxiliary field approach

**C.-J. David Lin**

**National Yang Ming Chiao Tung University**
國立陽明交通大學

Confinement 2024 @ Cairns
19/08/2024

# Outline

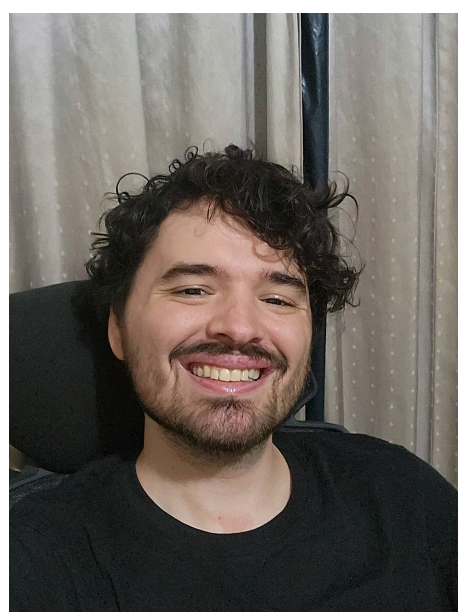
- ★ TMDPDFs and lattice QCD: what and how
- ★ Existing strategies and numerical results
- ★ Our approach
- ★ Outlook

# Collaboration

Anthony Francis
(NYCU)

Issaku Kanamori
(RIKEN)

C.-J. David Lin
(NYCU)



Wayne Morris
(NYCU)

Yong Zhao
(Argonne Nat'l Lab)

# Collaboration

Anthony Francis
(NYCU)

Issaku Kanamori
(RIKEN)

C.-J. David Lin
(NYCU)

Wayne Morris
(NYCU)

Yong Zhao
(Argonne Nat'l Lab)

# The long-term goal

Leading-twist TMDPDFs

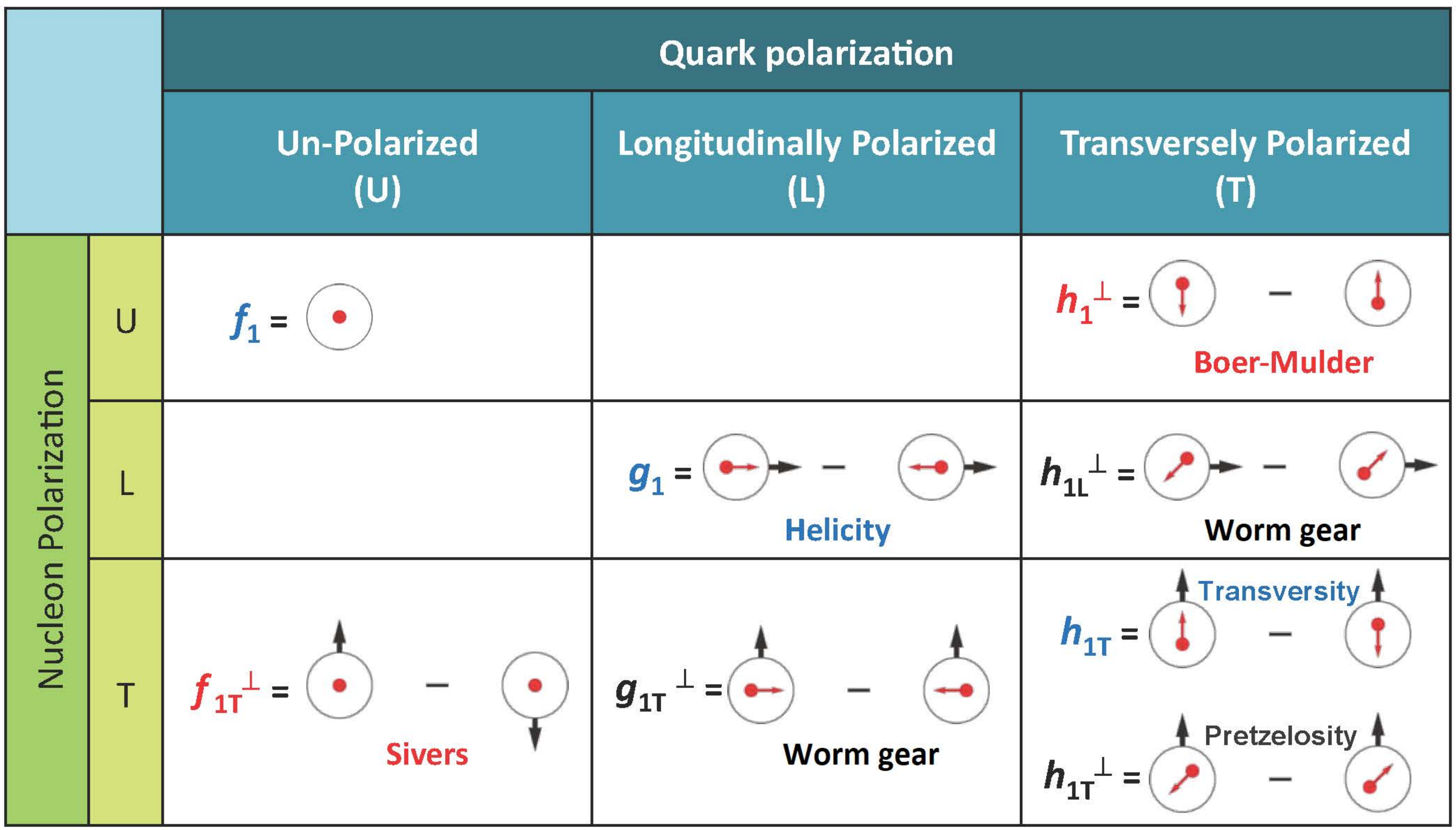
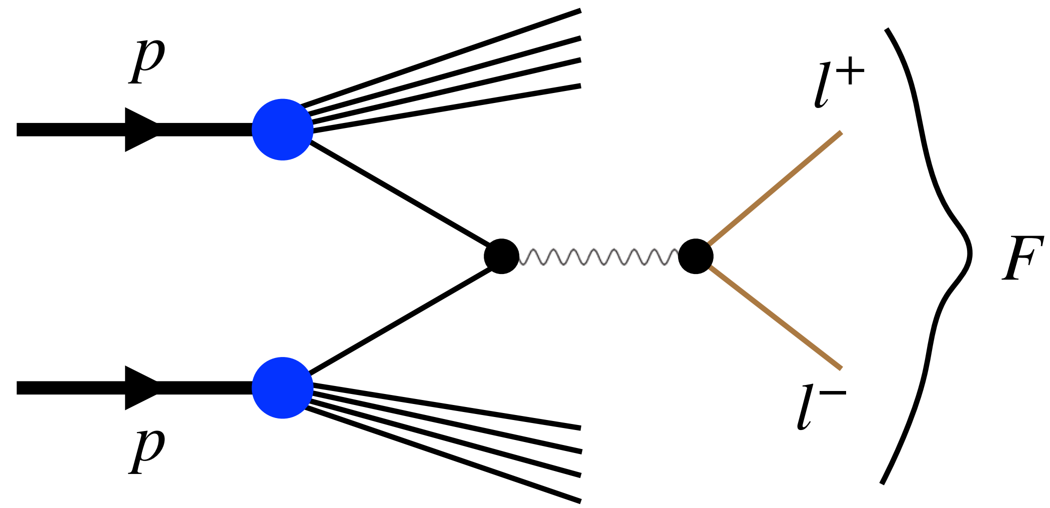




Figure from J. Arrington *et al*., arXiv:2022.13357

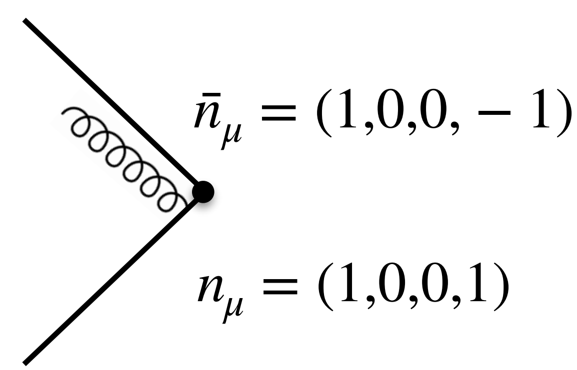
# Drell-Yan factorisation and TMDPDF




$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{ij} H_{ij}(Q,\mu) \int d^2b_T \; e^{i\vec{b}_T\cdot\vec{q}_T} \; f_i^{\mathrm{TMD}}(x_i,\vec{b}_T,\mu,\zeta_i) \; f_j^{\mathrm{TMD}}(x_j,\vec{b}_T,\mu,\zeta_j)$$

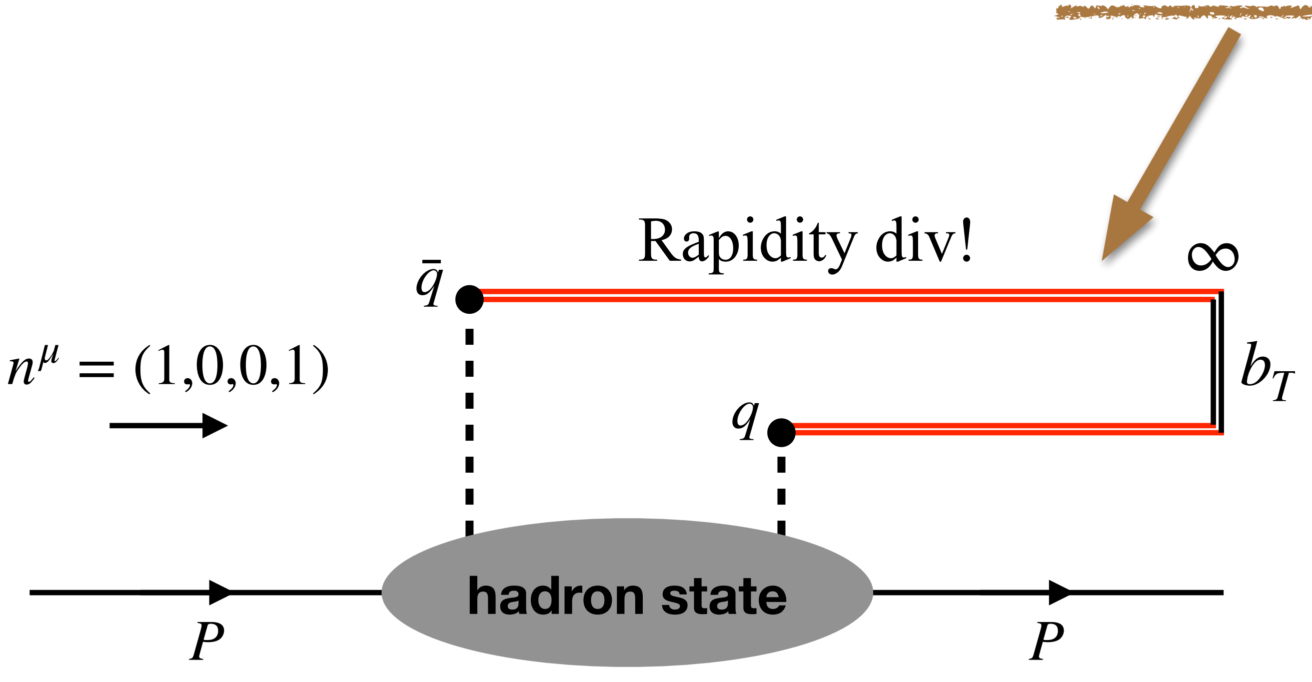
$$\times \left[ 1 + \mathcal{O}\left( \frac{q_T^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2} \right) \right]$$

$\zeta_{i,j}$ from "rapidity divergence" and $\zeta_i\zeta_j = Q^4$

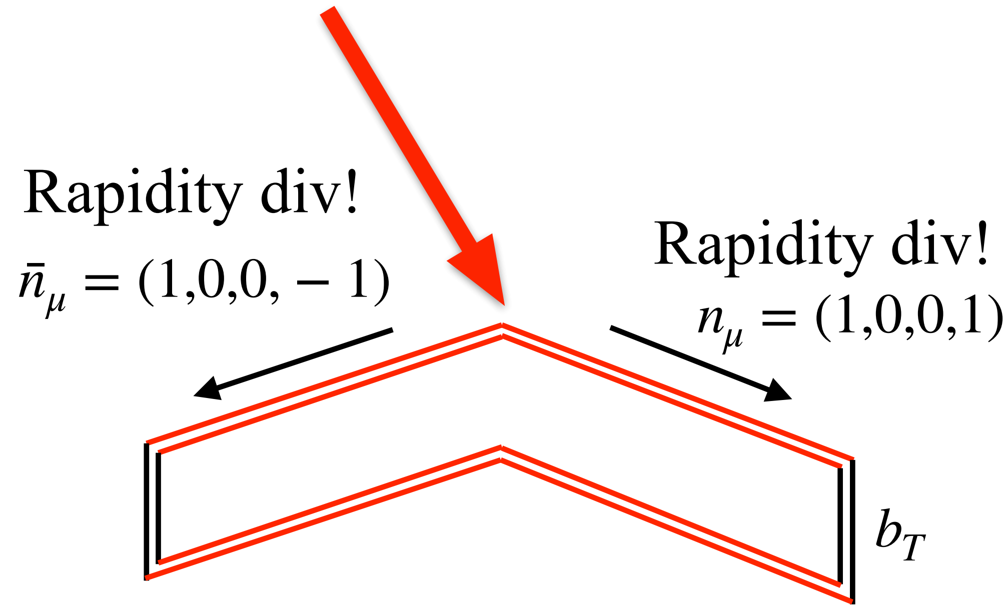


# Drell-Yan factorisation and TMDPDF

$$f^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) = B(x, \vec{b}_T, \mu, \zeta/\nu^2)\,\sqrt{\mathcal{S}(b_T, \mu, \nu)}$$




"Beam function"



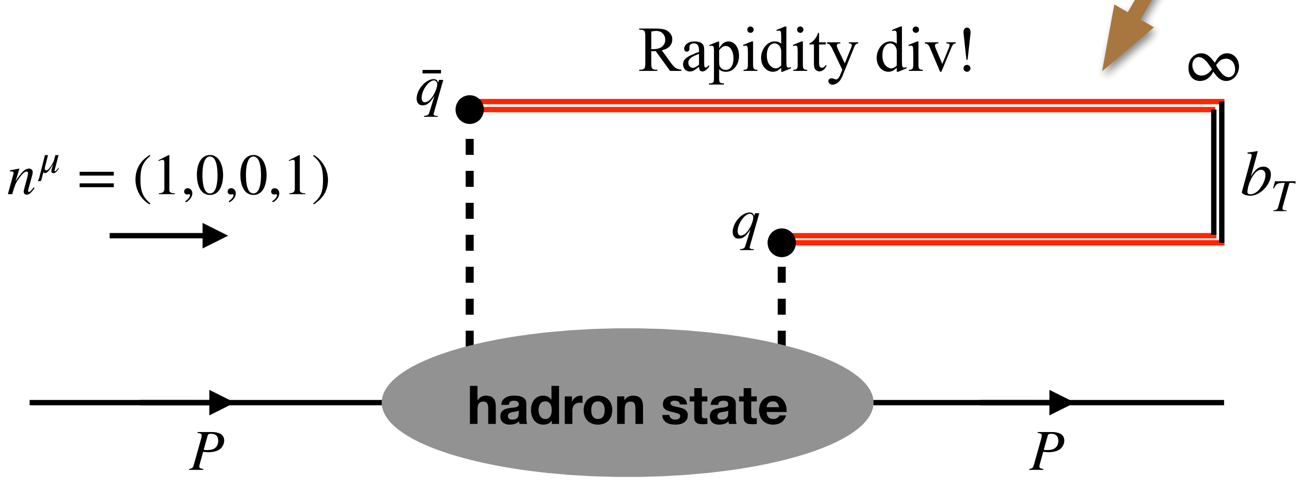

"Soft function"

And the "Collins-Super (CS) kernel" for evolution in $\nu$ ($\zeta$)

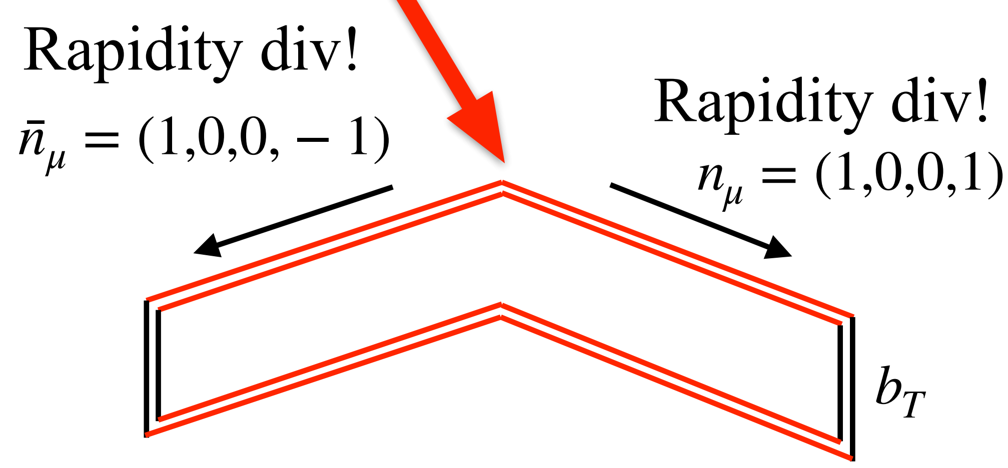
$\mathcal{S}(b_T, \mu, \nu) \Rightarrow \mathcal{S}_I(b_T, \mu), K(b_T, \mu) \Rightarrow$ both are *universal*

# Drell-Yan factorisation and TMDPDF

$$f^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) = B(x, \vec{b}_T, \mu, \zeta/\nu^2)\sqrt{\mathcal{S}(b_T, \mu, \nu)}$$




"Beam function"



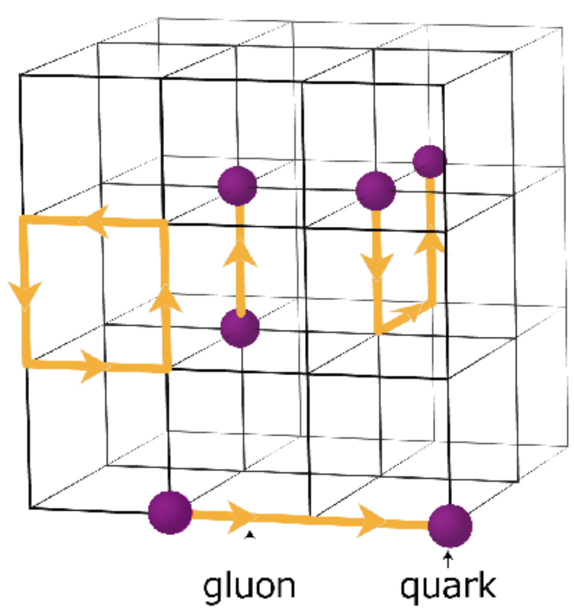

"Soft function"

And the "Collins-Super (CS) kernel" for evolution in $\nu$ ($\zeta$)

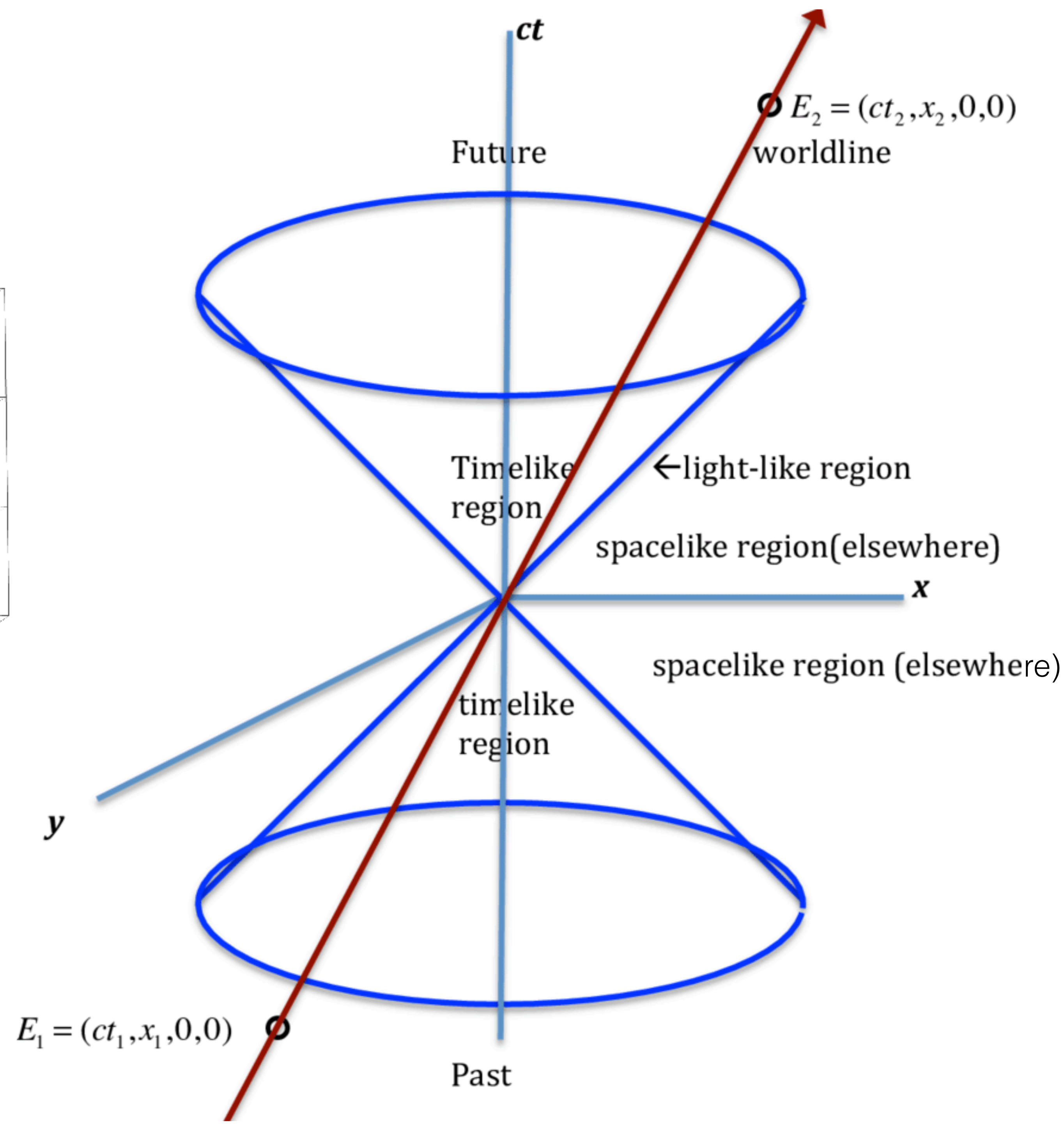
$\mathcal{S}(b_T, \mu, \nu) \Rightarrow \mathcal{S}_I(b_T, \mu), K(b_T, \mu) \Rightarrow$ both are *universal*

Focus of this talk

# Challenges in parton physics from lattice QCD




Euclidean space



# Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

$$\underline{\tilde{f}^{\rm TMD}(x, \vec{b}_T, \mu, P^z)} = \underbrace{C^{\rm TMD}(\mu, xP^z)}_{\text{pertub. theo.}} \, \underline{g_S(b_T, \mu)} \exp\left[\frac{1}{2}\underline{K(b_T, \mu)} \log\frac{(2xP^z)^2}{\zeta}\right] \times \underline{f^{\rm TMD}(x, \vec{b}_T, \mu, \zeta)} + \mathcal{O}\left(\frac{q_T^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

# Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

$$\tilde{f}^{\rm TMD}(x,\vec{b}_T,\mu,P^z) = \underbrace{C^{\rm TMD}(\mu,xP^z)}_{\text{pertub. theo.}}\, g_S(b_T,\mu)\exp\left[\frac{1}{2}K(b_T,\mu)\log\frac{(2xP^z)^2}{\zeta}\right]$$
$$\times f^{\rm TMD}(x,\vec{b}_T,\mu,\zeta) + \mathcal{O}\left(\frac{q_T^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

★ To obtain $f^{\rm TMD}$, one computes $\tilde{f}^{\rm TMD}$ with lattice QCD

# Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

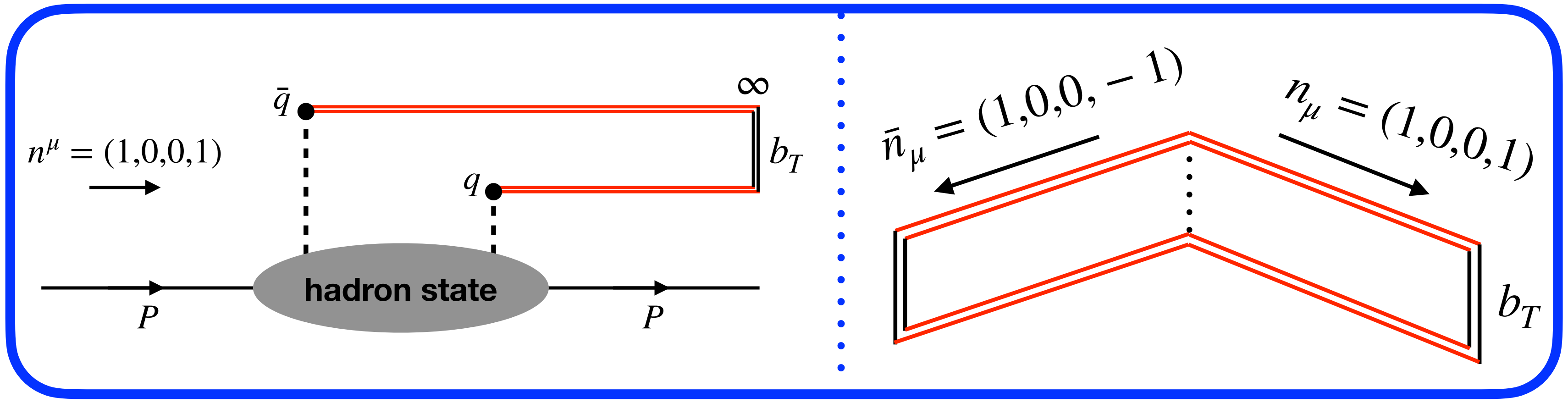
$$\underline{\tilde{f}^{\rm TMD}(x,\vec{b}_T,\mu,P^z)} = \underbrace{C^{\rm TMD}(\mu,xP^z)}_{\text{pertub. theo.}}\, \underline{g_S(b_T,\mu)}\, \exp\left[\frac{1}{2}\underline{K(b_T,\mu)}\log\frac{(2xP^z)^2}{\zeta}\right]$$
$$\times\, \underline{f^{\rm TMD}(x,\vec{b}_T,\mu,\zeta)} + \mathcal{O}\left(\frac{q_T^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

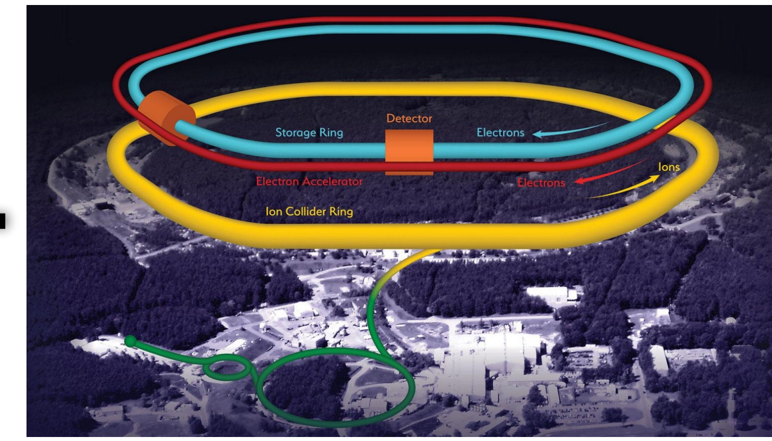
- ★ To obtain $f^{\rm TMD}$, one computes $\tilde{f}^{\rm TMD}$ with lattice QCD
- ★ Also need non-perturbative calculation of

  The Collins-Soper kernel, $K(b_T,\mu)$

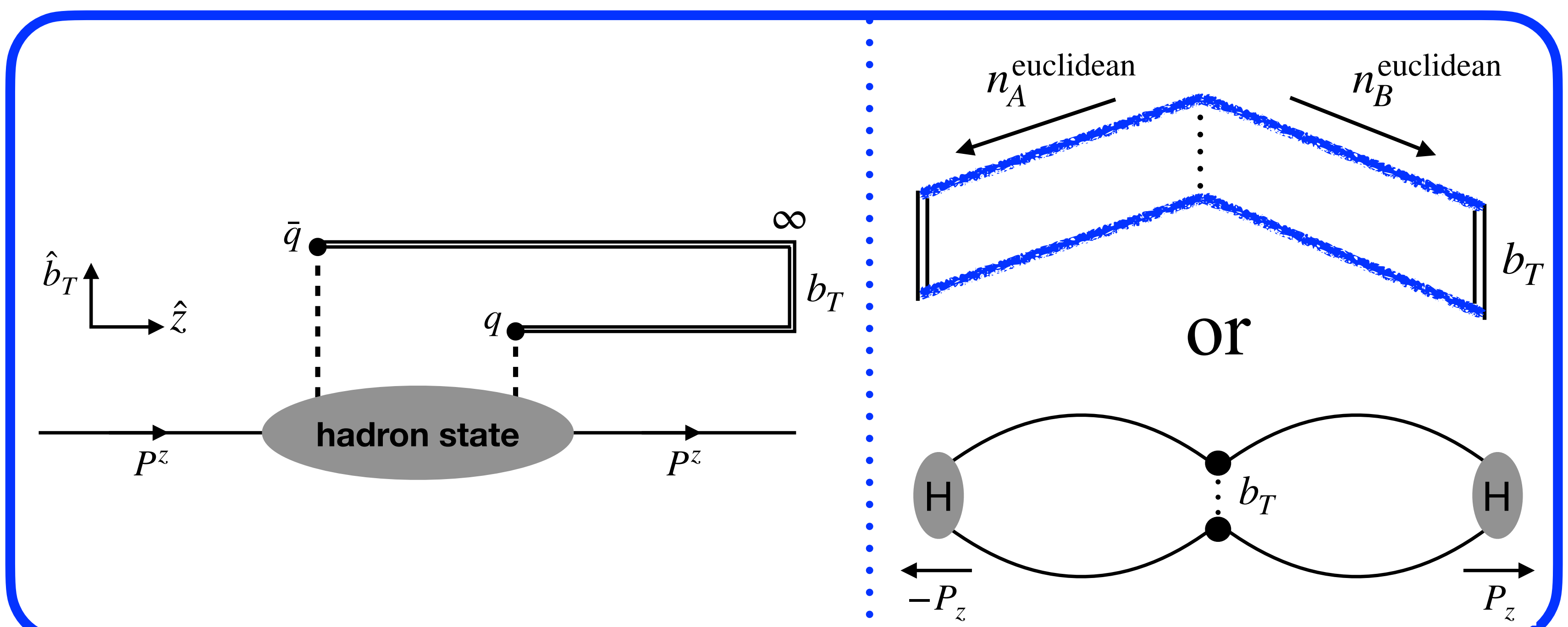
  The soft function, $g_S(b_T,\mu) \sim \sqrt{S_I(b_T,\mu)}$

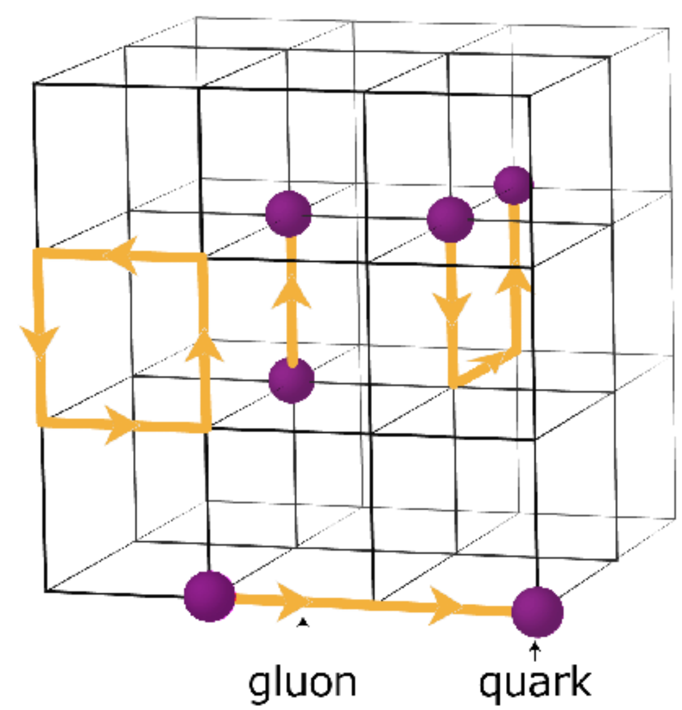
# TMDPDF from LQCD



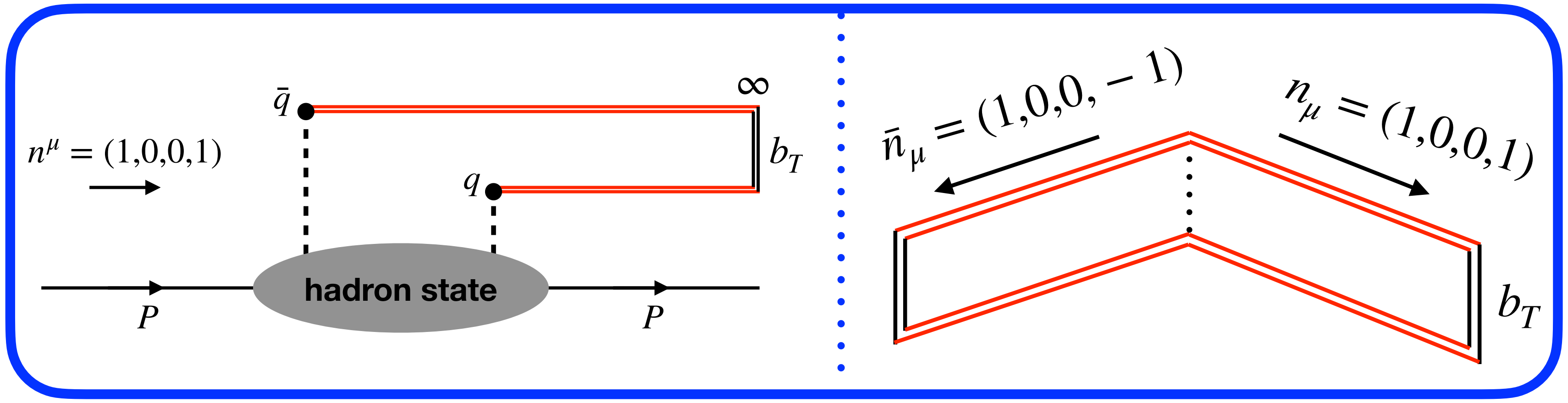

Minkowski, light-cone

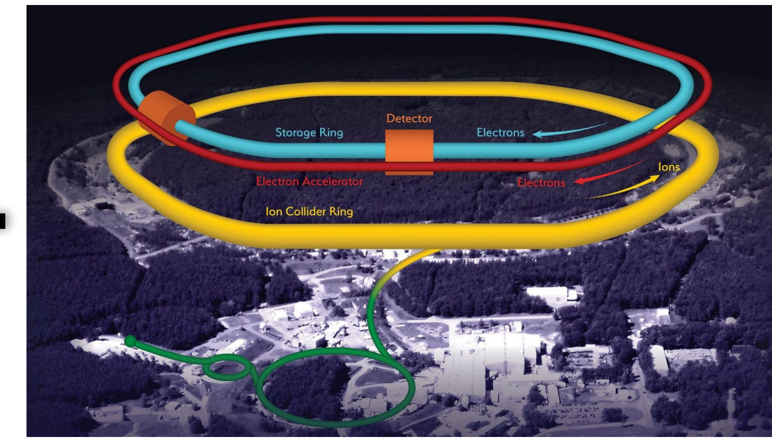



Euclidean, space-like

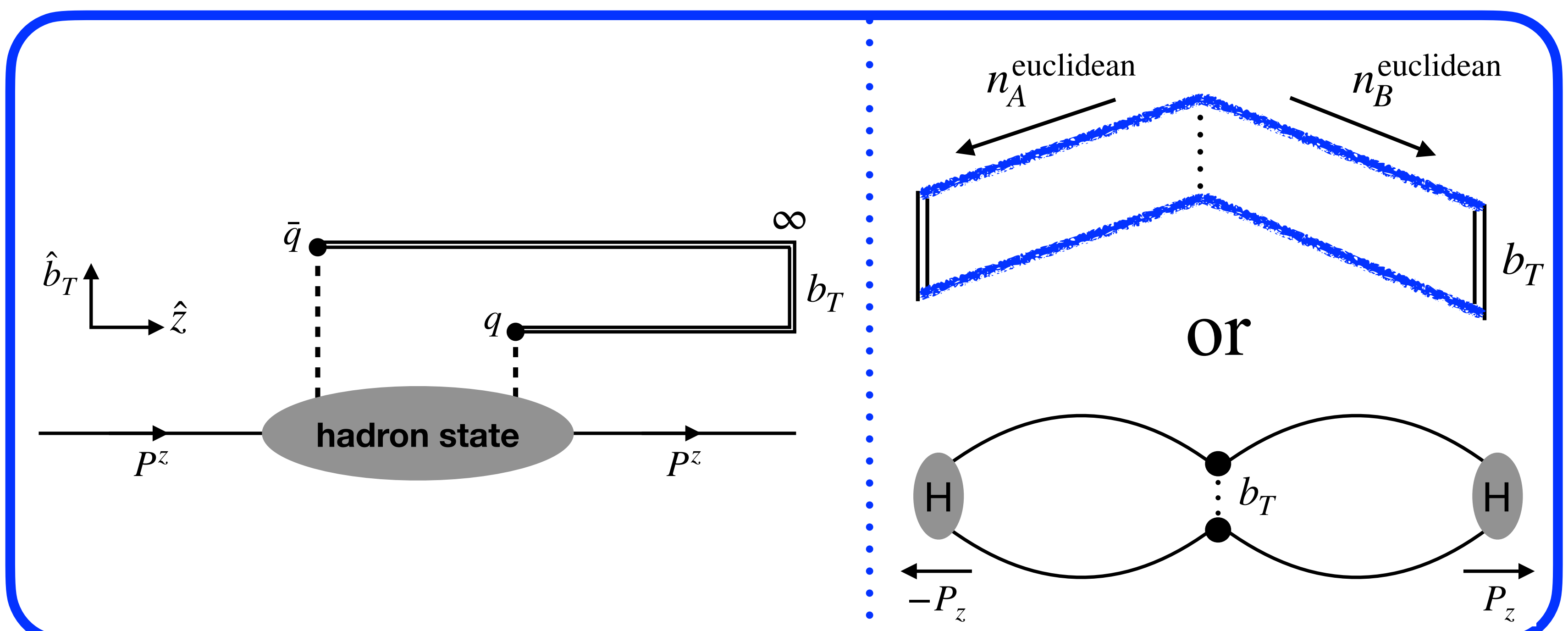
# TMDPDF from LQCD

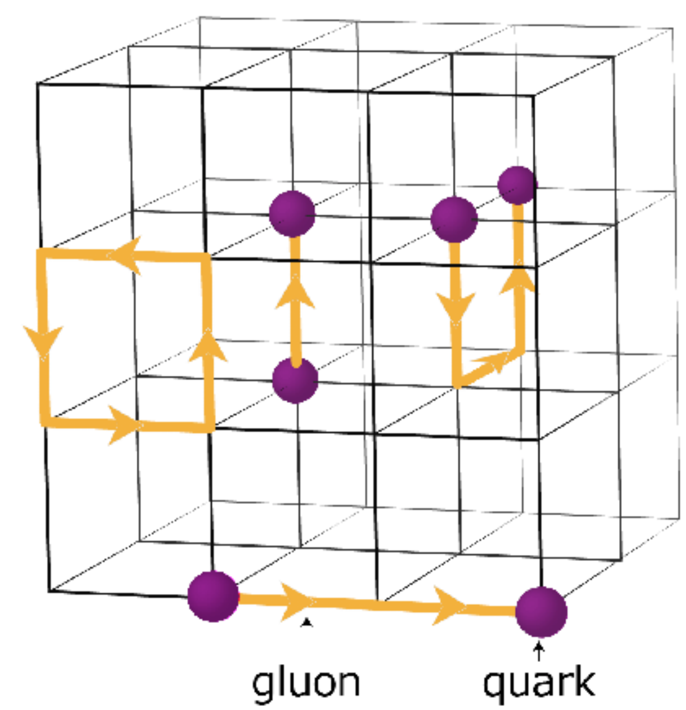



Minkowski, light-cone

Perturbation theory, $K(\mu, b_T)$, $S_I(\mu, b_T)$




Euclidean, space-like

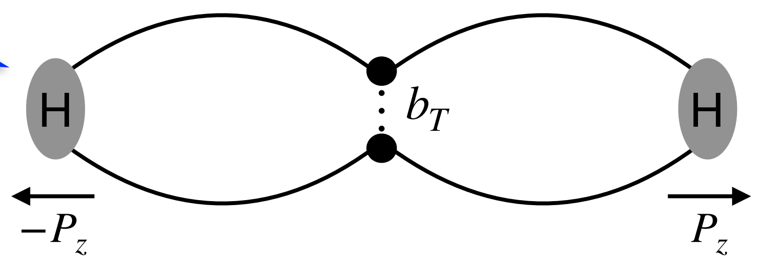
# Existing lattice results

# Intrinsic soft function from lattice QCD

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. **B955** (2020) 115054, Phys. Lett. **B811** (2020) 135946

★ Compute the form factor

$$F(b_T, P^z) = \langle \pi(-p^z) \,|\, \bar{u}\Gamma u(b_T)\ \bar{d}\Gamma d(0) \,|\, \pi(P^z) \rangle$$

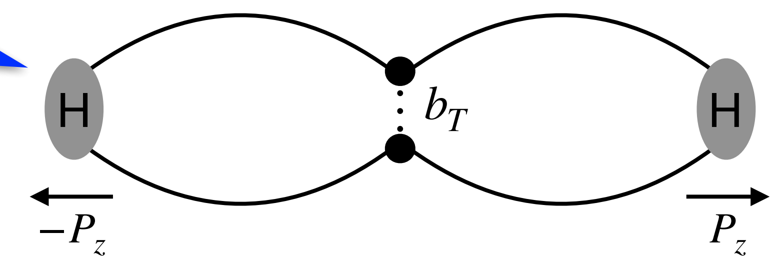


# Intrinsic soft function from lattice QCD

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. **B955** (2020) 115054, Phys. Lett. **B811** (2020) 135946

★ Compute the form factor

$$F(b_T, P^z) = \langle \pi(-p^z) \,|\, \bar{u}\Gamma u(b_T)\; \bar{d}\Gamma d(0) \,|\, \pi(P^z) \rangle$$

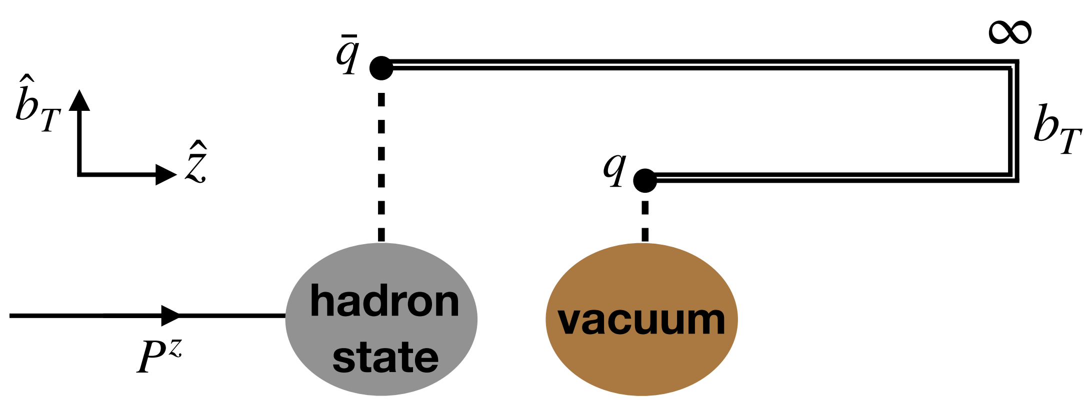



★ At large $P^z$ , it factorises to

$$F(b_T, P^z) = S_I(b_T, \mu) \int_0^1 dx dx' \, H_\Gamma(x, x', P^z, \mu)$$

perturbative

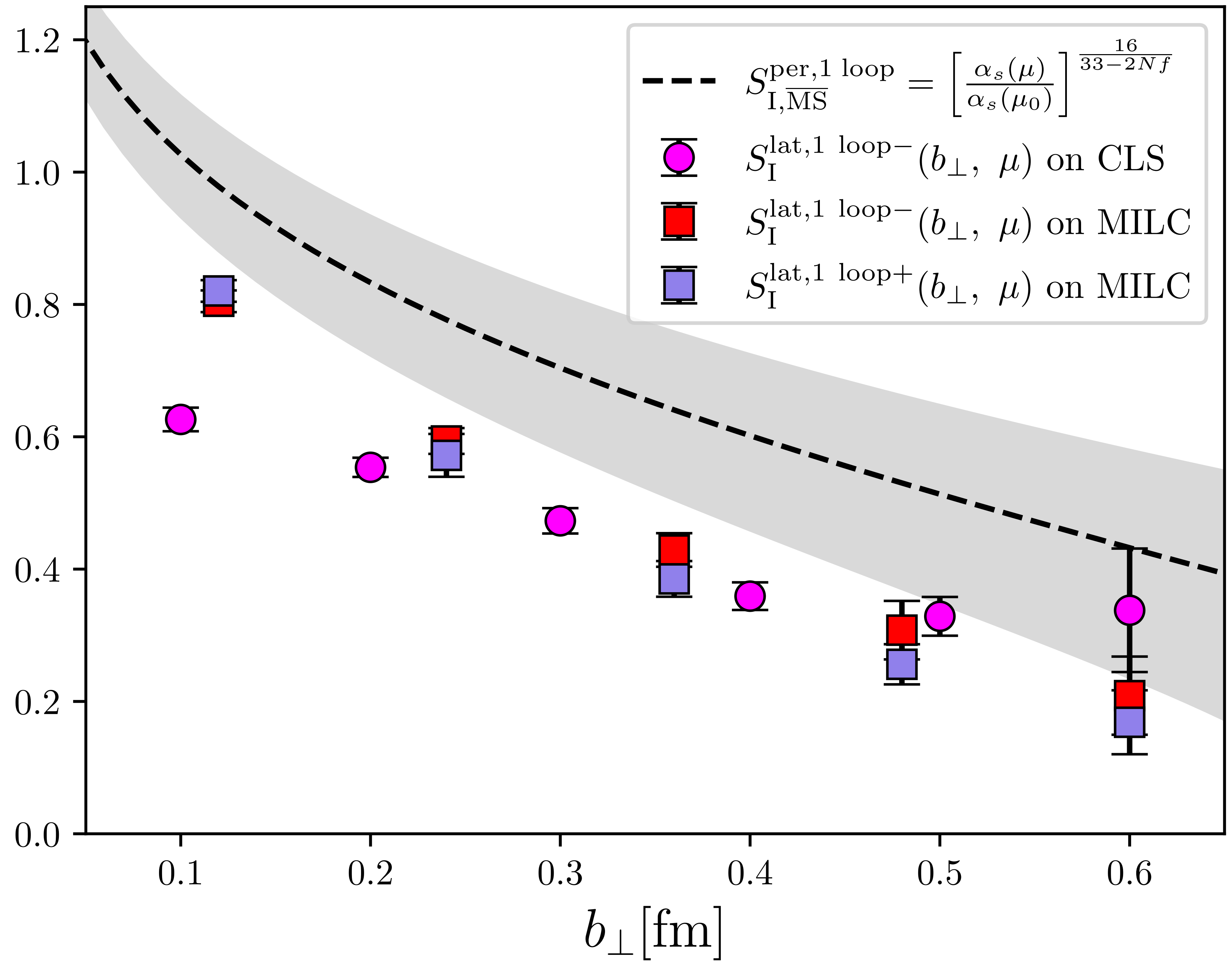
$$\times \Phi^\dagger(x', b_T, -P^z)\; \Phi(x, b_T, P^z) \;+\; \mathcal{O}\left(\frac{b_T}{P^z}, \frac{\Lambda_{\rm QCD}}{P^z}\right)$$




quasi pion TMD wave function

# Intrinsic soft function from lattice QCD

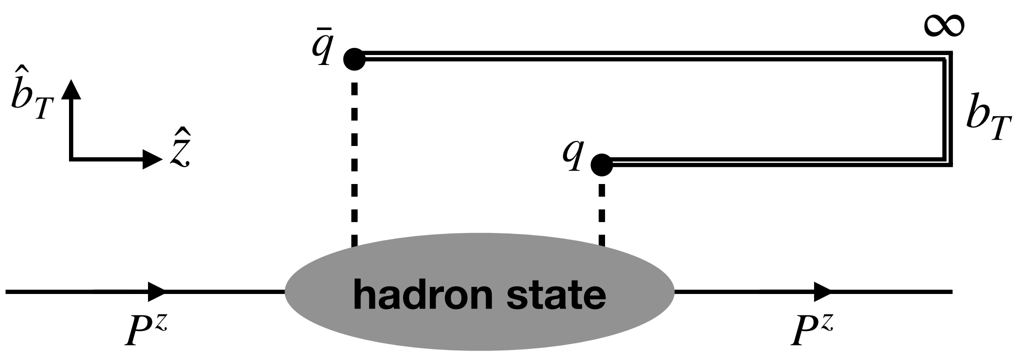
LPC Collaboration, JHEP **08** (2023) 172



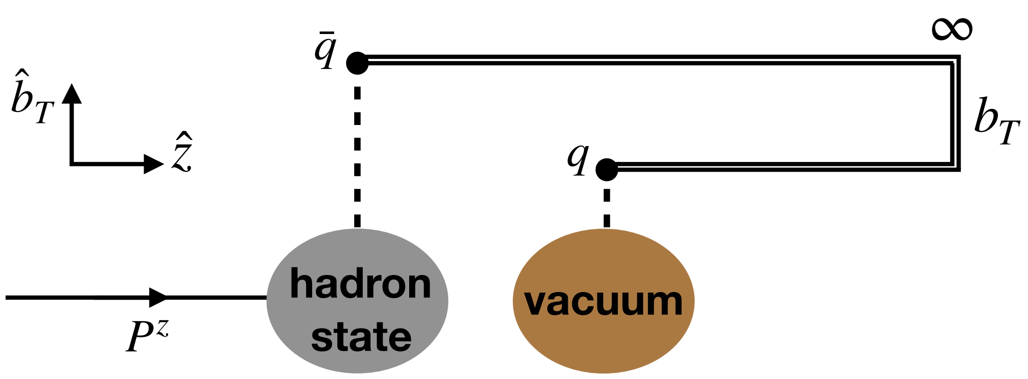
# CS kernel from lattice QCD

M. Ebert, I. Stewart, Y. Zhao, Phys. Rev., **D99** (2019) 034505

★ Compute qTMDPDF ($\tilde{f}^{\rm TMD}$) or qTMDWF ($\tilde{\Phi}^{\rm TMD}$)



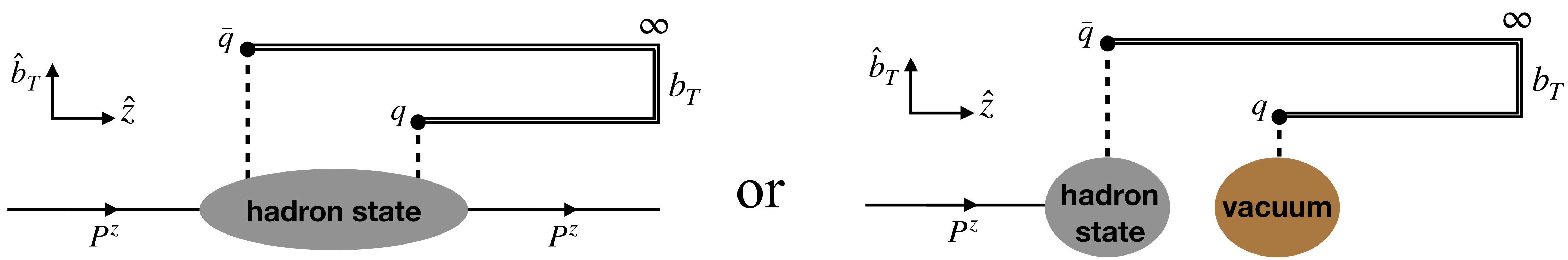

or



# CS kernel from lattice QCD

M. Ebert, I. Stewart, Y. Zhao, Phys. Rev., **D99** (2019) 034505

★ Compute qTMDPDF ($\tilde{f}^{\rm TMD}$) or qTMDWF ($\tilde{\Phi}^{\rm TMD}$)

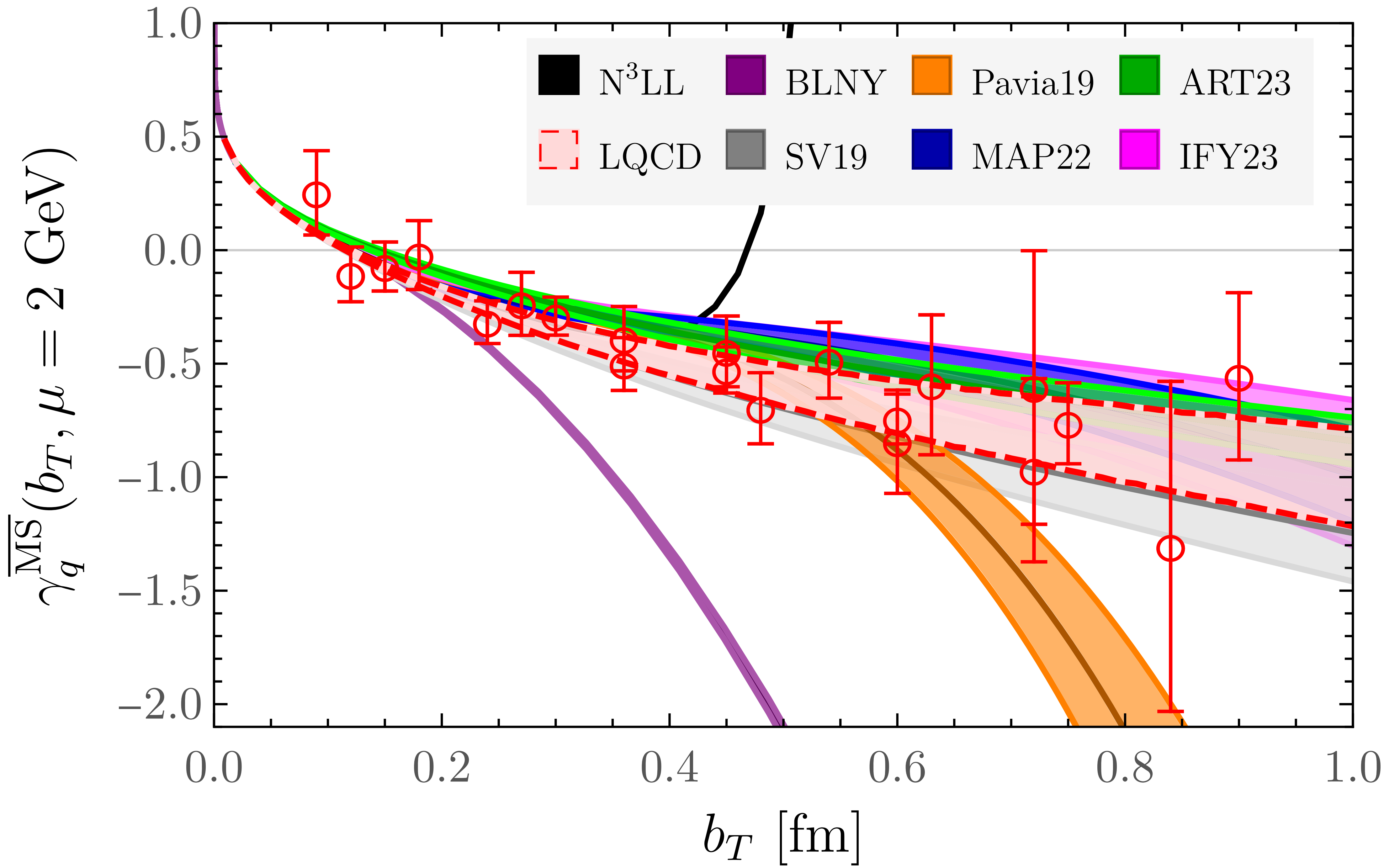



★ Determine the CS kernel from the ratio (at large $P^z$)

$$K(\mu, b_T) = \frac{1}{\log(P_1^z/P_2^z)} \log \frac{\underbrace{C^{\rm TMD}(\mu, xP_2^z)}\,\tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{\underbrace{C^{\rm TMD}(\mu, xP_1^z)}_{\text{perturbative}}\,\tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

# CS kernel from lattice QCD

A. Avhadiev, P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev. Lett. **132** (2024)



# Need of new approaches for Soft function and CS kernel

★ Recent, previous lattice calculations involve pion states

→ Universality?

# Need of new approaches for Soft function and CS kernel

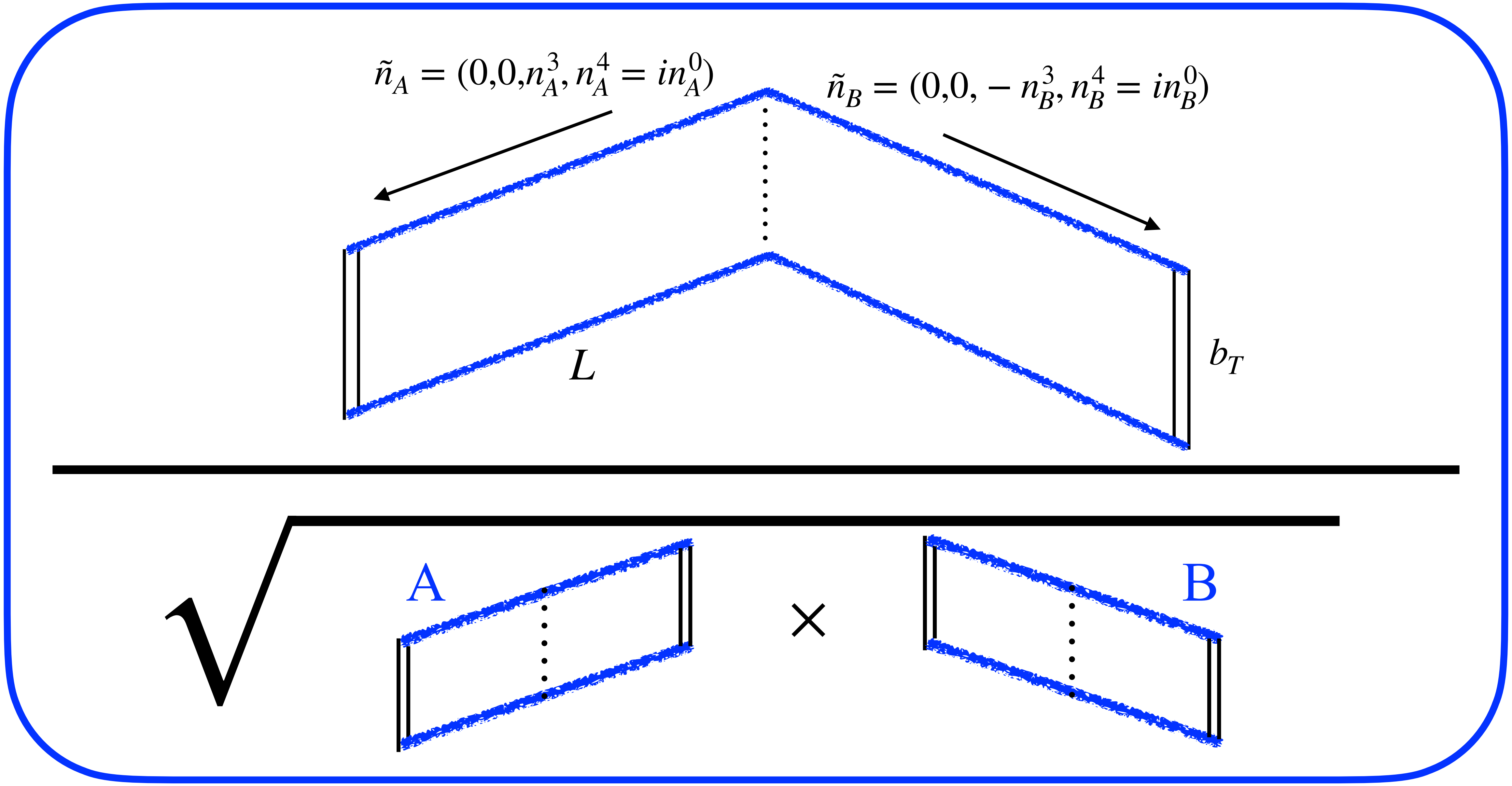
- ★ Recent, previous lattice calculations involve pion states
  - ➞ Universality?

- ★ ***Need calculations with other methods***

# Our approach

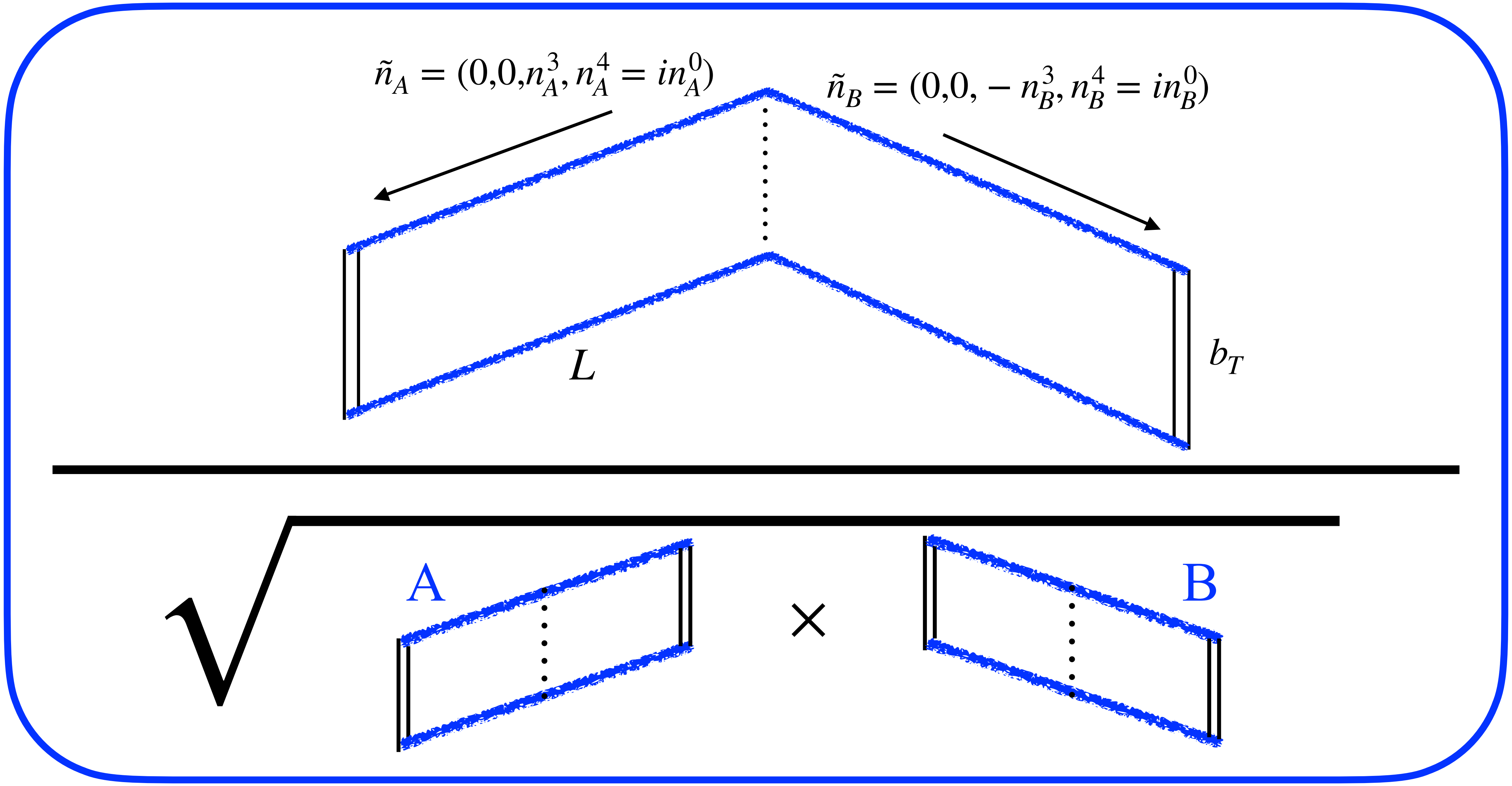
# Our approach:

## Soft function and CS kernel from Euclidean Wilson loops



# Our approach:

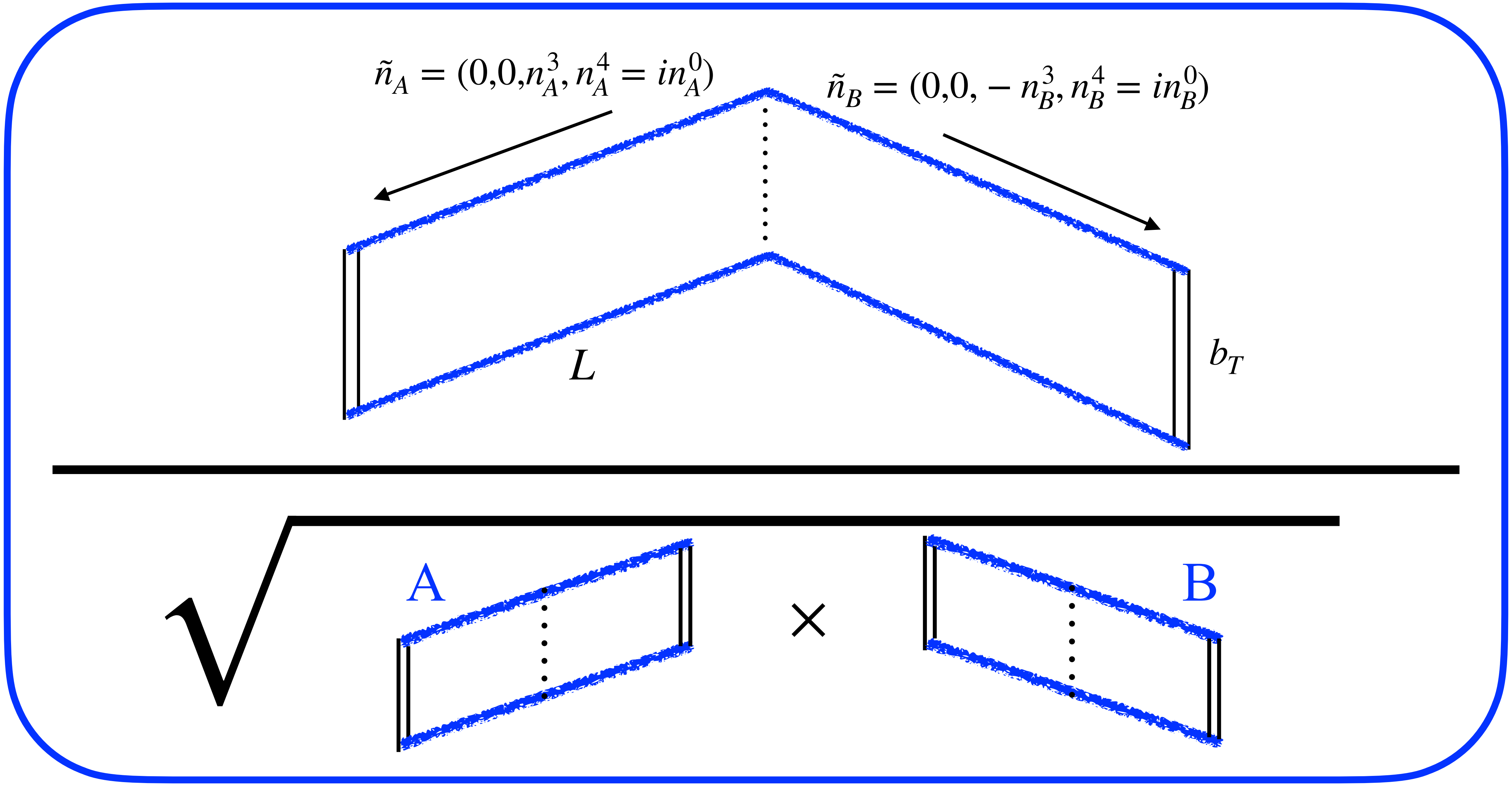
## Soft function and CS kernel from Euclidean Wilson loops




Gives the Collins soft function in *Minkowski* space

# Our approach:

## Soft function and CS kernel from Euclidean Wilson loops




Gives the Collins soft function in *Minkowski* space

Related to $S_I(b_T, \mu)$ and $K(b_T, \mu)$

# Our approach:
## Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

# Our approach:
## Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

→ Collins soft function with space-like regularisation can be obtained

# Our approach:
## Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

→ Collins soft function with space-like regularisation can be obtained

→ Rapidities are related to the directional vectors of the Wilson lines

$$r_{a,b} \equiv \frac{n^3_{A,B}}{n^0_{A,B}} = \frac{1 + e^{\pm y_{A,B}}}{1 - e^{\pm y_{A,B}}}$$

# Our approach:
## Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

➞ Collins soft function with space-like regularisation can be obtained

➞ Rapidities are related to the directional vectors of the Wilson lines

$$r_{a,b} \equiv \frac{n^3_{A,B}}{n^0_{A,B}} = \frac{1 + \mathrm{e}^{\pm y_{A,B}}}{1 - \mathrm{e}^{\pm y_{A,B}}}$$

➞ Finite-length effects are of $O(b_T^4/L^4)$ or smaller

# Our approach:
## Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

➞ Collins soft function with space-like regularisation can be obtained

➞ Rapidities are related to the directional vectors of the Wilson lines

$$r_{a,b} \equiv \frac{n^3_{A,B}}{n^0_{A,B}} = \frac{1 + \mathrm{e}^{\pm y_{A,B}}}{1 - \mathrm{e}^{\pm y_{A,B}}}$$

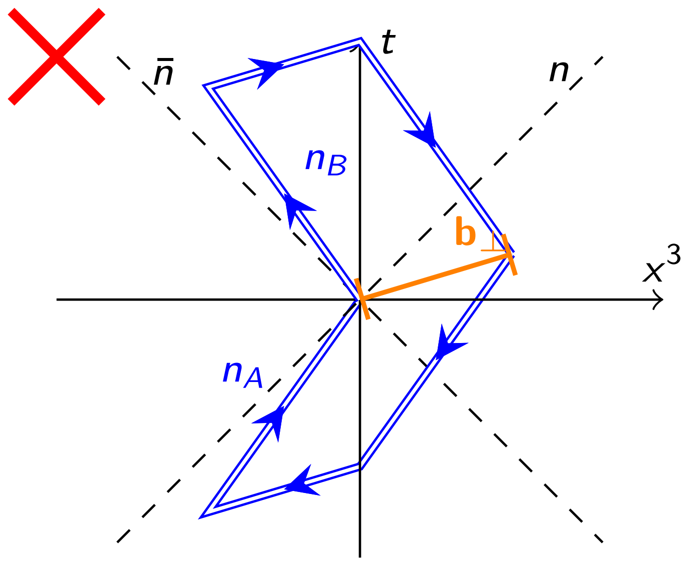
➞ Finite-length effects are of $O(b_T^4/L^4)$ or smaller

★ Determine $S_I(b_T, \mu)$ and $K(b_T, \mu)$ *via* varying $r_{a,b}$ and fitting to

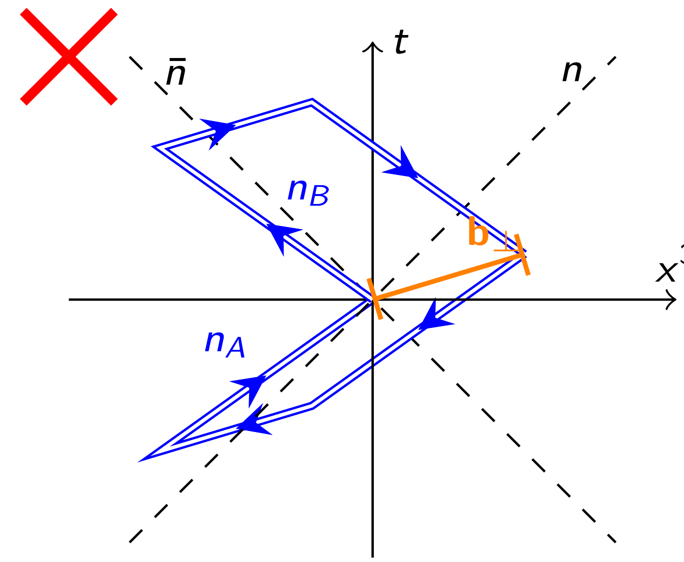
$$S_C(b_T, \mu, y_A, y_B) = S_I(b_T, \mu)\, \mathrm{e}^{2K(b_T,\mu)\times(y_A - y_B)}$$

# Rapidity regularisation in our approach

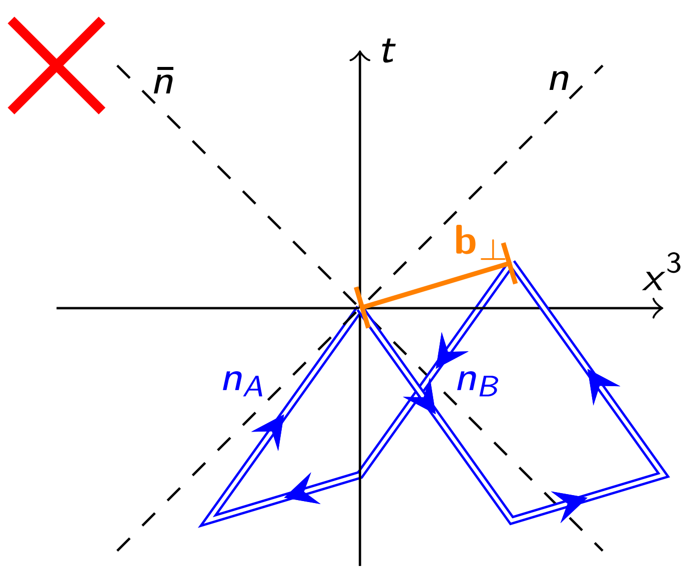
## Connection to Minkowski space



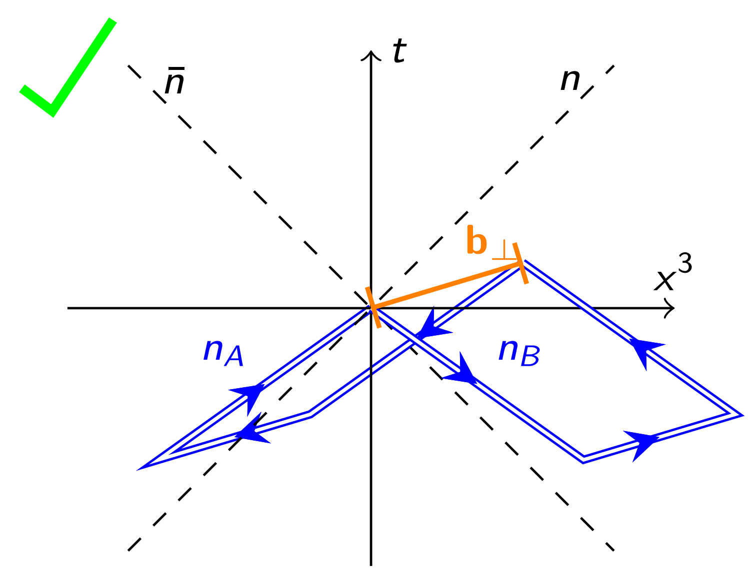

$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$




$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$




$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$




$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$

# Our approach:

## Auxiliary-field representation of Wilson lines

$$\mathscr{P} \exp\left[ig\int_{x(a)}^{x(b)} dx^\mu A_\mu(x)\right]$$

$$x^\mu(s) = x^\mu(a) + (s-a)n^\mu$$

$$= \frac{1}{Z_\psi}\int \mathscr{D}\psi\mathscr{D}\bar{\psi}\,\psi(b)\bar{\psi}(a)\exp\left\{ig\int_a^b ds\,\bar{\psi}(s)\left[\partial_s - n^\mu A_\mu(x(s))\right]\psi(s)\right\}$$

$\equiv D$

S. Samuel, NPB 149 (1979); I.Ya. Aref'eva, PLB93 (1980),…

➡ Computing Wilson line = Calculating auxiliary field propagator

$$-in\cdot D\, G(x) = \delta(x)$$

J. Mandura and M. Ogilvie, PRD 45 (1982); U. Aglietti, NPB 421 (1994)

Analogy to HQET

X. Ji, Y. Liu, Y.-S. Liu, NPB955 (2020)

# Our approach:
## Auxiliary-field propagator

$$G(\vec{x},\tau) = K(\tau)\, G(\vec{x},\tau-1)$$

$$K(\tau) = \left[1 - \frac{H_0|_\tau}{2n}\right]^n U_4^\dagger(\tau-1)\left[1 - \frac{H_0|_{\tau-1}}{2n}\right]^n$$

$$H_0\,\psi(x) = -\frac{i}{2}\sum_{\mu=1}^{3} v_\mu \left[U_\mu(x)\,\psi(x+\hat{\mu}) - U_{-\mu}(x)\,\psi(x-\hat{\mu})\right]$$

R.R. Horgan *et al.*, PRD 80 (2009)

# Numerical exploration of our method

At large Euclidean time, expect:

$$S_{\rm num} \overset{\tau\to\infty}{\sim} e^{2\tau(r_a+r_b)/a}/\tau^4$$

$$S_A \overset{\tau\to\infty}{\sim} e^{4(\tau r_a - iz)/a}/\tau^4$$

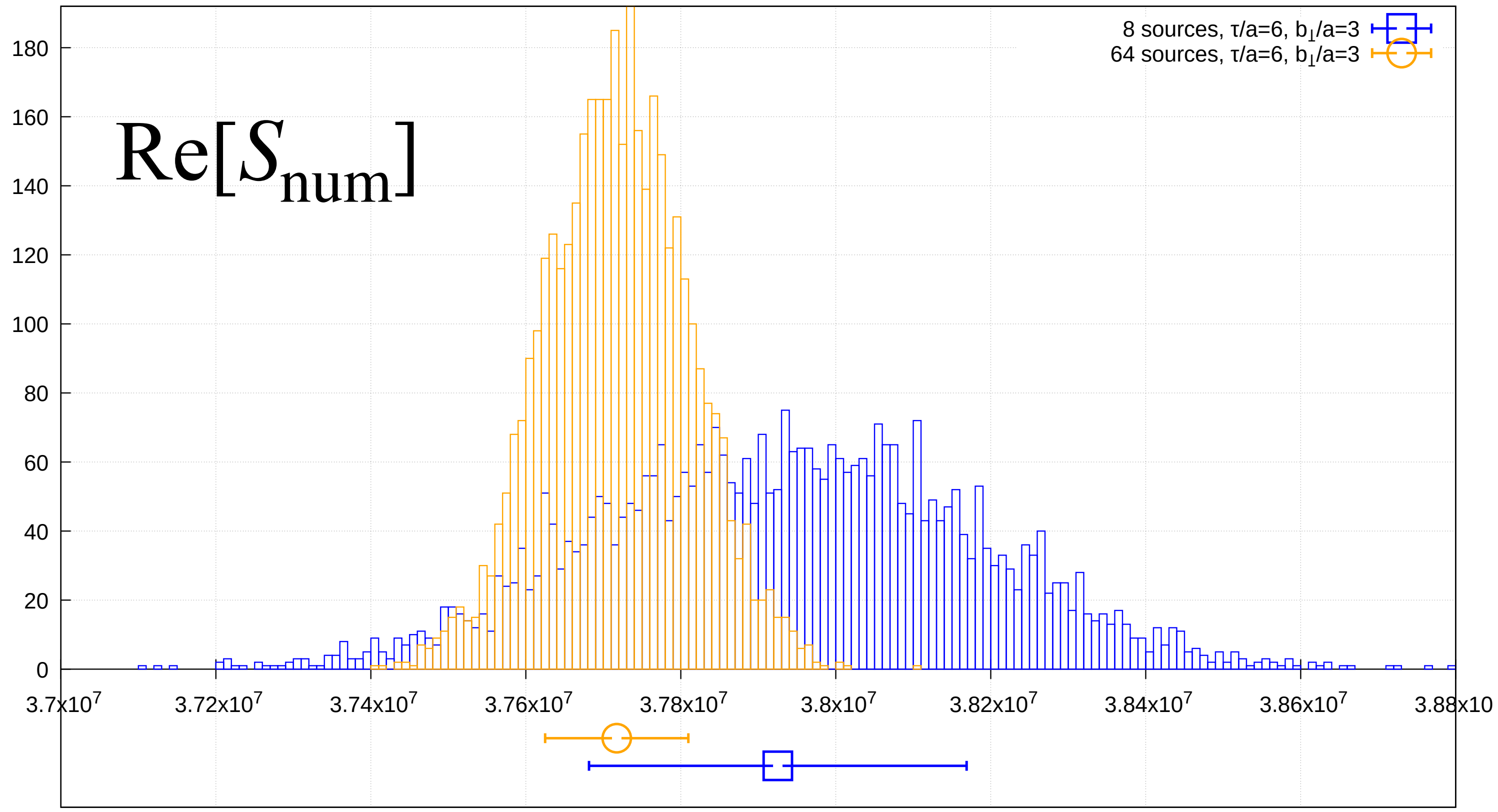
$$S_B \overset{\tau\to\infty}{\sim} e^{4(\tau r_b + iz)/a}/\tau^4$$

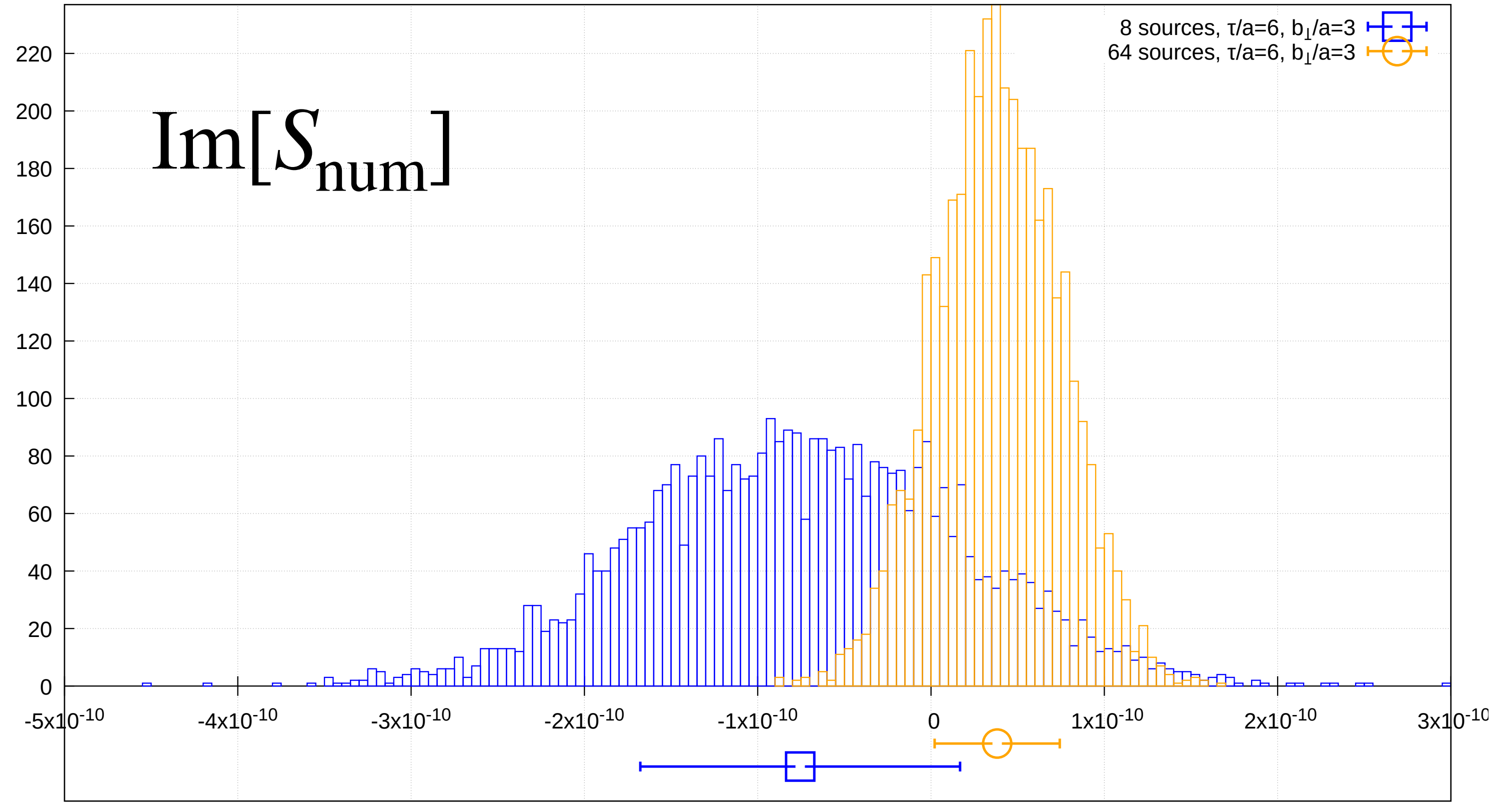
That is, $S_{\rm num}$ is real, $S_{A,B}$ are complex, but $S_A S_B$ is real

# Numerical exploration of our method

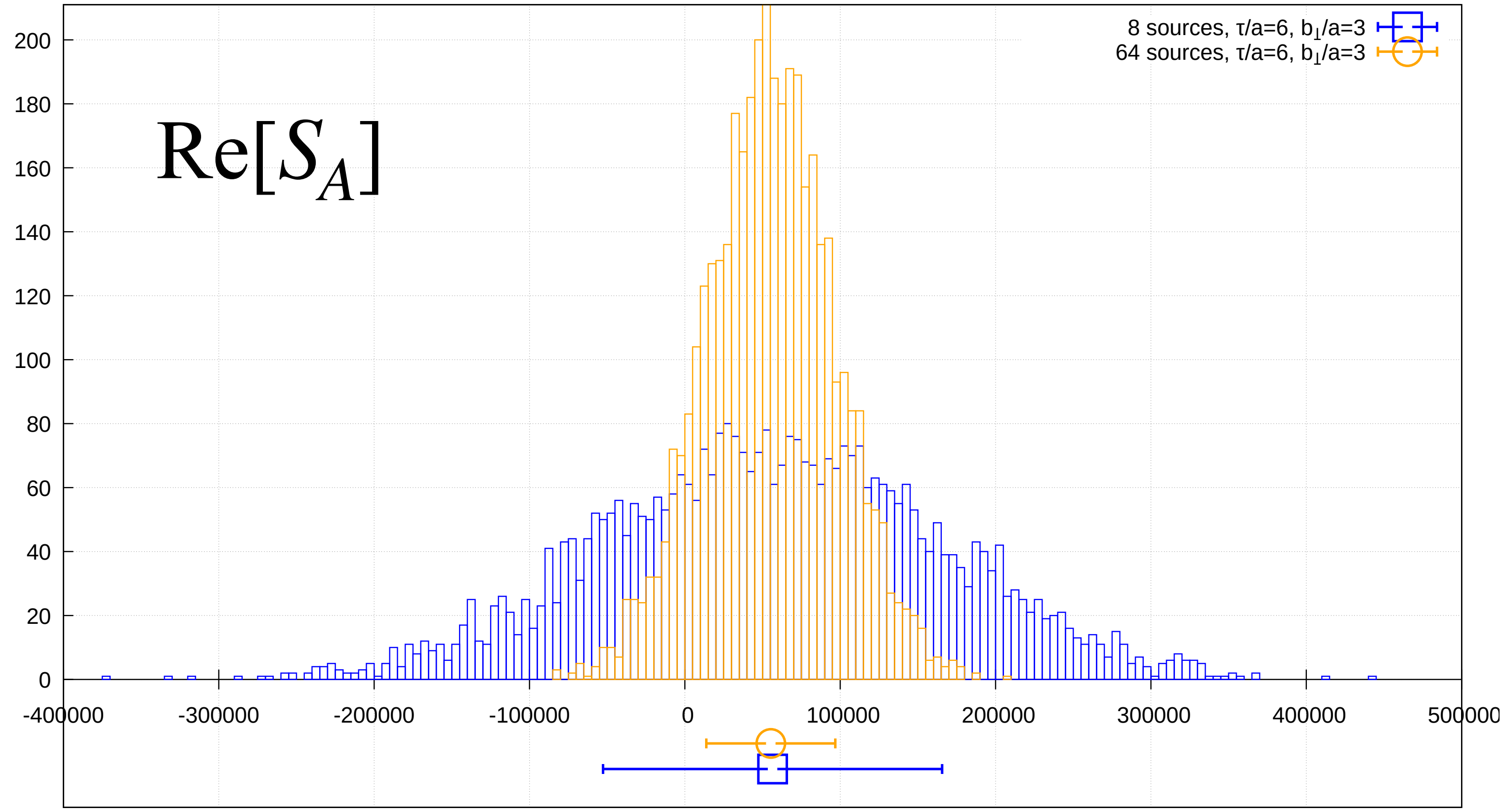
- Using $N_f = 2 + 1$ flavor PACS-CS configurations
- non-perturbatively $\mathcal{O}(a)$-improved Wilson quark action and Iwasaki gauge action
- $32^3 \times 64$ lattice with $a = 0.0907(13)$ fm
- 400 configurations
- thyp2 smearing
- Up to 32 sources per configuration
- Using GPT/GRID

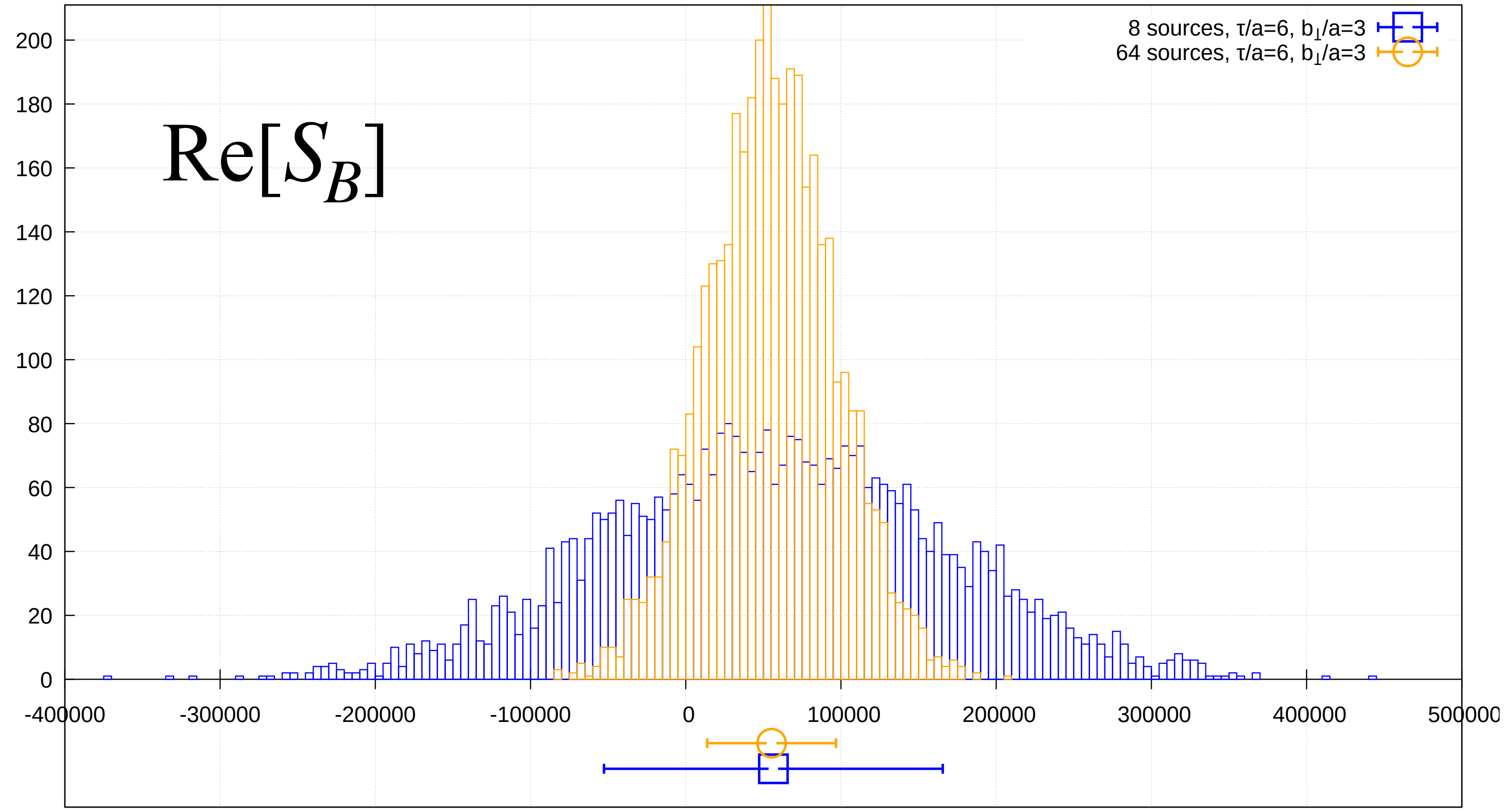
# Numerical exploration of our method



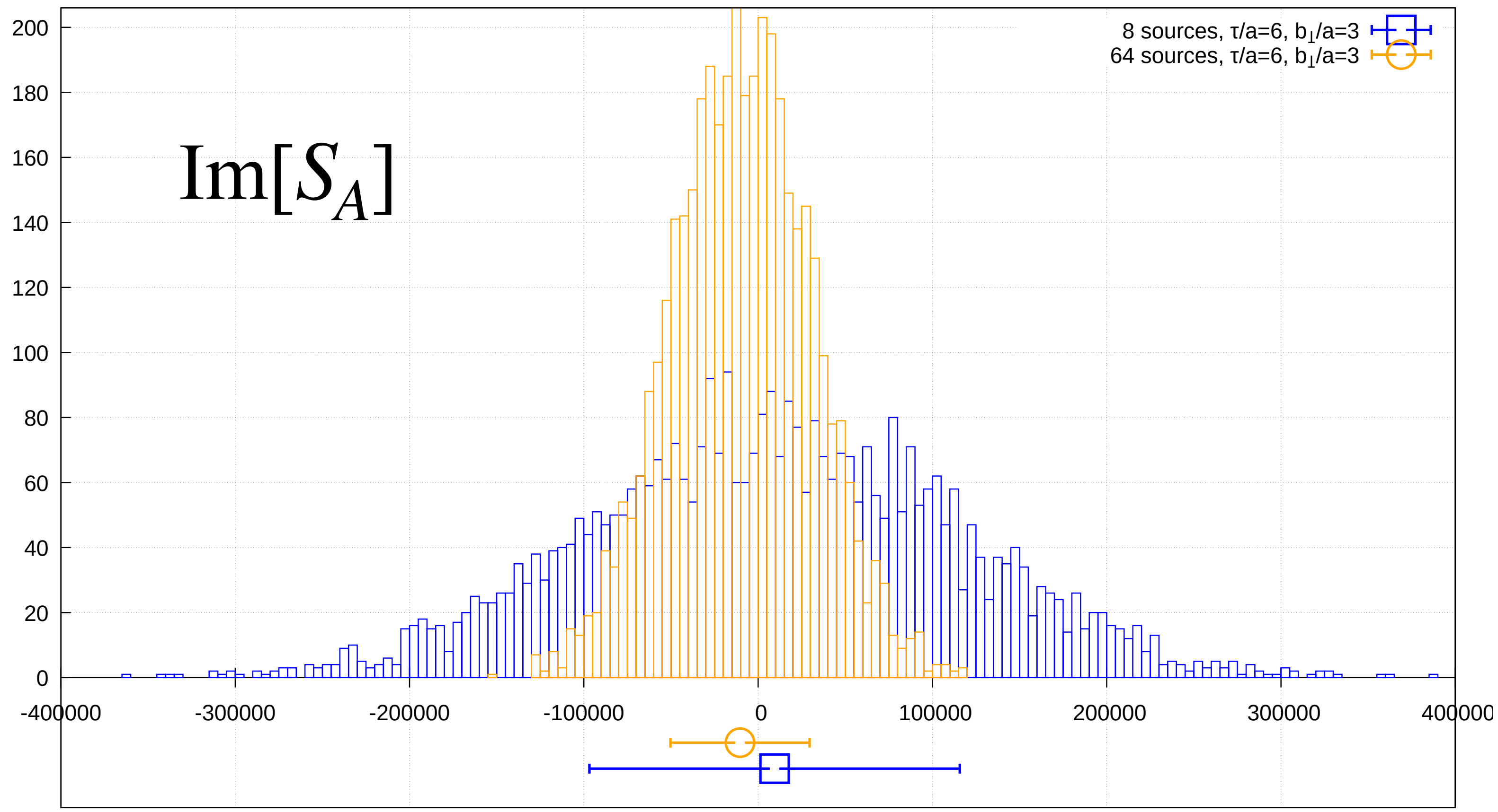



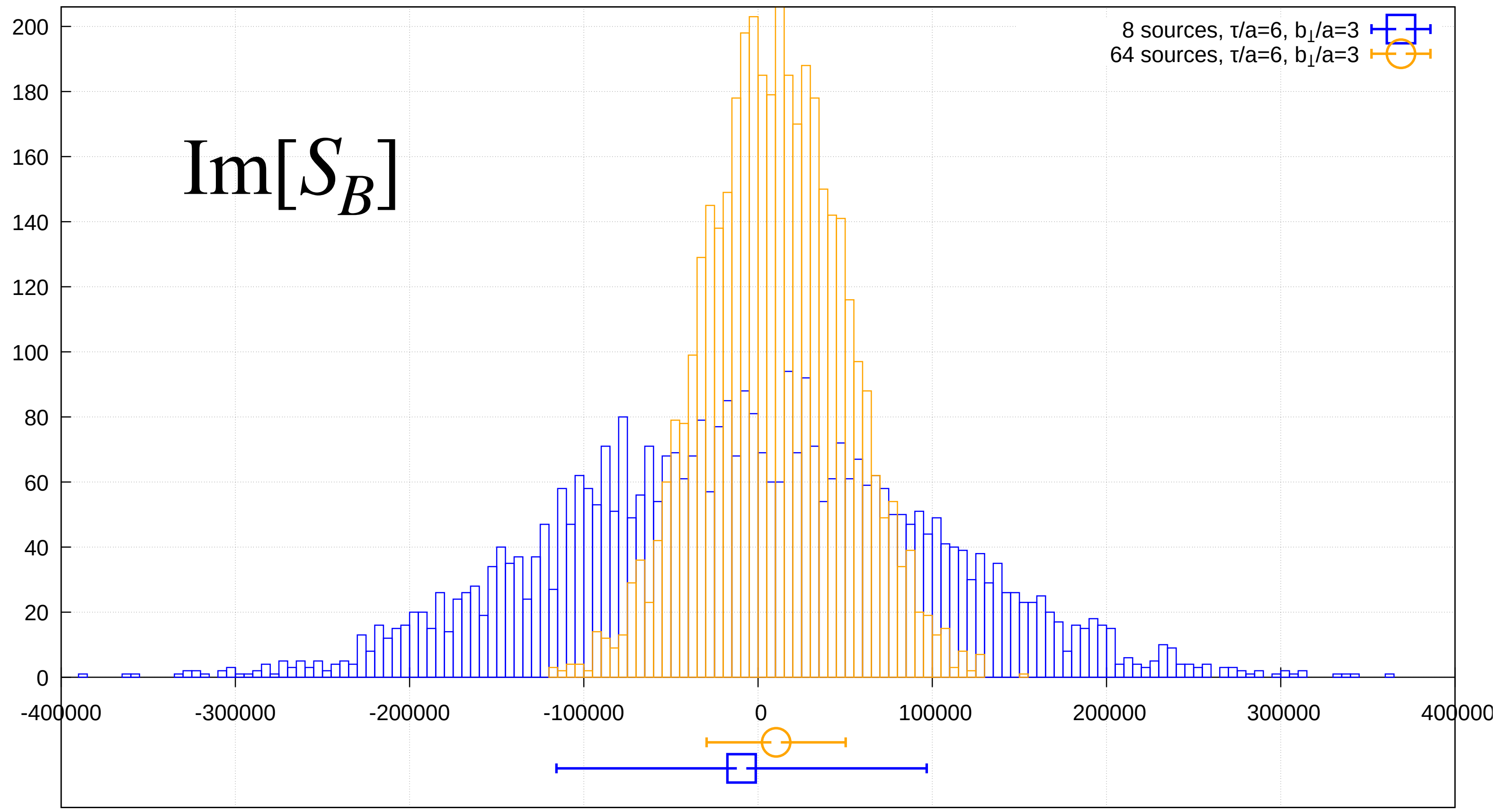
# Numerical exploration of our method



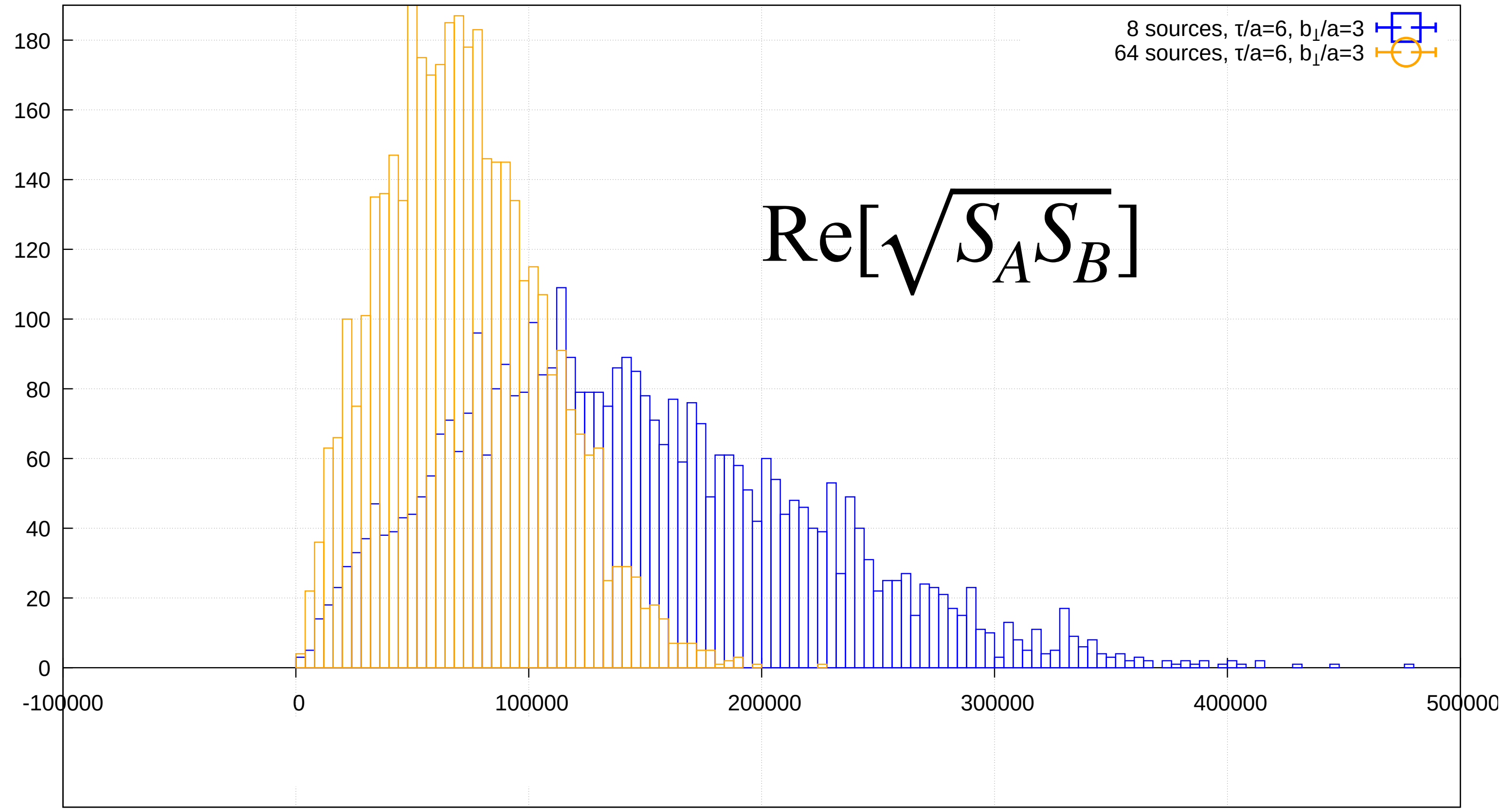



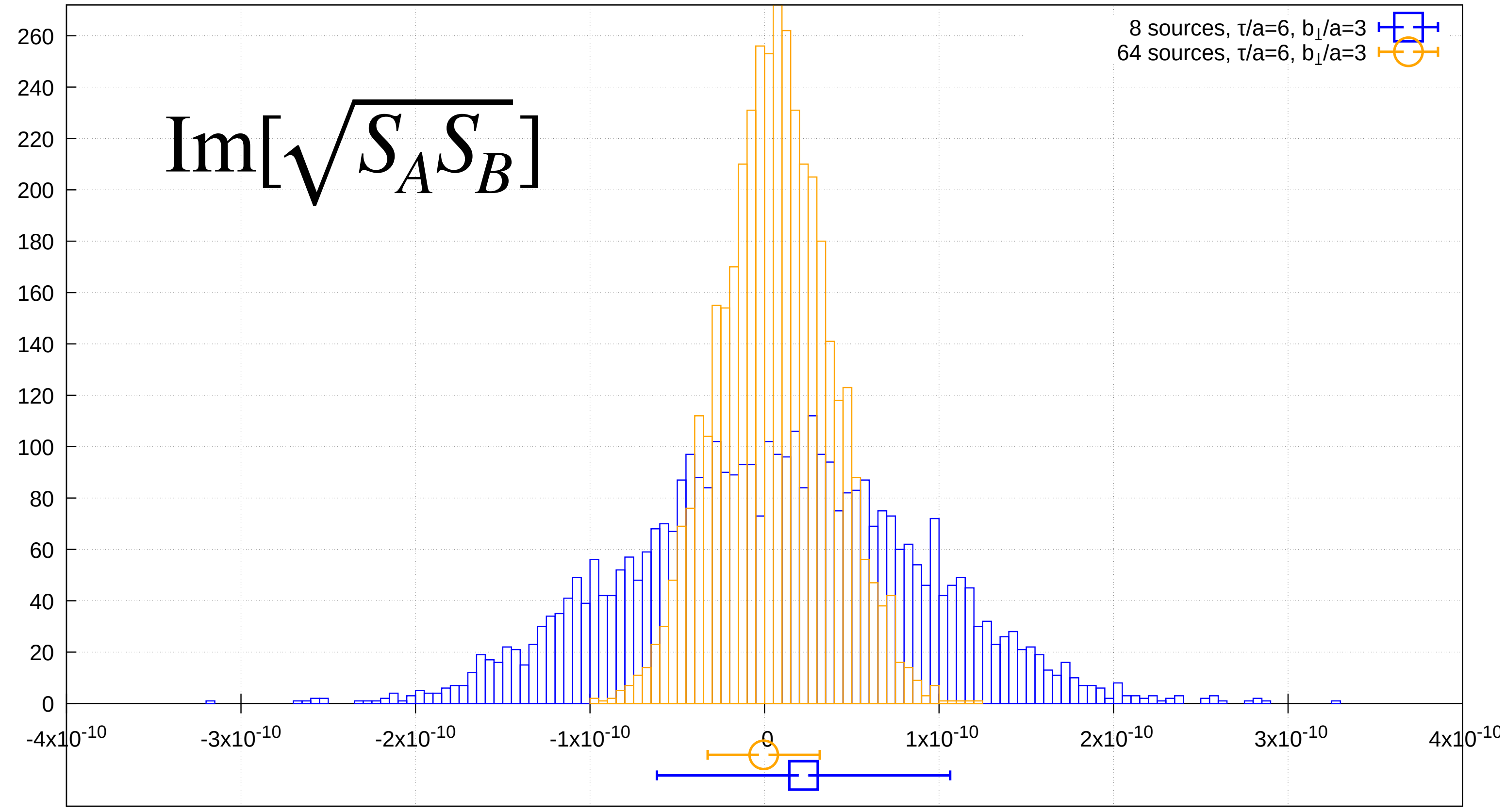
# Numerical exploration of our method



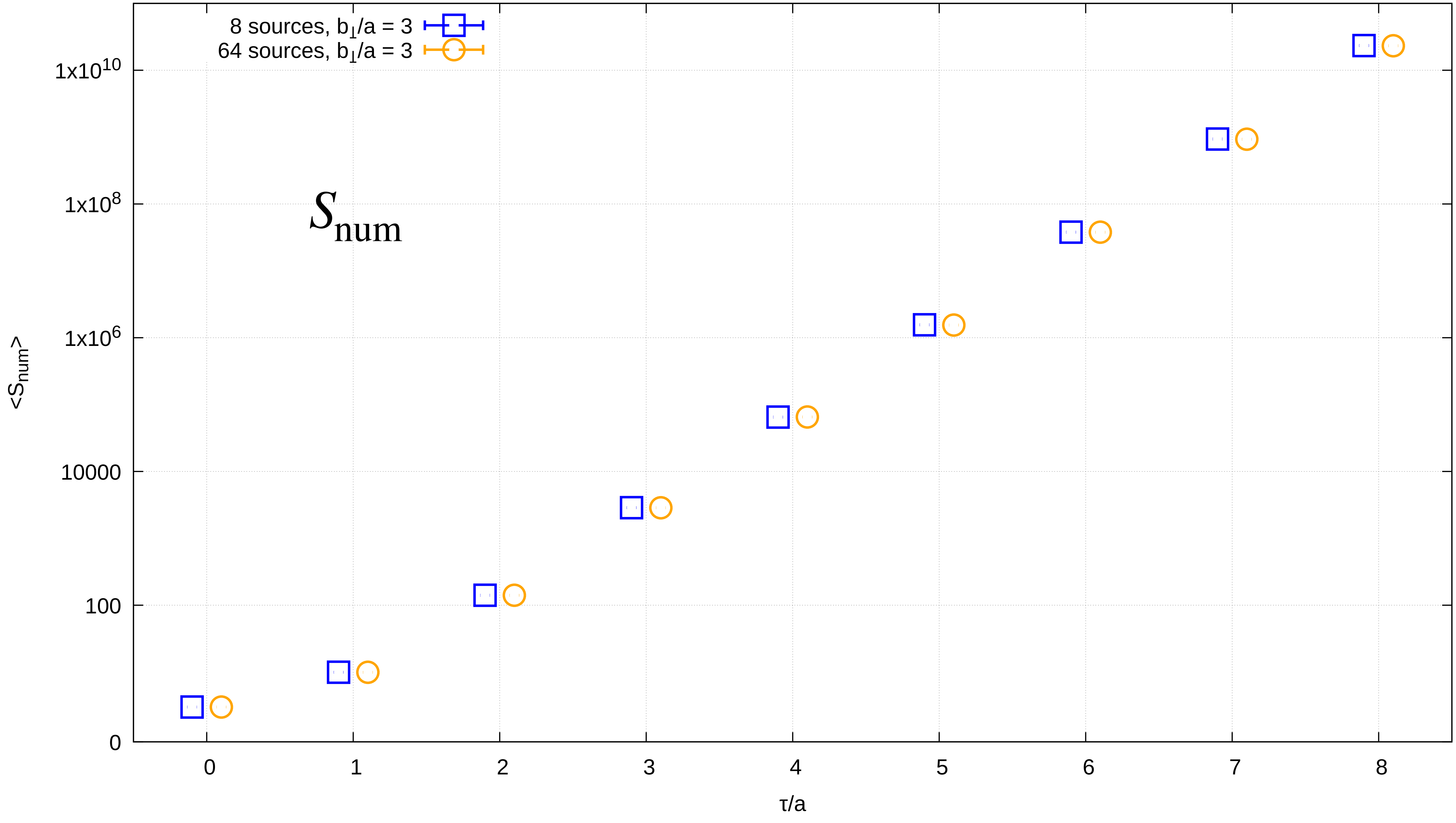



# Numerical exploration of our method

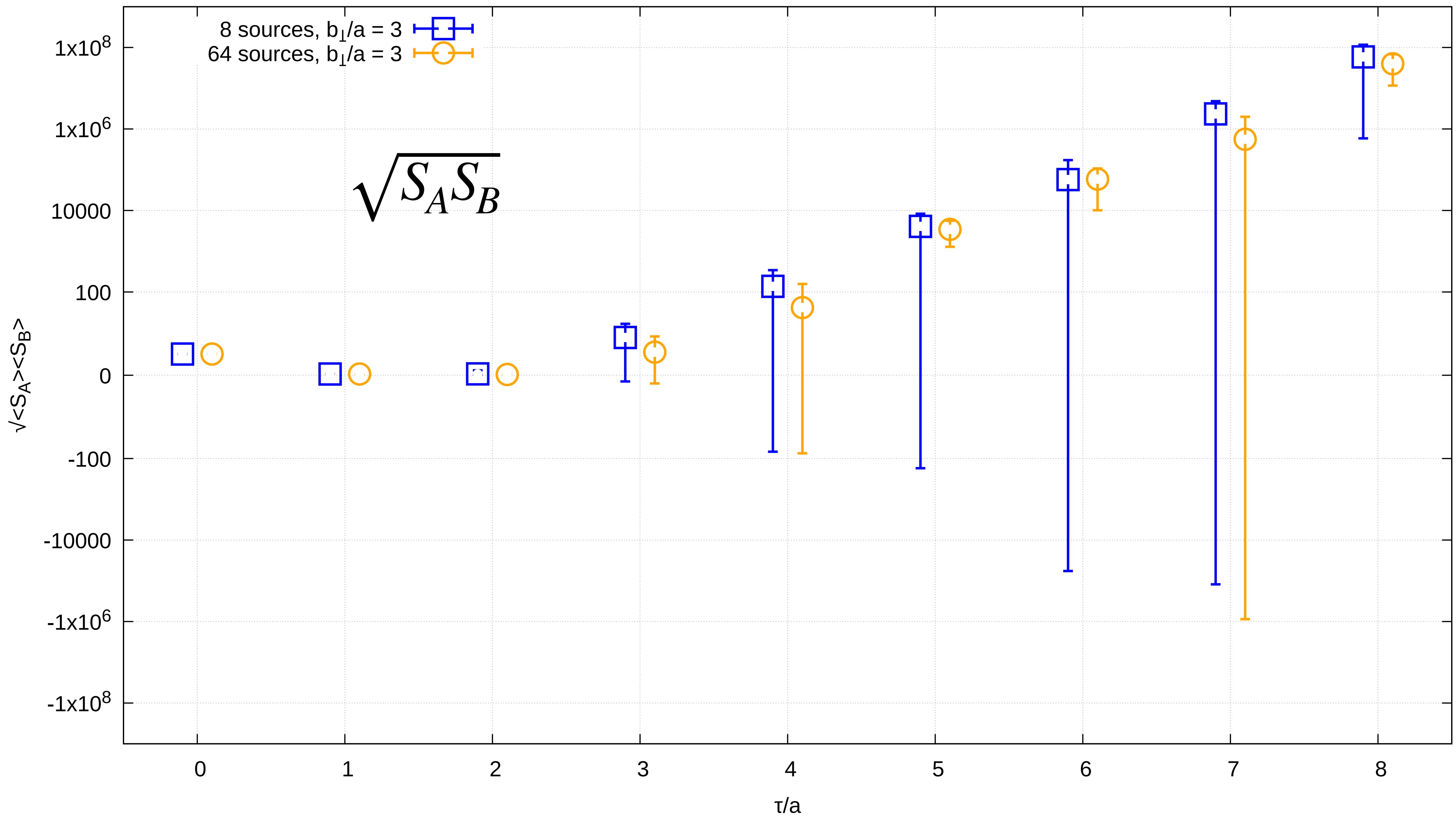


# Numerical exploration of our method



# Numerical exploration of our method



# Conclusion and outlook

★ Lattice QCD can contribute to TMD physics

→ Most calculations on the $K(b_T, \mu)$and $S_I(b_T, \mu)$

→ Existing results obtained from hadronic M.E.

*Need alternative methods to check universality*

★ We have proposed an alternative method

→ Does not involve hadronic M.E.

→ Complex-directional Wilson loops in Euclidean space

→ Numerical calculation on-going