The parity-odd structure function of the nucleon from the Compton amplitude in lattice QCD

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Granada, Lattice 2017

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Jordan Mckee
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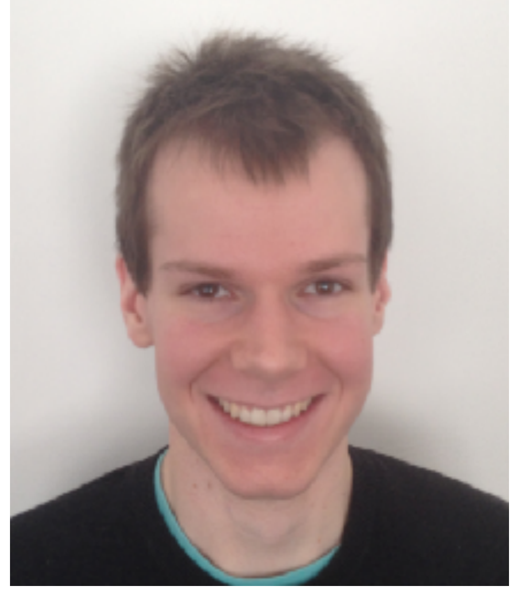

Alex Chambers
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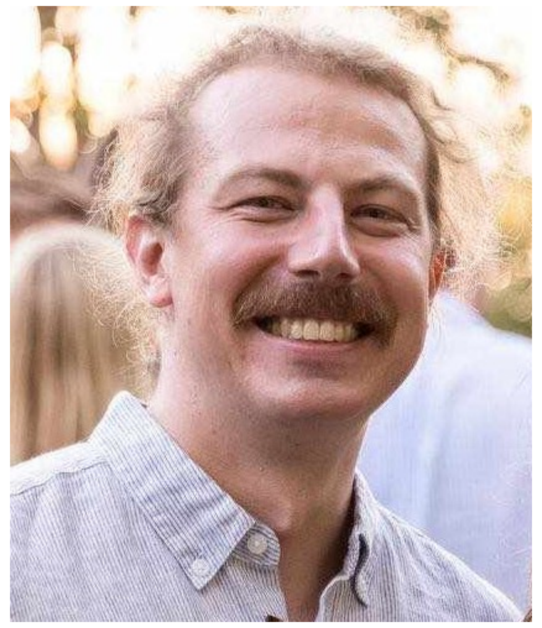
PhD 2024 (?)



Joshua Crawford
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PhD ongoing

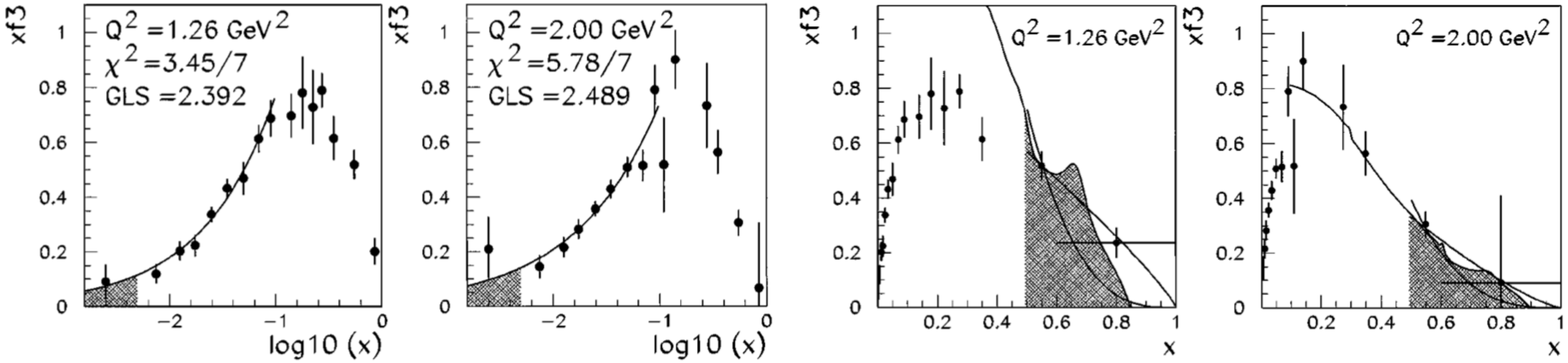
QCDSF Talks

- **James Zanotti** [Mon Session E (QCD and New Physics) @ 16:20]
  Constraining beyond the Standard Model nucleon isovector charges
- **Ross Young** [Mon Session B (Light Quarks) @ 17:00]
  Revealing the transverse force distributions in the nucleon from lattice QCD
- **Jordan Mckee** [Wed Session B (Light Quarks) @ 18:10]
  Compton Amplitude of the Pion using Feynman-Hellmann
- **Ian Van Schalkwyk** [Wed Poster @ 18:30]
  Calculation of the Compton Amplitude at High Momentum using Momentum Smearing
- **Nabil Humphrey** [Thur Session B (Light Quarks) @ 11:00]
  Multi-nucleon matrix elements on the lattice with e-graph optimised Wick contractions and the Feynman-Hellmann theorem
- **Thomas Schar** [Thur Session F (Nuclear and Astro-particle) @ 16:50]
  Reduction of discretisation artifacts in the lattice subtraction function calculation

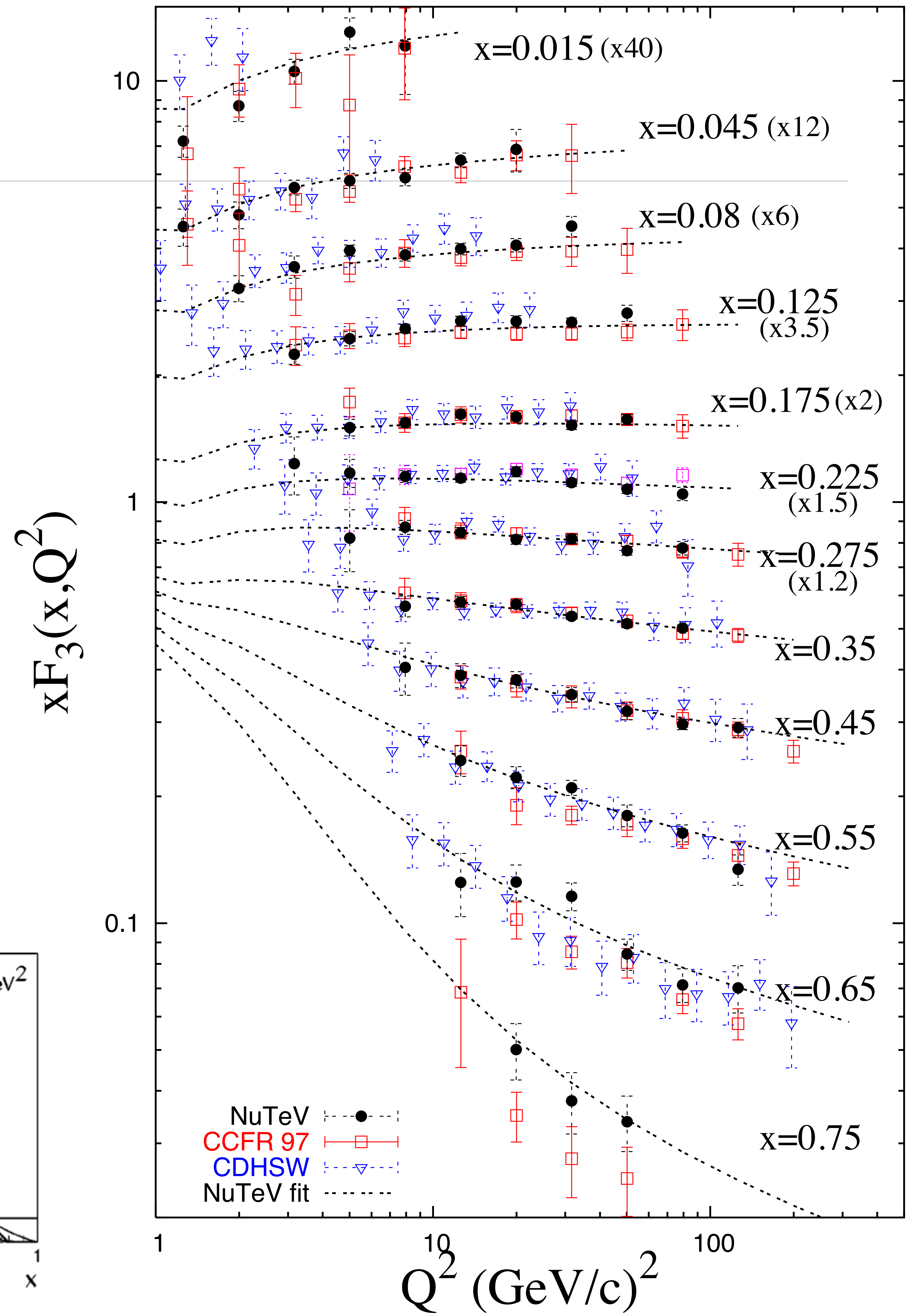
M. Tzanov et al. (NuTeV Collab.), PRD74 012008 (2006) [hep-ex/0509010]

Motivation

- Nucleon structure (leading twist)
  - PDFs from first principles
  - Understanding the behaviour in the high- and low-x regions
- World ν-N data:
  - **NuTev (Fermilab)**
  - **CHORUS (CERN)**
  - **CCFR (Fermilab) E744, E770, and older E180**
  - **BEBC (CERN) Gargamelle, WA25, and WA59**
  - **SKAT (Zeuthen)**



J. H. Kim et al., PRL81 (1998) 3595-3598 [hep-ex/9808015]



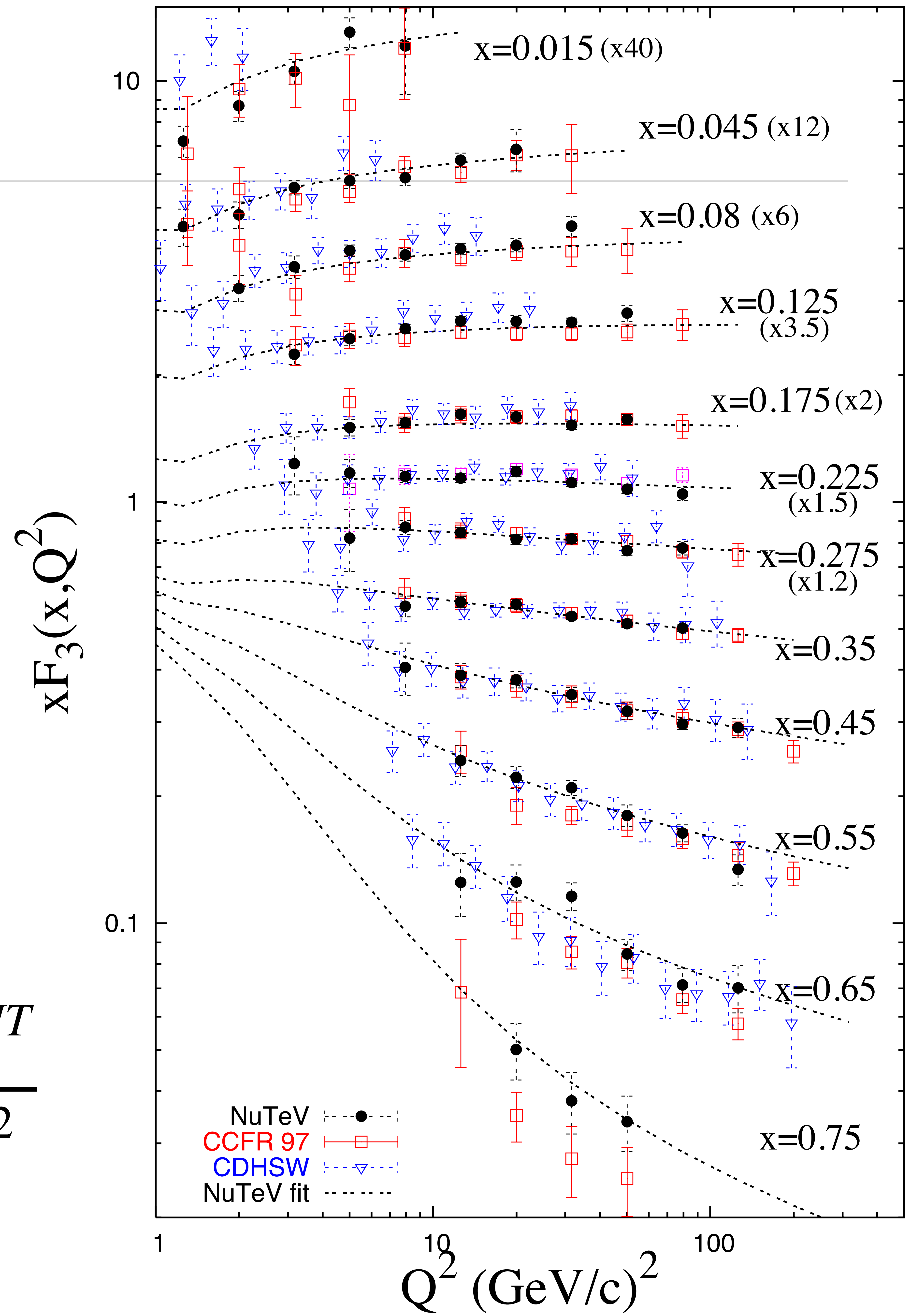
Motivation

- <u>Scaling</u>
  - Q^2 cuts of global QCD analyses
- <u>Power corrections / Higher twist effects</u>
  - Target mass corrections
  - Twist-4 contributions
  - GLS sum rule:

$$S^{GLS} = \int_0^1 dx F_3^{(\nu p + \bar{\nu} p)}(x, Q^2) = 3\left[1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots\right] - \frac{\Delta^{HT}}{Q^2}$$

  - $\Delta^{HT} \sim 0.15 - 0.5$ see X.-D. Huang et al., NPB969 (2021) 115466 [2101.10922]

M. Tzanov et al. (NuTeV Collab.), PRD74 012008 (2006) [hep-ex/0509010]



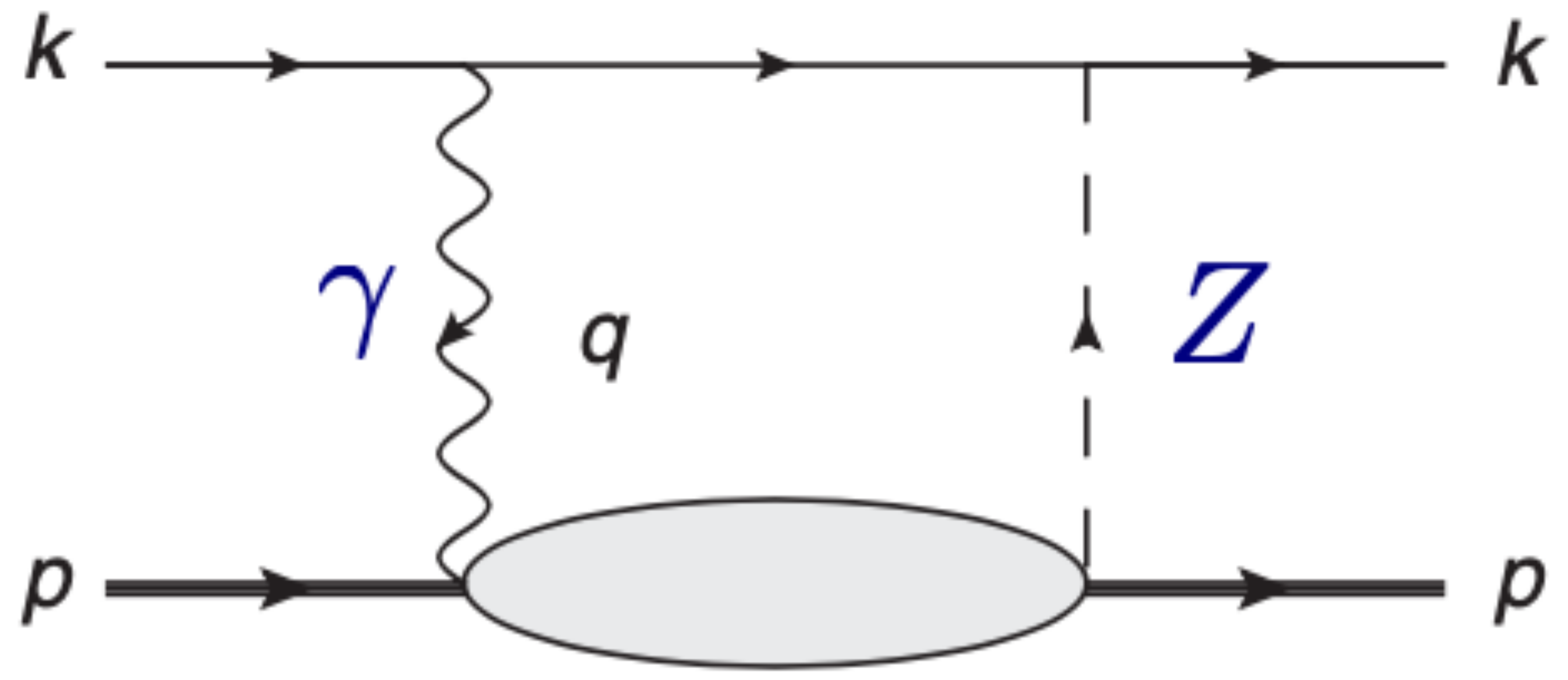
Motivation | EW Box

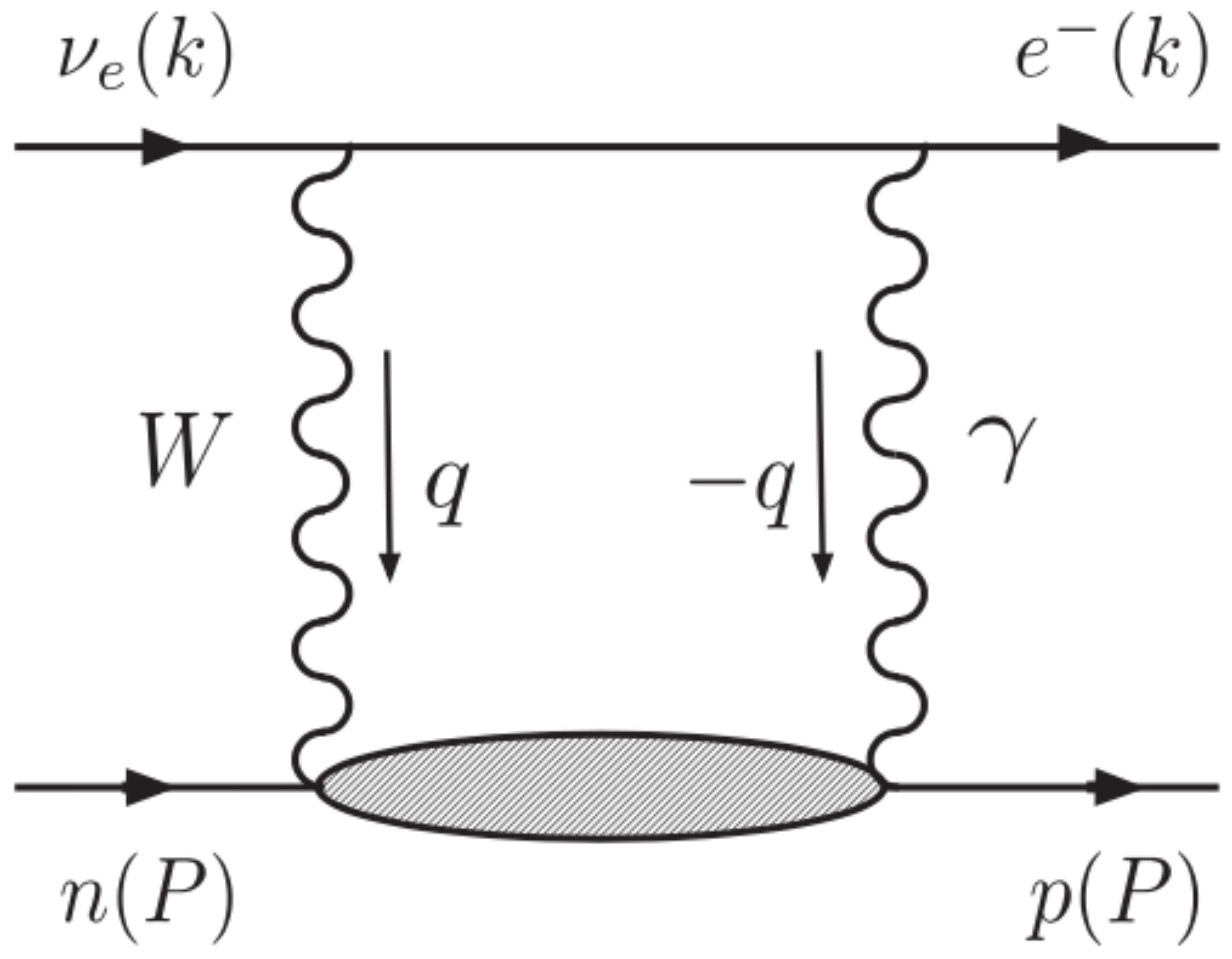
- Leading theoretical uncertainty in:
- Weak charge of the proton,

$$Q_W = (1 + \Delta_\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e')$$
$$+ \Box^{WW}_{AA} + \Box^{ZZ}_{AA} + \Box^{\gamma Z}_{VA}$$

- CKM matrix element extracted from superallowed β decays,

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathscr{F}t(1+\Delta_R^V)} \rightarrow \propto \Box^{\gamma W}_{VA}$$



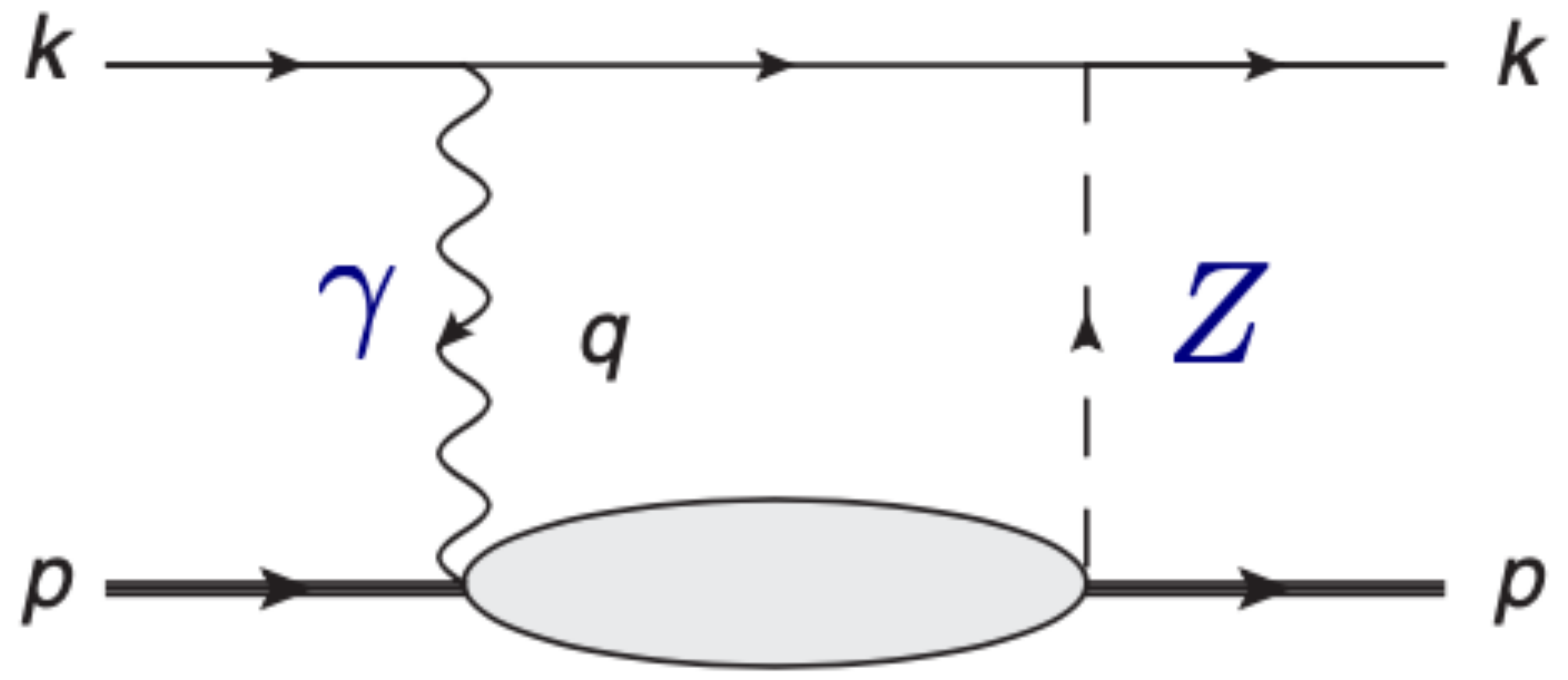


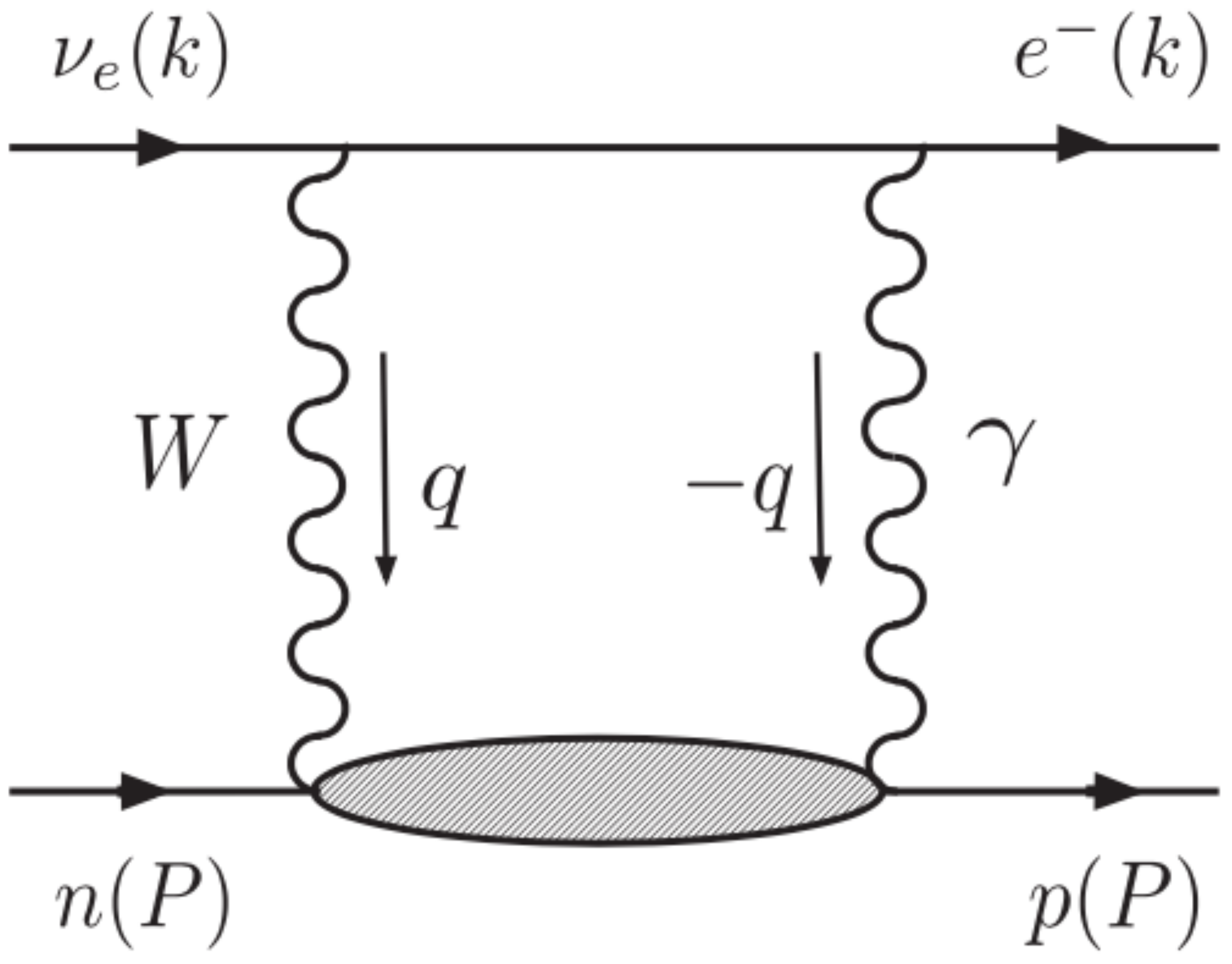
Motivation | EW Box

- Box diagrams proportional to an integral over the whole Q^2 range

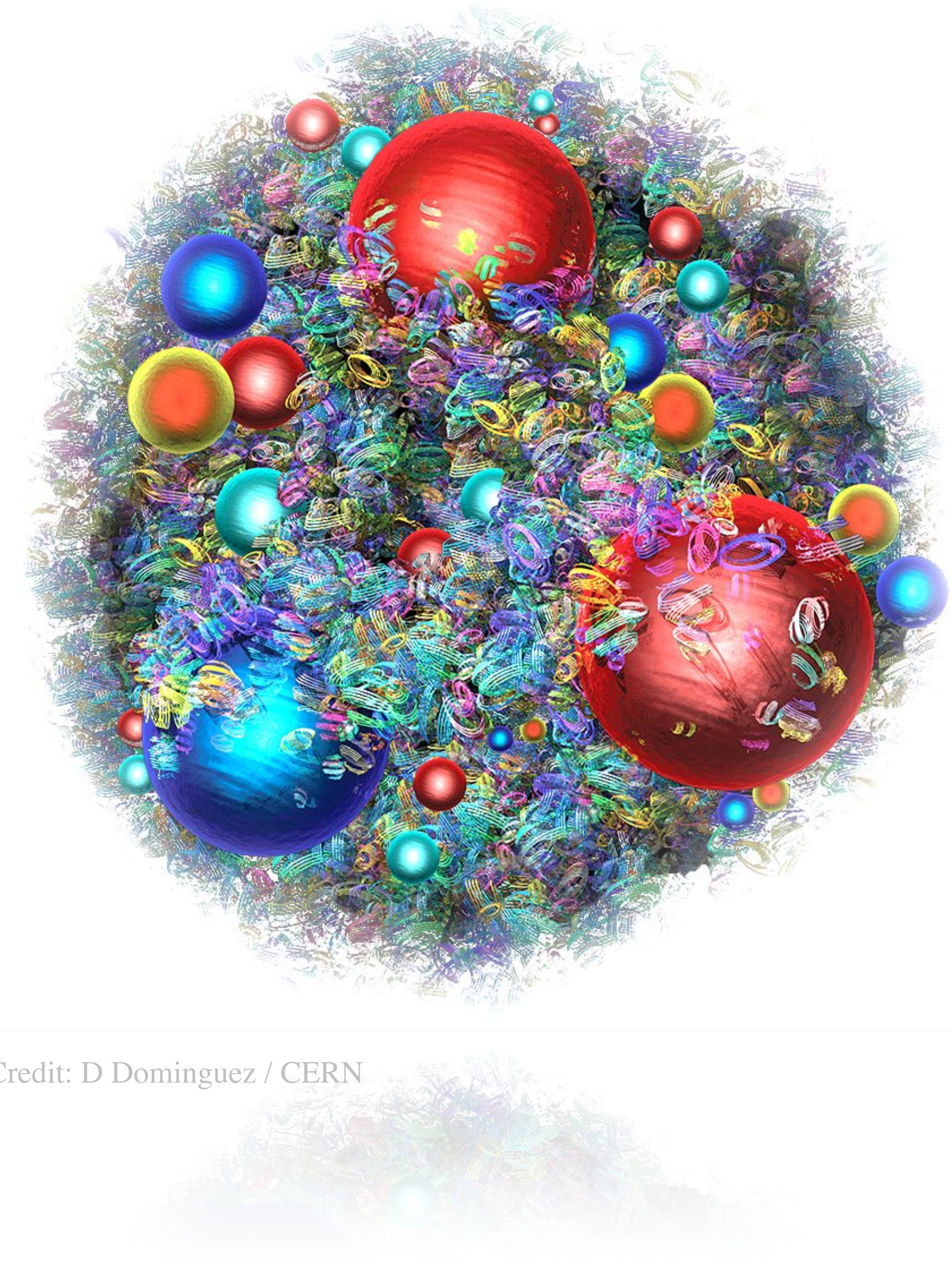
$$\Box_A^{\gamma Z/W} \propto \int_0^\infty \frac{dQ^2}{Q^2}\, \mu_1^{(3)}(Q^2)\, (\ldots)$$

- Low-Q^2 (non-perturbative) regime dominates the integral
- F_3 is experimentally poorly determined in low Q^2
- Lattice approach is ideal for a high-precision determination of moments



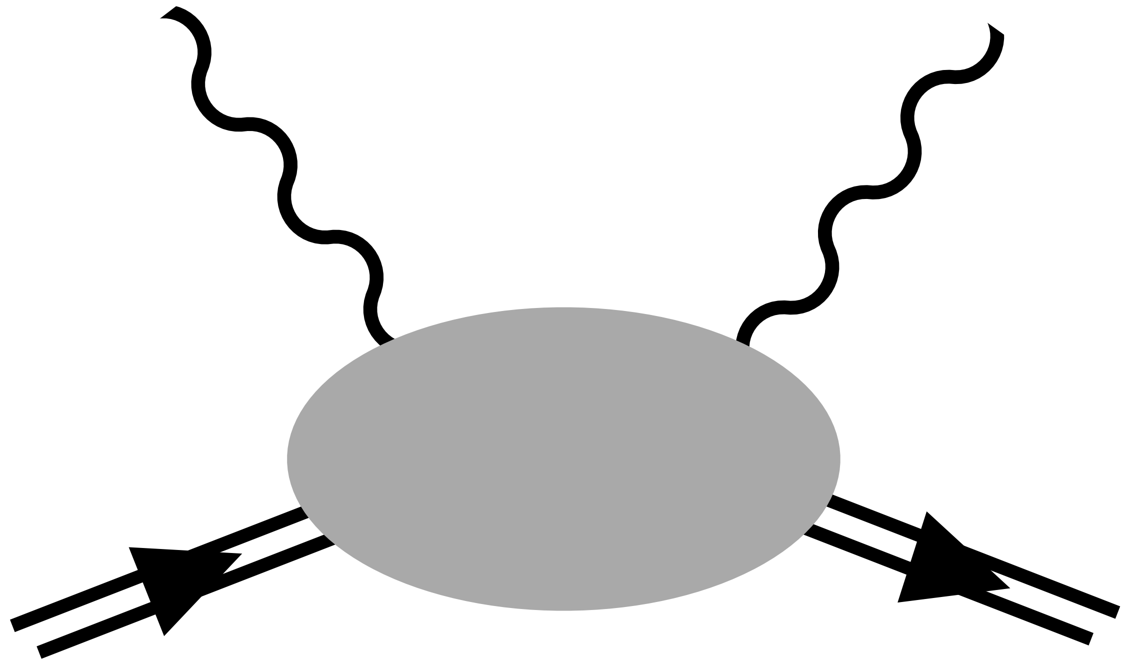


Outline



- Forward Compton Amplitude
- Feynman-Hellmann Theorem on the Lattice
- Parity-violating F_3
- Summary & Outlook

Forward Compton Amplitude



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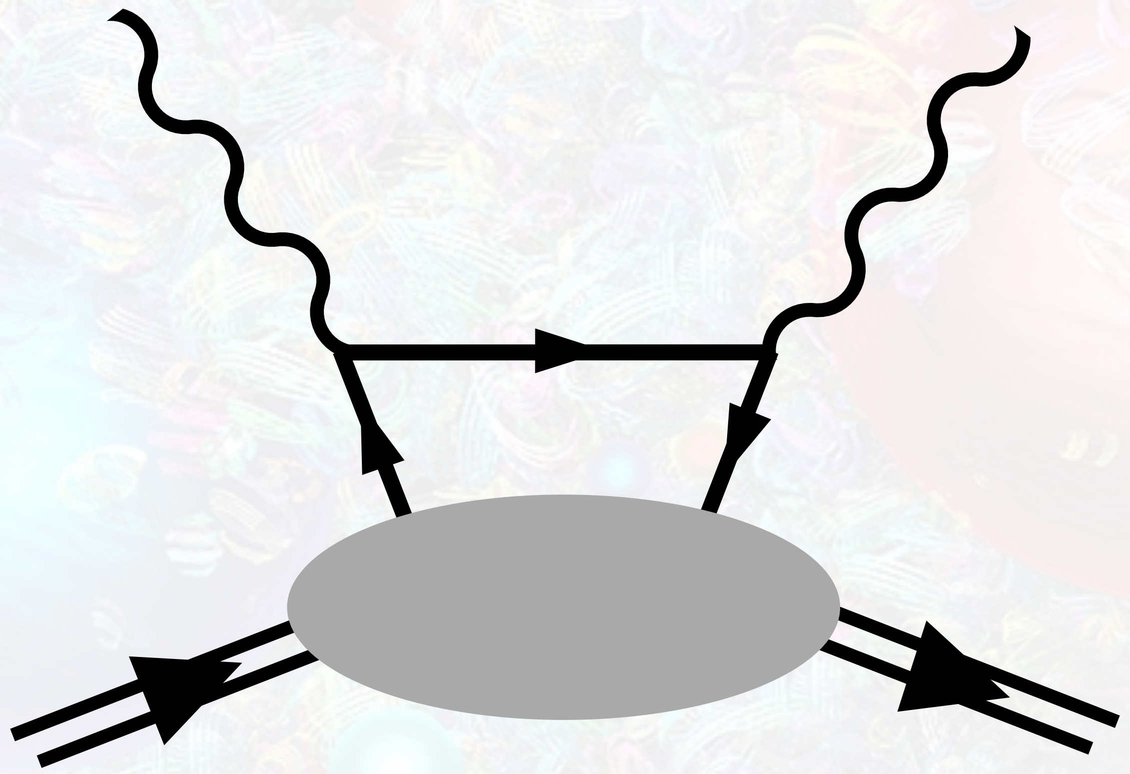


$$+\ \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q^2}, \frac{M_N^2}{Q^2}\right)$$

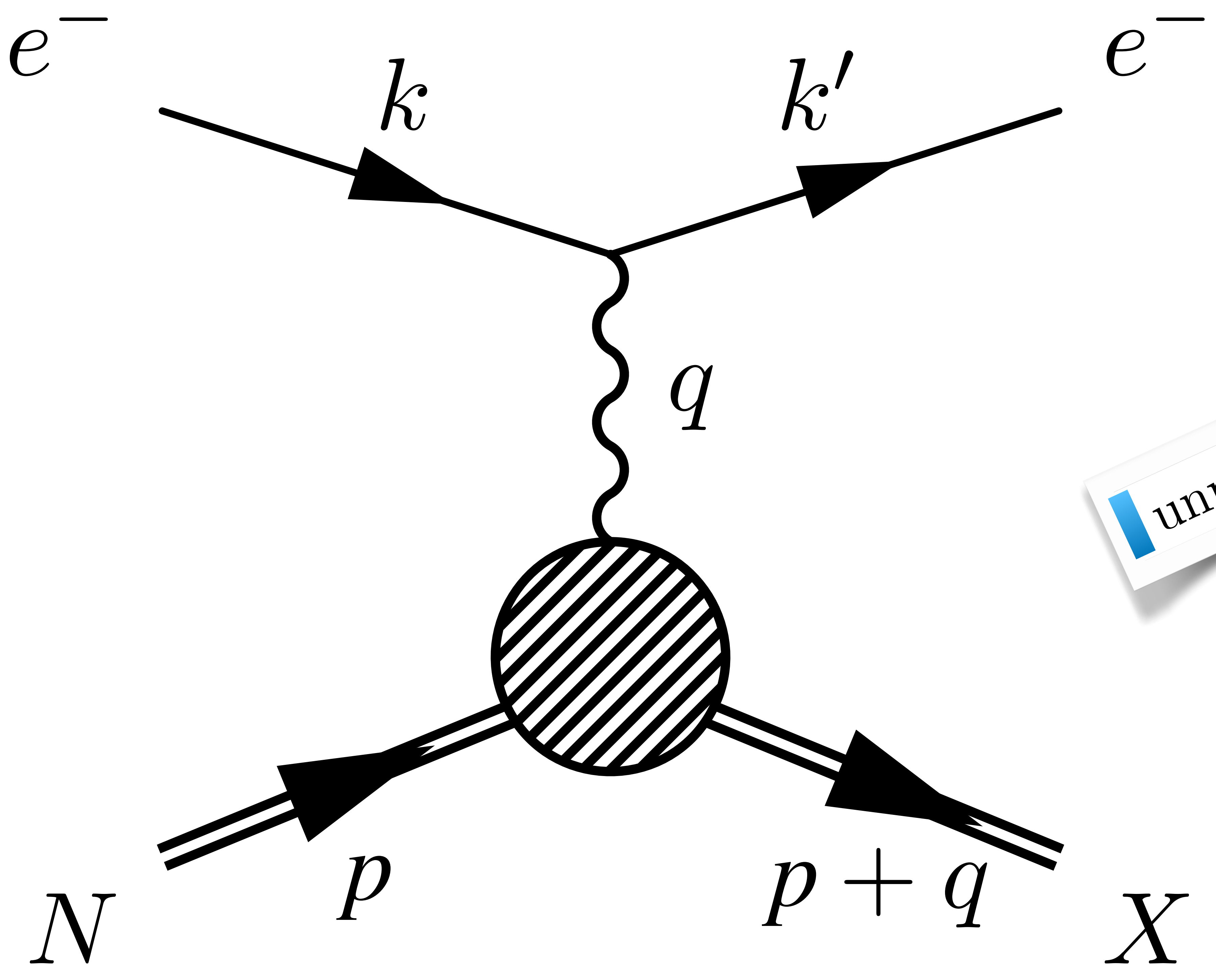
DIS and the Hadronic Tensor

Deep ($Q^2 \gg M^2$) inelastic ($W^2 \gg M^2$) scattering (DIS)

$$d\sigma \sim L_j^{\mu\nu} W_{\mu\nu}^j$$

$j = \gamma, Z$, and γZ (neutral) or W (charged)

leptonic tensor *hadronic tensor*



$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq\cdot z} \rho_{ss'} \langle p, s' | [J_\mu^V(z), J_\nu^V(0)] | p, s \rangle$$

$$\rho_{ss'} = \tfrac{1}{2}\delta_{ss'}$$

unpolarised

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) \frac{F_2(x, Q^2)}{p \cdot q}$$

Structure Functions

Parity Violating Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i \int d^4z\, e^{iq\cdot z} \rho_{ss'} \langle p, s' | \mathscr{T}\{J_\mu(z) J_\nu(0)\} | p, s \rangle$$, spin avg. $\rho_{ss'} = \frac{1}{2}\delta_{ss'}$

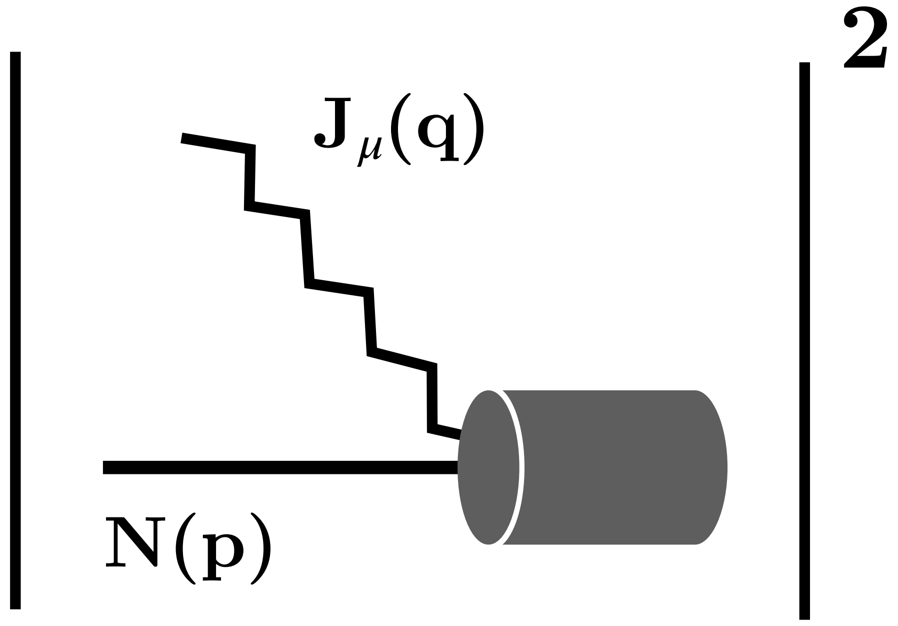
Same Lorentz decomposition as the Hadronic Tensor

$$= -g_{\mu\nu}\mathscr{F}_1(\omega, Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathscr{F}_2(\omega, Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathscr{F}_3(\omega, Q^2)$$

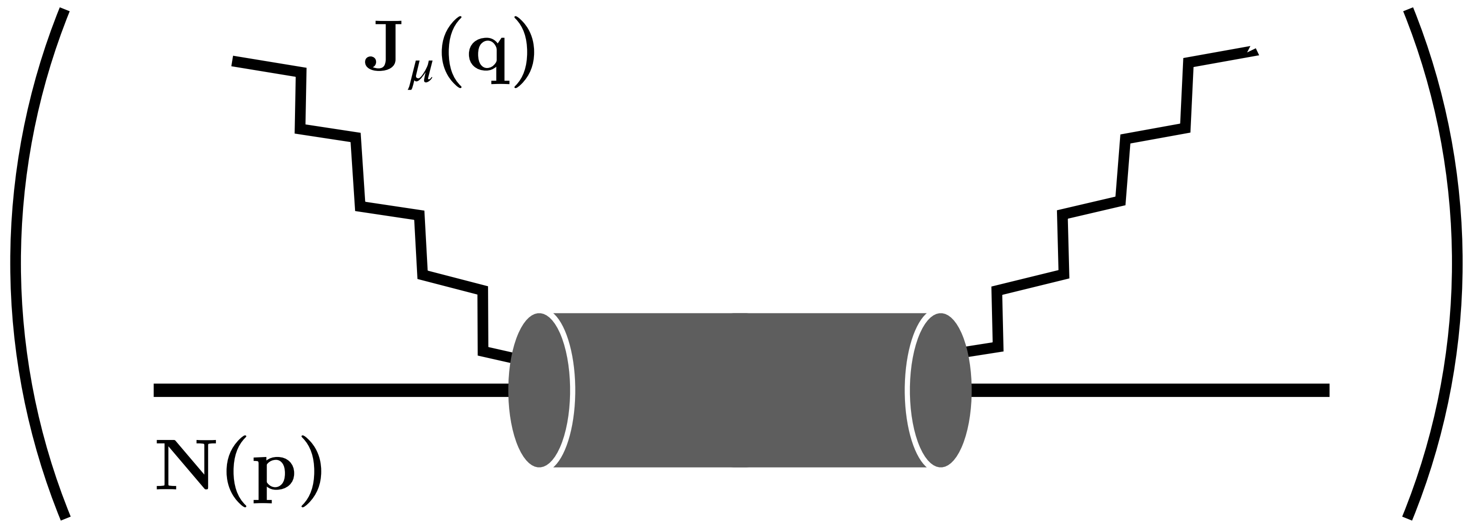
$$+\frac{q_\mu q_\nu}{p\cdot q}\mathscr{F}_4(\omega, Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathscr{F}_5(\omega, Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathscr{F}_6(\omega, Q^2)$$

allowed terms because parity is violated

Optical theorem



$\sim$ $2\,\mathrm{Im}$



$$\omega = \frac{2p\cdot q}{Q^2}$$

$$\varepsilon^{0123} = 1$$

DIS Cross Section ~ Hadronic Tensor

Forward Compton Amplitude ~ Compton Tensor

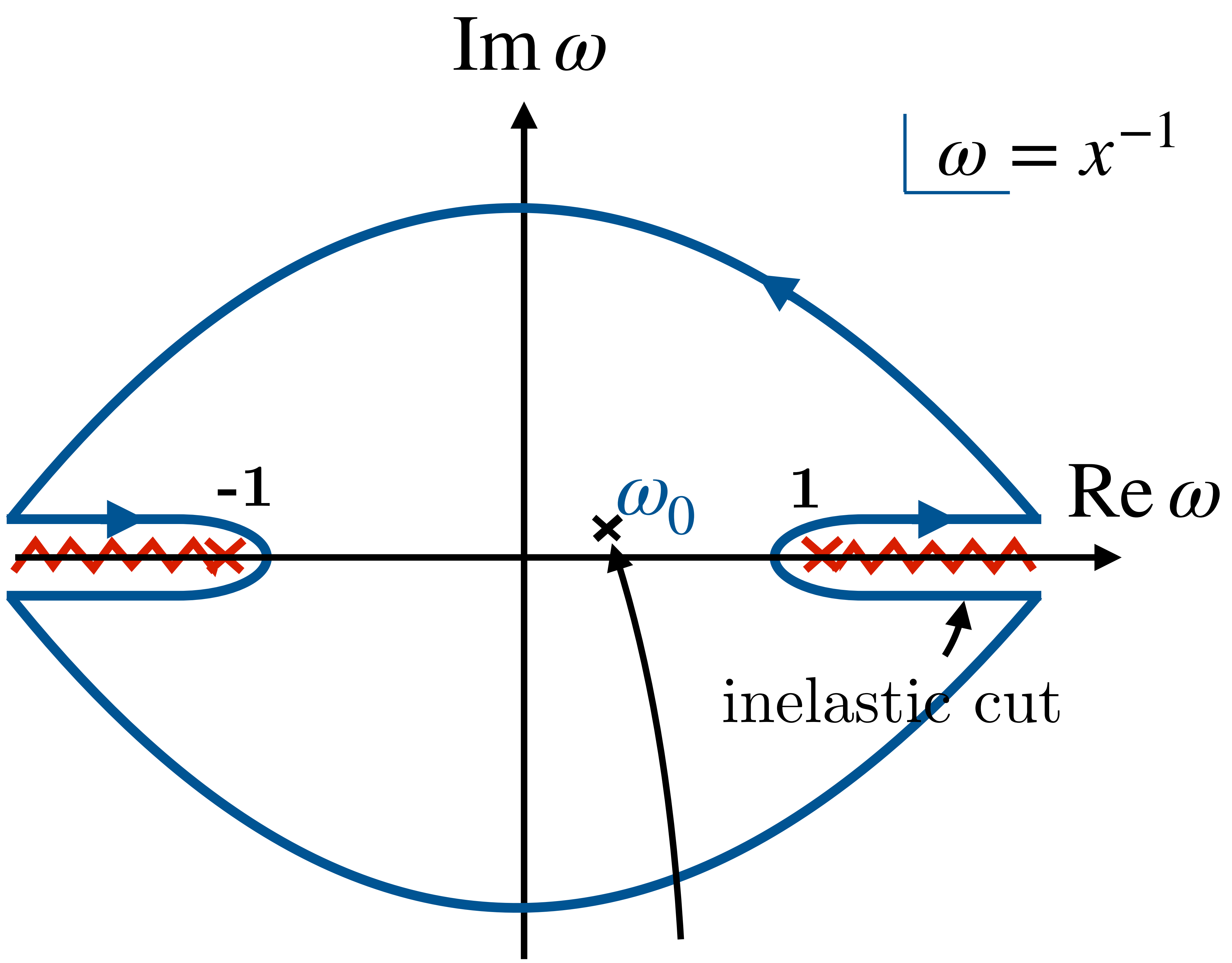
Nucleon Structure Functions

- **for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,**

$$T_{\mu\nu}(p,q) = i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathscr{F}_3(\omega,Q^2)$$

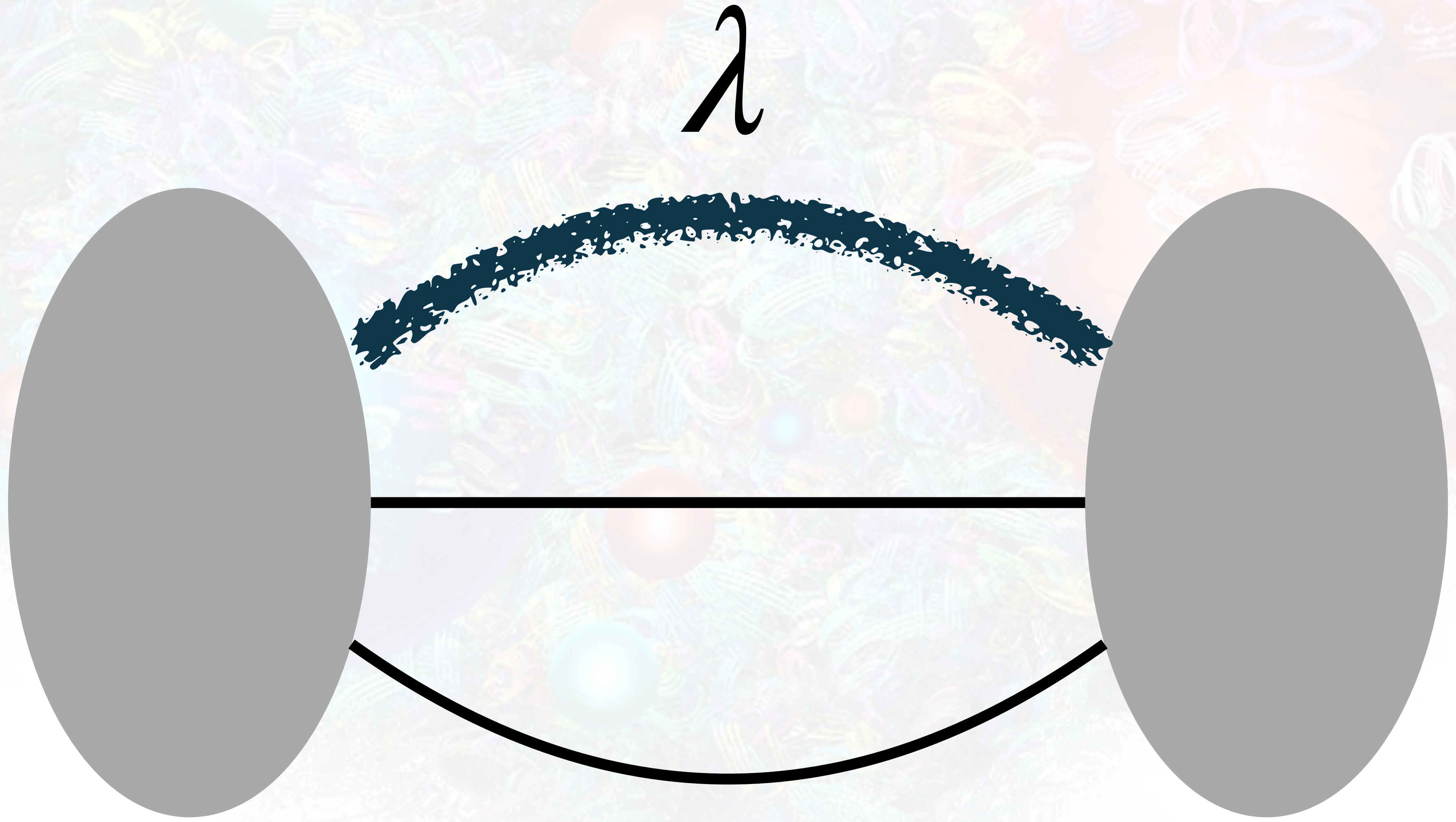
- **we can write down dispersion relations and connect Compton SFs to DIS SFs:**

$$\mathscr{F}_3(\omega,Q^2) = 4\omega\int dx\frac{F_3(x,Q^2)}{1-x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

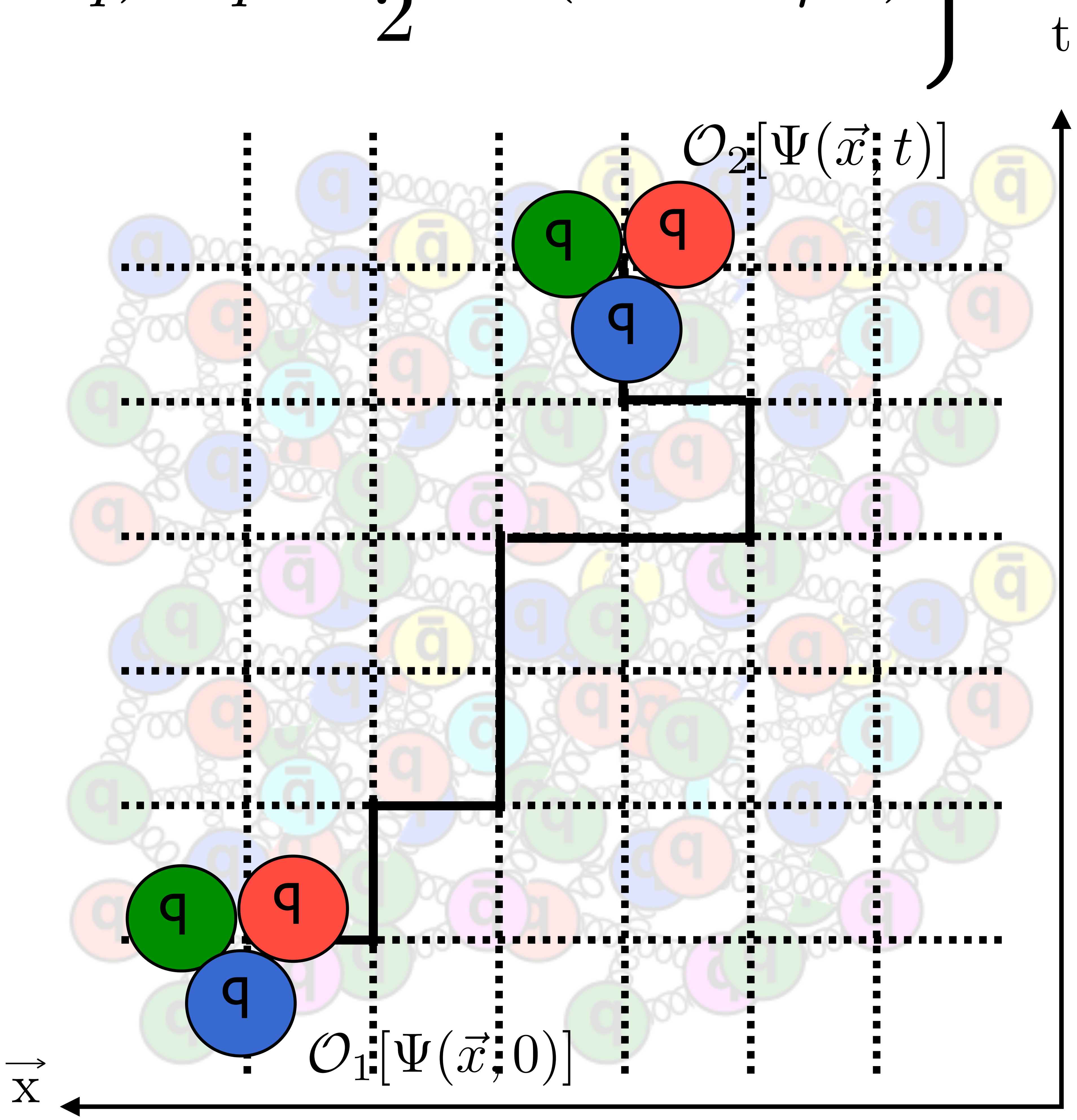
Feynman-Hellmann Theorem on the Lattice



Lattice QCD

$$\mathcal{S}_{QCD}[\psi,\bar{\psi},A] = \int d^4x \left\{ \sum_q \bar{\psi}_q \left(i\gamma^\mu D_\mu - m_q \right) \psi_q - \frac{1}{2}\mathrm{Tr}\left(G^{\mu\nu} G_{\mu\nu} \right) \right\}$$

$$\langle \hat{O}_2(t)\hat{O}_1(0)\rangle = \frac{\int D[\Psi] e^{-S_E[\Psi]} O_2[\Psi(x,t)] O_1[\Psi(x,0)]}{\int D[\Psi] e^{-S_E[\Psi]}}$$



- Discretise the space-time continuum: regularises the theory
- Compute the observables via supercomputer simulations
  - i.e. approximate the infinite dimensional path integral

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} \mathcal{O}[\Psi]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} = \lim_{N\to\infty} \sum_{n=1}^{N} \mathcal{O}[U_n]$$

- Take the appropriate limits to recover continuum physics
  - $a \to 0,\ m_\pi^{\mathrm{latt}} \to m_\pi^{\mathrm{phys}},\ V \to \infty$

FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

$$S \to S(\lambda) = S + \lambda \int d^4x\, \mathcal{O}(x) \xrightarrow{\text{e.g. local bilinear operator}} \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \dots\}$$

real parameter

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda}\langle 0 | \mathcal{O} | 0 \rangle$$

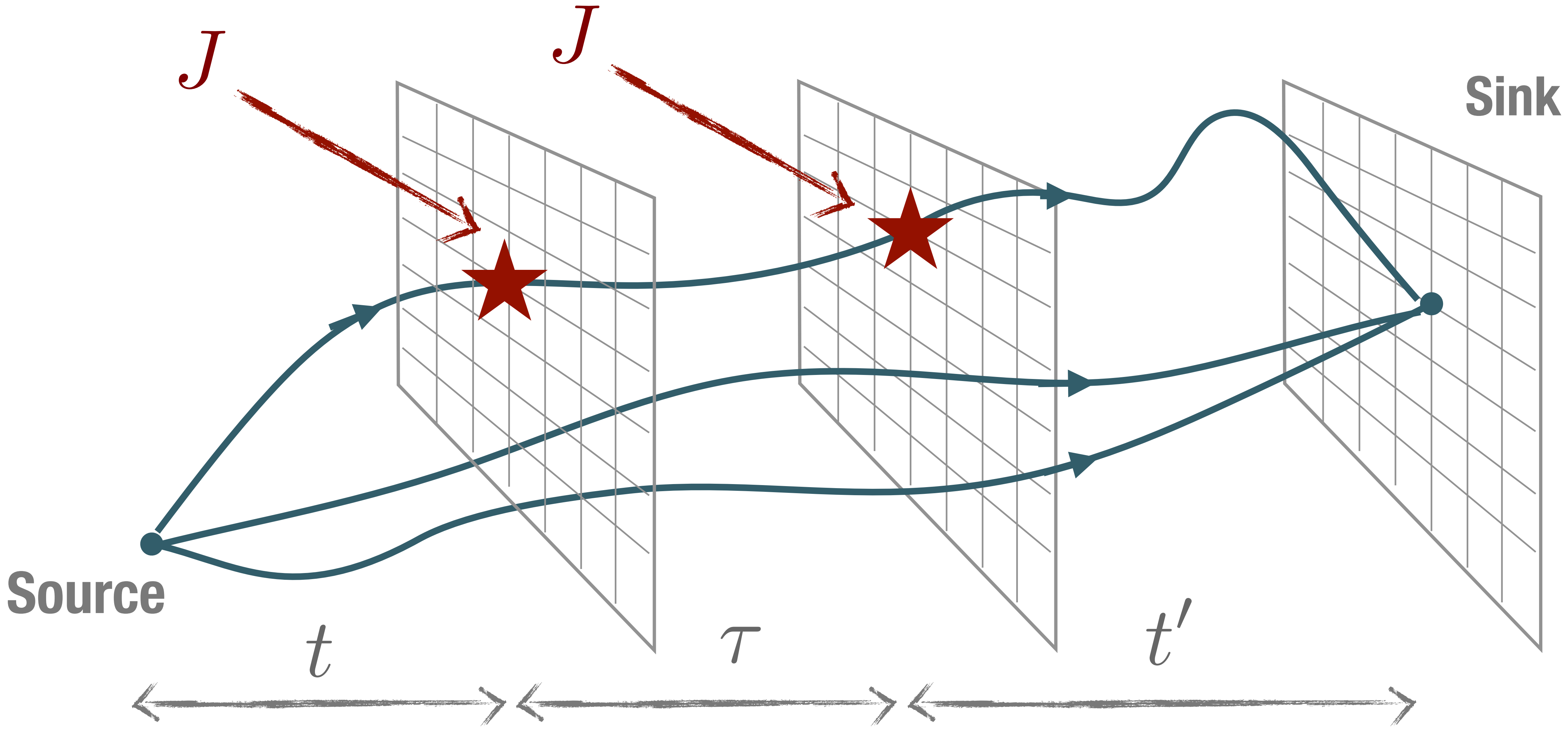
$E_\lambda \to$ spectroscopy, 2-pt function

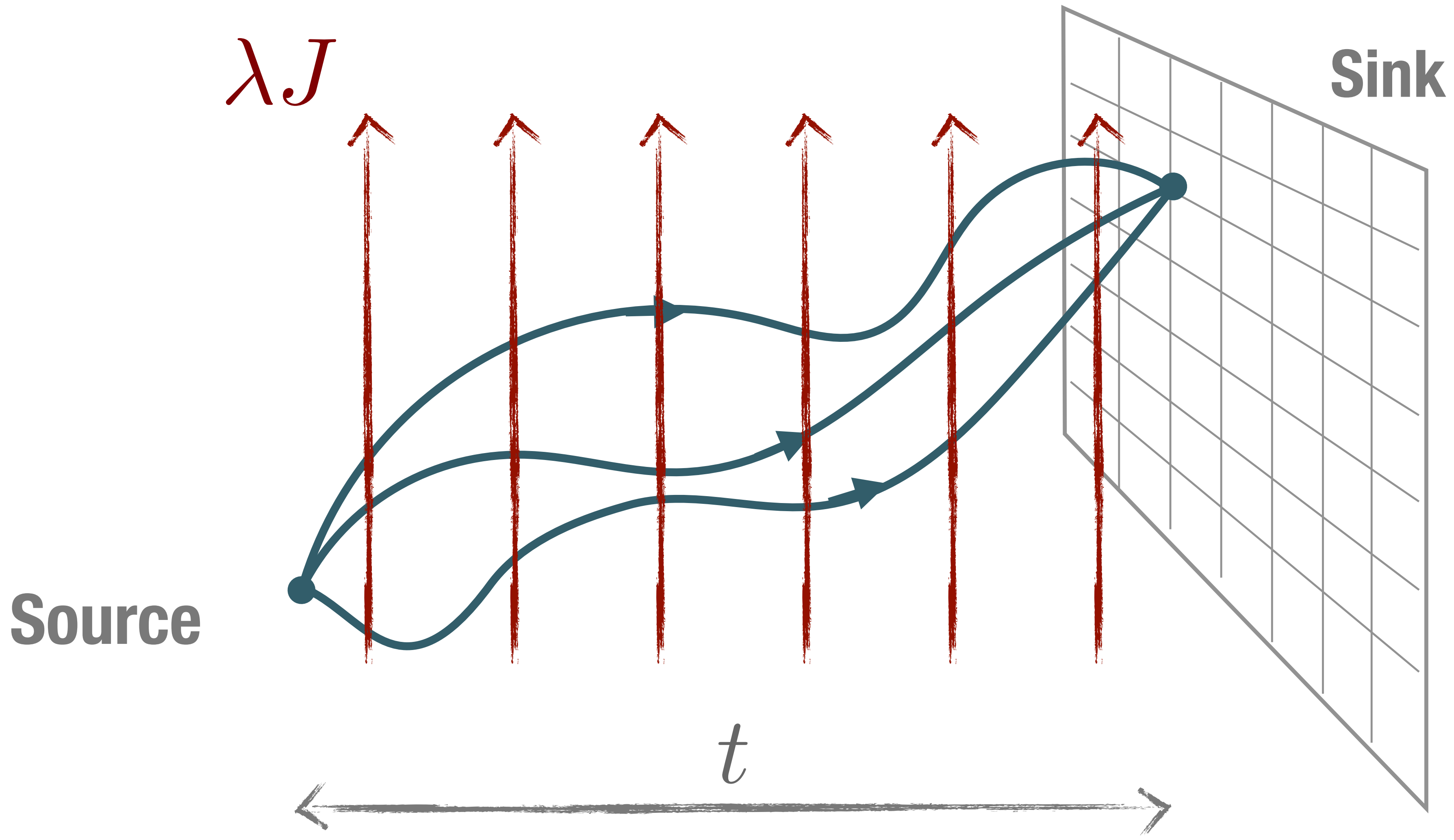
$\langle 0 | \mathcal{O} | 0 \rangle \to$ determine 3-pt

Applications:

- **σ - terms**
- **Form factors**

Compton amplitude





- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

energy gap to the lowest excitation

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N|J(\tau_E)J|N\rangle$$

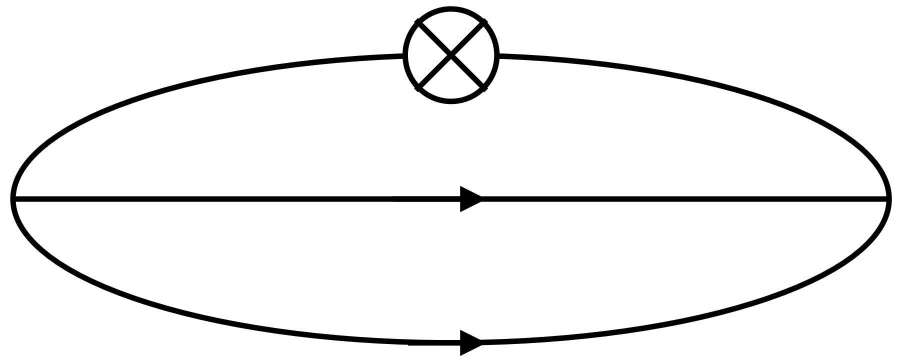
$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$

- **Feynman—Hellmann**

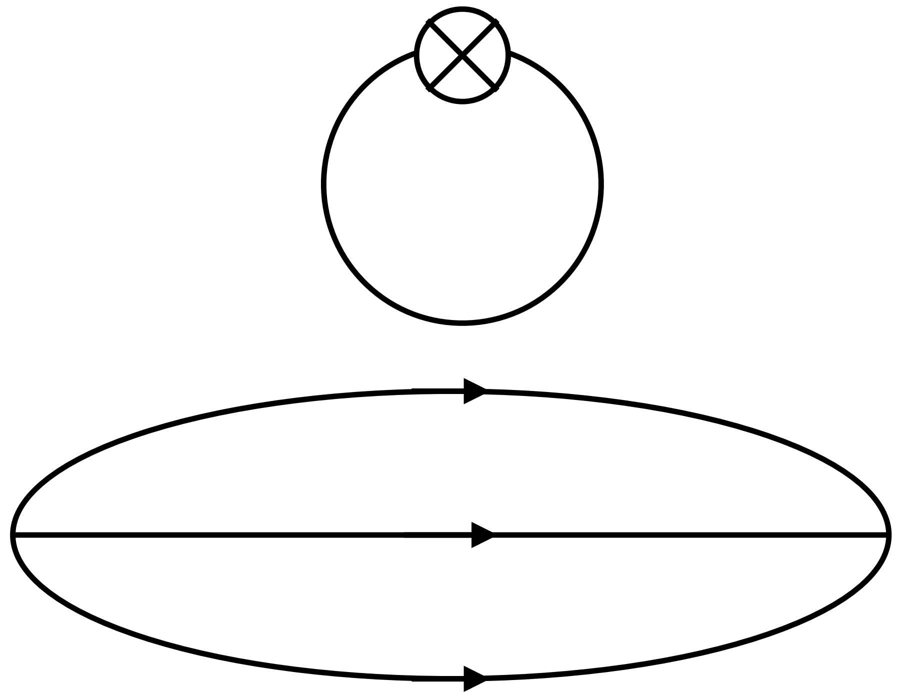
$$t \gg \frac{1}{\Delta E}, \qquad \left.\frac{\partial^2 E}{\partial\lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

QCDSF Applications of FH

- Can modify fermion action in 2 places:
  - quark propagators
  - fermion determinant



Connected



Disconnected

(Requires new gauge configurations)

g_A, $\Delta\Sigma$ [PRD90 (2014)]

NPR [PLB740 (2015)]

G_E,G_M [PRD96 (2017)]

$F_{1,2}(\omega, Q^2)$ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]

GPDs [PRD104 (2022), PRD110 (2024)]

$\Sigma \to n$ [PRD108 (2023) 3, 034507]

g_A, g_T, g_S [PRD108 (2023) 9, 094511]

$\langle x \rangle_g$ [PLB714 (2012)]

NPR [PLB740 (2015)]

Δs [PRD92 (2015)]

Parity-violating $\mathscr{F}_3$
and
the $\gamma - Z/W$ boxes

Parity Violating Forward Compton Amplitude

- **Expand the dispersion relation for small $\omega \rightarrow$ 1st Cornwall-Norton moment:**

$$M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x, Q^2) = \left.\frac{\mathscr{F}_3(\omega, Q^2)}{4\omega}\right|_{\omega=0}$$

allows for a test of the Gross-Llewellyn-Smith sum rule $\quad (a_s = \alpha_s(Q^2)/\pi)$

$$M_{1,uu}^{(3)}(Q^2) = \int_0^{1^-} dx F_3(x, Q^2) = 2\left(1 + \sum_{i=1}^{4} a_s^i \, c_i(n_f)\right) + \frac{\Delta_{HT}}{Q^2} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q^4}\right)$$

known coeffs.

Higher-twist

- **Also connected to the determination of the EW box diagram**

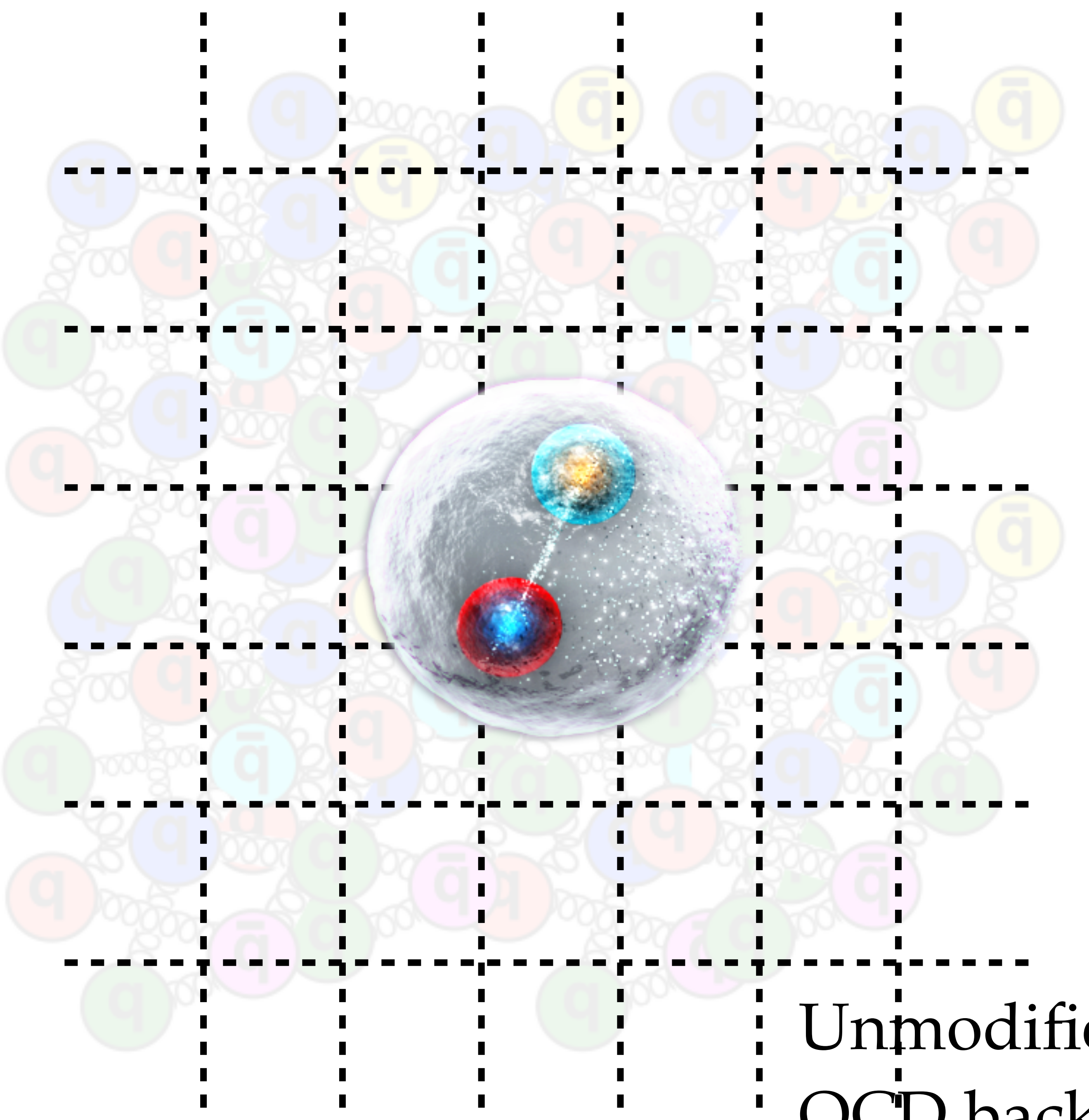
$$\square_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2 + Q^2}\mu_1^{(3)}(Q^2)$$

Calculation Details

QCDSF/UKQCD configurations
$48^3 \times 96$, 2+1 flavour (u/d+s)
$\beta = \begin{pmatrix} 5.65 \\ 5.95 \end{pmatrix}$, NP-improved Clover action
PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$m_\pi \sim 410$ MeV, ~SU(3) sym.

$$m_\pi L \sim \begin{bmatrix} 6.9 \\ 5.3 \end{bmatrix} \qquad a \sim \begin{bmatrix} 0.068 \\ 0.052 \end{bmatrix} \text{ fm}$$



- Local EM and axial current insertion, $J_\mu^{V[A]}(x) = Z_{V[A]}\bar{q}(x)\gamma_\mu[\gamma_5]q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Current momenta $0.1 \lesssim Q^2 \lesssim 10$ GeV^2
- Roughly 500 measurements
- Nucleon at rest: $\vec{p} = (0,0,0)$ thus $\omega = 0$, varying $\vec{q}$
- Connected 2-pt only, no disconnected since F_3 is non-singlet

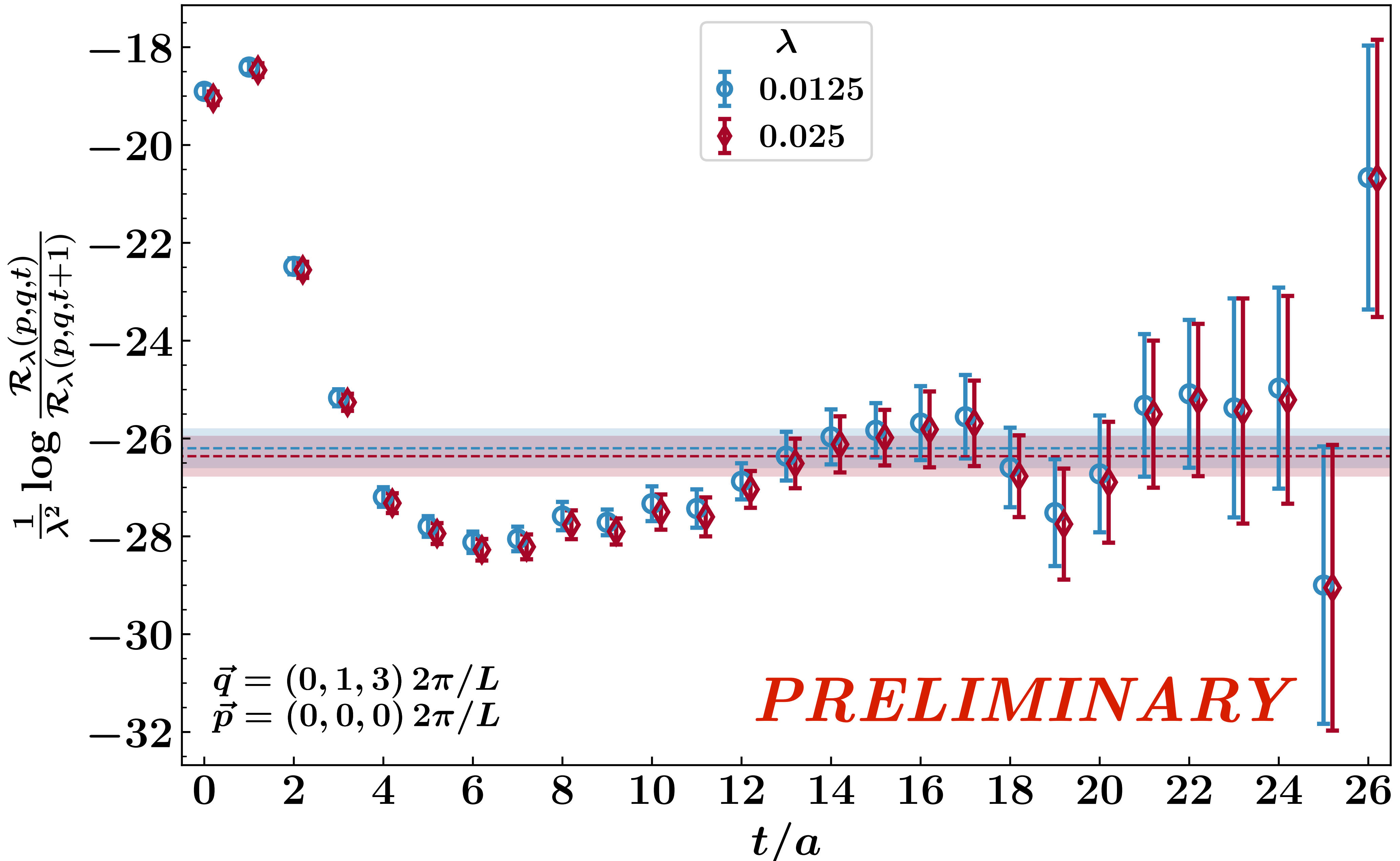
2^{nd} order energy shifts

- **Ratio of perturbed 2-pt functions**

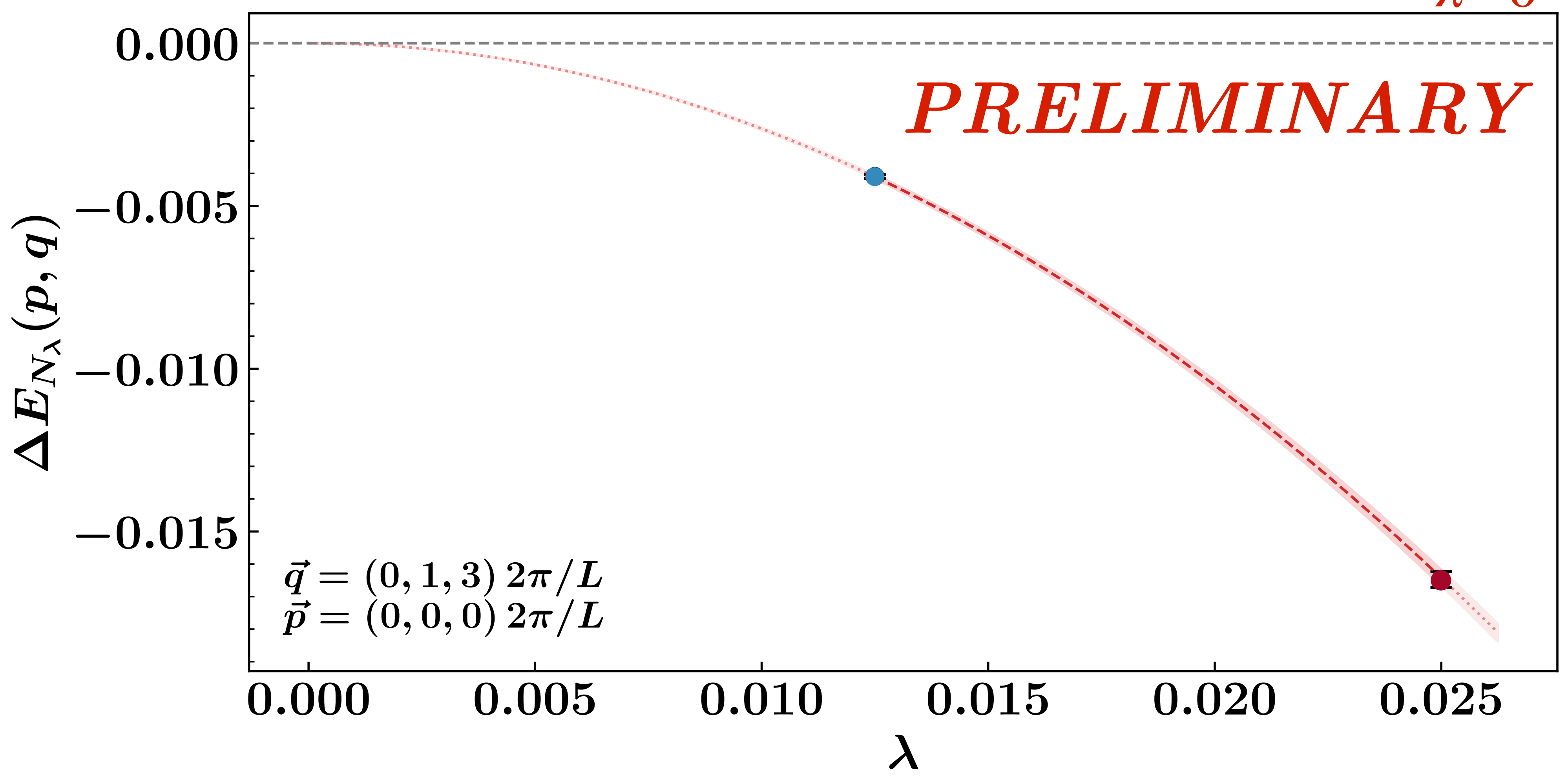
$$\mathcal{R}^{qq}_{\lambda}(p,t) \equiv \frac{G^{(2)}_{+\lambda_1^q,+\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,-\lambda_2^q}(p,t)}{G^{(2)}_{+\lambda_1^q,-\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,+\lambda_2^q}(p,t)}$$

$$\to A_\lambda e^{-4\Delta E_{N_\lambda}(p)\, t}$$

- **Extract energy shifts for each $|\lambda|$**



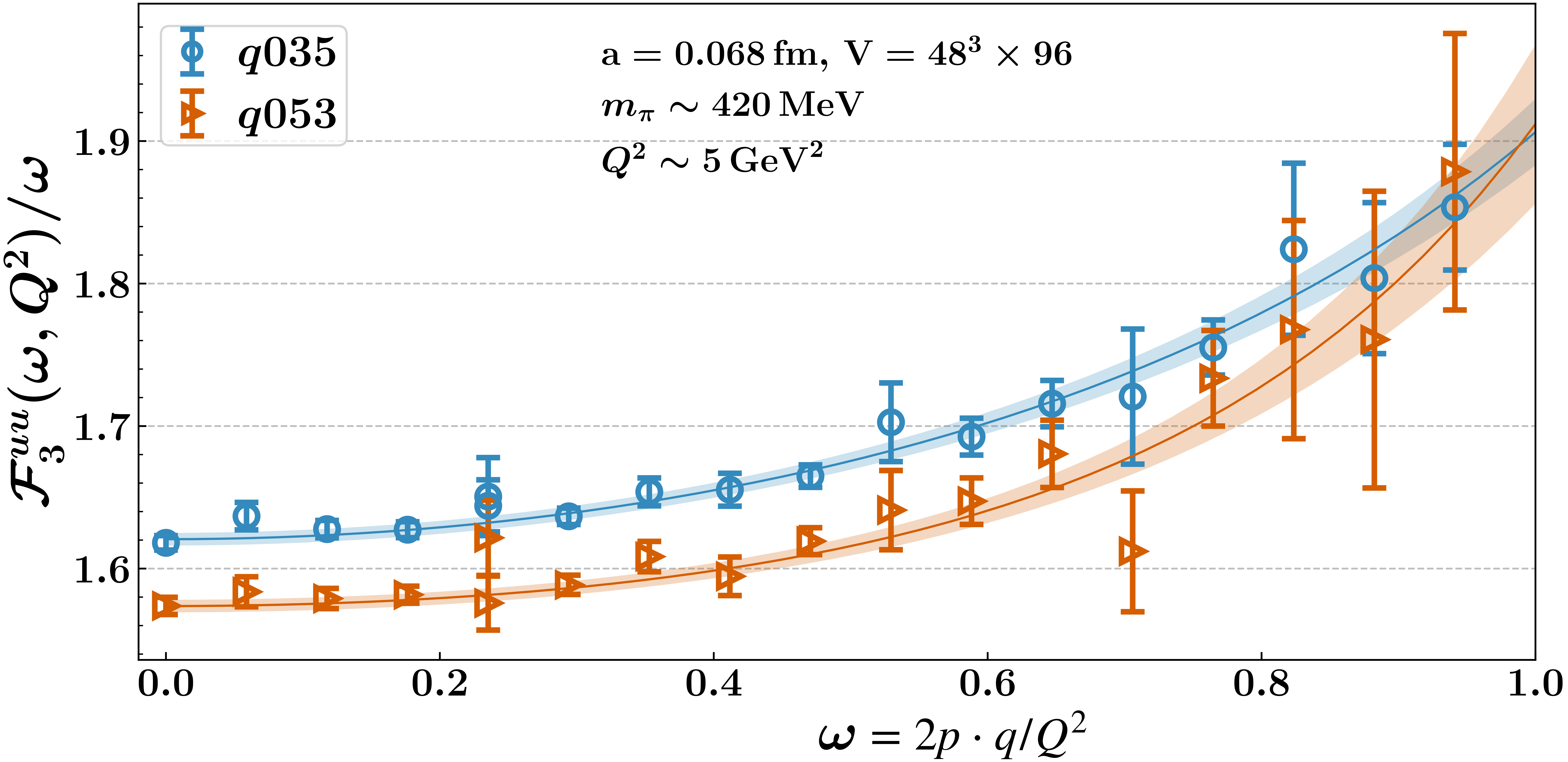
- **Get the 2nd order shift** $\left.\frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0}$



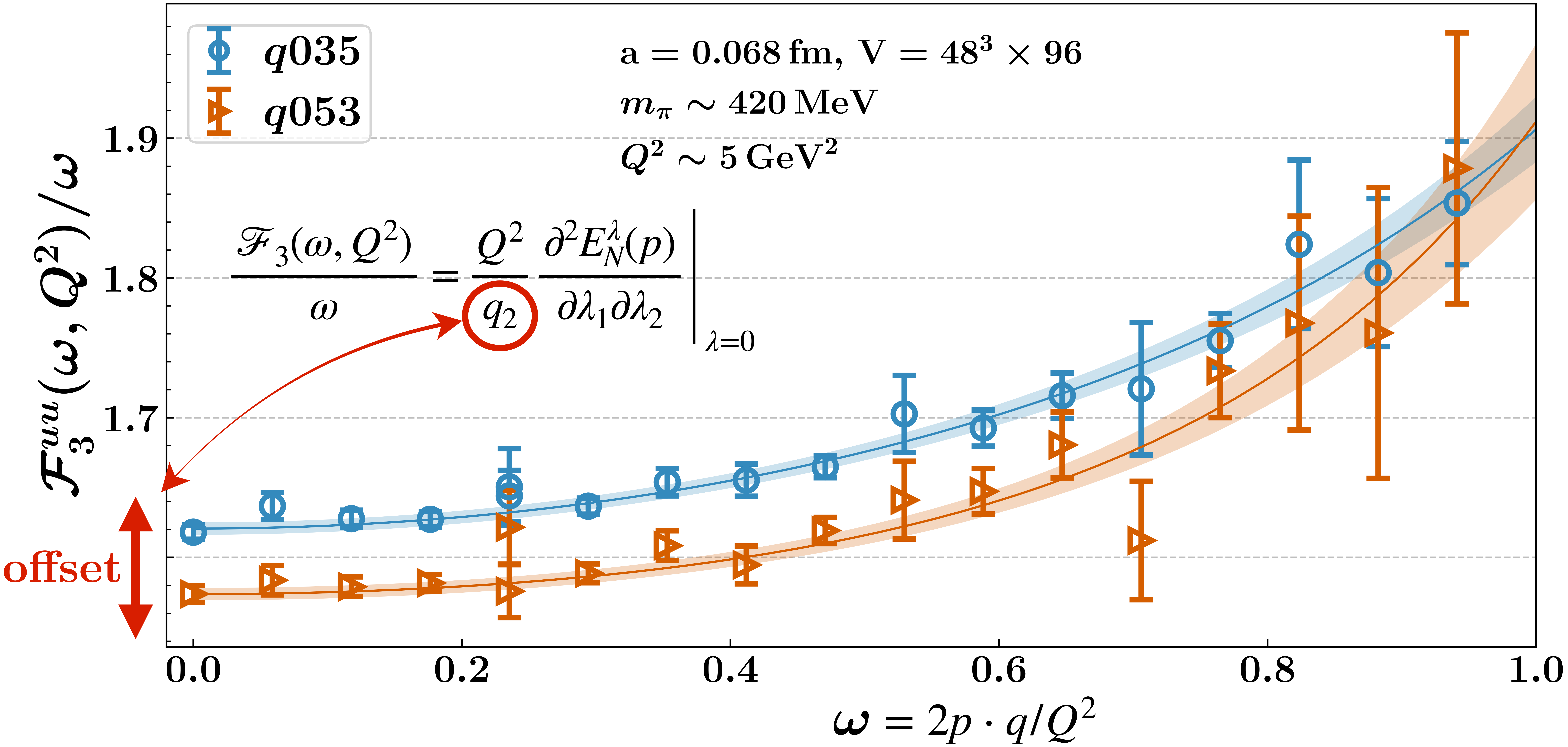
- **Energy shift is related to the amplitude**

$$\frac{\mathcal{F}_3(\omega,Q^2)}{\omega} = \frac{Q^2}{q_2}\left.\frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0}$$

$\mathscr{F}_3$ | unimproved



$\mathcal{F}_3$ | unimproved



Syst. 1: LPT improvement

Alec Hannaford Gunn
U.Adelaide
PhD 2023

Thomas Schar
U.Adelaide
MPhil ongoing

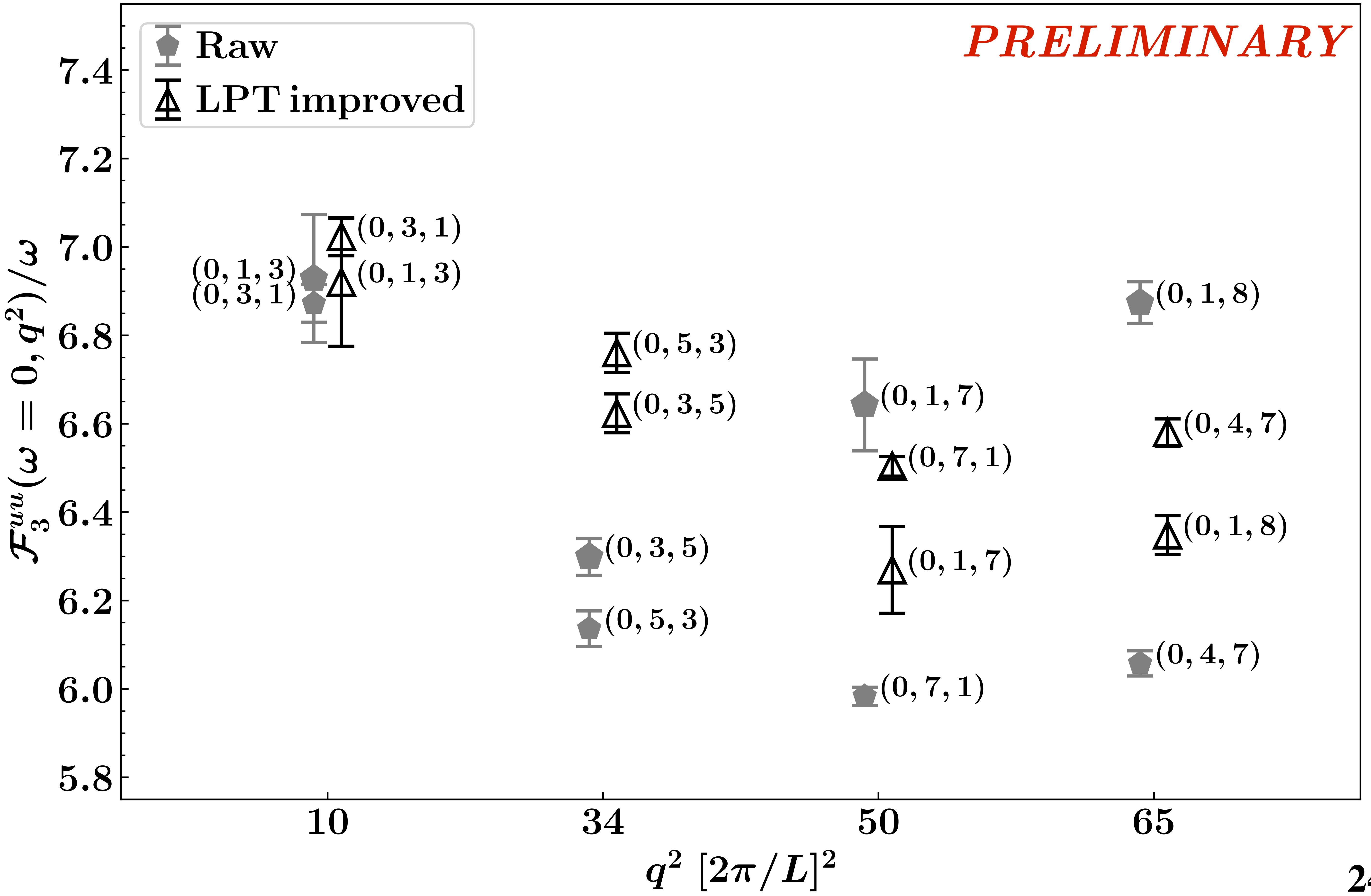
$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{q_2} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1 \partial\lambda_2}\bigg|_{\lambda=0}$$

introduces discretisation error due to broken rotational symmetry

- **Replace the kinematic factor by a lattice OPE motivated factor**

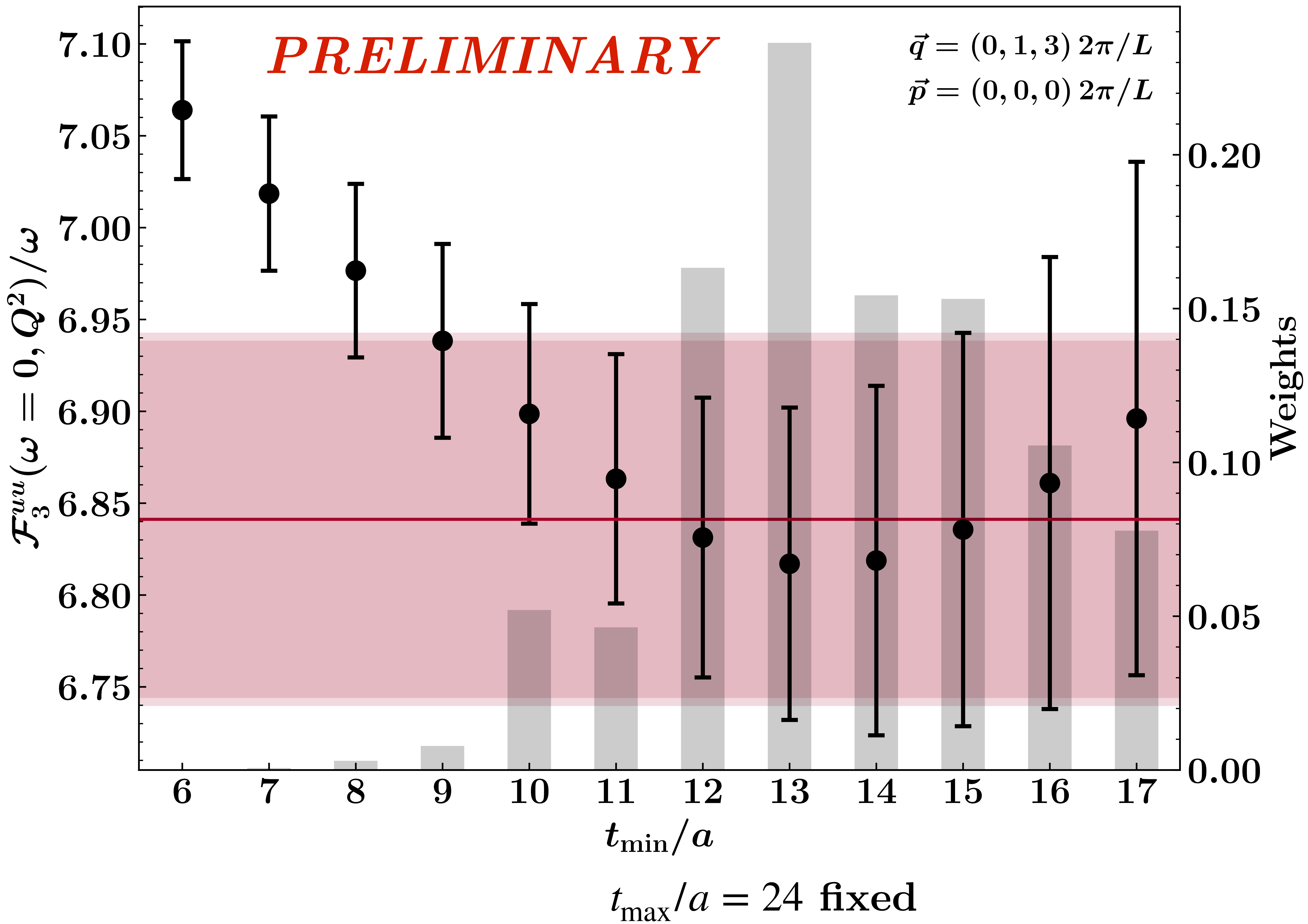
$$\frac{Q^2}{q_2} \to \frac{\sum_i \sin^2 q_i + \left[\sum_i (1-\cos q_i)\right]^2}{\sin q_2}$$

- **[Thomas Schar Thur@ 16:50] for details**



Syst. 2: Weighted averaging

Method: Beane et al. (NPLQCD/QCDSF), *PRD103 054504* (2021)



- **Red line (mean):** $\bar{\mathcal{O}} = \sum_f w^f \mathcal{O}^f$
- **Red band (total uncertainty):**

$$\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 = \sum_f w^f (\delta\bar{\mathcal{O}}^f)^2$$

$$\delta_{\mathbf{sys}}\bar{\mathcal{O}}^2 = \sum_f w^f (\mathcal{O}^f - \bar{\mathcal{O}})^2$$

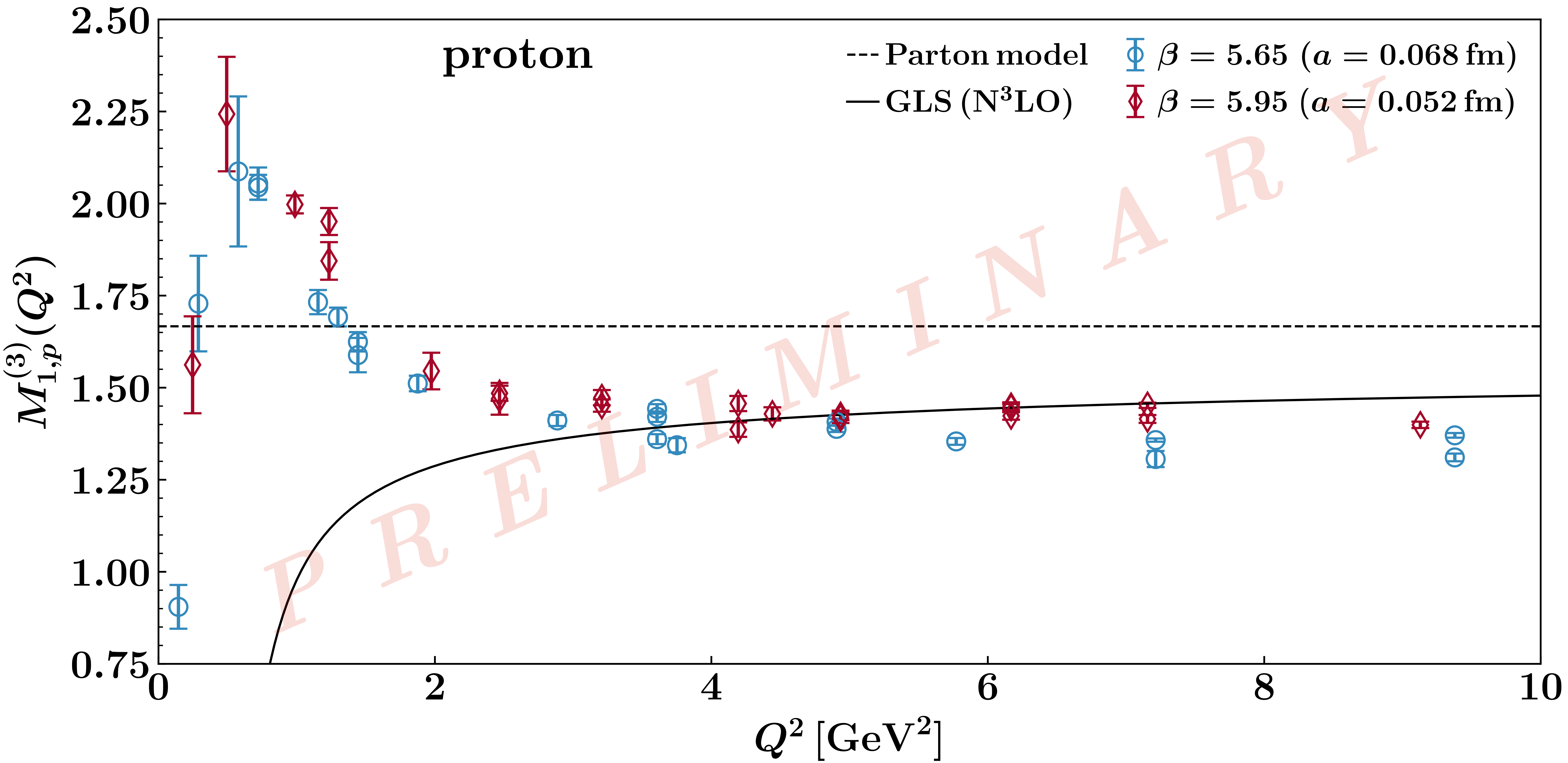
$$\delta\bar{\mathcal{O}} = \sqrt{\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 + \delta_{\mathbf{sys}}\bar{\mathcal{O}}^2}$$

- **Weights:** $w^f = \dfrac{p_f\,(\delta\mathcal{O}^f)^{-2}}{\sum_{f'} p_{f'}(\delta\mathcal{O}^{f'})^{-2}}$

  where p_f is the one sided p-value of the ratio fits

$\mathscr{F}_3^{\gamma Z}$ | First moment

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour



$\mathscr{F}_3^{\gamma Z}$ | First moment

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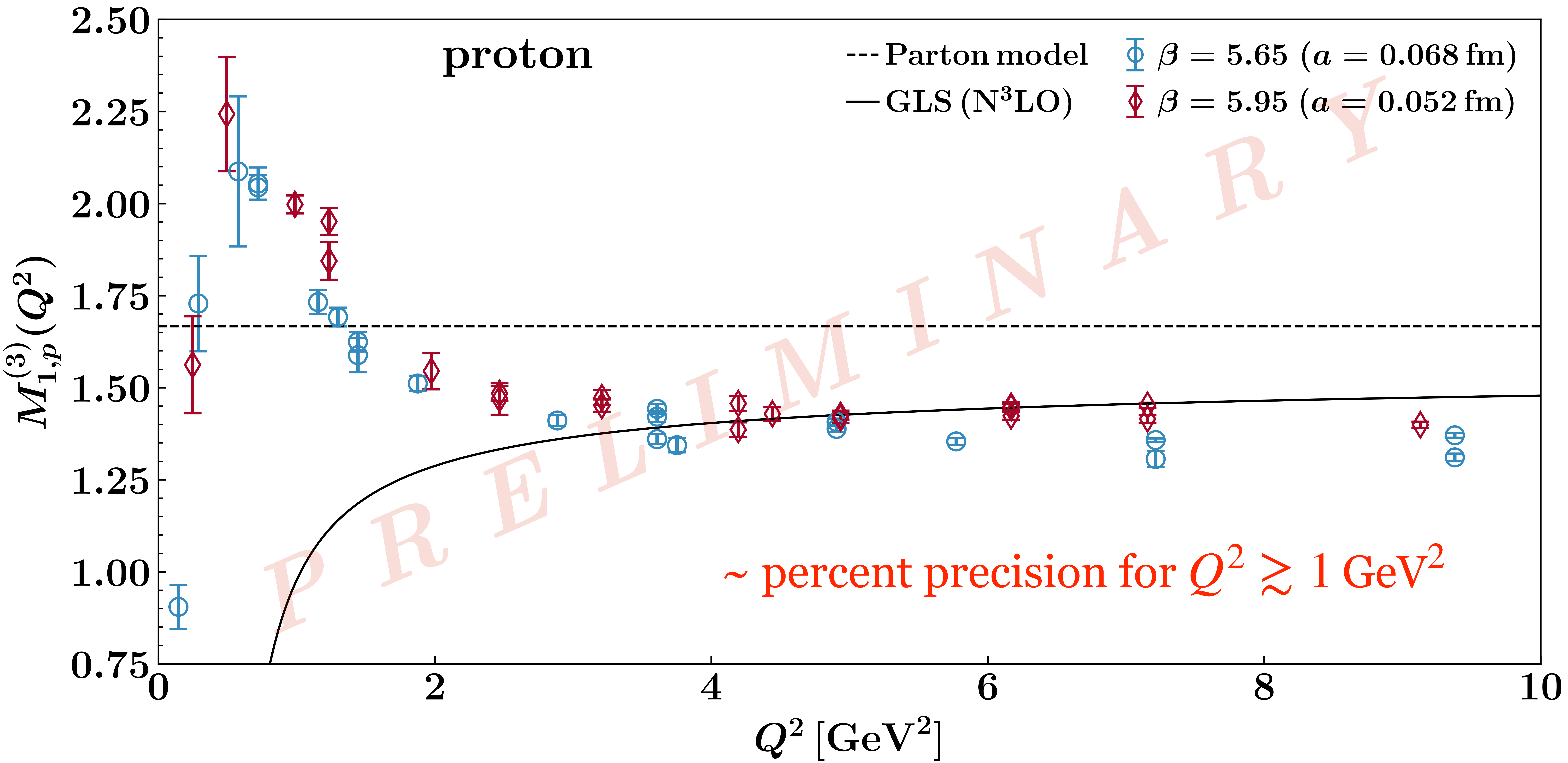
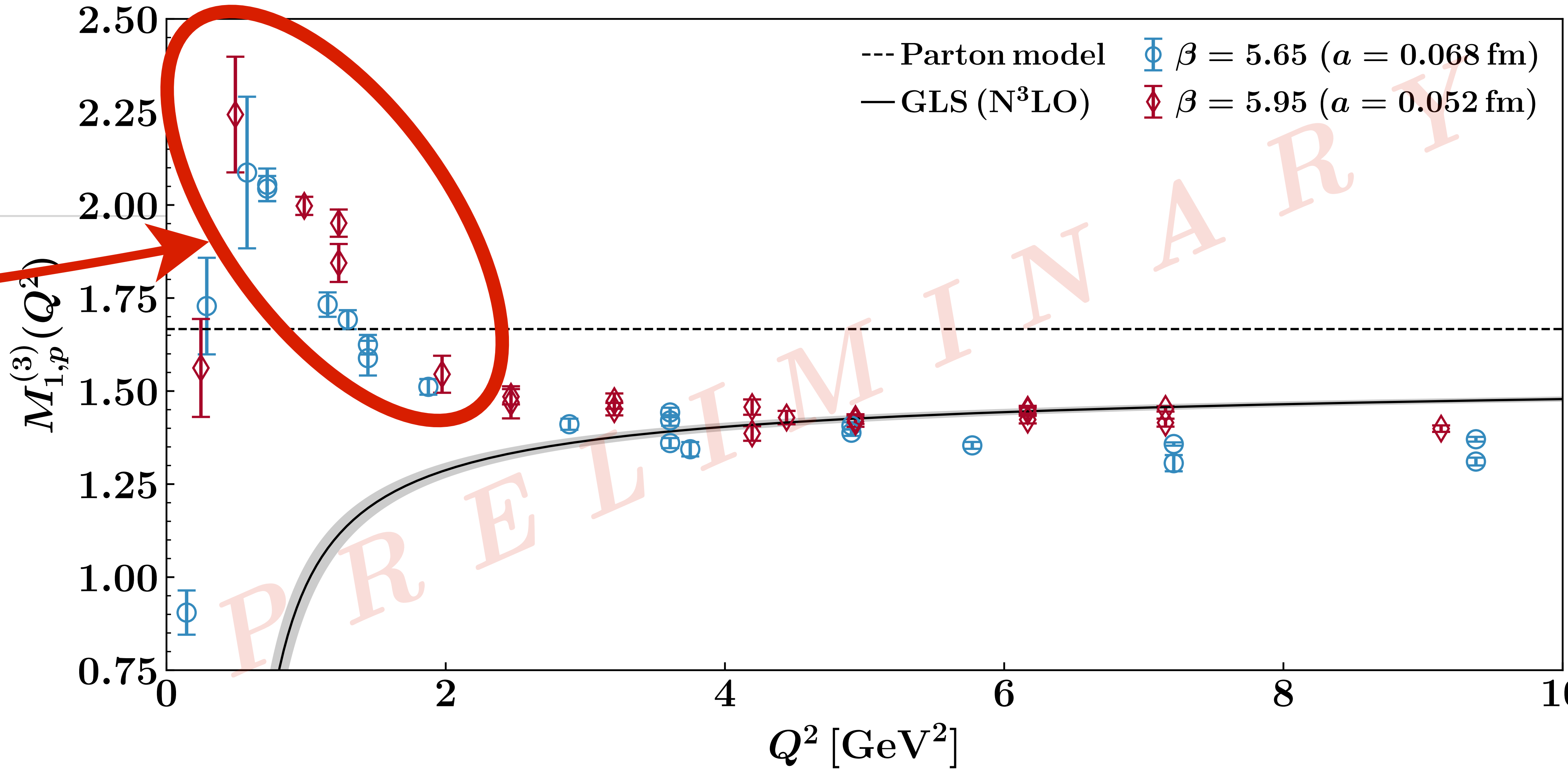


Elastic contribution

- **Peak is mostly elastic**
- subtract elastic contribution:

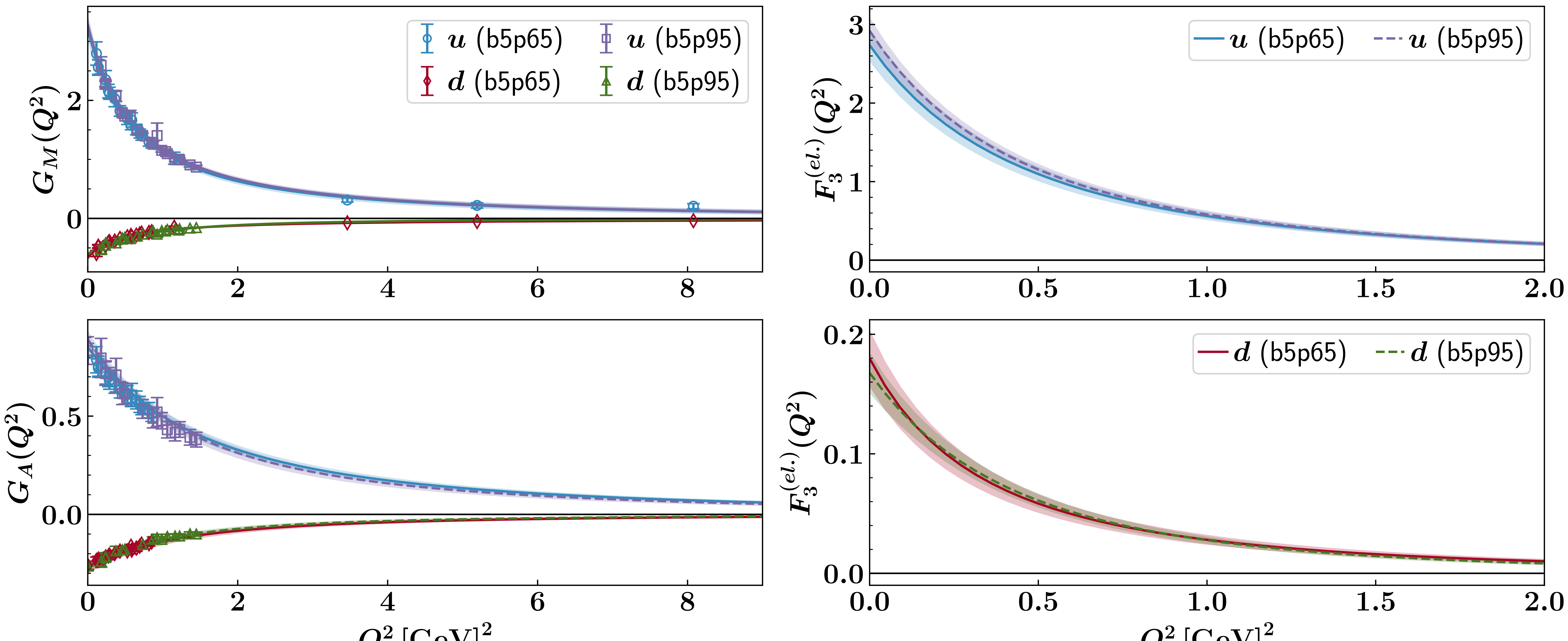
$$F_3^{(el.)} = -G_M(Q^2)G_A(Q^2)x\delta(1-x)$$

- provides insights into higher twist contributions



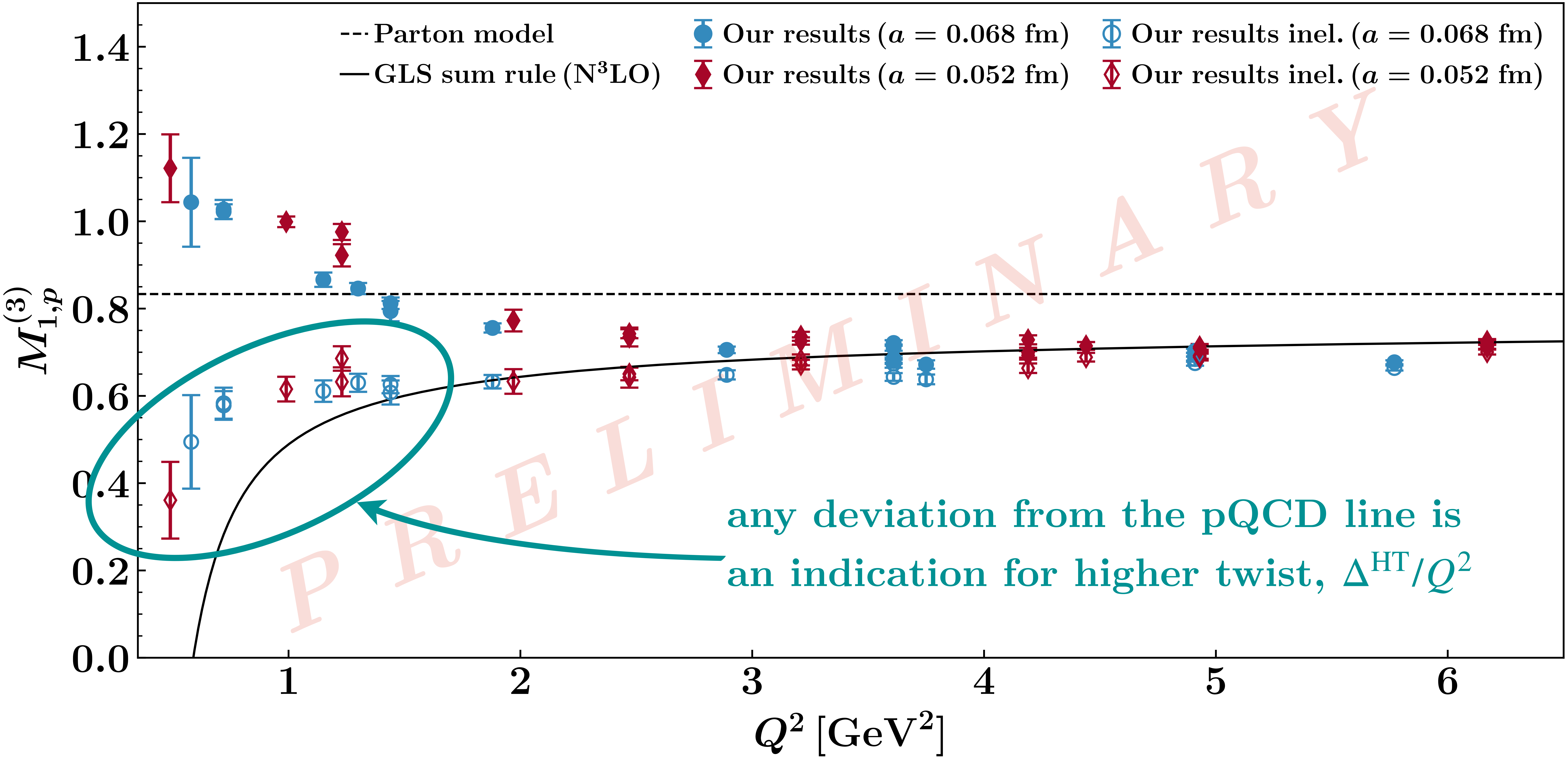
low-Q^2: 3-pt functions
high-Q^2: Feynman-Hellmann

low-Q^2: 3-pt functions
dipole parametrisation



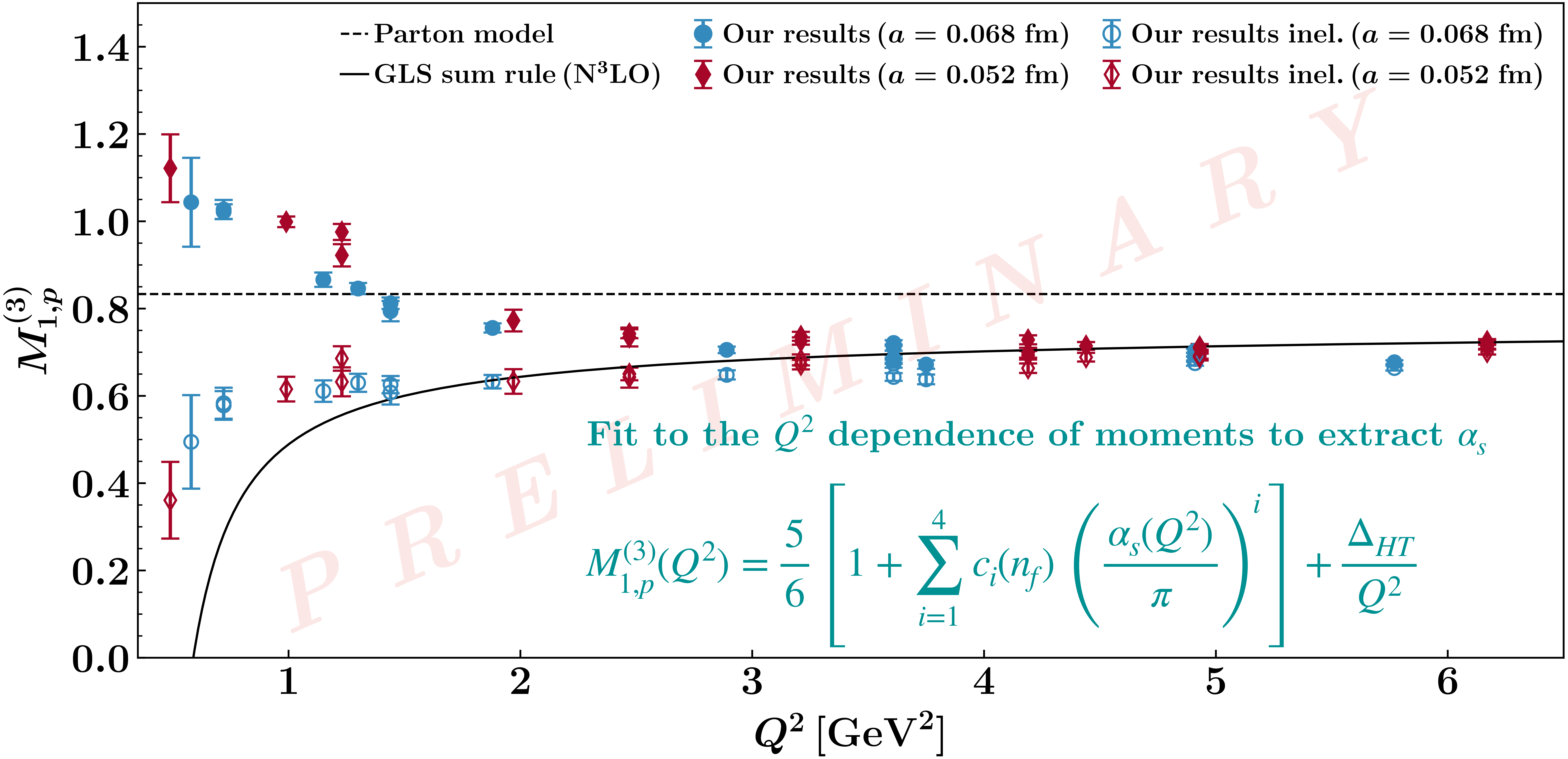
$\mathcal{F}_3^{\gamma Z}$ | Higher-twist

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour



$\mathscr{F}_3^{\gamma Z}$ | determining α_s

$a = 0.068, 0.052$ fm
$m_\pi \sim 410$ MeV
48^3x96, 2+1 flavour



Fit to the Q^2 dependence of moments to extract α_s

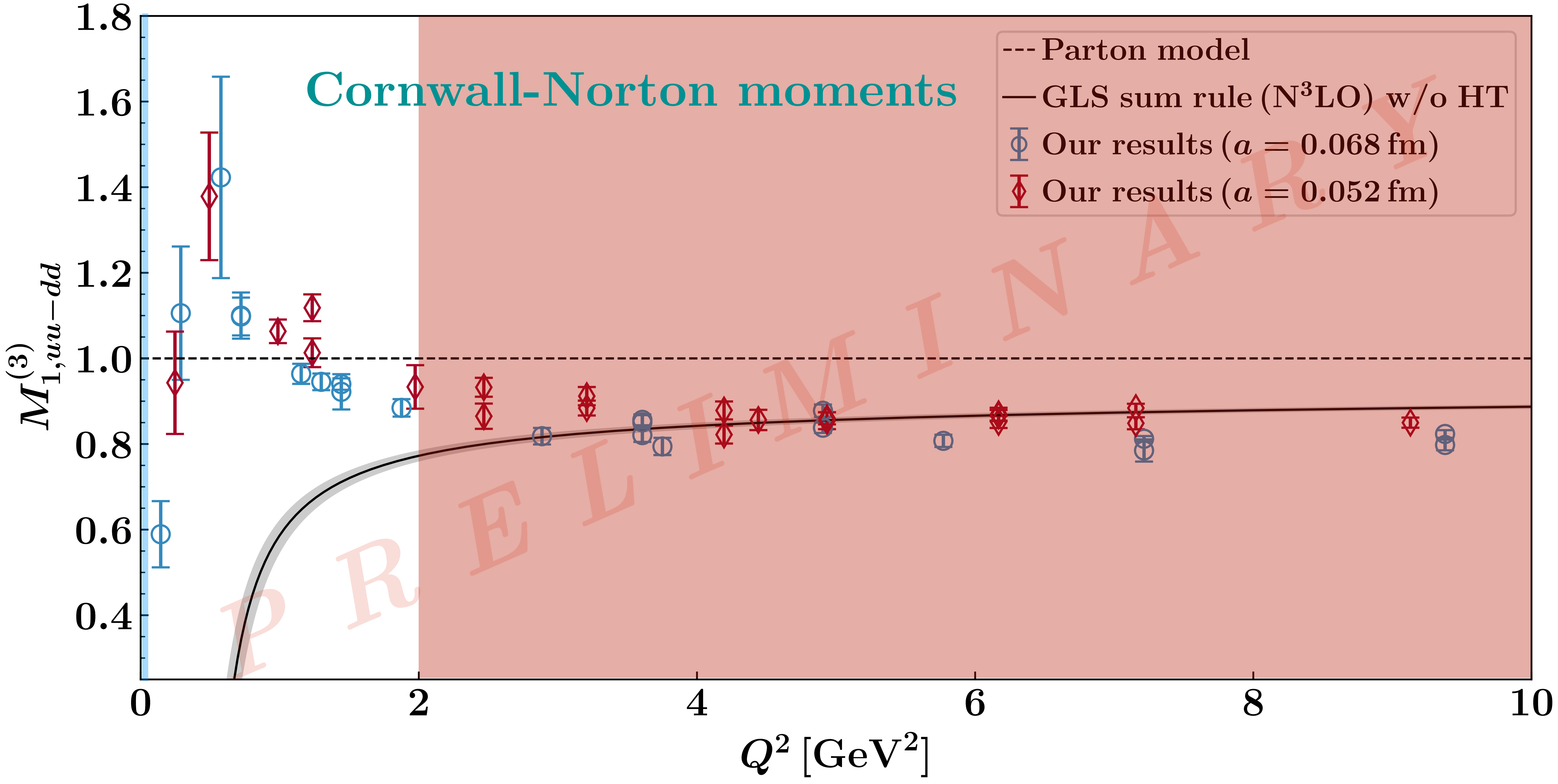
$$M_{1,p}^{(3)}(Q^2) = \frac{5}{6}\left[1 + \sum_{i=1}^{4} c_i(n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^i\right] + \frac{\Delta_{HT}}{Q^2}$$

$\mathscr{F}_3^{\gamma W}$ | EW box

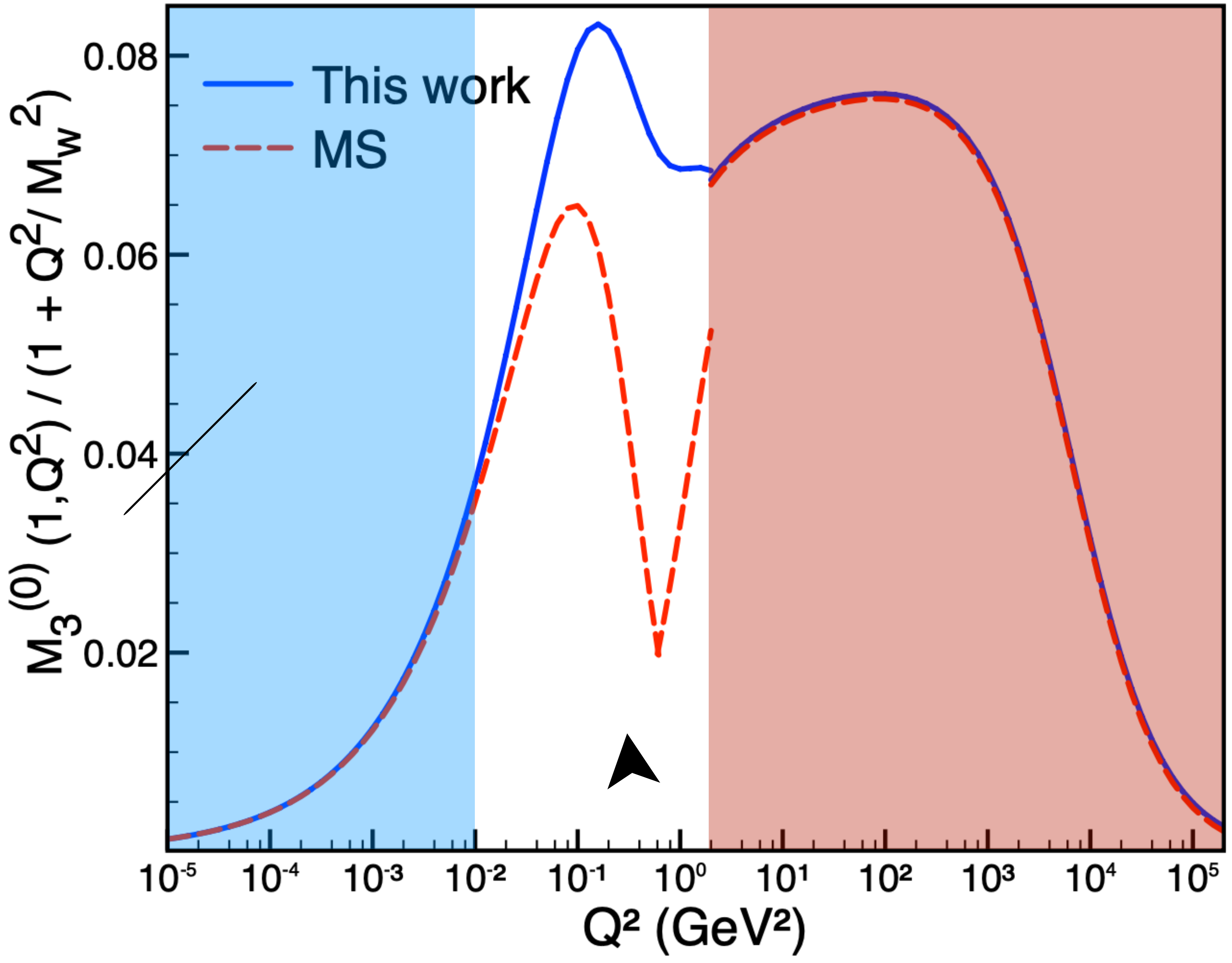
related talk [Xu Feng Mon @17:00]

- Electroweak box diagrams need Nachtmann moments

$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\, C_N(x,Q^2)\, F_3^{(0)}(x,Q^2)$$



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



form factors

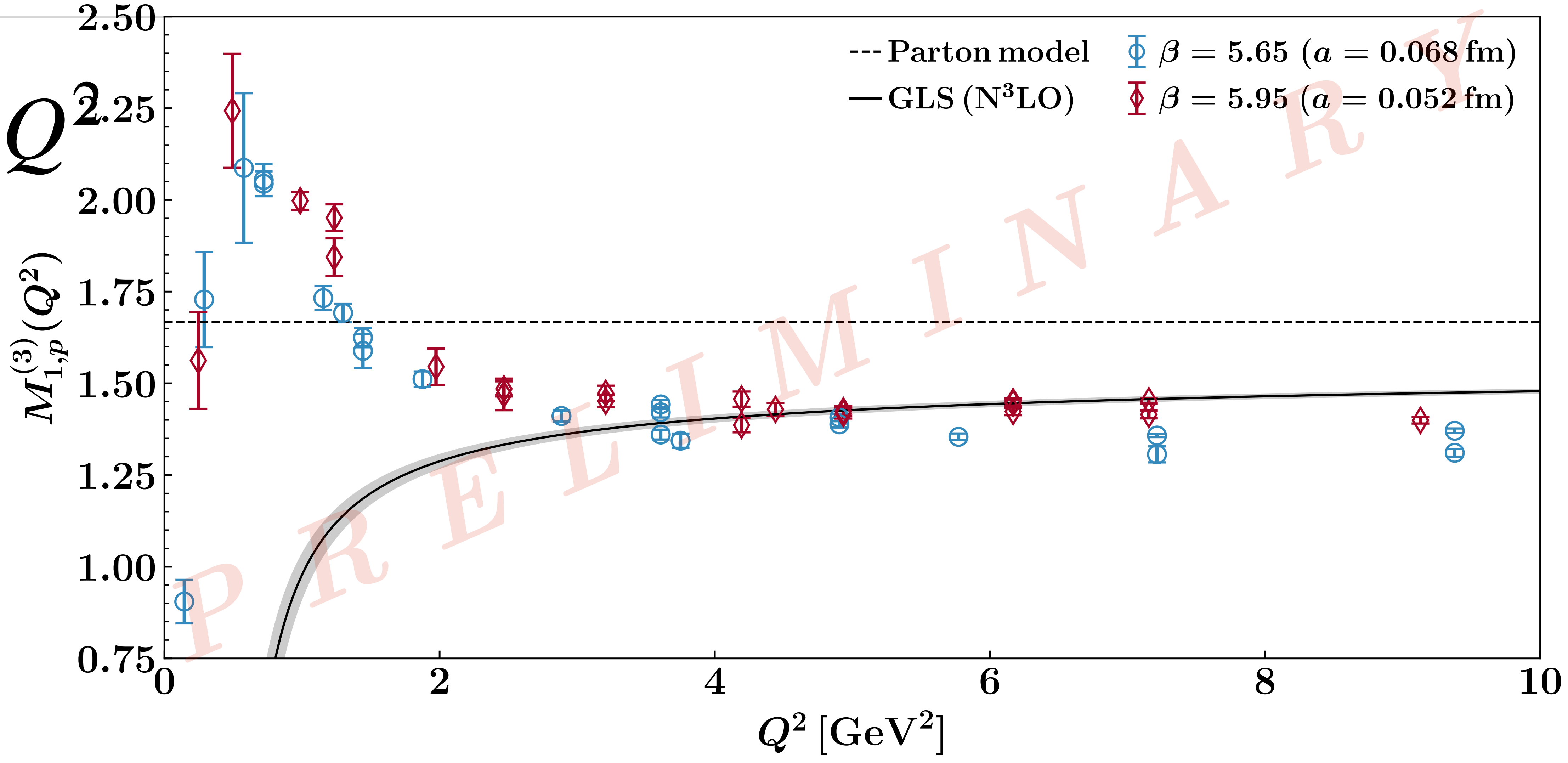
perturbation theory

input from LQCD required

$0.01 \lesssim Q^2 \lesssim 2$ GeV^2

Summary & Outlook

- ☑ Lowest moment of $F_3(x, Q^2)$ in a wide range of Q^2
- ☑ w/ Good statistical precision
- ☑ Clear indication of higher-twist and non-perturbative effects



- ☐ Full control over lattice artefacts, e.g. a, M_π, V dependence
- ☐ Utilise GLS sum rule to determine α_s
  - ☐ requires continuum extrapolation, $a = 0.082$ fm runs ongoing
- ☐ Estimate Nachtmann moments relevant for EW box diagrams, $\square^{\gamma W/Z}_{VA}$
  - ☐ requires at least lowest 3 Cornwall-Norton moments
  - ☐ we have them for $Q^2 \gtrsim 2\,\mathrm{GeV}^2$, on-going work for $Q^2 \lesssim 2\,\mathrm{GeV}^2$

Acknowledgements

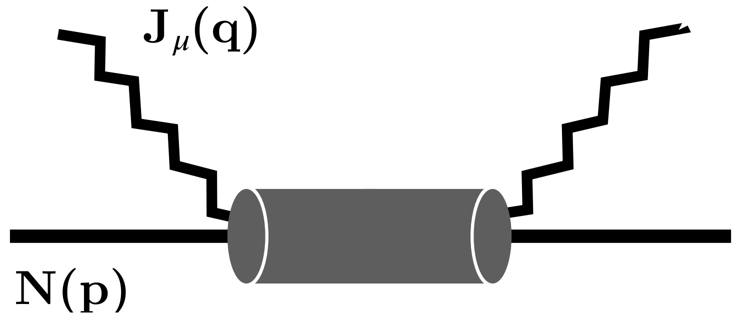
- The numerical configuration generation (using the BQCD lattice QCD program)) and data analysis (using the Chroma software library) was carried on the
  - DiRAC Blue Gene Q and Extreme Scaling (EPCC, Edinburgh, UK) and Data Intensive (Cambridge, UK) services,
  - the GCS supercomputers JUQUEEN and JUWELS (NIC, Jülich, Germany) and
  - resources provided by HLRN (The North-German Supercomputer Alliance),
  - the NCI National Facility in Canberra, Australia (supported by the Australian Commonwealth Government) and
  - the Phoenix HPC service (University of Adelaide).
- RH is supported by STFC through grant ST/P000630/1.
- PELR is supported in part by the STFC under contract ST/G00062X/1.
- KUC, RDY and JMZ are supported by the Australian Research Council grants DP190100297 and DP220103098.

Backup

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p,q) = \int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) \,|\, \mathcal{T}\{J_\mu(z) J_\mu(0)\} \,|\, N(p)\rangle$$



- **Action modification**

local EM current

$J_\mu(z) = \sum_q e_q \bar{q}(z)\gamma_\mu q(z)$

$$S \to S(\lambda) = S + \lambda \int d^4z \,(e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}})\, J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G_\lambda^{(2)}(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial\lambda^2} - tA(\mathbf{p})\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right) e^{-E_N(\mathbf{p})t}$$

from spectral decomposition

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z (e^{iq\cdot z} + e^{-iq\cdot z})\langle N(\mathbf{p})|\mathcal{T}\{\mathcal{J}(z)\mathcal{J}(0)\}|N(\mathbf{p})\rangle$$

from path integral

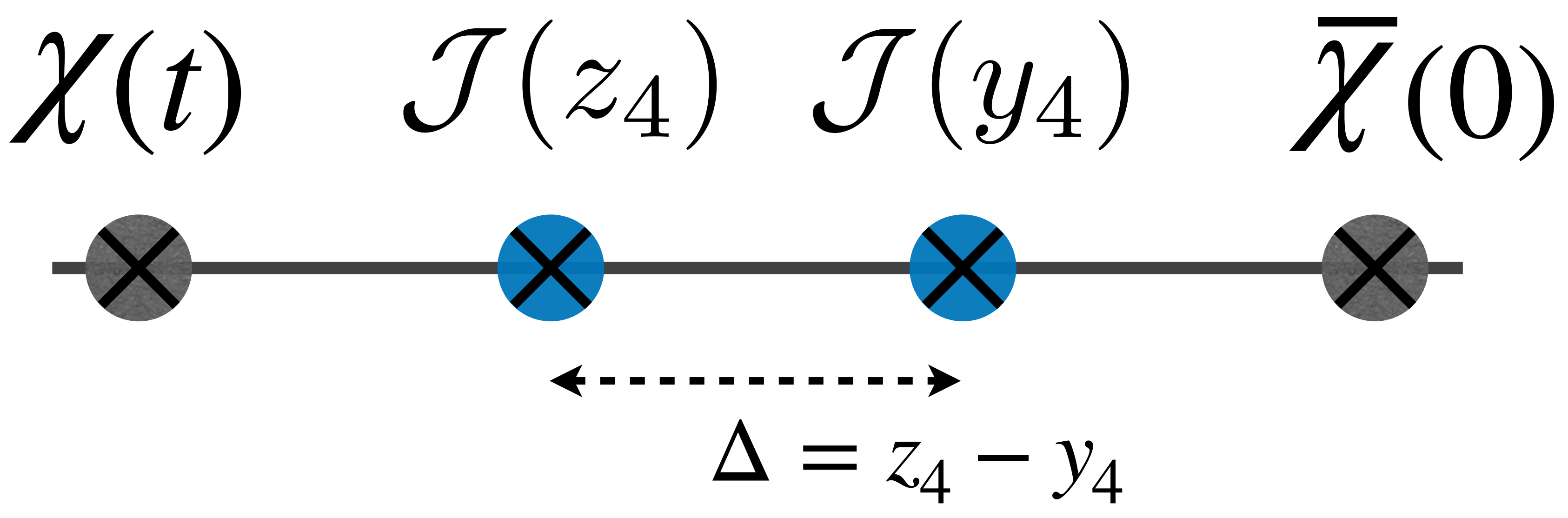
- **equate the time-enhanced terms:**

$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z (e^{iq\cdot z} + e^{-iq\cdot z})\langle N(\mathbf{p})|\mathcal{J}(z)\mathcal{J}(0)|N(\mathbf{p})\rangle}^{T_{\mu\mu}(p,q)} + (q \to -q)$$

<u>Compton amplitude is related to the second-order energy shift</u>

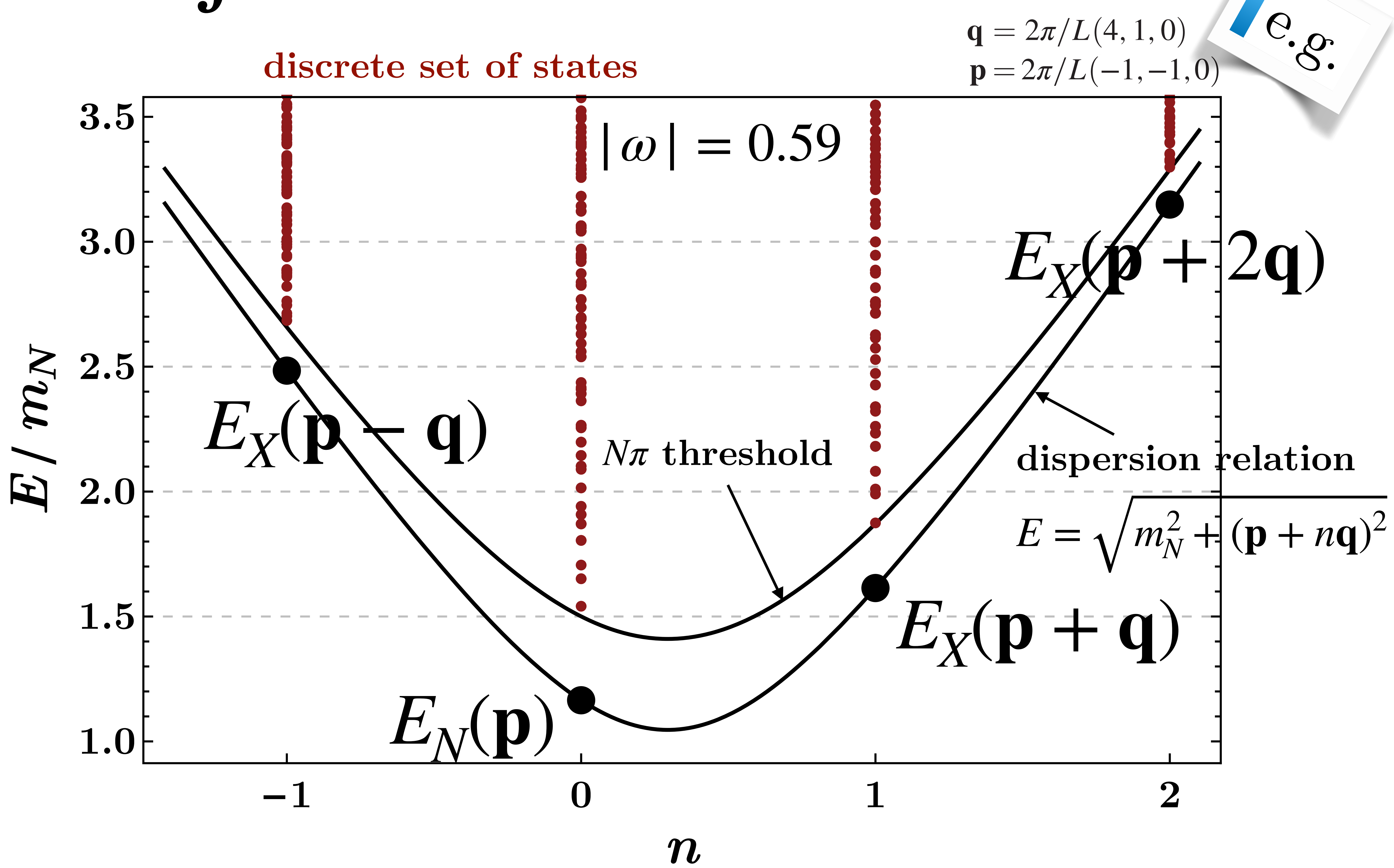
Compton amplitude via the FH relation at 2nd order

- **relevant contribution comes from the ordering where the currents are sandwiched**



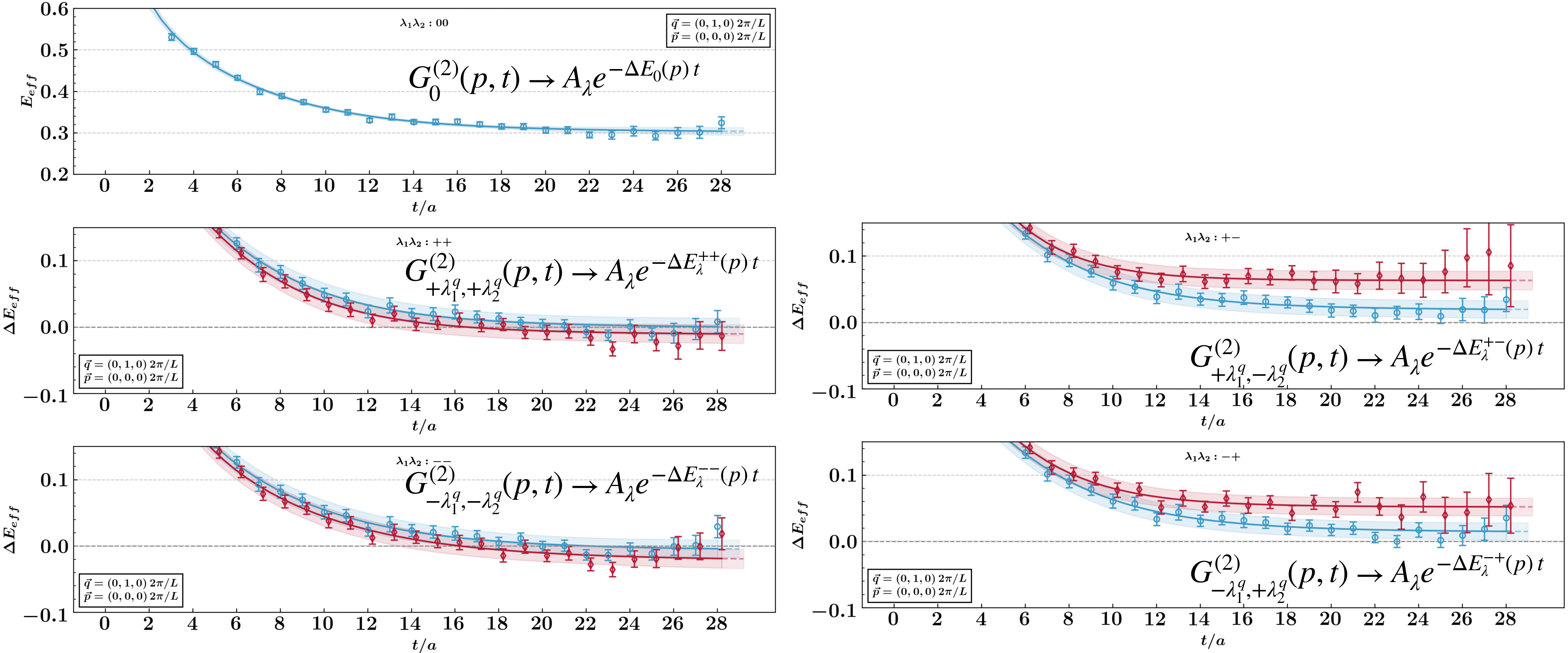
$$\sim e^{-E_N(\mathbf{p})t} \int^t d\Delta\, e^{-\left(E_X(\mathbf{p}+\mathbf{q}) - E_N(\mathbf{p})\right)\Delta} (t - \Delta)$$

- under the condition $|\omega| < 1$, $E_X(\mathbf{p} + n\mathbf{q}) \gtrsim E_N(\mathbf{p})$, so the intermediate states cannot go on-shell
- ground state dominance is ensured in the large time limit



Multi-exp fits ($Q^2 \lesssim 1\ \mathrm{GeV}^2$)

Second order energy shift: $\Delta E_{N_\lambda}(p) = \dfrac{1}{4}\left[\Delta E_\lambda^{++}(p) + \Delta E_\lambda^{--}(p) - \Delta E_\lambda^{+-}(p) - \Delta E_\lambda^{-+}(p)\right] - E_0(p)$



Future lattices

Currently thermalising/generating

- $64^3 \times 96$, $a = (0.068, 0.052)\,\mathrm{fm}$, $m_\pi = (220, 270)$ MeV *(completed - early 2024)*
- $80^3 \times 114$, $a = 0.068\,\mathrm{fm}$, $m_\pi = 150$ MeV *(still thermalising)*
- $96^3 \times 128$, $a = 0.052\,\mathrm{fm}$, $m_\pi = 140$ MeV *(thermalised + O(50) trajectories)*

Compton results so far

Using BQCD [EPJ Web Conf. 175 (2018) 14011]

on

- JUWELS (Jülich, Germany)
- CSD3 (Cambridge, UK)
- Tursa (Edinburgh, UK)