

# Charm mesons, charmonium resonances and exotics from lattice QCD

Christopher Thomas, University of Cambridge

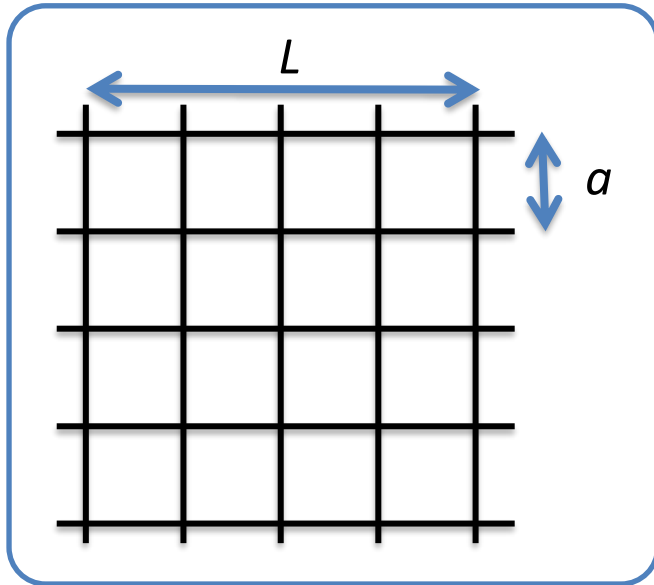
[c.e.thomas@damtp.cam.ac.uk](mailto:c.e.thomas@damtp.cam.ac.uk)

XVIth Quark Confinement and the Hadron Spectrum conference,  
Cairns, Australia, 19 – 24 August 2024



# Lattice QCD spectroscopy

Systematically-improvable  
first-principles calculations



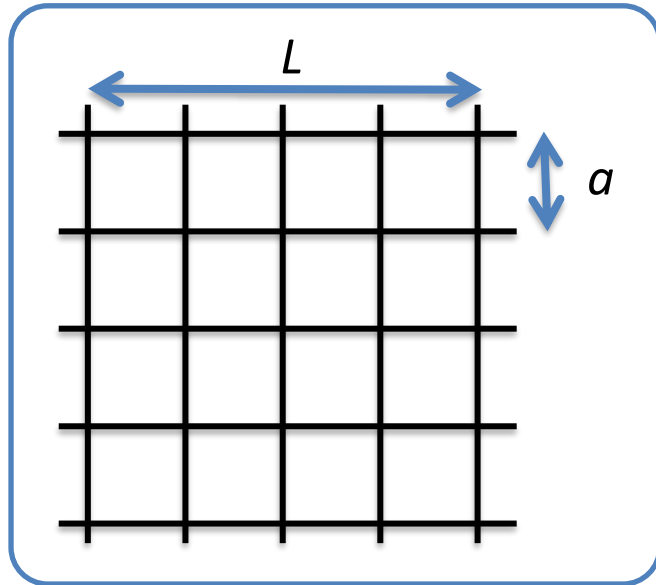
- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically  
(Euclidean time,  $t \rightarrow i t$ )

Note:

- Finite  $a$  and  $L$
- Possibly heavy u, d quarks  
( $\rightarrow$  unphysical  $m_\pi$ )

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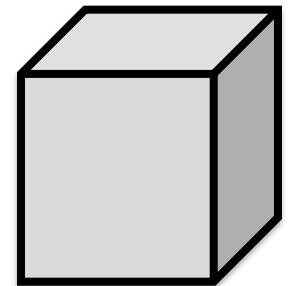
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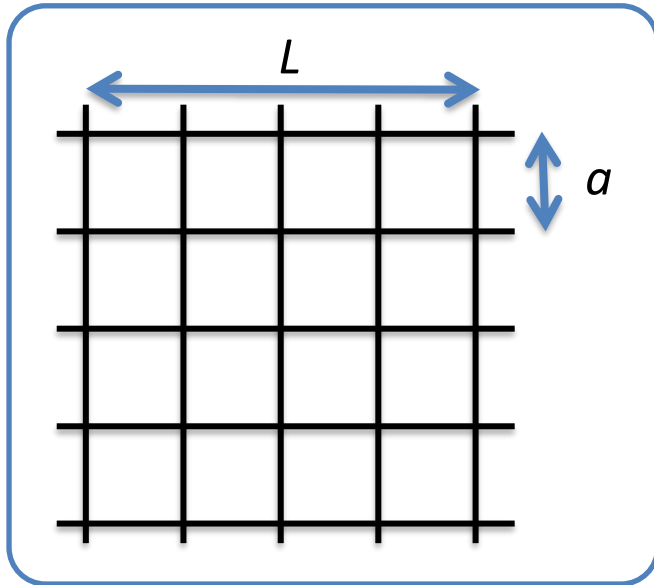
Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$



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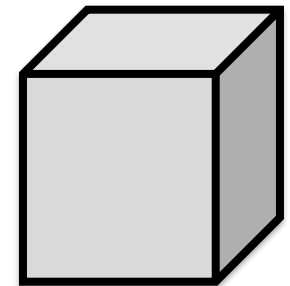
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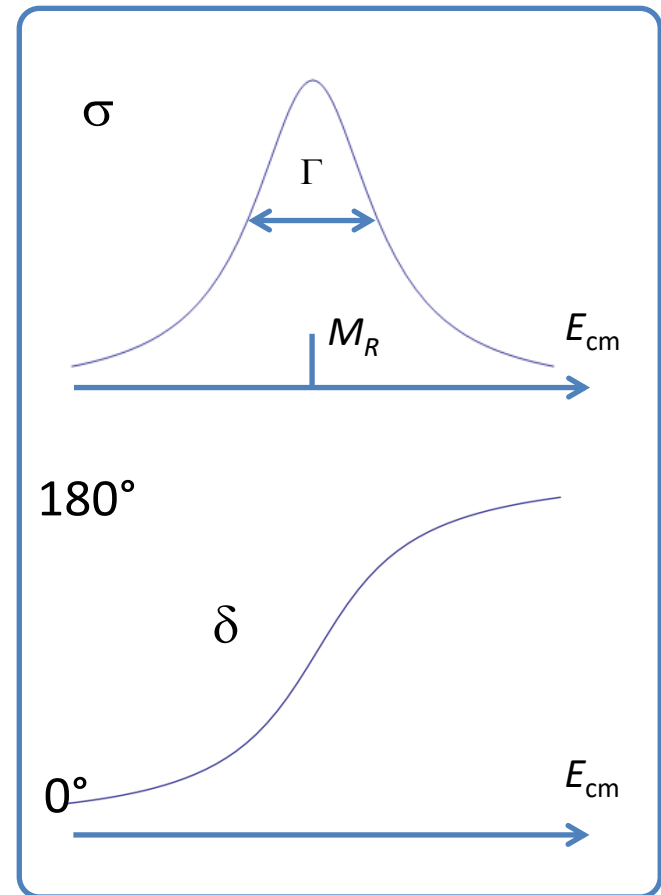
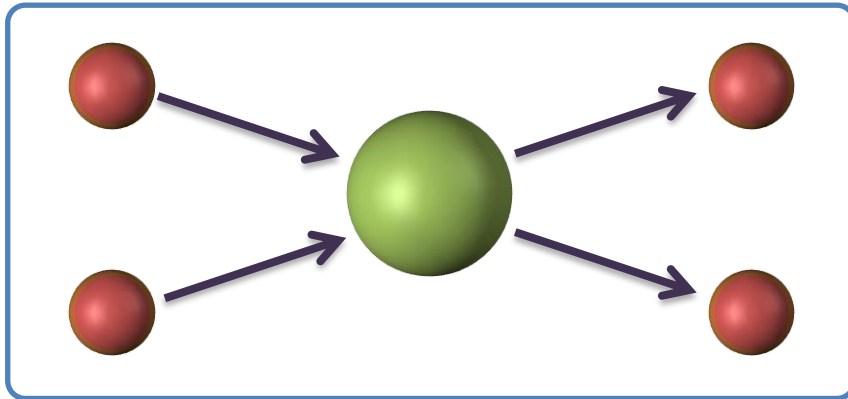
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Excited spectra: large bases of operators with appropriate structures

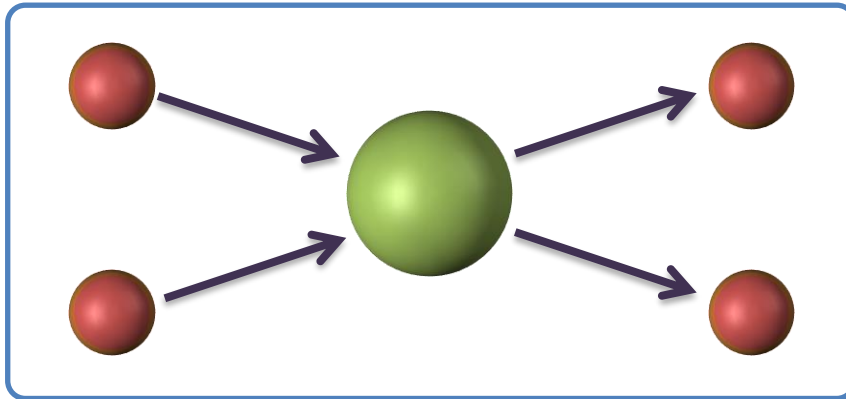
# Scattering and resonances

Most hadrons are resonances and decay strongly to lighter hadrons

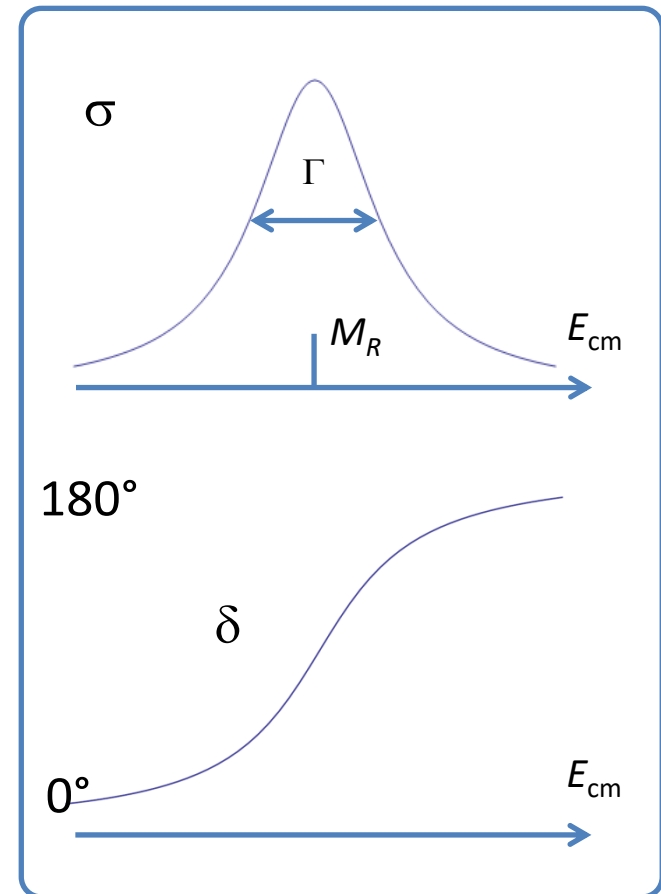
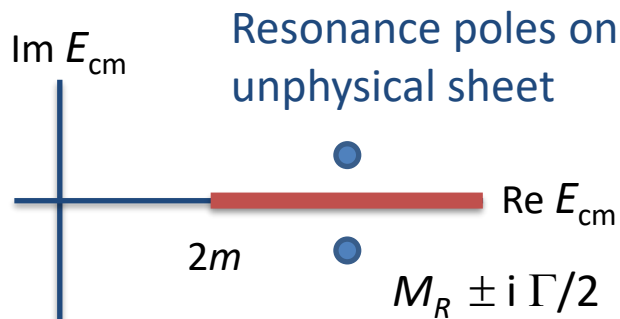


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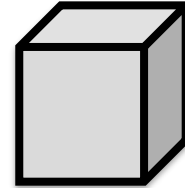
Singularity structure of scattering matrix (poles  $\rightarrow$  state content)



# Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

**Lüscher method** [NP B354, 531 (1991)]  
and extensions: relate discrete set of  
**finite-volume energy levels**  $\{E_{cm}\}$  to  
**infinite-volume scattering  $t$ -matrix**.

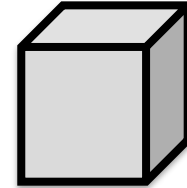


$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

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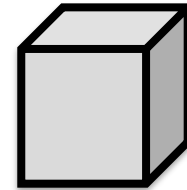
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Param.  $t(E_{\text{cm}})$  using various forms, e.g.  $K$ -matrix (unitarity)

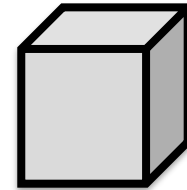
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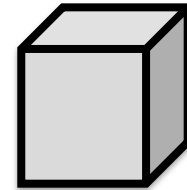
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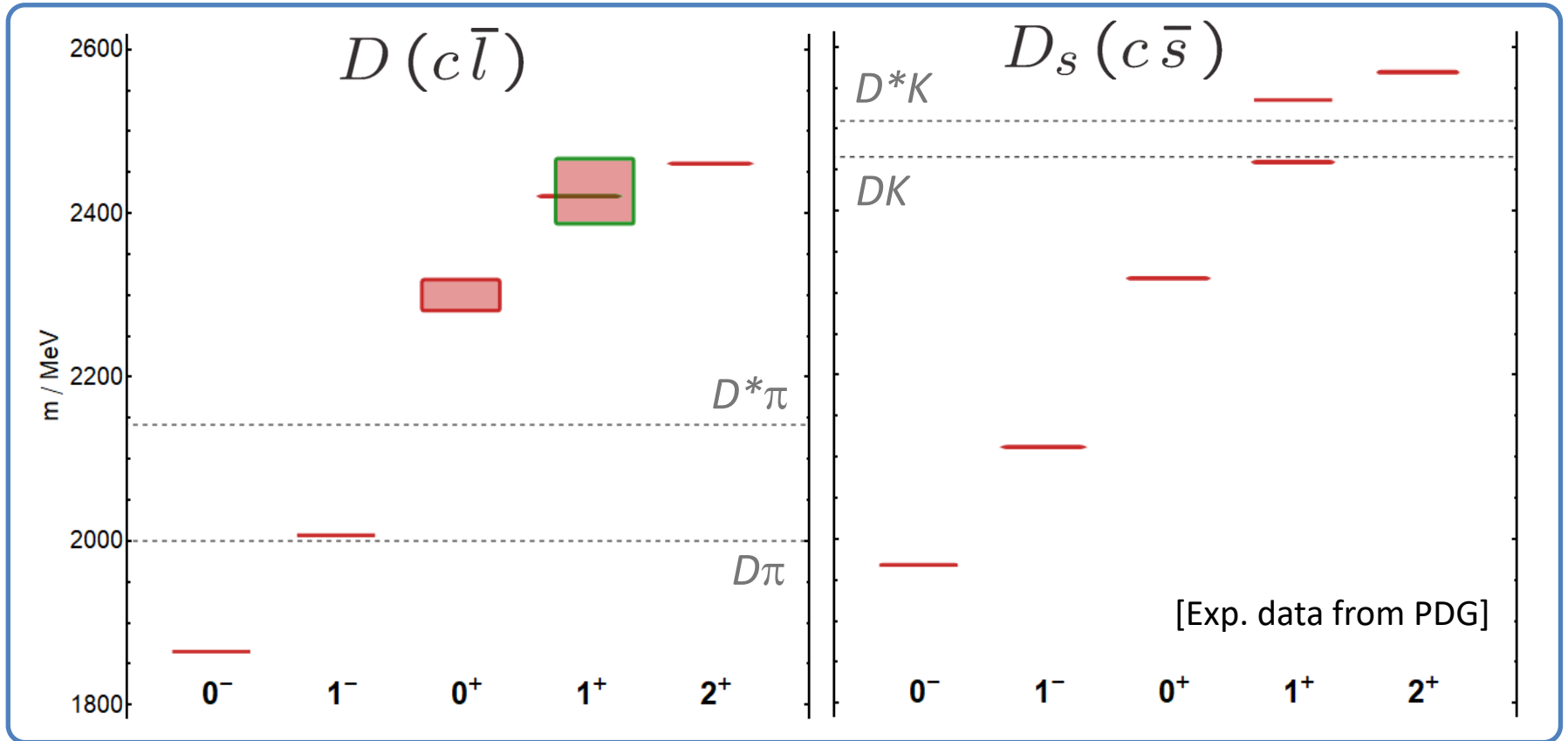
Analytically continue  $t(E_{\text{cm}})$  in complex  $E_{\text{cm}}$  plane, look for poles.

Demonstrated in calcs. of  $\rho$ , light scalars,  $b_1$ , charm mesons, ...

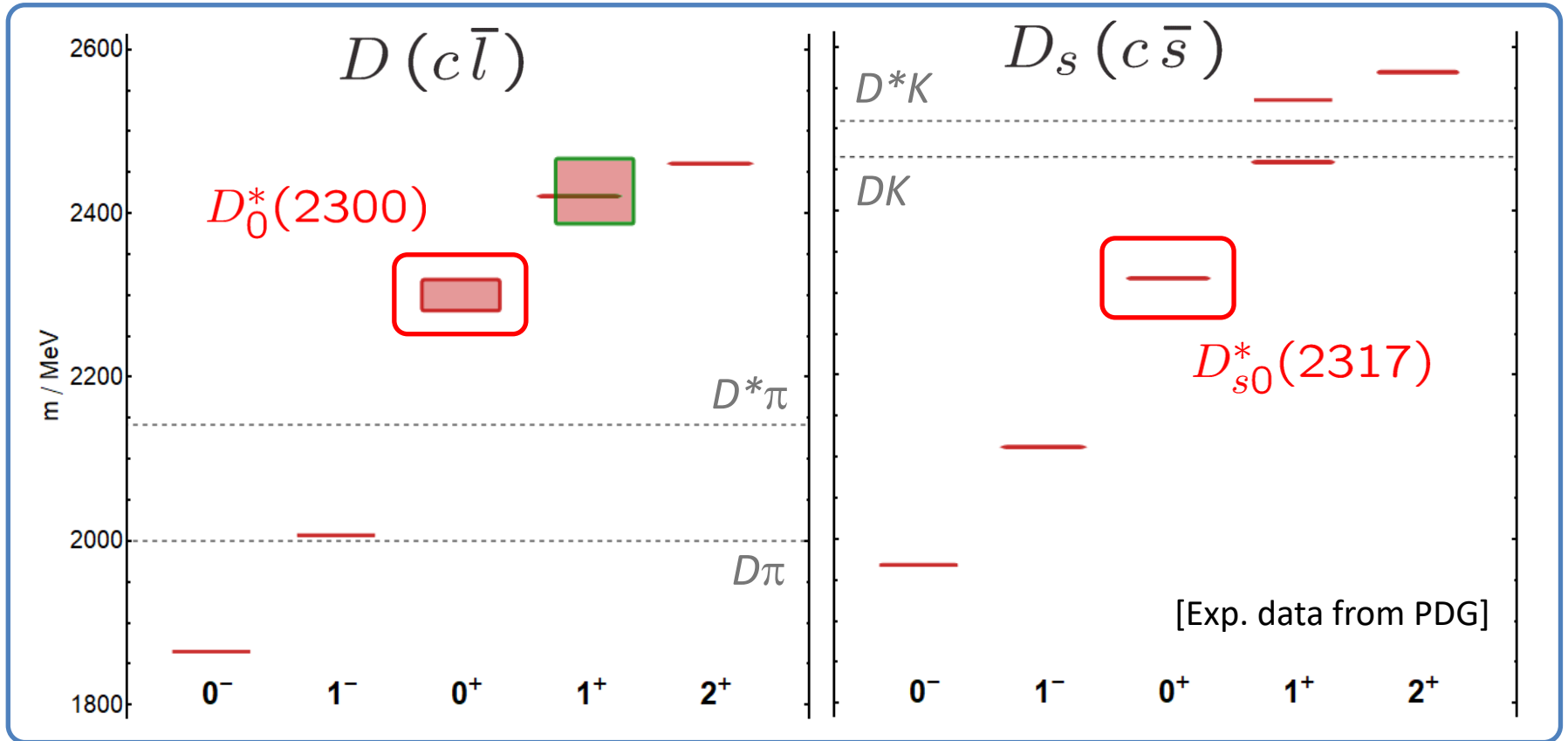
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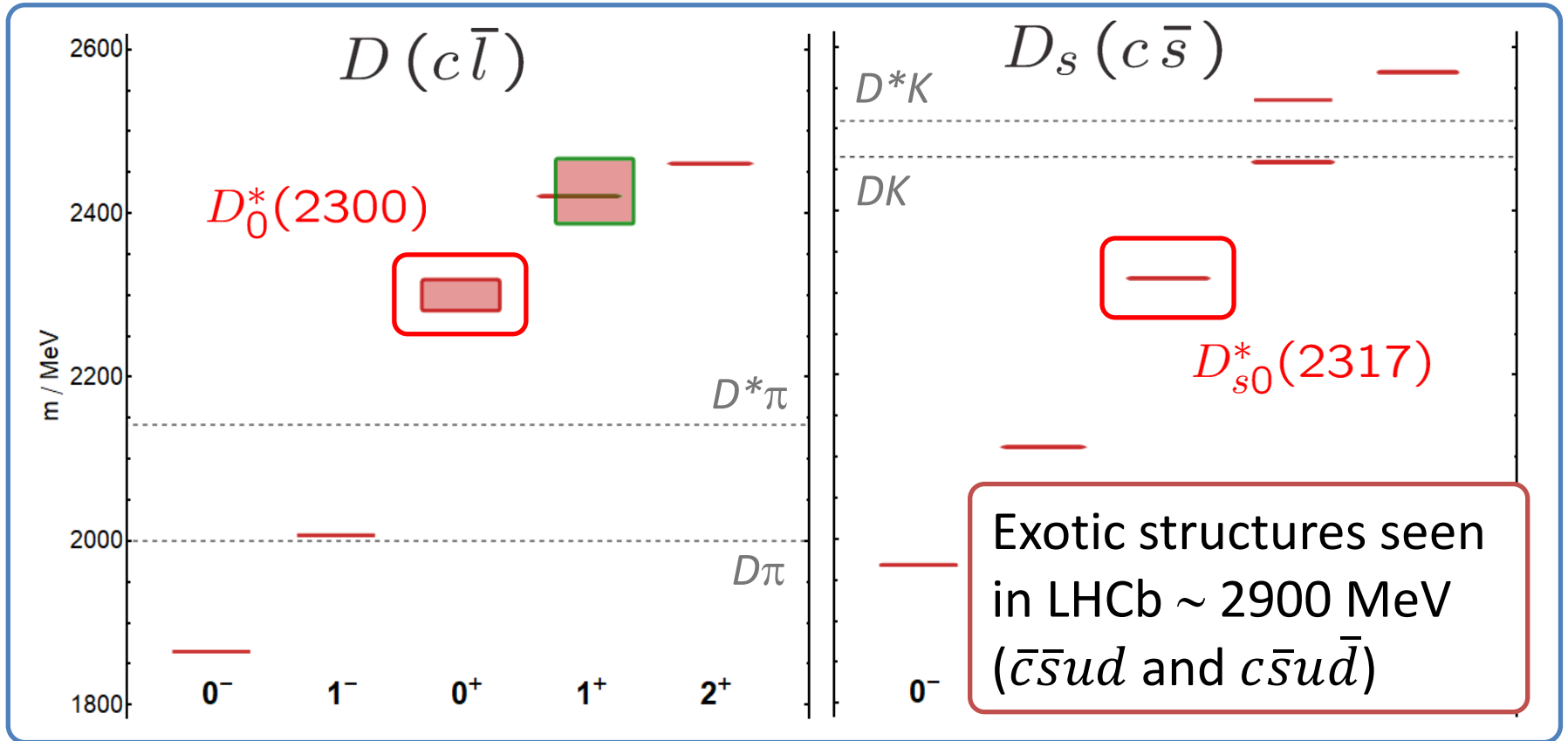
# Charm ( $D$ ) and charm-strange ( $D_s$ ) mesons



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# Charm ( $D$ ) and charm-strange ( $D_s$ ) mesons



## Other calculations

Some other lattice QCD work on  $DK$  and/or  $D\pi$  scattering:

- Mohler *et al* [PR D87, 034501 (2013), 1208.4059];
- Liu *et al* [PR D87, 014508 (2013), 1208.4535];
- Mohler *et al* [PRL 111, 222001 (2013), 1308.3175];
- Lang *et al* [PR D90, 034510 (2014), 1403.8103];
- Bali *et al* (RQCD) [PR D96, 074501 (2017), 1706.01247];
- Alexandrou *et al* (ETM) [PR D101 034502 (2020), 1911.08435];
- Gregory *et al* [2106.15391];
- Yan *et al* [2404.13479].

Also:

- Martínez Torres *et al* [JHEP 05 (2015) 153, 1412.1706];
- Albaladejo *et al* [PL B767, 465 (2017), 1610.06727];
- Du *et al* [PR D98, 094018 (2018), 1712.07957];
- Guo *et al* [PR D98, 014510 (2018), 1801.10122];
- Guo *et al* [EPJ C79, 13 (2019), 1811.05585];
- Lutz, Guo, Heo, Korpa [PR D106, 114038 (2022), 2209.10601];
- Korpa *et al* [PR D107, L031505 (2023), 2211.03508];
- Asokan *et al* [PR D107, L031505 (2023), 2212.07856];
- Gil-Domínguez, Molina [PRD , 2306.01848].



## $DK/\pi$ with $SU(3)_F$ sym

$SU(3)_F$  flavour symmetry ( $m_u = m_d = m_s$ )

$D_{\bar{3}}$  (D,  $D_s$ )  $\approx 1960$  MeV;  $\eta_8$  ( $\pi$ , K, ...)  $\approx 690$  MeV

$\eta_1 \approx 940$  MeV

Elastic  $D_{\bar{3}} \eta_8$  scattering:  $\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$   
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- Study dependence of  $DK/\pi$  scattering on light-quark mass
- Disentangle different  $SU(3)_F$  multiplets

( $I = 0$ )  $DK-D_s\eta$ :  $\bar{\mathbf{3}} \oplus \bar{\mathbf{15}}$

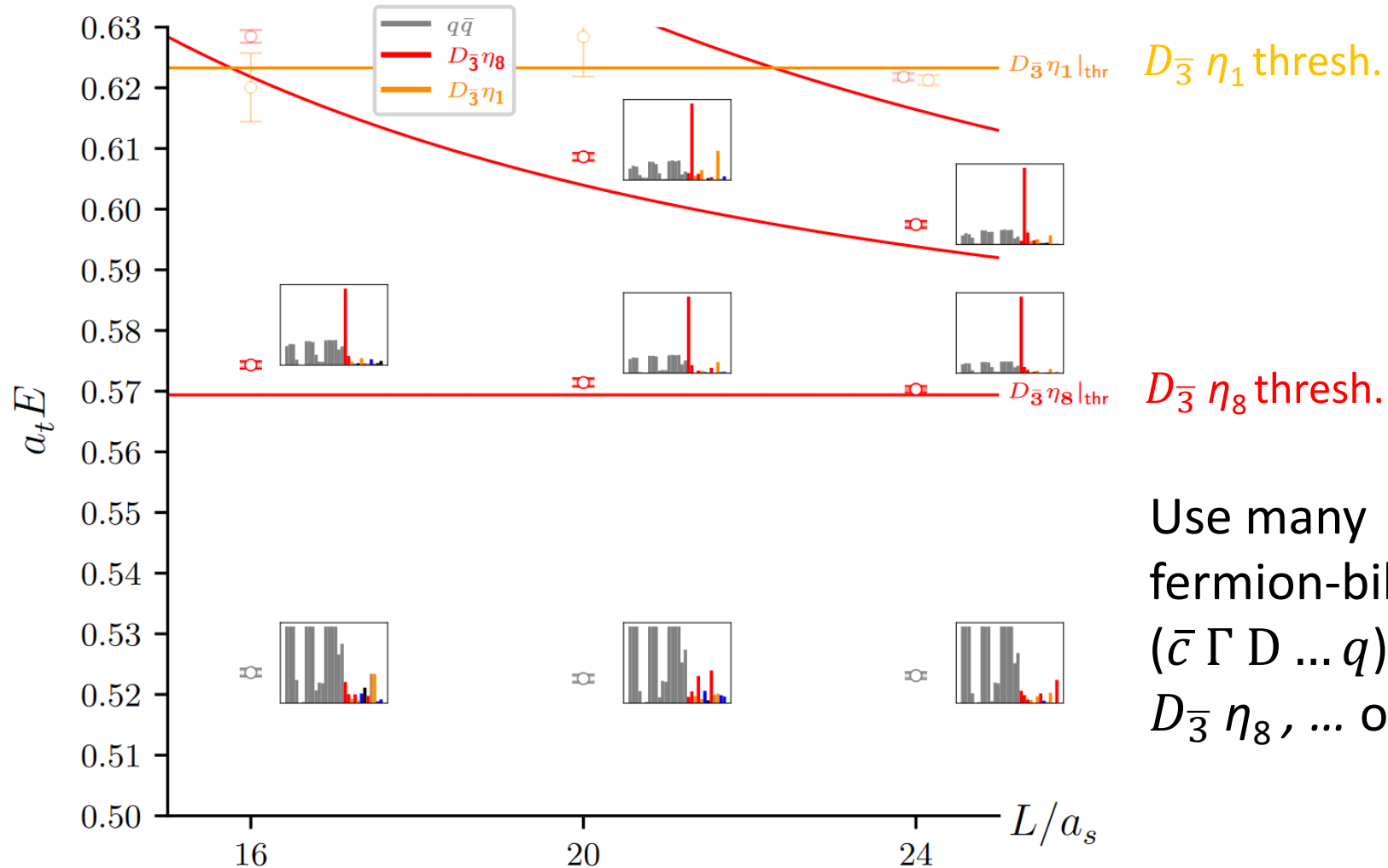
( $I = \frac{1}{2}$ )  $D\pi-D\eta-D_s\bar{K}$ :  $\bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$

[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); PR D98 014510 (2018) (1801.10122); EPJ C79, 13 (2019) (1811.05585); arXiv:2106.15391; PR D107, L031505 (2023)]

# $DK/\pi$ with $SU(3)_F$ sym – $\bar{3}$

[Yeo, Thomas, Wilson (HadSpec),  
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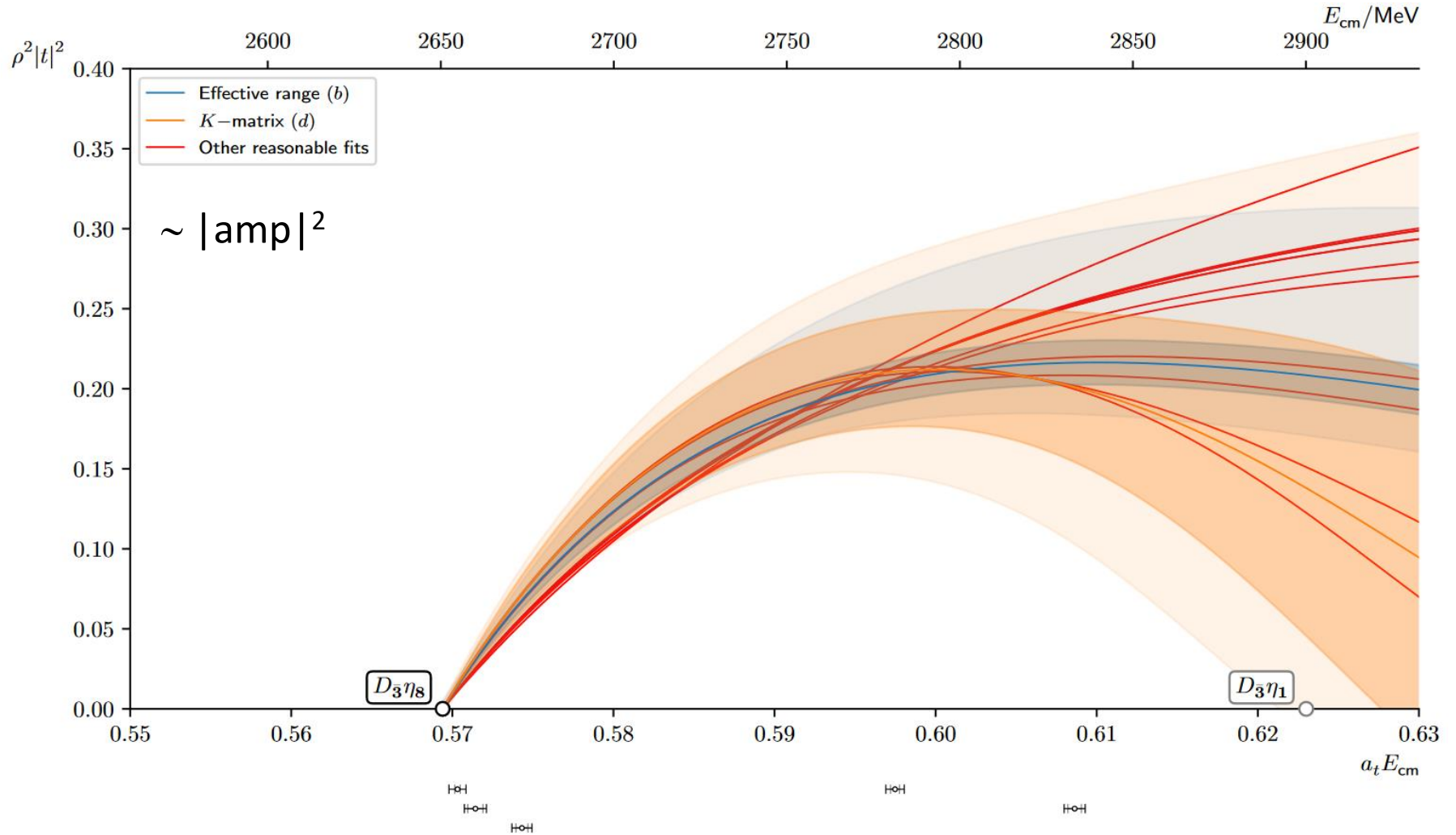
$$\mathbf{P} = [0,0,0] \quad J^P = 0^+, (4^+, \dots)$$



Use many  
fermion-bilinear  
( $\bar{c} \Gamma D \dots q$ ) and  
 $D_{\bar{3}}\eta_8, \dots$  operators

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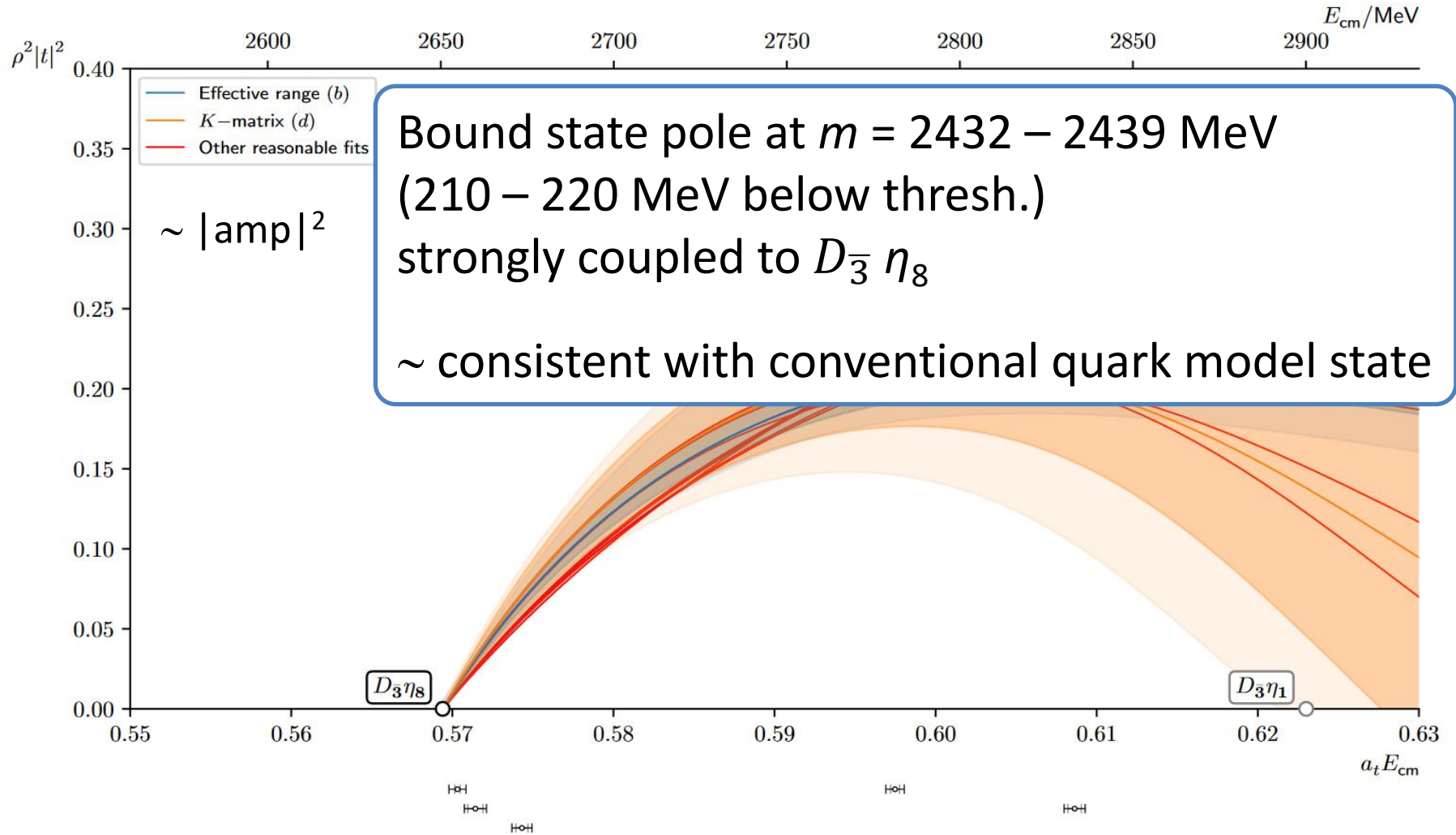
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Constrained by 8 energy levels

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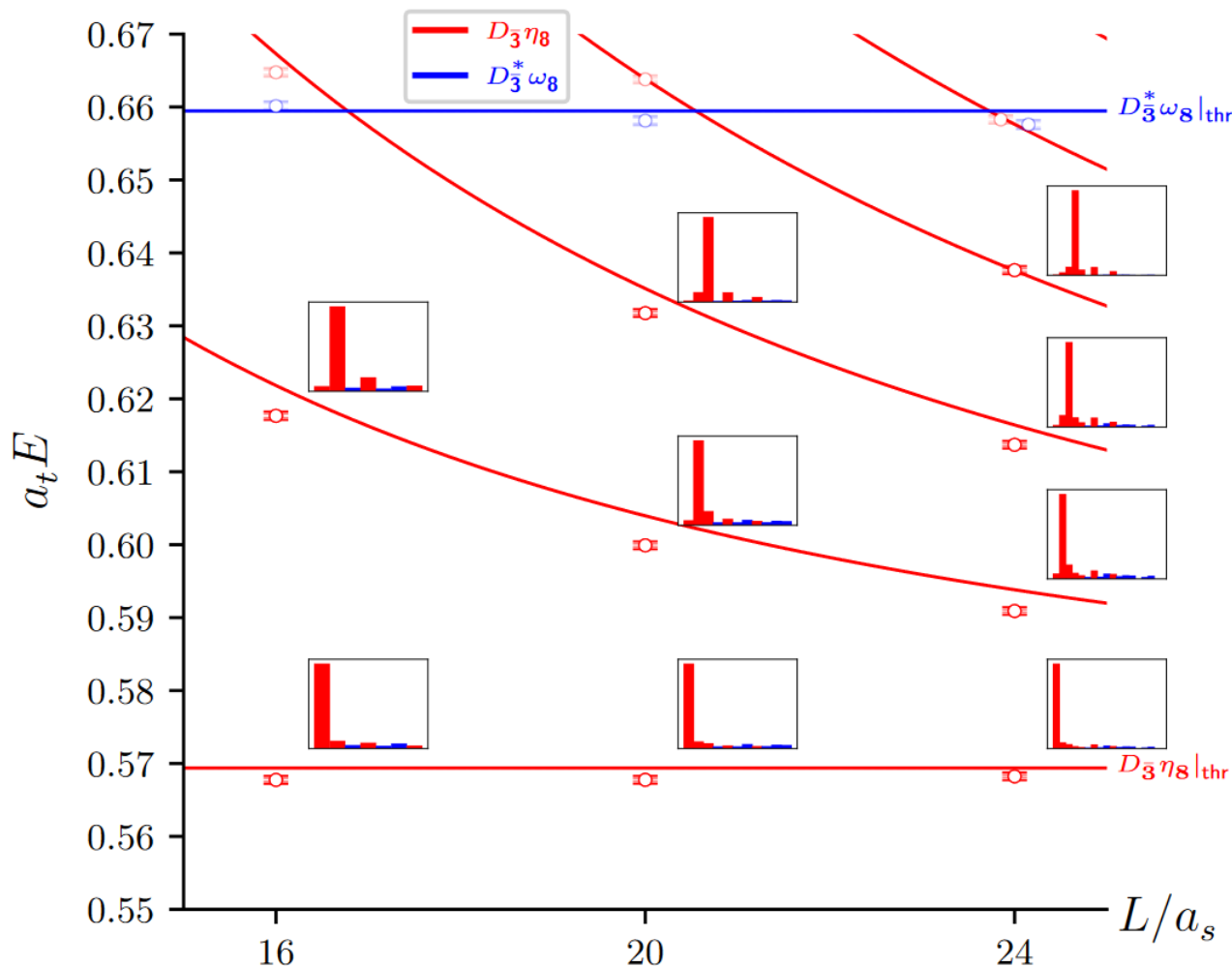


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# DK/ $\pi$ with $SU(3)_F$ sym – 6

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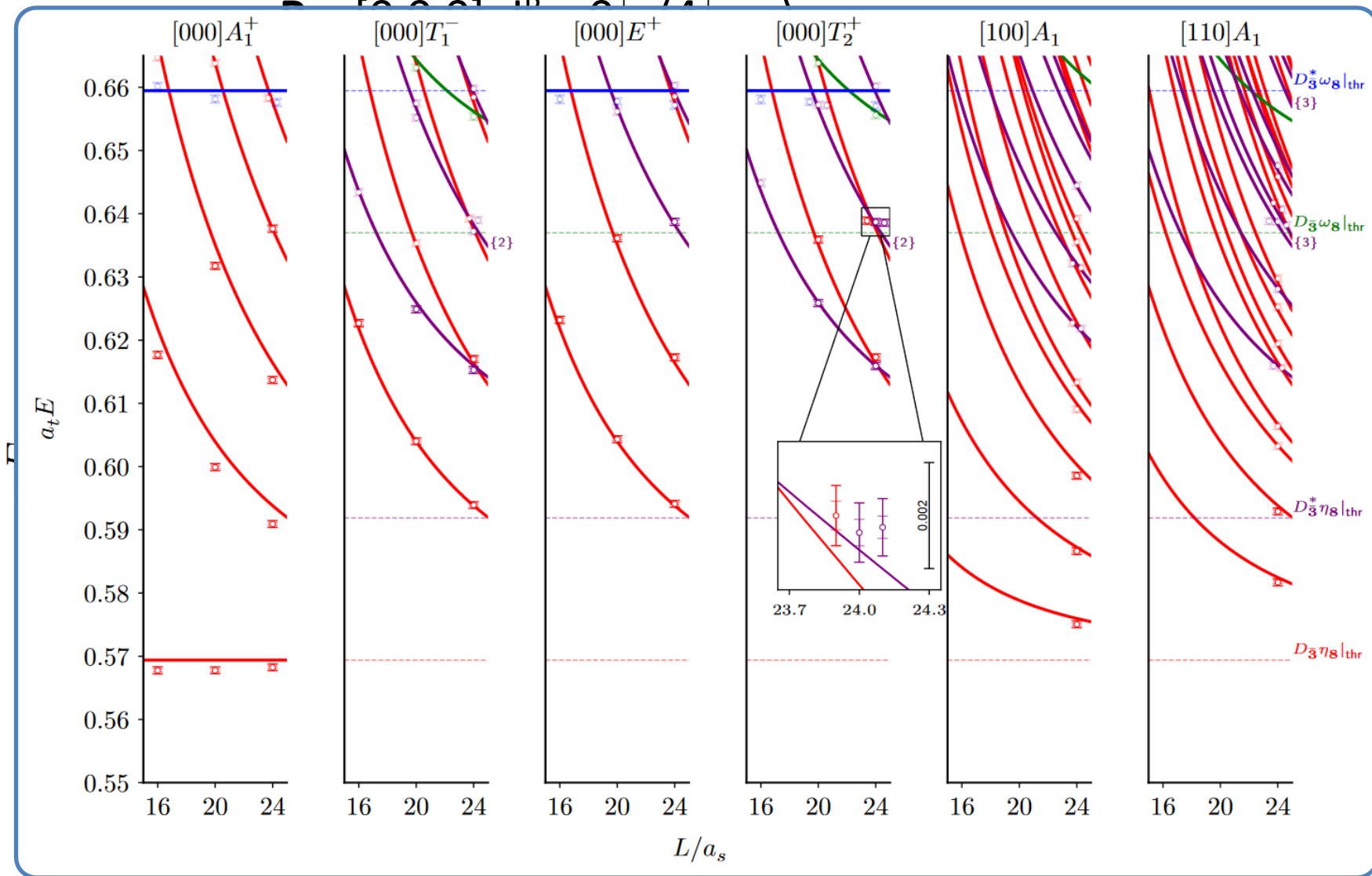
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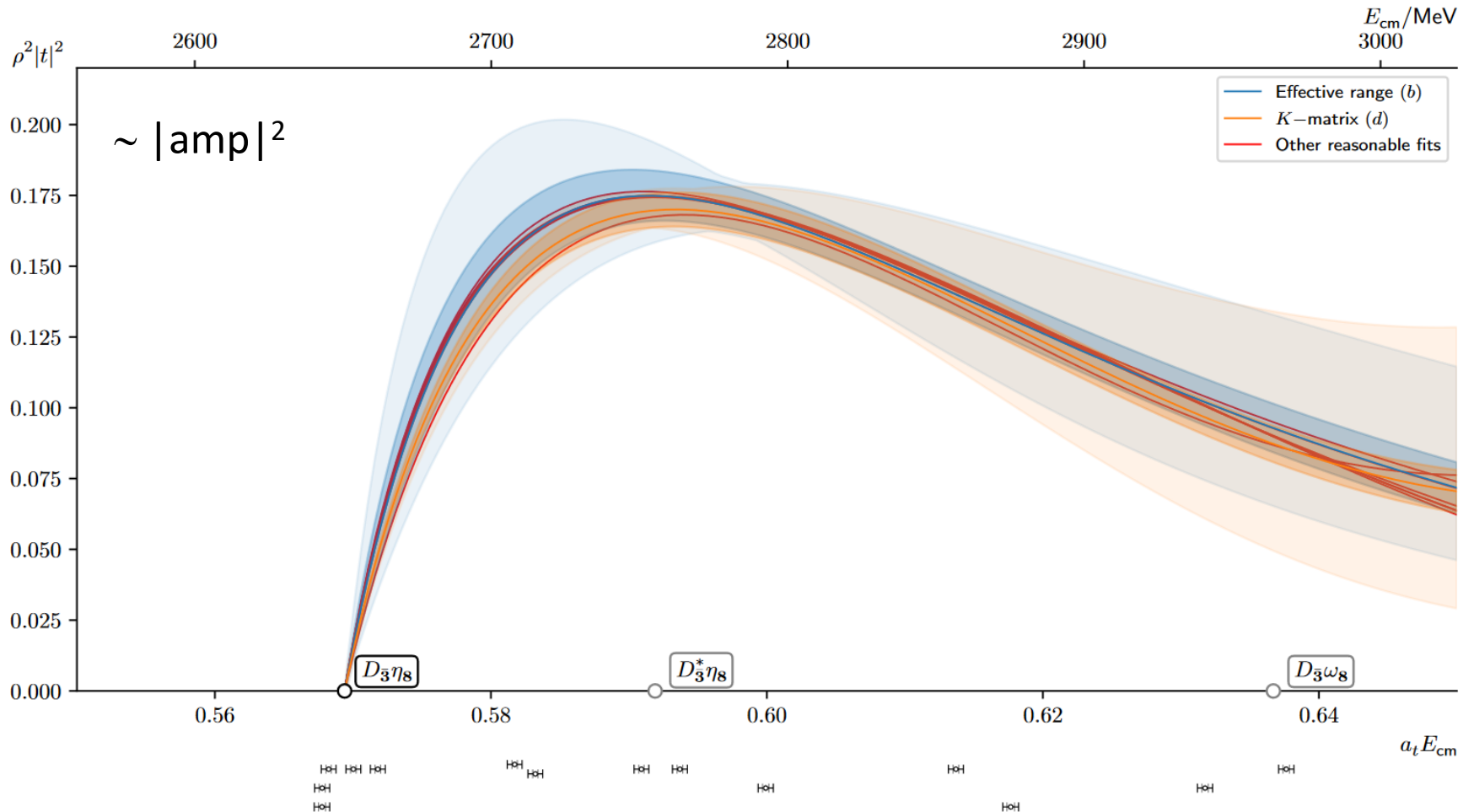
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# DK/π with SU(3)<sub>F</sub> sym – 6

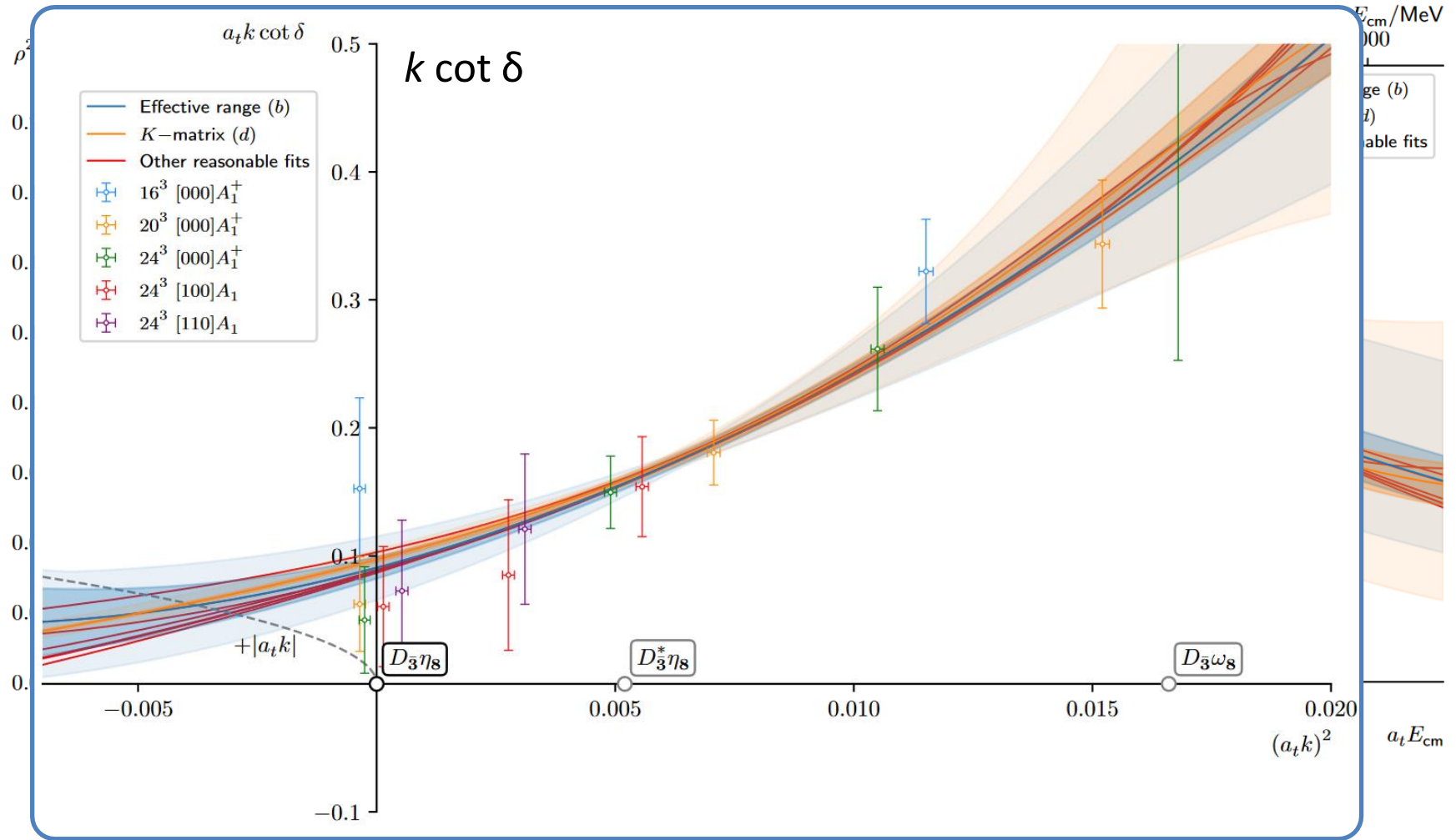
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Constrained by 14 energy levels

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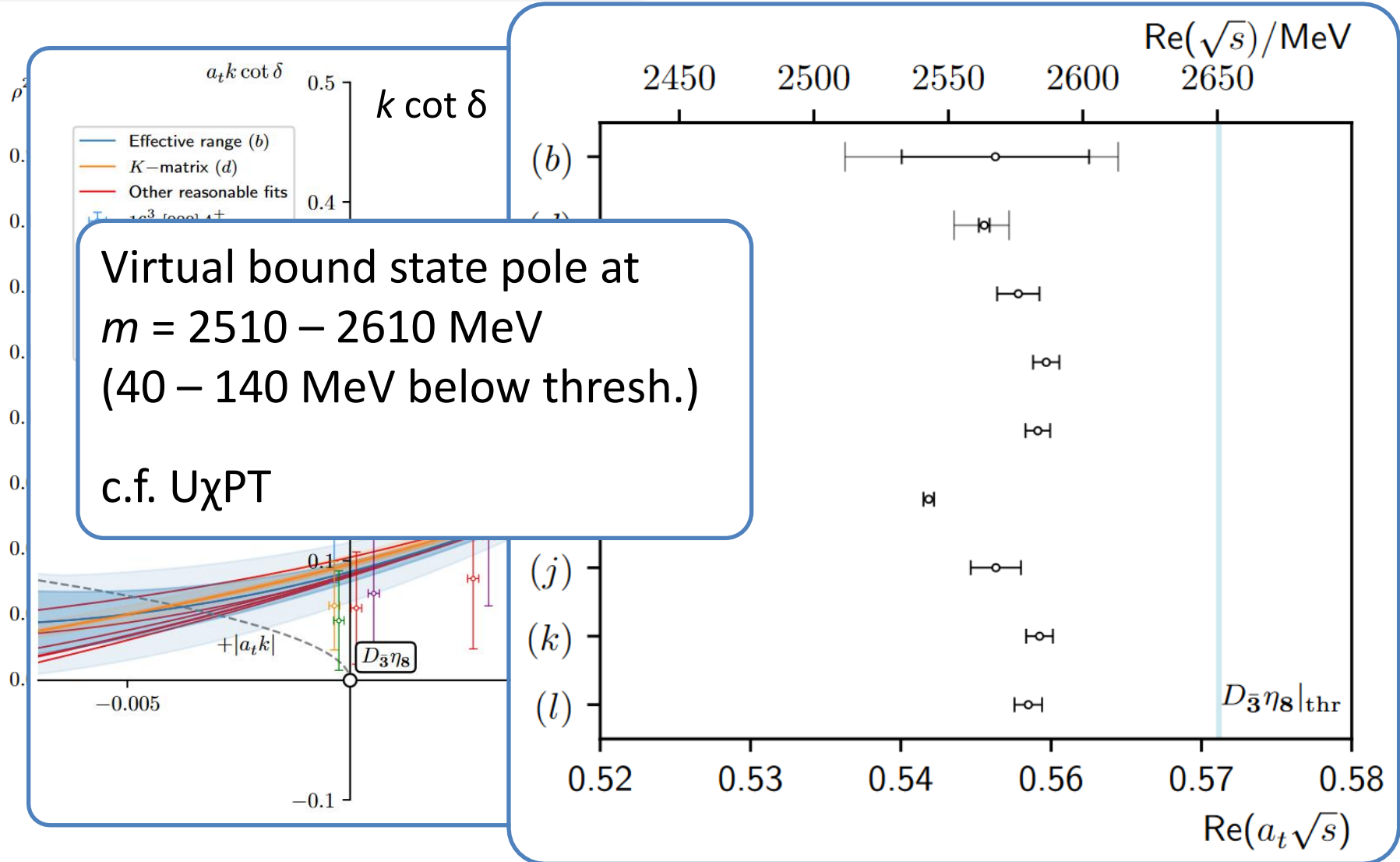
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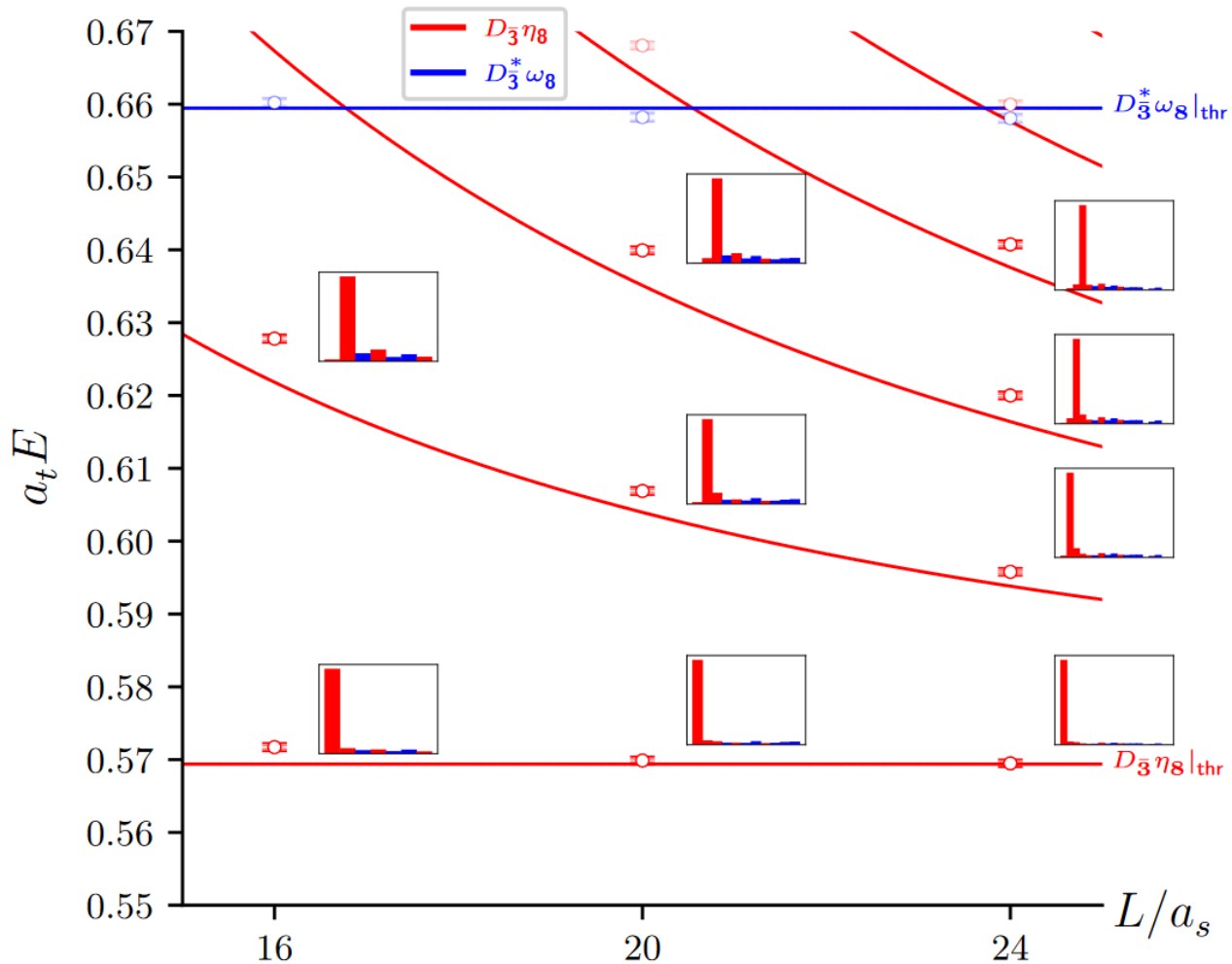
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# $DK/\pi$ with $SU(3)_F$ sym – $\overline{15}$

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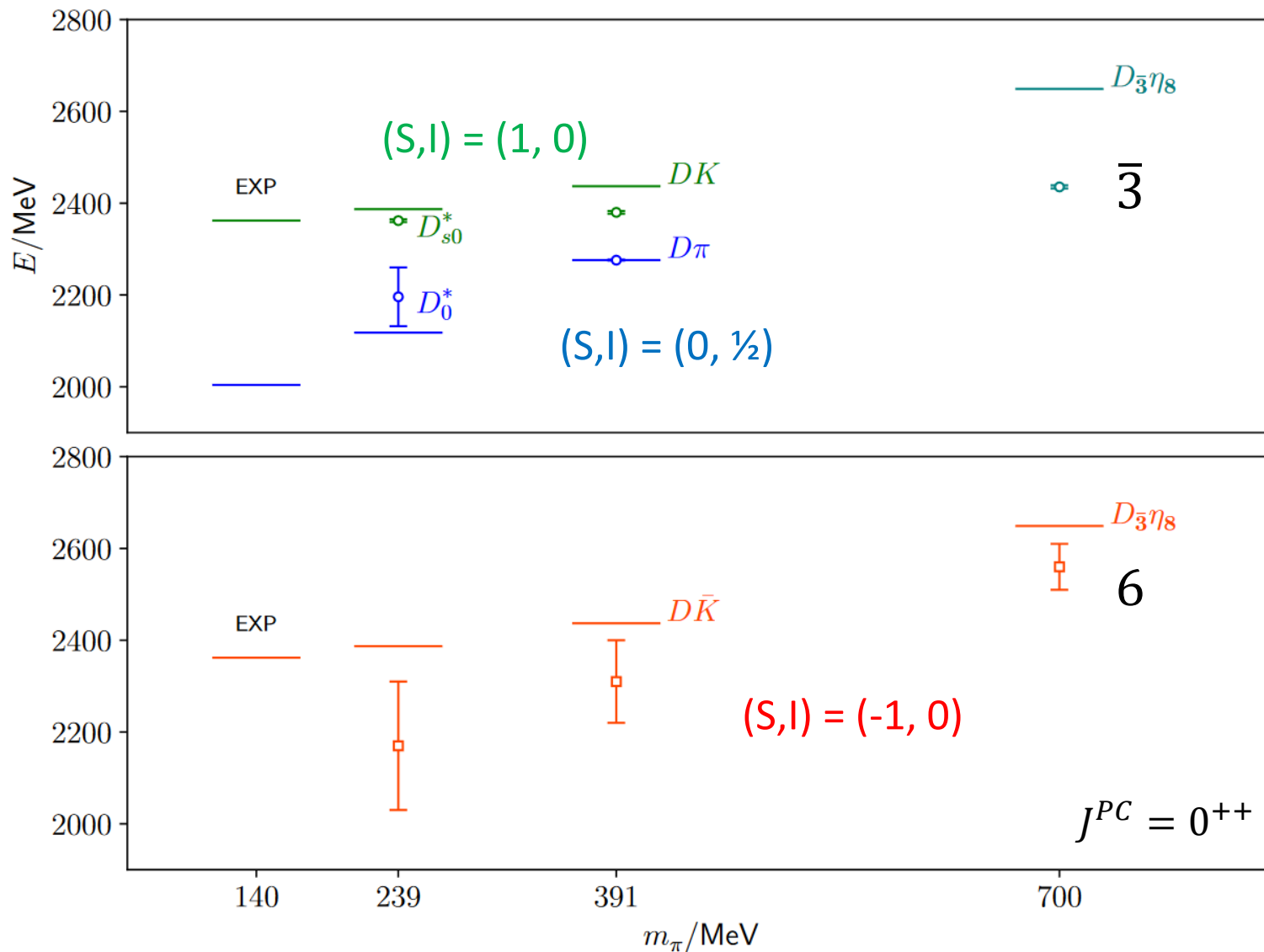


Use many  $D_{\overline{3}} \eta_8, \dots$   
operators

Weak repulsion.  
No poles in energy  
region considered

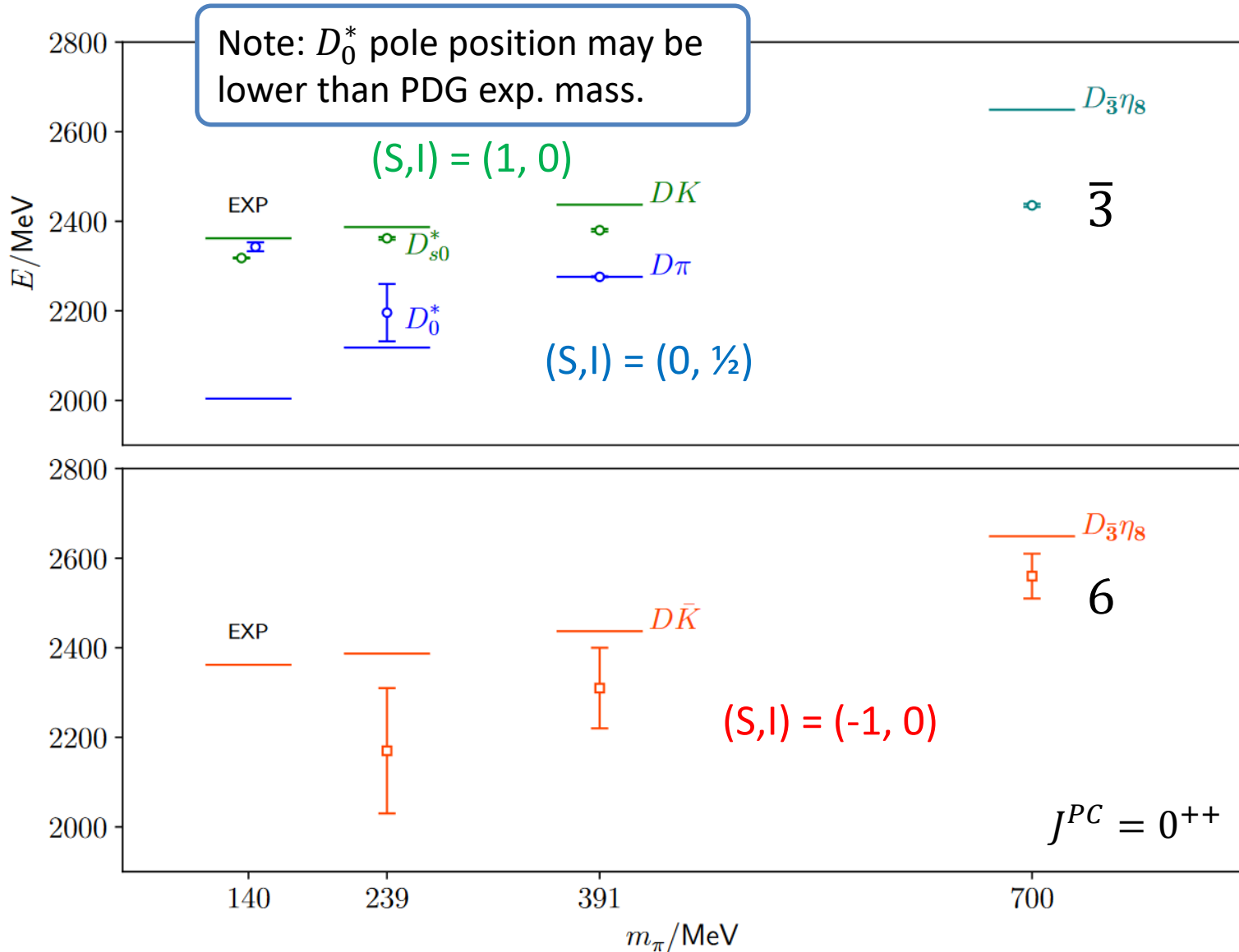
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[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



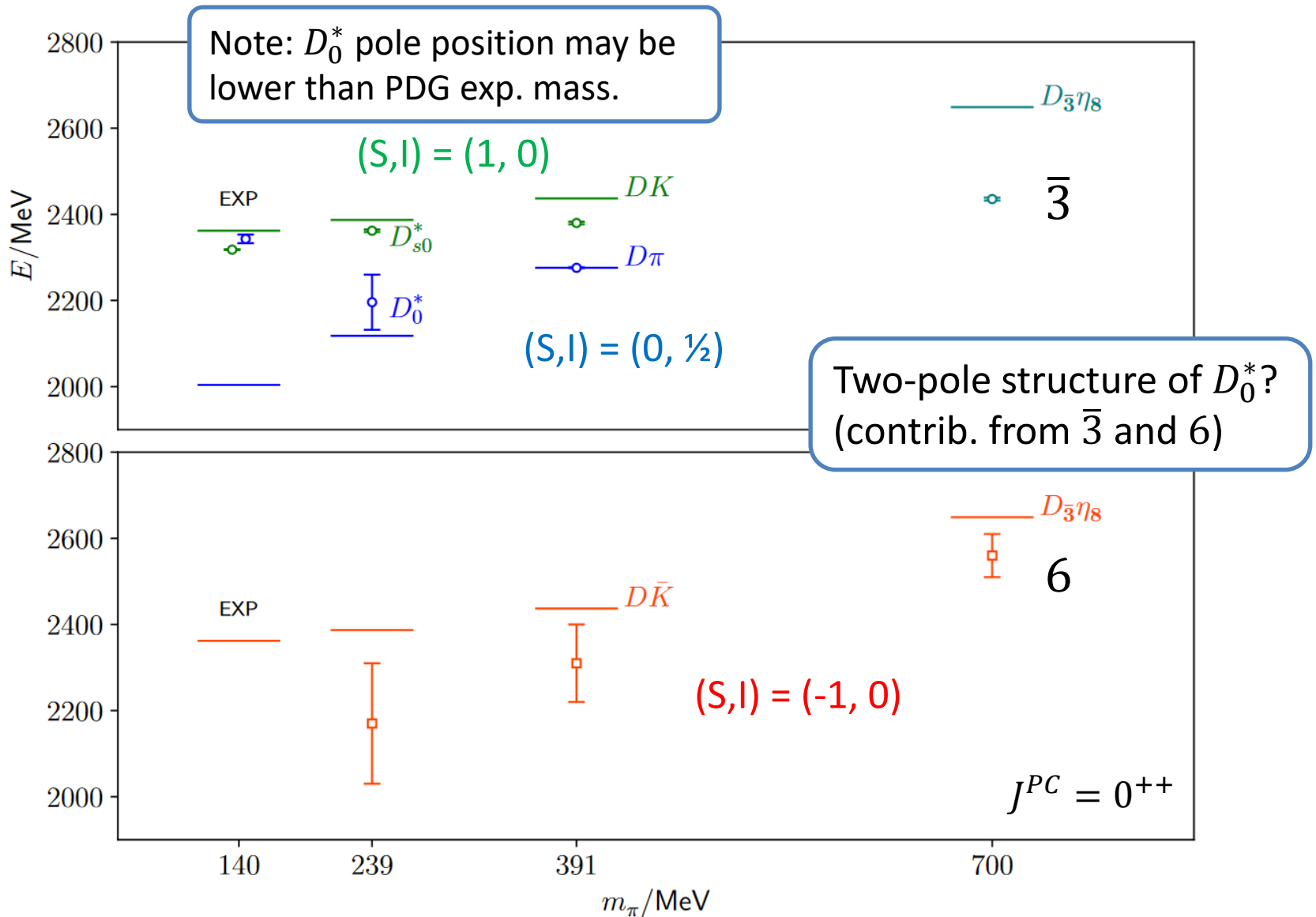
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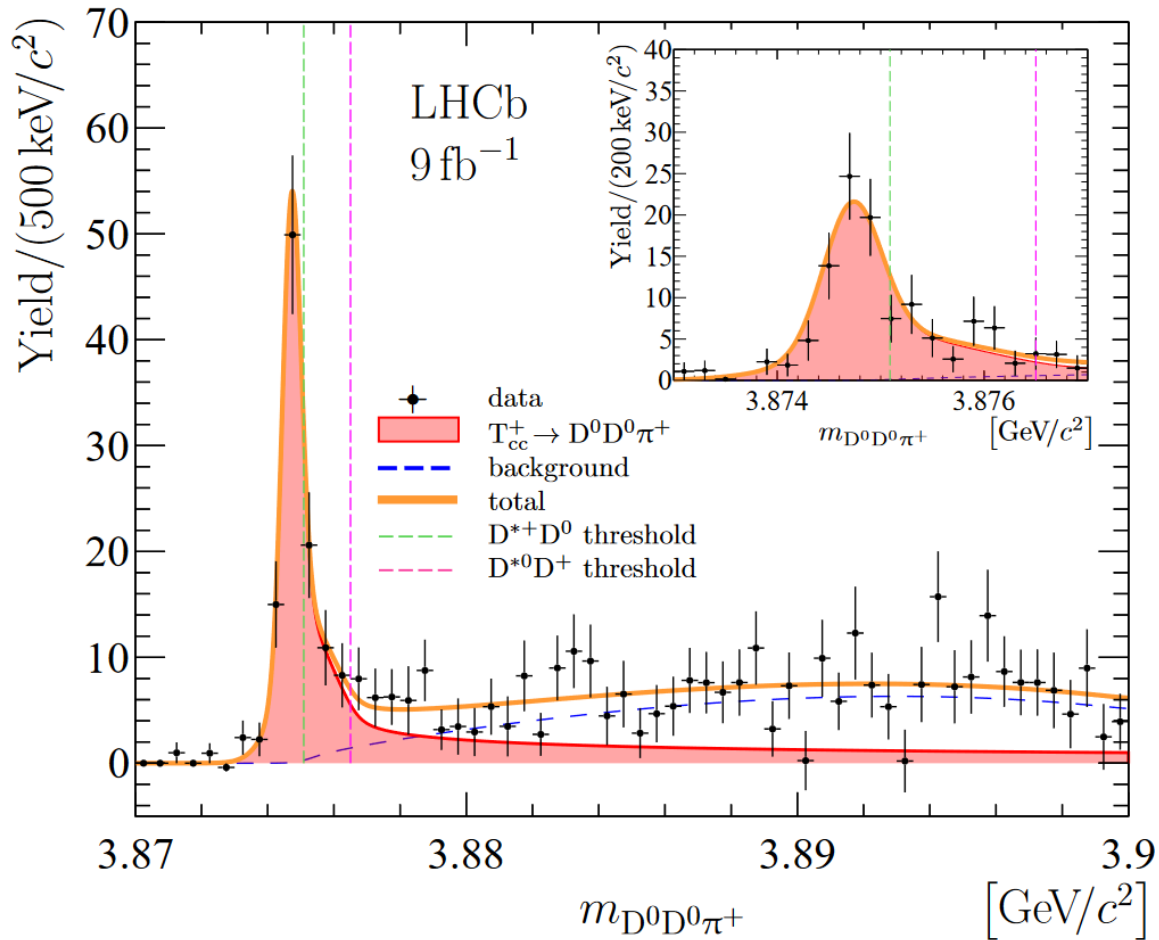






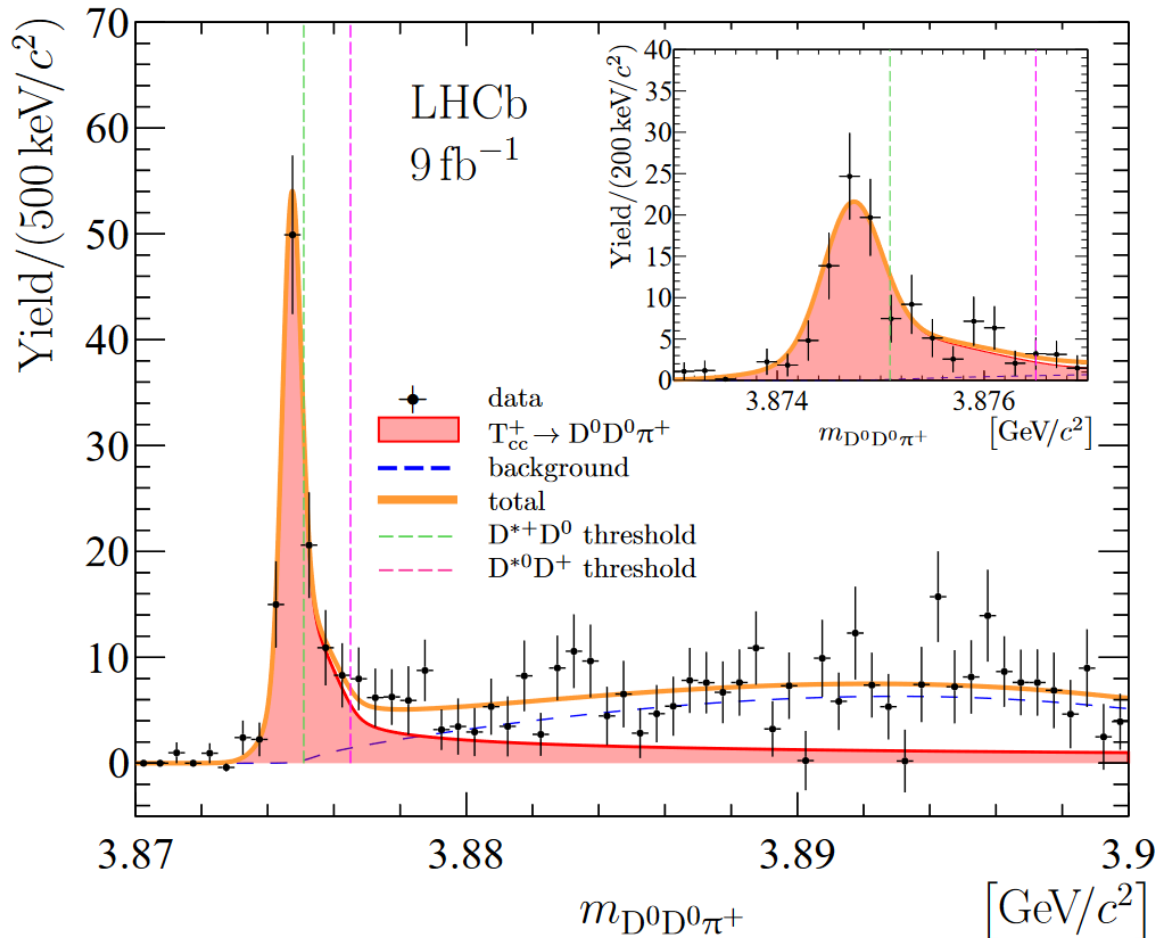
$T_{cc}$ 

$T_{cc}^+$  seen in  $D^0 D^0 \pi^+$  at LHCb [2109.01038, 2109.01056]  
Close to  $DD^*$  threshold,  $J^P=1^+$ ,  $I=0$ , exotic flavour ( $cc\bar{u}\bar{d}$ )



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Other lattice calcs:

- Padmanath & Prelovsek [2202.10110, PRL];
- Chen *et al* [2206.06185, PLB];
- Lyu *et al* (HAL QCD) [2302.04505, PRL];
- Collins, Nefediev, Padmanath, Prelovsek [2402.14715, PRD];

See also:

- Du *et al* [2303.09441, PRL];
- Meng *et al* [2312.01930, PRD].

What about higher energies (coupled  $DD^*$ ,  $D^* D^*$ )?

## Coupled $DD^*$ , $D^*D^*$ scattering

First lattice QCD calculation  
of coupled  $DD^*$ ,  $D^*D^*$   
scattering ( $l=0$ )

$m_\pi \approx 391$  MeV ( $D^*$  is stable),  
3 lattice volumes

Use many meson-meson-like  
 $DD^*$  and  $D^*D^*$  ops

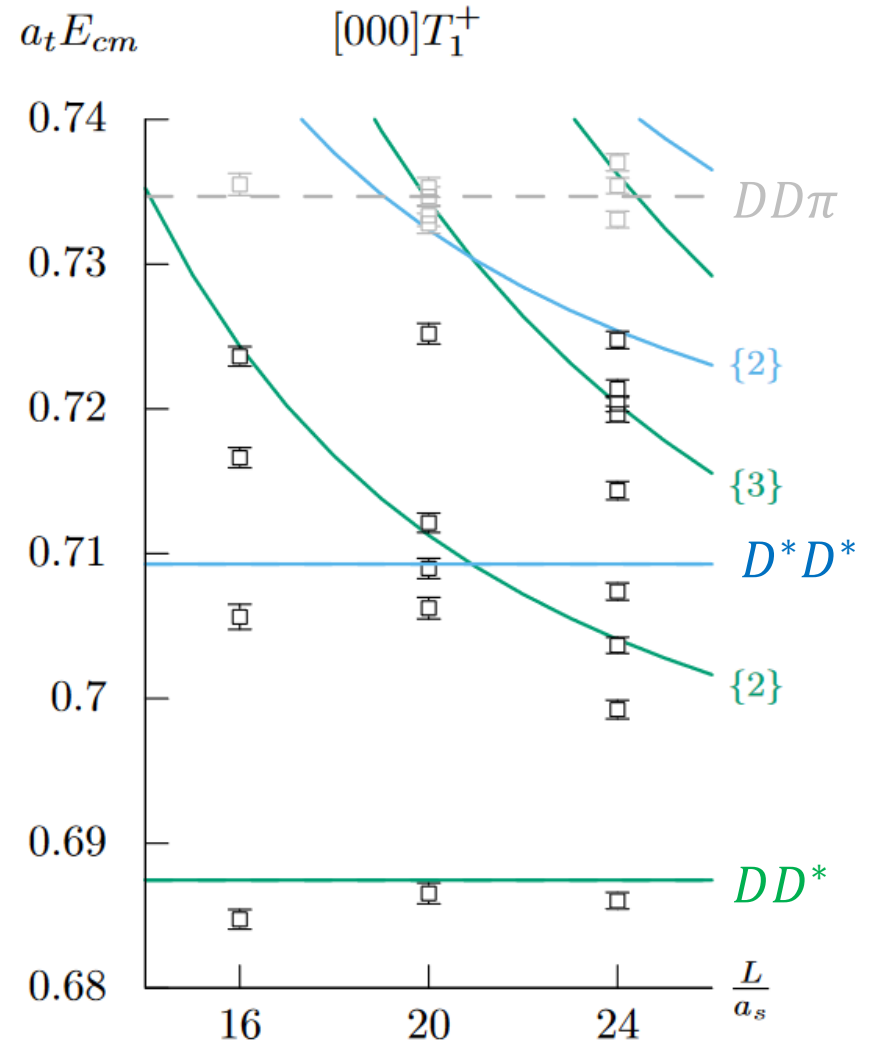
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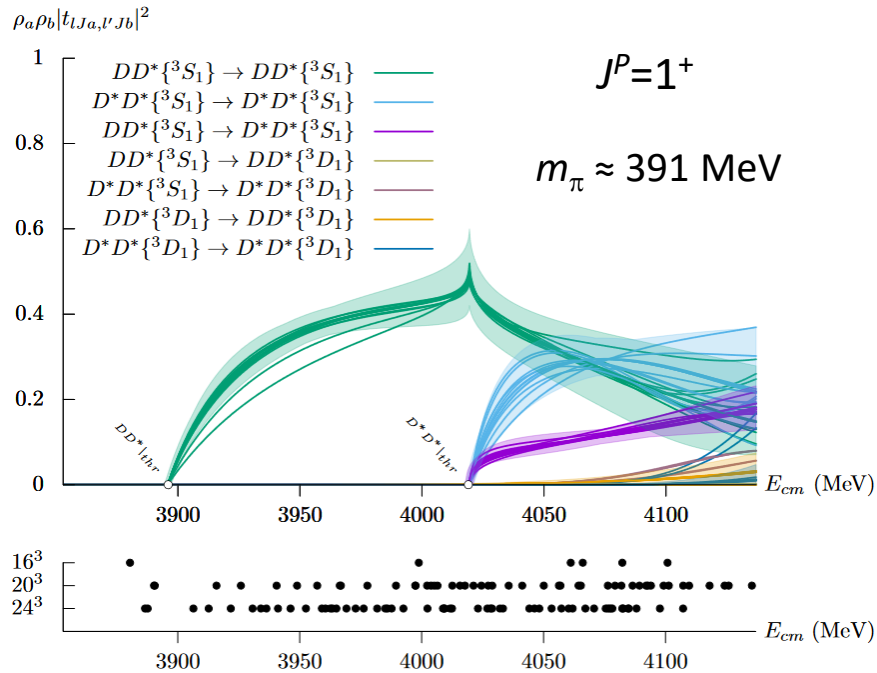
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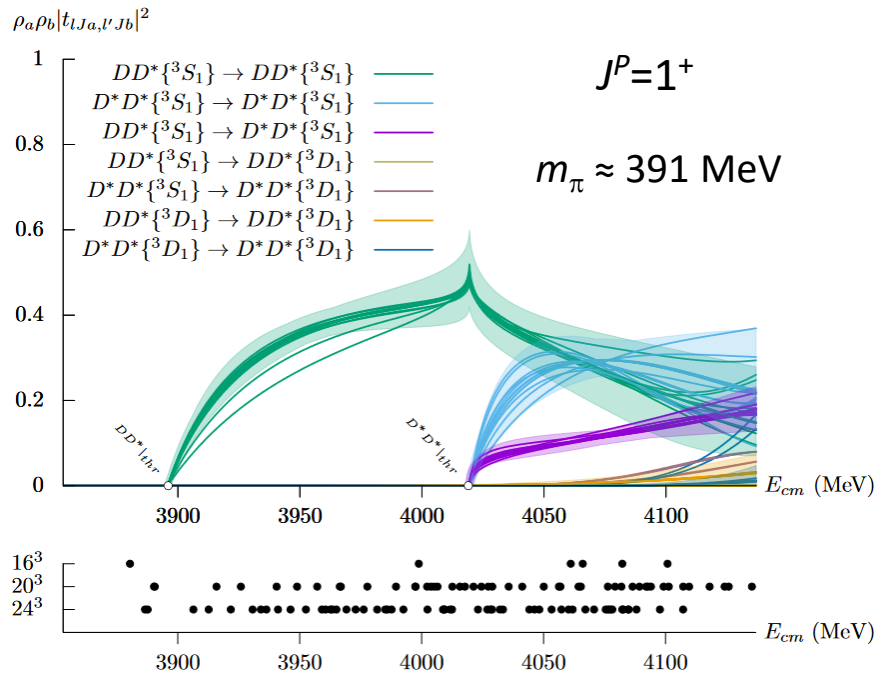


# $T_{CC}$ and $T'_{CC}$ in coupled $DD^*$ , $D^*D^*$ scattering

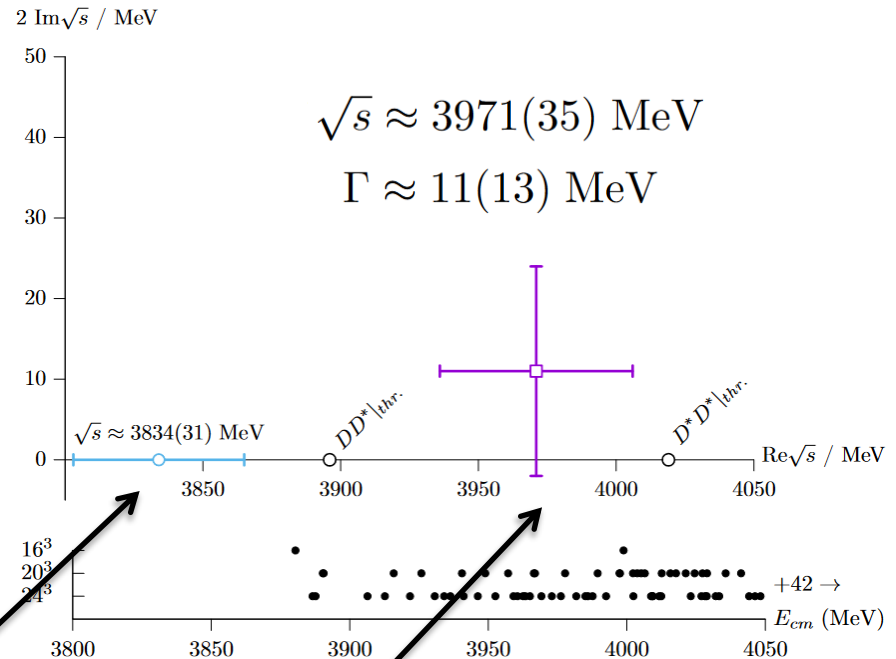


Use 109 energy levels

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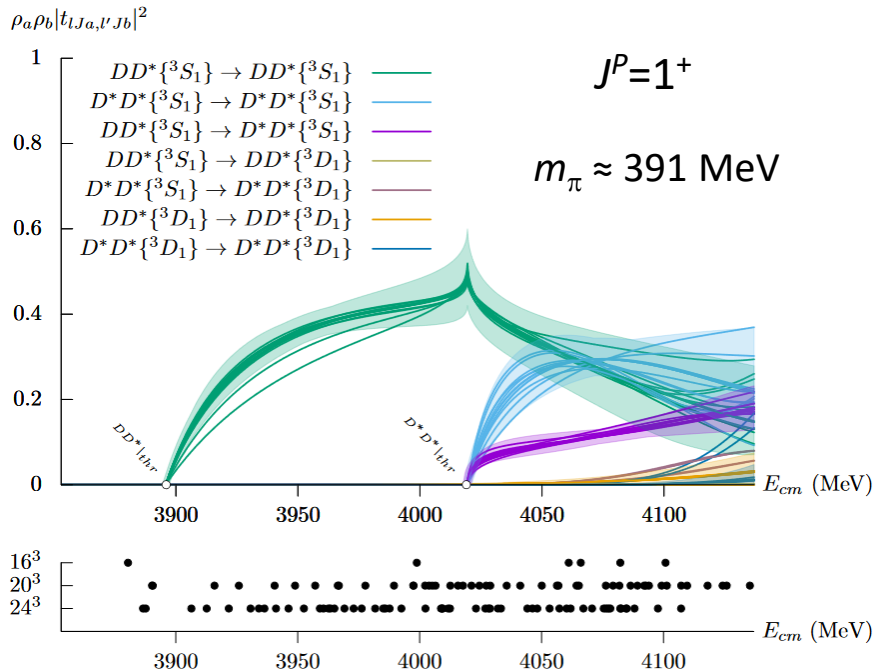


Virtual bound state ( $T_{CC}$ )  
below  $DD^*$  threshold  
 $\sqrt{s} \approx 3834(31) \text{ MeV}$

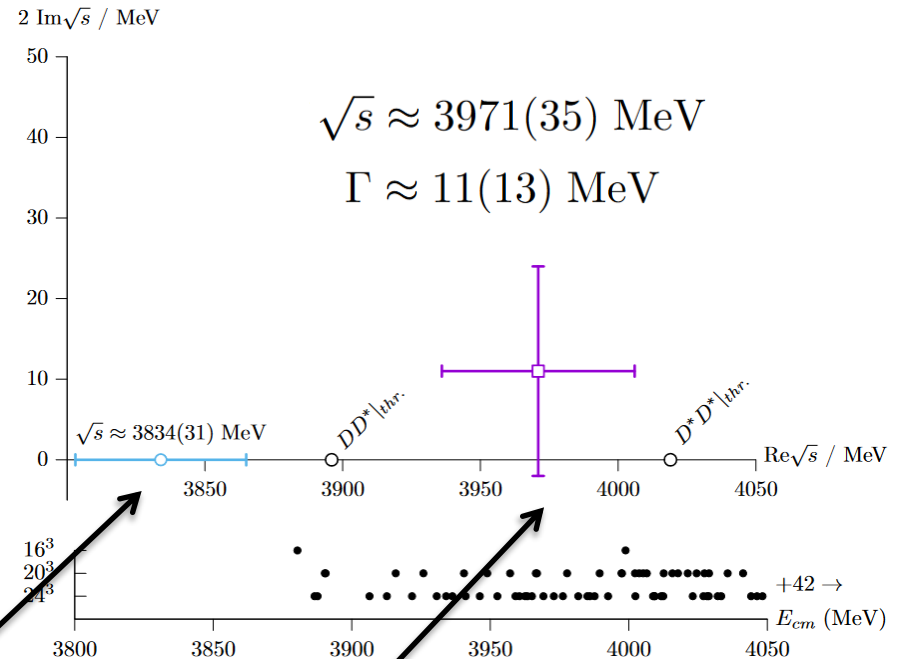


Resonance ( $T'_{CC}$ ) pole on (+, -)  
sheet below  $D^*D^*$  threshold  
(might be seen in  $DD$ ,  $DD\pi$ )

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Resonance ( $T'_{cc}$ ) pole on (+, -)  
 sheet below  $D^*D^*$  threshold  
 (might be seen in  $DD$ ,  $DD\pi$ )

Effect of left hand cut from  $\pi$  exchange  
 ( $\approx 18 \text{ MeV}$  below  $DD^*$  threshold)?





# Charmonium scalar ( $0^{++}$ ) and tensor ( $2^{++}$ ) resonances

Experimental situation:

- Ground state  $\chi_{c0}(1P)$  and  $\chi_{c2}(1P)$  below  $D\bar{D}$  threshold.  
Above that it is less clear...

# Charmonium scalar ( $0^{++}$ ) and tensor ( $2^{++}$ ) resonances

Experimental situation:

- Ground state  $\chi_{c0}(1P)$  and  $\chi_{c2}(1P)$  below  $D\bar{D}$  threshold. Above that it is less clear...
- $\chi_{c0}(3860) \rightarrow D\bar{D}$  (Belle). Not seen in  $B^+ \rightarrow D^+D^-K^+$  (LHCb). Theoretical reanalyses: may be from pole below  $D\bar{D}$  thresh.
- $X(3930) \rightarrow D\bar{D}$  (LHCb)
- $X(3960) \rightarrow D_s\bar{D}_s$  (LHCb)
- $X(3915) \rightarrow J/\psi\omega$  (Belle)
- $\chi_{c2}(3930) \rightarrow D\bar{D}$  (Belle, BABAR, LHCb)

# Charmonium $0^{++}$ and $2^{++}$ resonances

$m_\pi \approx 391$  MeV,  
3 lattice volumes

Use many  
fermion-bilinear ( $\bar{c} \Gamma D \dots c$ )  
and meson-meson-like ops  
( $\eta_c \eta$ ,  $D \bar{D}$ ,  $\eta_c \eta'$ ,  $D_s \bar{D}_s$ ,  $D \bar{D}^*$ ,  
 $D_s \bar{D}_s^*$ ,  $\psi \omega$ ,  $D^* \bar{D}^*$ ,  $\psi \phi$ ,  $\eta_c \sigma$ ,  
 $\chi_{c0,2} \sigma$ , ...)

First 'complete' lattice study  
of this energy region.

[**Wilson**, Thomas, Dudek, Edwards  
(HadSpec), 2309.14070 (PRL),  
2309.14071 (PRD)]

# Charmonium $0^{++}$ and $2^{++}$ resonances

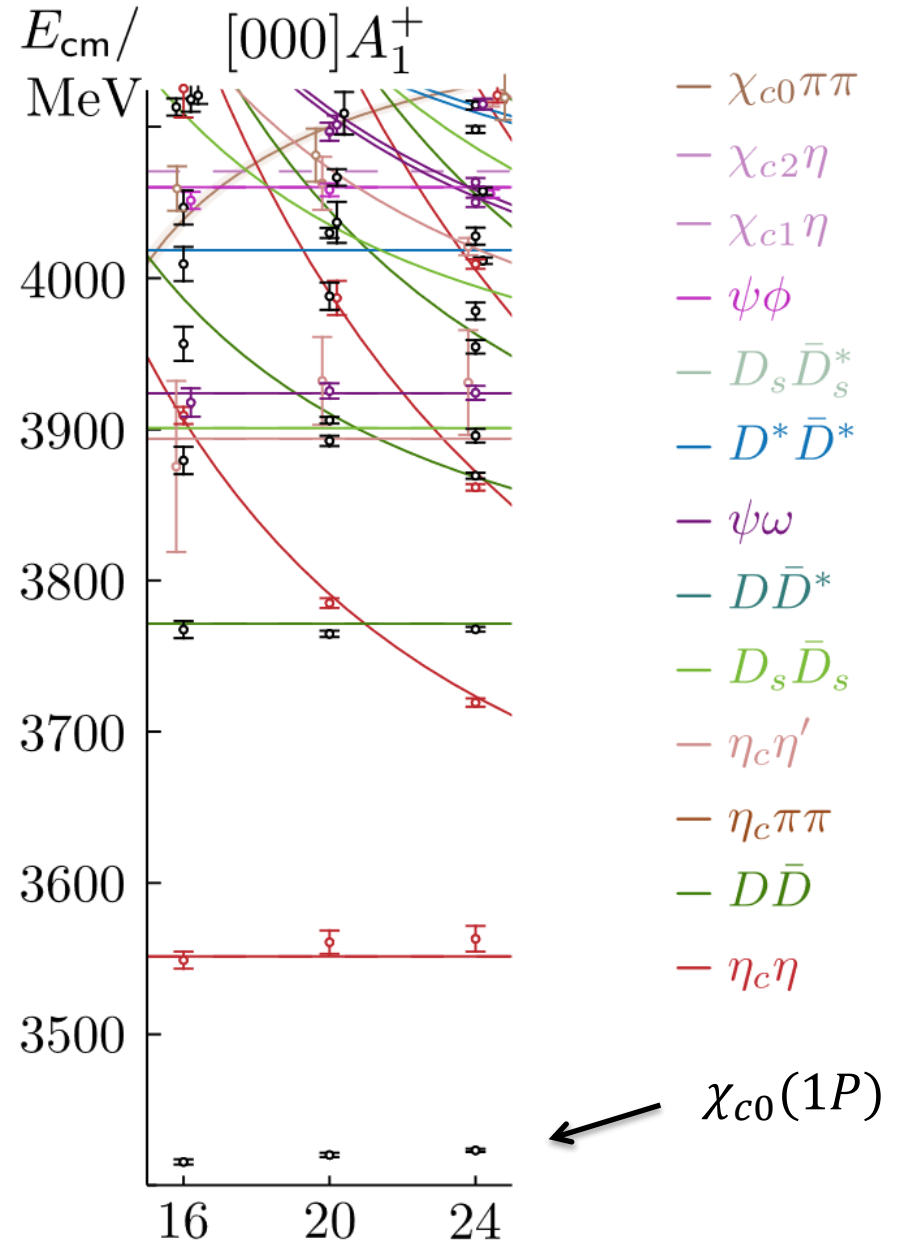
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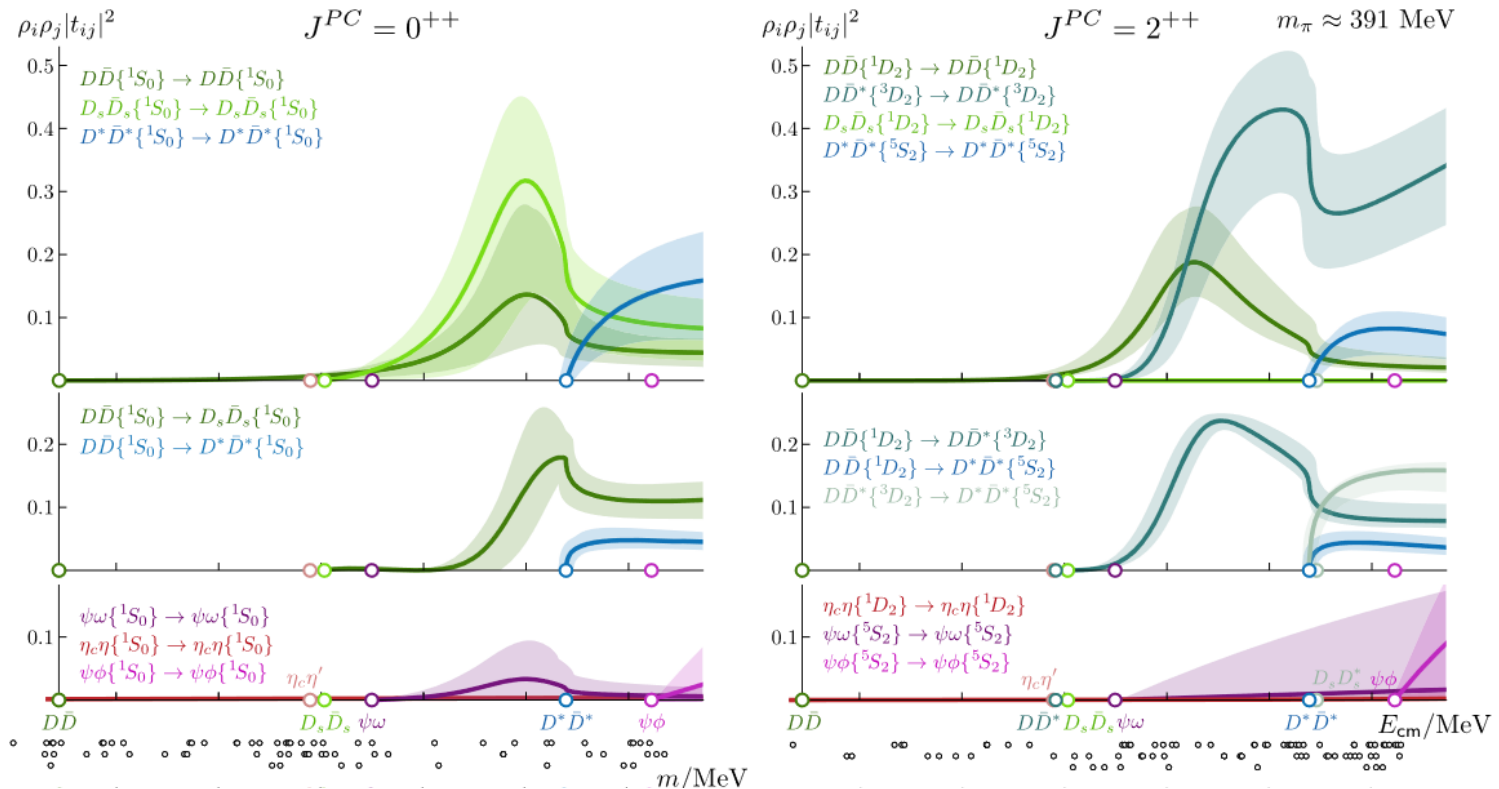
[Wilson, Thomas, Dudek, Edwards  
(HadSpec), 2309.14070 (PRL),  
2309.14071 (PRD)]

$$\mathbf{P} = [0,0,0] \quad J^P = 0^+, (4^+, \dots)$$



# Charmonium $0^{++}$ and $2^{++}$ resonances

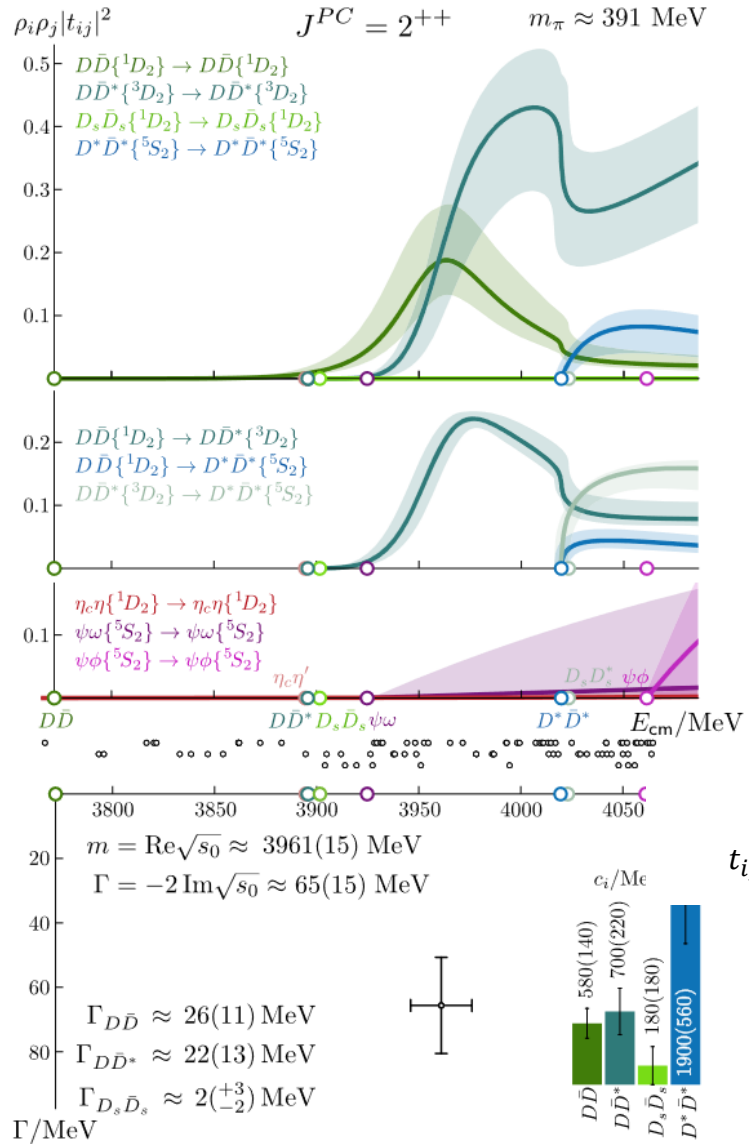
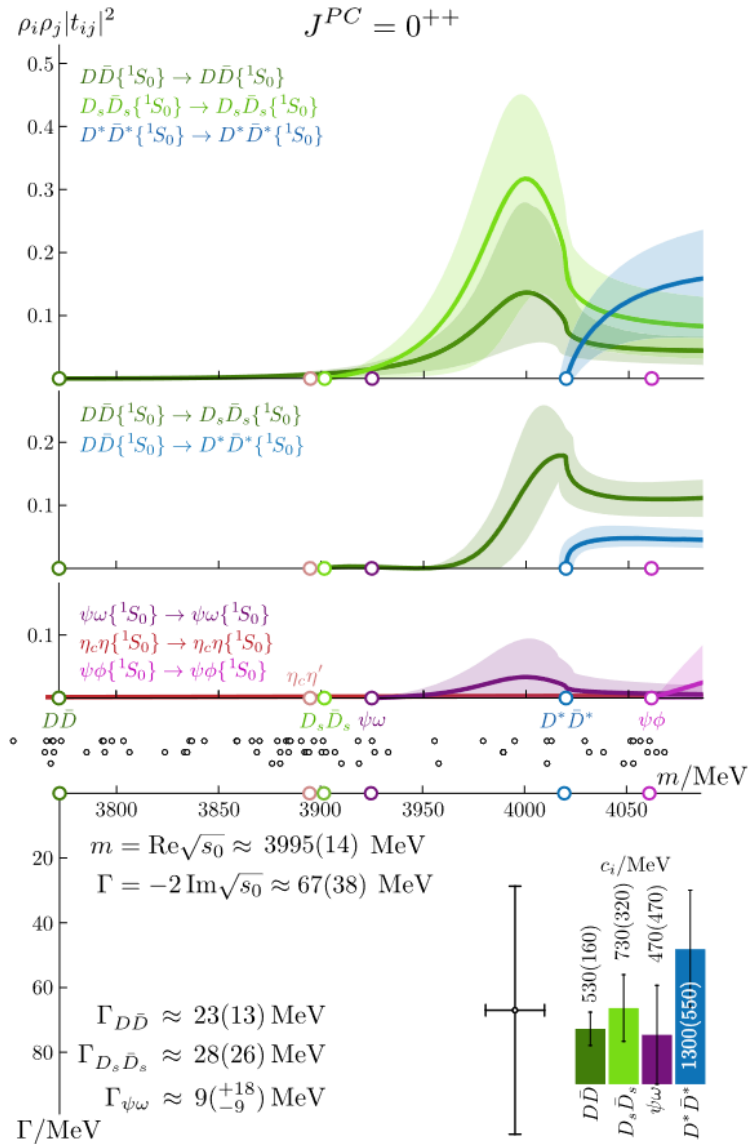
[2309.14070, 2309.14071]



Use more than 200 energy levels

# Charmonium $0^{++}$ and $2^{++}$ resonances

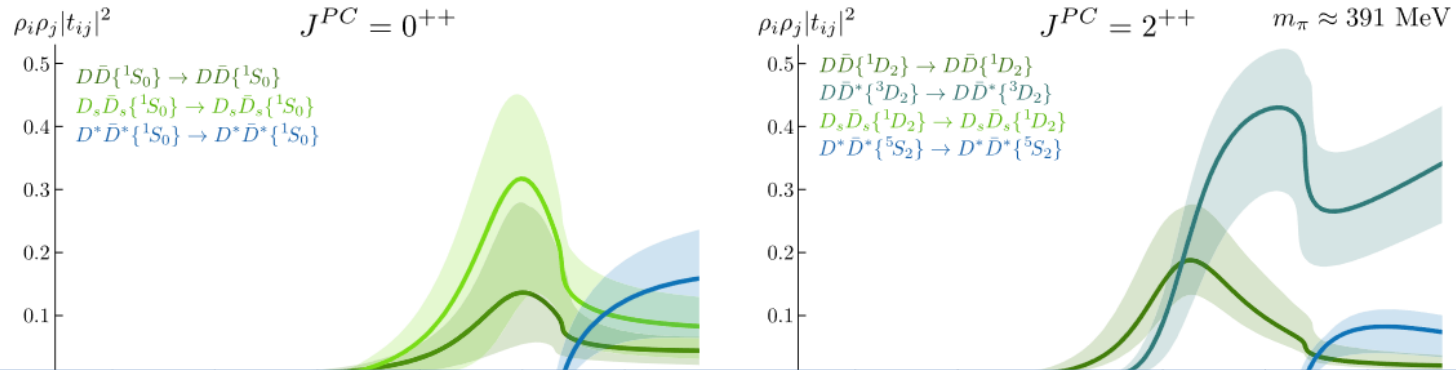
[2309.14070, 2309.14071]



$$t_{ij} \sim \frac{c_i c_j}{(s_0 - s)}$$

# Charmonium $0^{++}$ and $2^{++}$ resonances

[2309.14070, 2309.14071]



- Only one  $\chi_{c0}$  and one  $\chi_{c2}$  resonance up to  $\approx 4100$  MeV.
- No large scattering amps in channels with  $\bar{c}c$  + light meson
- Above ground state  $\chi_{c0}$  no other  $0^{++}$  bound states or near- $D\bar{D}$  /  $D_s\bar{D}_s$  threshold resonances.  
c.f. claims for an additional  $\chi_{c0}(3860)$  by Belle [1704.01872], lattice calc by Prelovsek *et al* [2011.02542], some models and some reanalysis of experimental data.
- (Also bound state in  $2^{-+}$  and narrow resonance in  $3^{++}$ .)

s)





## Summary

- A few examples of recent lattice QCD calculations of charm and charmonium(-like) mesons.
  - $DK/\pi$  at  $SU(3)_F$  sym. point – bound state in  $\bar{3}$  and exotic-flavour virtual bound state in 6.
  - $T_{cc}$  and  $T'_{cc}$  in coupled  $DD^*$ ,  $D^*D^*$  scattering.
  - Scalar and tensor charmonium resonances (only one of each in energy region investigated).
- Study evolution as vary light-quark masses
- Effect of left hand cut?
- Three (or more!?) hadron scattering
- Probe structure, e.g. transitions and form factors

# Acknowledgements



UNIVERSITY OF  
CAMBRIDGE



Science and  
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Facilities Council

DiRAC

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Jülich: Travis Whyte

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