Effective field theory for $\mu \to e$ conversion in nuclei

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Aug 20, 2024

XVIth Quark Confinement and the Hadron Spectrum Conference

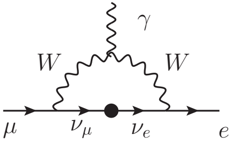
Cairns, Australia

MH, Menéndez, Noël PRL 130 (2023) 131902

Noël, MH JHEP 08 (2024) 052

Heinz, MH, Miyagi, Noël, Schwenk in preparation

Why lepton flavor violation?



- **Lepton flavor symmetry**
  - Lepton flavor conserved in SM with massless neutrinos
  - **Neutrino oscillations** sign of lepton flavor violation (LFV) in neutral sector
  - Propagates to charged sector via **mass insertions** in loops, but, e.g.,

$$\text{Br}[\mu \to e\gamma] \simeq \left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \simeq 10^{-50}$$

$\hookrightarrow$ unobservably small in SM!

  - Lepton flavor **"accidental" symmetry** of SM

  $\hookrightarrow$ LFV expected to occur for a wide range of BSM scenarios

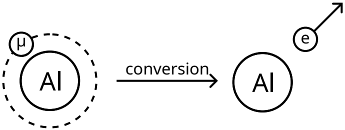
  - In practice: LFV highly sensitive null test

Why $\mu \to e$ conversion in nuclei?

LFV process	current limit on Br	(planned) experiments
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	MEG II
$\mu \to 3e$	$< 1.0 \times 10^{-12}$ SINDRUM	Mu3e
$\tau \to \ell\gamma, 3\ell, \ell P, \ldots$	$\lesssim 10^{-8}$ Belle, LHCb, ...	Belle 2, ...
$K \to \mu e, \mu e\pi, \mu e\pi\pi$	$\lesssim 10^{-11}$ KTeV, NA62, BNL	KOTO, LHCb
$\pi^0 \to \bar{\mu}e$	$< 3.6 \times 10^{-10}$ KTeV	
$\eta \to \bar{\mu}e$	$< 6 \times 10^{-6}$ SPEC	JEF, REDTOP (?)
$\eta' \to \bar{\mu}e$	$< 4.7 \times 10^{-4}$ CLEO II	
Au $\mu^- \to$ Au e^-	$< 7 \times 10^{-13}$ SINDRUM II	
Ti $\mu^- \to$ Ti e^-	$< 6.1 \times 10^{-13}$ SINDRUM II	
Al $\mu^- \to$ Al e^-	$\lesssim 10^{-17}$ (projected)	Mu2e, COMET

$\hookrightarrow$ major experimental improvements expected in coming years!

What is $\mu \to e$ conversion?



- What is $\mu \to e$ conversion? A theorist's perspective:
  - Muon bound in $1s$ level of nucleus
  - Muon converts to electron within Coulomb field of nucleus

$$\bar{e}\sigma^{\alpha\beta}\mu F_{\alpha\beta}, \qquad \bar{e}\mu\,\bar{q}q, \qquad \dots$$

  $\hookrightarrow$ both long- and short-range BSM mechanisms possible

  - Electron ejected with energy of muon–electron mass difference (minus binding energy)

  $\hookrightarrow$ **very clear experimental signature**

  - Background: muon decay in orbit $\mu \to e\nu_\mu\bar{\nu}_e$
  - Normalization: muon capture $\mu(A, Z) \to \nu_\mu(A, Z-1)$

Why effective field theory?

- For theory description, many different scales matter:
  - **BSM scale**: LFV operators $\Rightarrow$ Standard Model EFT (SMEFT)
  - **Electroweak scale**: integrate out W, Z $\Rightarrow$ low-energy EFT (LEFT)
  - **Hadronic scales**: $\Lambda_\chi \simeq 4\pi F_\pi \simeq m_N$ $\Rightarrow$ chiral perturbation theory
  - **Nuclear scales**: $M_\pi \simeq k_F \simeq \gamma$ $\Rightarrow$ chiral EFT, pionless EFT
  - **Atomic scales**: atomic binding $\Rightarrow$ Dirac equation, Coulomb distortions
- Objectives of EFT approach:
  - **Compare different probes of LFV**

    $\hookrightarrow$ example: $\mu \to e$ conversion in nuclei vs. $P \to \bar{\mu}e$ decays, $P = \pi^0, \eta, \eta'$
  - Discriminate among different underlying BSM operators
  - **Control theoretical uncertainties**

    $\hookrightarrow$ hadronic matrix elements, nuclear responses, Coulomb distortions
  - RG corrections

Master formulae

Very schematic master formula for $\mu \to e$ conversion

$$\Gamma[\mu(A,Z) \to e(A,Z)] \simeq \text{BSM Wilson coefficients} \otimes \text{hadronic matrix elements} \otimes \text{nuclear responses} \otimes \text{bound-state solution}$$

- Latter two traditionally combined into **overlap integrals** Kitano et al. 2002

$$S^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r) [g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) - f^{e}_{-1}(r) f^{\mu}_{-1}(r)] \qquad V^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r) [g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) + f^{e}_{-1}(r) f^{\mu}_{-1}(r)]$$

$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r) [g^{e}_{-1}(r)\, f^{\mu}_{-1}(r) + f^{e}_{-1}(r)\, g^{\mu}_{-1}(r)]$$

  $\hookrightarrow$ covers coherently enhanced **spin-independent** responses, $\Gamma_{\text{SI}} \propto \#N^2$

- Similarly, dominant **spin-dependent** contribution from nuclear responses finite for $q = 0$, e.g., for axial-vector operators $\Gamma_{\text{SD}} \propto g_A^2$ Davidson et al. 2018
- Further subleading responses from **multipole decomposition** Serot 1978

  $\hookrightarrow$ need to combine with solution of Dirac equation see below and following talk by Evan Rule

Application: indirect limits for $P \to \bar{\mu} e$

- Same operators probed in SD $\mu \to e$ conversion and $P \to \bar{\mu} e$ decays Gan et al. 2022

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2} \sum_{\substack{Y=L,R \\ q=u,d,s}} \Big[C_Y^{P,q} (\overline{e_Y} \mu)(\bar{q}\gamma_5 q) + C_Y^{A,q} (\overline{e_Y}\gamma^\mu \mu)(\bar{q}\gamma_\mu\gamma_5 q) \Big] + \frac{i\alpha_s}{\Lambda^3} \sum_{Y=L,R} C_Y^{G\tilde{G}} (\overline{e_Y}\mu) G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a + \text{h.c.}$$

$\hookrightarrow$ **axial-vector, pseudoscalar, $G\tilde{G}$ operators**

- Can use $\mu \to e$ conversion limits to derive indirect limits for $P \to \bar{\mu} e$
- In general, not the same linear combinations appear

  $\hookrightarrow$ consider first a single operator at a time, account for matrix elements

$\mu \to e$ (exp)	$P \to \bar{\mu} e$ (derived)	current limit
	$\text{Br}[\pi^0 \to \bar{\mu} e] \lesssim 4 \times 10^{-17}$	$< 3.6 \times 10^{-10}$
$\text{Br}[\mu\text{Ti} \to e\text{Ti}] < 6.1 \times 10^{-13}$	$\text{Br}[\eta \to \bar{\mu} e] \lesssim 5 \times 10^{-13}$	$< 6.0 \times 10^{-6}$
	$\text{Br}[\eta' \to \bar{\mu} e] \lesssim 7 \times 10^{-14}$	$< 4.7 \times 10^{-4}$

- General sensitivity orders of magnitude better, but could there be cancellations?

Indirect limits: impact of cancellations

- For a rigorous limit one needs to **scan over all Wilson coefficients**

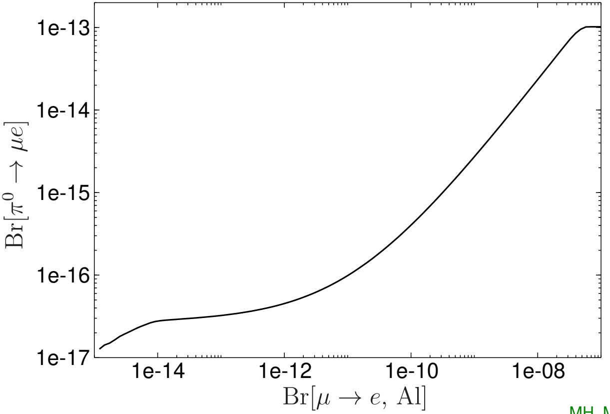
  $\hookrightarrow$ there are (fine-tuned) scenarios in which the $\mu \to e$ rate vanishes exactly
- For $\pi^0 \to \bar{\mu} e$: decay rate vanishes as well!

  rigorous limit: $\text{Br}[\pi^0 \to \bar{\mu} e] < 1.0 \times 10^{-13}$ (direct limit: $< 3.6 \times 10^{-10}$)
- For $\eta, \eta' \to \bar{\mu} e$: in principle, no strict limits, but required cancellations easily lifted by RG corrections
- RG produces SI operators, even when only starting with SD ones at high scale

  Cirigliano et al. 2017, Crivellin et al. 2017

$$C_Y^{V,q} \simeq -3 Q_q \frac{\alpha}{\pi} \log \frac{M_W}{m_N} C_Y^{A,q}$$

Future projection for $\pi^0 \to \bar{\mu}e$



MH, Menéndez, Noël 2023

- Combining Al and Ti limits, the single-operator sensitivity is restored

  ↪ **complementarity of different target nuclei**

Uncertainty quantification for nuclear responses

- So far: nuclear responses calculated in (phenomenological) **nuclear shell model**

  $\hookrightarrow$ uncertainty estimates difficult, especially for neutron responses
- **Ab-initio approaches**
  - Often uncertainties dominated by chiral Hamiltonian, not by many-body solution
  - Often correlations between different responses much more stable

    Hagen et al. 2015, Payne et al. 2019
- Need for **charge distributions with quantified uncertainties**
  - Solution of the Dirac equation
  - Input for correlation analysis in ab-initio approaches
- Charge distributions extracted from electron scattering **without uncertainties (!)**

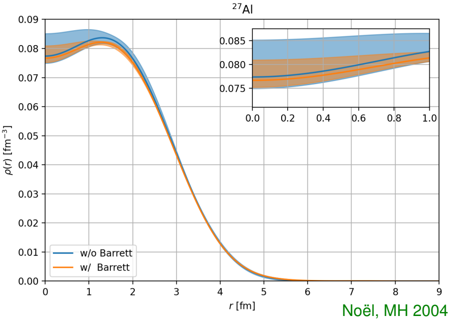
  $\hookrightarrow$ **Fourier–Bessel expansions** Dreher et al. 1974, de Vries et al. 1987

$$\rho(r) = \begin{cases} \sum_{n=1}^{N} a_n j_0(q_n r) & r \leq R \\ 0 & r > R \end{cases} \quad \text{with} \quad q_n = \frac{\pi n}{R}$$

Extracting charge distributions from electron scattering

- Practical challenges of re-analysis:
  - Most data taken in the 70s+80s
  - Many data sets not available at all ("private communication"), or only published in PhD theses
  - Documentation of uncertainties rudimentary
- Propagation of uncertainties computationally intensive
  $\hookrightarrow$ truncation errors in R, N
- Carried this program out for ^{27}Al, ^{40,48}Ca, and ^{48,50}Ti
- Results available as `python` notebook

2406.06677



Noël, MH 2004

Solving the Coulomb problem

- For a realistic description, need to resum **Coulomb phase shifts**

  $\hookrightarrow$ phase shift model

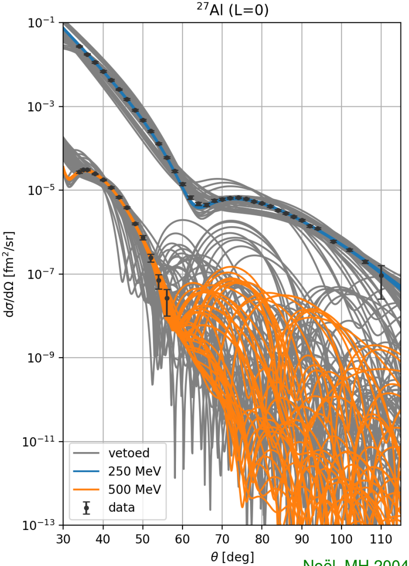
- Fits carried out over large grid of (N, R), using veto on oscillations and asymptotics to prevent overparameterization

- Constraints from **Barrett moments** measured in $2p \to 1s$ transitions of muonic atoms

$$\left\langle r^k e^{-\alpha r}\right\rangle = \frac{4\pi}{Z}\int_0^\infty dr\, r^{k+2}\rho(r)e^{-\alpha r}$$

- Similar approach also applies to Coulomb corrections elsewhere

  $\hookrightarrow$ parity-violating electron scattering



Noël, MH 2004

Results for dipole overlap integrals

Dipole overlap integrals

$D(^{27}\mathrm{Al}) = 0.0359(2)$ $D(^{40}\mathrm{Ca}) = 0.07531(5)$ $D(^{48}\mathrm{Ca}) = 0.07479(10)$

$D(^{48}\mathrm{Ti}) = 0.0864(1)$ $D(^{50}\mathrm{Ti}) = 0.0870(3)$

- Dipole overlap integrals

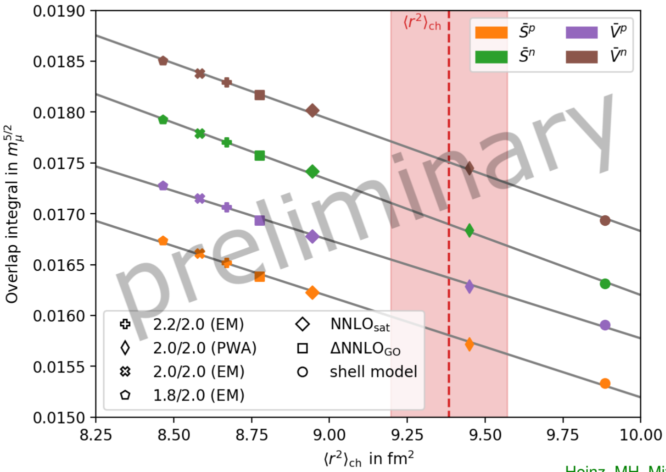
$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r) \big[g^e_{-1}(r)\, f^\mu_{-1}(r) + f^e_{-1}(r)\, g^\mu_{-1}(r) \big]$$

$\hookrightarrow$ only depends on charge distributions, electric field $E(r) = \frac{\sqrt{4\pi\alpha}}{r^2} \int_0^r dr'\, r'^2 \rho(r')$

- For the first time, **fully quantified uncertainties**
- For $S^{(N)}$, $V^{(N)}$ other nuclear responses contribute

$\hookrightarrow$ interplay with ab-initio methods

Towards ab-initio calculations for $\mu \to e$ conversion



Heinz, MH, Miyagi, Noël, Schwenk in preparation

- Calculations performed using VS-IMSRG for ^{27}Al see talk by Matthias Heinz, We, 14:00 (parallel E)
- Shell-model result also falls onto correlation

Full master formula: subleading nuclear responses

- To improve SD and even higher responses:
  - **Multipole decomposition** of one-body terms known with respect to fixed momentum transfer q Serot 1978
  - Here: need to combine with **bound-state physics**
    $\hookrightarrow$ perform Fourier transform of leptonic current numerically
  - Uncertainty quantification using ab-initio techniques in combination with data input
- **Two-body corrections**
  - Known to be important for some channels, e.g., **scalar operators** Cirigliano et al. 2022
  - So far evaluated within approximations ("normal ordering" in Fermi gas)
  - Techniques for multipole decomposition of two-body currents becoming available

Conclusions

- **EFT for $\mu \to e$ conversion in nuclei**
  - Discriminate LFV mechanisms
  - Controlled uncertainty estimates
- **Recent results**
  - Indirect limits for $P \to \bar{\mu} e$
  - Error propagation for charge densities
- **Outlook**
  - Overlap integrals from ab-initio calculations
  - Subleading nuclear responses
  - Two-body currents
  - Discriminatory power for different target nuclei and together with other LFV probes