CAPTURE, THERMALISATION AND ANNIHILATION OF DARK MATTER IN COMPACT OBJECTS

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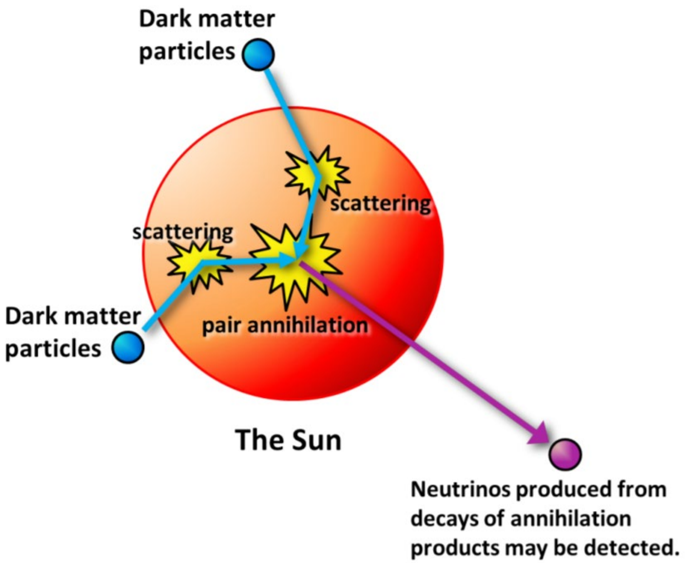
Outline

01 MOTIVATIONS

- Dark Matter Capture in Stars (Sun) well studied
- Capture driven by $\sigma_{\chi N}$
- Subsequent scatterings cause infall towards core
- Observable for the Sun: annihilation into neutrinos
- Relevant rates:
  - Capture rate
  - Thermalisation rate/time
  - Annihilation rate
  - (Evaporation rate)
- Other Stars recently investigated:
  - Neutron Stars (Nicole Bell's talk on Monday)
  - WD (this talk)



01 MOTIVATIONS

- Our works on Capture in compact stars:
- Neutron Stars:
  - JCAP 09 (2018) 018 (1807.02840)
  - JCAP 06 (2019) 054 (1904.09803)
  - JCAP 09 (2020) 028 (2004.14888)
  - JCAP 03 (2021) 086 (2010.13257)
  - Phys.Rev.Lett. 127 (2021) 11, 111803 (2012.08918)
  - JCAP 11 (2021) 056 (2108.02525)
  - JCAP 04 (2024) 006 (2312.11892)
- White Dwarfs:
  - JCAP 10 (2021) 083 (2104.14367)
  - JCAP 07 (2024) 051 (2404.16272) [THIS TALK]

01 MOTIVATIONS

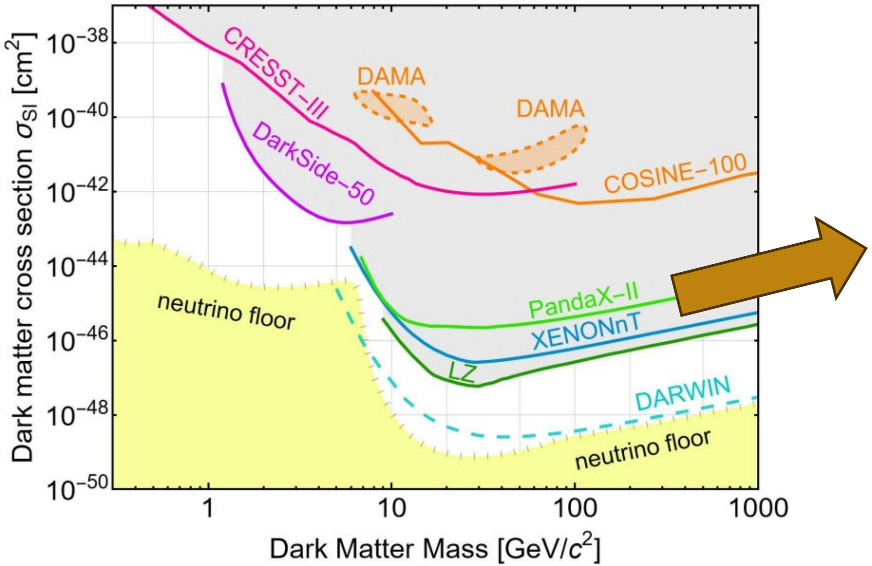
- More than 90% of the stars in the Galaxy are White Dwarfs (WDs)
- High density, extreme conditions and the existence of observational data

↓

- Powerful probe to test and constrain dark matter (DM) models
- This generally involves the accumulation of DM particles
  - Increase in their luminosity
  - DM-triggered supernova ignition/black hole formation

01 MOTIVATIONS

- WD made of Ions and degenerate electrons
  - Will consider only Ion targets here
- Capture rate will be driven by $\sigma_{\chi N}$
  - Parameter space is same as Direct Detection
- We will focus on very heavy DM: what happens on the right of this plot?



02 ASSUMPTIONS

- DM Capture usually based on Gould seminal work for Capture in the Earth:
- (i) DM trajectories are unaffected by collisions ✓
- (ii) Constant escape velocity ✗
- (iii) Constant iron (target) density ✗
- (iv) DM follows linear trajectories outside and inside the Earth's core, thereby neglecting gravitational focusing/gravity effects ✗

- Assumptions (ii-iv) are actually unnecessary
- We will see that the rate can be calculated (nearly) exactly

02 ASSUMPTIONS

- Additional approximations usually made:
  - Optical depth inside the star $Rn_T\sigma$, equal to the value at the center, for any point inside the star ✗
  - Differential cross section on target $d\sigma/dcos\theta \sim$constant ✗

- What we do:
  - Only assume (i) DM trajectories are unaffected by collisions ✓
  - Optical depth inside star depends on interaction rate
    - Same interaction rate used for Capture for consistency ✓
    - Optical depth depends on the point in the star ✓
    - Need to average over all trajectories ✓
  - Cross section very suppressed at large E_R (realistic) ✓
    - $\sigma_{\chi p} \sim const \rightarrow \sigma_{\chi T} \propto e^{-E_R/E_0}$

03 CAPTURE BY MULTIPLE SCATTERING

- This is the energy loss probability density distribution $f(E_R) = \frac{1}{\sigma_{T\chi}} \frac{d\sigma_{T\chi}}{dE_R}(E_R).$
- Probability to lose at least a certain energy after one scattering $\mathcal{F}_1(\delta E) = \int_{\delta E}^{\infty} dE_R f(E_R)$
- Similarly, after exactly N scatterings $\mathcal{F}_N(\delta E) = \int_0^{\delta E} dE_R \mathcal{F}_{N-1}(\delta E - E_R) f(E_R)$
- It is easy to find these functions using Laplace transform $\hat{\mathcal{F}}_N = \hat{\mathcal{F}}_{N-1} \hat{f}$

03 CAPTURE BY MULTIPLE SCATTERING

For simplicity, we assume that the DM-target cross section is well approximated by

$$\frac{d\sigma_{T\chi}}{d\cos\theta_{\rm cm}} \propto e^{-\frac{E_R}{E_0}}, \tag{3.26}$$

where E_R is the recoil energy and E_0 depends on the specific nuclear target. That is, we assume exponential nuclear form factors similar to the Helm approximation. This leads to

$$f(E_R) = \frac{\Theta(E_R)}{E_0} e^{-\frac{E_R}{E_0}}, \tag{3.27}$$

$$\mathcal{F}_1(\delta E) = e^{-\frac{\delta E}{E_0}}. \tag{3.28}$$

Defining the dimensionless quantity

$$\delta = \frac{\delta E}{E_0} = \frac{m_\chi u_\chi^2}{2E_0}, \tag{3.29}$$

and taking the Laplace transform of the $\mathcal{F}$ functions written in terms of δ, we find

$$\tilde{\mathcal{F}}_1(s) = \frac{1}{1+s}, \qquad \tilde{\mathcal{F}}_N(s) = \frac{1}{(1+s)^N}, \tag{3.30}$$

where the last expression corresponds to

$$\mathcal{F}_N(\delta) = \frac{e^{-\delta}\delta^{N-1}}{N-1!}. \tag{3.31}$$

03 CAPTURE BY MULTIPLE SCATTERING

- Probability to have interacted already N times is given by Poisson distribution

$$p_N(\tau_\chi) = e^{-\tau_\chi} \frac{\tau_\chi^N}{N!}$$

- Therefore the single scattering Capture rate can be written as

$$C_1 = \frac{\rho_\chi}{m_\chi} \int_0^{R_\star} dr 4\pi r^2 n_T(r) \sigma_{T\chi}(v_{\rm esc}(r)) v_{\rm esc}^2(r) \int_0^1 \frac{y dy}{\sqrt{1-y^2}} \int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi} p_0(\tau_\chi) \mathcal{F}_1(\delta)$$

- The Capture rate for exactly N scatterings can be obtained as

$$C_N = \frac{\rho_\chi}{m_\chi} \int_0^{R_\star} dr 4\pi r^2 n_T(r) \sigma_{T\chi}(v_{\rm esc}(r)) v_{\rm esc}^2(r) \int_0^1 \frac{y dy}{\sqrt{1-y^2}} \int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi} p_{N-1}(\tau_\chi) \mathcal{F}_N(\delta)$$

03 CAPTURE BY MULTIPLE SCATTERING

The total capture rate is given by the sum over all N collisions,

$$C = \sum_N C_N. \tag{3.35}$$

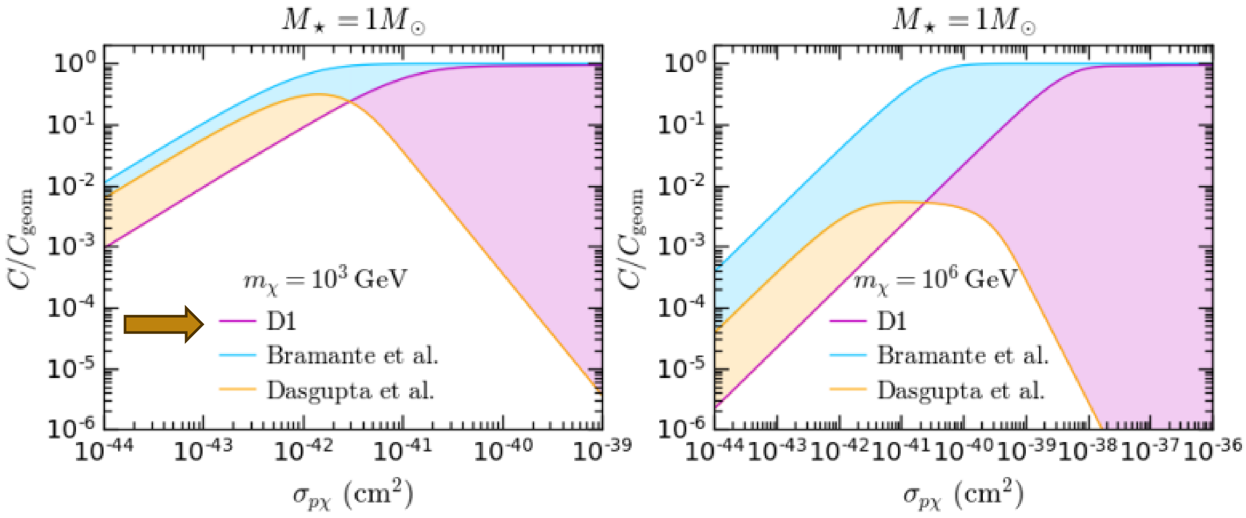
Next, instead of first evaluating the integrals in Eq. 3.34 and then summing over N, we sum the series first by introducing the response function, $G(\tau_\chi, \delta)$

$$G(\tau_\chi, \delta) \equiv \sum_{N=1}^{\infty} p_{N-1}(\tau_\chi)\mathcal{F}_N(\delta) = \sum_{N=1}^{\infty} \frac{e^{-\tau_\chi}\tau_\chi^{N-1}}{(N-1)!}\frac{e^{-\delta}\delta^{N-1}}{(N-1)!}$$
$$= e^{-\tau_\chi-\delta} I_0\left(2\sqrt{\tau_\chi\delta}\right), \tag{3.36}$$

- The resulting total Capture rate is

$$C = \frac{\rho_\chi}{m_\chi}\int_0^{R_\star} dr 4\pi r^2 n_T(r) v_{\rm esc}^2(r)\sigma_{T\chi}(v_{\rm esc}(r))\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}\tilde{G}\left(r, \frac{m_\chi u_\chi^2}{2E_0}\right)$$

02 CAPTURE RATE



04 MULTIPLE TARGETS

- Can expand the formalism to include multiple types of targets

First, as in the previous section, we consider the probability for DM to interact with a target i and lose energy of at least δE, while travelling a length $d\tau_\chi^i$, starting from a layer in the WD with optical depth τ_χ^i. This is given by the differential element $G(\tau_\chi^i, \delta_i) d\tau_\chi^i$, where

$$\delta_i = \frac{\delta E}{E_0^i}, \tag{3.43}$$

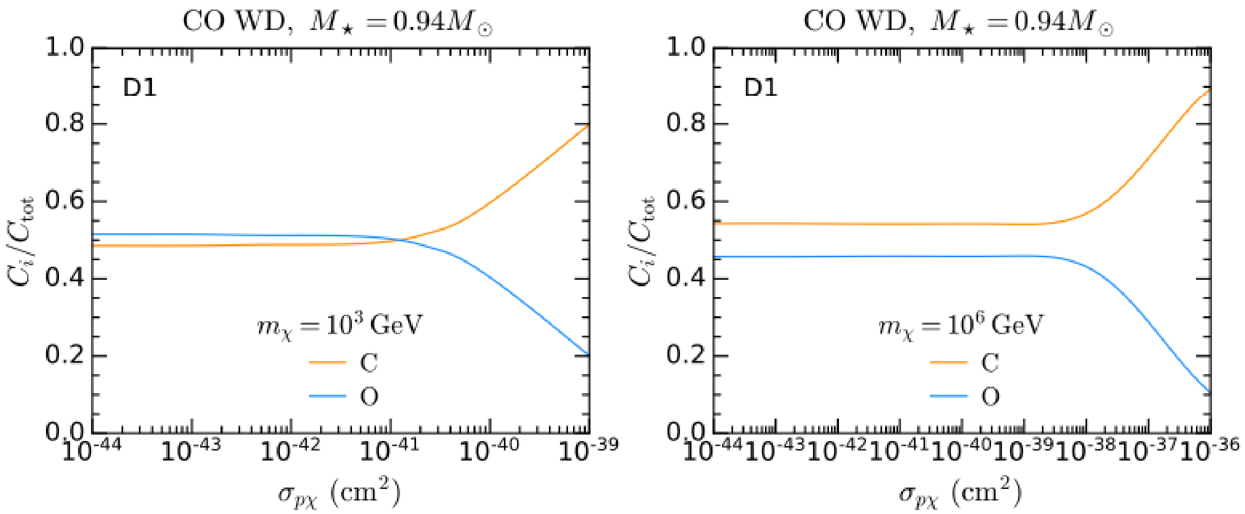
and the energy scale E_0^i depends on the target i and is calculated using Eq. 3.40. Thus, the probability to interact and lose the same amount of energy when DM travels a path-length τ_χ^i is simply the integral of the differential element over the trajectory, i.e.

$$\mathcal{G}(\tau_\chi^i, \delta_i) = \int_0^{\tau_\chi^i} d\tau \, G(\tau, \delta_i). \tag{3.44}$$

Next, we introduce a second target species. In the presence of these two ionic targets, the cumulative probability of DM to lose an energy δE after travelling an optical depth τ_χ^i in the target i and τ_χ^j in the second target j is found to be

$$\mathcal{G}_{2,ij}(\delta E) = \int_0^{\delta E / E_0^j} dz \, \mathcal{G}\left(\tau_\chi^i, \frac{\delta E - z E_0^j}{E_0^i}\right) \left[-\frac{\partial}{\partial z} \mathcal{G}(\tau_\chi^j, z)\right]. \tag{3.50}$$

04 MULTIPLE TARGETS



05 THERMALISATION

- Thermalisation time give by $$t_{\text{therm}} = \sum_{n=0}^{N} \frac{1}{\Omega^-(w(x_n))},$$
- Interaction rate $$\Omega_T^-(x) = \int_0^x dy\, R_T^-(x \to y)$$
- Large number of scatterings $$\frac{dx}{dt} = E(x) \qquad t_{\text{therm}} = \int_1^\infty \frac{dx}{E(x)}$$
- Energy loss per unit time $$E(x) = \int_0^x dy(x-y)R^-(x \to y)$$

05 THERMALISATION

- Two approximations are possible, they require 2 different assumptions
  - One needs relative speed to be dominated by either DM or Target ✓
  - The other one, for large DM mass, requires Target speed to be small ✗

- Zero temperature approximation (high energy DM)
  - Appropriate for capture
  - Both approximations can be applied

- Large mass approximation
  - Appropriate for Thermalisation of very heavy DM
    - Thermalisation time driven by last part where DM speed very low
  - Only one assumption is verified

05 THERMALISATION

In the large energy limit/zero temperature approx.:

$$E(x) \simeq \begin{cases} 2n_T(r)\sigma_T \left(\frac{x}{\mu}\right)^{m+3/2} v_T^{2m+1}, & d\sigma_{T\chi} \propto v_{\rm rel}^{2m} \\ \frac{4(m+1)}{m+2} n_T(r)\sigma_T v_T \left(\frac{x}{\mu}\right)^{m+3/2} \left(\frac{2m_T^2 v_T^2}{q_0^2}\right)^m, & d\sigma_{T\chi} \propto q_{\rm tr}^{2m} \end{cases} \tag{A.28}$$

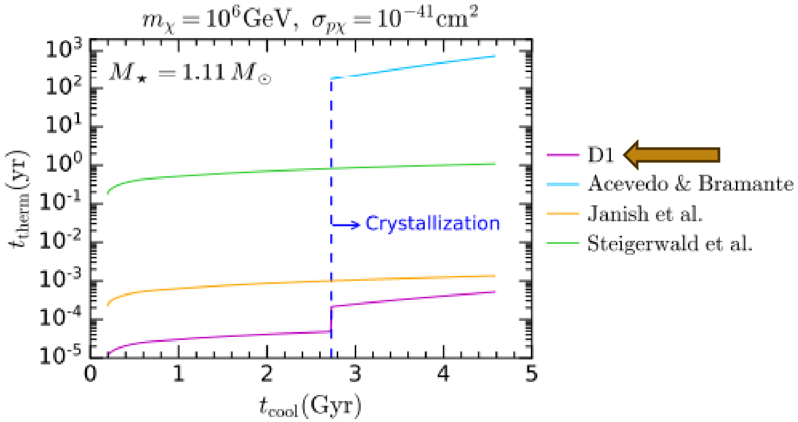
In the low energy regime, i.e. $x \sim 1$ we find for cross sections proportional to powers of the DM-ion relative velocity $v_{\rm rel}^{2m}$

$$E(x) \sim \Gamma\left(m + \frac{3}{2}\right) \frac{n_T(r)\sigma_T}{\sqrt{\pi}} \sqrt{\frac{x}{\mu}} v_T^{2m+1}, \tag{A.29}$$

while for differential cross sections proportional to $q_{\rm tr}^{2m}$ we have

$$E(x) \sim \Gamma\left(m + \frac{3}{2}\right) \frac{2(m+1)}{m+2} \frac{n_T(r)\sigma_T v_T}{\sqrt{\pi}} \sqrt{\frac{x}{\mu}} \left(\frac{2m_T^2 v_T^2}{q_0^2}\right)^m. \tag{A.30}$$

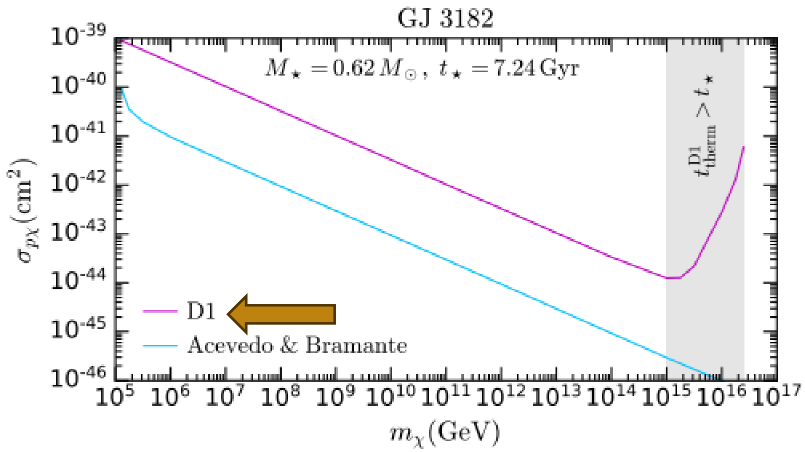
05 THERMALISATION

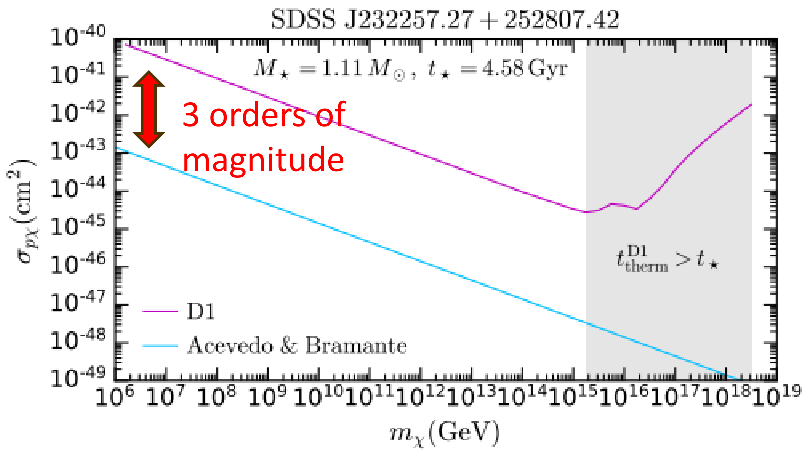


06 SELF GRAVITATION

- No DM self-repulsive forces
- DM collapses to isothermal sphere of radius $r_\chi = \sqrt{\frac{3T_\star}{2\pi G \rho_c m_\chi}}$
- Gravitational field due to DM at center can reach same order of magnitude of the one generated by ordinary matter
    - Requires $N_\chi(t) \geq \frac{4\sqrt{2}\pi^{3/2} r_\chi^3 \rho_c}{3\sqrt{3} m_\chi} = N_{\rm crit}$
- Star develops an instability and Core-Collapse is triggered

06 SELF GRAVITATION





07 CONCLUSIONS

- WD are interesting probe for DM
  - Capture, thermalisation (and annihilation) rates necessary to predict observables
  - Typical observables: luminosity, DM triggered collapse/Supernova
- Accurate calculation of Capture Rates in WD under minimal set of approximations
  - All assumptions used are well verified
  - Results differ by order of magnitude comparing to previous estimates in literature
- Accurate computation of thermalisation rates of DM in WD
  - High energy and low energy regimes
  - Analytical expressions verified numerically
- DM-induced WD collapse revisited using updates rates
  - Cross section required up to 3 orders of magnitude larger

THANK YOU

BACKUP

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02 CAPTURE BY SINGLE SCATTERING

- Assuming target at rest ($T = 0$)

$$\sqrt{u_\chi^2 + v_{\rm esc}^2(r)}$$

$$\int_0^1 \frac{y\,dy}{\sqrt{1-y^2}} e^{-\tau}.$$

y is the angular momentum (normalised to maximum)

$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_\star} dr 4\pi r^2 \int_0^\infty du_\chi \frac{w(r)}{u_\chi} f_{\rm MB}(u_\chi)\Omega_T^-(w)\eta(r)$$

This factor encodes the star opacity and the shape of trajectories given by the gravitational field

$$\mu = \frac{m_\chi}{m_T}, \qquad \mu_\pm = \frac{\mu \pm 1}{2}.$$

$$\Omega_T^-(w) = \frac{4\mu_+^2}{\mu w} n_T(r) \frac{E_R^{\max}(v_{\rm esc}, m_\chi, m_T)}{2m_\chi} \int_{E_R^{\min}}^{E_R^{\max}} dE_R \frac{d\sigma_{T\chi}}{dE_R}(v_{\rm esc}, E_R)$$

This factor encodes the energy loss probability

02 CAPTURE BY SINGLE SCATTERING

- Assuming target at rest ($T = 0$)

$$\sqrt{u_\chi^2 + v_{\rm esc}^2(r)}$$

$$\int_0^1 \frac{y\,dy}{\sqrt{1-y^2}} e^{-\tau_\chi}$$

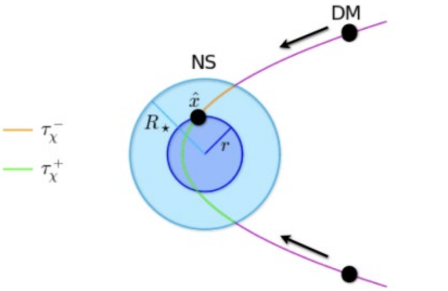
$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_\star} dr\, 4\pi r^2 \int_0^\infty du_\chi \frac{w(r)}{u_\chi} f_{\rm MB}(u_\chi)\Omega_T^-(w)\eta(r)$$

$$\Omega_T^-(w) = \frac{4\mu_+^2}{\mu w} n_T(r) \frac{E_R^{\max}(v_{\rm esc}, m_\chi, m_T)}{2m_\chi}\int_{E_R^{\min}}^{E_R^{\max}} dE_R \frac{d\sigma_{T\chi}}{dE_R}(v_{\rm esc}, E_R)$$

$$\mu = \frac{m_\chi}{m_T}, \qquad \mu_\pm = \frac{\mu \pm 1}{2}.$$

$Rn_T\sigma$

Maximum value assuming a straight line crossing the star across its center instead of



- Common approximations:
- Constant values, no averages
- (Differential) Cross section $d\sigma/dcos\theta \sim$constant

03 CAPTURE BY MULTIPLE SCATTERING

$$\sqrt{u_\chi^2 + v_{\rm esc}^2(r)}$$

$$\int_0^1 \frac{y\,dy}{\sqrt{1-y^2}}\, e^{-\tau}.$$

$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_\star} dr 4\pi r^2 \int_0^\infty du_\chi \frac{w(r)}{u_\chi} f_{\rm MB}(u_\chi)\Omega_T^-(w)\eta(r)$$

∞ → Our only approximation!

$$\Omega_T^-(w) = \frac{4\mu_+^2}{\mu w} n_T(r) \frac{E_R^{\max}(v_{\rm esc}, m_\chi, m_T)}{2m_\chi}\int_{E_R^{\min}}^{E_R^{\max}} dE_R \frac{d\sigma_{T\chi}}{dE_R}(v_{\rm esc}, E_R)$$

$$\mu = \frac{m_\chi}{m_T}, \qquad \mu_\pm = \frac{\mu \pm 1}{2}.$$

- Our approach:
- Assuming cross section very suppressed at large E_R (realistic)

$$\sigma_{\chi p} \sim const \rightarrow \sigma_{\chi T} \propto e^{-E_R/E_0}$$

- Opacity calculated from real trajectories

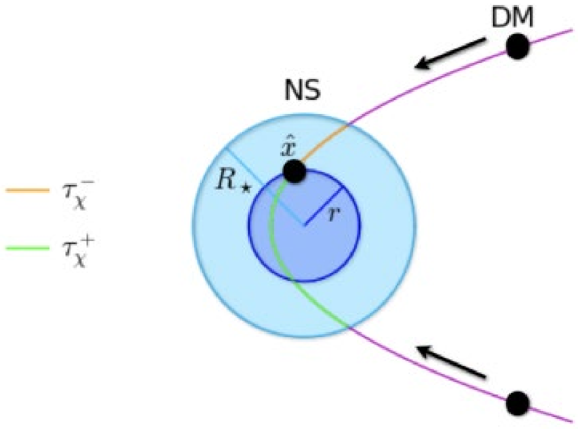
02 CAPTURE BY SINGLE SCATTERING

- Assuming target at rest ($T = 0$)

$$\sqrt{u_\chi^2 + v_{\rm esc}^2(r)}$$

$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_\star} dr 4\pi r^2 \int_0^\infty du_\chi \frac{w(r)}{u_\chi} f_{\rm MB}(u_\chi)\Omega_T^-(w)\eta(r)$$

$$\eta(r) = \frac{1}{2}\int_0^1 \frac{y dy}{\sqrt{1-y^2}}\left(e^{-\tau_-(r,y)} + e^{-\tau_+(r,y)}\right).$$



$$\tau_\chi^-(r,y) = \int_r^{R_\star} \frac{dx}{\sqrt{1 - y^2\frac{J_{\rm max}(r)^2}{J_{\rm max}(x)^2}}} \frac{\Omega^-(w(x))}{v_{\rm esc}(x)\sqrt{1-v_{\rm esc}^2(x)}},$$

$$\tau_\chi^+(r,y) = \int_r^{r_{\rm min}} + \int_{r_{\rm min}}^{R_\star} \frac{dx}{\sqrt{1 - y^2\frac{J_{\rm max}(r)^2}{J_{\rm max}(x)^2}}} \frac{\Omega^-(w(x))}{v_{\rm esc}(x)\sqrt{1-v_{\rm esc}^2(x)}} = 2\tau_\chi^-(r_{\rm min}, y) - \tau_\chi^-(r,y),$$

05 THERMALISATION

$$R_T^-(w \to v) = \int_0^\infty ds \int_0^\infty dt\, F(s,t) \frac{4\mu_+^2}{\mu} \frac{n_T(r) v}{w} \frac{d\sigma_{T\chi}}{d\cos\theta_{\text{cm}}}(s.t, w, v)\Theta(v - |t-s|), \quad \text{(A.1)}$$

$$F(s,t) = \frac{8\mu_+^2}{\sqrt{\pi}} k^3 t\mu\, e^{-k^2 u_T^2} \Theta\,(t+s-w)\,. \quad \text{(A.2)}$$

Next, we define the following functions

$$\delta_{\text{EXP}}(x, x_0, c) = c\, e^{-c(x-x_0)}\Theta(x - x_0), \quad \text{(A.3)}$$

$$\delta_{\text{G}}(x, x_0, c) = \frac{c}{\sqrt{\pi}} e^{-c^2(x-x_0)^2}, \quad \text{(A.4)}$$

where x, x_0, and c are generic variables. In the limit $c \to \infty$, these functions tend to delta functions, i.e.

$$\lim_{c\to\infty} \int_{-\infty}^{\infty} dx\, \delta_{\text{EXP}}(x, x_0, c) f(x) \to f(x_0), \quad \text{(A.5)}$$

$$\lim_{c\to\infty} \int_{-\infty}^{\infty} dx\, \delta_{\text{G}}(x, x_0, c) f(x) \to f(x_0), \quad \text{(A.6)}$$

where f is a generic function. Using the functions in Eqs. A.3 and A.4, we rewrite $F(s,t)$

$$F(s,t)\, ds\, dt = \delta_{\text{EXP}}(t^2, (w-s)^2, 2\mu\mu_+ k^2)\, dt^2 \delta_{\text{G}}\left(s, \frac{\mu w}{2\mu_+}, 2\mu_+ k\right) ds. \quad \text{(A.7)}$$

05 THERMALISATION

- Functions well approximated by delta function in some limits
  - One needs relative speed to be dominated by either DM or Target ✓
  - The other one, for large DM mass, requires Target speed to be small ✗

- Zero temperature approximation
  - Appropriate for capture
  - Both functions well approximated by delta functions

- Large mass approximation
  - Appropriate for Thermalisation of very heavy DM
  - Only one function well approximated by delta function