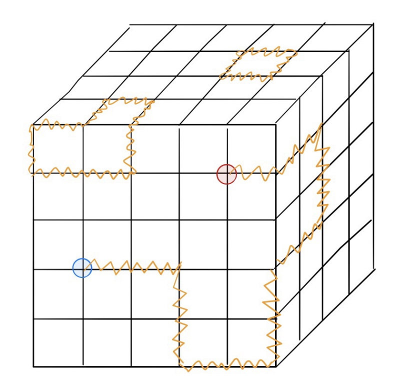
Quarkonium in lattice QCD

Brian Colquhoun

University of Glasgow

21 August 2024



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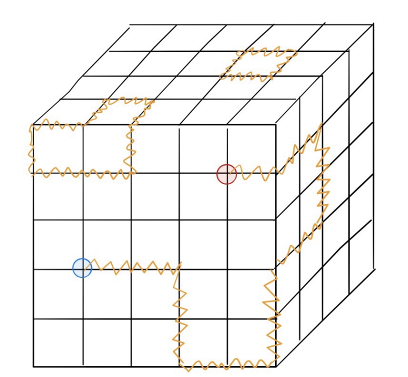
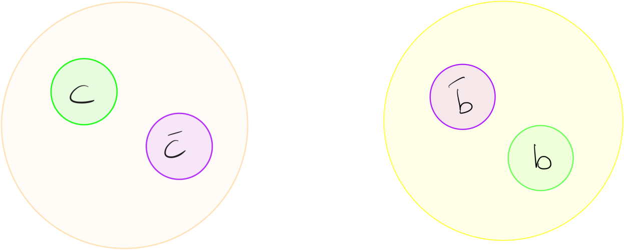
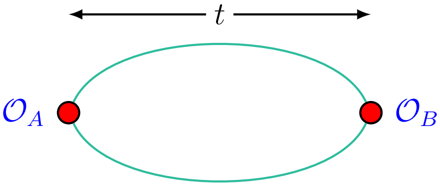


Introduction



Heavionium calculations presented here:

- ★ Precise HPQCD calculations using Highly Improved Staggered Quark (HISQ) action
  - Some now include QED effects
- ★ Realistic sea content in MILC HISQ configurations used here
  - Effect of 2+1+1 quarks
- ★ Note: comparisons true at time of publication
  - Some small changes, esp. to experimental results since



Fit these correlators

(Sometimes $a^2 = A$)

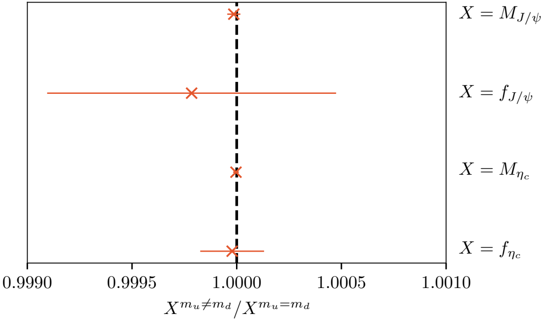
$$C_{\eta_h}(t) = \sum_n^N a_n^2 \left(e^{-E_n t} + e^{-E_n (N_t - t)}\right)$$

QED in these calculations

- The results here use *quenched* QED: the valence quarks see QED, but the sea quarks do not
- Randomly generated momentum space photon field $A_\mu(k)$ in Feynman gauge
  - Zero modes set to zero and then A_μ transformed to position space
  - $\exp(ieQA_\mu)$ (so they are $U(1)$) are added into gauge links

Isospin breaking effects

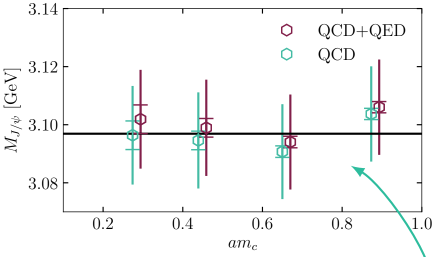
HPQCD [2005.01845]



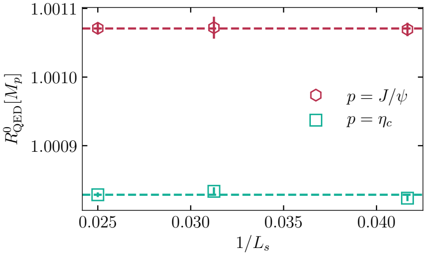
- ★ Check of effect of $m_u \neq m_d$
- ★ Indistinguishable effect at current level in charmonium calculations

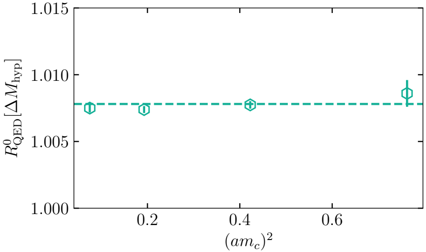
Meson masses

HPQCD [2005.01845]



Same (pure QCD) m_c – offset for clarity



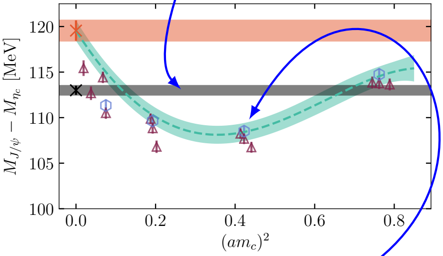


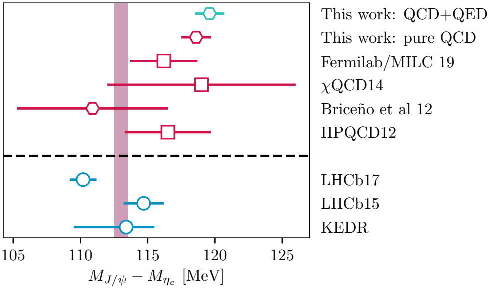
- ★ am_c has to be retuned downward when QED added
- ★ At fixed am_c meson masses & hyperfine splitting increase

Charmonium hyperfine splitting

HPQCD [2005.01845]

PDG





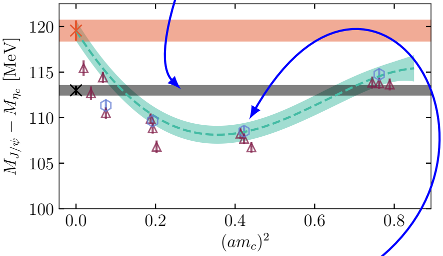
Includes lattice data with QED (blue hexagons)

$$M_{J/\psi} - M_{\eta_c}(\text{connected}) = 120.3(1.1)\ \text{MeV}$$

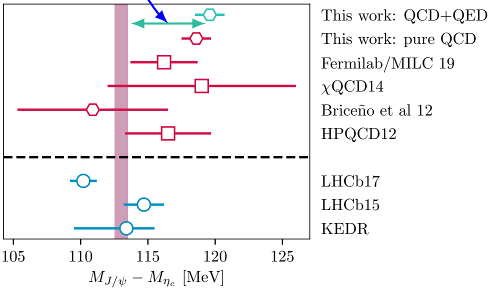
$$\Delta M_{\eta_c}^{\text{annih}} = +7.3(1.2)\ \text{MeV}$$

HPQCD [2005.01845]

PDG



$\Delta M_{\eta_c}^{\mathrm{annih}}$



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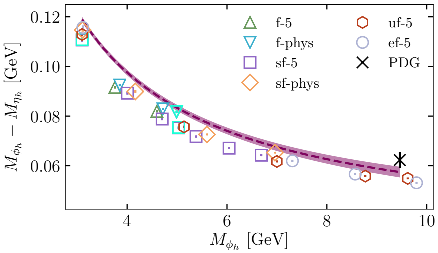
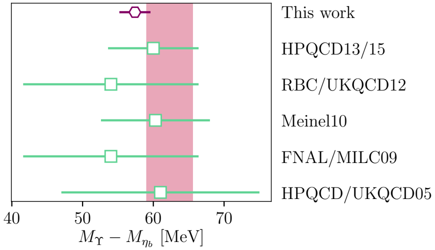
HPQCD [2101.08103]



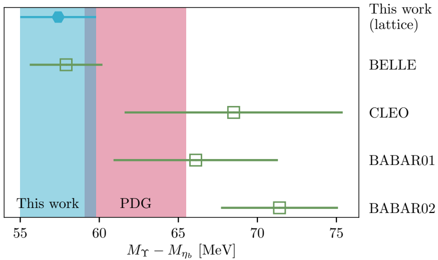


★ Fit of $M_\phi - M_{\eta_h}$ for $m_c \le m_h \le m_b$

Missing disconnected

$$M_\Upsilon - M_{\eta_b} = 57.5(2.3)(1.0) \text{ MeV}$$

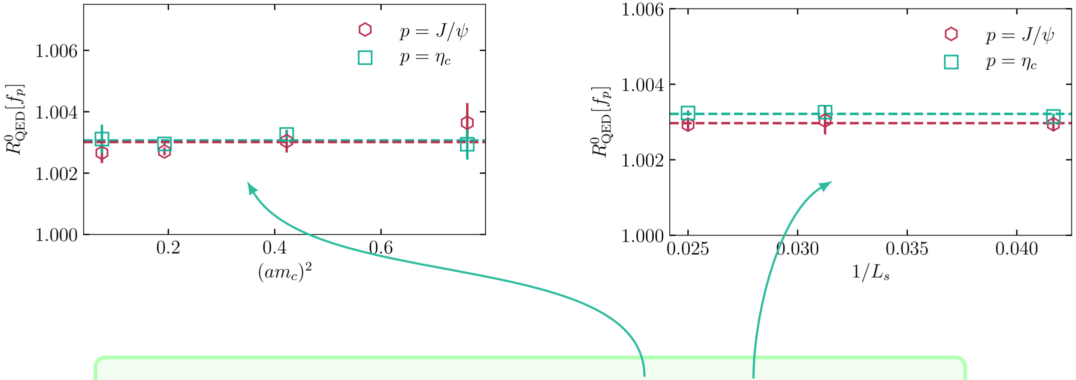
$$R_{\text{QED}}[\Delta M_{\text{hyp}}] = 1.0001(26)$$



Decay constants

Effect of QED on $f_{J/\psi}$ & f_{η_c}

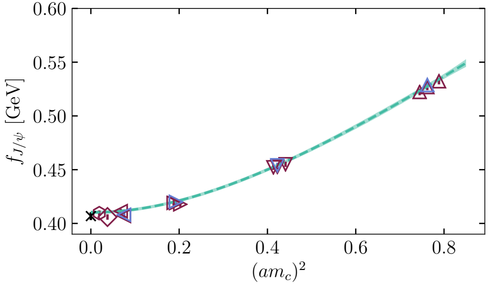
HPQCD [2005.01845]

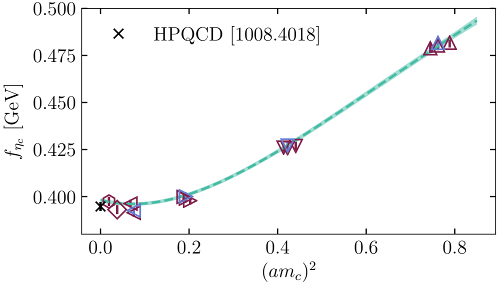


- ★ QED effect on decay constant as function of $(am_c)^2$ and $1/L_s$
  - ▶ No retuning of am_c to account for QED effect

$f_{J/\psi}$ & f_{η_c}

HPQCD [2005.01845]





$$f_{J/\psi} = Z_V \sqrt{\frac{2A_0^V}{E_0^V}}$$

$$f_{J/\psi,\mathrm{QCD}} = 409.6(1.6)\ \mathrm{MeV}$$

$$f_{J/\psi,\mathrm{QCD+QED}} = 410.4(1.7)\ \mathrm{MeV}$$

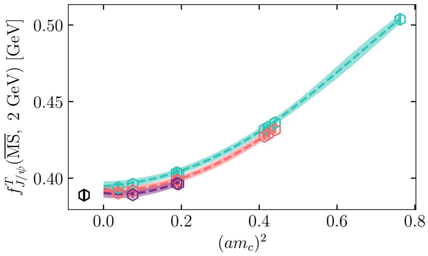
$$f_{\eta_c} = 2m_c \sqrt{\frac{2A_0^P}{(E_0^V)^3}}$$

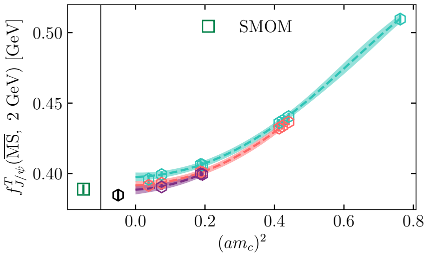
$$f_{\eta_c,\mathrm{QCD}} = 397.5(1.0)\ \mathrm{MeV}$$

$$f_{J/\psi,\mathrm{QCD+QED}} = 398.1(1.0)\ \mathrm{MeV}$$

J/ψ tensor decay constant

HPQCD [2008.02024]

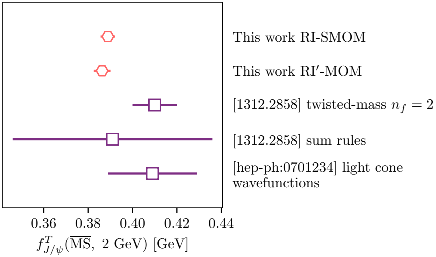




$$\langle 0|\bar{\psi}\sigma_{\alpha\beta}\psi|J/\psi\rangle = i f^T_{J/\psi}(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha)$$

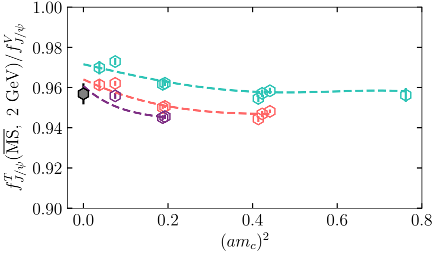
$$f^T_{J/\psi} = Z_T\sqrt{\frac{2A_0^T}{E_0^T}}$$

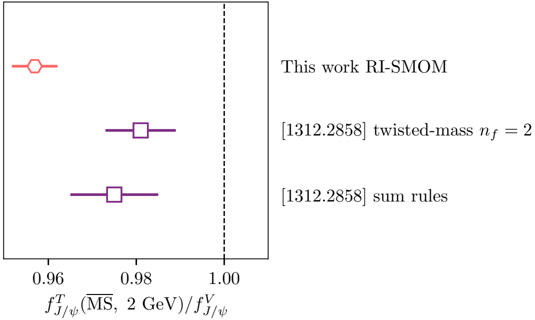
$$f^T_{J/\psi}\left(\overline{\text{MS}}, 2\text{ GeV}\right) = 0.3927(27)\text{ GeV (int. SMOM)}$$



$f^T_{J/\psi}\left(\overline{\text{MS}}, 2\text{ GeV}\right)/f^V_{J/\psi}$

HPQCD [2008.02024]





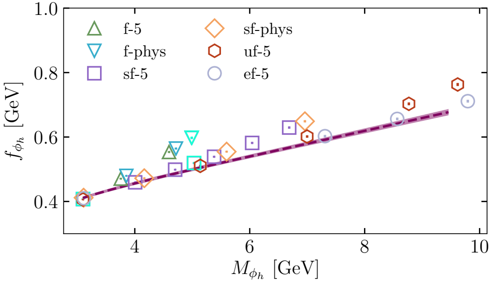
$$\frac{f^T_{J/\psi}\left(\overline{\text{MS}}, 2\text{ GeV}\right)}{f^V_{J/\psi}} = 0.9569(52)\ (\text{int. SMOM})$$

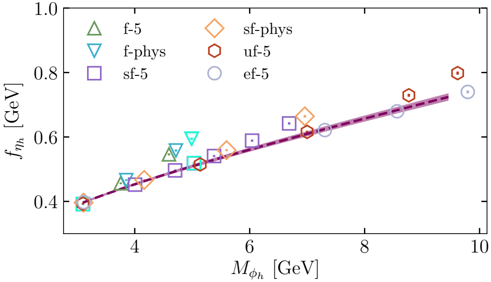
Bottomonium decay constants

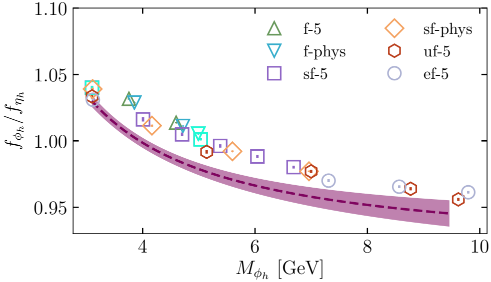
HPQCD [2101.08103]

$$f_{\phi_h} = Z_V \sqrt{\frac{2A_0^V}{E_0^V}}$$

$$f_{\eta_h} = 2m_h \sqrt{\frac{2A_0^P}{(E_0^P)^3}}$$







HPQCD [2101.08103]

$$f_\Upsilon = 677.2(9.7)\ \text{GeV}$$

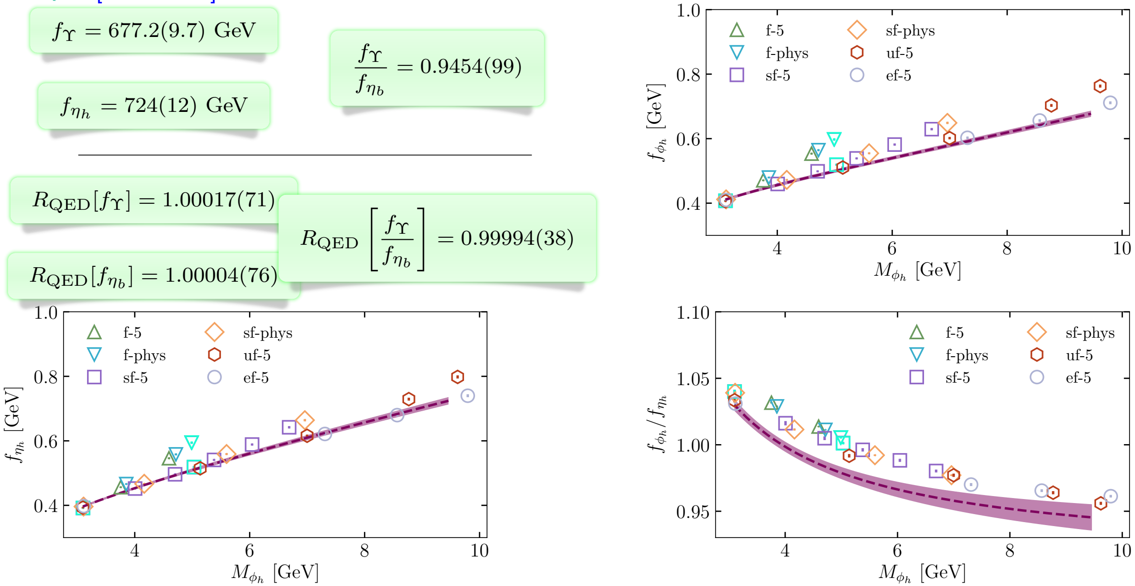
$$f_{\eta_h} = 724(12)\ \text{GeV}$$

$$\frac{f_\Upsilon}{f_{\eta_b}} = 0.9454(99)$$

$$R_{\text{QED}}[f_\Upsilon] = 1.00017(71)$$

$$R_{\text{QED}}[f_{\eta_b}] = 1.00004(76)$$

$$R_{\text{QED}}\left[\frac{f_\Upsilon}{f_{\eta_b}}\right] = 0.99994(38)$$



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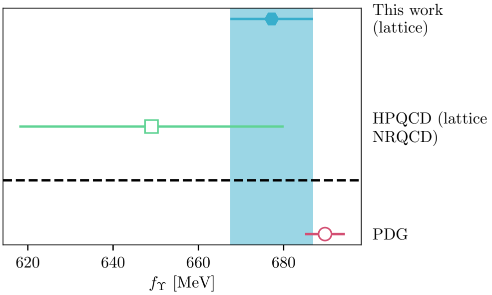
Leptonic width from f_Υ:

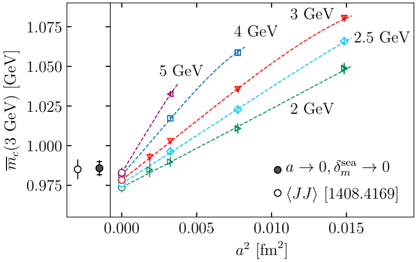
$$\Gamma(\Upsilon \to e^+e^-) = \frac{4\pi}{3}\alpha_{\text{QED}}^2 e_b^2 \frac{f_\Upsilon^2}{M_\Upsilon}$$

$$\Gamma(\Upsilon \to e^+e^-) = 1.292(37)(3)$$



Quark masses

HPQCD [2005.01845]

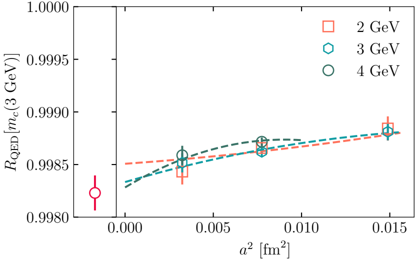


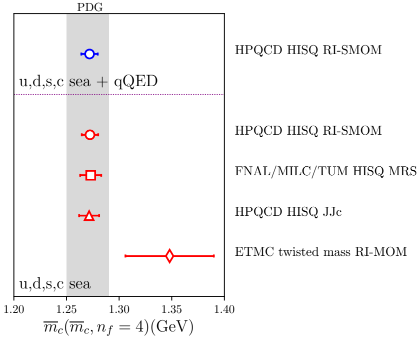
$$\overline{m}(\mu, a) = Z_m^{\overline{\text{MS}}/\text{SMOM}}(\alpha_s(\mu))\, Z_m^{\text{SMOM}}(\mu, a) m(a)$$

$$\overline{m}_c(3\text{ GeV})_{\text{QCD}} = 0.9858(51)\text{ GeV}$$

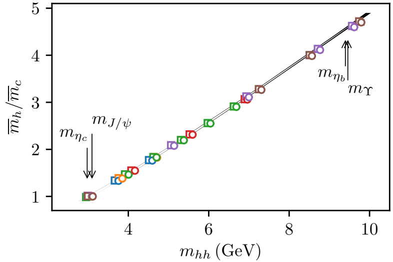
$$\overline{m}_c(3\text{ GeV})_{\text{QCD+QED}} = 0.9841(51)\text{ GeV}$$

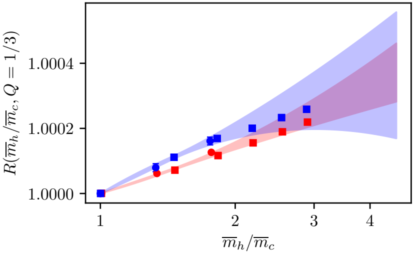
$$\overline{m}_c(\overline{m}_c)_{\text{QCD+QED}} = 1.2719(78)\text{ GeV}$$





HPQCD [2102.08103]





Solve:

$$f^P_{hh}(\overline{m}_b/\overline{m}_c) = m^{\mathrm{cont}}_{\eta_b}$$

$$f^V_{hh}(\overline{m}_b/\overline{m}_c) = m^{\mathrm{cont}}_{\Upsilon}$$

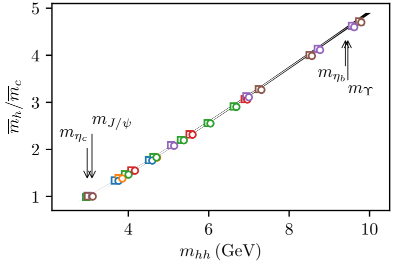
$$\overline{m}_b/\overline{m}_c = \begin{cases} 4.578(12) & \text{from } am^P_{hh} \\ 4.578(15) & \text{from } am^V_{hh} \end{cases}$$

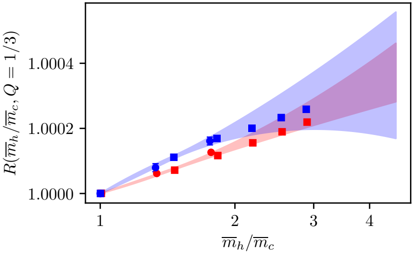
$$\left.\frac{\overline{m}_b(\mu)}{\overline{m}_c(\mu)}\right|_{\substack{\mathrm{QCD}\\ \mathrm{QED}}} = \frac{R(\overline{m}_b/\overline{m}_c, Q_{c,b} = 0 \to \frac{1}{3})}{R(\overline{m}_c, Q_c = \frac{1}{3} \to \frac{2}{3})} \times \left.\frac{\overline{m}_b}{\overline{m}_c}\right|_{\mathrm{QCD}}$$

$$R(\overline{m}_c(\mu), Q_c = \tfrac{1}{3} \to \tfrac{2}{3}) = \frac{\overline{m}_c(\mu) \text{ with } Q_c = \frac{2}{3}}{\overline{m}_c(\mu) \text{ with } Q_c = \frac{1}{3}}$$

$$R(\overline{m}_b/\overline{m}_c, Q_c = 0 \to \tfrac{1}{3}) = \frac{\overline{m}_b/\overline{m}_c \text{ with } Q_{c,b} = \frac{1}{3}}{\overline{m}_b/\overline{m}_c \text{ with } Q_{c,b} = 0}$$

HPQCD [2102.08103]





Solve:

$$f^P_{hh}(\overline{m}_b/\overline{m}_c) = m^{\rm cont}_{\eta_b}$$

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$$\overline{m}_b/\overline{m}_c = \begin{cases} 4.578(12) & \text{from } am^P_{hh} \\ 4.578(15) & \text{from } am^V_{hh} \end{cases}$$

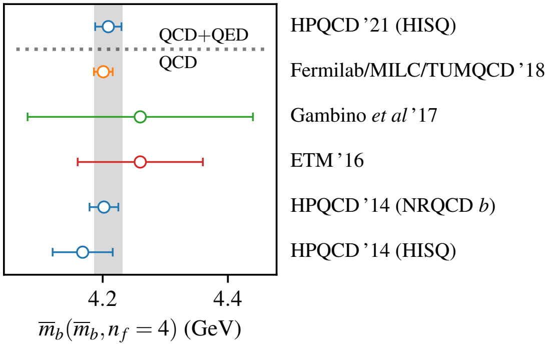
$$\left.\frac{\overline{m}_b(\mu)}{\overline{m}_c(\mu)}\right|_{\substack{\rm QCD\\ \rm QED}} = \frac{R(\overline{m}_b/\overline{m}_c, Q_{c,b} = 0 \to \frac{1}{3})}{R(\overline{m}_c, Q_c = \frac{1}{3} \to \frac{2}{3})} \times \left.\frac{\overline{m}_b}{\overline{m}_c}\right|_{\rm QCD}$$

$$\left.\frac{\overline{m}_b(3\text{ GeV})}{\overline{m}_c(3\text{ GeV})}\right|_{\substack{\rm QCD\\ \rm QED}} = \begin{cases} 4.586(13) & \text{from } am^P_{hh} \\ 4.586(15) & \text{from } am^V_{hh} \end{cases}$$

HPQCD [2101.08103]

$$\overline{m}_b(3\ \text{GeV}, n_f = 4)|_{\substack{\text{QCD}\\\text{QED}}} = 4.513(26)\ \text{GeV}$$

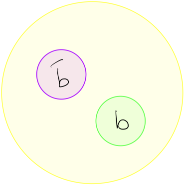
$$\overline{m}_b(\overline{m}_b)|_{\substack{\text{QCD}\\\text{QED}}} = \begin{cases} 4.209(21) & n_f = 4 \\ 4.202(21)\ \text{GeV} & n_f = 5 \end{cases}$$



Heavionium decays

Heavionium decays

- ★ Decays with photons can be used as tests of our understanding of internal structure of mesons from strong interaction physics
- ★ $\eta_c \to \gamma\gamma$ result vastly improved picture from the lattice
  - ▶ Experimental results give no clear consensus for $\Gamma(\eta_c \to \gamma\gamma)$
- ★ $\eta_b \to \gamma\gamma$ has not yet been seen; our result is a prediction for Belle II

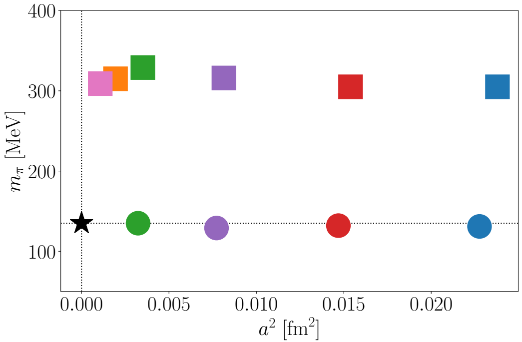


This work

- ★ Precise calculation by using Highly Improved Staggered Quark (HISQ) action
- ★ Calculate these decays with realistic sea
  - ▶ Effect of 2+1+1 quarks
- ★ $2 - 3\%$ uncertainties for $\Gamma(\eta_b \to \gamma\gamma)$
- ★ Combining our result with NRQCD calculation, can get new total η_b decay width

- ★ Full details of calculation for $\eta_c \to \gamma\gamma$ process & J/ψ radiative decays in Phys Rev **D** 108 (2023) [arXiv:2305.06231]

- $2+1+1$ HISQ gauge ensembles provided by MILC Collaboration
- Lattice spacings from ≈ 0.15 fm down to ≈ 0.03 fm depending on process
- Combination of $m_s/m_l = 5$ and physical m_l
- Valence heavy quarks $m_c \leq m_h \leq m_b$ also use HISQ formalism
- Tuned m_b to match η_b mesons



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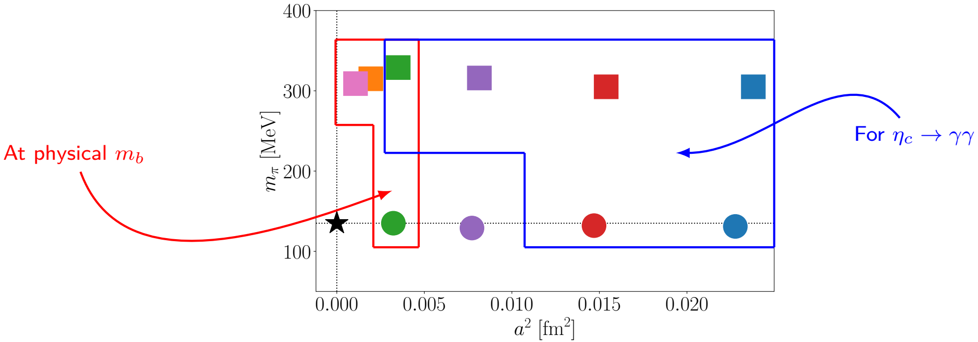
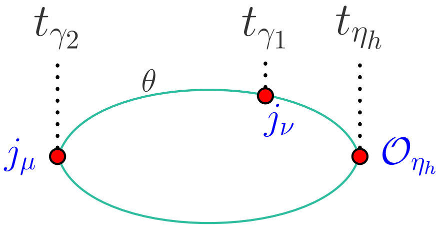


HPQCD [2305.06231]

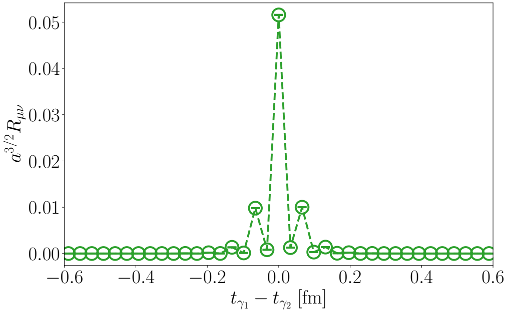


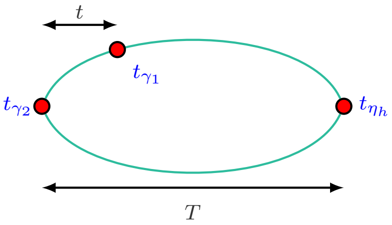
Ji & Jung [hep-lat/0101014] & [hep-lat/0103007]:

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = a \sum_{t_{\gamma_1}} e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

- ★ For on-shell photons: $\omega_1 = |\vec{q}_1| = |\vec{q}_2| = \frac{M_{\eta_h}}{2}$
- ★ Momentum twist, θ, to tune ω_1
- ★ Require component of momentum orthogonal to photon polarisations (also mutually orthogonal)
  - ▶ We chose to put all momentum in y direction (photons in x and z)
- ★ Vector currents require renormalisation; we use RI-SMOM scheme HPQCD '19 [1909.00756]

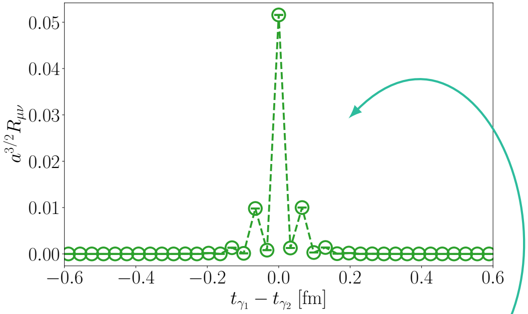
A look at the integrand

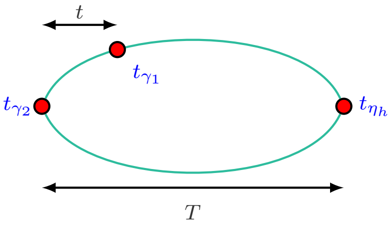




a [fm]	m_s/m_l	T
0.059	5	14, 17, 22, 25, 30
0.057	27.5	12, 17, 22, 27, 32, 37
0.044	5	22, 29, 36, 43
0.033	5	21, 30, 39, 48

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_{t_{\gamma_1}} a e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

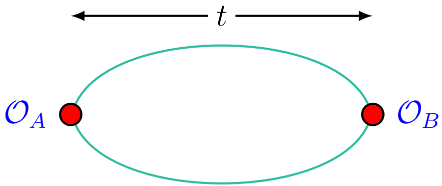




$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_{t_{\gamma_1}} a e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

a [fm]	m_s/m_l	T
0.059	5	14, 17, 22, 25, 30
0.057	27.5	12, 17, 22, 27, 32, 37
0.044	5	22, 29, 36, 43
0.033	5	21, 30, 39, 48

HPQCD [2305.06231]



Fit two sets of correlators:

$$C_{\eta_h}(t) = \sum_n^N a_n^2 \left(e^{-E_n t} + e^{-E_n (N_t - t)}\right)$$

and

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_n^N a_n b_n \left(e^{-E_n (t_{\gamma_2} - t_{\eta_h})} + e^{-E_n (N_t - t_{\gamma_2} + t_{\eta_h})}\right)$$

Ground state

Extract form factor $F_{\text{latt}}(0,0)$ by:

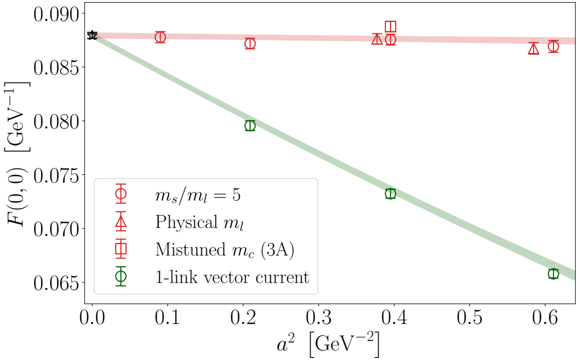
$$\frac{F_{\text{latt}}(0,0)}{a} = b_0 Z_V^2 \sqrt{\frac{2}{aM_{\eta_h}} \frac{L_s}{\theta\pi}},$$

which relates to the width for two on-shell photons:

$$\Gamma(\eta_h \to \gamma\gamma) = \pi \alpha_{\text{em}}^2 Q_h^4 M_{\eta_h}^3 F(0,0)^2.$$

$$\eta_c \to \gamma\gamma$$

HPQCD [2305.06231]



$$\frac{F^{(t)}_{\text{latt}}(0,q_2^2)}{a} = \frac{F(0,0)}{(1-\frac{q_2^2}{M^2_{\text{pole}}})}\left[1+\sum_{i=1}^{i_{\max}}\kappa^{(i,t)}_{a\Lambda}\left(a\Lambda^{(t)}\right)^{2i}+\kappa_{\text{val},c}\delta^{\text{val},c}+\kappa_{\text{sea},c}\delta^{\text{sea},c}\right.$$

$$\left.+\kappa^{(0)}_{\text{sea},uds}\delta^{\text{sea},uds}\left\{1+\kappa^{(1,t)}_{\text{sea},uds}(a\tilde{\Lambda})^2+\kappa^{(2,t)}_{\text{sea},uds}(a\tilde{\Lambda})^4\right\}\right]$$

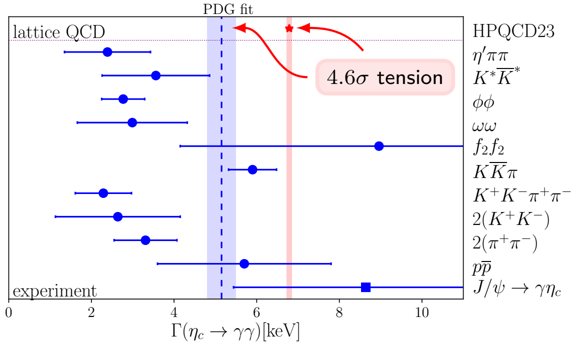
HPQCD [2305.06231]

Continuum result gives

$$F(0,0) = 0.08793(29)_{\rm fit}(26)_{\rm syst}\ \mathrm{GeV}^{-1}$$

From which we can determine the width:

$$\Gamma(\eta_c \to \gamma\gamma) = 6.788(45)_{\rm fit}(41)_{\rm syst}\ \mathrm{keV}$$



PDG fit: 5.15(35) keV ← $\chi^2 = 118$ for 81 d.o.f.; $p = 0.005$

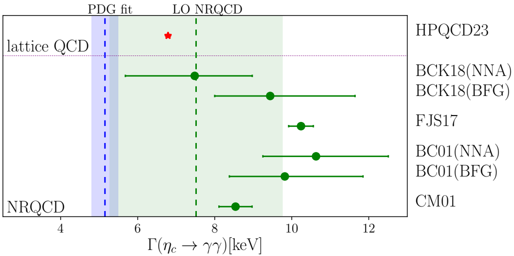
HPQCD [2305.06231]

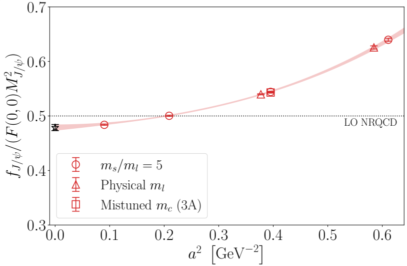
(Czarnecki & Melnikov '01 [hep-ph/0109054]):

Expectation in nonrelativistic limit:

$$\frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\eta_c \to \gamma\gamma)} \approx \frac{3}{4}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$

$$\frac{f_{J/\psi}}{F(0,0)M_{J/\psi}^2} = \frac{1}{2}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$



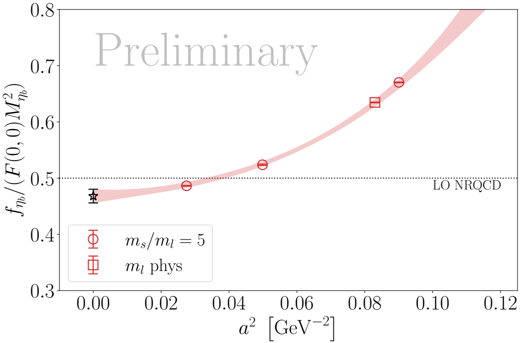


$$\text{Our result}: \frac{f_{J/\psi}}{F(0,0)M_{J/\psi}^2} = 0.4786(57)_{\text{fit}}(14)_{\text{syst}}$$

$M_{J/\psi}$, $f_{J/\psi}$ & $\Gamma(J/\psi \to e^+e^-)$ (for LO NRQCD central value) from HPQCD '20 [2005.01845]

$\eta_b \to \gamma\gamma$ RESULTS

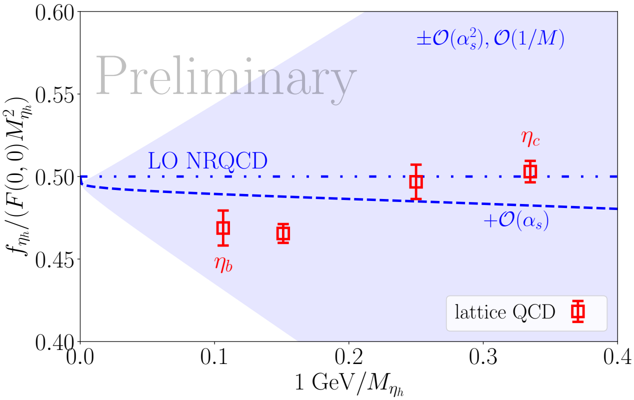
$\eta_b \to \gamma\gamma$ from ratio with decay constant



$$R^{\text{latt}}_{\eta_b} = R^{\text{phys}}_{\eta_b}\left[1 + \sum^{i_{\max}} \kappa^{(i)}_{a\Lambda}(a\Lambda)^{2i} + \kappa_{\text{val},b}\delta^{\text{val},b} + \kappa_{\text{sea},c}\delta^{\text{sea},c} + \kappa^{(0)}_{\text{sea},uds}\delta^{\text{sea},uds}\{1 + \kappa^{(1)}_{\text{sea},uds}(a\tilde{\Lambda})^2 + \kappa^{(2)}_{\text{sea},uds}(a\tilde{\Lambda})^4\}\right]$$

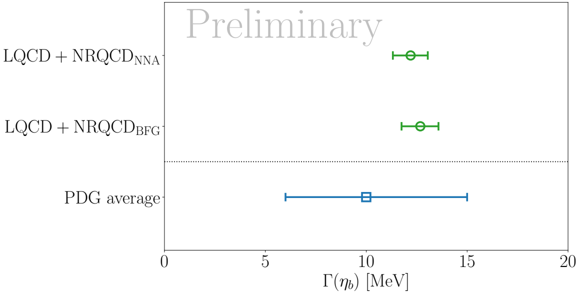
$$R^{\text{phys}}_{\eta_b} = 0.468(12); \quad F(0,0) = 0.01751(53)\ \text{GeV}^{-1}$$

$$\Gamma(\eta_b \to \gamma\gamma) = 0.526(32)\ \text{keV}$$



$$R_{\eta_h} \equiv \frac{f_{\eta_h}}{F_{\eta_h}(0,0)M_{\eta_h}^2} = \frac{1}{2}\left(1 + \mathcal{O}\left(\alpha_s\right) + \mathcal{O}\left(v^2/c^2\right)\right)$$

★ Can get full width $\Gamma(\eta_b)$ by combining our decay width with $\dfrac{\Gamma(\eta_b)}{\Gamma(\eta_b \to \gamma\gamma)}$ from NRQCD



$$\Gamma(\eta_b)_{\mathrm{NNA}} = 12.20 \left({}^{+42}_{-47}\right)_{\mathrm{NRQCD}} (74)_{\mathrm{LQCD}} \text{ MeV}$$

$$\Gamma(\eta_b)_{\mathrm{BFG}} = 12.68 \left({}^{+47}_{-53}\right)_{\mathrm{NRQCD}} (77)_{\mathrm{LQCD}} \text{ MeV}$$

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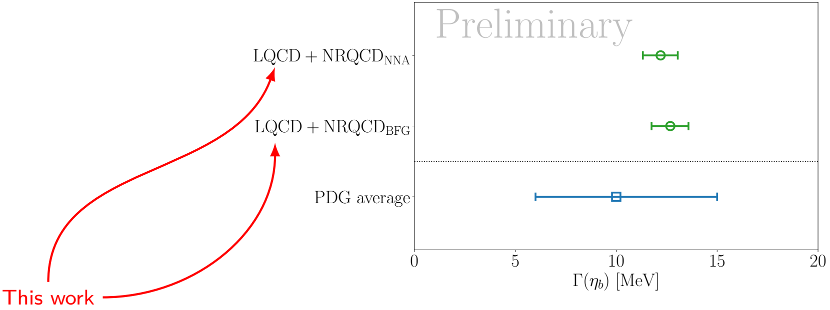

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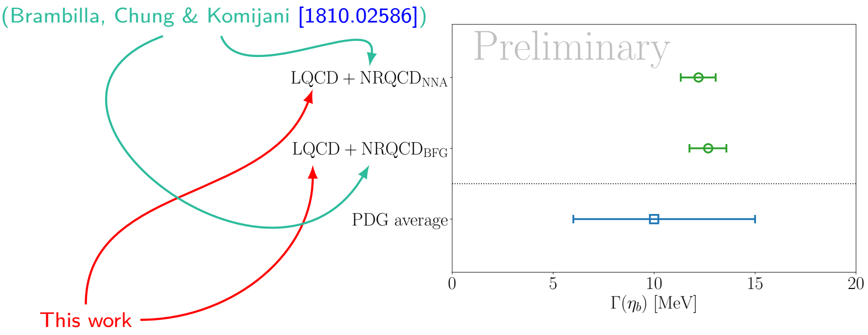

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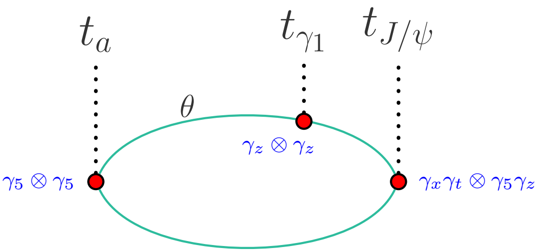
$$J/\psi \to \gamma a$$

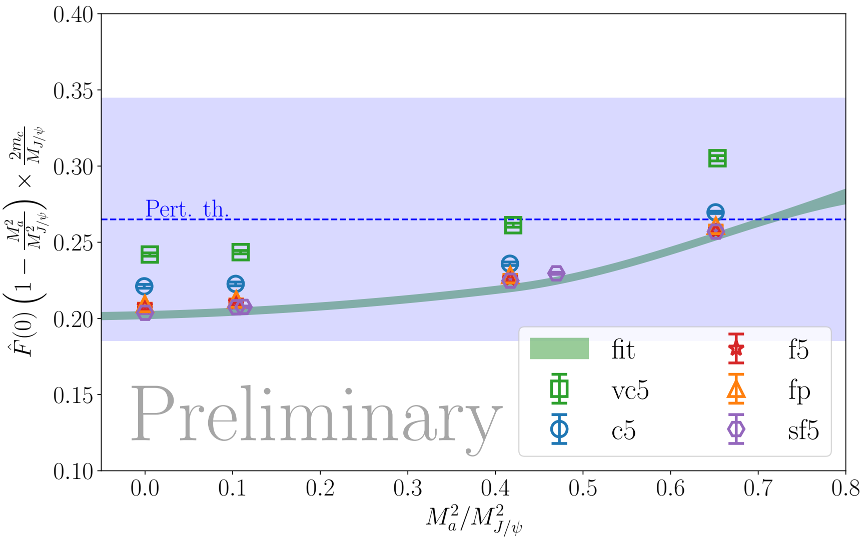
w/ Christine Davies, G. Peter Lepage & Sophie Renner

- ★ Analogously calculate process where γ and an axion-like particle couple to charm



$$\hat{F}(0) = b_0 Z_V \sqrt{2M_{J/\psi}} \frac{L_s}{\theta\pi}$$

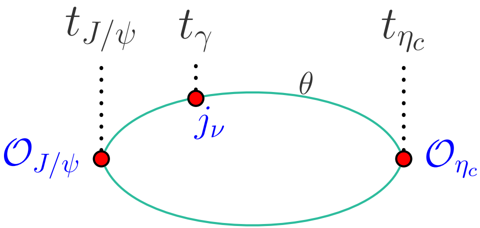
$$\Gamma(J/\psi \to \gamma a) = \frac{1}{24}\frac{C_{cc}^2}{f^2}\alpha_{\rm em} Q_c^2 M_{J/\psi}^3 \left(1 - \frac{M_a^2}{M_{J/\psi}^2}\right) \hat{F}(0)^2$$



0.40
0.35
0.30
0.25
0.20
0.15
0.10
$\hat{F}(0)\left(1-\frac{M_a^2}{M_{J/\psi}^2}\right)\times\frac{2m_c}{M_{J/\psi}}$
Pert. th.
Preliminary
fit
vc5
c5
f5
fp
sf5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$M_a^2/M_{J/\psi}^2$

$$J/\Psi \to \gamma \eta_c$$

HPQCD [2305.06231]

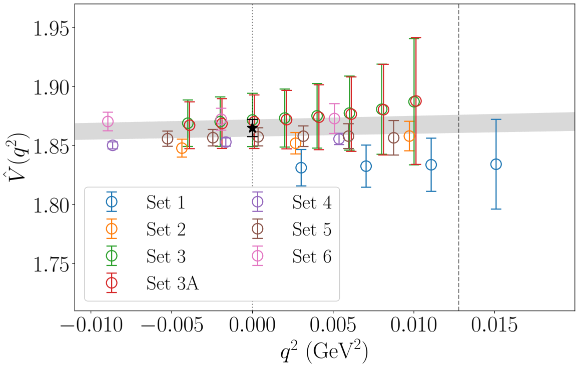


$$C_{3\mathrm{pt}}(t,T) = \sum_{i,j}^{N_n,N_n} a_i e^{-E_i t} V_{ij} b_j e^{E_j (T-t)}$$

$$\hat{V}(q^2) = \frac{M^{\mathrm{latt}}_{J/\psi} + M^{\mathrm{latt}}_{\eta_c}}{M^{\mathrm{latt}}_{J/\psi} q^y} Z_V \sqrt{2M^{\mathrm{latt}}_{J/\psi}} \sqrt{2E^{\mathrm{latt}}_{\eta_c}} V_{00}$$

$$\Gamma\left(J/\psi \to \gamma\eta_c\right) = \alpha Q_c^2 \frac{16}{3} \frac{|\mathbf{k}|^3}{\left(M_{\eta_c} + M_{J/\psi}\right)^2} |\hat{V}(0)|^2$$

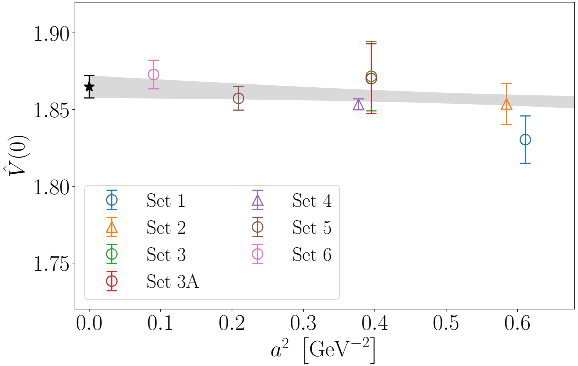
$\hat{V}(q^2)$



$$\hat{V}_{\text{latt}}(0) = 1.8649(73)_{\text{fit}}(75)_{\text{syst}}$$

$$\hat{V}_{\text{latt}}(q^2) = \sum_{k=0}^{2} A^{(k)} \left(\frac{q^2}{\left(M_{J/\psi}^{\text{latt}}\right)^2} \right)^k \left[1 + \sum_{i=1}^{i_{\max}} \kappa_{a\Lambda}^{(i,k)} (a\Lambda)^{2i} + \kappa_{\text{val},c}^{(k)} \delta^{\text{val},c} + \kappa_{\text{sea},c}^{(k)} \delta^{\text{sea},c} \right.$$
$$\left. + \kappa_{\text{sea},uds}^{(0,k)} \delta^{\text{sea},uds} \left\{ 1 + \kappa_{\text{sea},uds}^{(1,k)} (\tilde{\Lambda} a)^2 + \kappa_{\text{sea},uds}^{(2,k)} (\tilde{\Lambda} a)^4 \right\} \right]$$

$\hat{V}(0)$

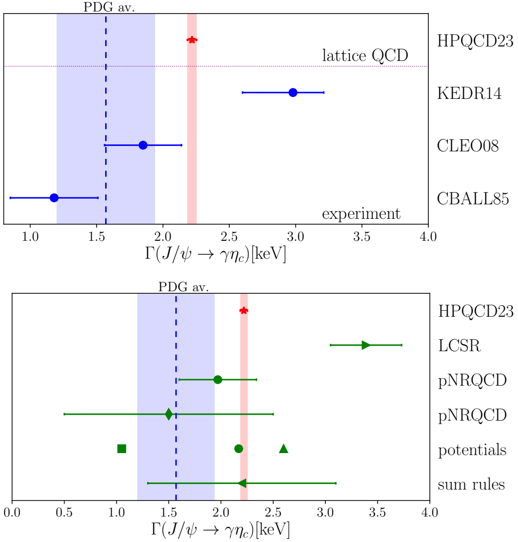


$$\hat{V}_{\text{latt}}(0) = 1.8649(73)_{\text{fit}}(75)_{\text{syst}}$$

$$\Gamma(J/\psi \to \gamma\eta_c) = 2.219(17)_{\text{fit}}(18)_{\text{syst}}(24)_{\text{expt}}(4)_{\text{QED}}\ \text{keV}$$

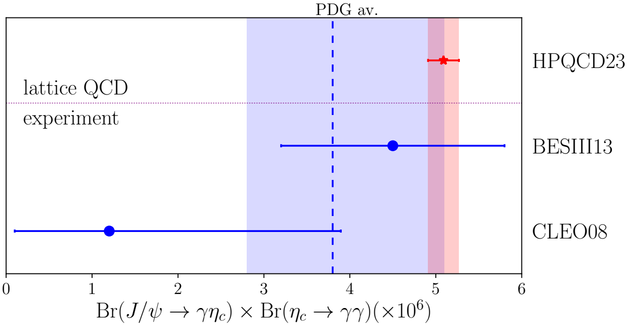
$$\text{Br}(J/\psi \to \gamma\eta_c) = 2.40(3)_{\text{latt}}(5)_{\text{expt}}\%$$

Summary results: $J/\psi \to \gamma\eta_c$



Product of fractions

★ Product of $J/\psi \to \gamma\eta_c$ and $\eta_c \to \gamma\gamma$ measured at by CLEO and BESIII



$$J/\Psi \to \eta_c e^+ e^-$$

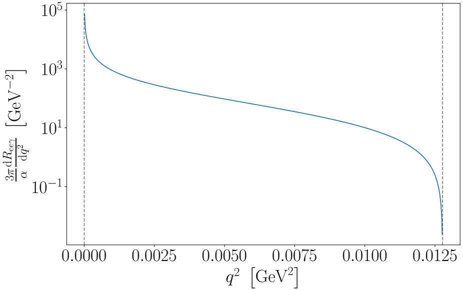
$J/\Psi \to \eta_c e^+ e^-$

HPQCD [2305.06231]

$$R_{ee\gamma} = \frac{\mathcal{B}(J/\Psi \to \eta_c e^+ e^-)}{\mathcal{B}(J/\Psi \to \gamma \eta_c)}; \quad \frac{dR_{ee\gamma}}{dq^2} = \frac{\alpha}{3\pi q^2} \left| \frac{\hat{V}(q^2)}{\hat{V}(0)} \right|^2 \left(1 - \frac{4m_e^2}{q^2}\right)^{\frac{1}{2}} \left(1 + \frac{2m_e^2}{q^2}\right)$$

$$\times \left(\left(1 + \frac{q^2}{M_{J/\psi}^2 - M_{\eta_c}^2}\right)^2 - \frac{4M_{J/\psi}^2 q^2}{(M_{J/\psi}^2 - M_{\eta_c}^2)^2} \right)^{\frac{3}{2}}$$

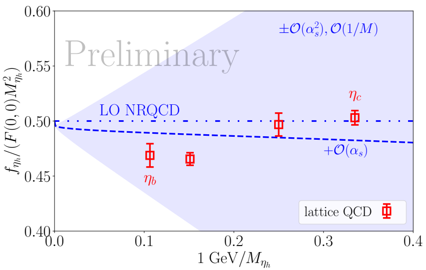
Landsberg '85 Phys. Rep. 128, 301

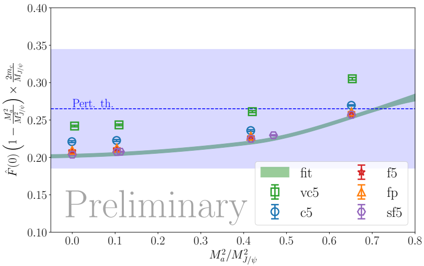


$$\Gamma(J/\Psi \to \eta_c e^+ e^-) = 0.01349(15)_{\text{latt}}(15)_{\text{expt}}(13)_{\text{QED}} \text{ keV}$$

$$\text{Br}(J/\Psi \to \eta_c e^+ e^-) = 1.457(16)_{\text{latt}}(15)_{\text{QED}}(31)_{\text{expt}} \times 10^{-4}$$

Summary



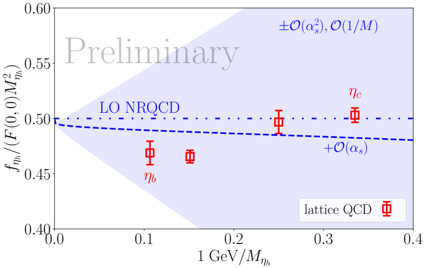


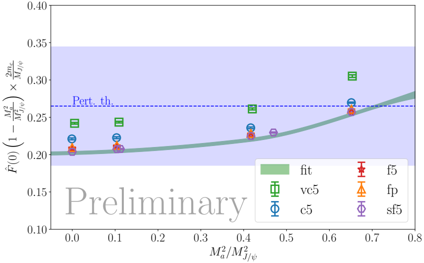
- ★ Many heavionium quantities measured to high precision, now incl. (quenched) QED in some cases
- ★ Vastly improved $\Gamma(\eta_c \to \gamma\gamma)$
- ★ We're hoping results from experiment for $\eta_b \to \gamma\gamma$ will be forthcoming

Belle II?

- ★ Similar analysis being performed for decay with axion-like particle: $J/\psi \to a\gamma$

Summary





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Thank you!