Electromagnetic and Axial-vector Structure of Singly Heavy Baryons in a Pion Mean Approach

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Baryon from the pion mean fields

Witten's seminal idea: Baryon in the large Nc

NPB, 149(1979)285

- Problem in low-energy QCD: Large value of the strong coupling constant

The number of color as an implicit expansion parameter

→ * A baryon can be viewed as a state of Nc quarks bound by mesonic **mean fields**.

- Its mass is proportional to *Nc*, while its width is of order *O(1)*.
- Mesons are weakly interacting

  (Quantum fluctuations are suppressed by 1/Nc: *O(1/Nc)*).

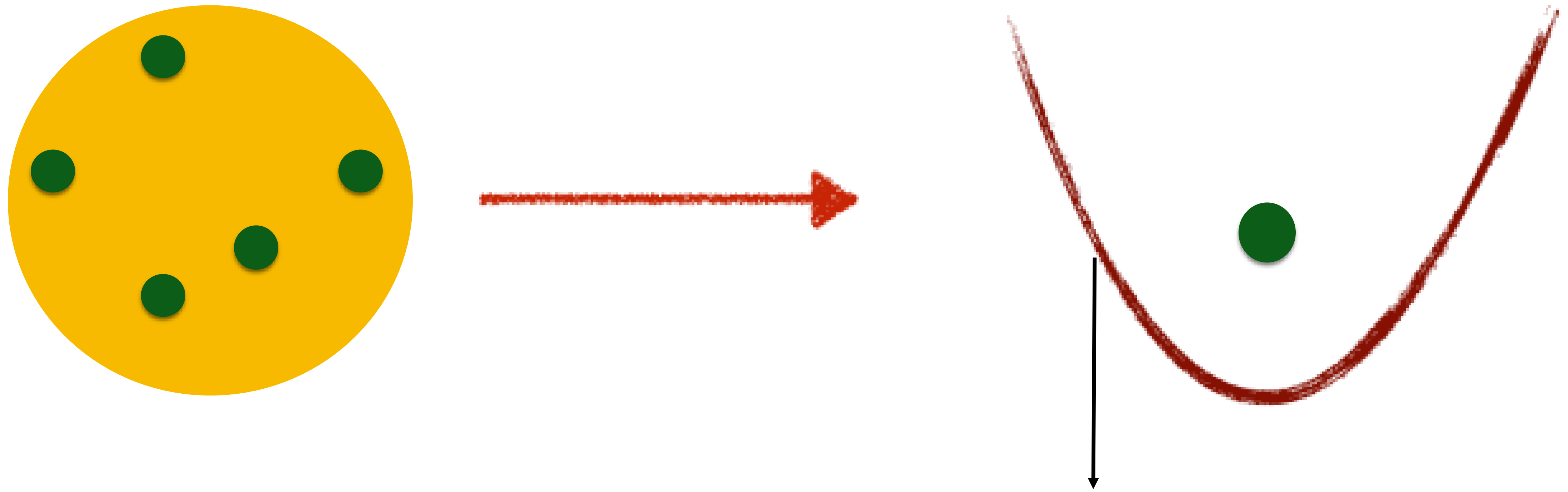
Extension

* A singly heavy baryon can be viewed as a state of Nc-1 quarks bound by mesonic **mean fields**.

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

This classical solution is regarded as a mean field.



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

* Baryons as a state of Nc quarks bound by mesonic mean fields.

Effective chiral action:

Blotz, HChK, Goeke et al. PPNP 37 (1996) 91

Diakonov, hep-ph/9802298

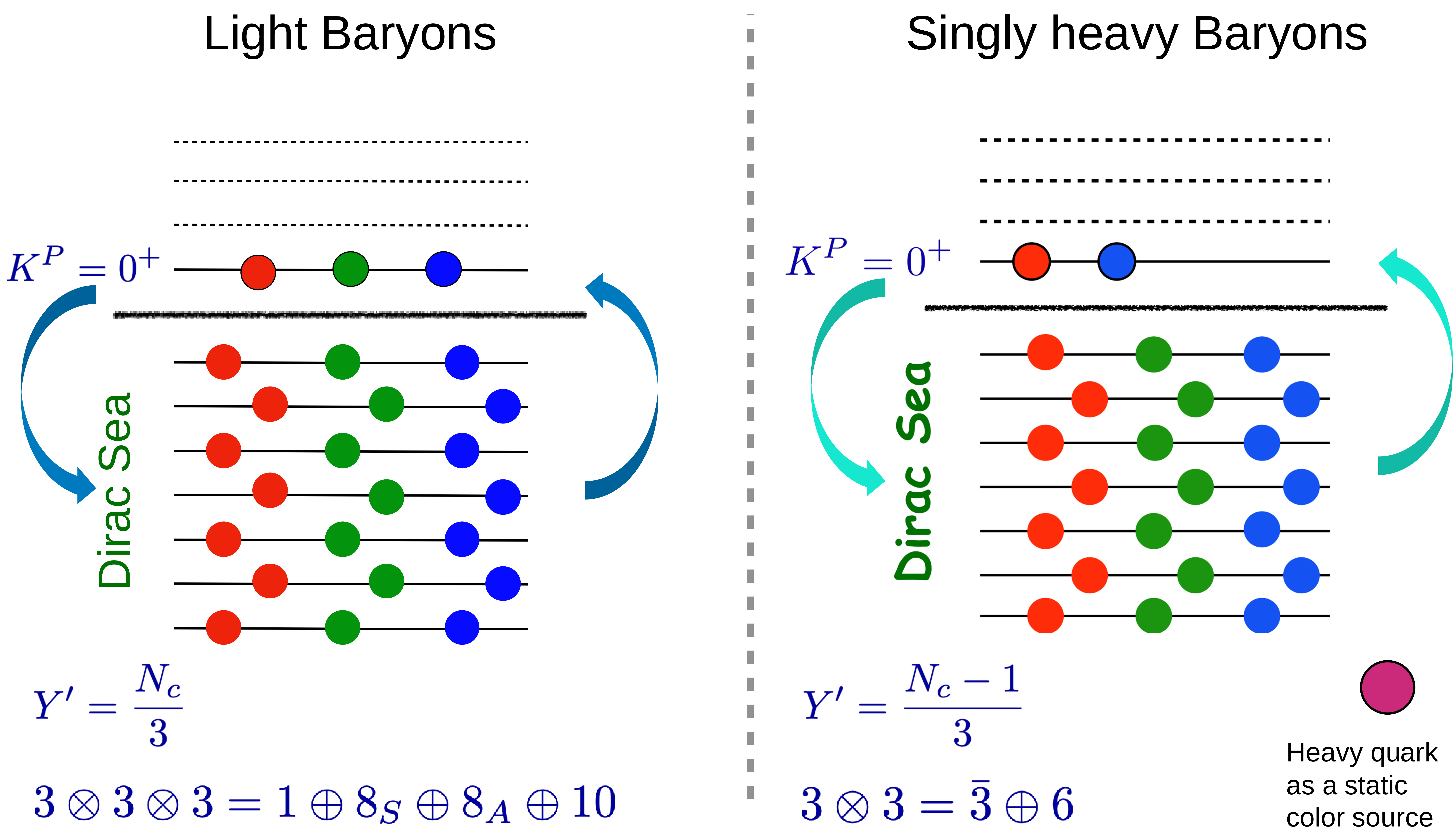
$$S_{\rm eff}[\pi^a] = -N_c {\rm Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

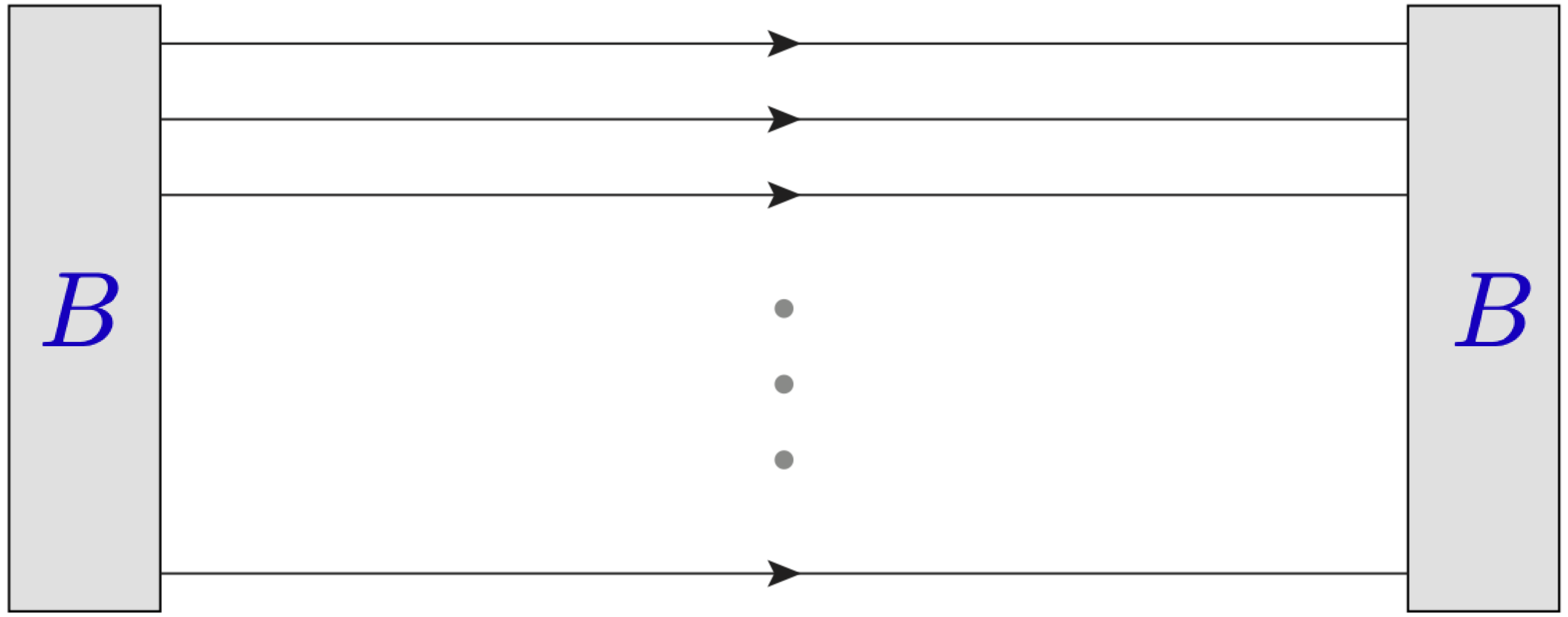
* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\ 2,\ 3 \\ 0, & a = 4,\ 5,\ 6,\ 7,\ 8. \end{cases}$$

It breaks spontaneously $\mathrm{SU}(3)_{\rm flavor} \otimes \mathrm{O}(3)_{\rm space} \to \mathrm{SU}(2)_{\rm isospin+space}$

Witten's trivial embedding $U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$


Light Baryons
Singly heavy Baryons
$K^P = 0^+$
$K^P = 0^+$
Dirac Sea
Dirac Sea
$Y' = \frac{N_c}{3}$
$Y' = \frac{N_c - 1}{3}$
$3 \otimes 3 \otimes 3 = 1 \oplus 8_S \oplus 8_A \oplus 10$
$3 \otimes 3 = \bar{3} \oplus 6$
Heavy quark as a static color source



HChK et al. PPNP 37 (1996) 91

Yang, HChK, Praszalowicz, Polyakov, PRD 94 (2016) R071502

Light baryon correlation function

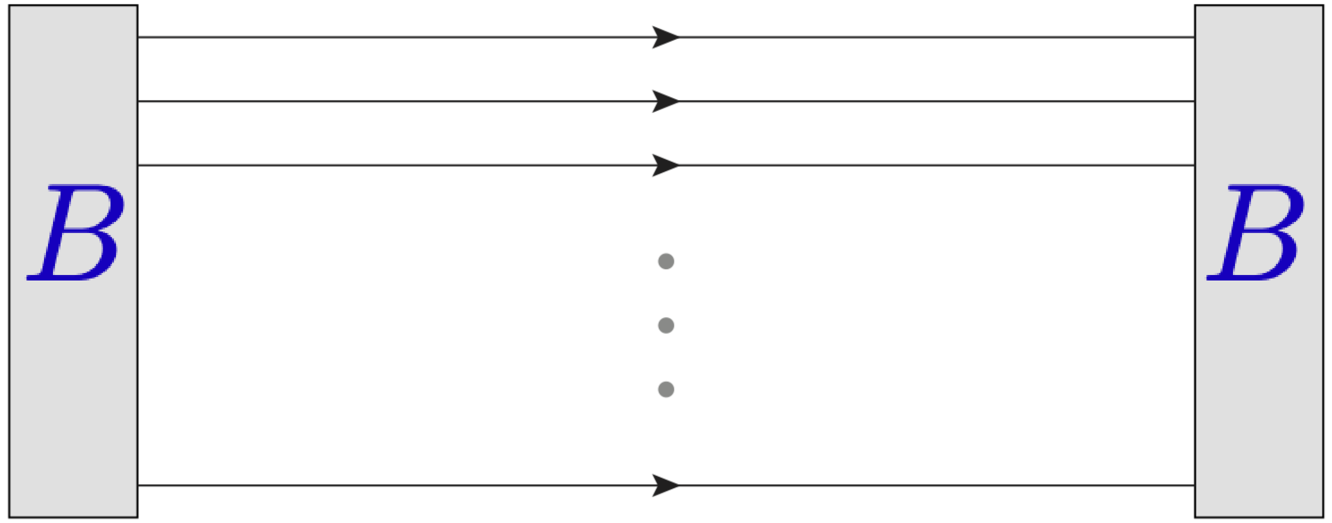
$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

Single heavy baryon correlation function

$$\langle J_{B_Q} J_{B_Q}^\dagger \rangle \sim e^{(N_c-1) E_{\rm val} T}$$

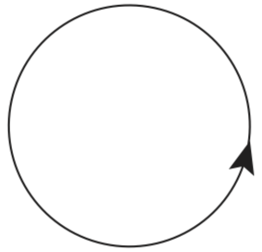
Presence of Nc (Nc-1) quarks will polarize the vacuum or create mean fields.

Nc (Nc-1) valence quarks → Vacuum polarization or meson mean fields



HChK et al. PPNP 37 (1996) 91

Yang, HChK, Praszalowicz, Polyakov,
PRD 94 (2016) R071502



$$\sim e^{-E_{\rm sea}T}$$

Light baryon classical mass

$$E_{\rm cl} = N_c E_{\rm val} + E_{\rm sea}$$

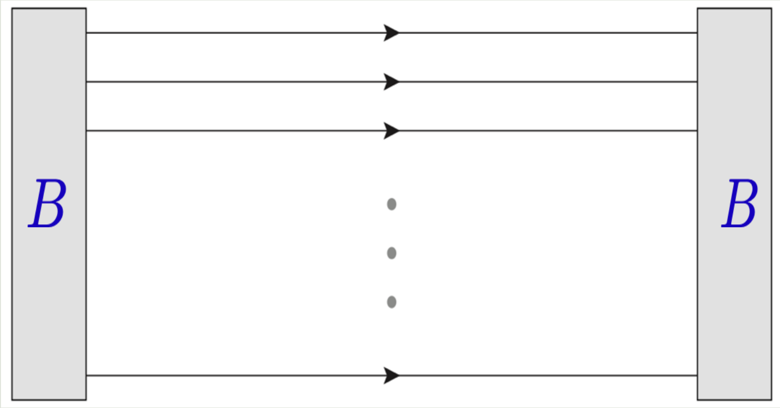
Single heavy baryon classical mass

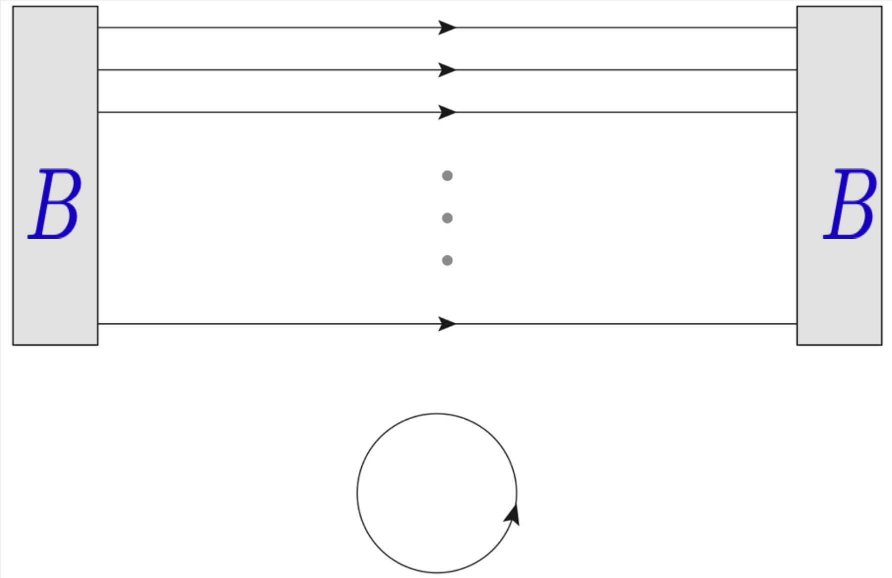
$$E_{Q.\rm cl} = (N_c - 1)E_{\rm val} + E_{\rm sea} + m_Q$$

$$\frac{\delta E_{cl}}{\delta U} = 0 \longrightarrow M_{\rm cl} \longrightarrow P(r)$$

P(r): Soliton profile function
or Soliton field

JY Kim, HChK, G.S. Yang, M. Oka, PRD.103 (2021) 074025





$$\Psi_h(x) = \exp(-im_Q v\cdot x)\tilde{\Psi}_h(x)$$

Heavy baryon states

$$|B,p\rangle = \lim_{x_4\to-\infty}\exp(ip_4x_4)\mathcal{N}(\boldsymbol{p})\int d^3x\,\exp(i\boldsymbol{p}\cdot\boldsymbol{x})(-i\Psi_h^\dagger(\boldsymbol{x},x_4)\gamma_4)J_B^\dagger(\boldsymbol{x},x_4)|0\rangle,$$

$$\langle B,p| = \lim_{y_4\to\infty}\exp(-ip'_4y_4)\mathcal{N}^*(\boldsymbol{p}')\int d^3y\,\exp(-i\boldsymbol{p}'\cdot\boldsymbol{y})\langle 0|J_B(\boldsymbol{y},y_4)\Psi_h(\boldsymbol{y},y_4)$$

Ioffe-type currents

$$J_B(x) = \frac{1}{(N_c-1)!}\epsilon_{\alpha_1\cdots\alpha_{N_c-1}}\Gamma^{f_1\cdots f_{N_c-1}}_{(TT_3Y)(JJ_3Y_R)}\psi_{f_1\alpha_1}(x)\cdots\psi_{f_{N_c-1}\alpha_{N_c-1}}(x),$$

$$J_B^\dagger(y) = \frac{1}{(N_c-1)!}\epsilon_{\alpha_1\cdots\alpha_{N_c-1}}\Gamma^{f_1\cdots f_{N_c-1}}_{(TT_3Y)(JJ'_3Y_R)}(-i\psi^\dagger(y)\gamma_4)_{f_1\alpha_1}\cdots(-i\psi^\dagger(y)\gamma_4)_{f_{N_c-1}\alpha_{N_c-1}}$$

Detailed expressions will be skipped.

$$\langle B(p', J'_3)|B(p, J_3)\rangle = \frac{1}{\mathcal{Z}_{\rm eff}}\mathcal{N}^*(p')\mathcal{N}(p) \lim_{x_4\to-\infty}\lim_{y_4\to\infty} \exp\left(-iy_4p'_4 + ix_4p_4\right)$$
$$\times \int d^3x d^3y \exp(-i\boldsymbol{p}'\cdot\boldsymbol{y} + i\boldsymbol{p}\cdot\boldsymbol{x}) \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}\tilde{\Psi}_h\mathcal{D}\tilde{\Psi}_h^\dagger J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x)$$
$$\times \exp\left[\int d^4z\left\{(\psi^\dagger(z))^f_\alpha\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)_{fg}\psi^{g\alpha}(z) + \Psi_h^\dagger(z)v\cdot\partial\Psi_h(z)\right\}\right]$$
$$= \frac{1}{\mathcal{Z}_{\rm eff}}\mathcal{N}^*(p')\mathcal{N}(p) \lim_{x_4\to-\infty}\lim_{y_4\to\infty} \exp\left(-iy_4p'_4 + ix_4p_4\right)$$
$$\times \int d^3x d^3y \exp(-i\boldsymbol{p}'\cdot y + i\boldsymbol{p}\cdot\boldsymbol{x})\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x)\rangle_0$$

$$\mathcal{Z}_{\rm eff} = \int \mathcal{D}U \exp(-S_{\rm eff})$$

Light-quark propagator

$$G(y,x) = \left\langle y\left|\frac{1}{i\partial\!\!\!/ + iMU^{\gamma_5} + i\overline{m}}(i\gamma_4)\right|x\right\rangle$$
$$= \Theta(y_4 - x_4)\sum_{E_n>0} e^{-E_n(y_4-x_4)}\psi_n(\boldsymbol{y})\psi_n^\dagger(\boldsymbol{x}) - \Theta(x_4 - y_4)\sum_{E_n<0} e^{-E_n(y_4-x_4)}\psi_n(\boldsymbol{y})\psi_n^\dagger(\boldsymbol{x})$$

Heavy-quark propagator $G_h(y,x) = \left\langle y\left|\frac{1}{\partial_4}\right|x\right\rangle = \Theta(y_4 - x_4)\delta^{(3)}(\boldsymbol{y} - \boldsymbol{x})$

Detailed expressions will be skipped.

Heavy-baryon two-point correlation function

$$\langle J_B(y)\Psi_h(y)(-i\Psi_h^\dagger(x)\gamma_4)J_B^\dagger(x)\rangle_0 \sim \exp\left[-\{(N_c-1)E_{\mathrm{val}}+E_{\mathrm{sea}}+m_Q\}T\right] = \exp[-M_B T]$$

$$B(p',J_3')|B(p,J_3)\rangle = 2M_B\delta_{J_3'J_3}(2\pi)^3\delta^{(3)}(\boldsymbol{p}'-\boldsymbol{p}) \;\; \text{in the large } N_c \text{ limit}$$

Classical mass of a singly heavy baryon

$$\boxed{M_B = (N_c-1)E_{\mathrm{val}}+E_{\mathrm{sea}}+m_Q}$$

Detailed expressions will be skipped.

Zero-mode (collective) quantization

- Rotational & Translational zero modes

$$\int \mathcal{D}U \mathcal{F}[U(\boldsymbol{x})] \to \int d^3\boldsymbol{X} \int \mathcal{D}A\, \mathcal{F}\left[TAU_{\rm cl}(R\boldsymbol{x})A^\dagger T^\dagger\right]$$

It naturally gives the 3D Fourier transform.

- Collective Hamiltonian & Wavefunctions in flavor SU(3) symmetry

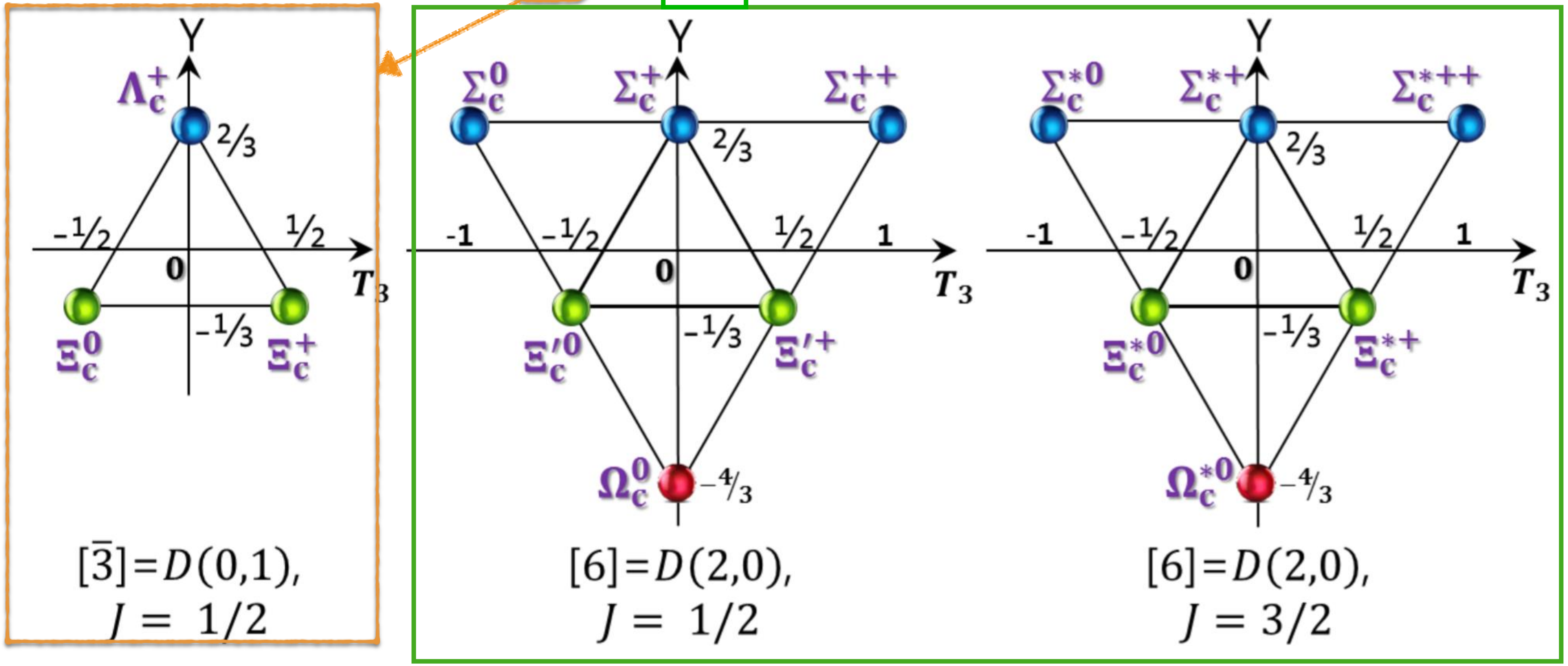
$$H_{\rm coll} = M_{\rm sol} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2$$

$$\Psi^{(\mu)}_{(YTT_3)(Y_RJJ_3)}(A) = \sqrt{\dim(\mu)}(-1)^{J_3-Y_R/2}D^{(\mu)*}_{(YTT_3)(Y_RJ-J_3)}(A)$$

Heavy Quark Symmetry

- In the heavy quark mass limit, a heavy quark spin is conserved, so light-quark spin is also conserved: Heavy-quark spin symmetry
- In this limit, heavy baryons are independent of heavy-quark flavors: Heavy-quark flavor symmetry
- In this limit, a heavy quark can be considered as **a static color source**.
- Dynamics is governed by light quarks.

$$\mathbf{3} \otimes \mathbf{3} = \overline{\mathbf{3}} \oplus \mathbf{6}$$



Nc-1 quarks represent heavy-baryon spectra.

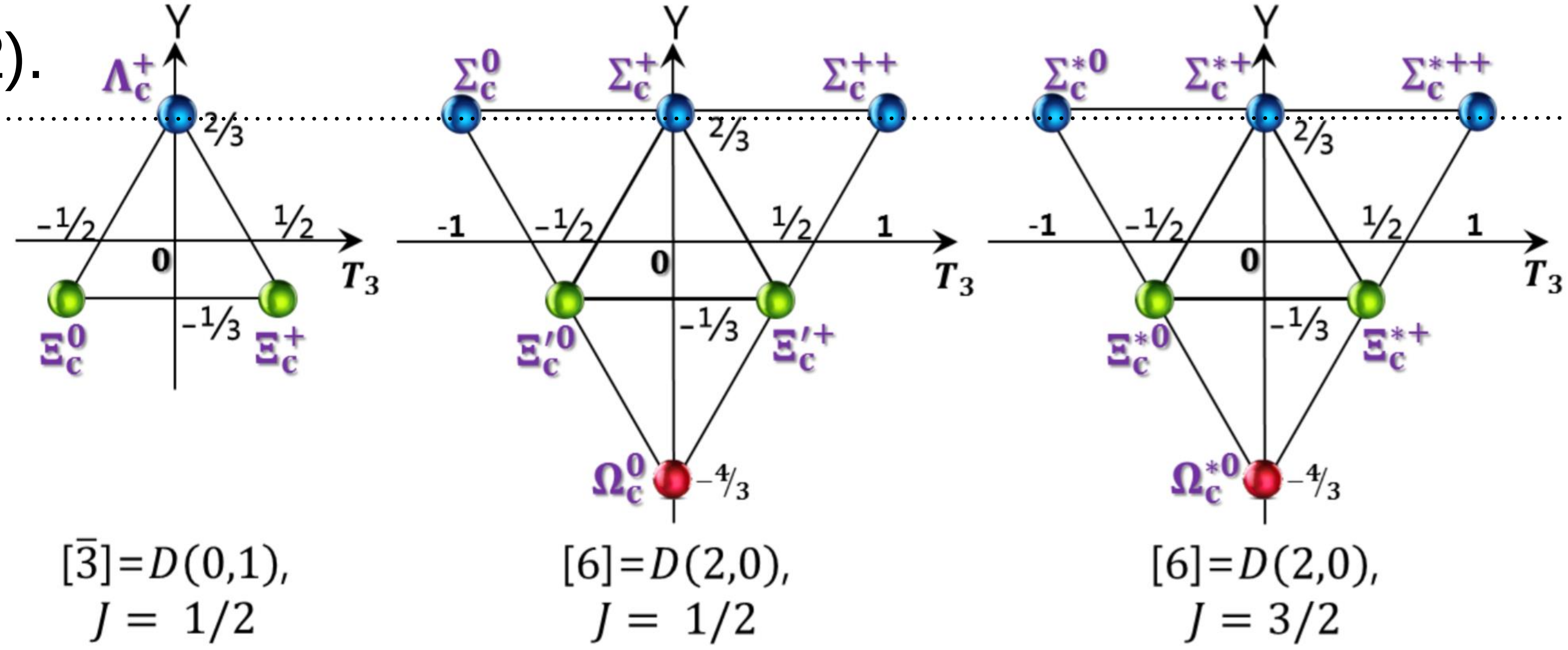
Yang, HChK, Praszalowicz, Polyakov, PRD 94 (2016) R071502

$$Y' = \frac{N_c - 1}{3}$$

Grand spin: $K = 0 \to T = J$

- The lowest rotationally excited states $\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6}$

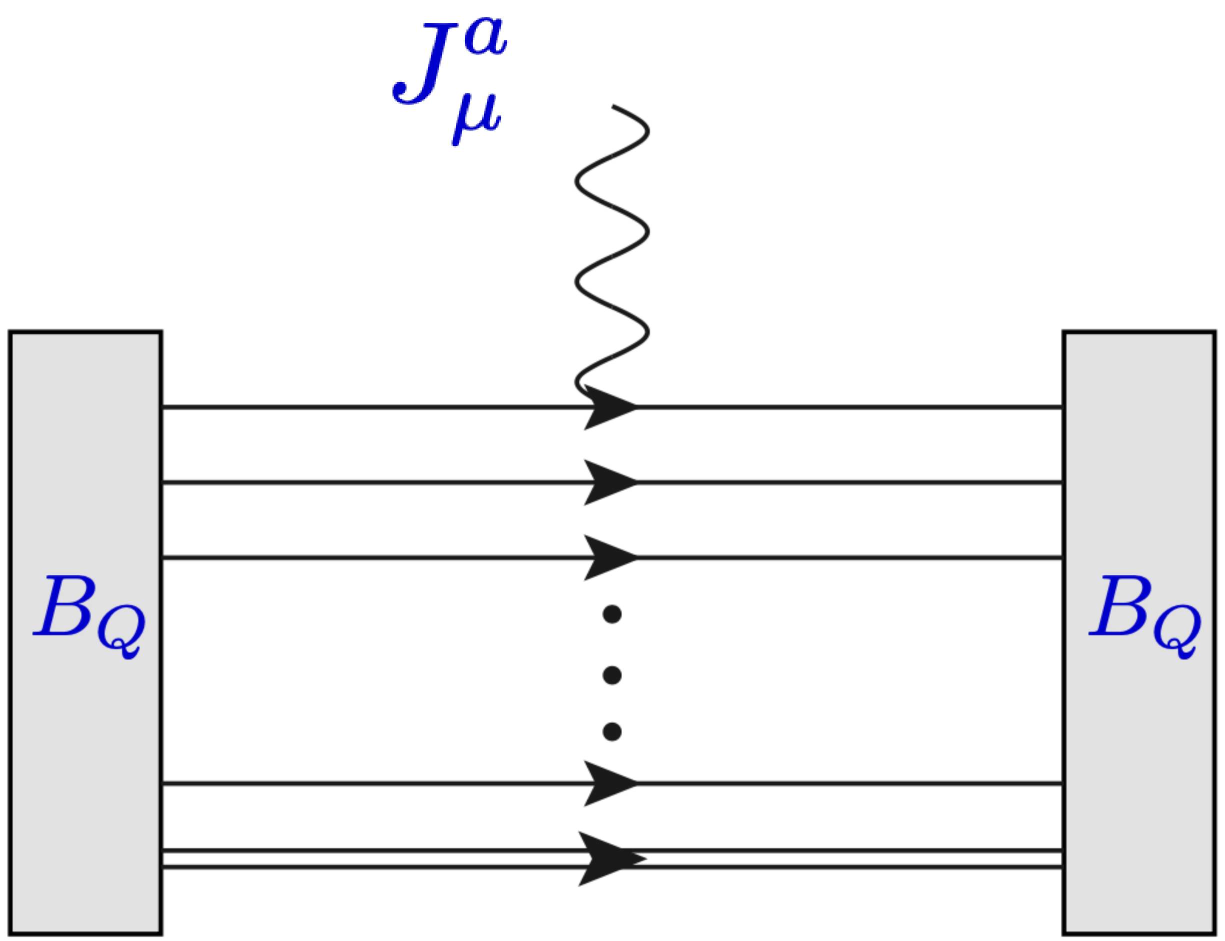
* T=0 for a anti-triplet: J=0 for it. Combining a charm quark with spin 1/2, we have one anti-triplet.
* T=1 for a sextet: J=1. We have two sextets with a charm quark. (1/2, 3/2).

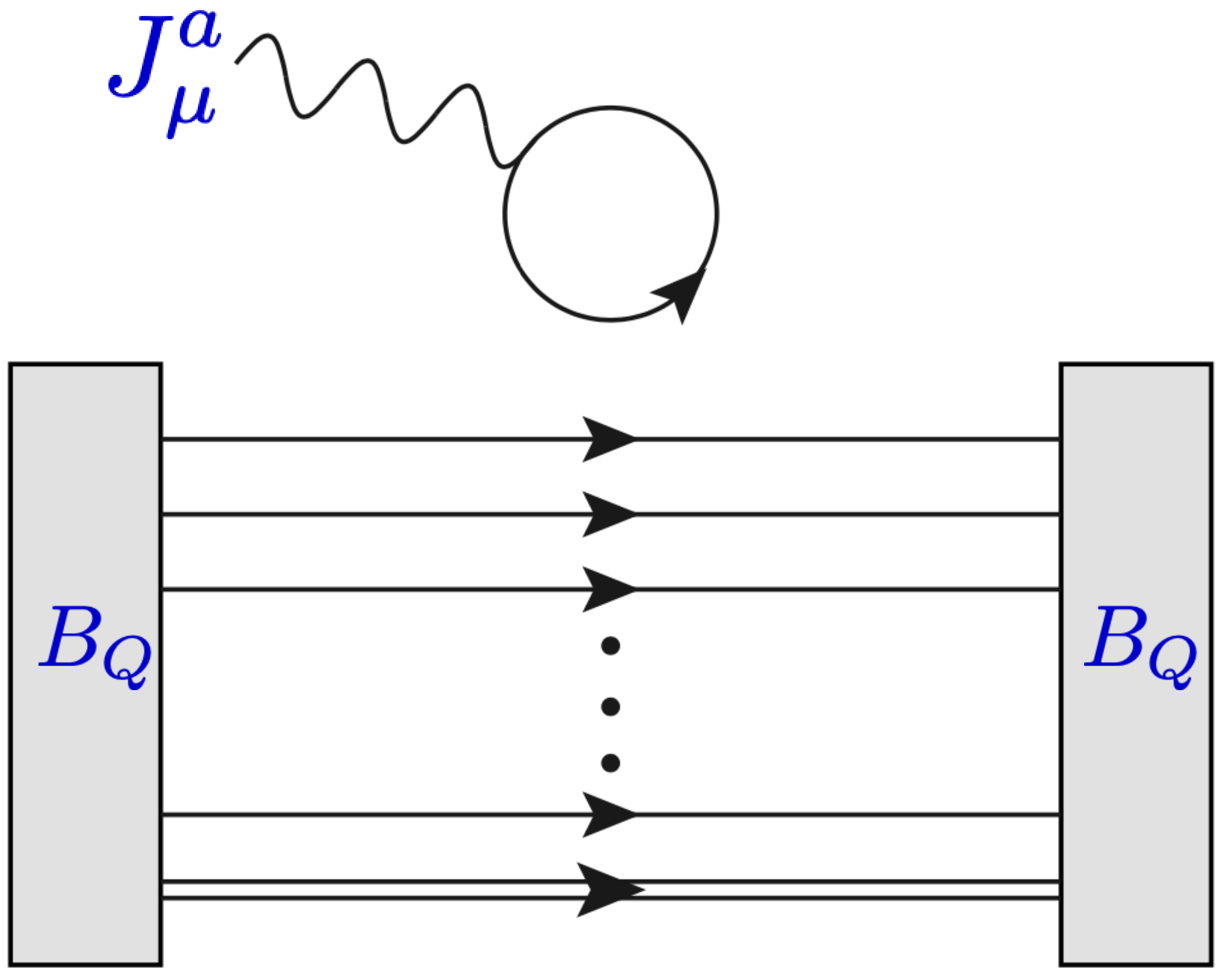


$Y' = 2/3$

Constrained by the valence quarks

Observables for singly heavy baryons





$$\langle B_Q(p', s')|J_\mu^a|B_Q(p, s)\rangle \sim \langle J_{B_Q} J_\mu^a J_{B_Q}^\dagger \rangle_0 \sim \int D\psi D\psi^\dagger D\pi^a J_{B_Q} J_\mu^a J_{B_Q}^\dagger e^{-S}$$

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Heavy-quark current

P. L. Cho and H. Georgi, PLB 296, 408 (1992); 300, 410(E) (1993)

$$-i\Psi_h^\dagger(x)\gamma^\mu Q_h\Psi_h(x) = -i\exp(-im_Q v\cdot x)\tilde{\Psi}_h^\dagger(x)\left[v_\mu + \frac{i}{2m_Q}(\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu) + \frac{1}{2m_Q}\sigma_{\mu\nu}(\overleftarrow{\partial}_\mu + \overrightarrow{\partial}_\mu)\right]Q_h\tilde{\Psi}_h(x)$$

$$\approx -i\exp(-im_Q v\cdot x)\tilde{\Psi}_h^\dagger(x)v_\mu Q_h\tilde{\Psi}_h(x)$$

A heavy quark inside a singly heavy baryon only gives a constant contribution to its electric form factor in the infinitely heavy-quark mass limit.

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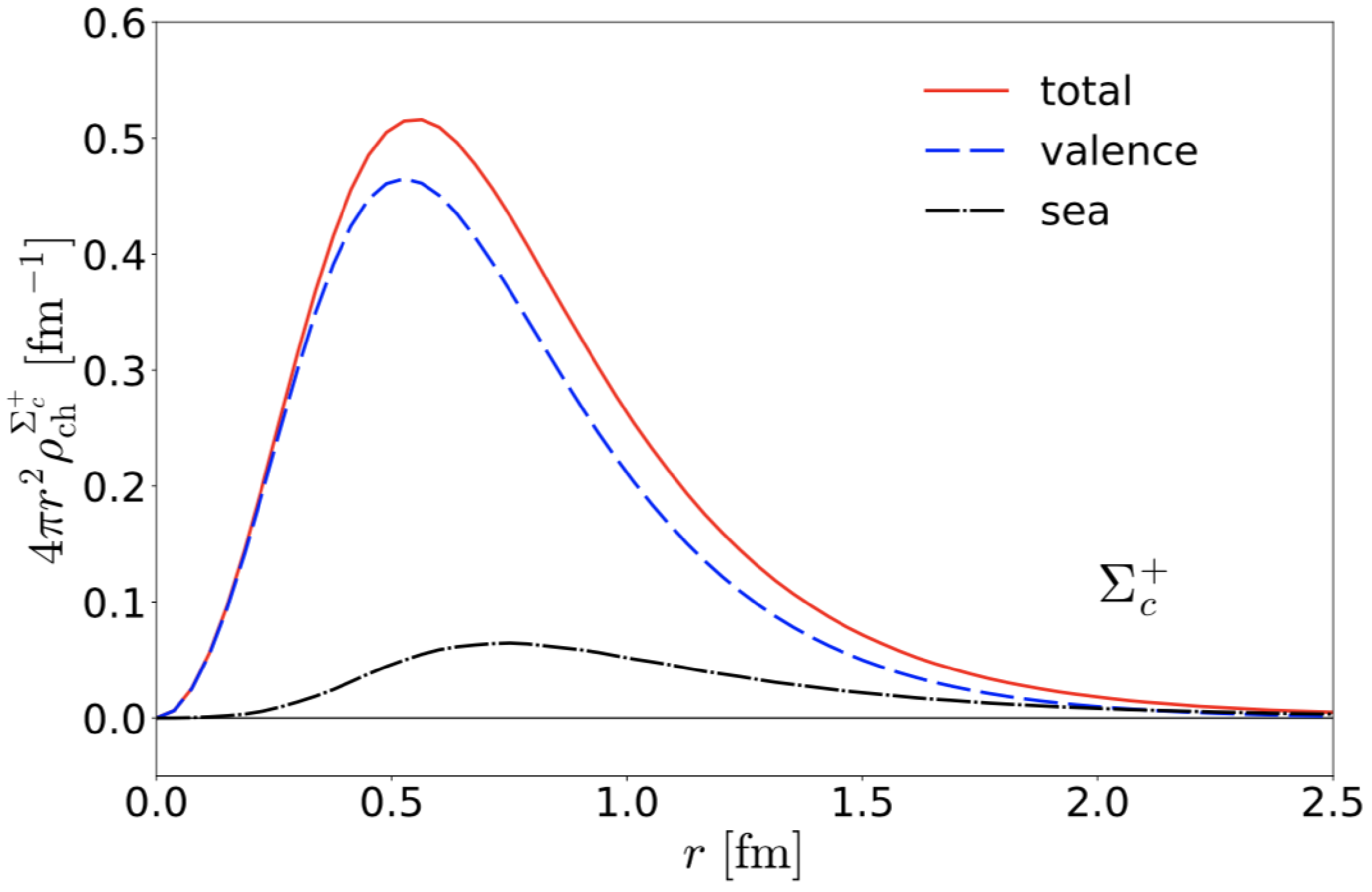
Electromagnetic Structure
of
Singly heavy baryons

Electromagnetic form factors

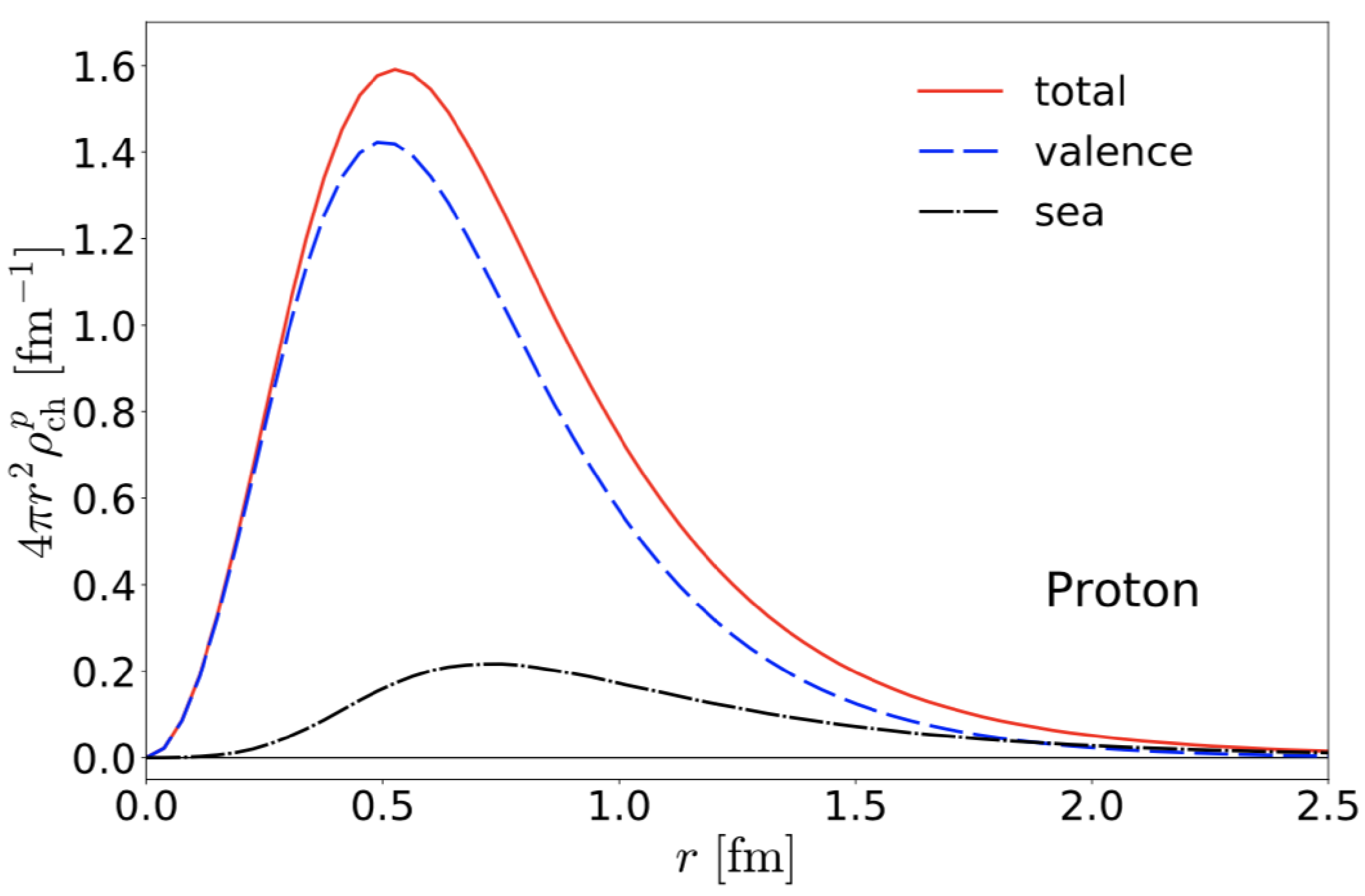
$$J_\mu(x) = \bar{\psi}(x)\gamma_\mu \hat{\mathcal{Q}}\psi(x) + e_Q \bar{\Psi}(x)\gamma_\mu \Psi(x)$$ - Heavy quark: point-like structure $(m_Q \to \infty)$

Electric charge densities

$\Sigma_c^+ \; (udc)$



proton (uud)

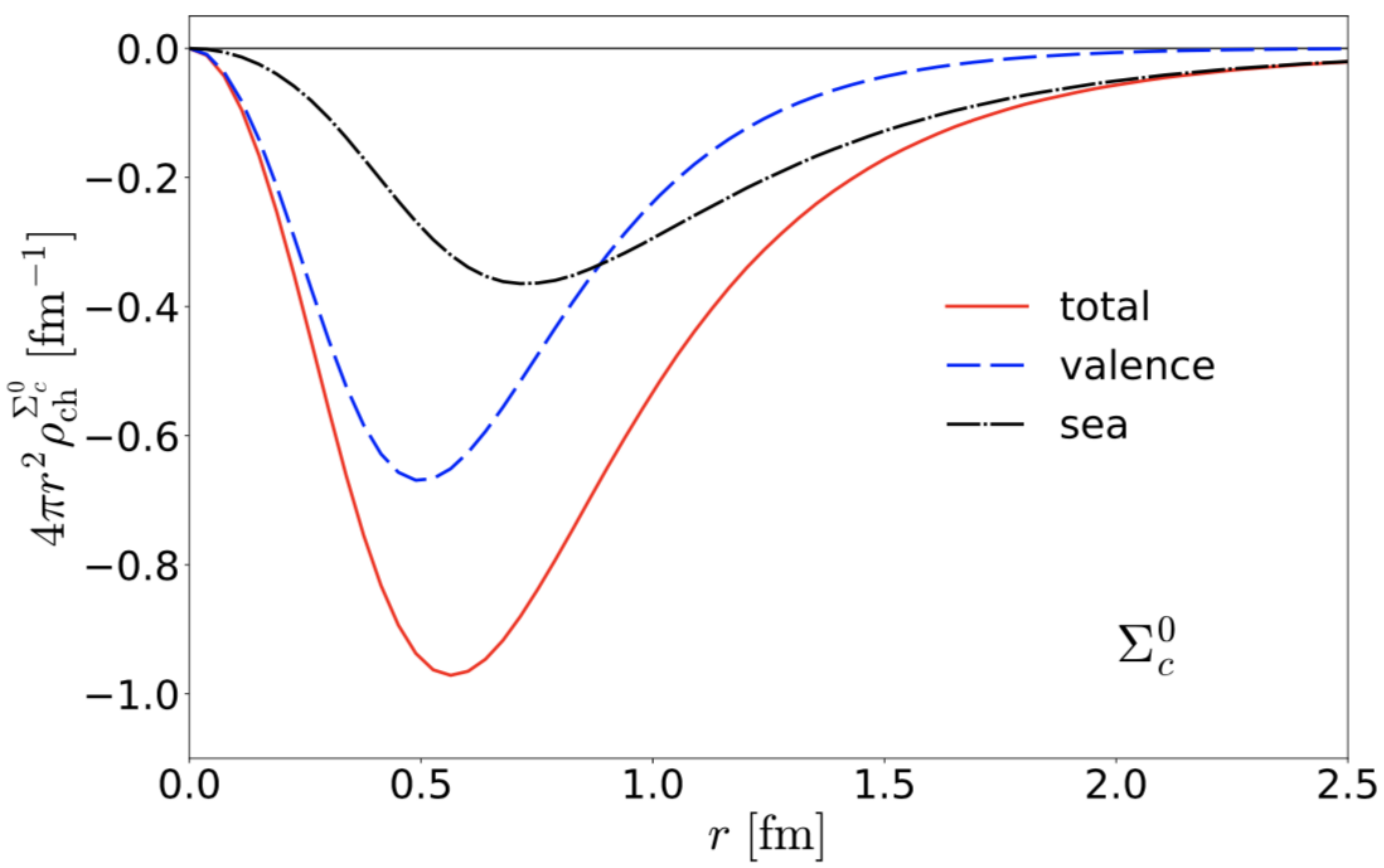


J.Y. Kim and HChK, PRD D97, 114009 (2018).

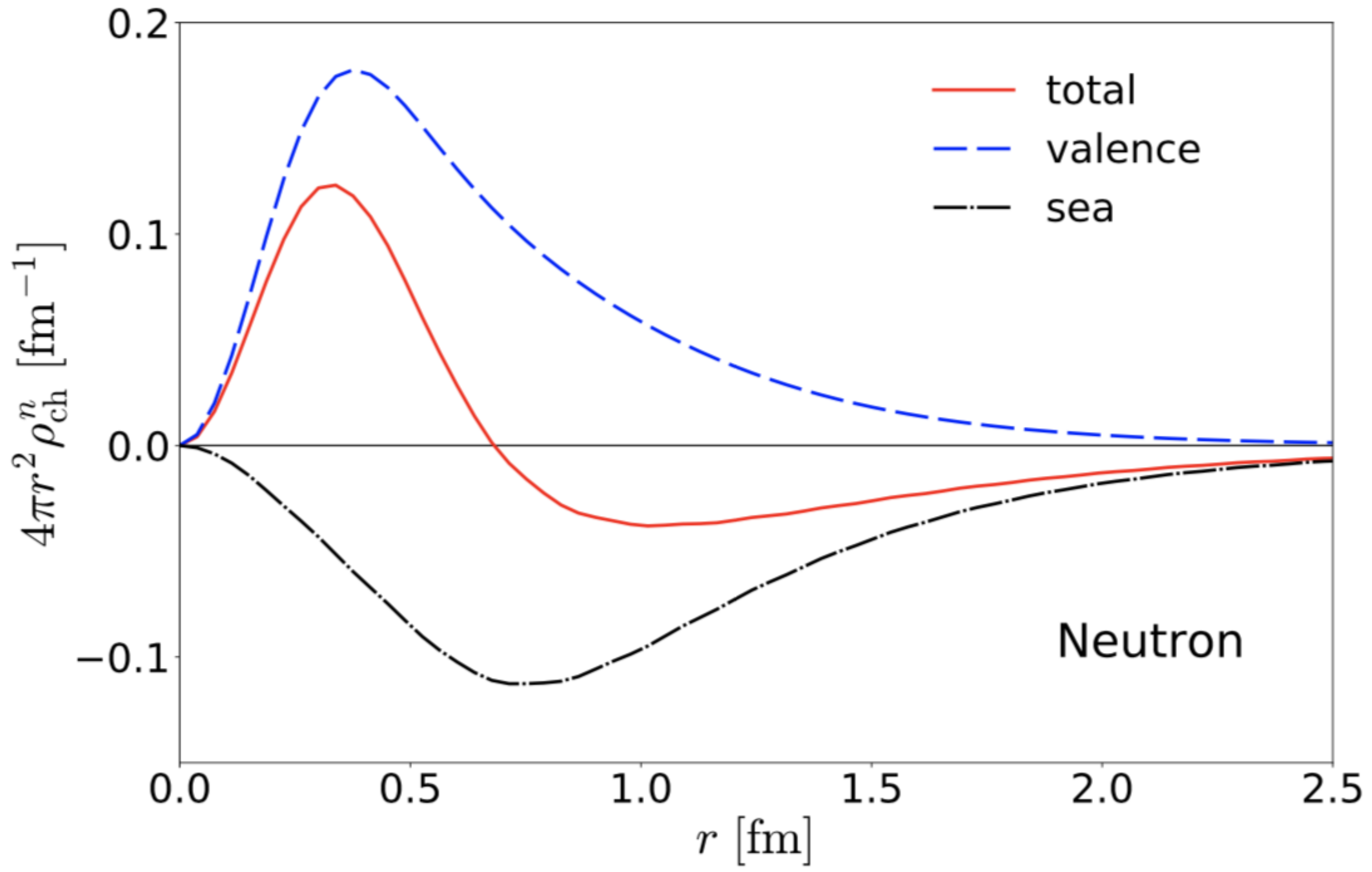
Electromagnetic form factors

Electric charge densities

$\Sigma_c^0\,(ddc)$



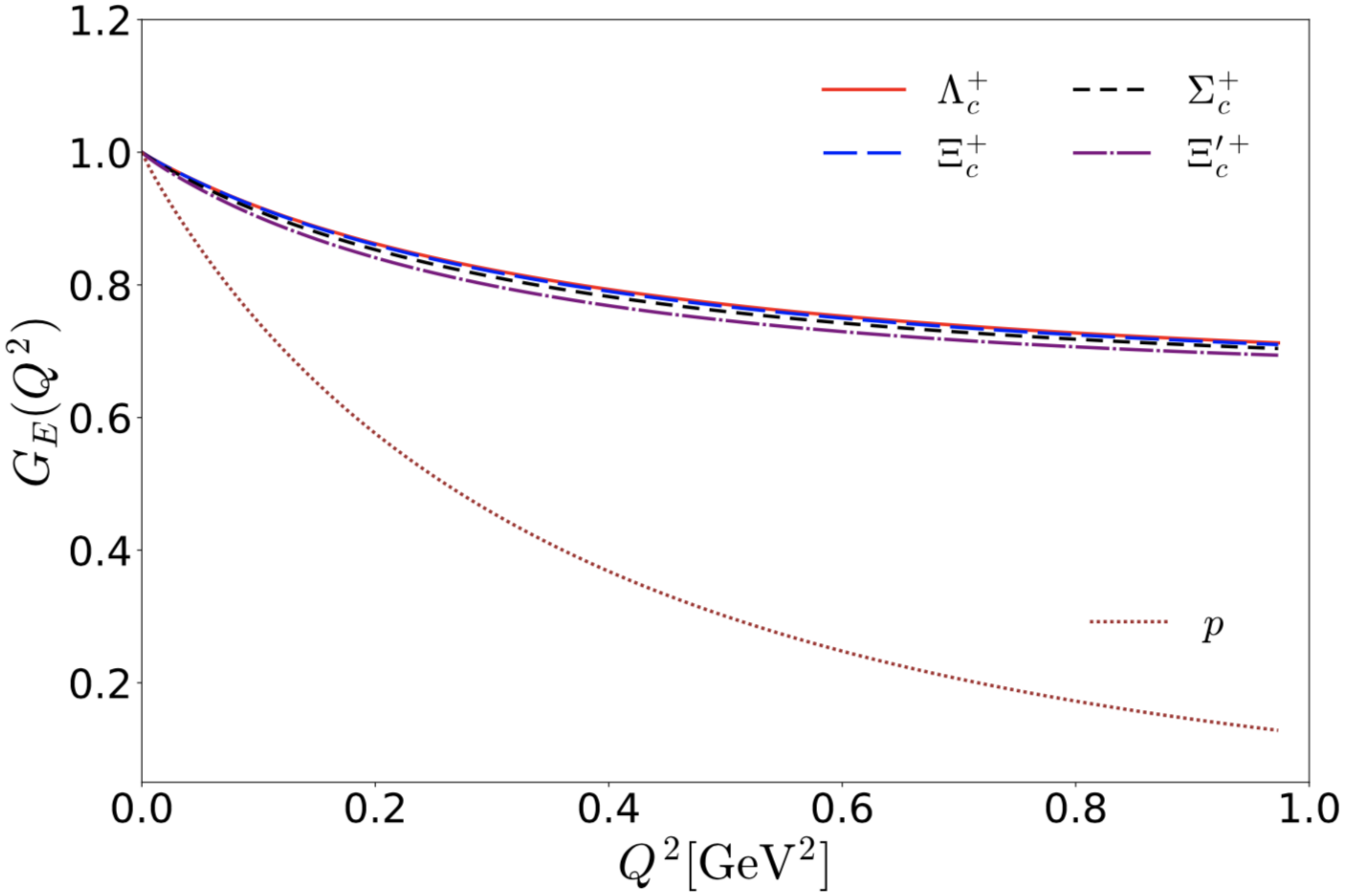
Neutron (udd)



J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Electric form factors



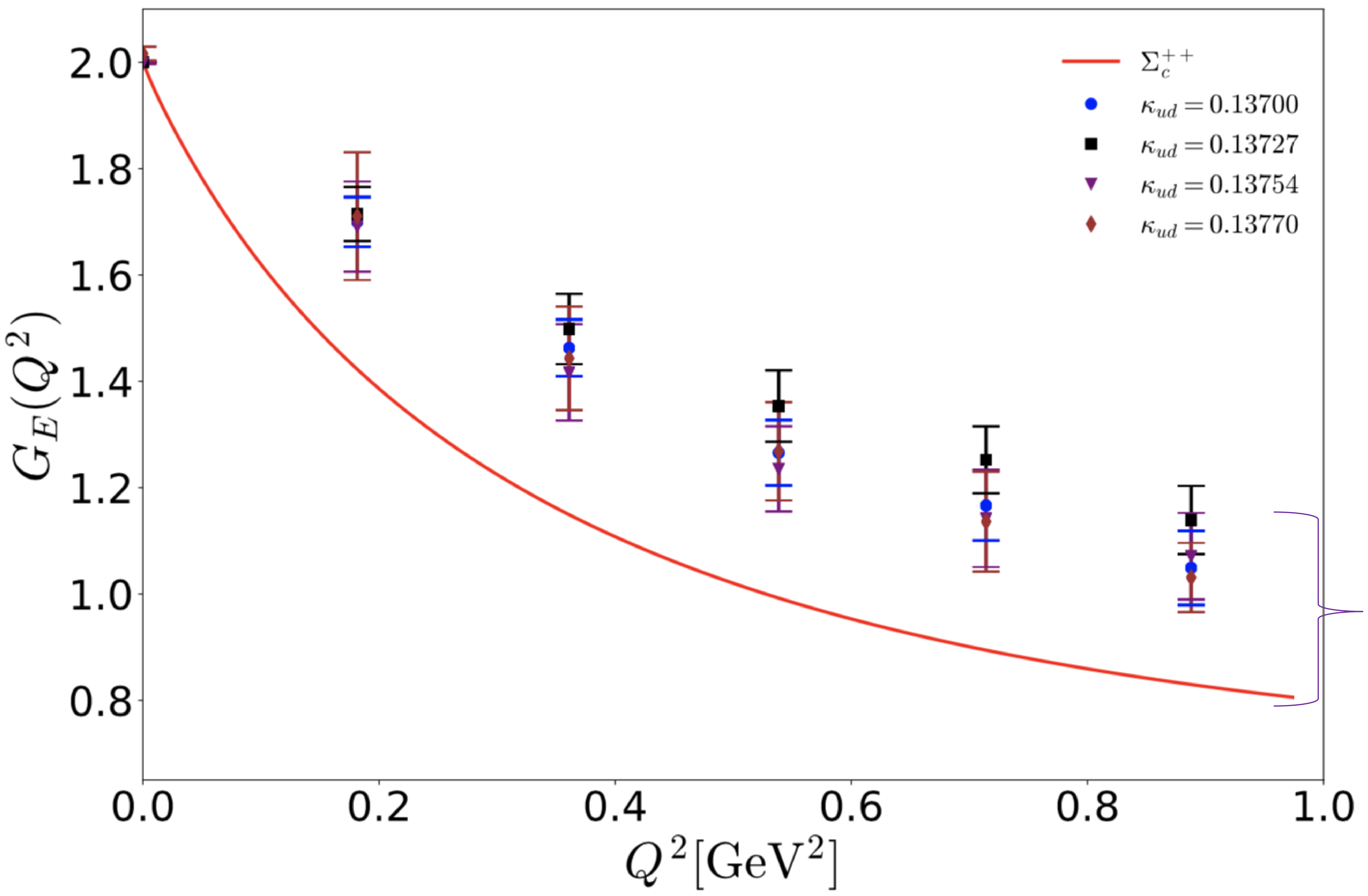
Heavy baryons are electrically more compact than the proton!

J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Electric form factors

Lattice data: K. U. Can et al., JHEP 05 (2014) 125.

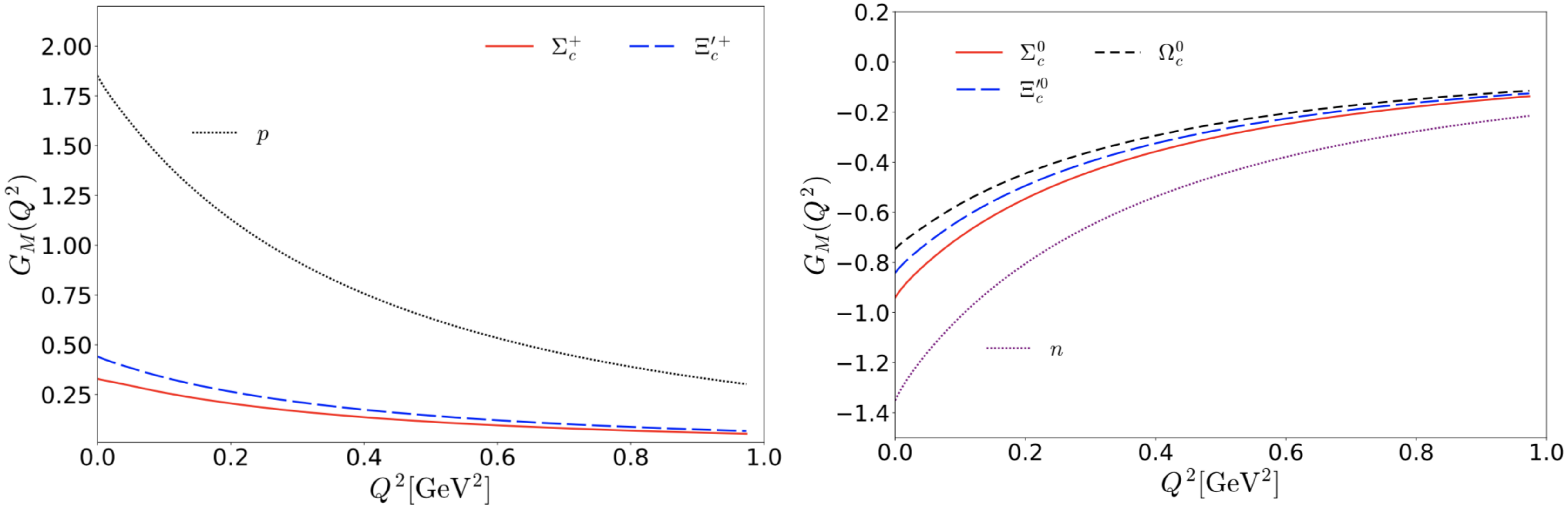


Due to the different values of the pion mass.

J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Magnetic form factors



The singly heavy baryons are less magnetized than the proton and the neutron.

J.Y. Kim and HChK, PRD D97, 114009 (2018).

Magnetic moments of heavy baryons

- Collective operators for the magnetic moments

$$\hat{\mu}^{(0)} = w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3,$$

$$\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{8q} + w_5 \left(D^{(8)}_{Q3} D^{(8)}_{88} + D^{(8)}_{Q8} D^{(8)}_{83} \right) + w_6 \left(D^{(8)}_{Q3} D^{(8)}_{88} - D^{(8)}_{Q8} D^{(8)}_{83} \right)$$

- The parameter wi's are determined by the experimental data on the magnetic moments of the baryon octet.

No additional free parameter!

- Results of the magnetic moments of the baryon sextet with spin 1/2

$\mu\left[6_1^{1/2},\, B_c\right]$	$\mu^{(0)}$	$\mu^{(\text{total})}$	Oh et al. [17]	Scholl and Weigel [18]	Faessler et al. [19]	Lattice QCD [20,22]
Σ_c^{++}	2.00 ± 0.09	2.15 ± 0.1	1.95	2.45	1.76	2.220 ± 0.505
Σ_c^{+}	0.50 ± 0.02	0.46 ± 0.03	0.41	0.25	0.36	–
Σ_c^{0}	-1.00 ± 0.05	-1.24 ± 0.05	-1.1	-1.96	-1.04	-1.073 ± 0.269
$\Xi_c^{\prime +}$	0.50 ± 0.02	0.60 ± 0.02	0.77	–	0.47	0.315 ± 0.141
$\Xi_c^{\prime 0}$	-1.00 ± 0.05	-1.05 ± 0.04	-1.12	–	-0.95	-0.599 ± 0.071
Ω_c^{0}	-1.00 ± 0.05	-0.85 ± 0.05	-0.79	–	-0.85	-0.688 ± 0.031

Yang & HChK, PLB 781, 601 (2018).

Magnetic moments of heavy baryons

- Baryon Sextet with spin 3/2

$\mu\left[6_1^{3/2},\ B_c\right]$	$\mu^{(0)}$	$\mu^{(\mathrm{total})}$	Oh et al. [17]	Lattice QCD [21]
Σ_c^{*++}	3.00 ± 0.14	3.22 ± 0.15	3.23	–
Σ_c^{*+}	0.75 ± 0.04	0.68 ± 0.04	0.93	–
Σ_c^{*0}	-1.50 ± 0.07	-1.86 ± 0.07	-1.36	–
Ξ_c^{*+}	0.75 ± 0.04	0.90 ± 0.04	1.46	–
Ξ_c^{*0}	-1.50 ± 0.07	-1.57 ± 0.06	-1.4	–
Ω_c^{*0}	-1.50 ± 0.07	-1.28 ± 0.08	-0.87	-0.730 ± 0.023

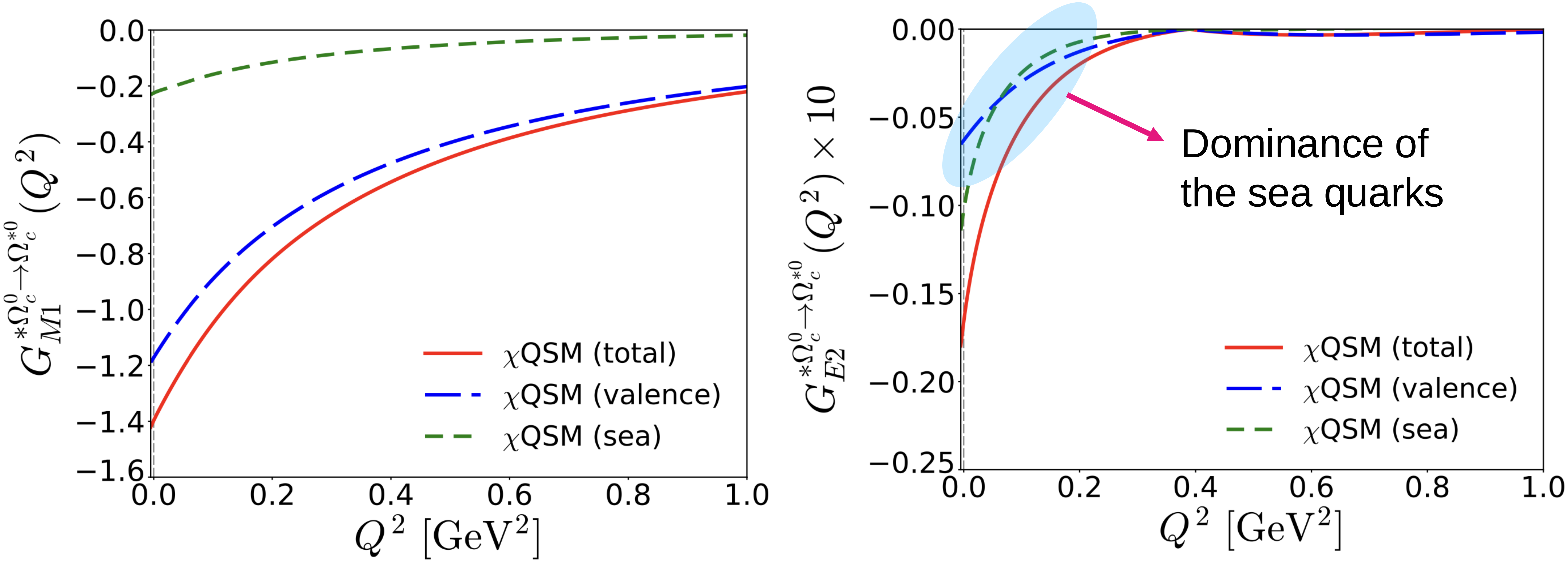
No additional free parameter!

Yang & HChK, PLB 781, 601 (2018).

Electromagnetic transitions
of
Singly heavy baryons with spin 3/2

Electromagnetic transitions

We will show only the Omega-c transitions. For other transitions, refer to the reference.



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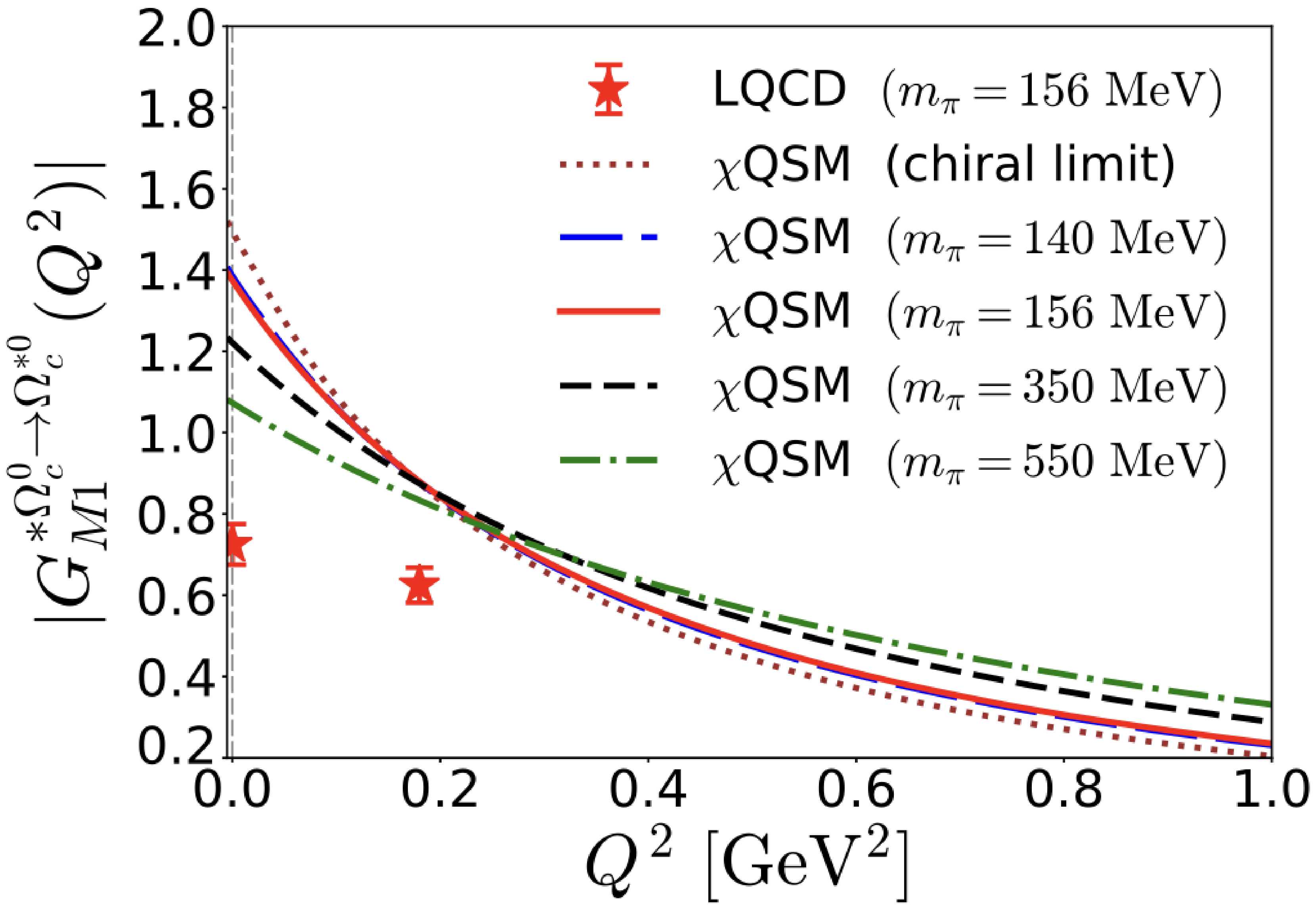
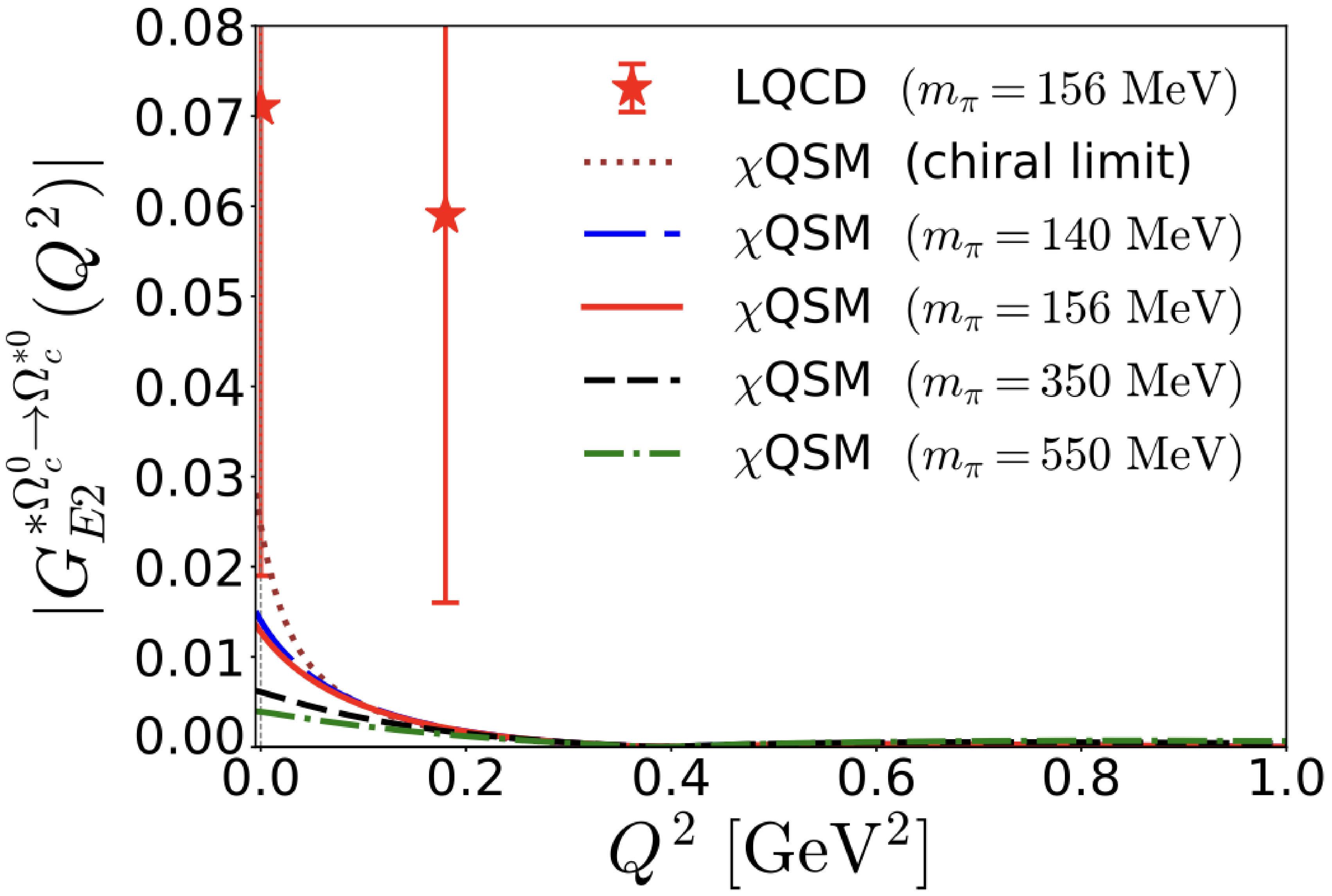
Electromagnetic transitions

$\Omega_c^0 \gamma \to \Omega_c^{*0}$

Lattice Data: H Bahtiyar, KU Can, G Erkol, M Oka, PLB 747, 281 (2015)





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Electromagnetic transitions

$\Gamma(B_c\gamma \to B_c^*)$	χQSM ($m_s = 0$ MeV)	χQSM ($m_s = 180$ MeV)	χSM [35]	LQCD [20]	Bag [51]	χPT [12]	QCDSR [16, 17]	QM [15]
$\Lambda_c^+\gamma \to \Sigma_c^{*+}$	63.37	69.76	191.13 ± 15.15	–	126	161.8	130(45)	151(4)
$\Xi_c^+\gamma \to \Xi_c^{*+}$	34.14	31.97	55.77 ± 5.22	–	44.3	21.6	52(25)	54(3)
$\Xi_c^0\gamma \to \Xi_c^{*0}$	0	0.08	1.61 ± 0.42	–	0.908	1.84	0.66(32)	0.68(4)
$\Sigma_c^{++}\gamma \to \Sigma_c^{*++}$	1.12	1.08	2.41 ± 0.22	–	0.826	1.20	2.65(1.20)	–
$\Sigma_c^+\gamma \to \Sigma_c^{*+}$	0.07	0.06	0.11 ± 0.02	–	0.004	0.04	0.40(16)	0.140(4)
$\Sigma_c^0\gamma \to \Sigma_c^{*0}$	0.28	0.30	0.80 ± 0.06	–	1.08	0.49	0.08(3)	–
$\Xi_c^{\prime +}\gamma \to \Xi_c^{*+}$	0.09	0.09	0.21 ± 0.02	–	0.011	0.07	0.274	–
$\Xi_c^{\prime 0}\gamma \to \Xi_c^{*0}$	0.35	0.34	0.64 ± 0.05	–	1.03	0.42	2.142	–
$\Omega_c^0\gamma \to \Omega_c^{*0}$	0.38	0.34	0.49 ± 0.08	0.074	1.07	0.32	0.932	–

[35] G. S. Yang and HChK, PLB 801 (2020) 135142

JY Kim, HChK, G.S. Yang, M. Oka, PRD.103 (2021) 074025

Strong decays
of
Singly heavy baryons

Strong decay rates

- Collective operator for the strong vertices in SU(3) symmetric case

$$\mathcal{O}_\varphi = \frac{3}{M_1 + M_2} \sum_{i=1,2,3} \left[G_0 D^{(8)}_{\varphi\, i} - G_1\, d_{ibc} D^{(8)}_{\varphi\, b} \hat{S}_c - G_2 \frac{1}{\sqrt{3}} D^{(8)}_{\varphi\, 8} \hat{S}_i \right] p_i$$

- Decay widths

$$\Gamma_{B_1 \to B_2 + \varphi} = \frac{1}{2\pi} \overline{\langle B_2 \,| \mathcal{O}_\varphi |\, B_1 \rangle^2} \frac{M_2}{M_1} p$$

$$G_0 = -\frac{M + M'}{6 f_\varphi} a_1$$

$$G_{1,2} = \frac{M + M'}{6 f_\varphi} a_{2,3}$$

a_1	a_2	a_3
-3.509 ± 0.011	3.437 ± 0.028	0.604 ± 0.030

G. Yang and HChK, PRC **92**, 035206 (2015)

No additional free parameter!

$$f_\pi = 93\,\mathrm{MeV}, \quad f_K = 1.2 f_\pi$$

- These parameters a_i have been determined by the hyperon semileptonic decays.

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Strong decays of heavy baryons

- Decay widths of the charm baryon sextet

#	decay	this work	exp.
1	$\Sigma_c^{++}(\mathbf{6}_1, 1/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	1.93	$1.89^{+0.09}_{-0.18}$
2	$\Sigma_c^{+}(\mathbf{6}_1, 1/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^0$	2.24	< 4.6
3	$\Sigma_c^{0}(\mathbf{6}_1, 1/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	1.90	$1.83^{+0.11}_{-0.19}$
4	$\Sigma_c^{++}(\mathbf{6}_1, 3/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	14.47	$14.78^{+0.30}_{-0.19}$
5	$\Sigma_c^{+}(\mathbf{6}_1, 3/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^0$	15.02	< 17
6	$\Sigma_c^{0}(\mathbf{6}_1, 3/2) \to \Lambda_c^{+}(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	14.49	$15.3^{+0.4}_{-0.5}$
7	$\Xi_c^{+}(\mathbf{6}_1, 3/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	2.35	2.14 ± 0.19
8	$\Xi_c^{0}(\mathbf{6}_1, 3/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	2.53	2.35 ± 0.22

Experimental data are taken from the PDG.

No additional free parameter!

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Strong decays of heavy baryons

- Decay widths of the bottom baryon sextet

#	decay	this work	exp.
1	$\Sigma_b^+(\mathbf{6}_1, 1/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	6.12	$9.7^{+4.0}_{-3.0}$
2	$\Sigma_b^-(\mathbf{6}_1, 1/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	6.12	$4.9^{+3.3}_{-2.4}$
3	$\Xi_b'(\mathbf{6}_1, 1/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	0.07	< 0.08
4	$\Sigma_b^+(\mathbf{6}_1, 3/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	10.96	11.5 ± 2.8
5	$\Sigma_b^-(\mathbf{6}_1, 3/2) \to \Lambda_c^0(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	11.77	7.5 ± 2.3
6	$\Xi_b^0(\mathbf{6}_1, 3/2) \to \Xi_b(\overline{\mathbf{3}}_0, 1/2) + \pi$	0.80	0.90 ± 0.18
7	$\Xi_b^-(\mathbf{6}_1, 3/2) \to \Xi_b(\overline{\mathbf{3}}_0, 1/2) + \pi$	1.28	1.65 ± 0.33

Experimental data are taken from the PDG.

No additional free parameter!

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Quark spin content
of
Singly heavy baryons

X. Ji, PRL 78 (1997) 610

- Spin of a baryon

$$J_{1/2} = \frac{1}{2}\boxed{\Delta\Sigma} + L_q + J_G = \frac{1}{2}$$

$$J_{3/2} = \frac{3}{2}\boxed{\Delta\Sigma} + L_q + J_G = \frac{3}{2}$$

Understanding the spin structure of the nucleon is one of the EIC missions.

- Axial-vector current

$$A^{\chi}_{\mu}(x) = \bar{\psi}(x)\gamma_{\mu}\gamma_5\frac{\lambda^{\chi}}{2}\psi(x) + \bar{\Psi}(x)\gamma_{\mu}\gamma_5\Psi(x)$$

Light-quark part   Heavy-quark part

- Flavor decomposition of the axial-vector constants

$$g_A^{(0)} = \sum_{q=u,d,s,c} \Delta q = \Delta\Sigma,$$

(Gluons were integrated out.)

$$g_A^{(3)} = \Delta u - \Delta d,$$

$$g_A^{(8)} = \frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s)$$

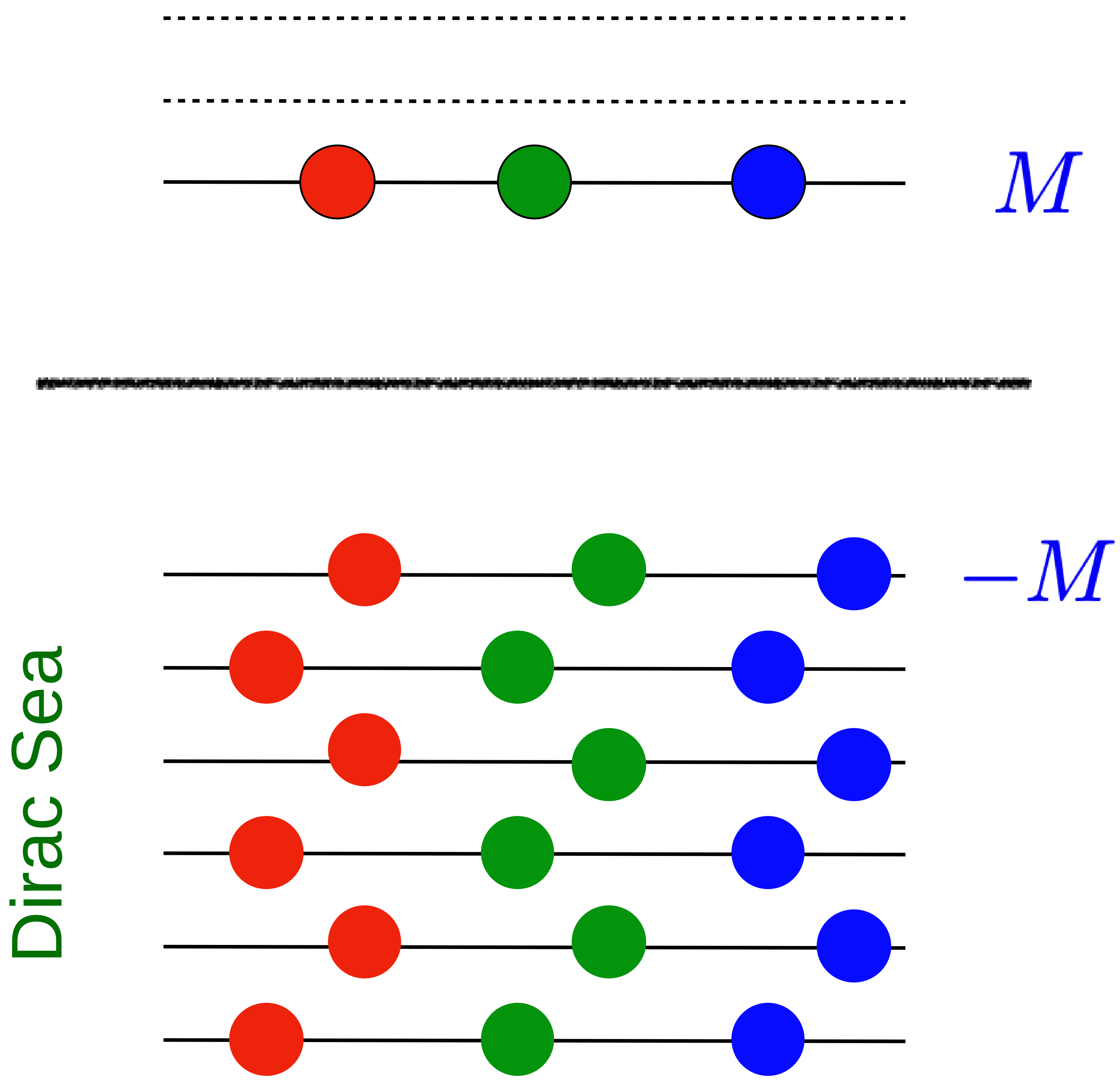
Limit to the Nonrelativistic Quark Model (NRQM)

$R \to 0$ (R: Soliton Size)

$a_1 \to -(N_c + 2)$

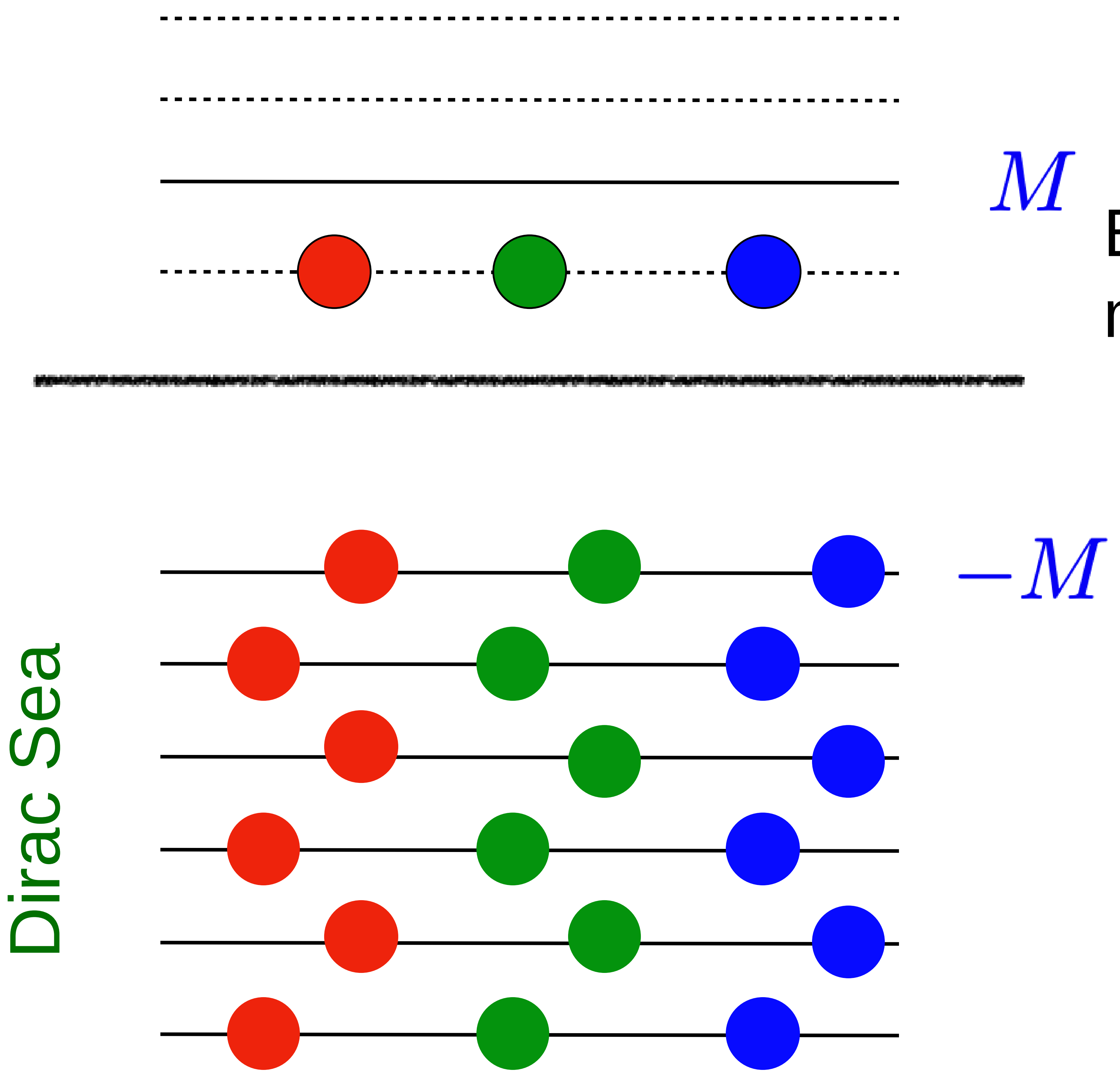
$a_2 \to 4$

$a_3 \to 2$



XQSM

$R \sim 1\,\mathrm{fm}$ (R: Soliton Size)

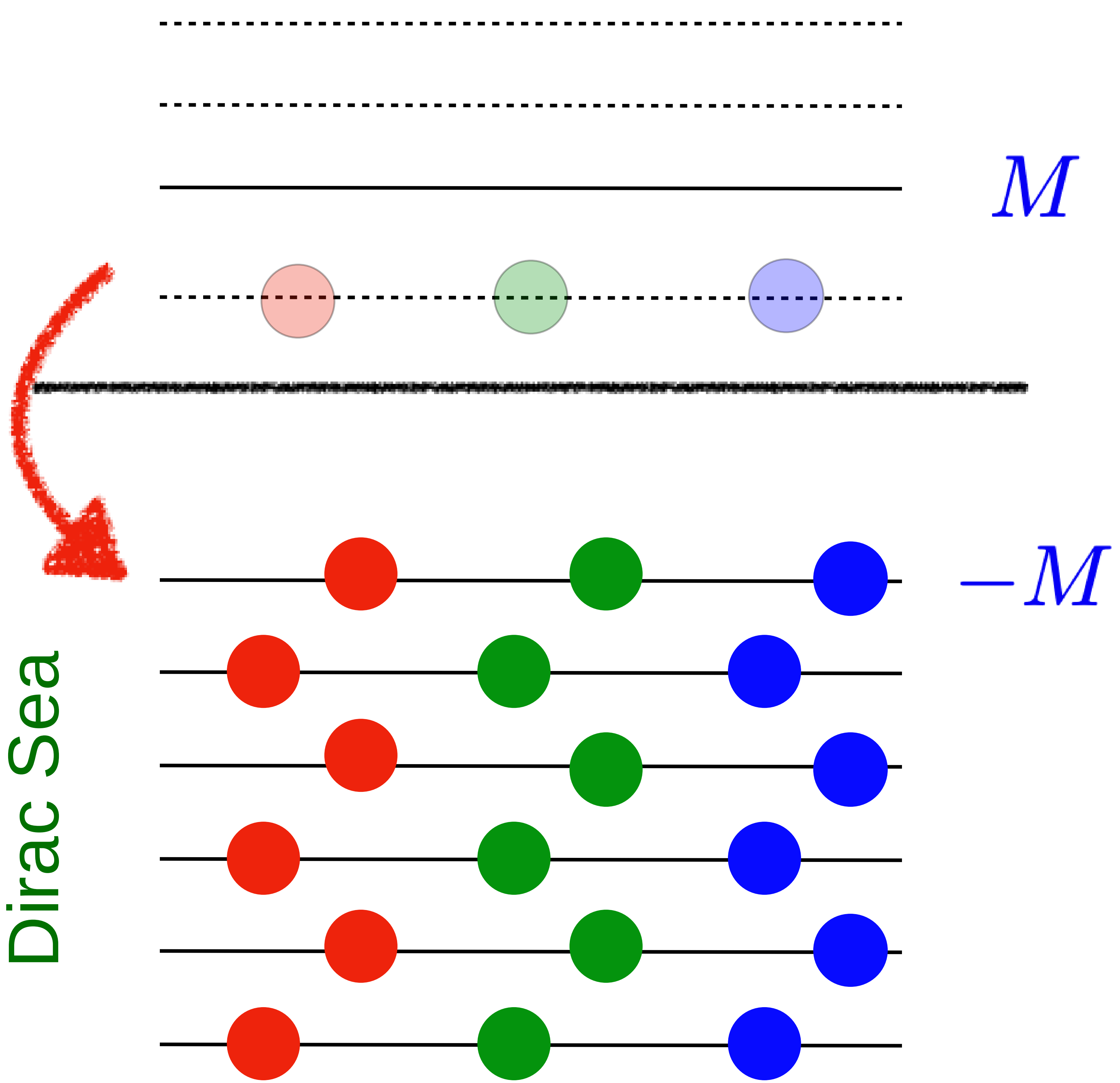


Bound by the pion mean field

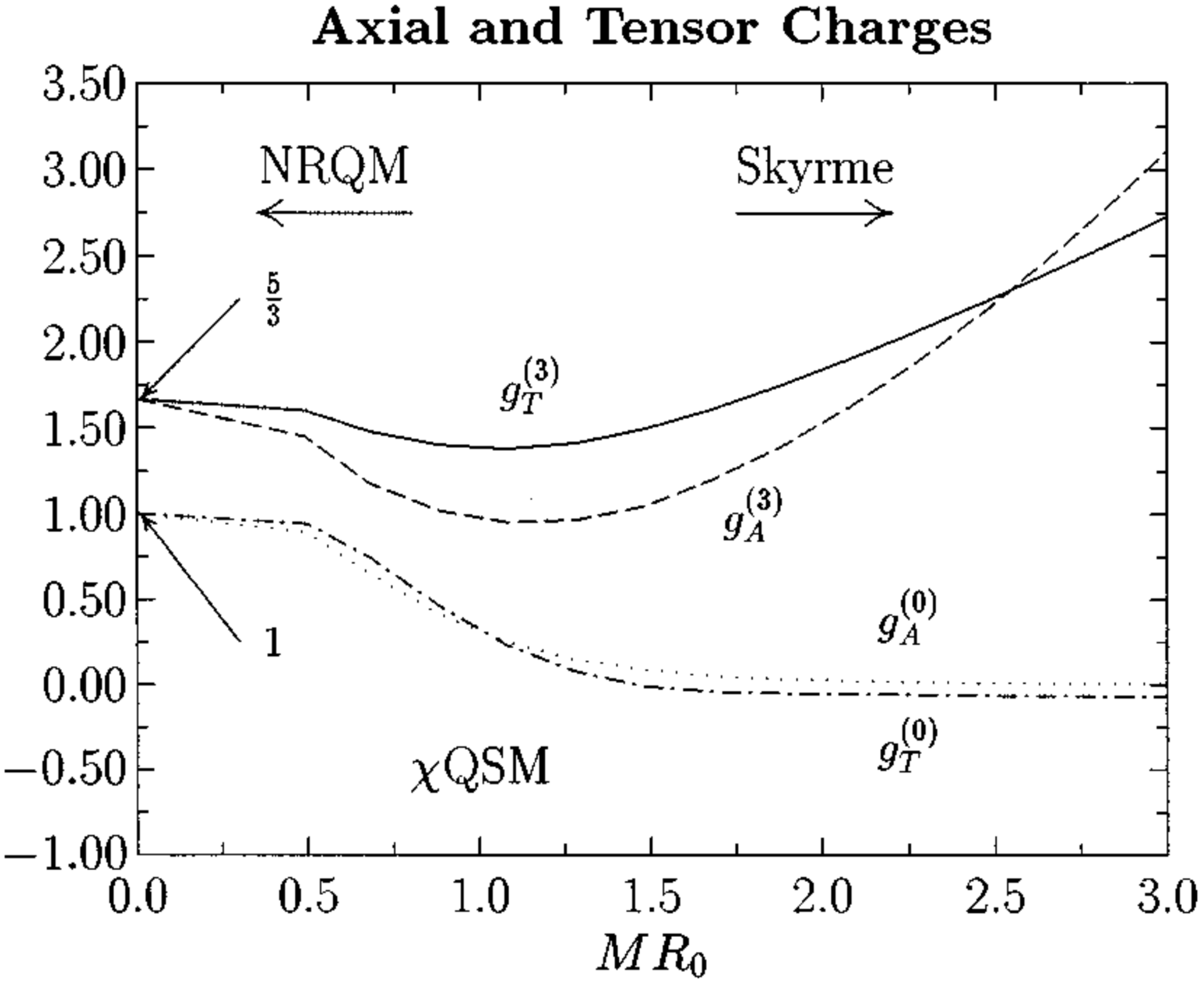
Limit to the Skyrme Model

$R \to \infty$ (R: Soliton Size)

- As R further increases, the quarks cross the border M=0 and the soliton acquires a winding number =1 and then dives into the Dirac sea.



Interpolation between the NRQM and the Skyrme model



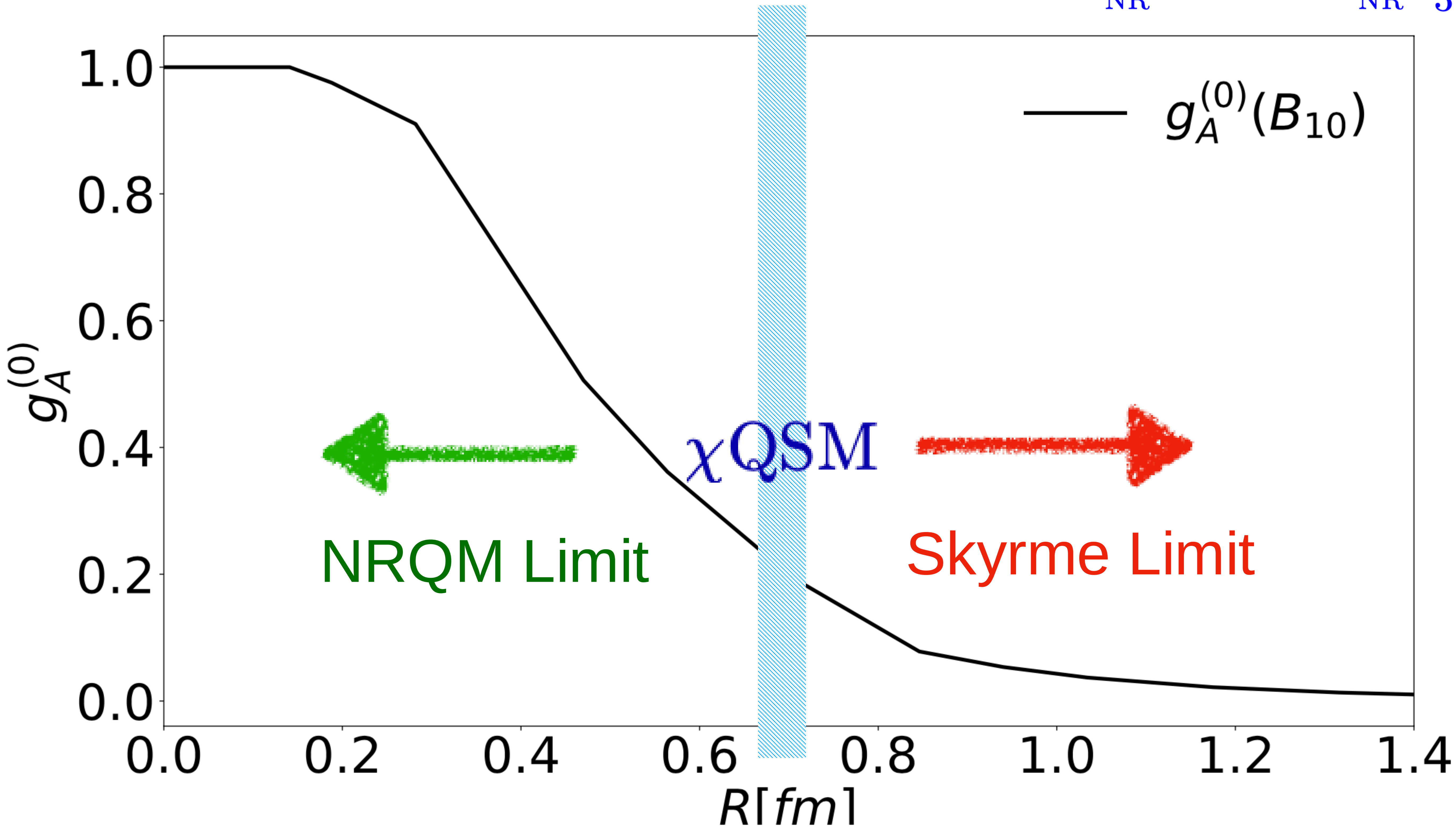
$$g_A^{(3)} \sim (MR)^2$$

$$g_A^{(0)} \sim \frac{1}{(MR)^4}$$

HChK, M.V. Polyakov, PRD 53 (1996) 4715

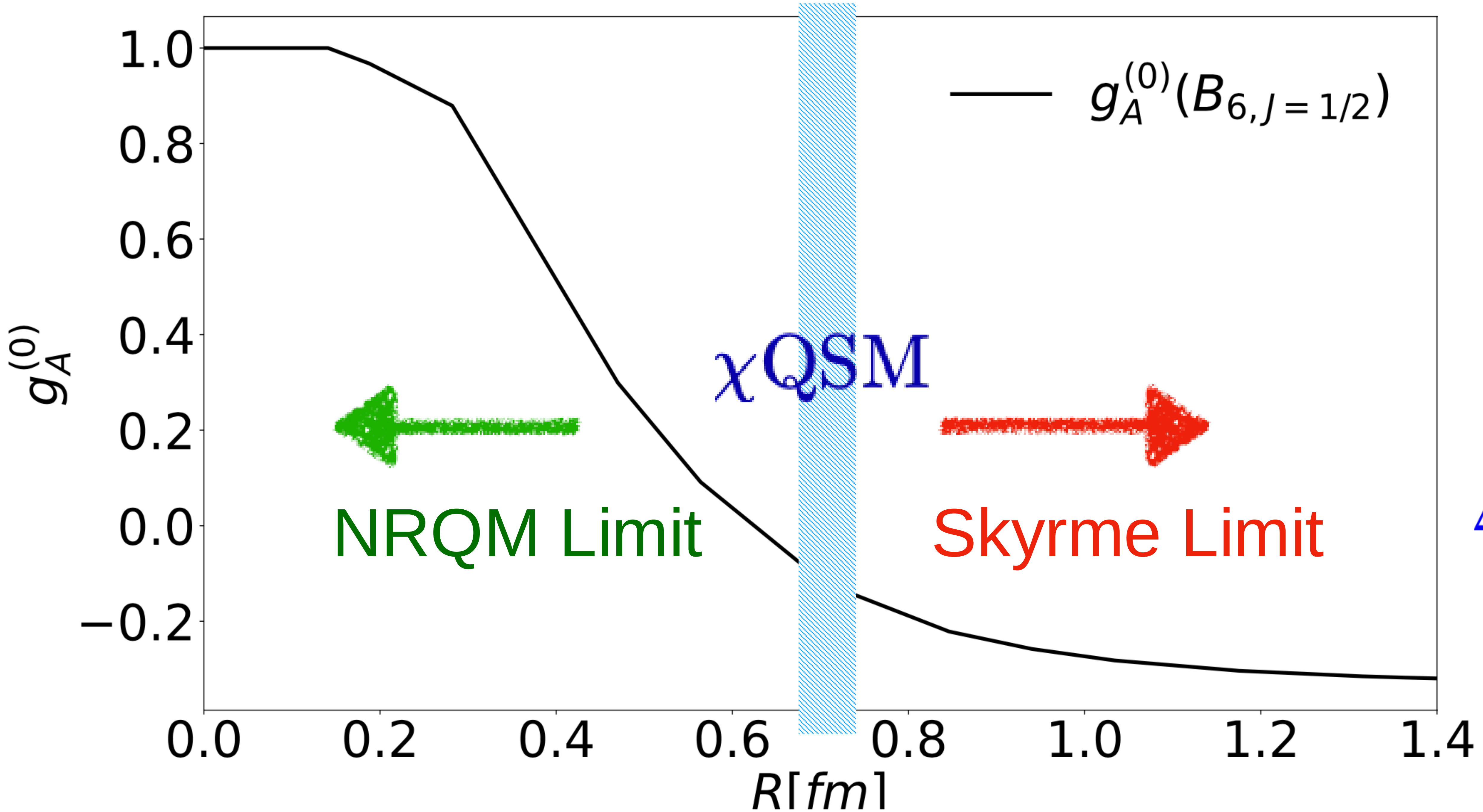
$$g_A^{(0),\Delta^+} \xrightarrow[\text{NR}]{} 1, \quad g_A^{(3),\Delta^+} \xrightarrow[\text{NR}]{} \frac{1}{3}, \quad g_A^{(8),\Delta^+} \xrightarrow[\text{NR}]{} \frac{1}{\sqrt{3}}$$

$$g_A^{(0),p} \xrightarrow[\text{NR}]{} 1, \quad g_A^{(3),p} \xrightarrow[\text{NR}]{} \frac{5}{3}, \quad g_A^{(8),p} \xrightarrow[\text{NR}]{} \frac{1}{\sqrt{3}}$$



$$g_A^{(0),\Sigma_c^{++}} \xrightarrow[\text{NR}]{} 1, \quad g_A^{(3),\Sigma_c^{++}} \xrightarrow[\text{NR}]{} \frac{4}{3}, \quad g_A^{(8),\Sigma_c^{++}} \xrightarrow[\text{NR}]{} \frac{4}{3\sqrt{3}}$$

$$g_A^{(0),\Sigma_c^{*++}} \xrightarrow[\text{NR}]{} 1, \quad g_A^{(3),\Sigma_c^{*++}} \xrightarrow[\text{NR}]{} \frac{2}{3}, \quad g_A^{(8),\Sigma_c^{*++}} \xrightarrow[\text{NR}]{} \frac{2}{3\sqrt{3}}$$



$$\Delta u = \Delta d = \Delta s = 0$$

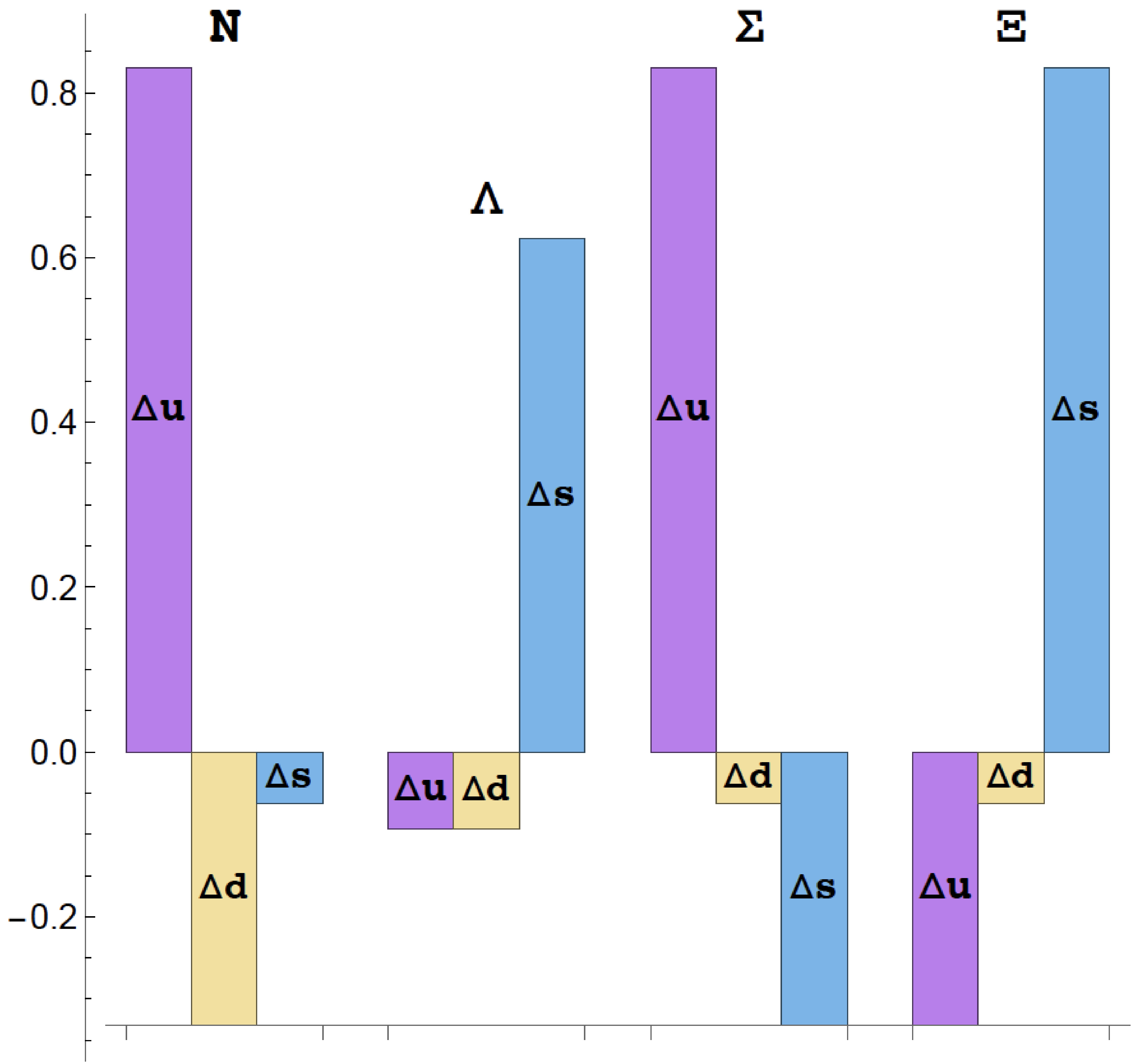
$$\Delta c = -\frac{1}{3}$$

- The quark spin content of the baryon octet (Flavor SU(3) symmetry)

$J_3 = 1/2$	$g_A^{(0)}$	$g_A^{(3)}$	$g_A^{(8)}$	Δu	Δd	Δs
p	0.437	1.163	0.360	0.831	−0.332	−0.062
n	0.437	−1.163	0.360	−0.332	0.831	−0.062
Λ	0.437	0.000	−0.827	−0.093	−0.093	0.623
Σ^+	0.437	0.893	0.827	0.831	−0.062	−0.332
Σ^0	0.437	0.000	0.827	0.384	0.384	−0.332
Σ^-	0.437	−0.893	0.827	−0.062	0.831	−0.332
Ξ^+	0.437	−0.270	−1.187	−0.332	−0.062	0.831
Ξ^0	0.437	0.270	−1.187	−0.062	−0.332	0.831

J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

- The quark spin content of the baryon octet



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The quark spin content of the baryon octet

Works	$g_A^{(0)}$	$g_A^{(3)}$	$g_A^{(8)}$	Δu	Δd	Δs
Present work	0.437	1.163	0.360	0.831	−0.332	−0.062
COMPASS [1] ($Q^2 = 3\,\mathrm{GeV}^2$)	[0.26, 0.36]	1.22(5)(10)	$\cdots$	[0.82, 0.85]	[−0.45, −0.42]	[−0.11, −0.08]
χQCD [2]	0.405(25)(37)	1.216(31)(7)	0.510(27)(39)	0.847(18)(32)	−0.407(16)(18)	−0.035(6)(7)
Green et al. [3]	0.494(11)(15)	1.206(7)(21)	0.565(11)(13)	0.863(7)(14)	−0.345(6)(9)	−0.0240(21)(11)
Cyprus group [4]	0.402(34)(10)	1.216(31)(7)	0.526(39)(10)	0.830(26)(4)	−0.386(16)(6)	−0.042(10)(2)
de Florian et al. [5] ($Q^2 = 10\,\mathrm{GeV}^2$)	$0.366^{+0.042}_{-0.062}$	$\cdots$	$\cdots$	$0.793^{+0.028}_{-0.034}$	$-0.416^{+0.035}_{-0.025}$	$-0.012^{+0.056}_{-0.062}$

[1] COMPASS Coll., PLB 753 (2016) 18.

[2] XQCD, PRD 98 (2018) 074505.

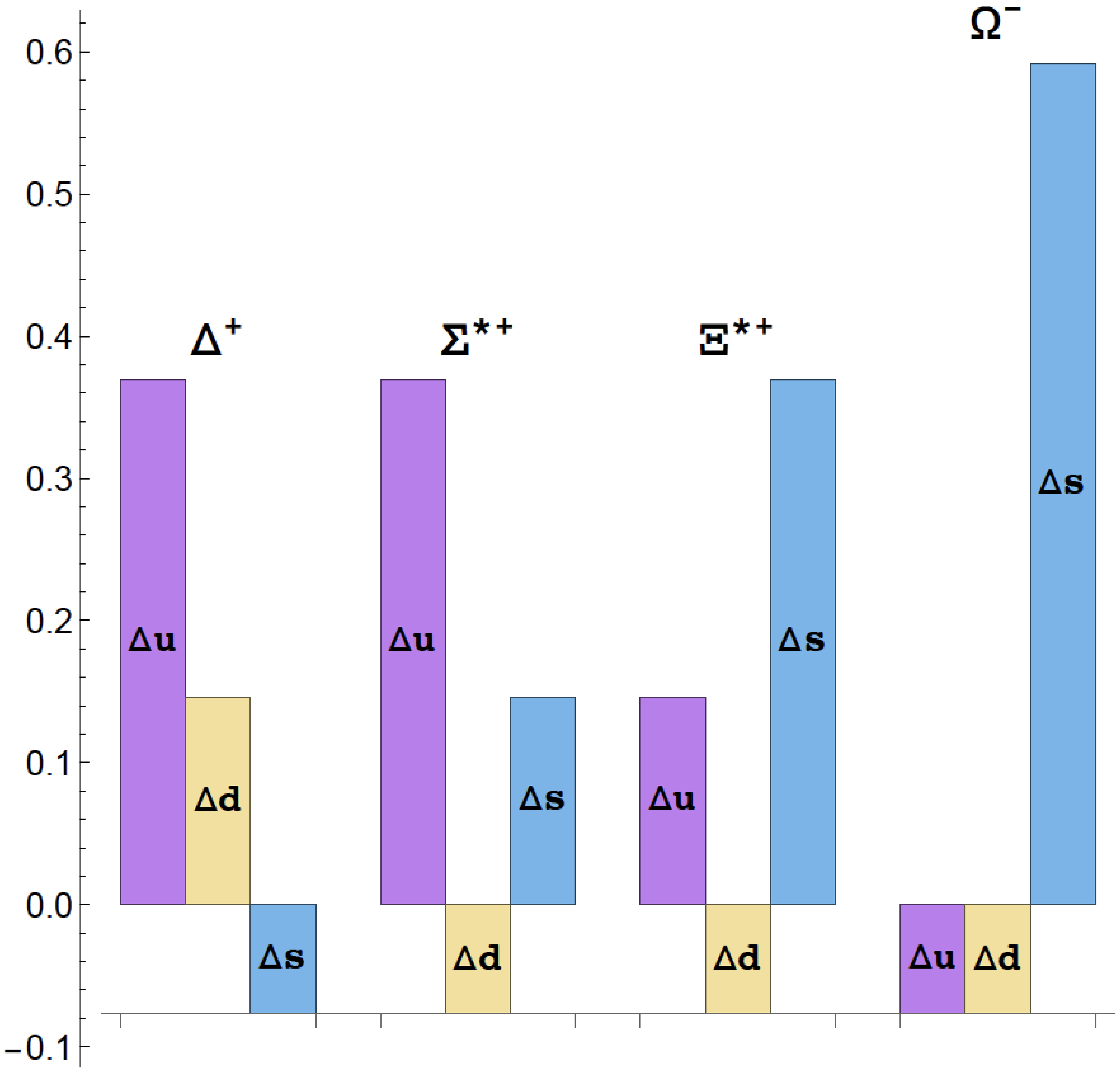
[3] Green et al., PRD 95 (2017) 114502.

[4] Alexandrou et al., PRL 119 (2017) 142002.

[5] de Florian et al., PRD 80 (2009) 034030.

J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

The quark spin content of the baryon decuplet



J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

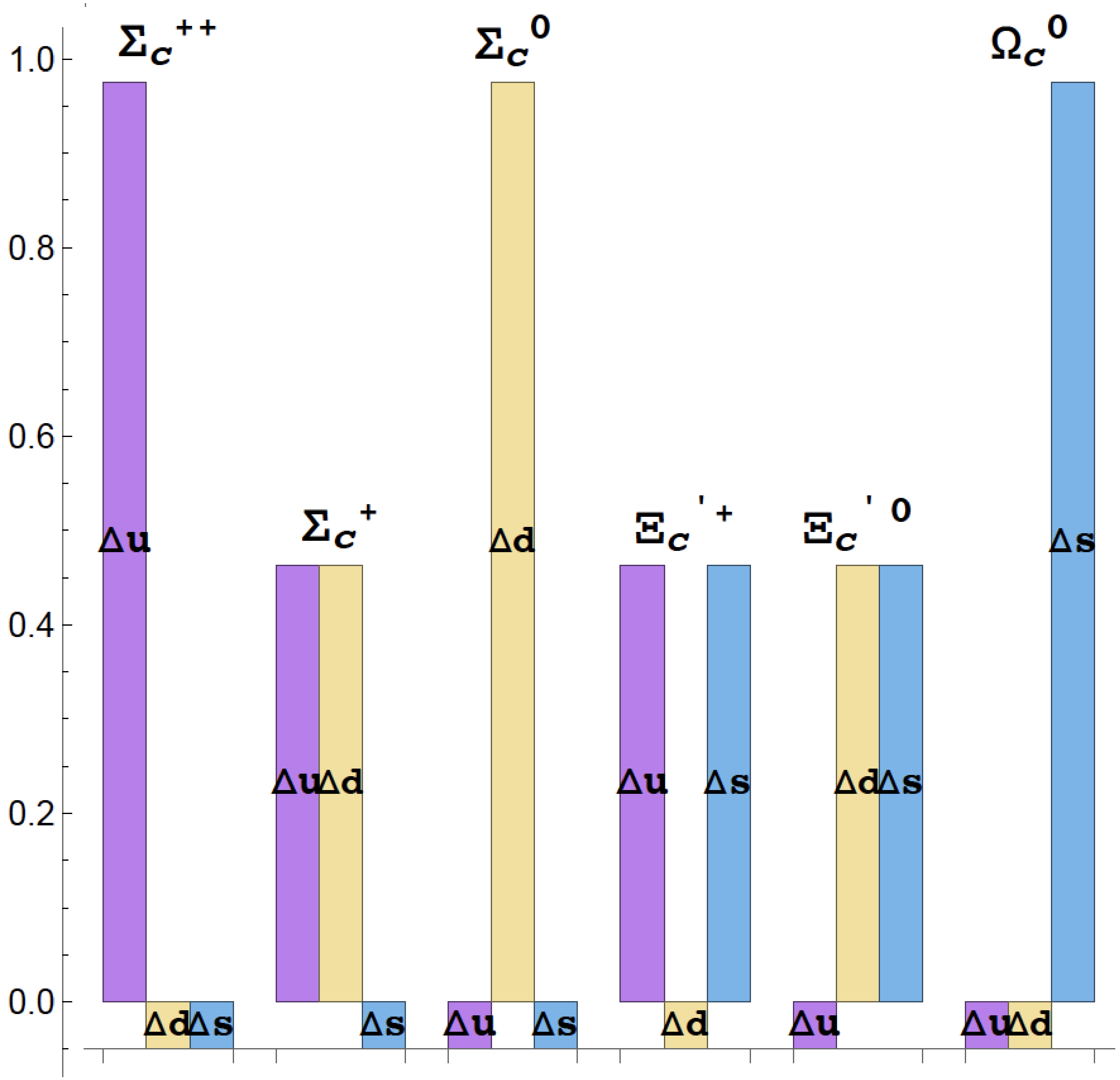
- The quark spin content of the baryon sextet with J=1/2

	$g_A^{(0)}$	$g_A^{(3)}$	$g_A^{(8)}$	Δu	Δd	Δs	Δc
Σ_c^{++}	0.543	1.026	0.592	0.976	−0.050	−0.050	−0.333
Σ_c^{+}	0.543	0.000	0.592	0.463	0.463	−0.050	−0.333
Σ_c^{0}	0.543	−1.026	0.592	−0.050	0.976	−0.050	−0.333
Σ_c [38]	0.4094 ± 0.0199	–	–	0.7055 ± 0.0191	–	–	-0.2970 ± 0.0113
$\Xi_c'^{+}$	0.543	0.513	−0.296	0.463	−0.050	0.463	−0.333
$\Xi_c'^{0}$	0.543	−0.513	−0.296	−0.050	0.463	0.463	−0.333
Ξ_c' [38]	0.4872 ± 0.0127	–	–	0.3433 ± 0.0085	–	0.4539 ± 0.0055	-0.3133 ± 0.0069
Ω_c^{0}	0.543	0.00	−1.185	−0.050	−0.050	0.976	−0.333
Ω_c^{0} [38]	0.5428 ± 0.0118	–	–	–	–	0.8554 ± 0.0117	-0.3125 ± 0.0054

[38] Alexandrou et al., PRL 119 (2017) 142002

J. M. Suh, J.Y. Kim, G.S. Yang, HChK,
PRD 106, 054032 (2022)

- The quark spin content of the baryon sextet with J=1/2



J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

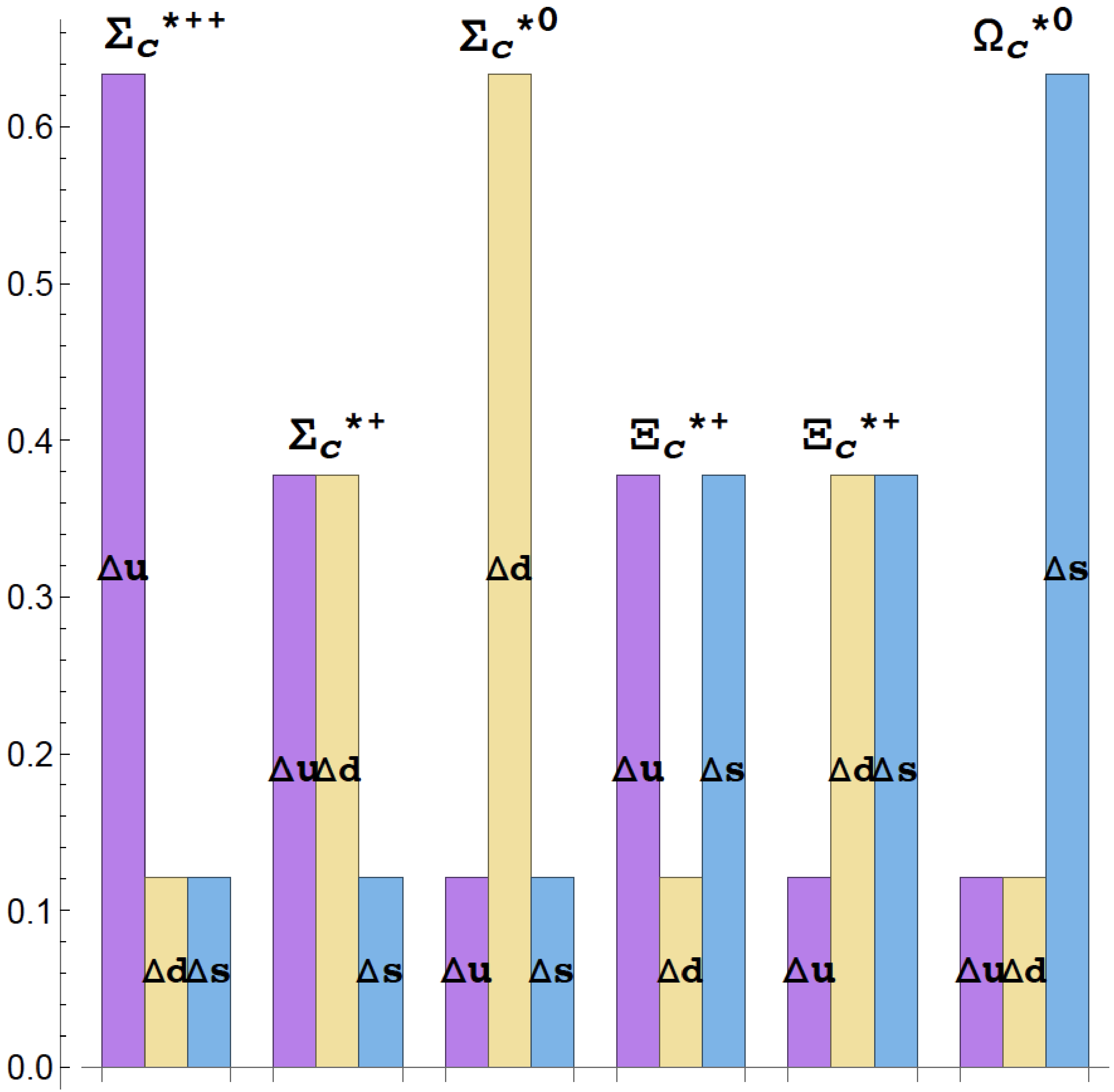
The quark spin content of the baryon sextet with J=3/2

	$g_A^{(0)}$	$g_A^{(3)}$	$g_A^{(8)}$	Δu	Δd	Δs	Δc
Σ_c^{*++}	0.543	0.513	0.296	0.634	0.121	0.121	-0.333
Σ_c^{*+}	0.543	0.000	0.296	0.378	0.378	0.121	-0.333
Σ_c^{*0}	0.543	-0.513	0.296	0.121	0.634	0.121	-0.333
Σ_c^{*} [38]	$(2.0004 \pm 0.0346)/3$	–	–	$(1.0899 \pm 0.0308)/3$	–	–	$(0.9043 \pm 0.0090)/3$
Ξ_c^{*+}	0.543	0.257	-0.148	0.378	0.121	0.378	-0.333
Ξ_c^{*0}	0.543	-0.257	-0.148	0.121	0.378	0.378	-0.333
Ξ_c^{*} [38]	$(2.1192 \pm 0.0254)/3$	–	–	$(0.5466 \pm 0.0150)/3$	–	$(0.6587 \pm 0.0104)/3$	$(0.9103 \pm 0.0075)/3$
Ω_c^{*0}	0.543	0.000	-0.592	0.121	0.121	0.634	-0.333
Ω_c^{*0} [38]	$(2.1961 \pm 0.0261)/3$	–	–	–	–	$(1.2904 \pm 0.0204)/3$	$(0.9026 \pm 0.0090)/3$

[38] Alexandrou et al., PRL 119 (2017) 142002

J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

The quark spin content of the baryon sextet with J=3/2



All positively polarized!

J. M. Suh, J.Y. Kim, G.S. Yang, HChK, PRD 106, 054032 (2022)

Outlook: 1/m_Q corrections and Gluons

Nonlocal Chiral theory of baryons from the instanton vacuum

Y.W. Choi &HChK, in preparation

$$\mathcal{Z} = \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \mathcal{D}\sigma \mathcal{D}\pi^a \, \exp(-\mathcal{S}[\psi^\dagger, \psi, \sigma, \pi^a])$$

$$= \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \mathcal{D}\sigma \mathcal{D}\pi^a \, \exp\Big[- \int d^4x \, \Big\{ \psi_f^\dagger(x)(-i\partial\!\!\!/)\psi_f(x) + \big(\sigma^2(x) + \pi^2(x)\big)$$

$$- i\psi^\dagger(x) \overleftarrow{F} M_0\big(\sigma(x) + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\big) \overrightarrow{F} \psi(x) \Big\} \Big]$$

$\vec{F}$ Momentum-dependent quark ff

This new theory open a way of dealing with gluon fields for the baryons effectively.

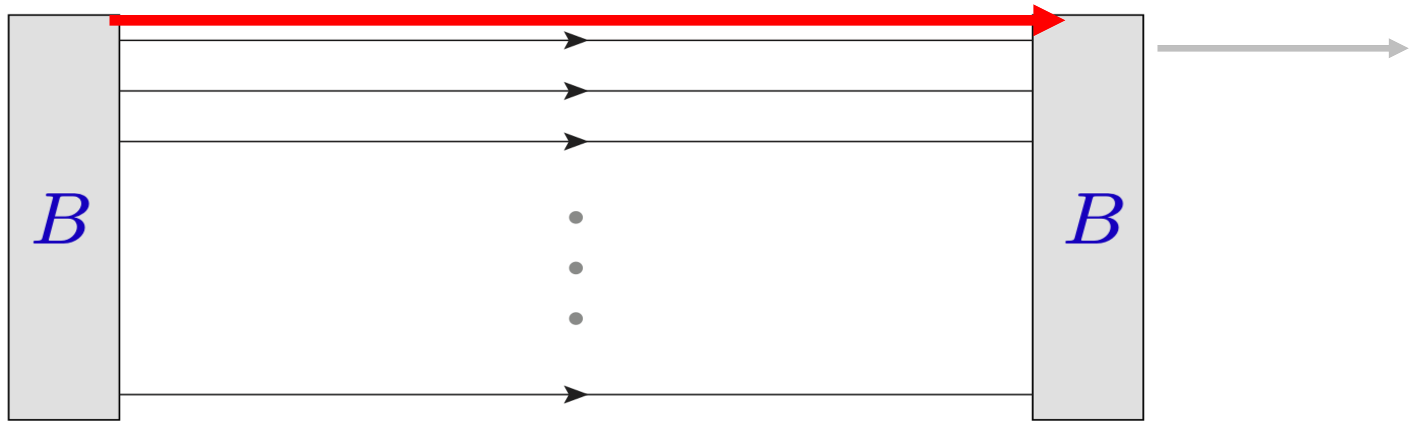
Gluon fields ➡ Instantons ➡ Effective quark operator ➡ Pion mean fields

D. Diakonov, M. V. Polyakov, Ch. Weiss, NPB 480 (1996) 341

Heavy-quark propagator

Heavy-quark effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = -h^\dagger i(v\cdot D)h - \frac{i}{2m_Q}h^\dagger\left(D_\perp^2 - \frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}\right)h$$



Heavy-quark propagator

$$G_h(y,x) = \left\langle y\left|\frac{1}{\partial_4}\right|x\right\rangle = \Theta(y_4-x_4)\delta^{(3)}(\boldsymbol{y}-\boldsymbol{x})$$

$$\Rightarrow\quad G_h(y,x) = \left\langle y\left|\frac{1}{i\not{D}}\right|x\right\rangle + \frac{1}{m_Q}\frac{1}{f(E,B)}$$

⇒ This can be expressed in terms of the pion mean fields.

⇒ A heavy quark can finally interact with the light quarks via the pion mean fields.

⇒ $1/m_Q$ corrections to the structure of the singly heavy baryons

Summary

- We introduced a pion mean-field approach, which describe both light and singly heavy baryons on an equal footing.
- Electromagnetic properties of singly heavy baryons were well explained.
- The axial properties of singly heavy baryons (low-lying states) were discussed.
- Notable finding: All the light quarks inside a singly heavy baryon with spin 3/2 are positively polarized.
- Outlook: 1/m_Q corrections from the gluon fields

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

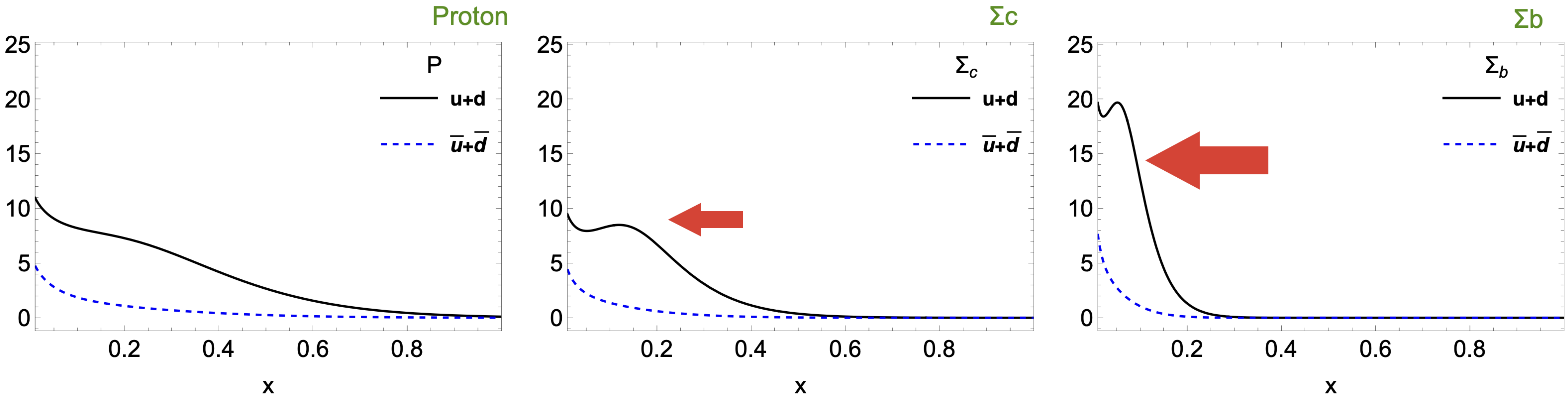
by Shakespeare

Thanks to my Collaborators!
J.-Y. Kim, H.-D. Son, J.-M. Suh, Gh.-S. Yang

Thank you very much for the attention!

Parton distribution functions in a Singly heavy baryon

$u(x) + d(x)$

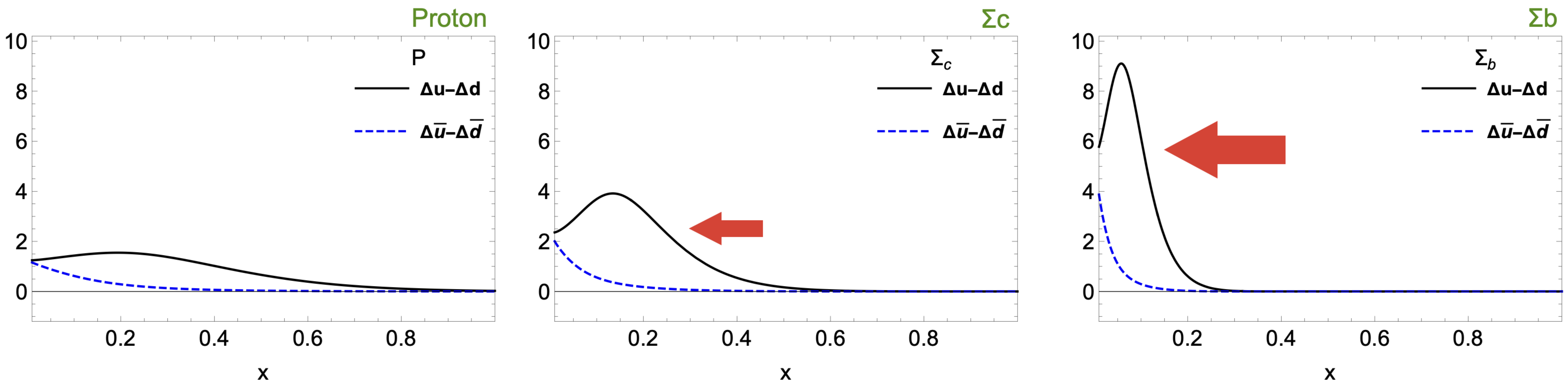


Momentum sum-rule

$$\int_0^1 dx\,x\,[\underbrace{u(x) + d(x) + \bar{u}(x) + \bar{d}(x)}_{M_{sol}/M_h} + \underbrace{Q(x)}_{M_Q/M_h}] = 1$$

H.D. Son and HChK, PTEP 2023, 033D02

$\Delta u(x) - \Delta d(x)$



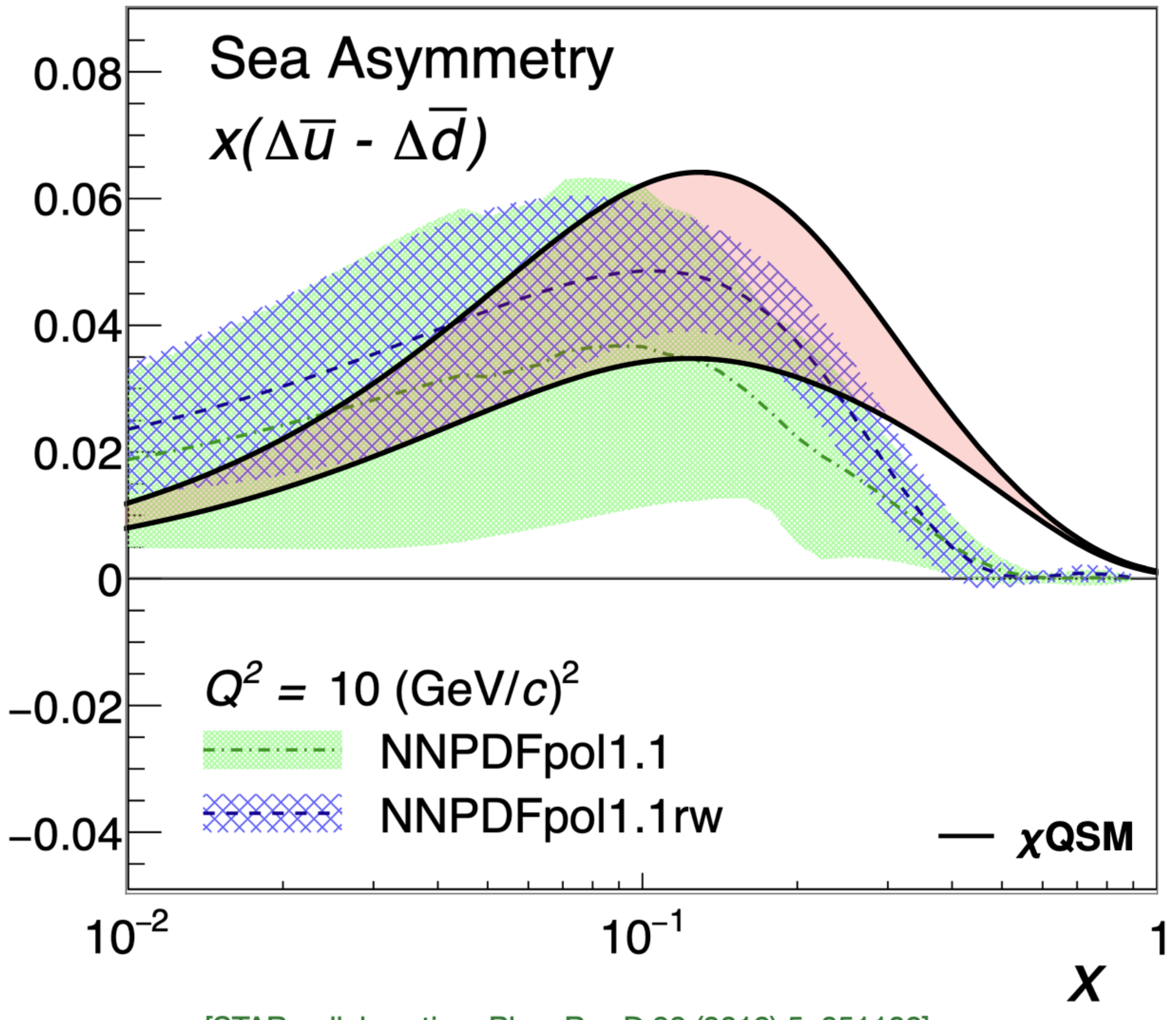
Similar behavior as the isoscalar unpolarized distribution, squeezed into small x

Spin sum-rule $\int_0^1 dx[\Delta u(x) - \Delta d(x) + \Delta\bar{u}(x) - \Delta\bar{d}(x)]$ is **identical for Σ_c and Σ_b**

Numerically, $\int_0^1 dx[\Delta u(x) - \Delta d(x) + \Delta\bar{u}(x) - \Delta\bar{d}(x)]$ = **1.4** (T_3=+1). (Δc=-1/3, NR)

H.D. Son and HChK,
PTEP 2023, 033D02

Polarized antiquark flavor asymmetry: model case



[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

Band: Model systematic uncertainty

fixed ρ~1/(600MeV), in the chiral limit

M [MeV]	330	420
M_N [MeV]	1161	1077
ρ/R	0.32	0.37
F_π [MeV]	77	90

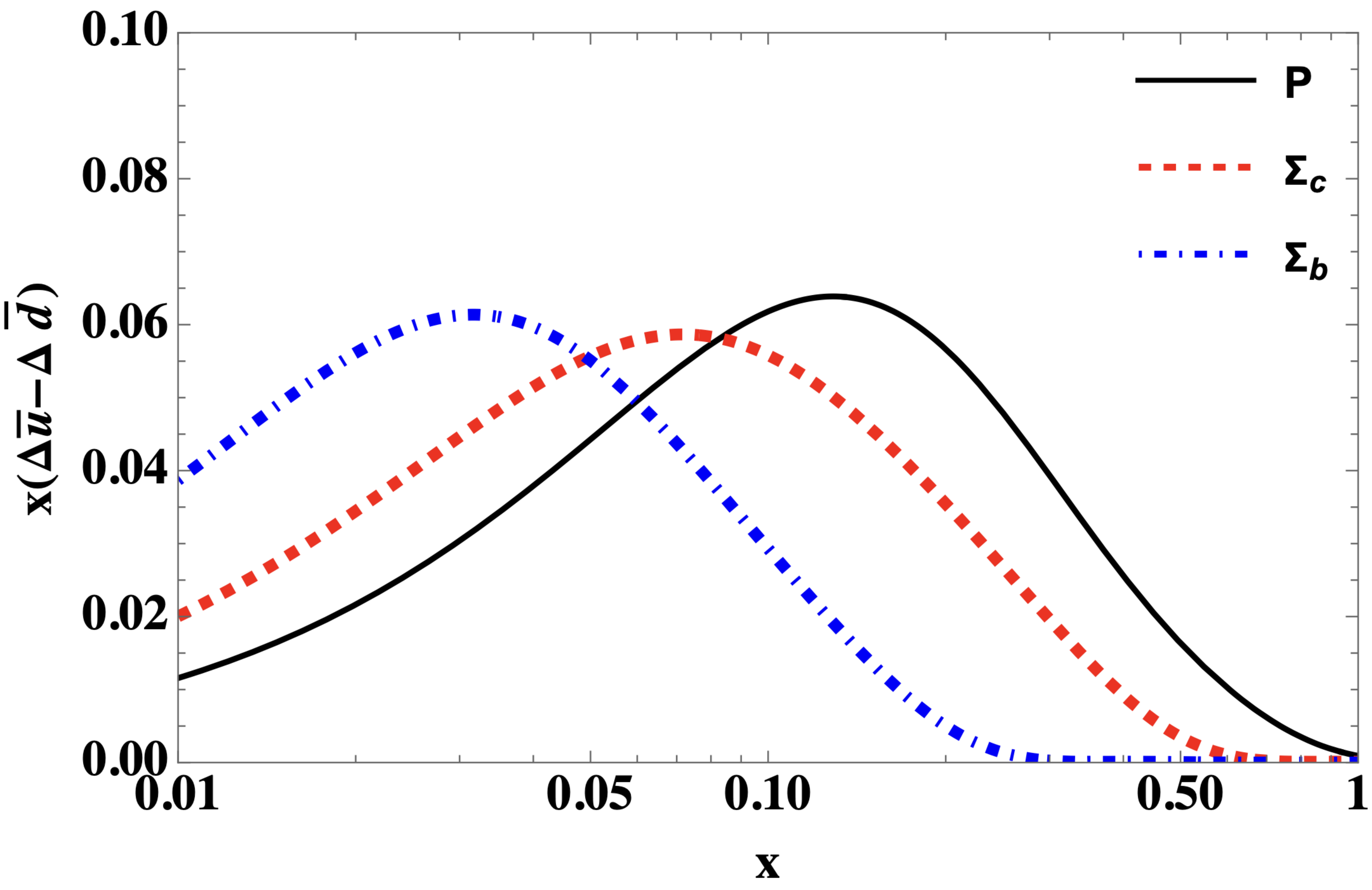
Continuum contribution (Polarized vacuum) is crucial

Softness: quark virtuality (momentum dep. mass)

1/Nc correction can enhance the PDF ~30%

Scale evolution

Antiquark flavor asymmetry: heavy baryon



H.D. Son and HChK, PTEP 2023, 033D02