Polyakov loop, QCD thermodynamics and observables

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XVIth Quark Confinement and the Hadron Spectrum

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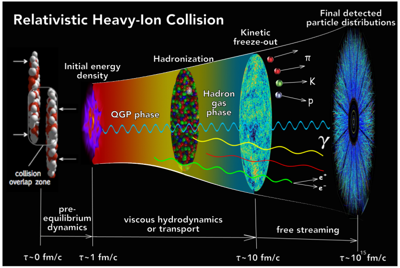
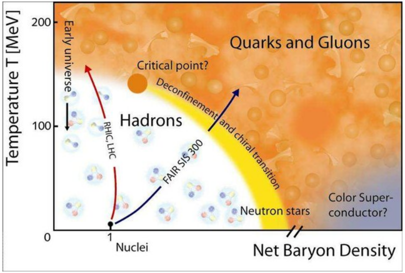
- **Thermodynamics** of QCD – quark-gluon matter in heavy-ion collisions, evolution of the early Universe, the origin of visible mass, etc.
- QCD phase structure – continuous efforts from first-principles calculations; running and planned experiment at RHIC-STAR, FAIR-GSI, NICA, HIAF...
- **Observables** to connect theory and experiment – particle yields, fluctuations, polarisation, (collective) flows, jets, · · ·

The Present and Future of QCD, QCD Town Meeting White Paper, 2023.

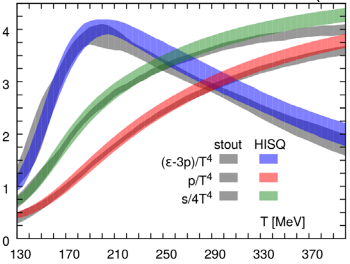
G. Aarts, J. Phys. Conf. Ser. 706: 022004 (2016)



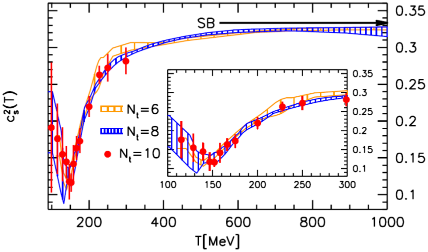


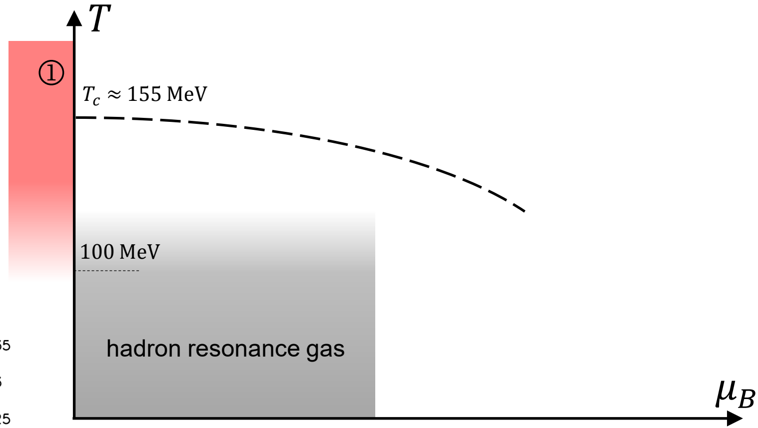
QCD equation of state (EoS)

① hotQCD Collab. PRD 90: 094503 (2014)



Borsanyi et al. JHEP 11, 077 (2010)



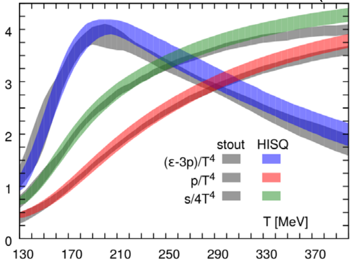


QCD pressure; at zero μ_B:

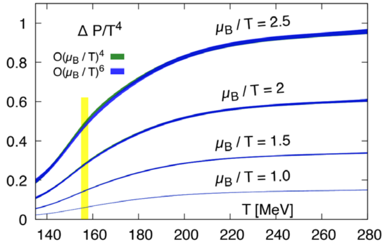
$$P(T,0)$$

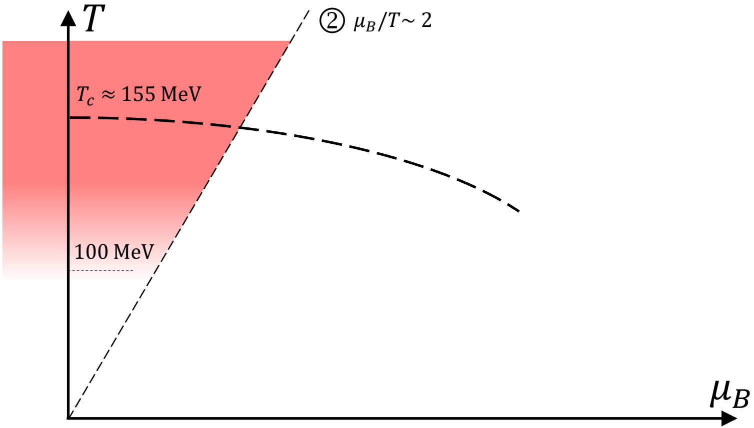
QCD equation of state (EoS)

hotQCD Collab. PRD 90: 094503 (2014)



② **Taylor expansion,** e.g.:
hotQCD Collab. PRD 105: 074511 (2022)

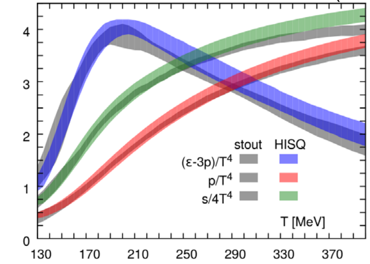




finite $\hat{\mu}_B = \mu_B/T$:

$$P(T, \hat{\mu}_B) = P(T, 0) + \frac{c_2(T)}{2!}\hat{\mu}_B^2 + \frac{c_4(T)}{4!}\hat{\mu}_B^4 + \cdots$$

hotQCD Collab. PRD 90: 094503 (2014)



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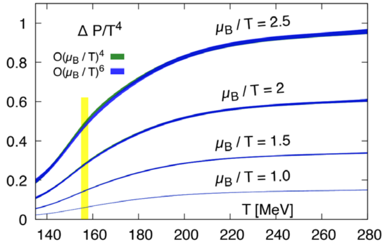
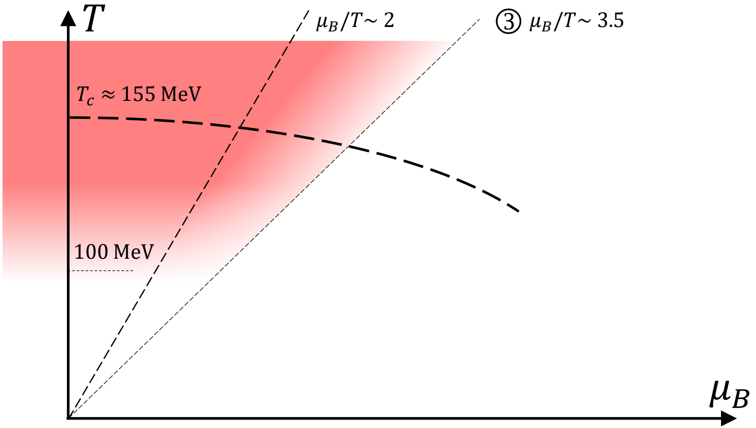
hotQCD Collab. PRD 105: 074511 (2022)





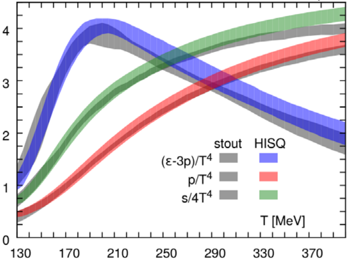
③ **T' expansion:** Borsanyi et al. (WB Collab.) PRL 126: 232001 (2021)
Pade: hotQCD Collab. PRD 105: 074511 (2022)

$$P(T,\hat{\mu}_B) = P(T,0) + \Delta P(T,\hat{\mu}_B)$$

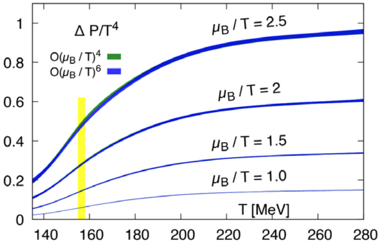
methods with improved converges radius.

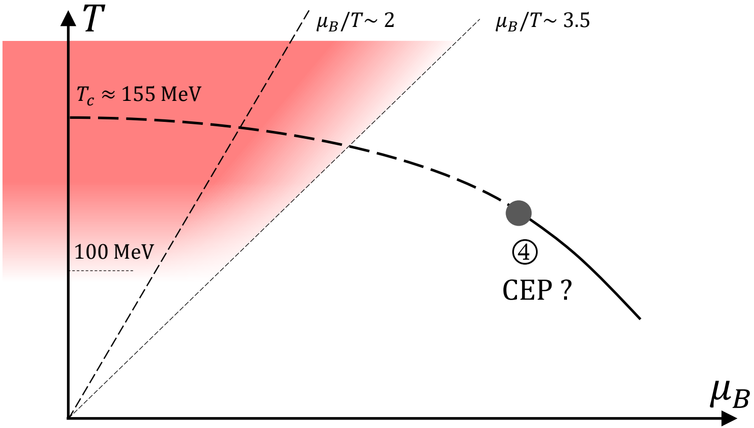
QCD equation of state (EoS)

hotQCD Collab. PRD 90: 094503 (2014)



Taylor expansion, e.g.:
hotQCD Collab. PRD 105: 074511 (2022)





T' expansion: Borsanyi et al. (WB Collab.) PRL 126: 232001 (2021)
Pade: hotQCD Collab. PRD 105: 074511 (2022)

④ **Estimated CEP** still afar at μB ~ **600 MeV**: (see also in F. Rennecke's talk) **discrete (lattice)** - Clarke et al., 2405.10196, and **continuum (functional QCD)** – Fu et al, PRD (2019), Gao et al., PLB (2020), Gunkel et al. PRD (2021)

Thermodynamics at finite temperature and density

$$P(T,\mu_B) = P(T,0) + \int_0^{\mu_B} n_B(T,\mu)\, d\mu, \qquad n_B = \frac{1}{3}(n_u + n_d + n_s + \cdots).$$

General thermodynamics: $n_q = \langle \bar{q}\gamma_4 q\rangle = -T\sum_{\omega_p}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\mathrm{tr}_{C,D}\left[\gamma_4 G_q(\tilde{p})\right].$

Quark propagator in nonperturbative QCD ($\tilde{p} = (\boldsymbol{p}, \omega_p + \mathrm{i}\mu_q)$):

$$G_q^{-1}(\tilde{p}) = \mathrm{i}(\omega_p + \mathrm{i}\mu_q + gA_4)\gamma_4\, Z_q^E(\tilde{p}^2) + \mathrm{i}\boldsymbol{\gamma}\cdot\boldsymbol{p}\, Z_q^M(\tilde{p}^2) + Z_q^E M_q(\tilde{p}^2),$$

universal in both discrete and continuum approaches.

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Quark propagator in nonperturbative QCD ($\tilde{p} = (\boldsymbol{p}, \omega_p + \mathrm{i}\mu_q)$):

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universal in both discrete and continuum approaches.

- chiral phase transition: quark dynamical mass $M_q(\tilde{p}^2)$ (essentially $\langle\bar{q}q\rangle$);
- confinement - deconfinement: A_4, i.e. the **Polyakov loop**:

$$\mathcal{L}[A_4] = \frac{1}{N_c}\mathrm{tr}\,\mathcal{P}\exp\left(\mathrm{i}g\int_0^{\beta} dx_4\, A_4\right).$$

I. Phenomenological method
– combine with phase transition studies

YL, F. Gao, B. C. Fu, H. C. Song, Y. X. Liu, Phys. Rev. D 109: 114031 (2024).

Consider the nonperturbative regime of the momentum:

$$Z_q^{E,M}(\tilde{p}^2) = Z_q^{(0)} + \mathcal{O}(\tilde{p}^2), \qquad M_q(\tilde{p}^2) = M_q^{(0)} + \mathcal{O}(\tilde{p}^2),$$

the leading contribution suggests a phenomenological construction of G_q in terms of O_X:

$$O_\chi = M_q^{(0)}, \quad O_A = \mathcal{L},$$

$$\left[G_q^{(0)}(\tilde{p})\right]^{-1} = Z_q^{(0)} \left(\mathrm{i}(\omega_p + \mathrm{i}\mu_q + gA_4[O_A])\gamma_4 + \mathrm{i}\gamma \cdot \boldsymbol{p} + M_q^{(0)}[O_\chi]\right);$$

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With frequency-independent $M_q^{(0)}$, the (renormalised) thermodynamic quantities become analytic functions in terms of the order parameters, e.g.:

$$n_q(T, \mu_q) = 2N_c T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f_q(M_q^0, \mathcal{L}, \mathcal{L}^\dagger; \boldsymbol{p}, T, \mu_q).$$

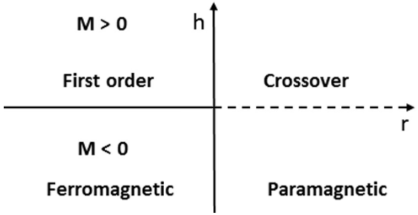
Knowledge on the phase structure $\rightarrow$ thermodynamics.

Combine with phase transition studies - M_q

Phenomenology around CEP is captured by 3D Ising parametrisation - Z(2) [1,2]:

$$\mathcal{M}_{\text{Ising}} = \mathcal{M}_0 R^{\beta}\theta,$$
$$h = h_0 R^{\beta\delta}\theta(1 + a\theta^2 + b\theta^4),$$
$$r = R(1-\theta^2).$$

Map $\mathcal{M}_{\text{Ising}} \in (-1,1)$ to $M_q^{(0)} \in (M_q^{vac.}, 0)$: linear.



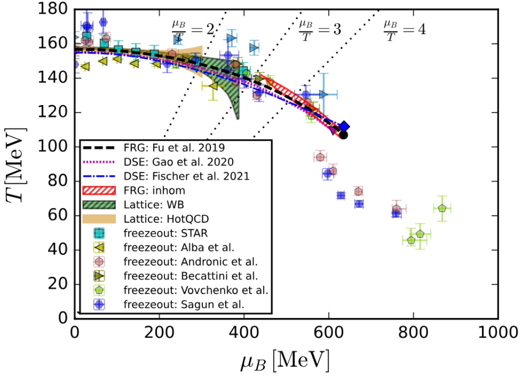
[1] Parotto et.al. PRC 101: 034901 (2020).

[2] Kahangirwe et.al. PRD 109: 094046 (2024).

Modified (non-linear) map $(r,h) \leftarrow (T,\mu_B)$:

constrained by the up-to-date phase transition line: $T_c/T_0 = 1 - \kappa_2 (\mu_B/T_0)^2 + \kappa_4 (\mu_B/T_0)^4 + \cdots$.

(Plot: Fu, Commun. Theor. Phys. 74: 097304 (2022).)



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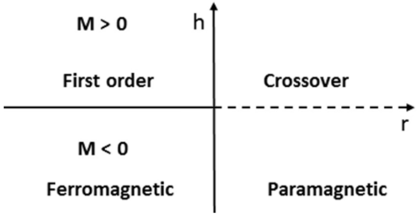

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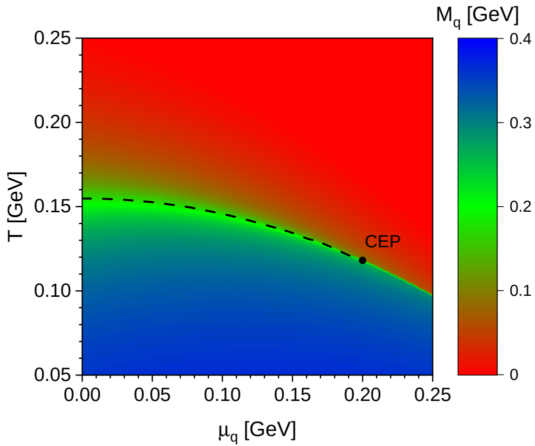
[2] Kahangirwe et.al. PRD 109: 094046 (2024).

Modified (non-linear) map $(r, h) \leftarrow (T, \mu_B)$:

constrained by the up-to-date chiral phase transition line.



Not all crystal clear, especially at finite density... preliminary study from known inputs.

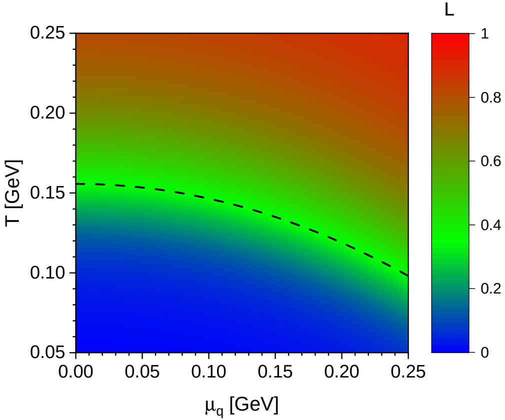
Zero μ_B: data taken from functional RG study [1].

Finite μ_B: T' parametrisation [2]

$$T'/T_0 = T/T_0 + \kappa_2(\mu_B/T_0)^2,$$
$$\mathcal{L}(T,\mu_B) = \mathcal{L}^\dagger(T,\mu_B) = \mathcal{L}(T',0);$$

κ_2: close connection between the chiral and deconfinement phase transition (see e.g. [3]);

consequence of taking $\mathcal{L} \simeq \mathcal{L}^\dagger$ is reflected in [4].



[1] Fu and Pawlowski, PRD 92: 116006 (2015).

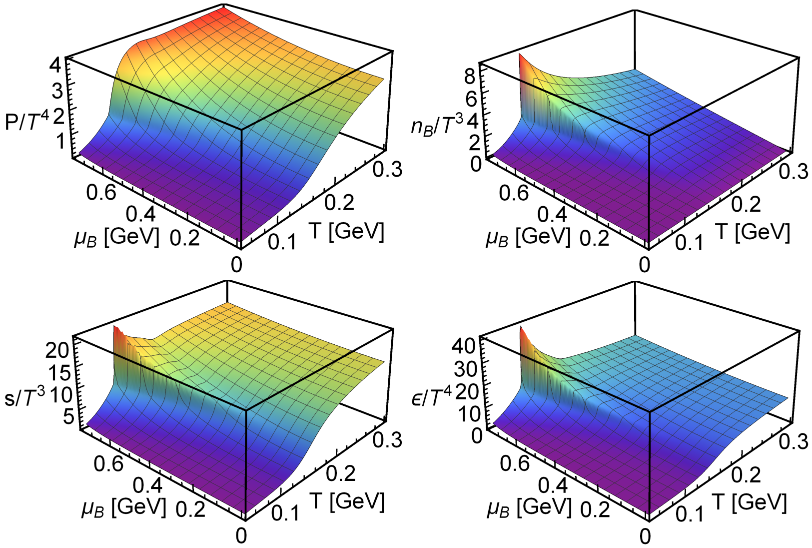
[2] Borsanyi et.al. (WB Collab.), PRL 126: 232001 (2021).

[3] Fischer et al., PRD 90: 034022 (2014).

[4] Fu et al., PRD 101: 054032 (2020).

2+1-flavor QCD EoS from the order parameters

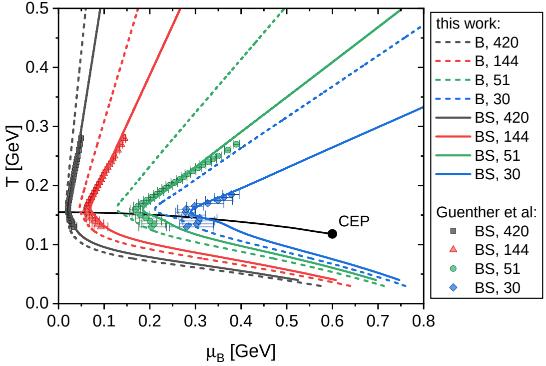
Github: EoS-PhaseDiagramMap

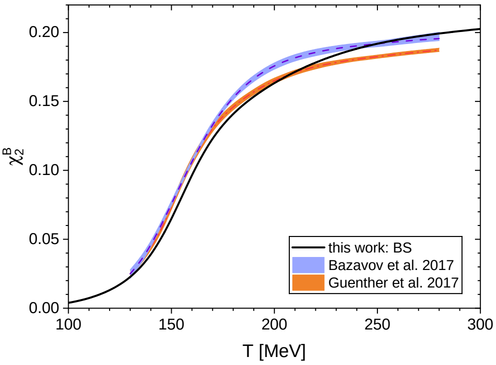


Isentropic trajectories: $s/n_B = const.$, compare with lattice QCD (left); compare with hadron resonance gas (HRG) model at low T: Zheng et al., 2407.03795.

Baryon number fluctuations: $\chi_k^B = \frac{\partial^k (P/T^4)}{\partial(\mu_B/T)^k} = \frac{\partial^{k-1}(n_B/T^3)}{\partial(\mu_B/T)^{k-1}}$,

provides quantitative check for the limitations: $\partial_T^i \partial_{\mu_B}^j P$ up to $i + j \leq 2$.

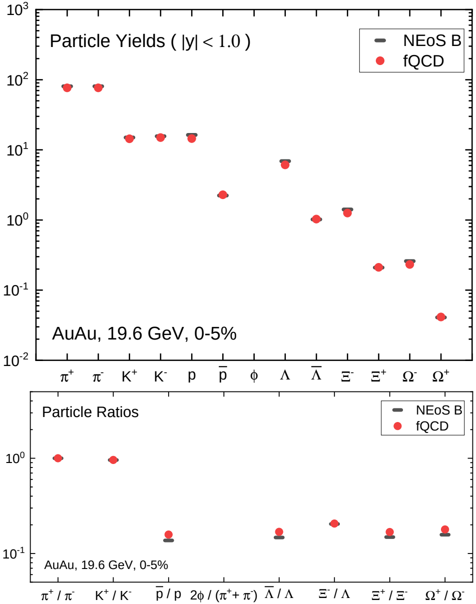




II. Impacts of Polyakov loop on the observables in quark-gluon matter

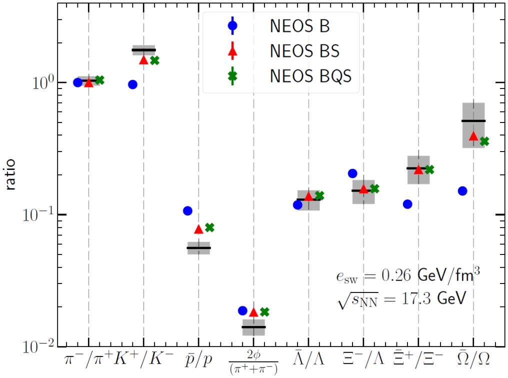
YL, F. Gao, B. C. Fu, H. C. Song, Y. X. Liu, Phys. Rev. D 109: 114031 (2024).

F. Gao, J. Harz, C. Hati, **YL**, I. Oldengott and G. White, 2309.00672 and 2407.17549.



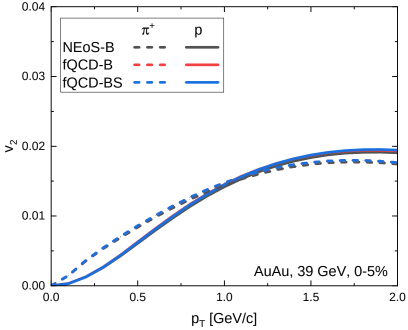
fQCD-based EoS as input of hydrodynamic simulation - viscous hydro. (MUSIC).

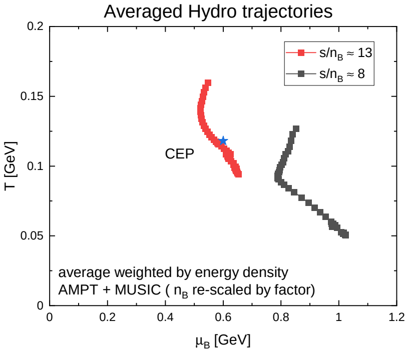
Compare with NEoS [1] (HRG + lattice QCD) and experiment; right plot from [1].



[1] Monnai et al., Phys. Rev. C 100, 024907 (2019).

More on the elliptic flow v_2 and dynamical evolution - ideal hydro. preliminary.
A glimpse of 1st-order phase transition - with AMPT initial profile at 7.7 GeV.





Going beyond CEP and towards even higher density: need to separate L and $L^\dagger$.

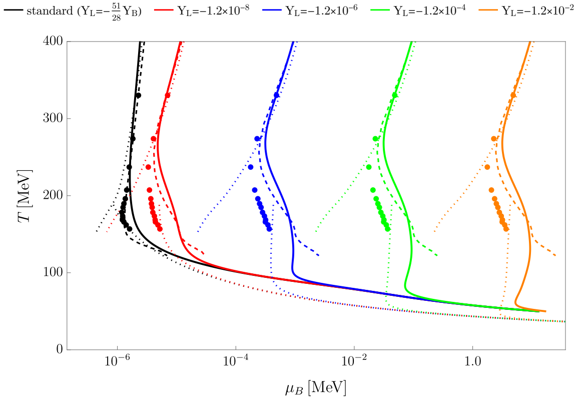
Lepton (flavour) asymmetries (Y_L) and cosmological evolution:

$$Y_{L_\alpha} = \frac{n_{L_\alpha}}{s} = \frac{n_\alpha + n_{\nu_\alpha}}{s}, \quad \alpha = e, \mu, \tau,$$
$$Y_B = \frac{n_B}{s} = \sum_i \frac{b_i n_i}{s},$$
$$Y_Q = \frac{n_Q}{s} = 0 \quad \Leftrightarrow \quad n_Q = \sum_i q_i n_i = 0.$$

Further applications on the cosmological gravitational wave signal ($\mathcal{O}(10^{-6})$ Hz):

Gao et al., 2309.00672 and 2407.17549.
Zheng et al., 2407.03795.

Impacts of Polyakov loop on the trajectories:
solid: with $\mathcal{L}$; dashed: w/o. $\mathcal{L}$; dotted: HRG;



III. Polyakov loop and glue dynamics

YL, F. Gao, Y. X. Liu and J. Pawlowski, Phys. Rev. D 110, 014036 (2024)

YL, F. Gao, Y. X. Liu and J. Pawlowski, in preparation.

The nonperturbative quark propagator:

$$G_q^{-1}(\tilde{p}) = \mathrm{i}(\omega_p + \mathrm{i}\mu_q + gA_4)\gamma_4 Z_q^E(\tilde{p}^2) + \mathrm{i}\gamma\cdot\boldsymbol{p}\, Z_q^M(\tilde{p}^2) + Z_q^E M_q(\tilde{p}^2),$$

A_4 is essentially the gluon 1-point function (background field); for SU(3):

$$A_4 = \frac{2\pi T}{g}(\varphi_3\lambda_3 + \varphi_8\lambda_8)\,, \qquad \mathcal{L} = \frac{1}{3}\left[e^{-\mathrm{i}\frac{2\pi\varphi_8}{\sqrt{3}}} + 2e^{\mathrm{i}\pi\frac{\varphi_8}{\sqrt{3}}}\cos\pi\varphi_3\right].$$

The eigenvalues $\varphi_{3,8}$ correspond to the representations of gluons / quarks, e.g.:

$$\varphi_3 = \{\pm\varphi, \pm\varphi/2, \pm\varphi/2, 0, 0\}\,, \qquad \text{adjoint rep.,}$$

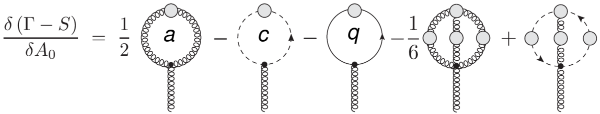
$$\varphi_3 = \{\pm\varphi/2, 0\}\,, \qquad \text{fundamental rep.}$$

φ_8 reflects the difference between the conjugated Polyakov loops $\mathcal{L}$ and $\mathcal{L}^\dagger$;

Polyakov loop potential and glue dynamics

Polyakov loop potential $V[A_4]$ is the effective glue potential in terms of background field A_4;

Dyson-Schwinger equation (DSE) for $V'[A_4] = \delta V[A_4]/\delta A_4$ [1,2]:



Alternatively, within fRG approach [3].

We apply DSE with one-loop truncation [1]:

$$V[A_4] = V_{\rm glue}[A_4] + V_q[A_4],$$

$$V_{\rm glue}[A_4] = \frac{1}{2} V_{\rm Weiss}[A_4] + V_a[A_4] + V_c[A_4],$$

V_a, V_c and V_q determined by the propagators (the 2-point functions);

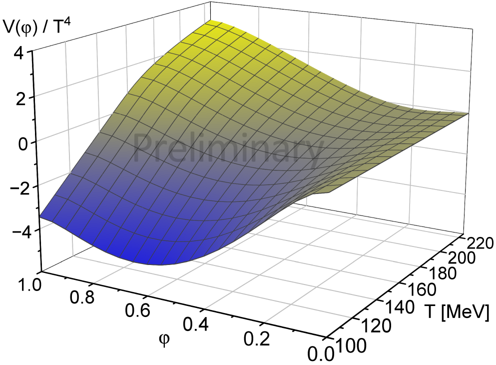
DSEs for the propagators - see G. Eichmann's talk; we refer to a recent DSE calculation [4].

[1] Fischer et al., Phys. Lett. B 732: 273 (2014). [2] Fister and Pawlowski, Phys. Rev. D 88: 045010 (2013).

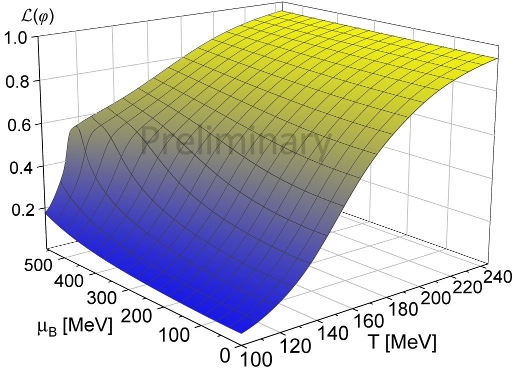
[3] Braun et al., Phys. Lett. B 684: 262 (2010). [4] **YL**, et al., Phys. Rev. D 110: 014036 (2024).

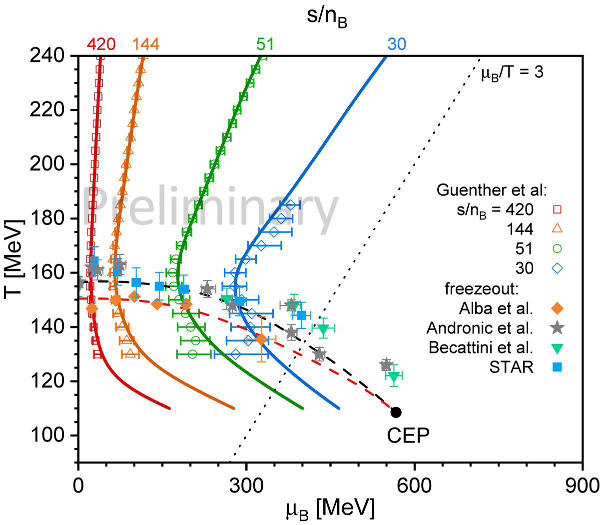
Preliminary study on the "center average" $\varphi_8 \simeq 0$, i.e. $\mathcal{L} \simeq \mathcal{L}^\dagger$.

Full SU(3) potential at $\mu_B = 0$ as a function of φ_3 and T:



Polyakov loop at finite (T, μ_B), and the deconfinement phase transition:





QCD isentropic trajectories via DSE:

$s/n_B = 420$, 144, 51 and 30 agree with lattice QCD [2]; meet with the freezeout points at $\sqrt{s_{\rm NN}} = 200$, 62.4, 19.6 and 11.5 GeV, respectively.

[1] Fischer et al., Phys. Lett. B 732: 273 (2014).

[2] Guenther et al (WB Collab.) Nucl. Phys. A 967: 720-723 (2017).

Summary and outlook

Progress on:

- Order parameter framework, as a functional-QCD-based model for EoS.
- Relevance of Polyakov loop on QCD thermodynamics in hadronic phase; impacts on particle yields and evolution trajectories.
- Probing glue dynamics at finite T and μ_B via functional QCD approaches.

In the future:

- Separation of $\mathcal{L}$ and $\mathcal{L}^\dagger$: investigation on φ_8 at finite μ_B.
- Higher order fluctuations and critical phenomena.
- QCD thermodynamics from inhomogeneous effect/phase structure.

Thanks for your attention!

Back-up

From Green's functions to order parameters

Why simplifications:

discrete approach: difficulties at high density; towards critical end point (CEP);

continuum approach: ongoing quests to extract observables from full QCD dynamics.

see e.g. Isserstedt et al., PoS FAIRness 2022 (2023) 024.

Partition function $Z = Z[G_X]$ in the quantum field theory is a functional of Green functions $\{G_X\}$; QCD involves X = quarks, gluons and their coupling (3-pt func.), etc.

Alternative variables: $Z = Z[O_X]$, where O are the **order parameters**, there are connections between the order parameters and the quantum field:

$O_\chi = \langle \bar{q}q \rangle \leftrightarrow M_q$ (mass function), for chiral phase transition,

$O_A = \langle A_4 \rangle \leftrightarrow \mathcal{L}$ (Polyakov loop), for deconfinement phase transition.

Mapping the phase transition line

Modified r dependence of T, based on the commonly used linear mapping:

$$\frac{\mu_B}{\mu_B^E} - 1 = -r\omega\rho\cos\alpha_1 - h\omega\cos\alpha_2,$$
$$\frac{T}{T^E} - 1 = f_{\rm PT}(r) + h\omega\sin\alpha_2.$$

Phase transition line corresponds to $h = 0$; constraint on the map function $f_{\rm PT}$:

$$f_{\rm PT}(r) = \frac{T_c(\mu_B)}{T^E} - 1 \quad \text{and} \quad \mu_B = \mu_B^E\left(1 - r\omega\rho\cos\alpha_1\right),$$

from the transition line: $\frac{T_c}{T_0} = 1 - \kappa_2\left(\frac{\mu_B}{T_0}\right)^2 + \kappa_4\left(\frac{\mu_B}{T_0}\right)^4 + \cdots;$

up-to-date benchmarks: $T_0 = 155\,\mathrm{MeV}$, $\kappa_2 \simeq 0.016$, $\kappa_4 \simeq 0$;

and estimates: $\mu_B^E = 600\,\mathrm{MeV}$; $T^E = T_c(\mu_B^E) = 118\,\mathrm{MeV}$.

WB Collab., PRL (2020), hotQCD Collab., PLB, (2019),
Fu et al., PRD (2019), Gao et al., PLB (2020), Gunkel et al. PRD (2021).

φ_8 and finite μ_B

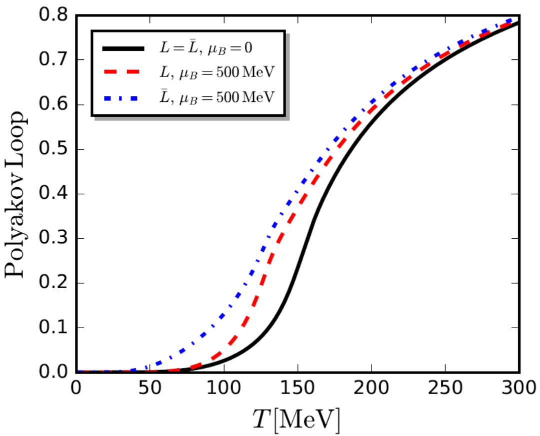
φ_8 manifests the difference between the conjugated Polyakov loops $\mathcal{L}$ and $\mathcal{L}^\dagger$:

$$\mathcal{L} = \frac{1}{3}\left[e^{-\mathrm{i}\frac{2\pi\varphi_8}{\sqrt{3}}} + 2e^{\mathrm{i}\pi\frac{\varphi_8}{\sqrt{3}}}\cos\pi\varphi_3\right].$$

Typically, $\varphi_8 = 0$ is the true minimum of the Polyakov potential at $\mu_B = 0$.

Preliminary investigation shows that for 2+1-flavour QCD, such difference is small up to $\mu_B \approx 500\,\mathrm{MeV} \approx 0.8\,\mu_B^{\mathrm{CEP}}$.

Fu et al., Phys. Rev. D 101: 054032 (2020).

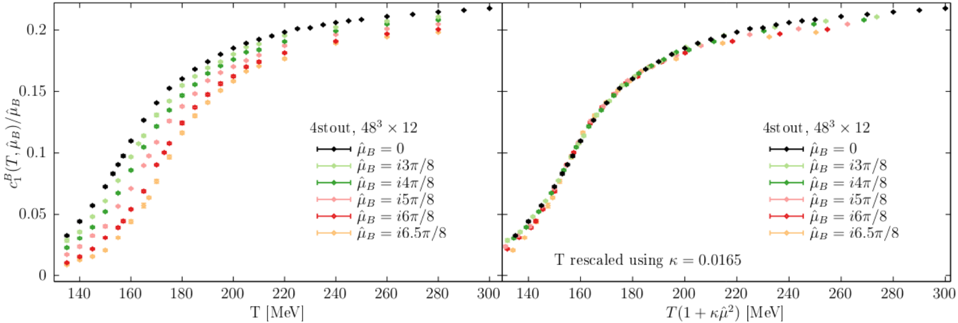


T' expansion

$$\frac{f(T,\hat{\mu}_B)}{\bar{f}(\hat{\mu}_B)} = \frac{f(T',0)}{\bar{f}(0)}, \qquad \text{Boltzmann limit:} \quad \bar{f}(\hat{\mu}_B) = \lim_{T\to\infty} f(T,\hat{\mu}_B);$$

$$T' = T + \kappa_2^f(T)\,\hat{\mu}_B^2 + \kappa_4^f(T)\,\hat{\mu}_B^4 + \cdots .$$

Leading expansion with constant $\kappa_2^f(T) = \kappa$ already shows remarkably well result;
see: Borsanyi et.al. (WB-Collab.), PRL 126: 232001 (2021).

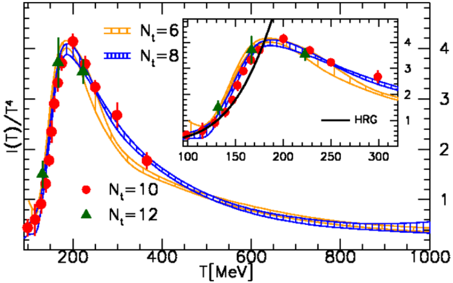


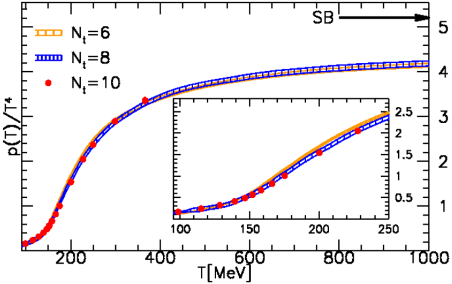
More on QCD EoS at zero μ_B

Details of the T dependence - trace anomaly $I(T) = \epsilon(T) - 3P(T)$ [1,2]:

$$P(T,\mathbf{0})/T^4 = \int_0^T \mathrm{d}T'(I(T')/T'^5).$$

Parametrisation of continuum extrapolated lattice QCD data is available [1].



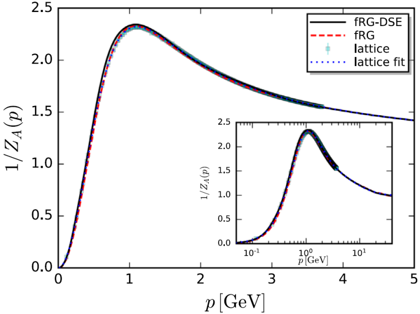


[1] S. Borsanyi, G. Endrodi, Z. Fodor, et al., JHEP 11, 077 (2010)

[2] Bazavov et al. (HotQCD Collab.), Phys. Rev. D 90: 094503 (2014)

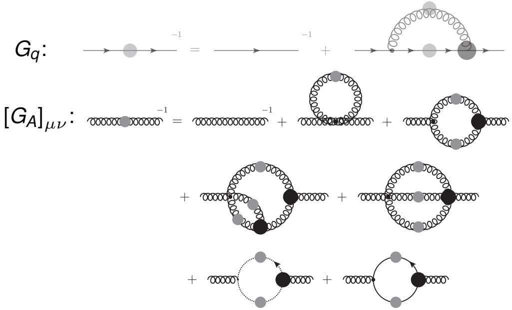
"Minimal" Dyson-Schwinger equations (DSEs)

- Self-consistent solutions for quark and gluon propagators at finite (T, μ_B);
- Optimised tensor structures of the quark-gluon vertex;



Vacuum full gluon propagator:

$$p^2[G_A]_{\mu\nu}(p) = 1/Z_A(p^2).$$



$$[G_A]^{-1}_{\mu\nu}(k)\Big|^{T,\mu_B}_{0,0} = \Pi^{\rm gauge}_{\mu\nu}(k)\Big|^{T,\mu_B}_{0,0} + \Pi^{\rm qrk}_{\mu\nu}(k)\Big|^{T,\mu_B}_{0,0}.$$

YL et al., Phys. Rev. D 110: 014036 (2024)

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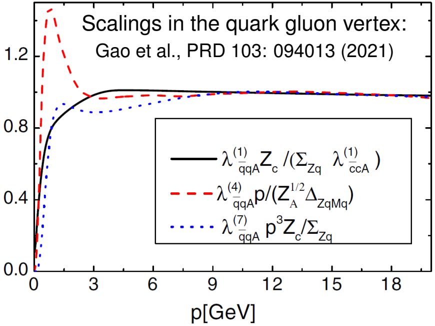

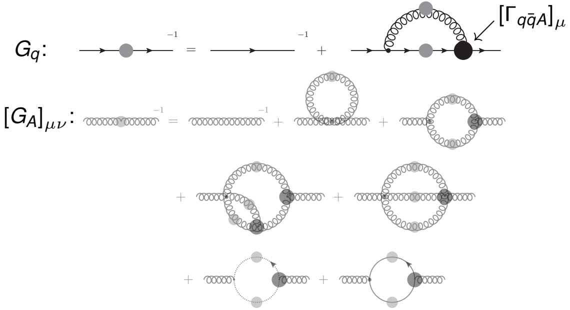

$$[\Gamma_{q\bar{q}A}]_\mu = \lambda^{(1)}_{q\bar{q}A}\,\gamma_\mu + \lambda^{(4)}_{q\bar{q}A}\,\sigma_{\mu\nu}k^\nu .$$

$$G_q^{-1}(p) = Z_q(p^2)\left[\mathrm{i}\gamma\cdot p + M_q(p^2)\right].$$

Emphasis on the Pauli term $\lambda^{(4)}$: Qin et al. PLB (2013), Williams, EPJA (2015), Cyrol et al. PRD (2018)

"Minimal" Dyson-Schwinger equations (DSEs)

Benckmarks in: vacuum quark mass function (left);
finite temperature and density: chiral phase transition line (right).