Delineation of cold and dense two-color QCD with linear sigma model

Daiki Suenaga (KMI, Nagoya U.)

Suenaga-Murakami-Itou-Iida; Phys.Rev.D 107, 054001 (2023)
Kawaguchi-**Suenaga**; JHEP 08, 189 (2023)
Suenaga-Murakami-Itou-Iida; Phys.Rev.D 109, 074031 (2024)
Kawaguchi-**Suenaga**; Phys. Rev. D 109, 096034 (2024)

Introduction

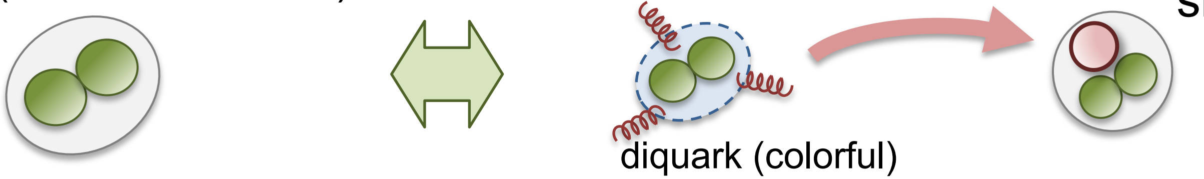
- **What is two-color QCD (QC_2D)?** = Strong interaction with $N_c = 2$

then

- Diquarks turn to be color-singlet baryons → well-defined!

diquark (hadron for $N_c = 2$)

for $N_c = 3$



singly heavy baryon (SHB) as a hadron

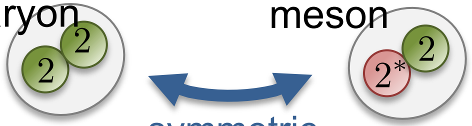
diquark (colorful)

→ Exploring diquark in QC_2D is expected to provide us with hints of unveiling SHB nature in our world

- Diquark baryons and mesons are treated in a unified way

diquark baryon

meson



$2 \simeq 2^*$:pseudoreality of color SU(2)

→ Chiral symmetry (flavor structure) is extended to $SU(2N_f)$ from $SU(N_f)_L \times SU(N_f)_R$

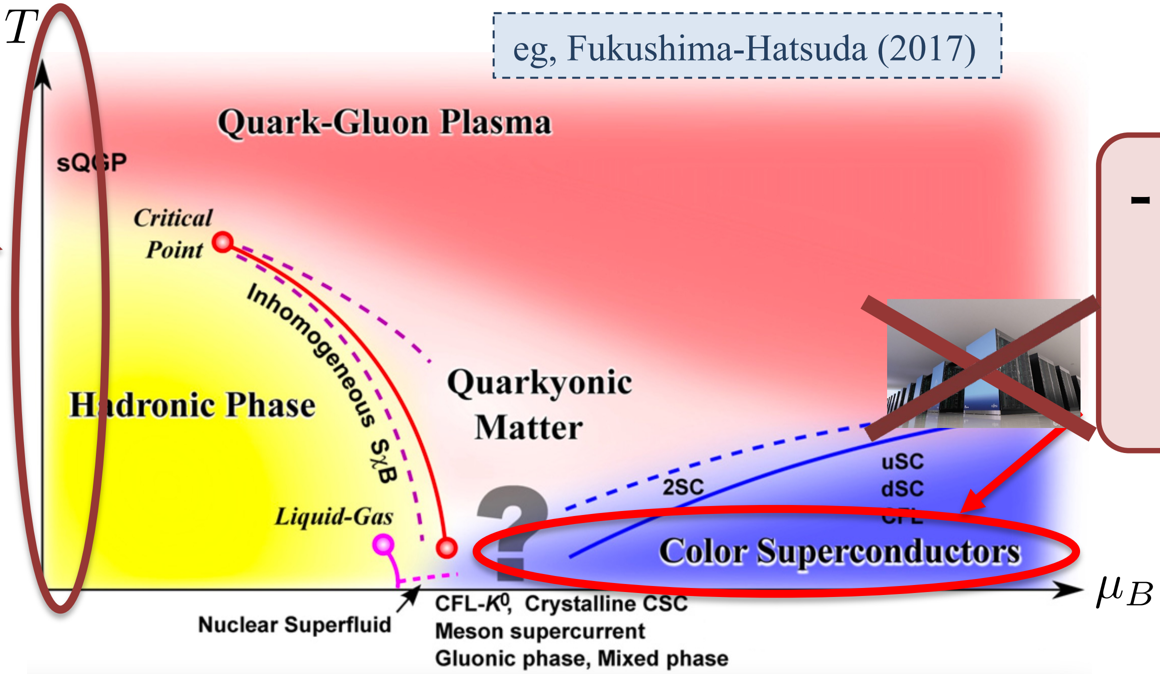
⋮

Introduction

• Why two-color QCD (QC_2D)?

- The first-principles lattice simulation is straightforwardly applicable **in cold and dense regime**

Lattice QCD



- Lattice QCD cannot apply due to the *sign problem* for $N_c = 3$

⇨ **frontier of QCD**

eg, G Aarts (2016)
K. Nagata (2022).

↔ Important for EoS of neutron stars

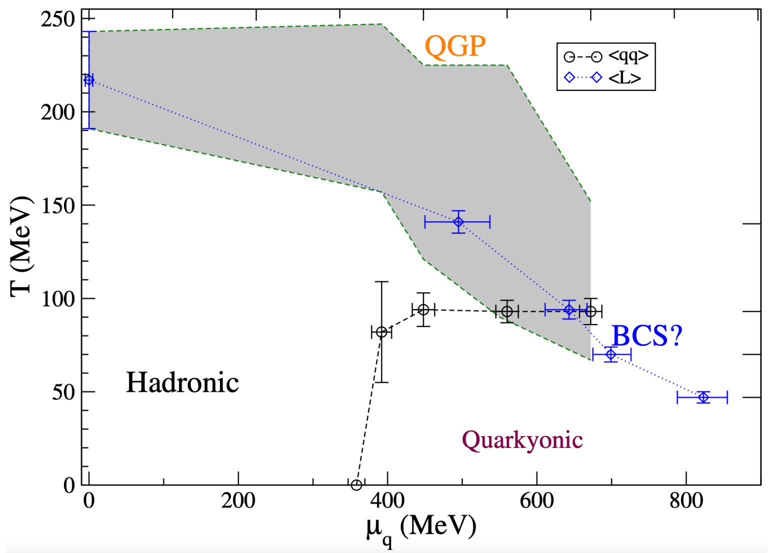
⇔ In QC_2D world, the lattice simulation is possible thanks to the pseudoreality of $SU(2)_c$

= noteworthy advantage of QC_2D

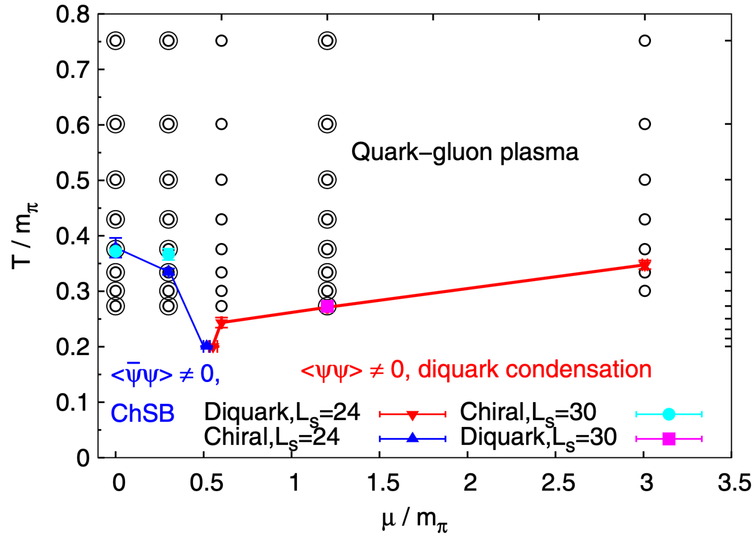
Introduction

• Phase diagram in QC_2D

- Examples of simulation results of phase diagram in QC_2D



Boz-Cotter-Fister-Mehta-Skullerud (2013)



Buividovich-Smith-Smekal (2020)

. . .

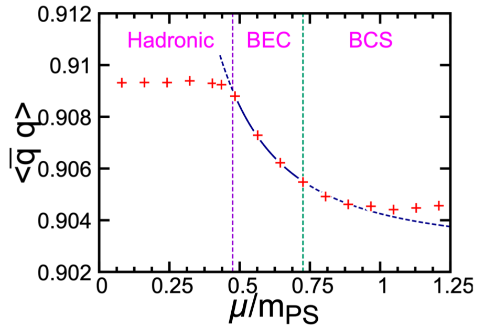
- Ireland/UK group (Hands, Skullerud, …)
- Russian group (Bornyakov, ...)
- UK group (Buividovich, ...)
- Japanese group (Iida-san, Itou-san, …),

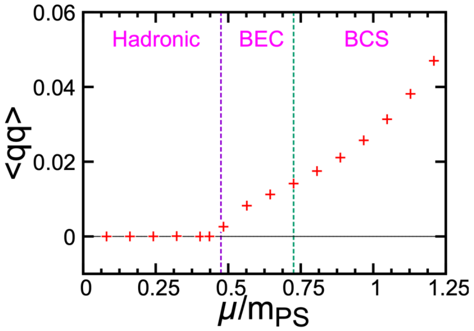
etc.

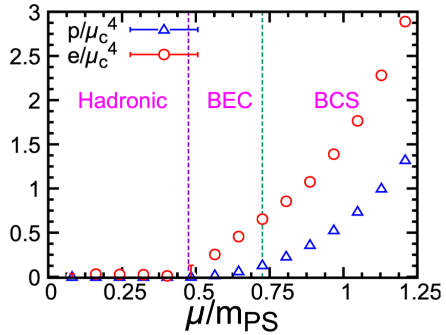
Introduction

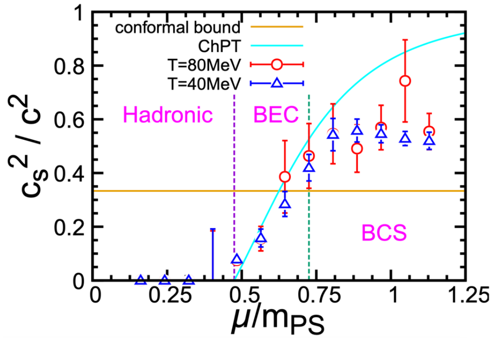
- **Lattice results**
  - In addition to phase diagram, hadron mass spectrum, gluon propagator, transport coefficient, EoS, sound velocity, $\langle\bar{\psi}\psi\rangle, \langle\psi\psi\rangle, \langle L\rangle$, etc. have been simulated

eg, Japanese group









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My approach

– (i) Regard QC_2D lattice simulations as useful "numerical experiments" of cold and dense QCD, then (ii) give interpretation from symmetry viewpoints based on effective models

My publications on QC_2D

Gluon propagator: **Suenaga**-Kojo(2019), Kojo-**Suenaga**(2021), CSE effect: **Suenaga**-Kojo(2021), Sound velocity: Kojo-**Suenaga**(2022), Kawaguchi-**Suenaga**(2024), Topological susceptibility: Kawaguchi-**Suenaga**(2023), Hadron mass: **Suenaga**-Murakami-Itou-Iida (2023, 2024), and in-preparations.

Model construction

- **Pauli-Gursey SU(4) symmetry**

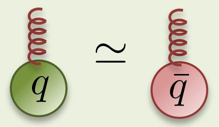
  - Pseudo reality of $SU(2)_c$ allows us to rewrite QC_2D Lagrangian with massless quarks as

$$\mathcal{L}_{\mathrm{QC_2D}} = \bar{\psi} i \not{\partial} \psi - g_s \bar{\psi} \not{A}^a T_c^a \psi = \Psi^\dagger i \partial_\mu \sigma^\mu \Psi - g_s \Psi^\dagger A_\mu^a T_c^a \sigma^\mu \Psi$$

pseudoreality: $\sigma^2 \sigma^a \sigma^2 = -(\sigma^a)^*$

In two-flavor: $\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$ with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$

Four-dimensional Pauli matrix: $\sigma^\mu = (1, \sigma^i)$



gluons are blind

  - $\mathcal{L}_{\mathrm{QC_2D}}$ is obviously invariant under $\Psi \to g\Psi$ $[g \in SU(4)]$

⇨ Pauli-Gursey SU(4) symmetry as the extended chiral symmetry   Pauli (1957), Gursey (1958)

  - $\bar{\psi}\psi = \frac{1}{2}\left(\Psi^T \sigma^2 \tau_c^2 E^T \Psi + \Psi^\dagger \sigma^2 \tau_c^2 E \Psi^*\right)$ is NOT generally invariant under SU(4) transformation, but only invariant under $\Psi \to h\Psi$ satisfying $h^T E h = E$ $[\text{then } h \in Sp(4)]$

symplectic matrix

$$E = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

⇨ chiral symmetry breaking pattern reads $SU(4) \to Sp(4)$

Model construction

• Spin-0 hadron field

- Let us introduce the following quark-bilinear 4×4 matrix

spin-singlet
color-singlet

$$\Sigma_{ij} \sim \Psi_j^T \sigma^2 \tau_c^2 \Psi_i = \begin{pmatrix} 0 & d_R^T \sigma^2 \tau^2 u_R & u_L^\dagger u_R & d_L^\dagger u_R \\ -d_R^T \sigma^2 \tau^2 u_R & 0 & u_L^\dagger d_R & d_L^\dagger d_R \\ -u_L^\dagger u_R & -u_L^\dagger d_R & 0 & d_L^\dagger \sigma^2 \tau^2 u_L^* \\ -d_L^\dagger u_R & -d_L^\dagger d_R & -d_L^\dagger \sigma^2 \tau^2 u_L^* & 0 \end{pmatrix}_{ij}$$

with $\Psi = (\psi_R, \tilde{\psi}_L)^T$
$= (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$

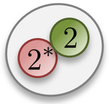
- Assignment of hadron fields

$$B \sim -\frac{i}{\sqrt{2}} \psi^T C \gamma_5 \tau_c^2 \tau_f^2 \psi \quad B' \sim -\frac{1}{\sqrt{2}} \psi^T C \tau_c^2 \tau_f^2 \psi \quad \sigma \sim \bar{\psi}\psi$$

$$a_0^a \sim \bar{\psi} \tau_f^a \psi \quad \eta \sim \bar{\psi} i \gamma_5 \psi \quad \pi^a \sim \bar{\psi} i \gamma_5 \tau_f^a \psi$$

Hadron	J^P	Quark number	Isospin
σ	0^+	0	0
a_0	0^+	0	1
η	0^-	0	0
π	0^-	0	1
B $(\bar{B})$	0^+	$+2(-2)$	0
B' $(\bar{B}')$	0^-	$+2(-2)$	0

mesons (σ, a_0, η, π)



baryons (B, B')

- 4×4 matrix Σ reads

$$\Sigma = \frac{1}{2} \begin{pmatrix} 0 & -\frac{B' - iB}{2\sqrt{2}} & \frac{\sigma - i\eta + a_0^0 - i\pi^0}{4} & \frac{a_0^+ - i\pi^+}{2\sqrt{2}} \\ \frac{B' - iB}{2\sqrt{2}} & 0 & \frac{a_0^- - i\pi^-}{2\sqrt{2}} & \frac{\sigma - i\eta - a_0^0 + i\pi^0}{4} \\ -\frac{\sigma - i\eta + a_0^0 - i\pi^0}{4} & -\frac{a_0^- - i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}' - i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+ - i\pi^+}{2\sqrt{2}} & -\frac{\sigma - i\eta - a_0^0 + i\pi^0}{4} & \frac{\bar{B}' - i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix}$$

$$\Sigma \to g \Sigma g^T \quad [g \in SU(4)]$$

cf, $\Sigma = \sigma + i\pi^a \tau^a$
for $N_c = 3$

Model construction

- **Lagrangian of Linear sigma model (LSM)**

  - (approximately) $SU(4)$-invariant LSM Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] - m_0^2 \mathrm{tr}[\Sigma^\dagger \Sigma] - \lambda_1 \left(\mathrm{tr}[\Sigma^\dagger \Sigma]\right)^2 - \lambda_2 \mathrm{tr}[(\Sigma^\dagger \Sigma)^2] + \mathrm{tr}[H^\dagger \Sigma + \Sigma^\dagger H] + c(\det\Sigma + \det\Sigma^\dagger)$$

$$D_\mu \Sigma = \partial_\mu \Sigma - i\mu_q \delta_{\mu 0}\{J, \Sigma\} \quad \text{with } J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

<u>chemical potential effect</u>

$$H = h_q E \quad \text{with } E = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

<u>current-quark mass effect</u>

$U(1)_A$ anomaly

- Advantage of LSM
  → we can see mass relation between parity partners

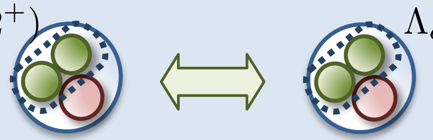
<u>parity partner</u>

$\eta, \pi \leftrightarrow \sigma, a_0$

$B(\bar{B}) \leftrightarrow B'(\bar{B}')$

<u>in $N_c = 3$ world</u>

$\Lambda_c(1/2^+)$ $\Lambda_c(1/2^-)$



(observed) (unobserved)

<u>My hope</u>

Hints from QC_2D analysis for the unobserved HQS-singlet $\Lambda_c(1/2^-)$?

Results of mean fields

- **Mean field**

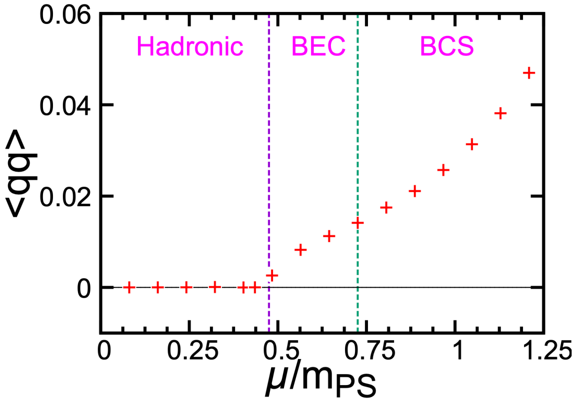
- The mean fields are $\sigma_0 \equiv \langle \sigma \rangle$ and $\Delta \equiv \left\langle \frac{B+\bar{B}}{\sqrt{2}} \right\rangle$

$\sigma_0 \sim \langle \bar{\psi}\psi \rangle$: chiral condensate

$\Delta \sim -\frac{i}{2}\langle \psi^T C\gamma_5 \tau_c^2 \tau_f^2 \psi \rangle + \mathrm{h.c.}$: diquark condensate

diquark cond. by lattice

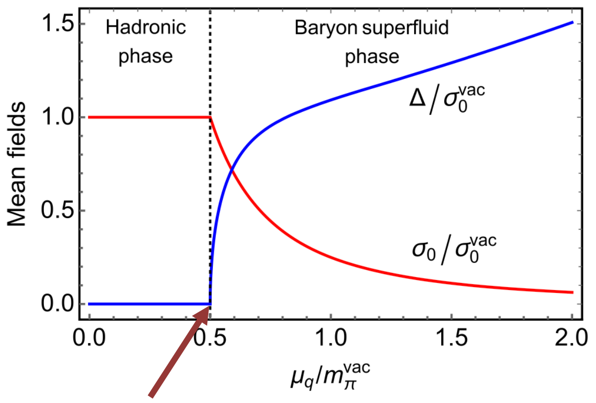
Iida et al, 2405.20566



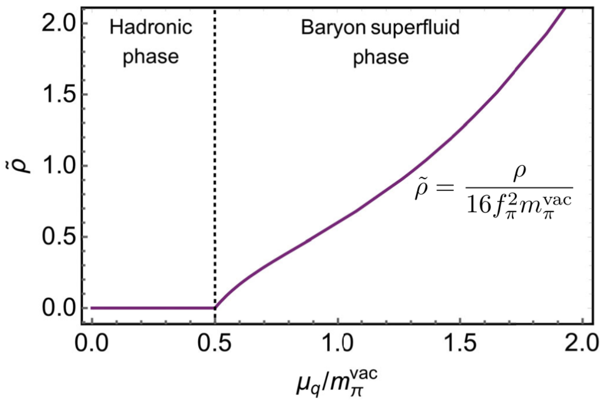
$\langle qq \rangle = 0$: hadronic phase

$\langle qq \rangle \neq 0$: baryon superfluid phase

σ_0 and Δ vs μ_q



density ρ vs μ_q



2nd order phase transition at $\mu_q = m_\pi^{vac}/2$

Input here

$\sigma_0^{vac} = 250\,\mathrm{MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{vac} = 738\,\mathrm{MeV}$

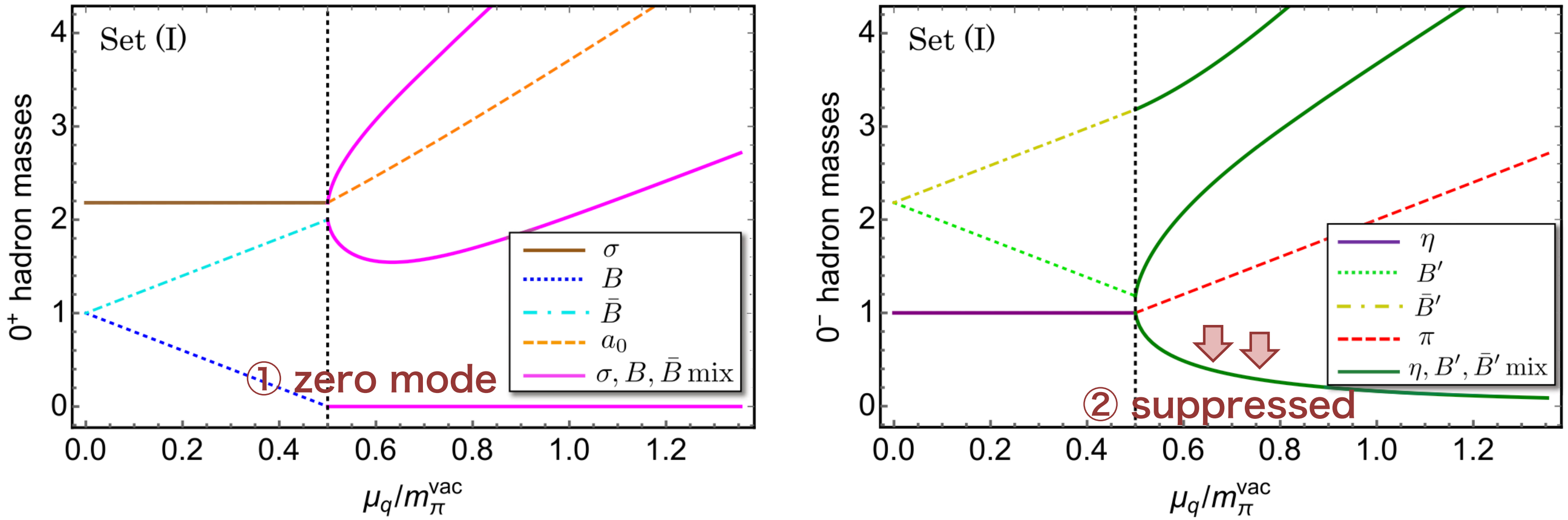
$m_{a_0}^{vac}/m_\pi^{vac} = 2.18$

} lattice, Murakami et al

Results of hadron masses

- **Results on hadron mass at finite μ_q (T=0)**

(normalized by $m_\pi^{\rm vac}$)



Input here

$\sigma_0^{\rm vac} = 250\,{\rm MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\rm vac} = 738\,{\rm MeV}$ and $m_{a_0}^{\rm vac}/m_\pi^{\rm vac} = 2.18$ } lattice, Murakami et al

- Baryon number violation in superfluid phase

$\sigma \leftrightarrow B \leftrightarrow \bar{B}$ mixing (0+ sector)

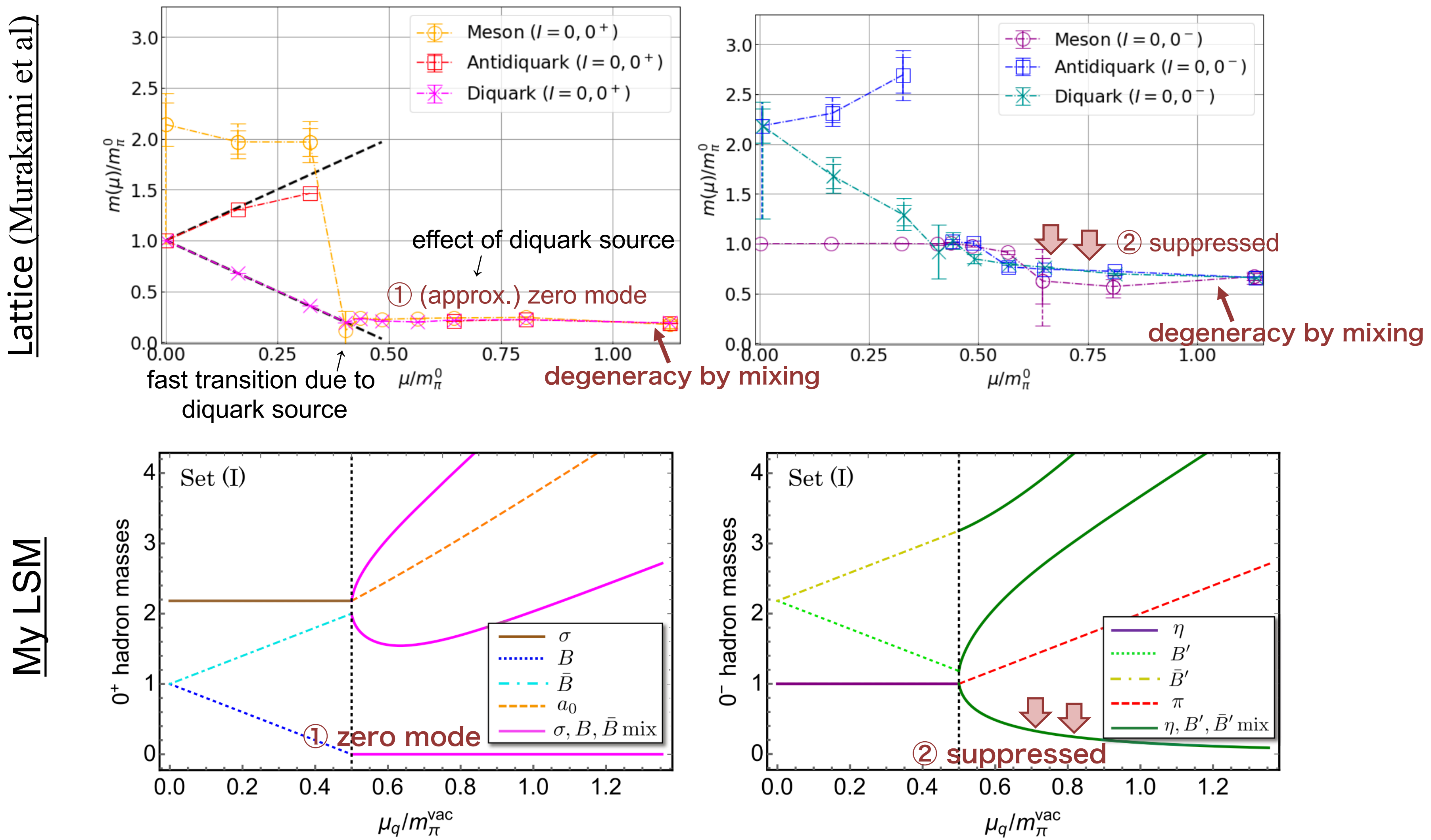
$\eta \leftrightarrow B' \leftrightarrow \bar{B}'$ mixing (0- sector)

① zero mode (NG mode of $U(1)$ baryon-number breaking)

② nonlinear suppression of " η " mass due to the mixing

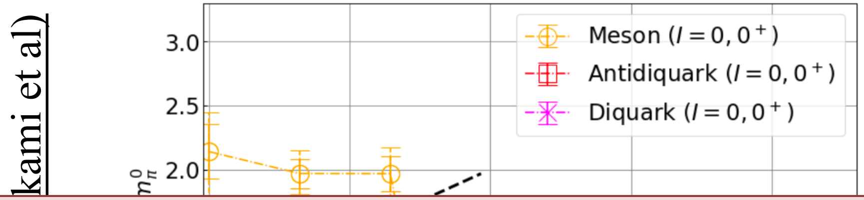
Results of hadron masses

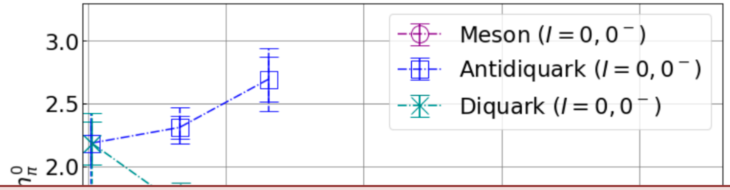
- Comparison with lattice



Results of hadron masses

- Comparison with lattice





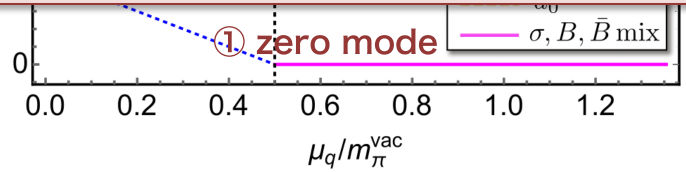
Succeeded in qualitative understanding!

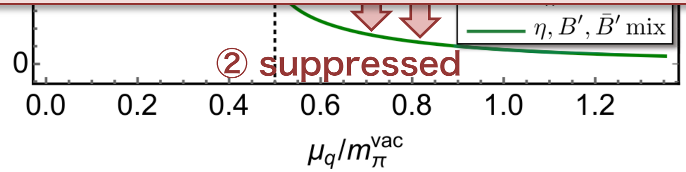
→ **Usefulness of my LSM as a tool to explore QCD-inspired symmetry aspects at density was proven**

+ Possibility of enhancement of $U(1)_A$ anomaly effects at density was discussed
[for detail: Suenaga et al. Phys. Rev. D 107, 054001 (2023)]

⇩

Now we are ready to apply my LSM to other quantities measured on the lattice



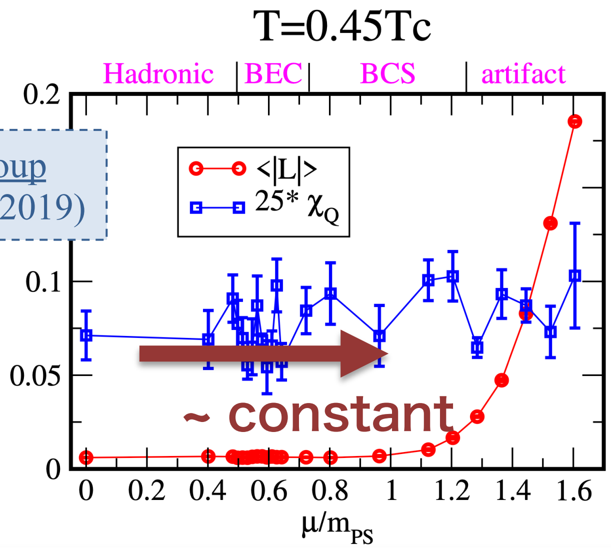


Application 1: Top. susceptibility

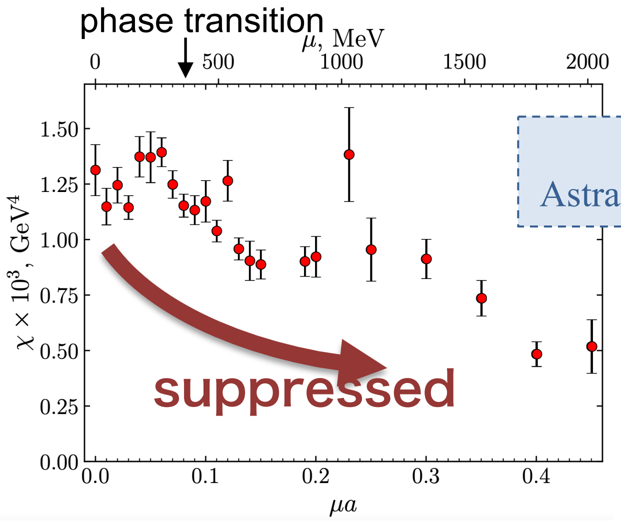
- **Topological susceptibility**

- Lattice results of topological susceptibility by two groups look inconsistent even at qualitative level

Japanese group
Iida-Itou-Lee (2019)



inconsistent



Russian group
Astrakhantsev et al (2020)

Definition of topological susceptibility

cf, Instanton, 'tHooft (1986)

$$\chi_{\rm top} = -\int d^4x \frac{\delta^2 \Gamma_{\rm QC_2D}}{\delta\theta(x)\delta\theta(0)}\bigg|_{\theta=0} = -i\int d^4x \langle Q(x)Q(0)\rangle$$ with topological operator $Q = \frac{g^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$

- We applied LSM to theoretically explore fate of $\chi_{\rm top}$ in dense QC_2D

Application 1: Top. susceptibility

- **Theoretical background of $\chi_{\rm top}$**

  - QC$_2$D generating functional with a θ-term is

$$Z_{\rm QC_2D} = \int [d\bar{\psi} d\psi][dA] \exp\left[i \int d^4x \left(\bar{\psi}(i\not{D} - m_l)\psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \theta \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}\right)\right]$$

⇩ U(1)$_A$ axial transformation $\psi \to \exp(i\theta/4\gamma_5)\,\psi$

  - θ dependence is absorbed into quark mass term via Fujikawa's method

$$Z_{\rm QC_2D} = \int [d\bar{\psi} d\psi][dA] \exp\left[i \int d^4x \left(\bar{\psi} i\not{D}\psi - m_l \bar{\psi} \exp(i\theta/2\gamma_5)\,\psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}\right)\right]$$

⇨ current quark mass

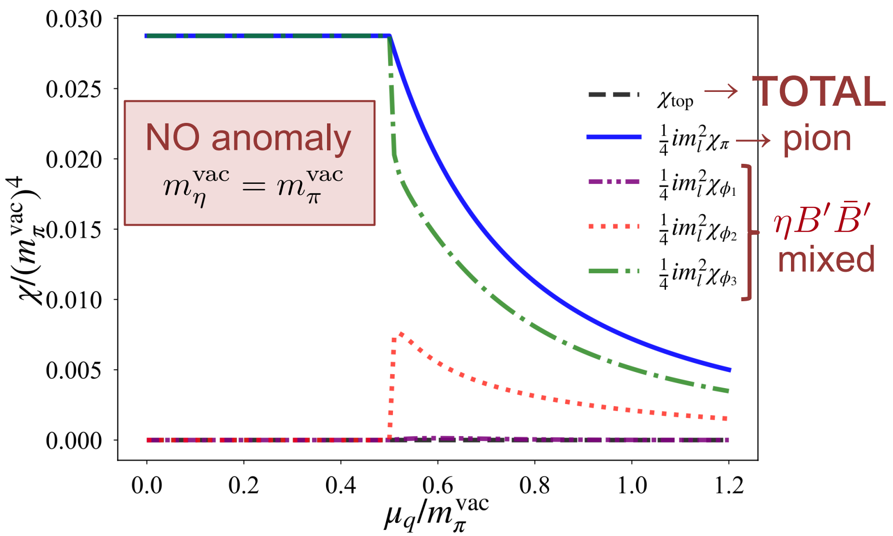
$$\chi_{\rm top} = -\int d^4x \frac{\delta^2 \Gamma_{\rm QC_2D}}{\delta\theta(x)\delta\theta(0)}\bigg|_{\theta=0} = \frac{i m_l^2}{4}(\chi_\pi - \chi_\eta) = \frac{f_\pi^2 m_\pi^2}{2}\left(1 - \frac{\chi_\eta}{\chi_\pi}\right)$$

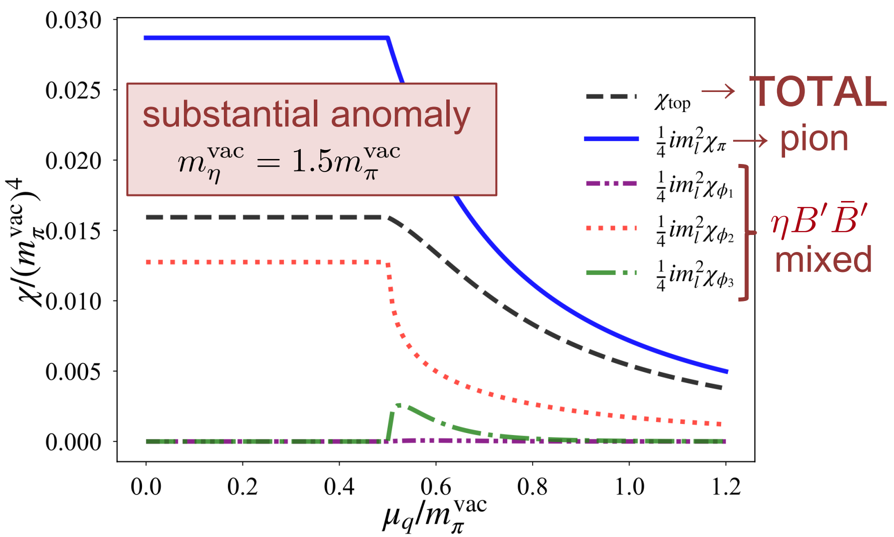
$$\begin{cases} \chi_\pi \delta^{ab} = \int d^4x \langle (\bar{\psi} i\gamma_5 \tau_f^a \psi)(x)(\bar{\psi} i\gamma_5 \tau_f^b \psi)(0)\rangle \\ \chi_\eta = \int d^4x \langle (\bar{\psi} i\gamma_5 \psi)(x)(\bar{\psi} i\gamma_5 \psi)(0)\rangle \end{cases}$$

- Matching $Z_{\rm QC_2D} = Z_{\rm LSM}$ enables us to evaluate χ_π and χ_η within LSM

Application 1: Top. susceptibility

- **Results**
  - $\chi_{\rm top}$ within LSM for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.0, 1.5$ reads



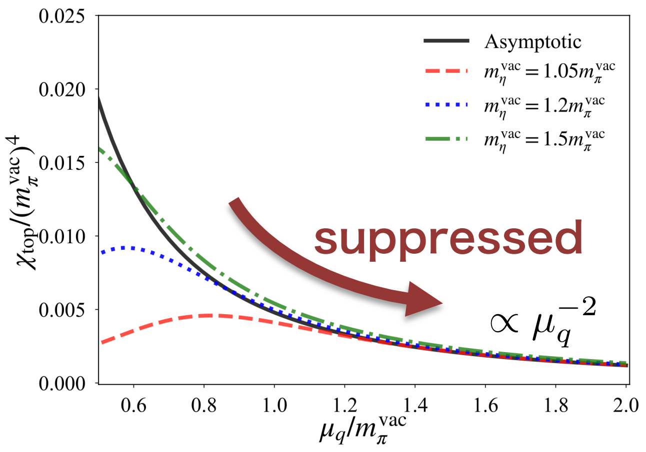


- Anomaly effect is absent $(m_\eta^{\rm vac} = m_\pi^{\rm vac})$ → $\chi_{\rm top}$ is always vanishing
- Anomaly effect is present $(m_\eta^{\rm vac} > m_\pi^{\rm vac})$ → $\chi_{\rm top}$ is positively induced
  - For $\mu_q \to \infty$, topological susceptibility asymptotically approaches zero

Application 1: Top. susceptibility

• Asymptotic behavior

- Asymptotic behavior of $\chi_{\rm top}$ for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.05,\ 1.2,\ 1.5$



- Black curve is analytic solution for large μ_q

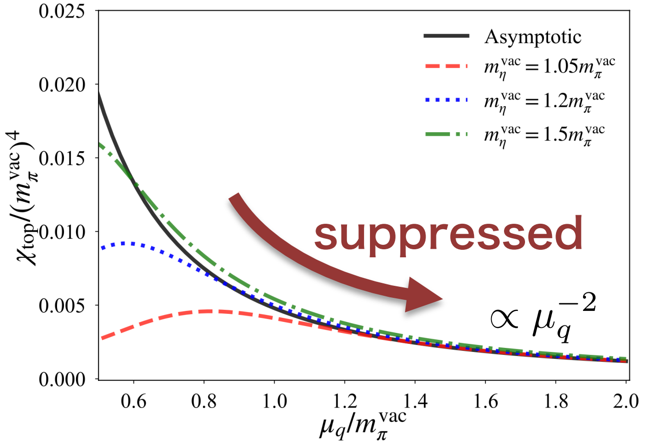
$$\chi_{\rm top} = -\frac{m_l\langle\bar\psi\psi\rangle}{4}\left(1-\frac{\chi_\eta}{\chi_\pi}\right) \to \frac{(f_\pi^{\rm vac})^2(m_\pi^{\rm vac})^4}{12}\underline{\mu_q^{-2}}$$

essentially from the chiral restoration $\sigma_0 \propto \mu_q^{-2}$

Application 1: Top. susceptibility

- **Asymptotic behavior**

- Asymptotic behavior of $\chi_{\rm top}$ for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.05,\ 1.2,\ 1.5$



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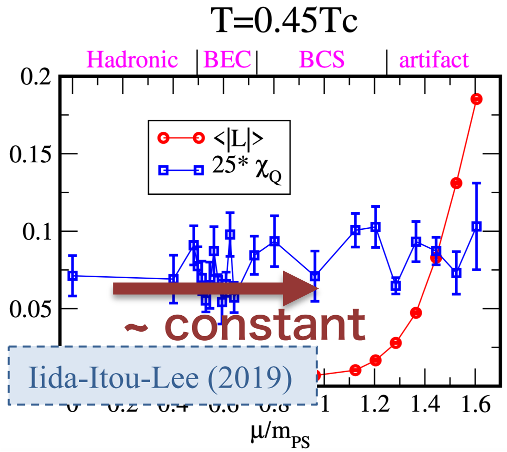
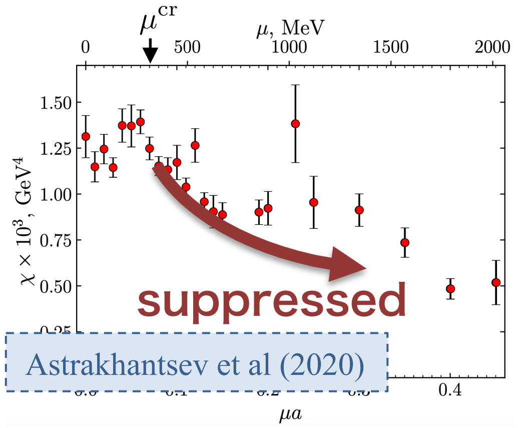
essentially from the chiral restoration $\sigma_0 \propto \mu_q^{-2}$

comparison with lattice





- Russian group result shows μ_q^{-2} behavior?
- Japanese group result could suggest enhancement of U(1) anomaly at finite density?
- When $m_\eta^{\rm vac}/m_\pi^{\rm vac}$ is not so large, my LSM result does not yield the sizable suppression

Application 2: Sound velocity

- **Sound velocity peak from chiral partners**

Universal structure: $(\text{ChPT result}) + (1/\delta\bar{m}^2_{\sigma-\pi}\ \text{contribution})$

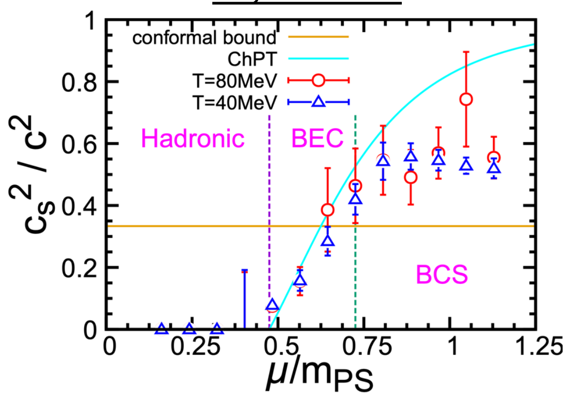
$\delta\bar{m}^2_{\sigma-\pi} = (m_\sigma^2 - m_\pi^2)/\mu_{\rm cr}^2$ with $\bar{\mu} = \mu/\mu_{\rm cr} = 2\mu/m_\pi$

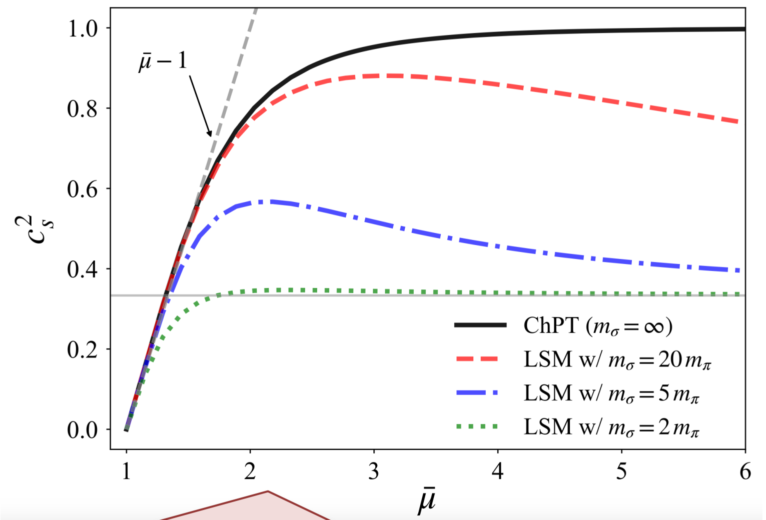
pressure: $p = f_\pi^2 m_\pi^2 \left(\bar{\mu}^2 + \frac{1}{\bar{\mu}^2}\right) + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(\bar{\mu}^2 - 1)^2\right]$

energy: $\epsilon = f_\pi^2 m_\pi^2 \left[\frac{(\bar{\mu}^2+3)(\bar{\mu}^2-1)}{\bar{\mu}^2}\right] + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(3\bar{\mu}^2+1)(\bar{\mu}^2-1)\right]$

sound velocity: $c_s^2 = \frac{(1-1/\bar{\mu}^4) + 8(\bar{\mu}^2-1)/\delta\bar{m}^2_{\sigma-\pi}}{(1+3/\bar{\mu}^4) + 8(3\bar{\mu}^2-1)/\delta\bar{m}^2_{\sigma-\pi}}$

cf, lattice

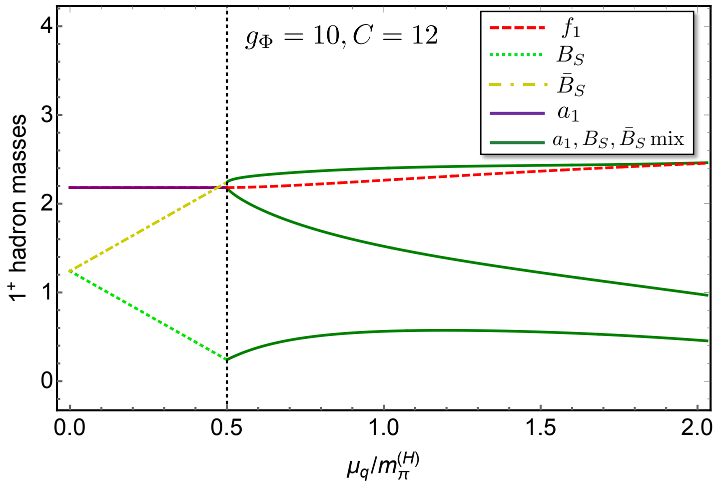


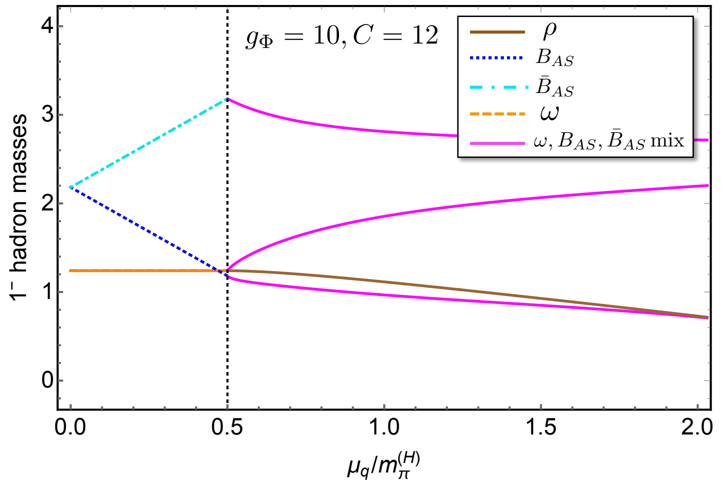


Sound velocity peak is induced thanks to the presence of σ meson

Extension of my LSM

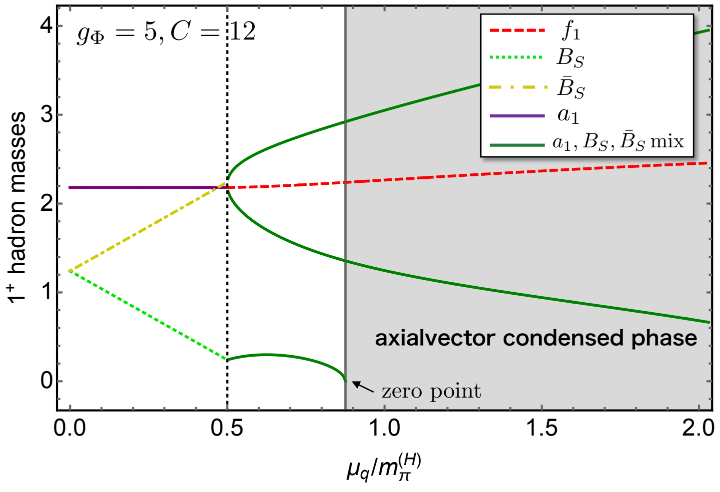
- **Extension of LSM to incorporate spin-1 hadrons**

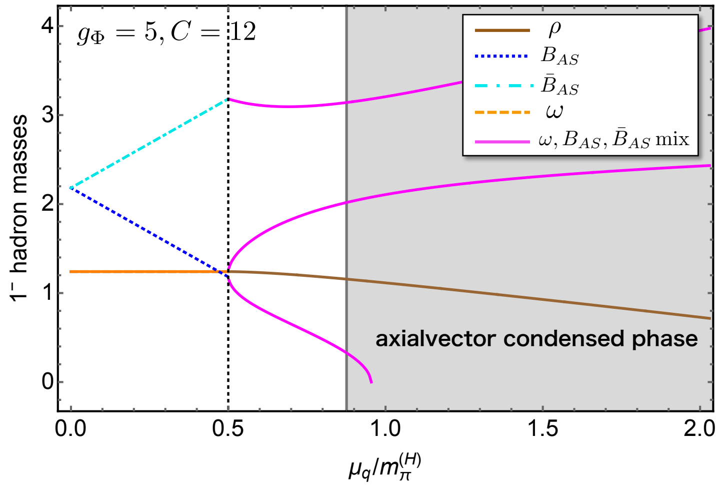




C ~ mixing strength between spin-0 and spin-1 hadrons
g_Φ ~ coupling strength among spin-1 hadrons

ρ mass reduction
→ consistent with lattice





Possibility of
vector condensate?
axialvector condensate?

cf, Lenaghan-Sannino-Splittorff (2002)

Conclusions

- I constructed Linear sigma model (LSM) in QC_2D and studied masses of spin-0 hadrons including parity partners at finite μ_q

comparison with lattice (qualitatively good)

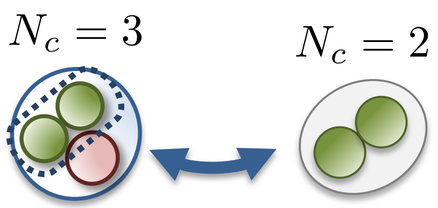
- My LSM can be regarded as a useful tool to explore cold and dense matter from symmetry viewpoint
  - (- Possibility of $U(1)_A$ anomaly enhancement at large μ_q)
  - Theoretical indication of suppression of the topological susceptibility at large μ_q
  - The sound velocity peak from chiral-partner structure
  - Model extension with spin-1 hadrons

Another direction/extension

no need to access dense medium

- Extension to 2+1 flavor at zero/finite temperature

⇨ Pursue nature of SHB in $N_c = 3$ world via diquark analysis in $N_c = 2$ world



MESSAGE: QC_2D lattice is a useful numerical experiment to focus on!