Quark confinement for exotic hadrons in the quark model

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XVI-th Quark Confinement and the Hadron Spectrum

Cairns, Australia

Ground-State Hadrons

- **Quark model prediction: GS=(no orbital excitation, $0\hbar\omega$)**

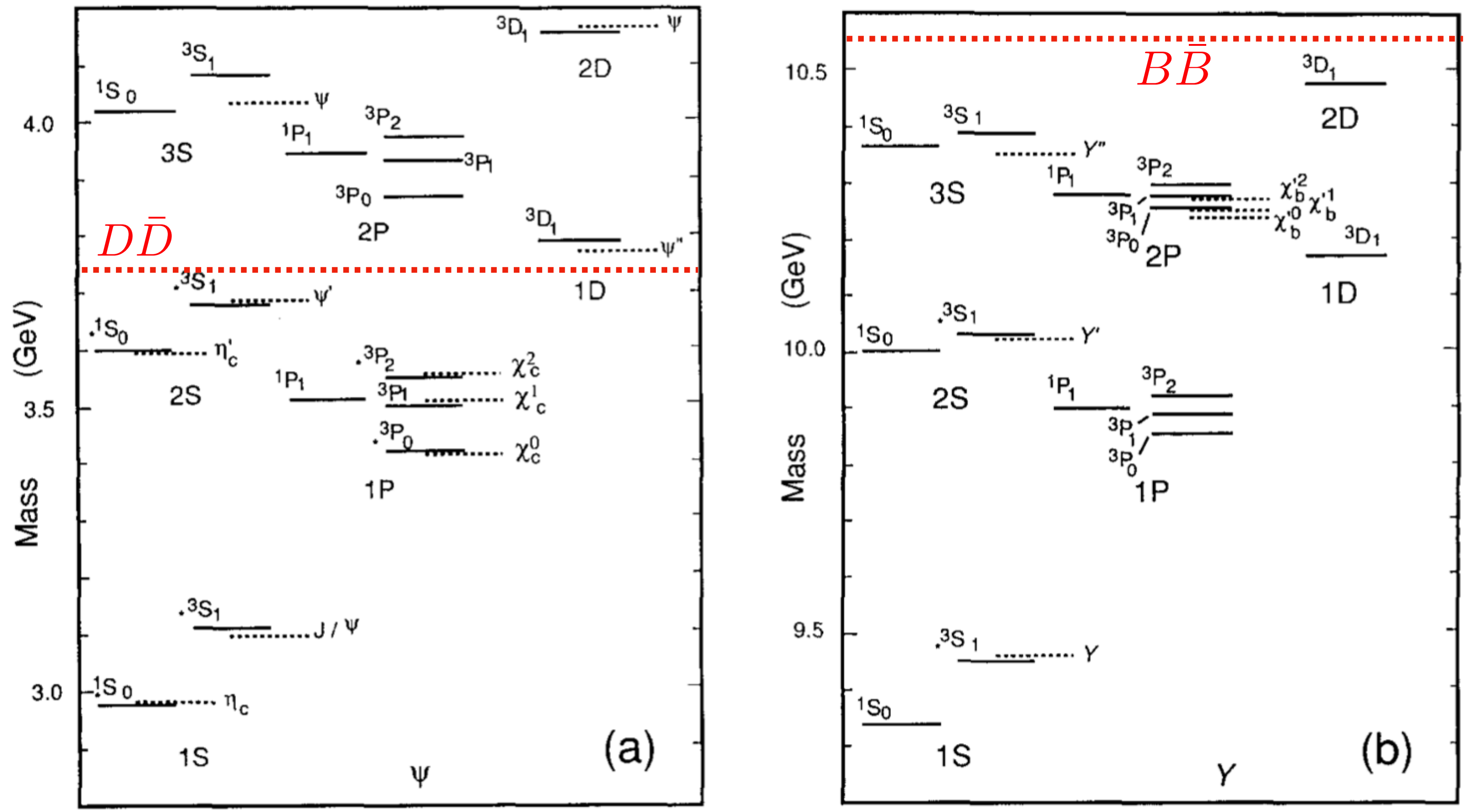
30 Mesons (0^-, 1^-)	q=(u,d,s)	c	b
$\bar{q}$=($\bar{u}$,$\bar{d}$,$\bar{s}$)	9+9	3+3	3+3
$\bar{c}$	3+3	1+1	1+1 $\bar{B}_c^*$ *not yet observed*
$\bar{b}$	3+3	1+1 B_c^*	1+1

75 Baryons flavor spin	qqq	Qqq	QQq	QQQ
S=3/2	10	12 Ω_b^*	9 Ξ_{cc}	4
S=1/2	8	18	12	2

- **The masses and quantum numbers of all the observed 29 mesons and 48 baryons are consistent with the quark model prediction.**
- **The missing GS meson (B_c^*) and baryons (Ω_b^*, . .) should be found as predicted.**

Excited states

- **The quark model works well for excited heavy mesons *below the two-hadron threshold.***

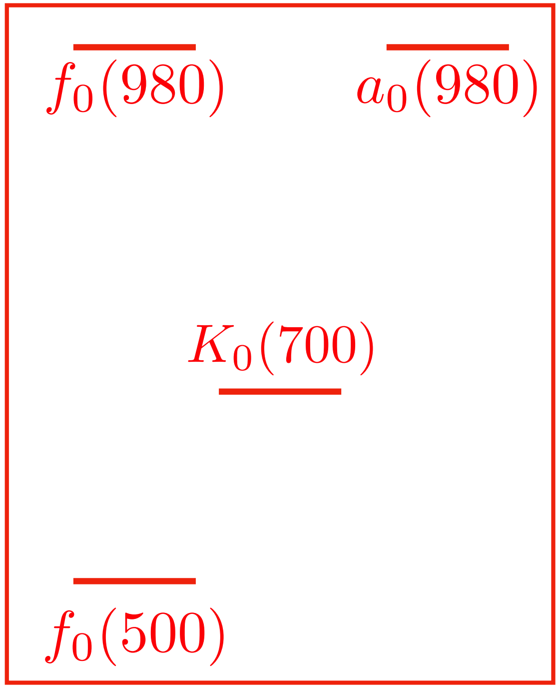


S.N. Mukherjee et al., Phys. Rep. 231 (1993)

Excited states (scalar nonet)

The hierarchy problem of the lowest scalar nonet

$$m(f_0) < m(K_0) < m(a_0) \sim m(f_0')$$



inconsistent with $q\bar{q}$ nonet

solved by Jaffe's tetra-quark picture,
R.L. Jaffe, PRD15, 267 (1977)

$f_0 = ud\bar{u}\bar{d}$ **no *s* (ideal mixing)**

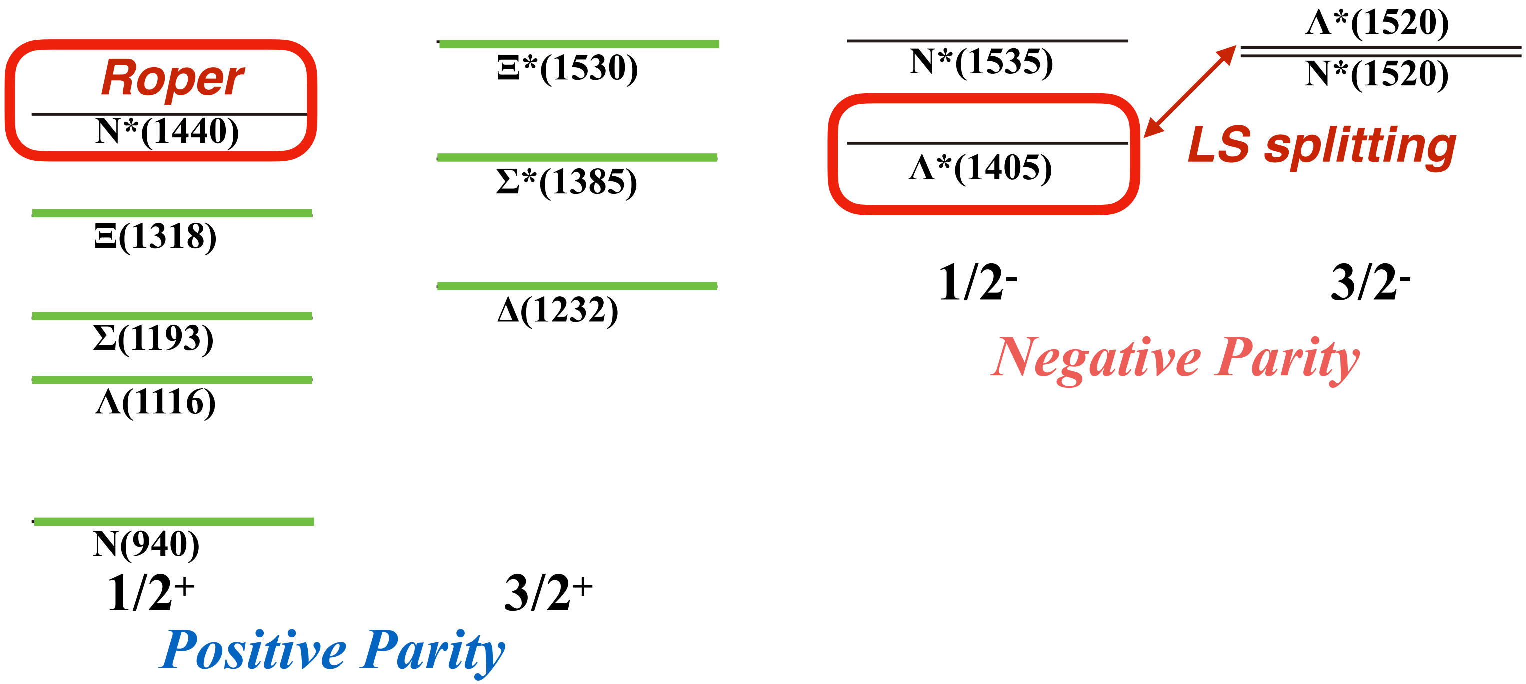
$K_0^0 = ud\bar{u}\bar{s}$ $K_0^+ = ud\bar{d}\bar{s}$ **1 *s***

$a_0^0 = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$ $a_0^+ = us\bar{d}\bar{s}$ $a_0^- = ds\bar{u}\bar{s}$ **2 *s***

$f_0' = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} + ds\bar{d}\bar{s})$ **2 *s***

Light Baryons: Excited states

- The lowest negative-parity baryon is Λ(1405).
  The LS splitting is abnormally large.
- The 1st positive-parity excited state N(1440) lower than N(1535)



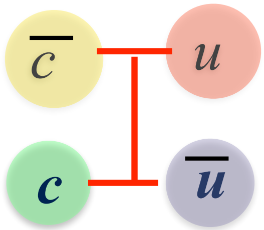
- Molecular structures, $\Lambda(1405) \sim N\bar{K}$, $N^*(1440) \sim N\sigma$, may explain the non-QM behaviors of the excited states.

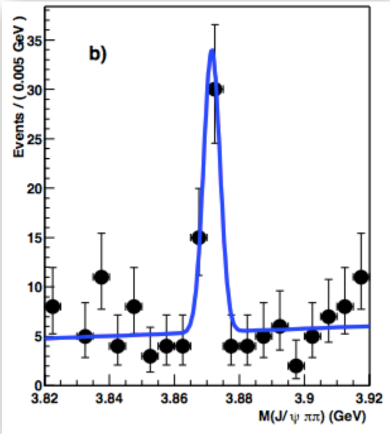
Exotic Multiquark Hadrons

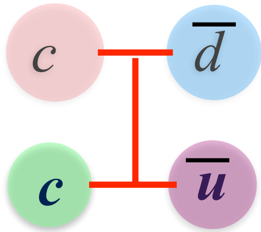
- Hadrons that do not fit the simple quark model picture

X(3872) *Belle*

PRL 91 (2003)

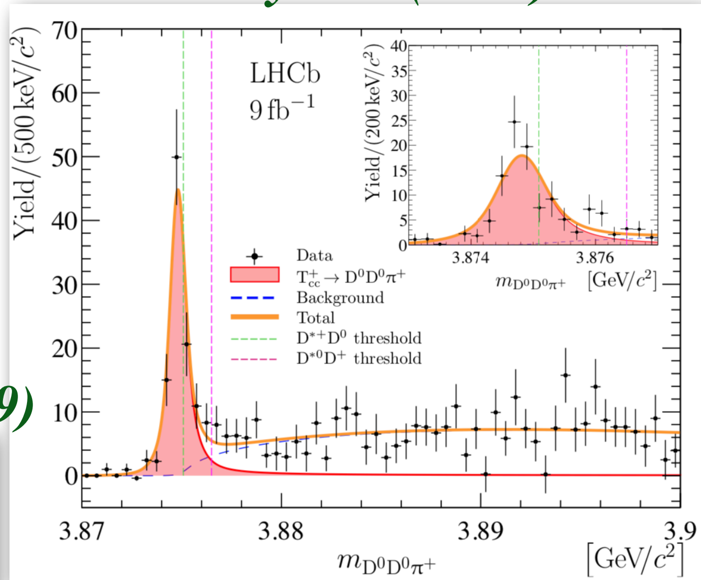






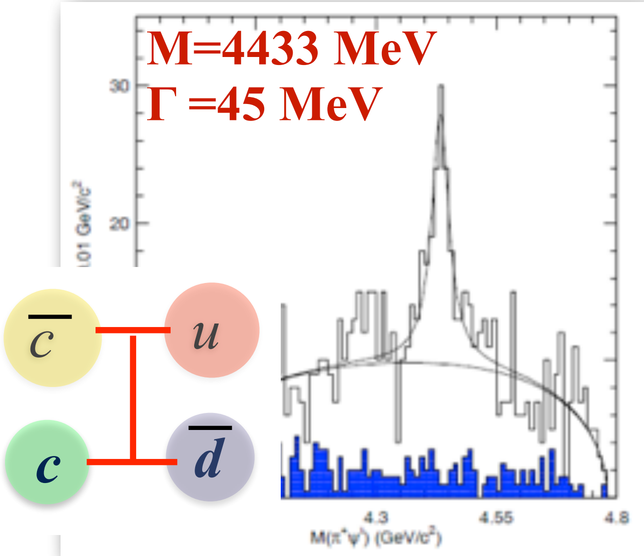
T_{cc} *LHCb*

Nature Phys. 18 (2022)751



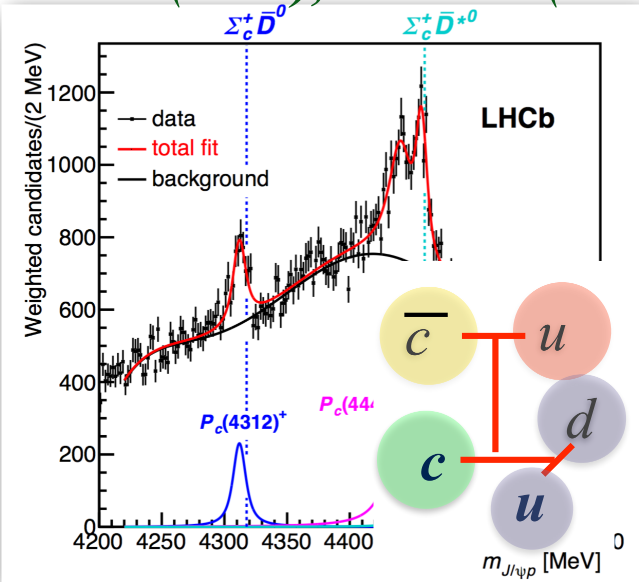
Z_c^+(4430) *Belle*

PRL 100 (2008) 142001



P_c (4312) (4440) (4457) *LHCb*

PRL 115 (2015), PRL 122 (2019)

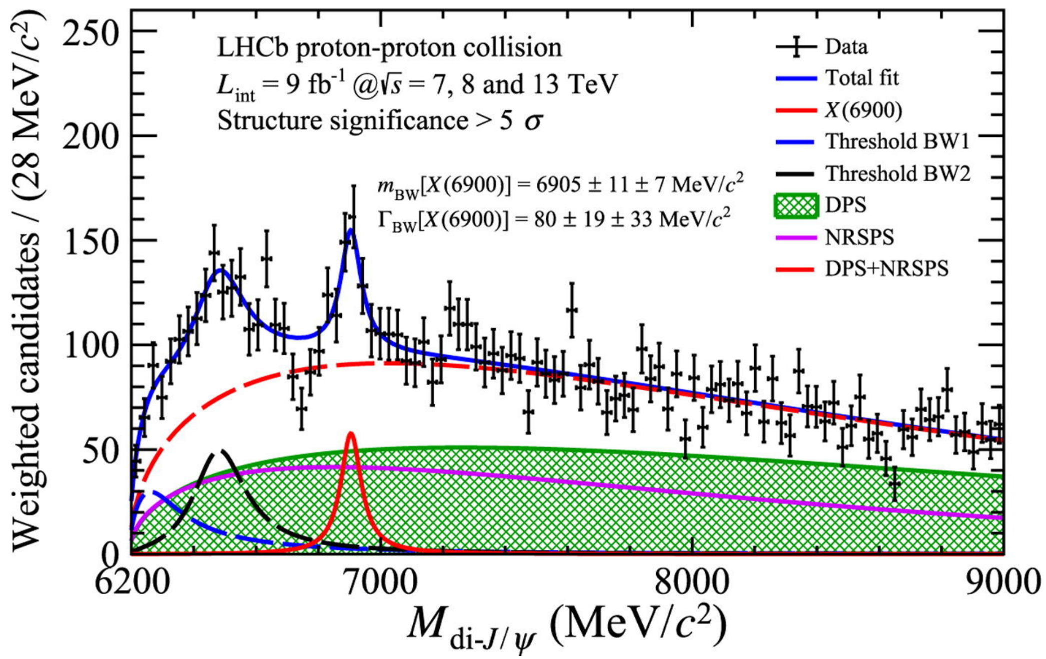


Exotic Multiquark Hadrons

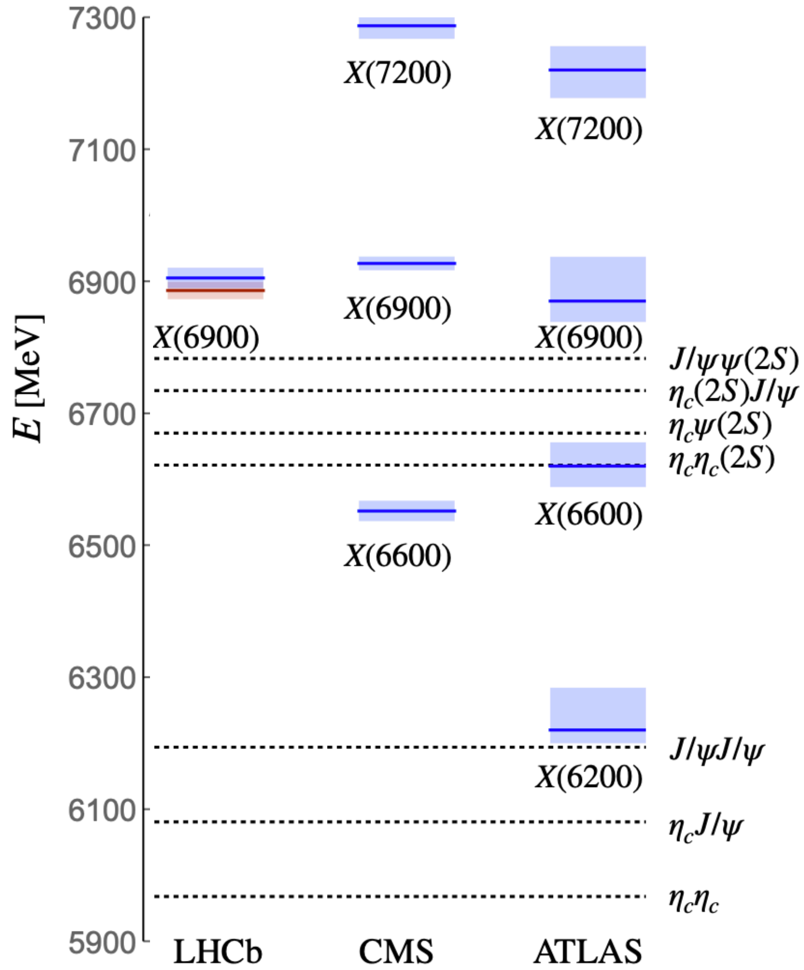
- ***Fully charmed tetra-quark resonances*** $X(cc\bar{c}\bar{c}) \rightarrow J/\psi + J/\psi$

observed at LHC (quantum numbers unknown)

$$m_{BW}[X(6900)] = 6905 \pm 11 \pm 7 \text{ MeV}/c^2$$
$$\Gamma_{BW}[X(6900)] = 80 \pm 19 \pm 33 \text{ MeV}/c^2$$



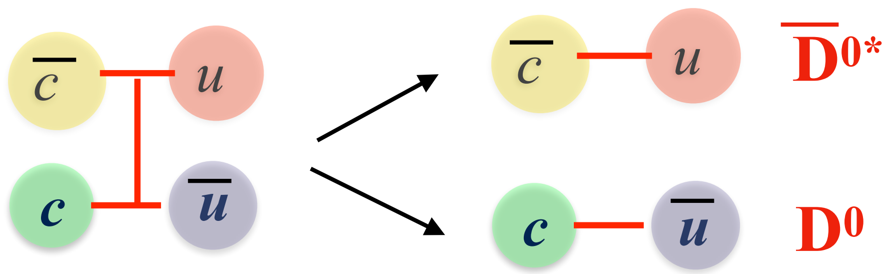
Two J/ψ spectrum



Current status summary

- **Quark model works excellently for the ground states and excited states below the two-hadron threshold.**
- **Multi-quark and/or molecular components play key roles for the excited/exotic states. The other ingredients (constituent gluon, diquark, . .) may be significant.**
- **Couplings to (S-wave) thresholds for two- (or more-) hadrons are critically important in understanding hadron spectra and structures.**
- **We focus on the *Confinement Mechanism of Multi-Quark System* and the couplings to two-hadron scattering states.**

X(3872)

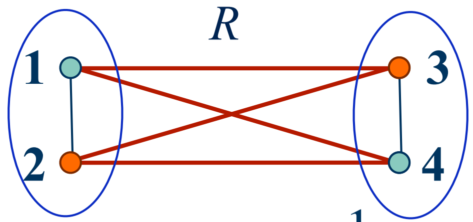


Confinement of Quarks in Multi-Quark States

Quark confinement for tetraquarks

- **Conventional quark model: sum of two-body confinement**

$$V = \sum_{i<j} (\lambda_i \cdot \lambda_j)(-a r_{ij}) \qquad (\lambda_i \cdot \lambda_j) = \sum_{\alpha} \lambda_i^\alpha \lambda_j^\alpha$$



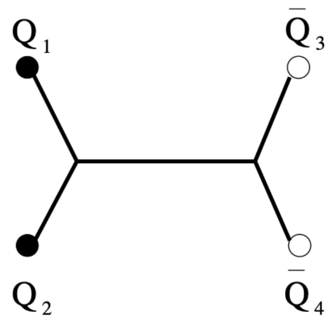
- satisfy color saturation

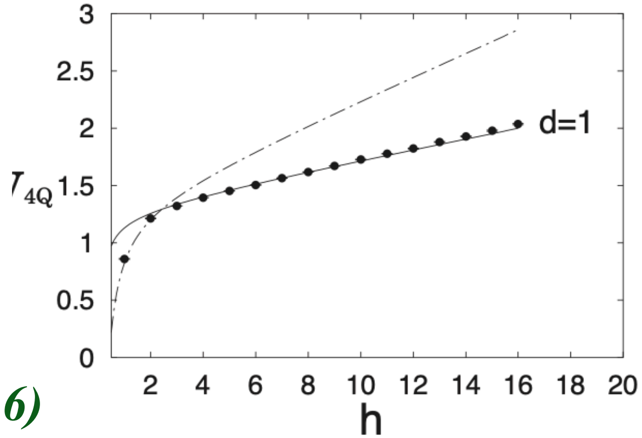
- induce long-range color van der Waals force

$$\text{cvdW} \propto -\frac{1}{R^3}$$

T. Appelquist, W. Fischler, Phys. Lett. B77, 405 (1978)
R.S. Willey, Phys. Rev. D18, 270 (1978)
S. Matsuyama, H. Miyazawa, Prog. Theor. Phys. 61, 942 (1979)

- **Confinement on the lattice**
  connect quarks by strings with the minimal lengths
  - need string rearrangements to allow color singlet mesons split away





Lattice QCD Wilson loop for tetra quarks
F. Okiharu, et al., J. Mod. Phys. 7 ,774–789 (2016)

Quark confinement for tetraquarks

- **Reconsider the quark confinement potential for the quark model from the QCD viewpoints.**
  Sum of two-body confinements v.s. String-like confinement
- **We propose to extend the color configuration space of the conventional quark model with a hidden-color $|hc\rangle\rangle$ state.**
  Its mixing induces extra short-range attraction among the multi-quark systems.
- **Test the new model by applying it to the fully charmed tetraquarks.**

PHYSICAL REVIEW D **108,** L071501 (2023)

Letter

Quark confinement for multiquark systems: Application to fully charmed tetraquarks

Guang-Juan Wang,[1,2,*] Makoto Oka,[3,2,†] and Daisuke Jido[4,‡]

"Check" the confinement potentials

- **Fully heavy tetraquarks, such as $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$, are ideal objects to check the validity of the quark model and confinement mechanism of multiquarks.**
- **Possible J^{PC} quantum numbers for the S-wave $cc\bar{c}\bar{c}$ or $bb\bar{b}\bar{b}$ states are $0^{++}, 1^{+-}, 2^{++}$.**
- **We apply the Gaussian expansion method (GEM) to the full 4-quark calculation. The AL1 potential model is emplyed.**
- **We use the complex scaling method to distinguish resonance states from scattering states in the finite size basis calculation, *i.e.*, rotating the variables into complex plane as $\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}, \boldsymbol{p} \to \boldsymbol{p}e^{-i\theta}$.**

Qi Meng, Guang-Juan Wang, MO, ArXiv:2404.01238
G.J. Wang, Qi Meng, MO, Phys. Rev. D106, 096005 (2022)
Qi Meng, et al., Phys. Lett. B 846, 138221 (2023)
Qi Meng et al., Phys. Lett. B814, 136095 (2021)

Quark Model Hamiltonian

- **Non-relativistic (NR) quarks with constituent masses**
  Linear + Color Coulomb + Color Magnetic interactions
- **"AL1" potential by Silvestre-Brac, Few-Body Syst. 20, 1 (1996)**

$$H = \sum_i \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) - K_G + \sum_{i<j} \frac{(\lambda_i \cdot \lambda_j)}{4} V_{ij}$$

$$V_{ij} = -\frac{3}{4} \left(\sigma r_{ij} - \frac{\alpha}{r_{ij}} + \frac{2\pi\alpha'}{3m_i m_j} f(r_{ij}, r_{0ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) - \Lambda \right)$$

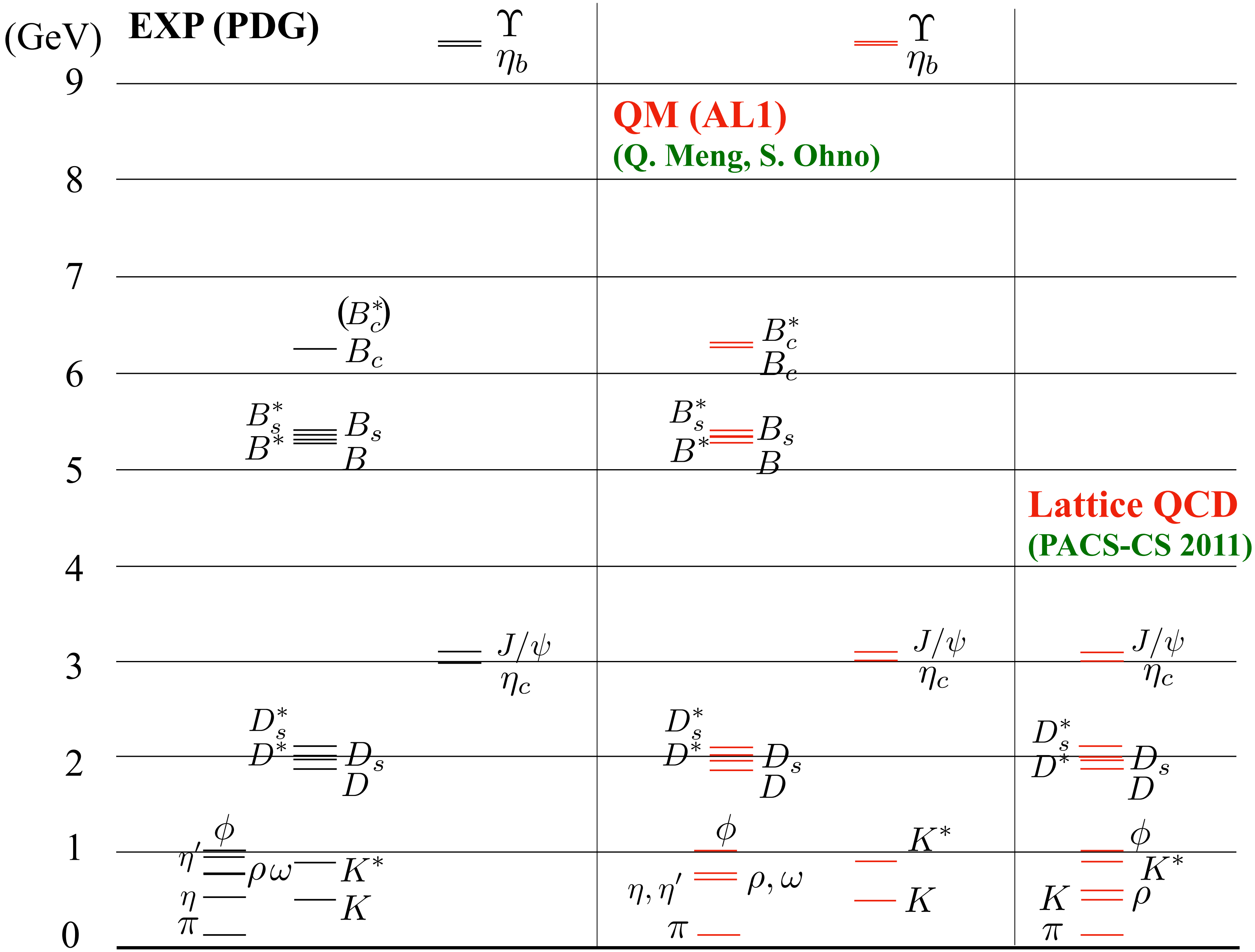
$$f(r, r_0) = \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \qquad r_{0ij} = A \left(\frac{2m_i m_j}{m_i + m_j} \right)^{-B}$$

$$m_{u/d} = 0.315\text{GeV}, m_s = 0.577\text{GeV}, m_c = 1.836\text{GeV}, m_b = 5.227\text{GeV}$$

$$\sigma = 0.1653\text{GeV}^2, \alpha = 0.5069, \alpha' = 1.8609$$

$$B = 0.2204\text{GeV}, A = 1.6553\text{GeV}^{B-1}, \Lambda = 0.8321\text{GeV}$$

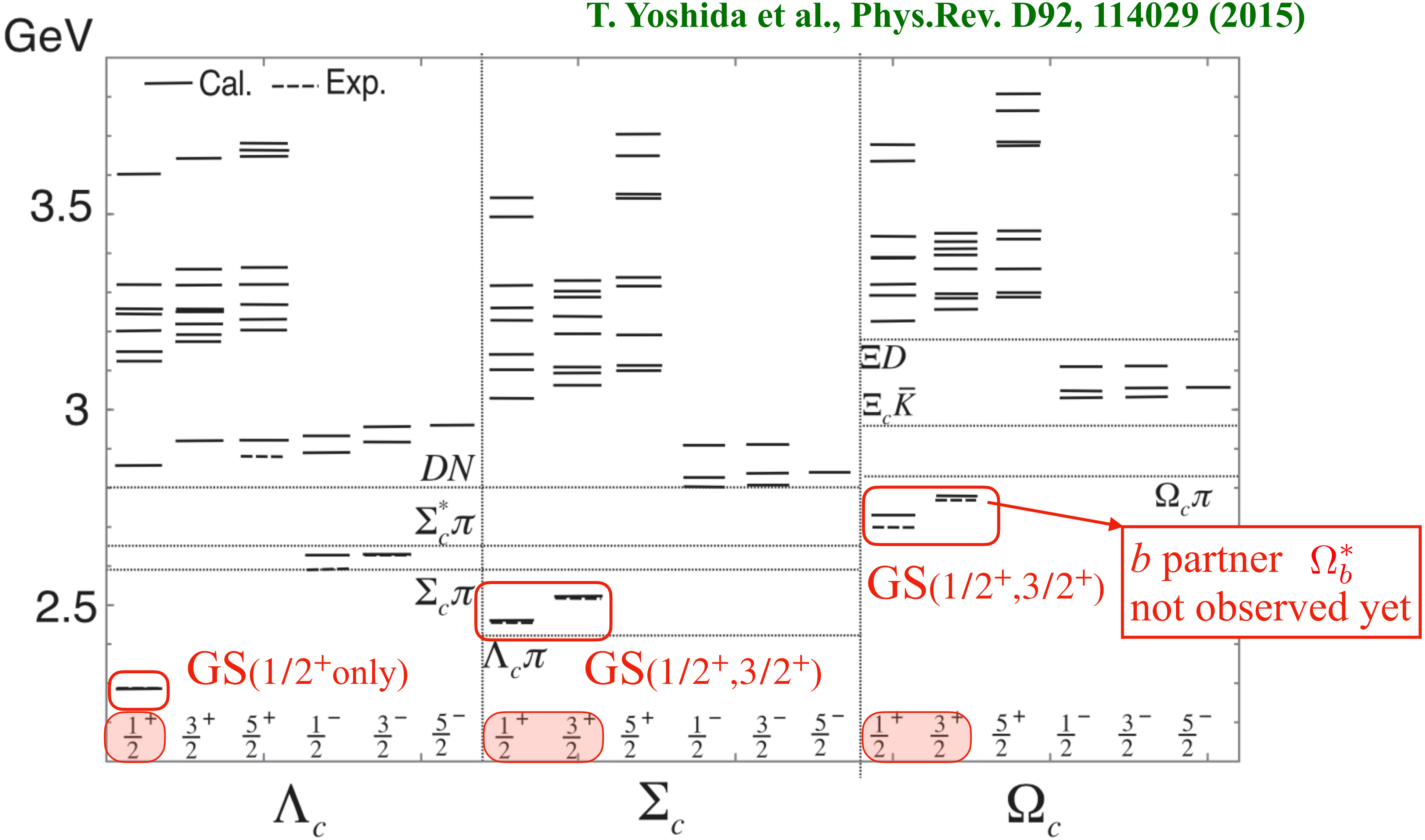
GS Meson Spectrum



Heavy Baryon Spectrum

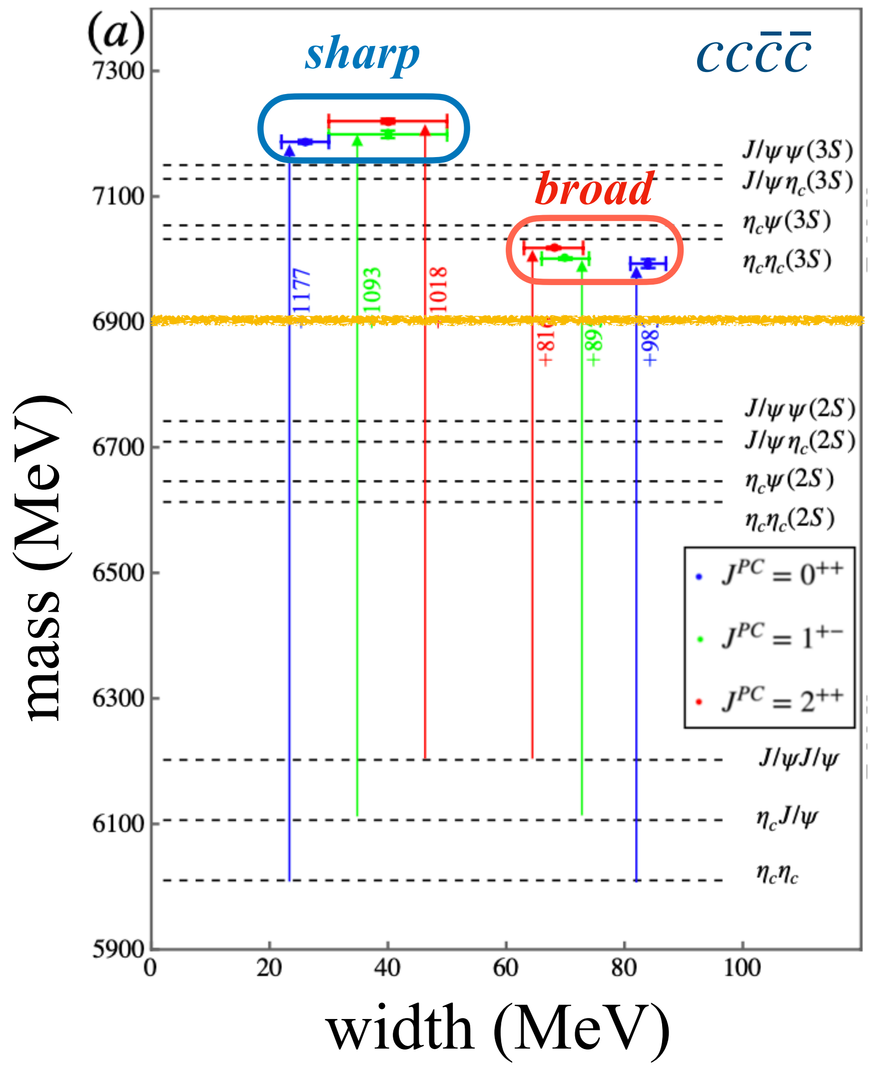
Quark model

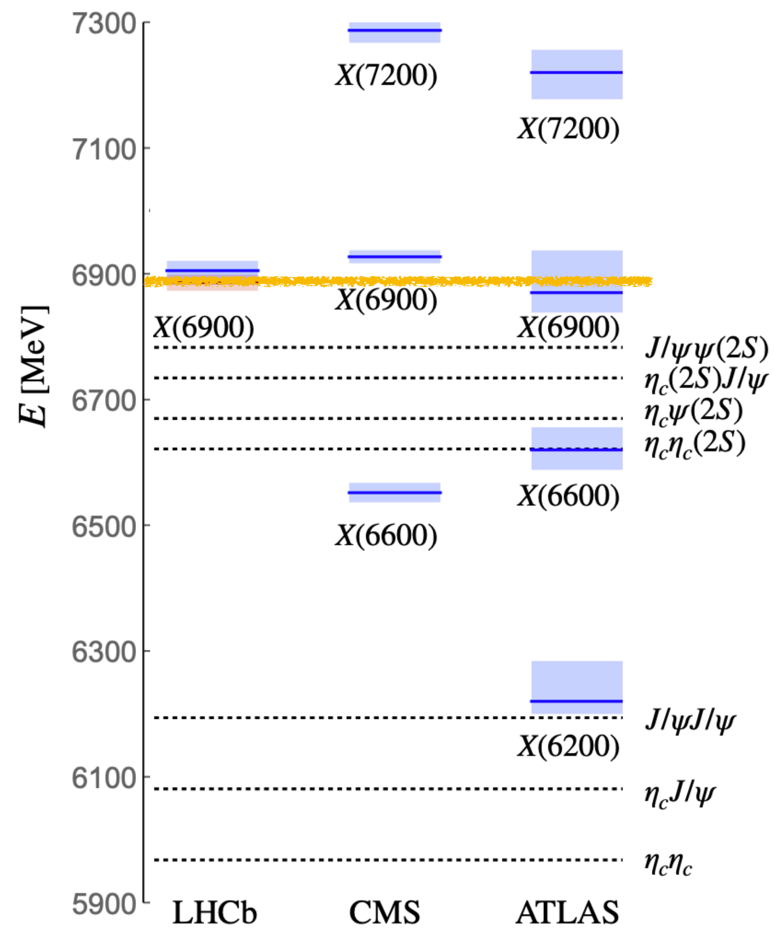
T. Yoshida et al., Phys.Rev. D92, 114029 (2015)



$cc\bar{c}\bar{c}$ spectrum for conventional confinement potential

Qi Meng, Guang-Juan Wang, MO, ArXiv:2404.01238

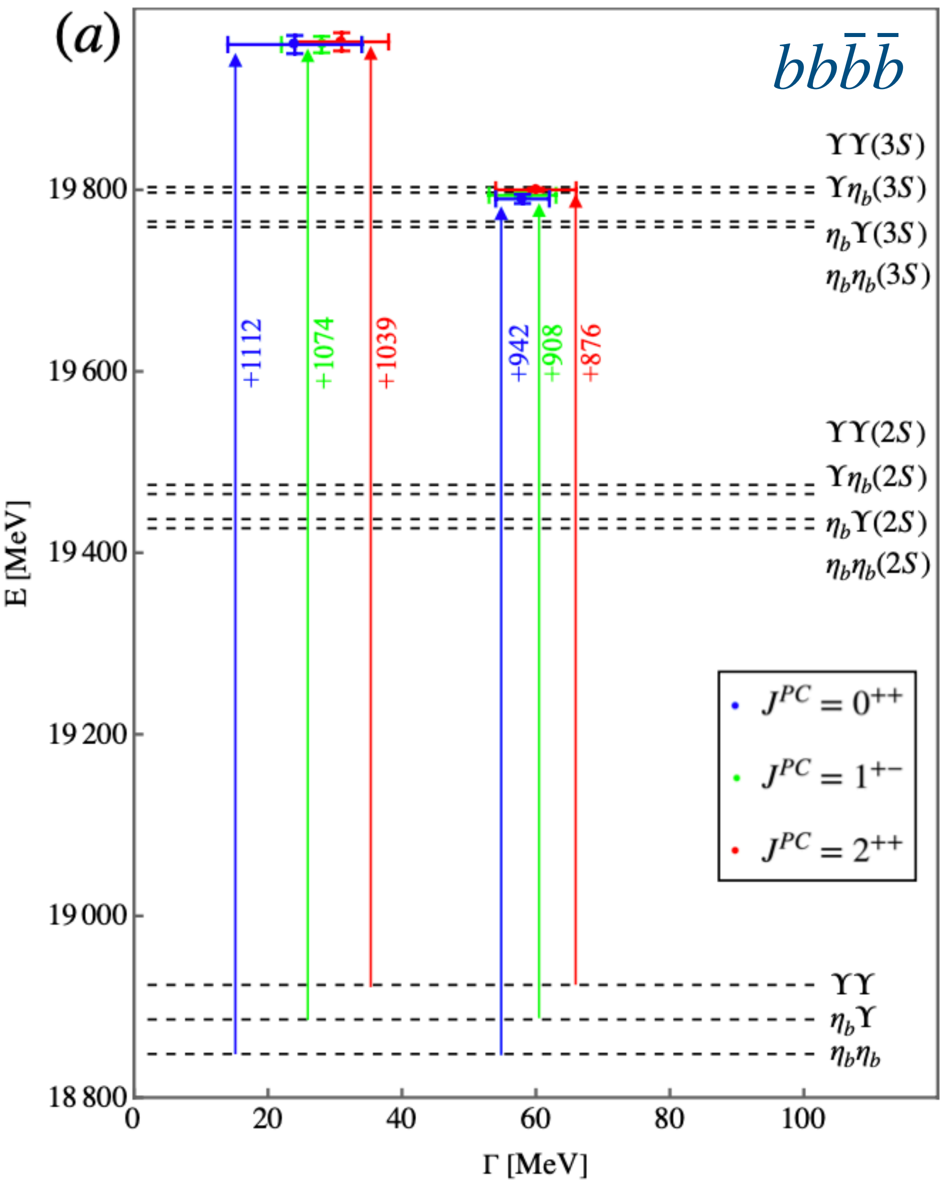


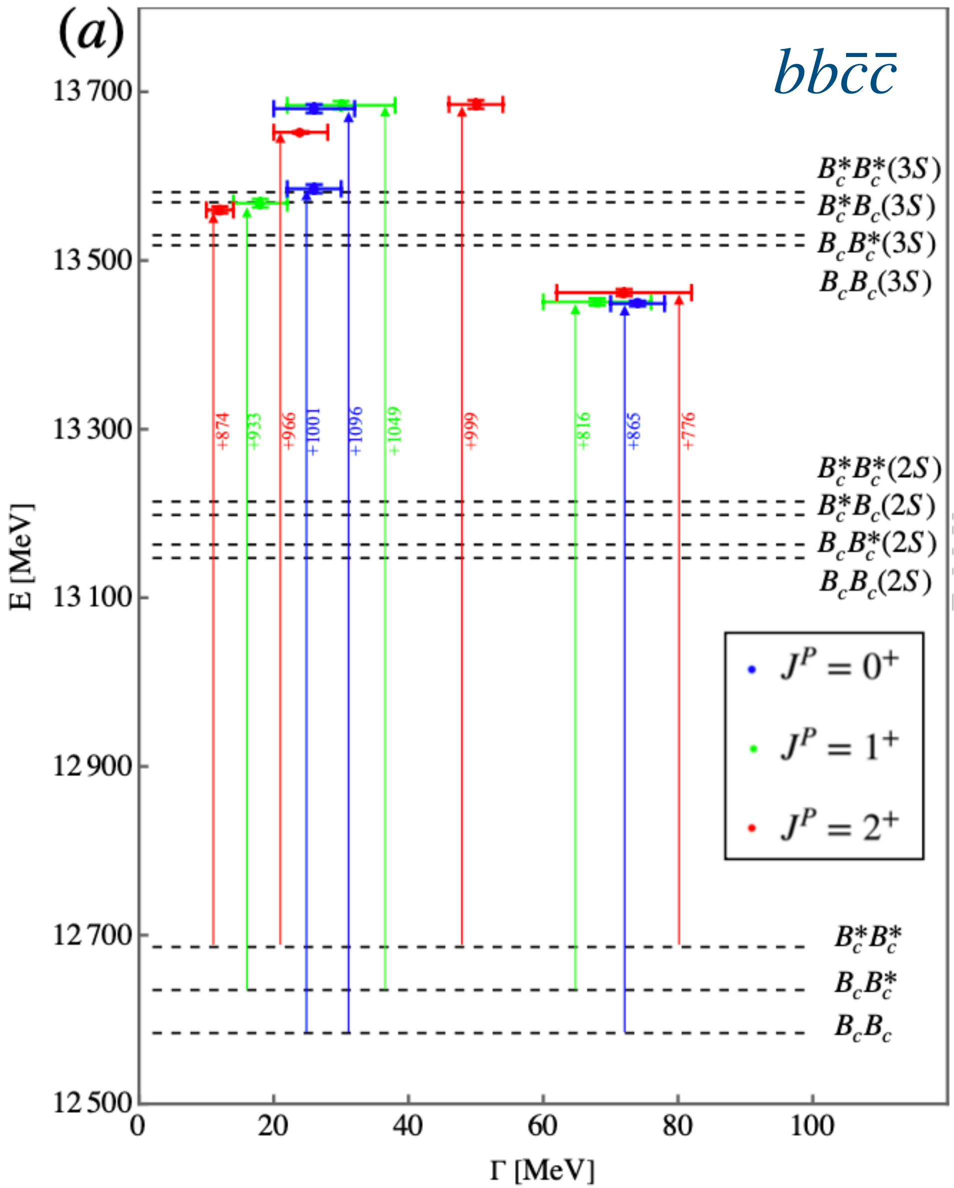


- No bound state, 6 resonances: 3 $(0^{++}, 1^{+-}, 2^{++})$ broad resonances around 7 GeV, and 3 sharp ones around 7.2 GeV.
- They are more than 100 MeV above the observed resonances.

$bb\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$ spectrum (no exp. data)

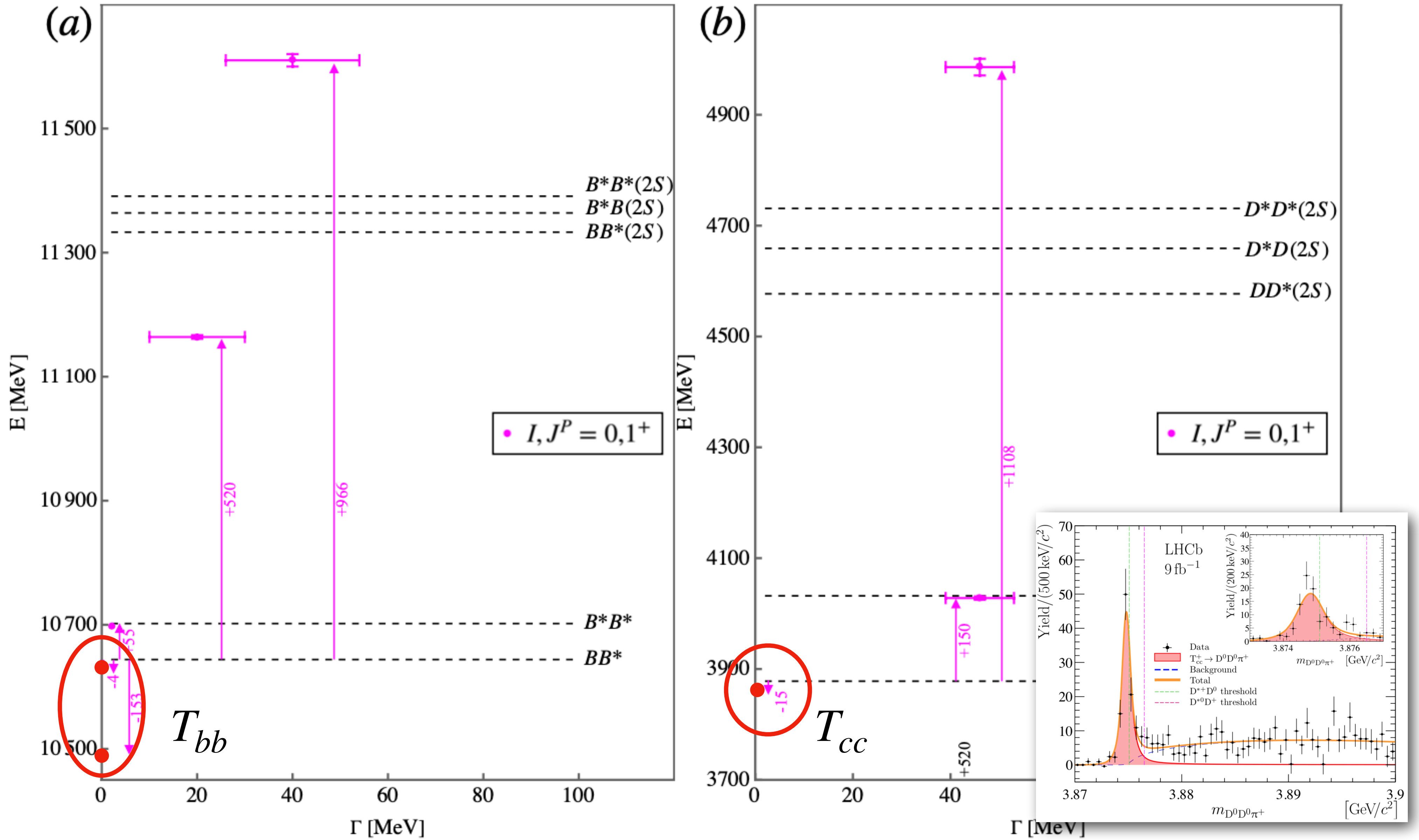
Similar resonance structures are seen but not low-lying resonances.





The same Hamiltonian applied to the doubly heavy tetraquarks, $T_{QQ} = QQ\bar{q}\bar{q}$ (I=0, 1$^+$), and gives 2 $bb\bar{u}\bar{d}$ and 1 $cc\bar{u}\bar{d}$ bound states.

Qi Meng et al., Phys. Lett. B814, 136095 (2021)

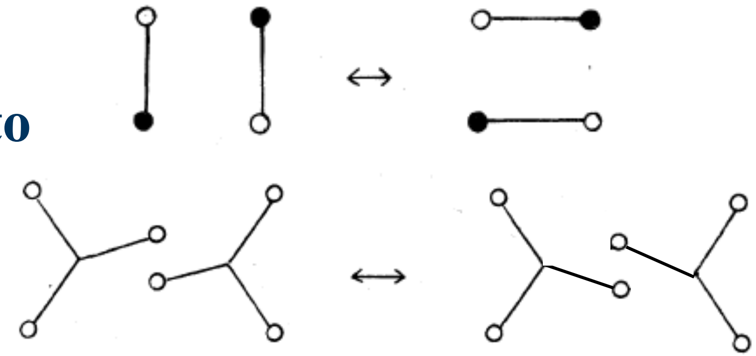


String confinement model

- “Reconnection of strings and quark matter”,
  H. Miyazawa, Phys. Rev. D20, 2953 (1979)

  “String Flip-Flop” model
  with reconnections of strings according to the spatial configuration of the quarks

  $$V_{\text{string}} = \sigma \times \underset{\text{links}}{\text{Min}} \sum r_{\text{link}}$$

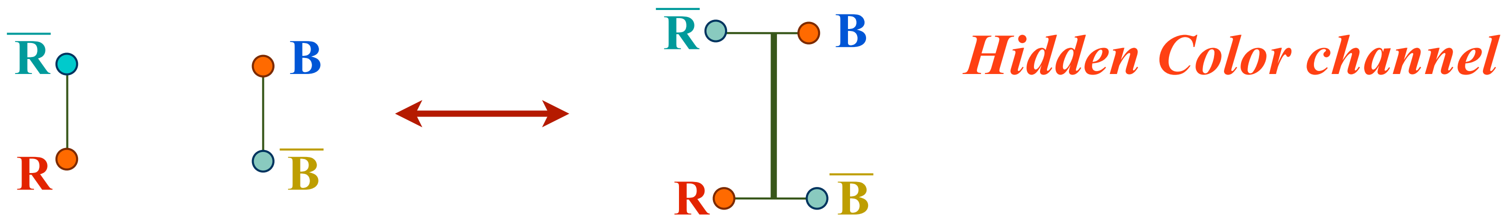


- Similar “string-type” confinement potential models for multi-quark systems were discussed by

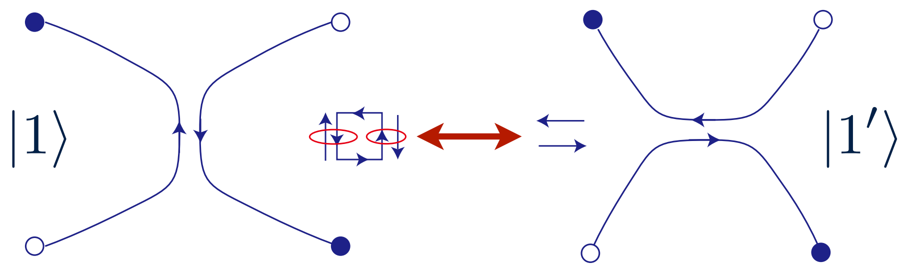
  O.W. Greenberg, J. Hietarinta, Phys. Lett. B 86, 309 (1979)
  N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
  M. Oka, Phys. Rev. D 31, 2274 (1985).
  F. Lenz, et al., Annals Phys. 170, 65 (1986).
  Y. Koike et al., Nucl. Phys. A449, 635 (1986), PTP S137, 21 (2000).
  G.A. Miller, Phys. Rev D37, 2431 (1998).
  J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 85, 014019 (2012).

String flip-flop model

- The string FF model works well for the U(1) charge, but for the color SU(3) theory, color recombination becomes nontrivial. Coupling of hidden color channels is unavoidable.



- QCD predicts flip-flop in the *strong coupling expansion*



- Two states are independent and transferred dynamically. This "string dynamics" can *not* be implemented in the conventional quark model.

Quark model - color configurations

- **Only two independent color states are allowed in Quark Model.**

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = 2 \times \mathbf{1} \oplus 4 \times \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

- **Color singlet $Q_1\ Q_2\ Q_3^{bar}\ Q_4^{bar}$ system is described by**

  Two singlets (mesons) states:

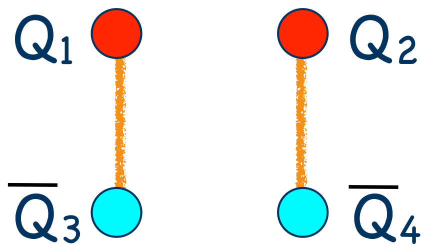
  $$|1\rangle = |(Q_1\bar{Q}_3)_\mathbf{1}(Q_2\bar{Q}_4)_\mathbf{1}\rangle \quad |1'\rangle = |(Q_1\bar{Q}_4)_\mathbf{1}(Q_2\bar{Q}_3)_\mathbf{1}\rangle$$

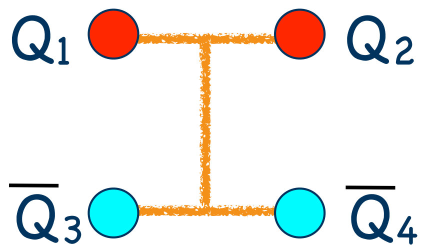
  Singlet + hidden color states:

  $$|1\rangle = |(Q_1\bar{Q}_3)_\mathbf{1}(Q_2\bar{Q}_4)_\mathbf{1}\rangle \quad |8\rangle = |(Q_1\bar{Q}_3)_\mathbf{8}(Q_2\bar{Q}_3)_\mathbf{8}\rangle$$

  Diquarks with color 3^{bar} and 6:

  $$|3\rangle = |(Q_1Q_2)_{\bar{\mathbf{3}}}(\bar{Q}_3\bar{Q}_4)_\mathbf{3}\rangle \quad |6\rangle = |(Q_1Q_2)_\mathbf{6}(\bar{Q}_3\bar{Q}_4)_{\bar{\mathbf{6}}}\rangle$$





- ***These bases are all equivalent.***

$$|1\rangle = \sqrt{\frac{1}{3}}|3\rangle + \sqrt{\frac{2}{3}}|6\rangle \qquad |8\rangle = -\sqrt{\frac{2}{3}}|3\rangle + \sqrt{\frac{1}{3}}|6\rangle$$

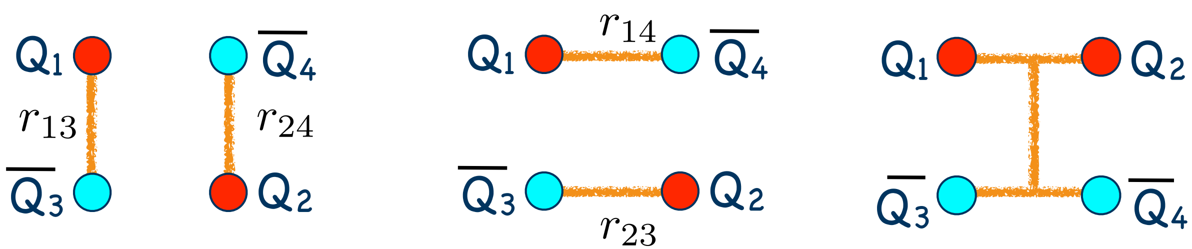
- **Two-meson states are not orthogonal.** $\langle 1|1'\rangle = \frac{1}{3}$

Novel string-like potential

- **If the quarks are the only carriers of color charges, the quark model does not have enough *freedom for color configurations*.**
- **We propose to extend the color Hilbert space of the quark model that can incorporate the color dynamics for multiquark systems.**

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

- **For a tetra-quark systems, we choose 3 color basis states:**



$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_1 (Q_2 \to \bar{Q}_4)_1\rangle$$

$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_1 (Q_2 \to \bar{Q}_3)_1\rangle$$

orthogonal bases

$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{3}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_3\rangle$$

Novel string-like potential

- **The string confinement potential**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}\rangle\rangle = \sigma(r_{13}+r_{24}),$$
$$\langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \sigma(r_{14}+r_{23}).$$

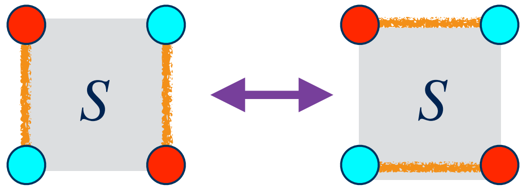
σ: **string tension**

- **Transitions by quantum tunneling filled the area by gauge field**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \kappa e^{-\sigma S}$$

S: **Minimal surface area**

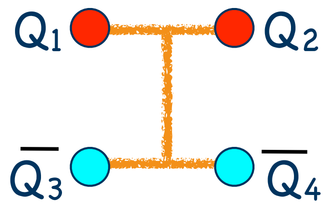
Y. Koike, O. Morimatsu, K. Yazaki, PTP S137, 21 (2000)



- **Confinement in $|\mathbf{hc}\rangle\rangle$ channel**

$$\langle\langle \mathbf{hc}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \sigma\left[\frac{r_{13}+r_{24}+r_{14}+r_{23}}{4}+\frac{r_{12}+r_{34}}{2}\right]$$
$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \pm\kappa' \exp(-\sigma S)$$

$\kappa' = \sqrt{8}\kappa$

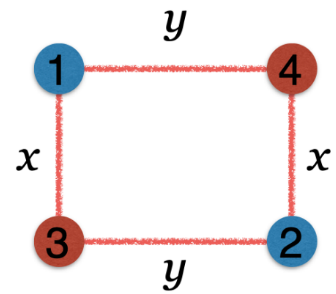


- **3-channel confinement potential (full 4-body potential)**

$$V_{\rm ST} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix} \begin{matrix} |\mathbf{1}\rangle\rangle \\ |\mathbf{1}'\rangle\rangle \\ |\mathbf{hc}\rangle\rangle \end{matrix}$$

Born-Oppenheimer effective potential

- **A rectangle configuration of $QQ\bar{Q}\bar{Q}$**



- **QM confinement for the non-orthogonal $|\mathbf{1}\rangle$, $|\mathbf{1}'\rangle$ bases**

$$V = \begin{pmatrix} 2\sigma x & (2/3)\sigma(x+y-\sqrt{x^2+y^2}) \\ (2/3)\sigma(x+y-\sqrt{x^2+y^2}) & 2\sigma y \end{pmatrix}$$

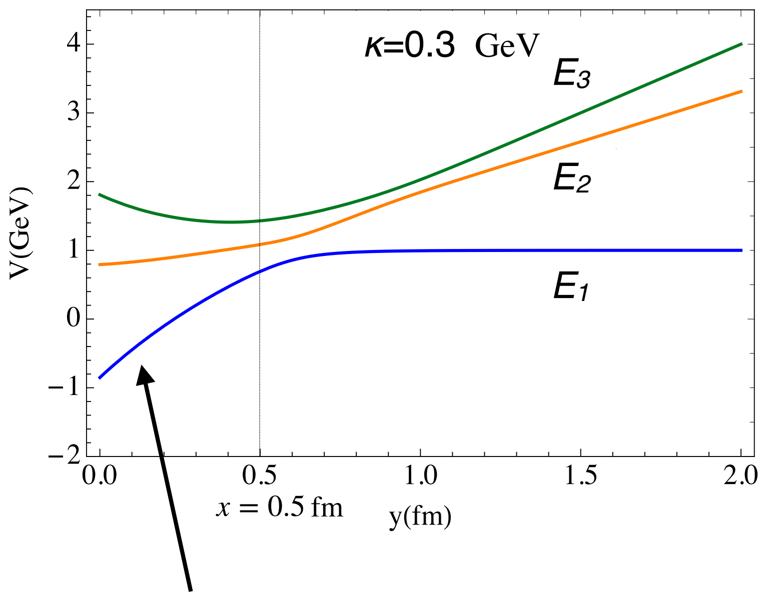
- **String confinement for the orthogonal $|\mathbf{1}\rangle\rangle$, $|\mathbf{1}'\rangle\rangle$, $|\mathbf{hc}\rangle\rangle$ bases**

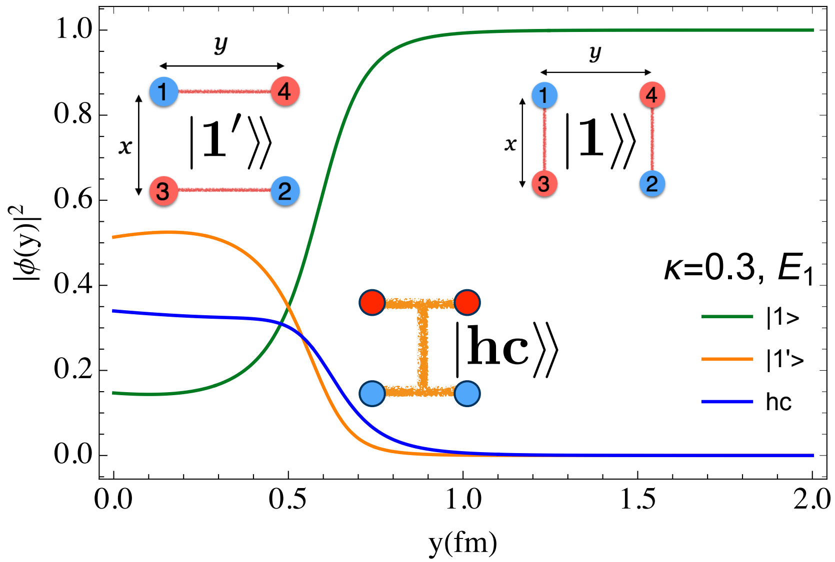
$$V_{\mathrm{ST}}(x,y) = \begin{pmatrix} 2\sigma x & \kappa e^{-\sigma xy} & \kappa' e^{-\sigma xy} \\ \kappa e^{-\sigma xy} & 2\sigma y & -\kappa' e^{-\sigma xy} \\ \kappa' e^{-\sigma xy} & -\kappa' e^{-\sigma xy} & \sigma(\frac{x+y}{2}+\sqrt{x^2+y^2}) \end{pmatrix}$$

One free parameter κ ($\kappa' = \sqrt{8}\kappa$), which is not determined from the meson spectrum.

Born-Oppenheimer effective potential

- **Novel String-like confinement: adiabatic potential at fixed *x***



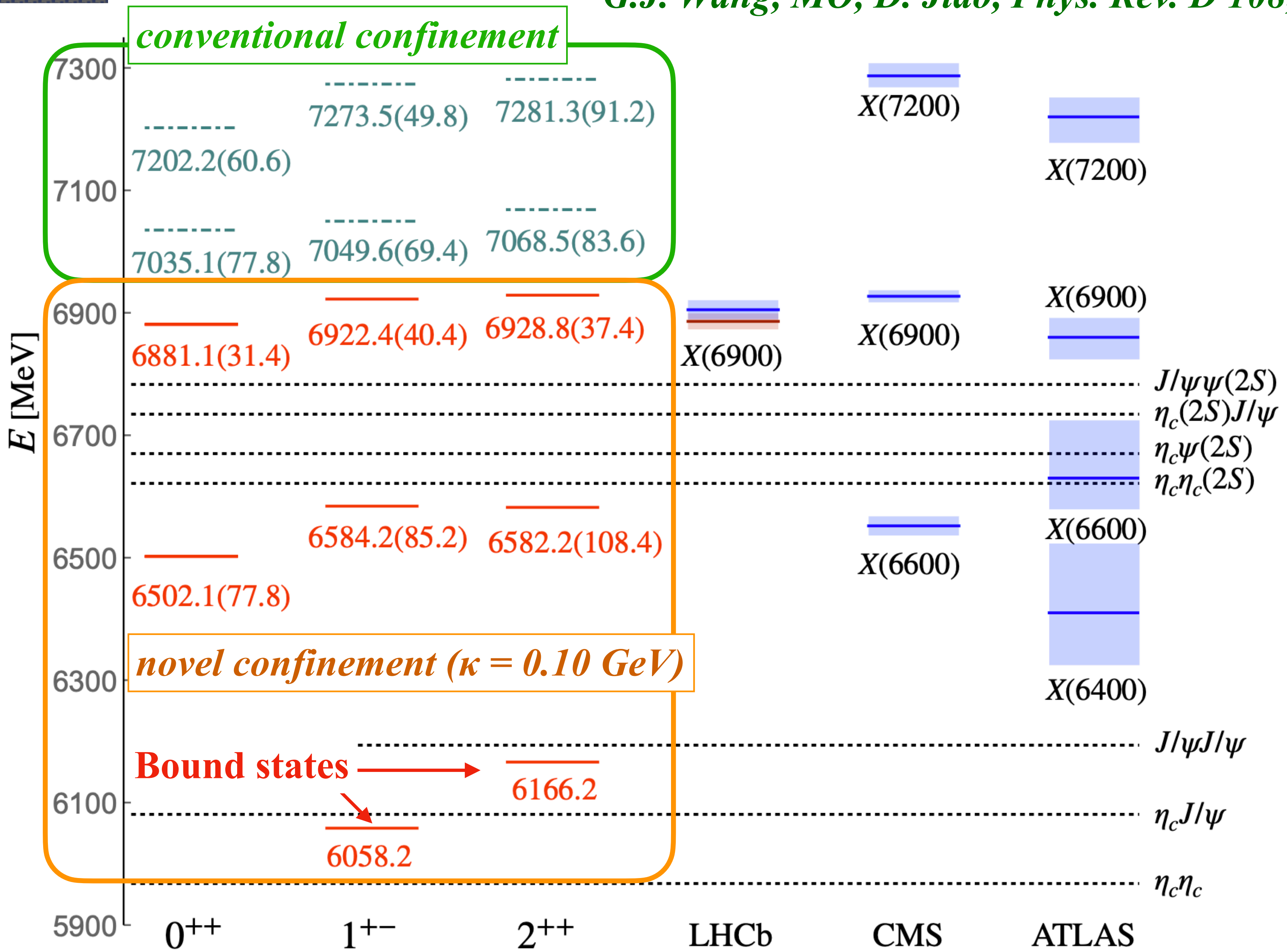


- **A short-range attraction due to the large mixing of the hidden color (hc) state. Compact tetra-quark configurations become important at short distances.**
- **The only free parameter *κ* will take a value in**

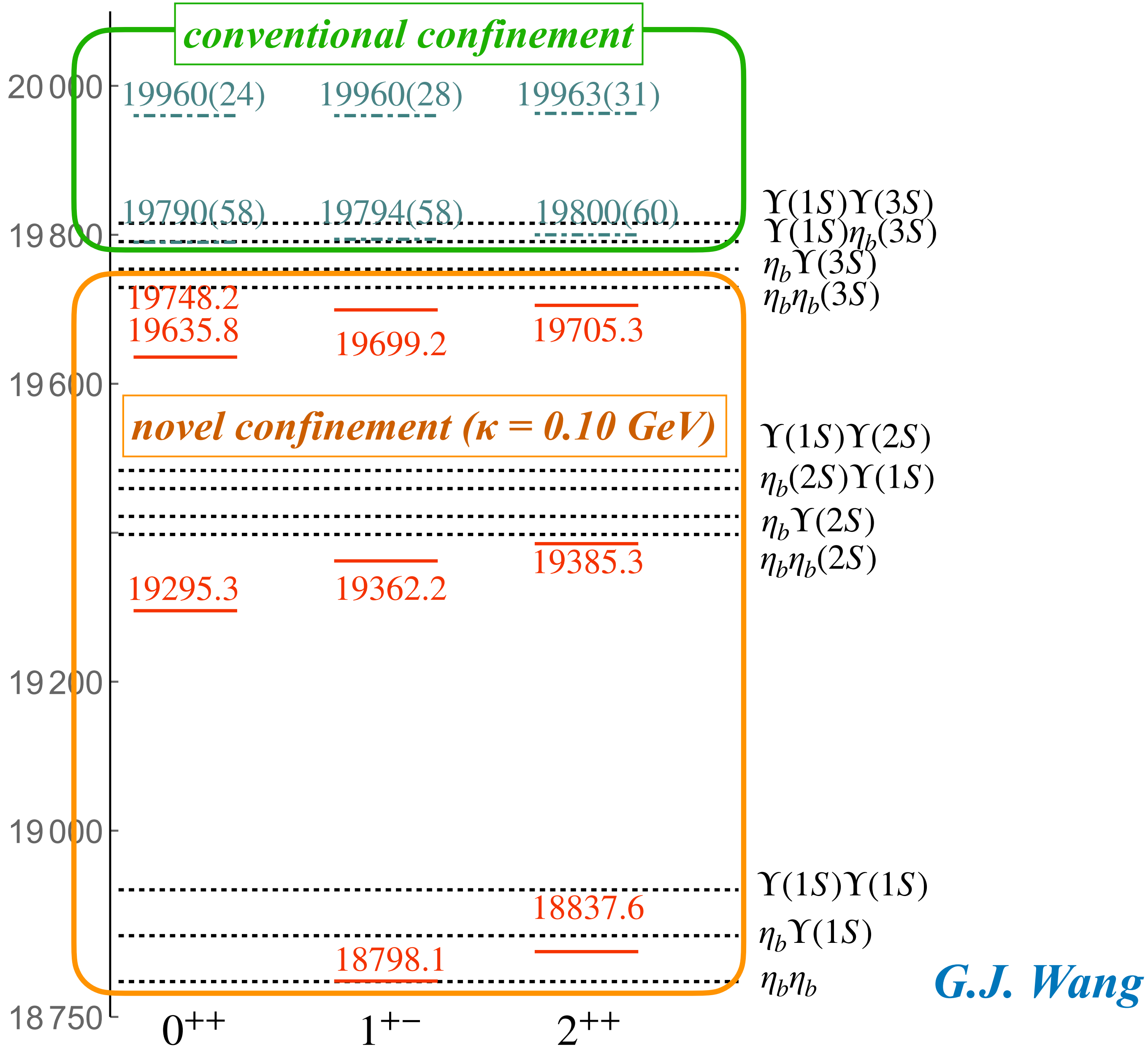
$$0 \le \kappa' = \sqrt{8}\kappa \le 2\sigma a \sim 2\sqrt{\sigma} \sim 1\text{GeV} \longrightarrow \kappa \le 0.3\text{GeV}$$

$cc\bar{c}\bar{c}$ spectrum with novel confinement

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)



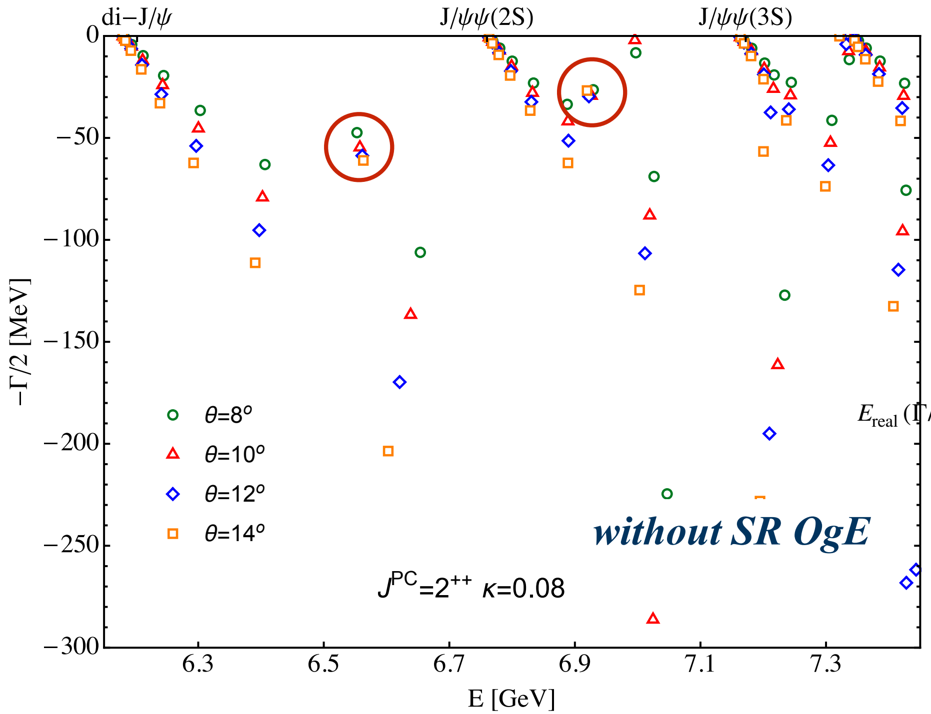
$bb\bar{b}\bar{b}$ spectrum with novel confinement

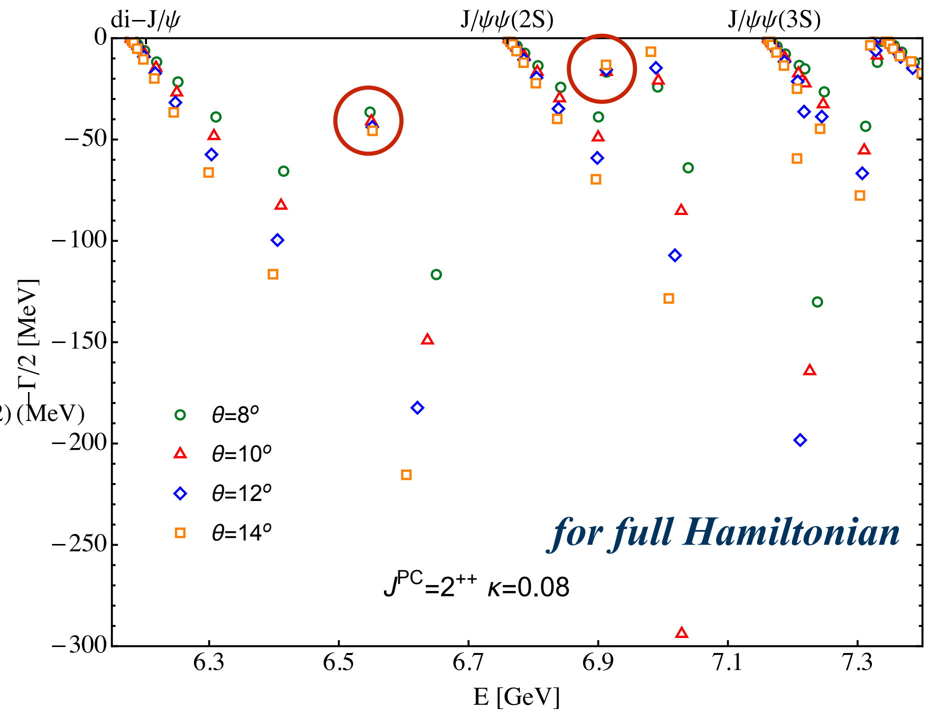


$cc\bar{c}\bar{c}$ spectrum with novel confinement

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

Complex scaling spectra of the spin-aligned 2^{++} states



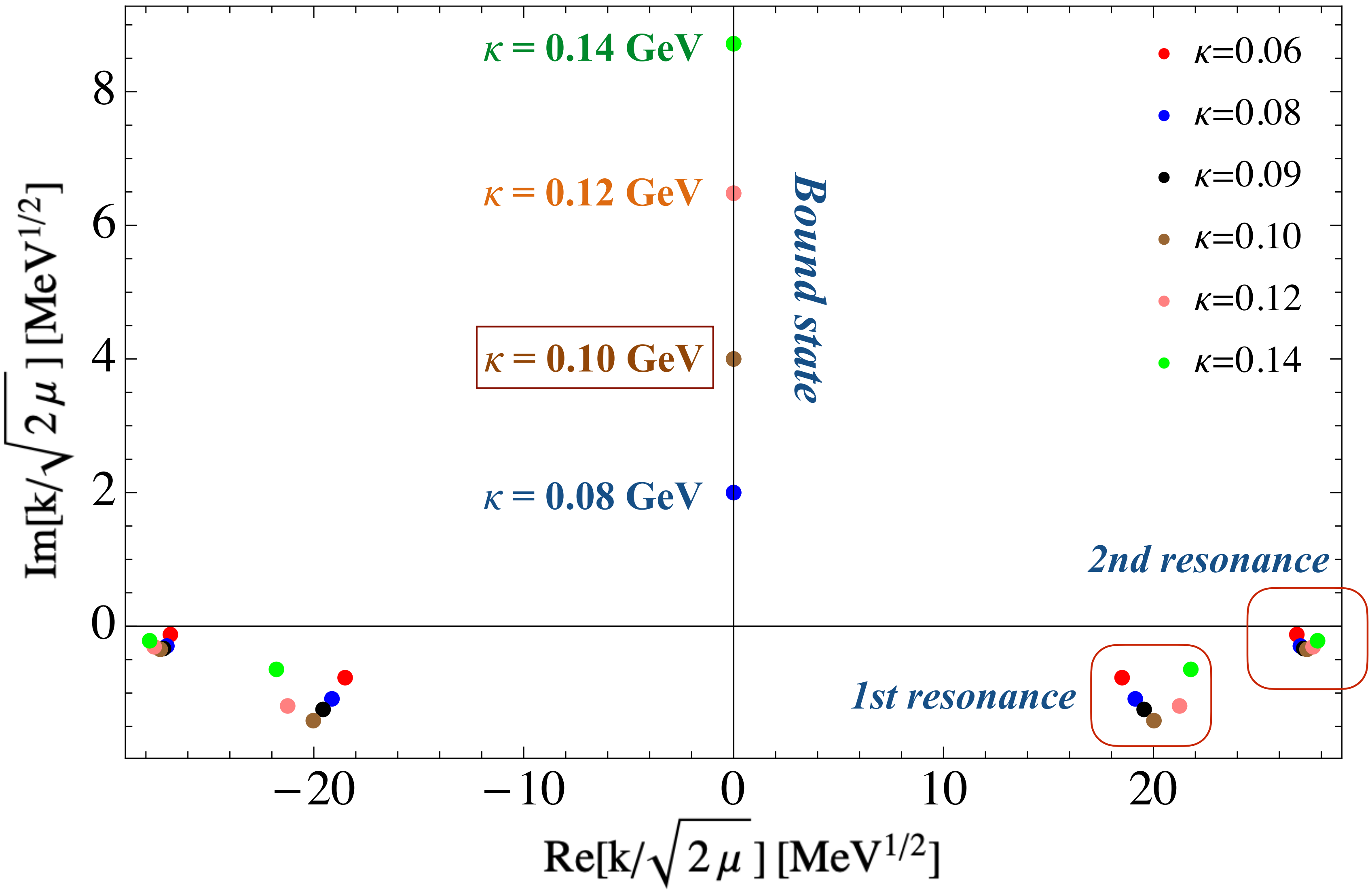


2^{++} bound and resonance states

κ (GeV)	Bound state (MeV)	Binding Ene. (MeV)	1st Res E (Γ/2) (MeV)	2nd Res E (Γ/2) (MeV)
0.08	6180	4	6550 (41)	6913 (17)
0.10	6166	16	6582 (54)	6929 (19)
0.12	6140	42	6631 (51)	6944 (16)

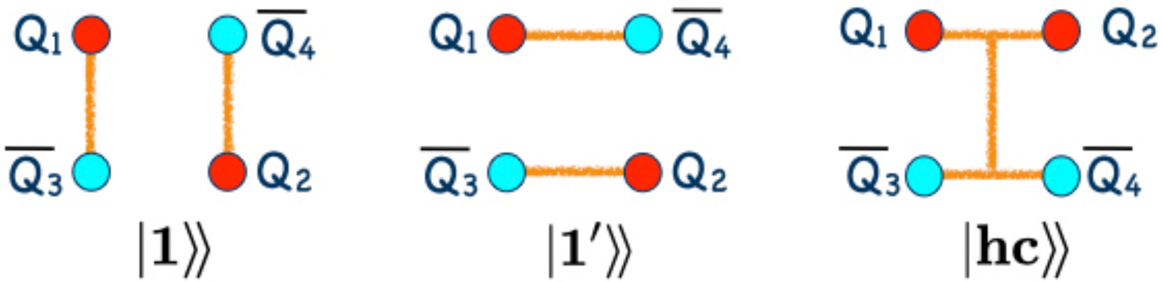
$cc\bar{c}\bar{c}$ spectrum with novel confinement

- **Pole positions of the 2^{++} charmed tetraquark in complex k plane**



Summary

- Confinement in the tetra(multi)-quark system is not trivial nor well established from the quark model viewpoints.
- String-like confinement potential is proposed by extending the color configuration space of the conventional quark model with a compact hidden-color state.



- Mixing of the $|\mathrm{hc}\rangle\rangle$ state induces strong attraction among the multi-quark systems.
- The model is applied to the fully-charmed tetra-quark system. A bound state appears due to the attraction of HC. Complex scaling method provides us with two or more resonances that seem consistent with experiment.