Universidad de Huelva

# *Pion and Kaon Electromagnetic and Gravitational Form Factors*

FÍSICA
MATEMÁTICAS
COMPUTACIÓN

*J. Rodríguez-Quintero*

*Y-Z Xu, K. Raya, C. D. Roberts, J.R-Q; Eur. Phys. J. C84 (2023) 191*

*Z-Q Yao, D. Binosi, C. D. Roberts, … [preliminary results]*

XVIth Quark Confinement and the Hadron Spectrum Conference
Cairns Convention Centre, Cairns, Queensland, Australia
19-24 August 2024 (inclusive)

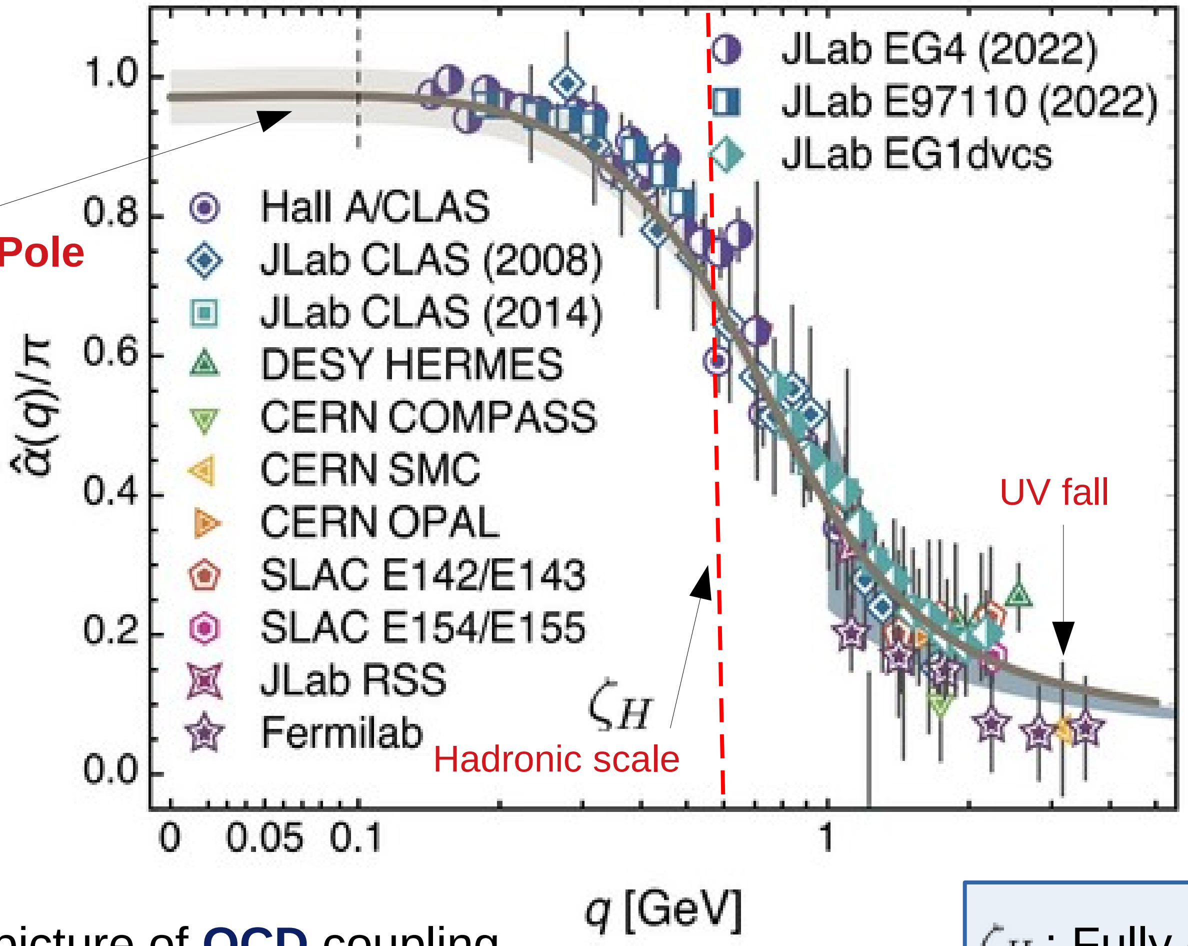
# QCD: Basic Facts

- **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.

'Effective Charge'

**No Landau Pole**

Enhancement



$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$

$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu\,,$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

**Modern** picture of **QCD** coupling.

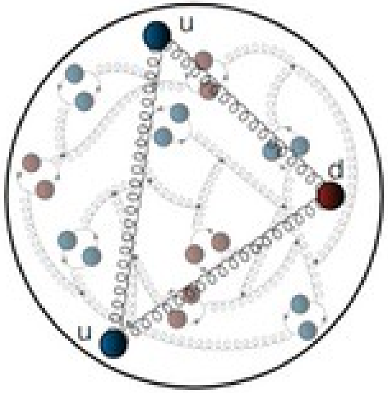
Combined continuum + QCD lattice analysis

$\zeta_H$ : Fully **dressed valence** quarks express all hadron's properties

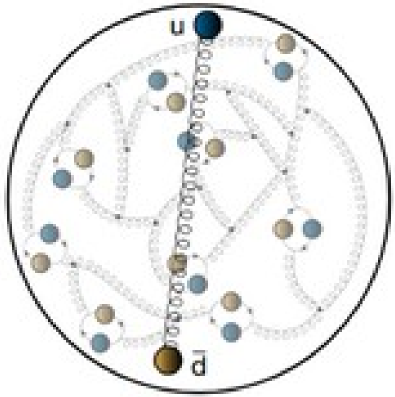
# Why pions and kaons?: understanding EHM

- **Pions** and **kaons** emerge as (pseudo)-**Goldstone** bosons of **DCSB**.

(besides being ‘simple’ bound states)



$m_p \approx 0.940$ GeV

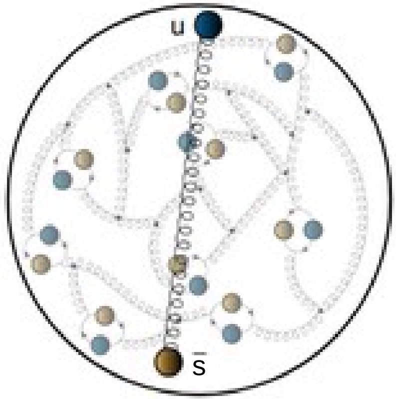


$m_\pi \approx 0.140$ GeV

→ Their study is crucial to understand the **EHM** and the ***hadron structure***:

- Dominated by **QCD** dynamics

  Simultaneously explains the mass of the proton and the *masslessness* of the pion



$m_K \approx 0.490$ GeV

- Interplay between **Higgs** and **strong** mass generating mechanisms.

**‘Higgs’ masses**

$m_{u/d} \approx 0.004$ GeV

$m_s \approx 0.095$ GeV

# CSM: the DSE approach

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions
  - **Infinite** tower of coupled equations.
  - ✗ Systematic **truncation** required

✓ **No assumptions** on the **coupling** for their derivation.

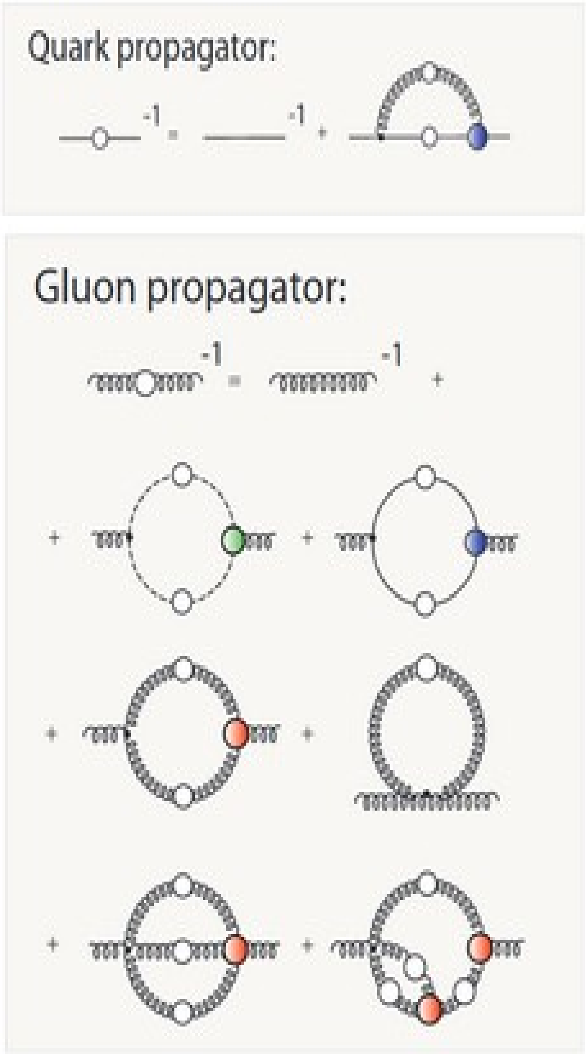
- ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**

✓ **Not limited** to a certain domain of current **quark masses**

✓ Maintain a **traceable connection** to QCD.

C.D. Roberts and A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

Example DSEs



Eichmann:2009zx

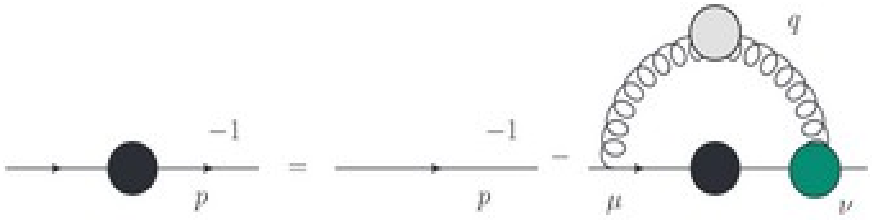
# CSM: the DSE approach

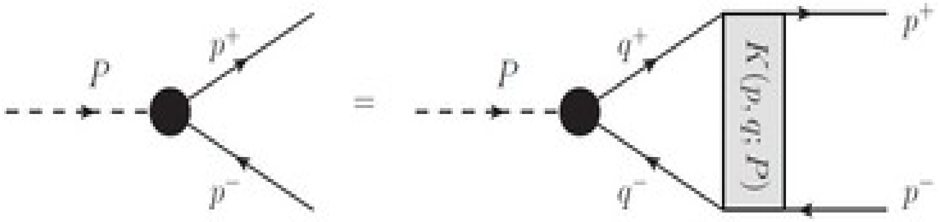
- **BSWF**: sandwich of the Bethe-Salpeter amplitude and quark propagators:

$$\chi_H(k^H;P_H) = S_q(k)\Gamma_H(k^H;P_H)S_{\bar{q}}(k-P_H)\ ,\ k^H = k - P_H/2\ .$$

$P^2 = -m_H^2$ : **meson's mass**; $\Gamma_H$ **BS amplitude**; $S_{q(\bar{q})}$ **quark (antiquark) propagator**

- Quark **propagator** and **BSA** should come from solutions of:





## Quark DSE

## Meson BSE

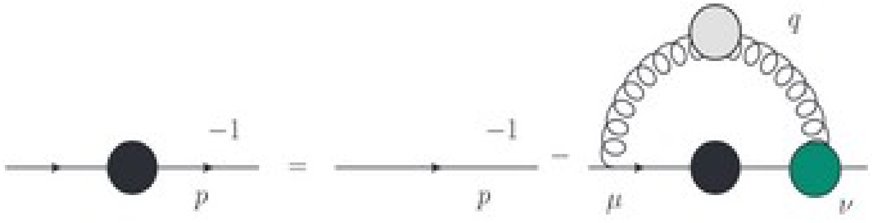
- Relates the quark propagator with **QGV** and **gluon propagator**.
- Contains **all interactions** between the quark and antiquark

# CSM: the DSE approach

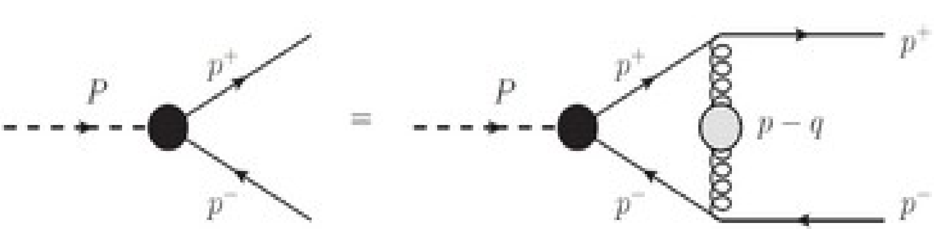
- For the ground-state pseudoscalar and vector mesons, it is typical to employ the so called Rainbow-Ladder (**RL**) truncation:

Y-Z Xu et al., PRD 100 (2019) 11, 114038.

K. Raya et al., PRD 101 (2020) 7, 074021.



**Quark DSE**



**Meson BSE**

- It preserves a QCD key symmetry in the chiral limit, manifested by the **Goldstone's Theorem** and whose most fundamental expression is captured in:

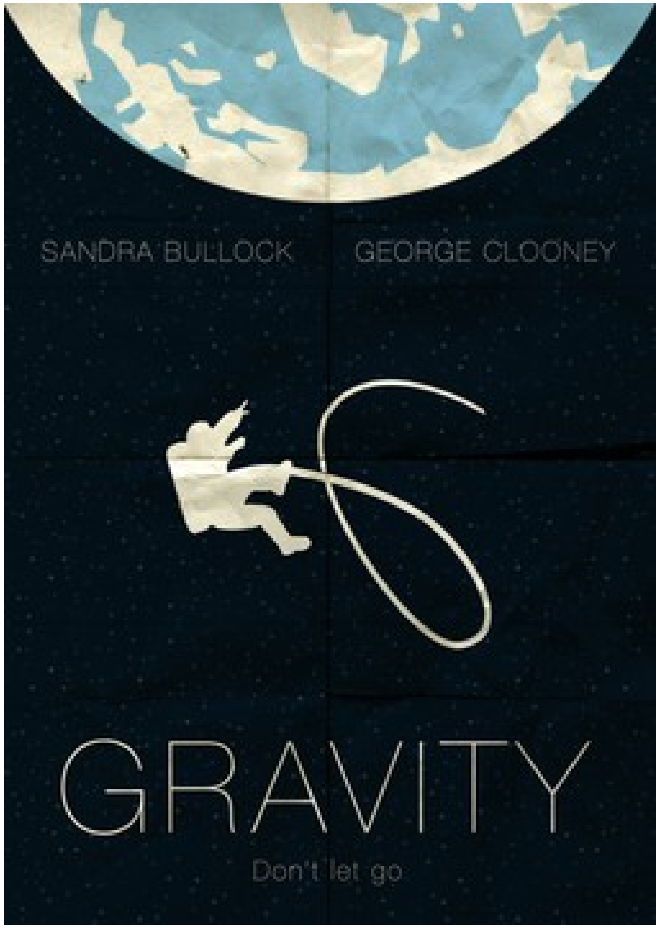
$$f_\pi E_\pi(k;P=0) = B(k^2)$$

**Leading** BSA

"***Mass*** Function"

> "**Pions** exists, if and only if, **DCSB** occurs."

# Electromagnetic and Gravitational
# Form Factors



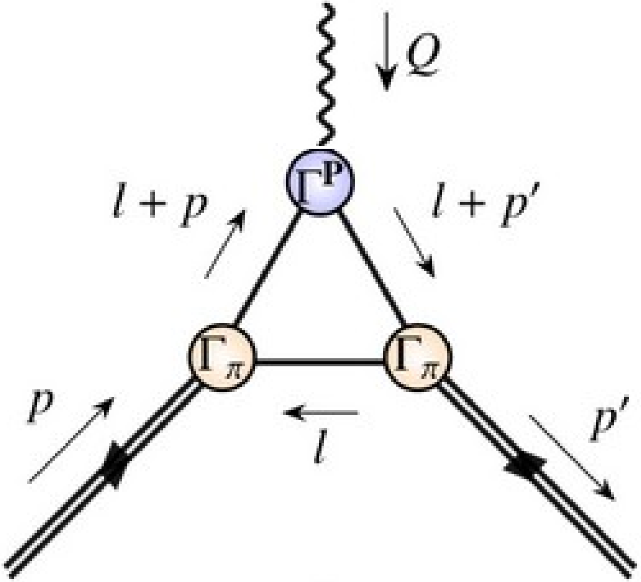
# EFFs and GFFs

- The five-point Schwinger function defining the elastic electromagnetic form factor for a charged pion takes the following form (within RL truncation)

$$\Lambda_\nu^{\gamma\pi}(P,Q) = 2N_c \mathrm{tr}_D \int \frac{d^4 l}{(2\pi)^4} \Gamma_\nu^\gamma(l+p', l+p) L(l,P,Q)$$

$$S(l+p)\Gamma_\pi(l+p/2;p)S(l)\bar{\Gamma}_\pi(l+p'/2;-p')S(l+p')$$

with: $2P = p'+p,\, Q = p'-p,\, p'\cdot p' = -m_\pi^2 = p\cdot p, P\cdot Q = 0.$



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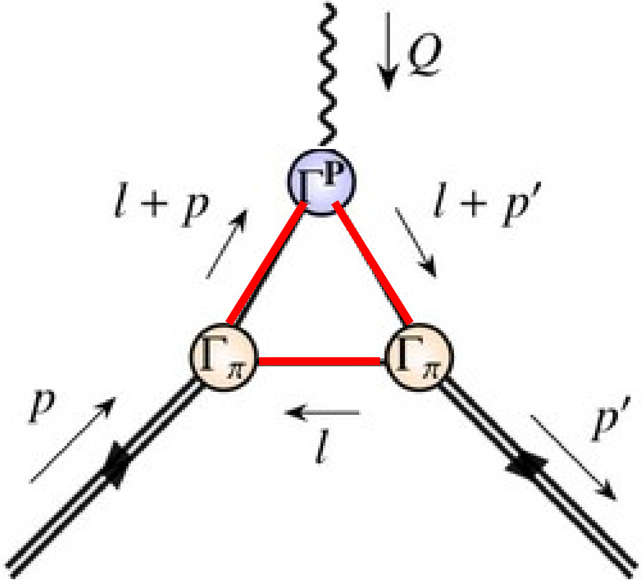

$$S(k) = 1/[i\gamma \cdot k\, A(k^2) + B(k^2)]$$

dressed quark propagator, obtained by solving the Gap equation in Rainbow truncation:

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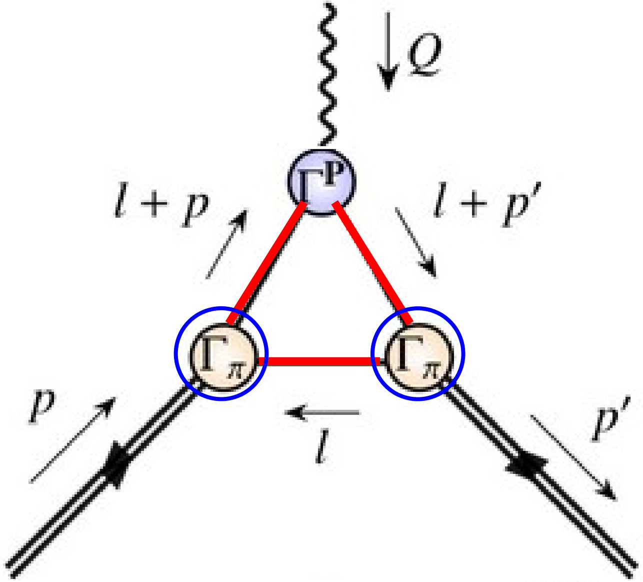

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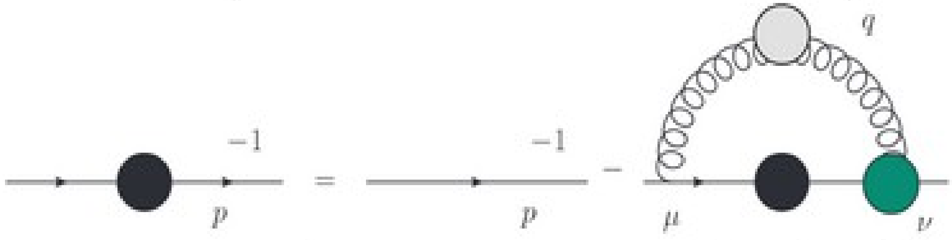
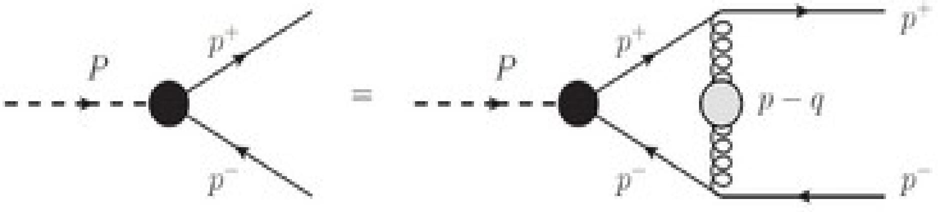


$\Gamma_\pi$ is the pion amplitude resulting from the correspondign Bethe-Salpeter equation within Rainbow-ladder (RL) truncation:



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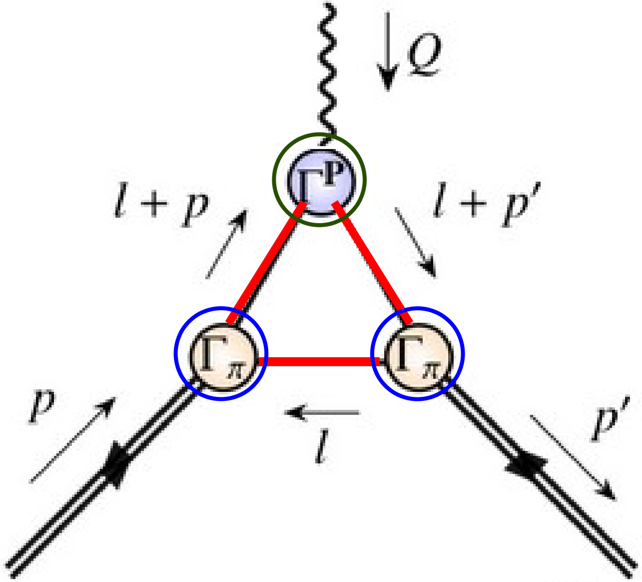

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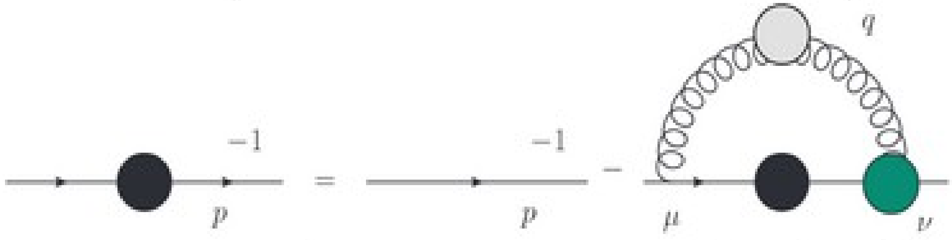

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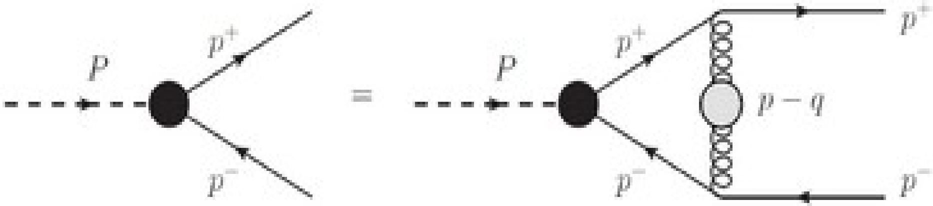

$\Gamma_\nu^\gamma$ is the dressed photon-quark vertex

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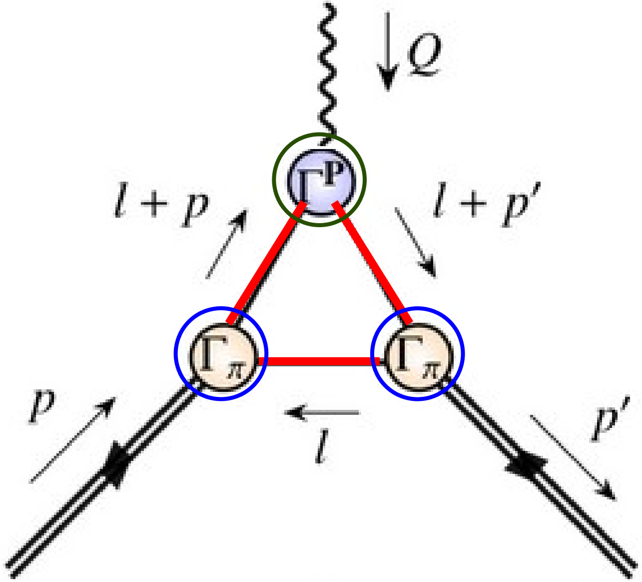

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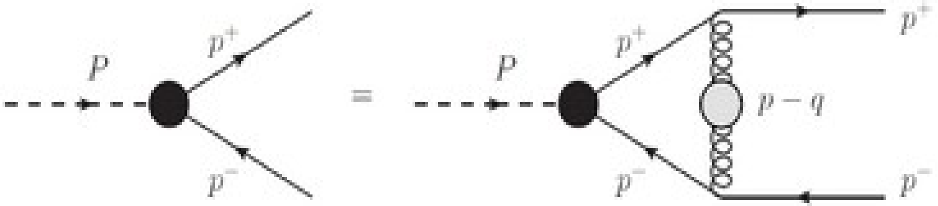

$\Gamma^\gamma_\nu$ is the dressed photon-quark vertex

- As RL guarantees Electromagnetic current conservation:

Electromagnetic form factor

$$Q_\nu \Lambda^{\gamma\pi}_\nu(P,Q) \equiv 0 \longrightarrow \Lambda^{\gamma\pi}_\nu(P,Q) = 2P_\nu F_\pi(Q^2)$$

# EFFs and GFFs: CSM elements

- The five-point Schwinger function defining the elastic **gravitational** form factor for a charged pion takes the following form (within RL truncation)

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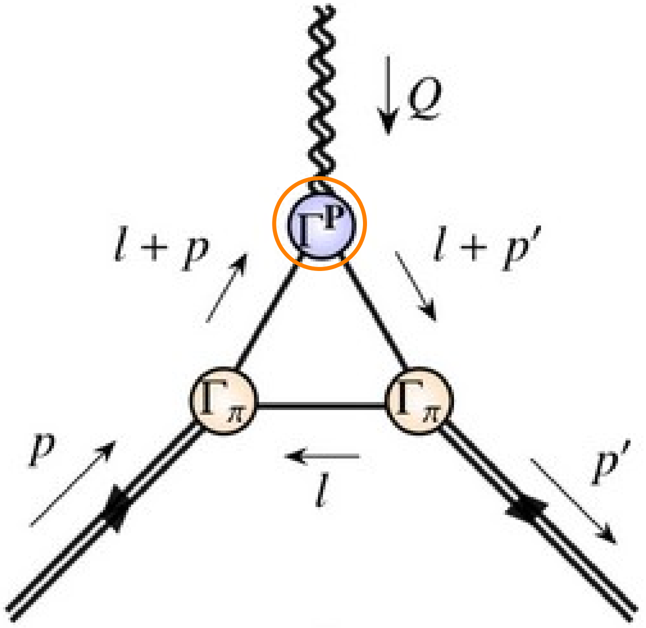

( . . . )

$\Gamma^g_{\mu\nu}$

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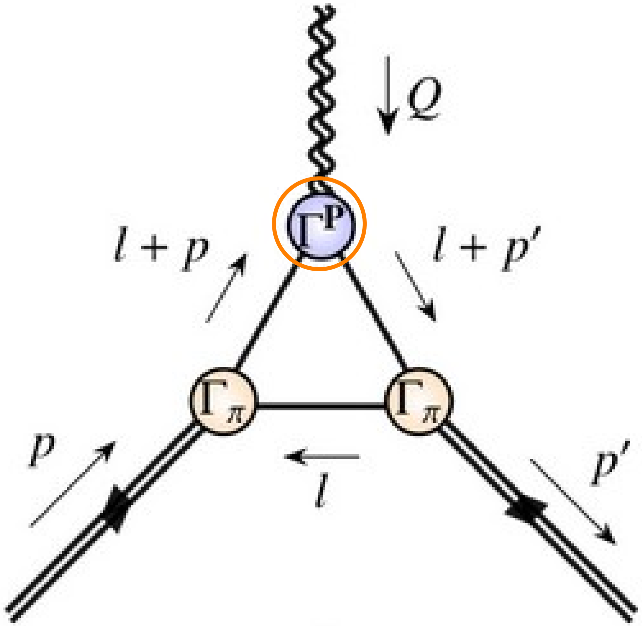

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$\Gamma^g_{\mu\nu}$ is the dressed graviton-quark vertex, entailed by replacing the electromagnetic with the gravitational current.

Then: $$\underbrace{\Lambda^g_{\mu\nu}(P,Q)}_{\langle P_f|T_{\mu\nu}(0)|P_i\rangle} = 2P_\mu P_\nu \theta_2^\pi(Q^2) + \frac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi(Q^2)$$

EMT’s spin-0 meson matrix element

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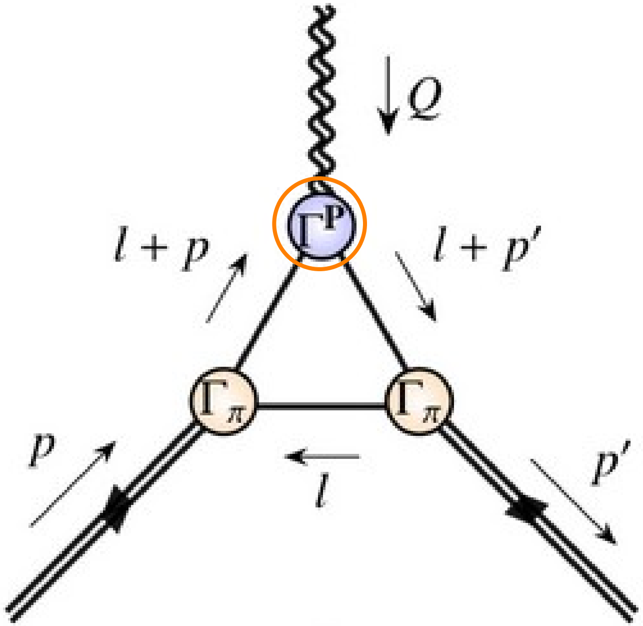

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- $\theta_{1.2}(Q^2)$ can be extracted by the projectors:

$$\mathcal{P}^{\theta_2}_{\mu\nu} = \frac{1}{4P^2}[3L_{\mu\nu}(P) + L_{\mu\nu}(Q) - \delta_{\mu\nu}]\,,\quad L_{\mu\nu}(P) = P_\mu P_\nu/P^2$$

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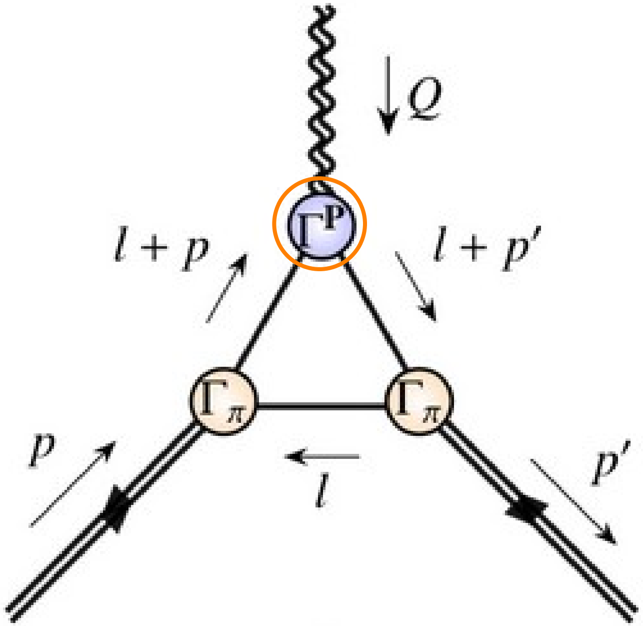

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$L_{\mu\nu}(P) = P_\mu P_\nu / P^2$

$\theta_1(Q^2)$ connected with the **mechanical** properties of the hadron

$\theta_2(Q^2)$ connected with the **mass** distribution inside the hadron

M. Polyakov, Phys.Lett. B555 (2003) 56-62
M. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025

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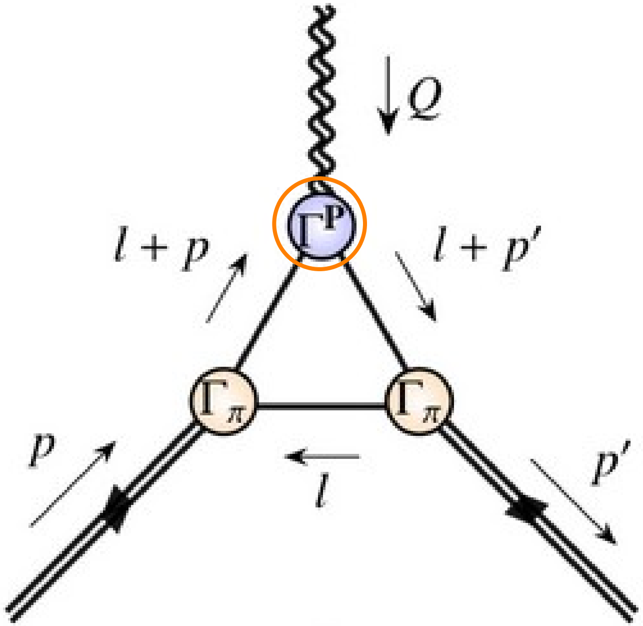

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- From fundamental symmetries, one knows: $\theta_2^\pi(0) = 1\,, \quad \theta_1^\pi(0) \overset{m_\pi^2=0}{=} 1\,, \quad \bar{c}^\pi(Q^2) \equiv 0\,.$

M. Polyakov, Phys. Lett. B555 (2003) 56-62
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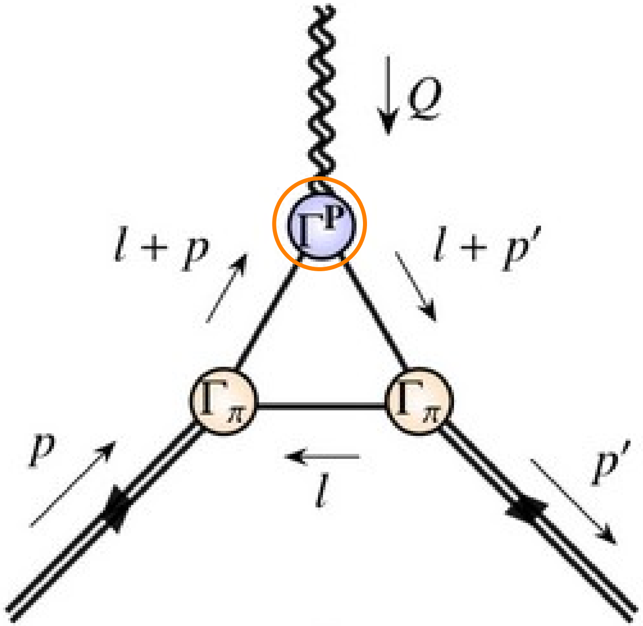

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  - Normalization relying on mass conservation

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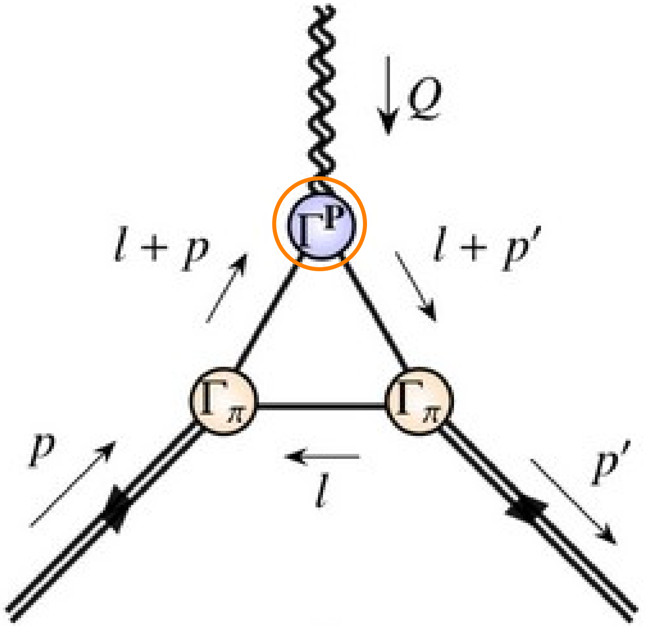

$$\Lambda^{g}_{\mu\nu}(P,Q) = 2N_c \mathrm{tr}_D \int \frac{d^4 l}{(2\pi)^4} \Gamma^{g}_{\mu\nu}(l+p',l+p) L(l,P,Q)$$

$$S(l+p)\Gamma_\pi(l+p/2;p)S(l)\bar{\Gamma}_\pi(l+p'/2;-p')S(l+p')$$

with: $2P = p'+p,\ Q = p'-p,\ p'\cdot p' = -m_\pi^2 = p\cdot p, P\cdot Q = 0.$

( . . . )

$\Gamma^{g}_{\mu\nu}$ is the dressed graviton-quark vertex, entailed by replacing the electromagnetic with the gravitational current.

Gravitational form factors

Then: $\Lambda^{g}_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2^\pi(Q^2) + \frac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi(Q^2)$

- From fundamental symmetries, one knows: $\theta_2^\pi(0) = 1\,, \quad \theta_1^\pi(0) \overset{m_\pi^2=0}{=} 1\,, \quad \bar{c}^\pi(Q^2) \equiv 0\,.$
  - Normalization relying on mass conservation
  - Manifestation of CSB and EHM through the soft-pion theorem

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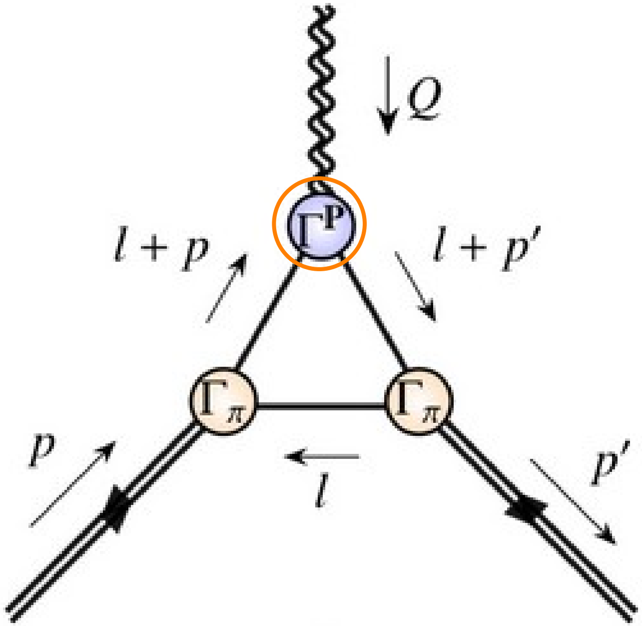

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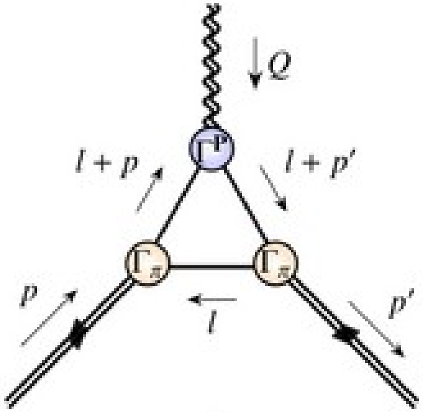
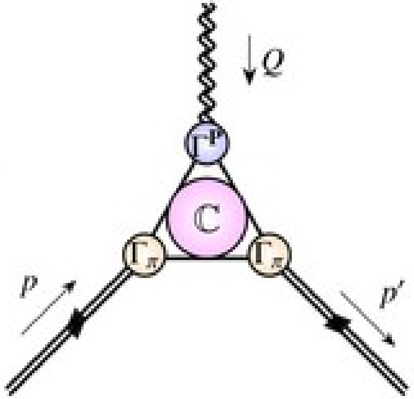


For any dressed quark-graviton vertex obeying the tensor WGTI, this diagram produces a Q-longitudinal contribution



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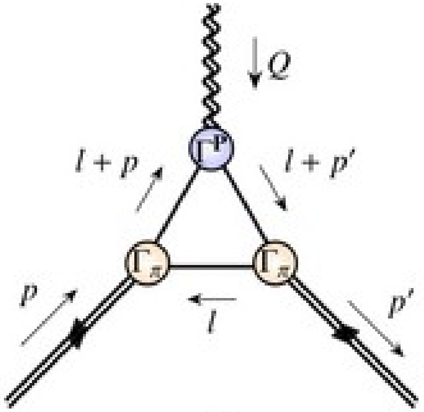

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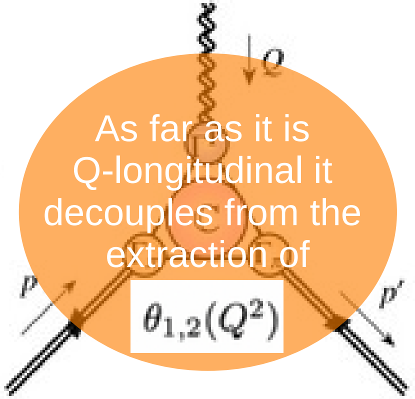

As far as it is Q-longitudinal it decouples from the extraction of $\theta_{1,2}(Q^2)$

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$$iQ_\nu \Gamma^\gamma_\nu(l'_+, l_+) = S^{-1}(l'_+) - S^{-1}(l_+) \qquad (l_+^{(\prime)} = l + p^{(\prime)})$$

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- The remaining transverse terms are obtained by solving the inhomogeneous vector BSE

$$[\Gamma^\gamma_\nu]_{\rho_1\rho_2}(k_+, k_-) = \underbrace{Z_2}_{\text{Wave function renormalization}}[\gamma_\nu]_{\rho_1\rho_2} + Z_2^2 \int_{dl}^{\Lambda} \mathcal{K}^{\rho'_2\rho_2}_{\rho_1\rho'_1}(k-l)[S(l_+)\Gamma^\gamma_\nu(l_+, l_-)S(l_-)]_{\rho'_1\rho'_2}$$

$$\mathcal{K}^{\rho'_2\rho_2}_{\rho_1\rho'_1}(\ell) = \tilde{\mathcal{G}}(s = \ell^2)\tfrac{4}{3}[i\gamma_\mu]_{\rho_1\rho'_1}[i\gamma_\nu]_{\rho'_2\rho_2} T_{\mu\nu}(\ell)$$

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Wave function renormalization

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The interaction is phenomenologically fixed such that: $m_\pi = 0.14, m_K = 0.49, f_\pi = 0.095, f_K = 0.116\,[GeV]$ but it has been seen to be consistent with an effective interaction relying on the PI effective charge.

# EFFs and GFFs: symmetry-preserving dressed vertices

- The new key ingredient is the **quark-graviton vertex**, which obeys its corresponding tensor WGT identity

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Wave function renormalization

Chiral limit mass renormalization

Same quark-gluon phenomenological interaction

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$$i\Gamma^g_{\mu\nu}(k_+,k_-) = Z_2[i\gamma_\mu k_\nu - \delta_{\mu\nu}(i\gamma\cdot k + Z^0_m m^\zeta)] + Z_2^2\int_{dl}^{\Lambda} \mathcal{K}(k-l)[S(l_+)i\Gamma^g_{\mu\nu}(l_+,l_-)S(l_-)]$$

Wave function renormalization

Chiral limit mass renormalization

Same quark-gluon phenomenological interaction

- Our approximation: $\Gamma^{gT}_{\mu\nu}(k,Q) = T_{\mu\nu}(Q)\Gamma_{\mathbb{1}}(k;Q)$

A Lorentz scalar embedded in the Dirac space defined by $\{\mathbf{1}, \gamma{\cdot}k, \gamma{\cdot}Q, \sigma_{\alpha\beta}k_\alpha Q_\beta\}$

# EFFs and GFFs: symmetry-preserving dressed vertices

- The new key ingredient is the **quark-graviton vertex**, which obeys its corresponding tensor WGT identity

$$Q_\mu i\Gamma^g_{\mu\nu}(k,Q) = S^{-1}(k_+)k_{-\nu} - S^{-1}(k_-)k_{+\nu}$$

$$\left\{ \begin{array}{l} k_\pm \; = \; k \pm Q/2 \\ k = (l'_+ + l_+)/2 \end{array} \right\}$$

- It can be built according to the following minimal Ansatz

$$i\Gamma^{g_M}_{\mu\nu}(k,Q) = i\Gamma^{\rm BC}_\mu(k,Q)k_\nu - \tfrac{1}{2}\delta_{\mu\nu}[S^{-1}(k_+) + S^{-1}(k_-)]$$
$$+ iT_{\mu\alpha}(Q)T_{\nu\beta}(Q)4\hat{\Gamma}^2_{\alpha\beta}(k_+,k_-)$$

It contains the Q-longitudinal part of the quark-photon vertex satisfying the vector WGTI, thus contributing to the saturation of the tensor WGTI.

This is a transverse term, not contributing to the tensor WGTI, cast to ensure the absence of kinematic singularities but obtained from the inhomogeneous tensor BSE and dominated by the dynamical singularity at the position of the lowest mass I=0 tensor meson.

- The complete dressed vertex generally contains more transverse structures, namely

$$\Gamma^g_{\mu\nu}(k,Q) = \Gamma^{g_M}_{\mu\nu}(k,Q) + \Gamma^{gT}_{\mu\nu}(k,Q)$$

Satisfying $Q_\mu \Gamma^{gT}_{\mu\nu}(k,Q) = 0$ and representing all non-included transverse contributions

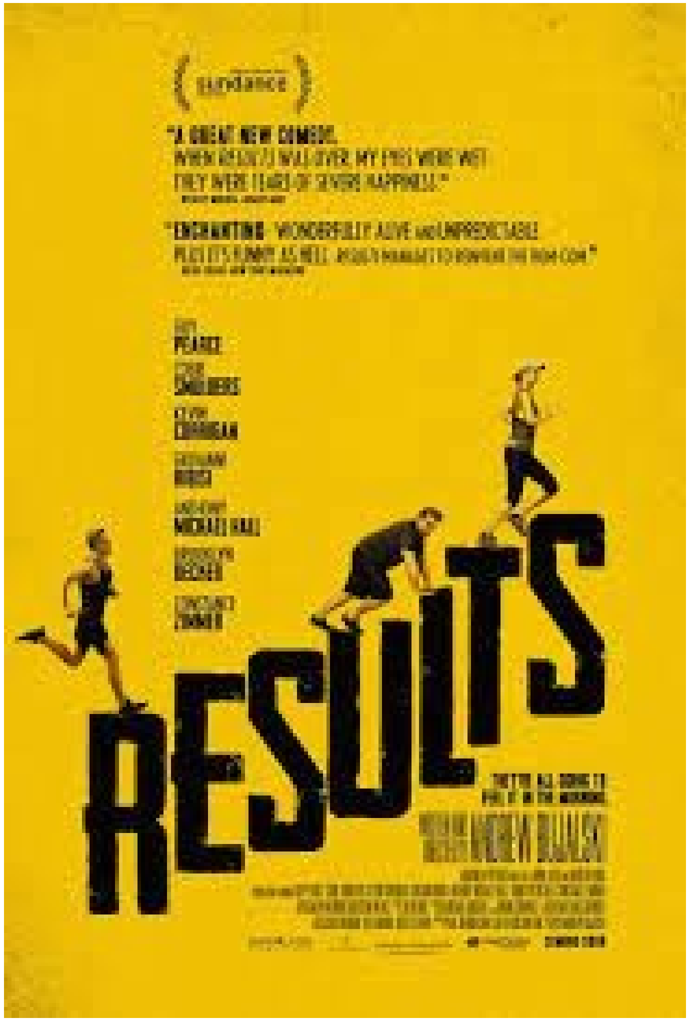
- And obtained through (RL):

$$\mathrm{tr}_D \mathcal{P}^j_{\mu\nu}(k,Q)\Gamma^{gT}_{\mu\nu}(k_+k_-) = \mathrm{tr}_D \mathcal{P}^j_{\mu\nu}(k,Q)Z_2^2 \int_{dl}^{\Lambda} \mathcal{K}(k-l)S(l_+)\{\Gamma^{g_M}_{\mu\nu}(l_+,l_-) + T_{\mu\nu}(Q)\Gamma_{\mathbb{I}}(l_+;l_-)\}S(l_-)$$

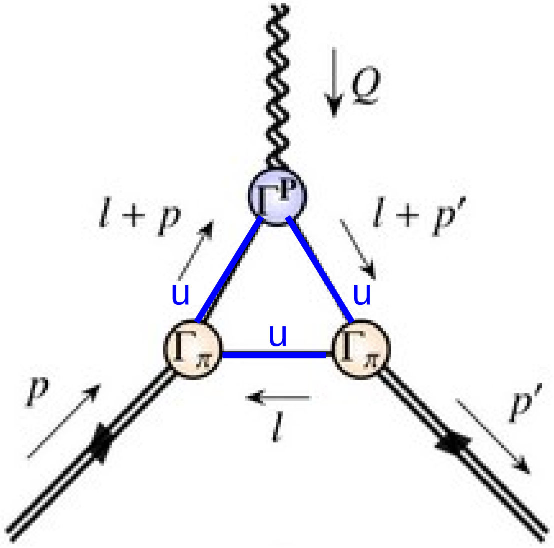
An inhomogeneous BSE exhibiting a pole at each scalar resonance generated by the interaction

- Our approximation: $\Gamma^{gT}_{\mu\nu}(k,Q) = T_{\mu\nu}(Q)\Gamma_{\mathbb{I}}(k;Q)$

A Lorentz scalar embedded in the Dirac space defined by $\{\mathbf{1}, \gamma{\cdot}k, \gamma{\cdot}Q, \sigma_{\alpha\beta}k_\alpha Q_\beta\}$


sundance
"A GREAT NEW COMEDY.
WHEN RESULTS WAS OVER, MY EYES WERE WET.
THEY WERE TEARS OF SEVERE HAPPINESS."
"ENCHANTING. WONDERFULLY ALIVE AND UNPREDICTABLE.
PLUS IT'S FUNNY AS HELL. RESULTS MANAGES TO REINVENT THE ROM-COM."
PEARCE
SMULDERS
CORRIGAN
ROSSI
MICHAEL HALL
BECKER
ZINNER
RESULTS
ANDREW BUJALSKI

# Results: Pion’s EFFs and GFFs



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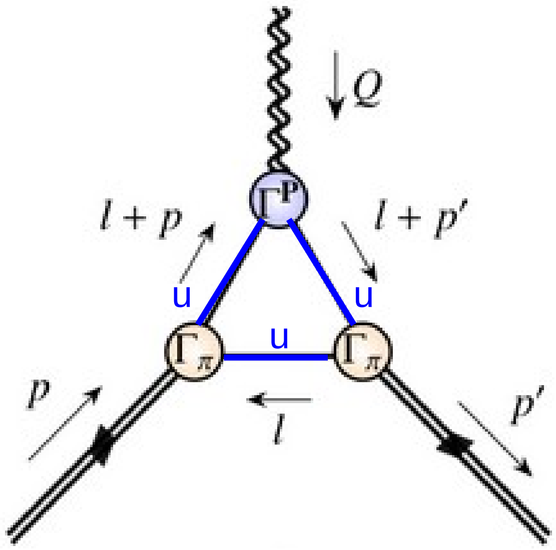

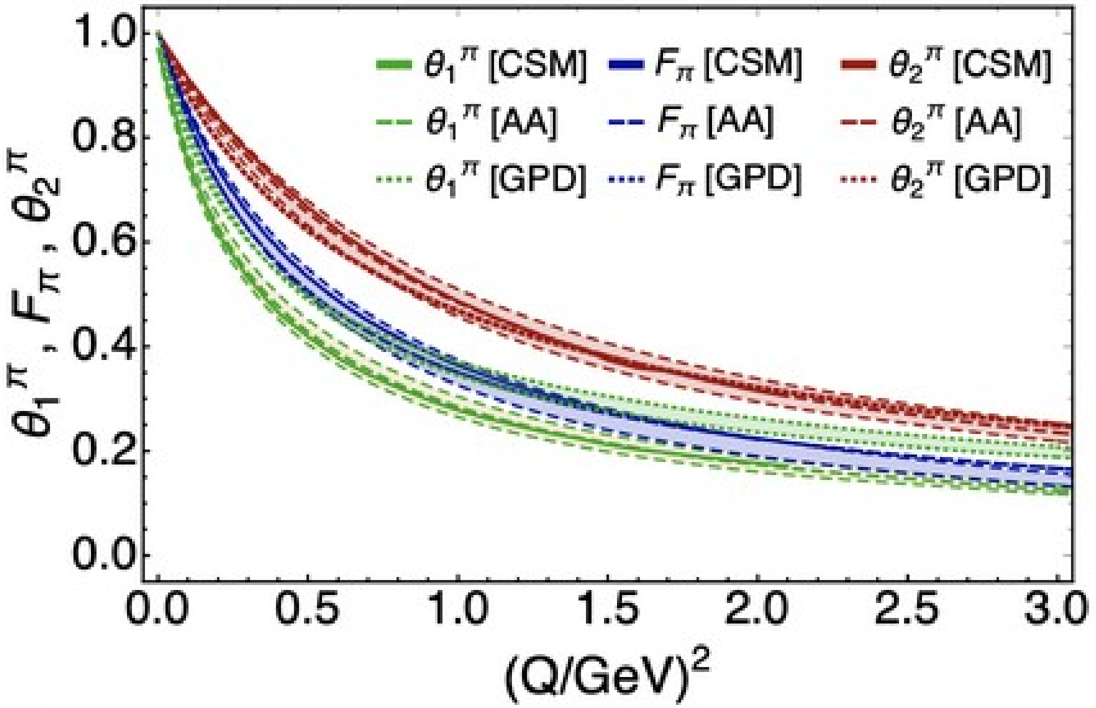

| | $\theta_1^{\mathcal{P}}(0)$ | $r_{\mathcal{P}}^{\theta_1}$ (fm) | $r_{\mathcal{P}}^{F}$ (fm) | $r_{\mathcal{P}}^{\theta_2}$ (fm) |
|---|---|---|---|---|
| $\pi_{\rm CSM}$ | 0.97 | 0.81 | 0.64 | 0.47 |
| $\pi_{\rm AA}$ | 0.97 | 0.80(4) | 0.64(3) | 0.49(3) |
| $\pi_{\rm GPD}$ | | 0.81 | 0.69 | 0.56 |

$$r^2 = -6(d/dt)\ln \mathcal{F}(t)|_{t=0}$$

Recent analyses of data yield $r_\pi^{\theta_2} = 0.51(2)$ fm , $r_\pi^F = 0.64(2)$ fm
$\theta_1^\pi(0) = 0.97(1)$

Y.-Z. Xu et al., Chin. Phys. Lett. Exp. 40 (2023) 041201.
Z.-F. Cui et al., Phys. Lett. B822 (2021) 136631.
M. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

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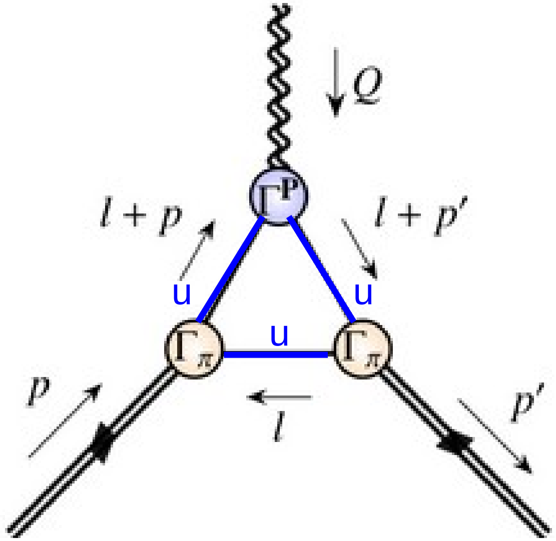

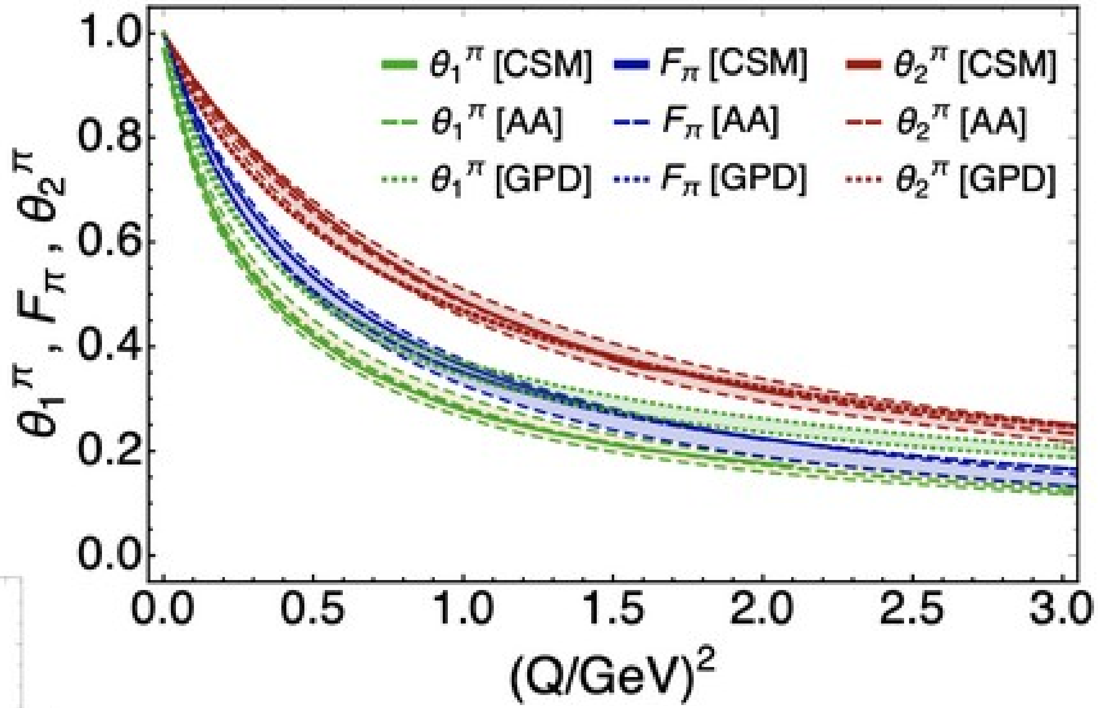

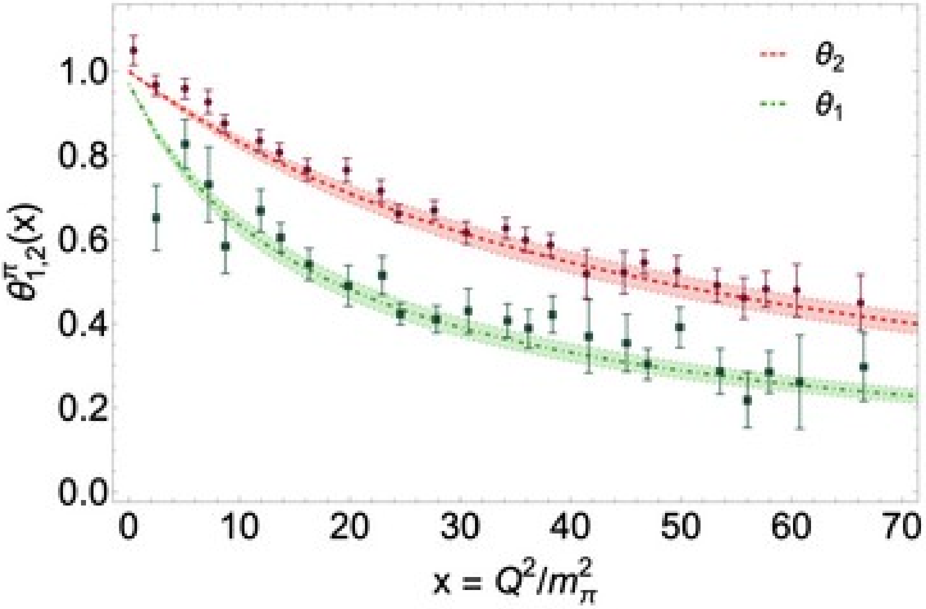

D.C. Hacket et al., Phys. Rev. D108 (2023) 114504.

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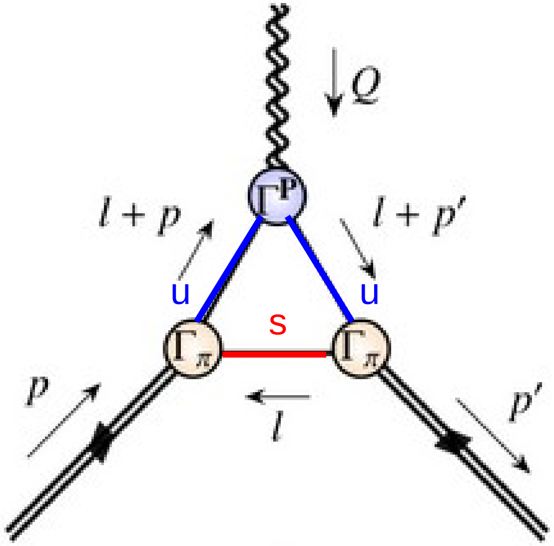

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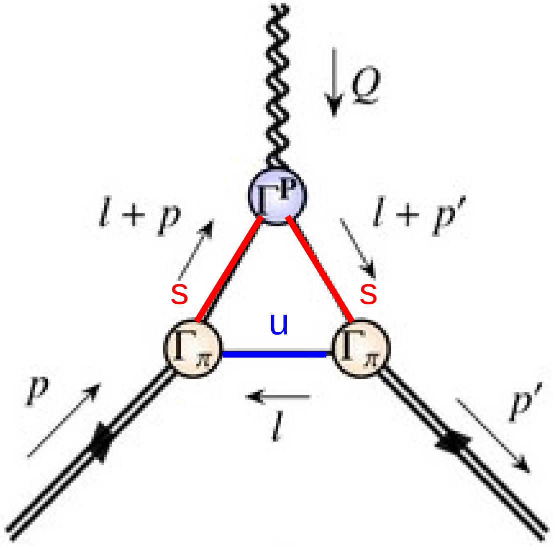

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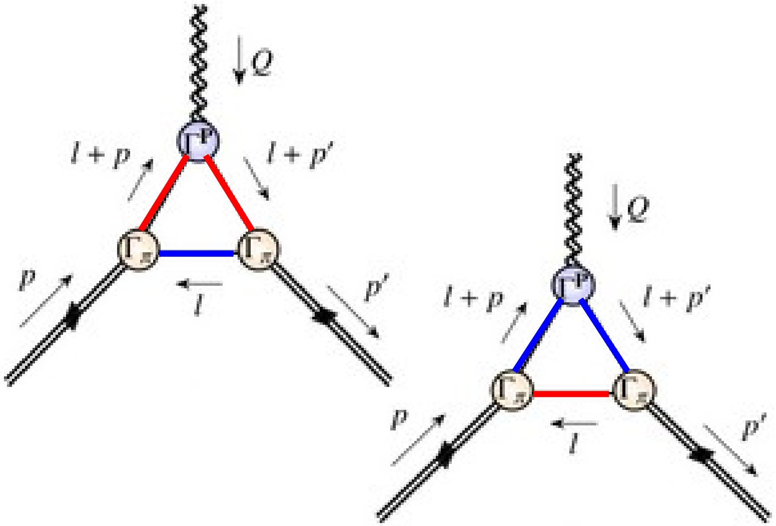

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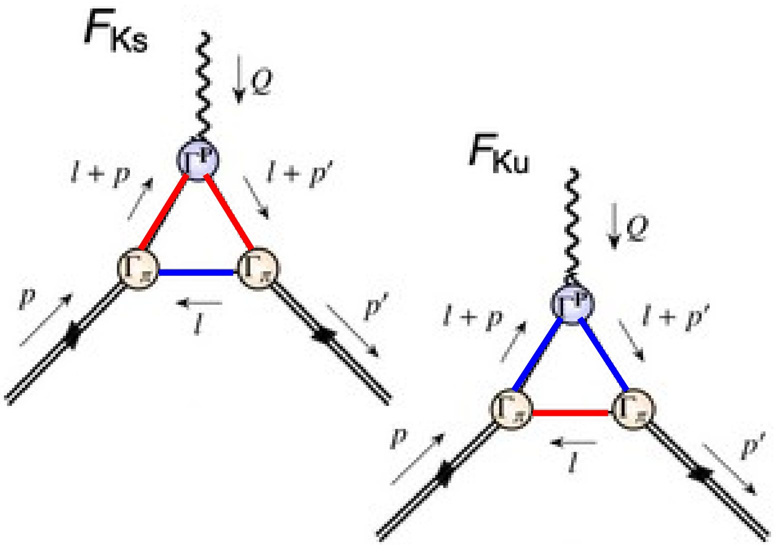

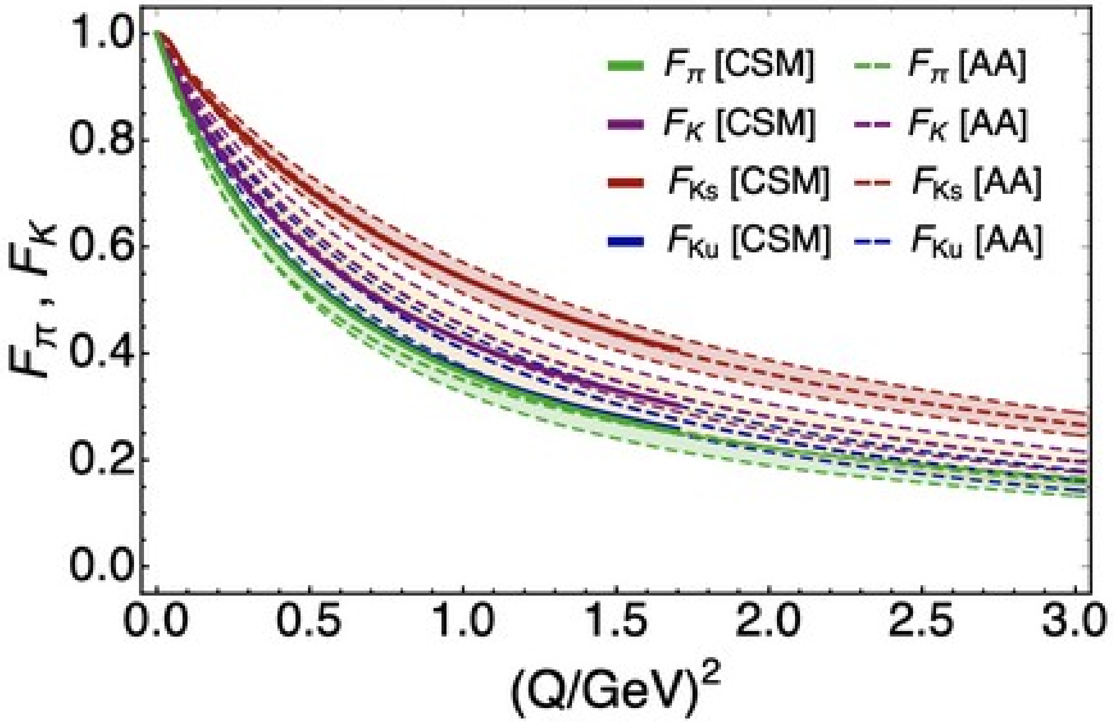

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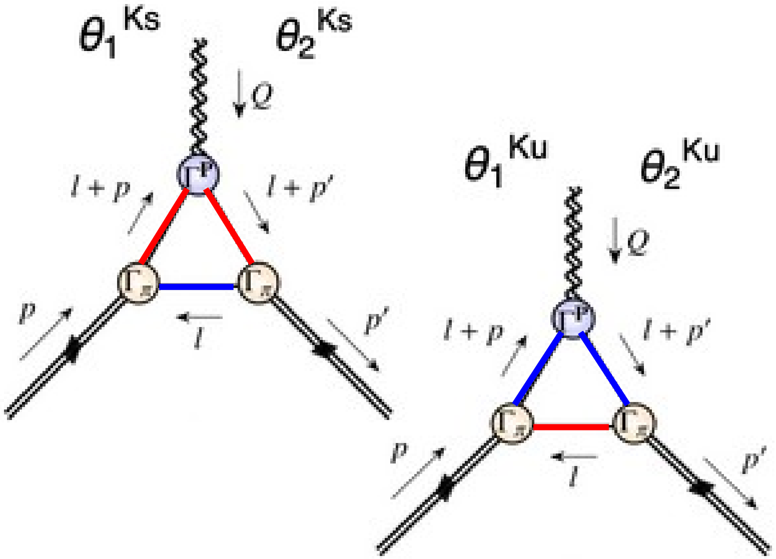

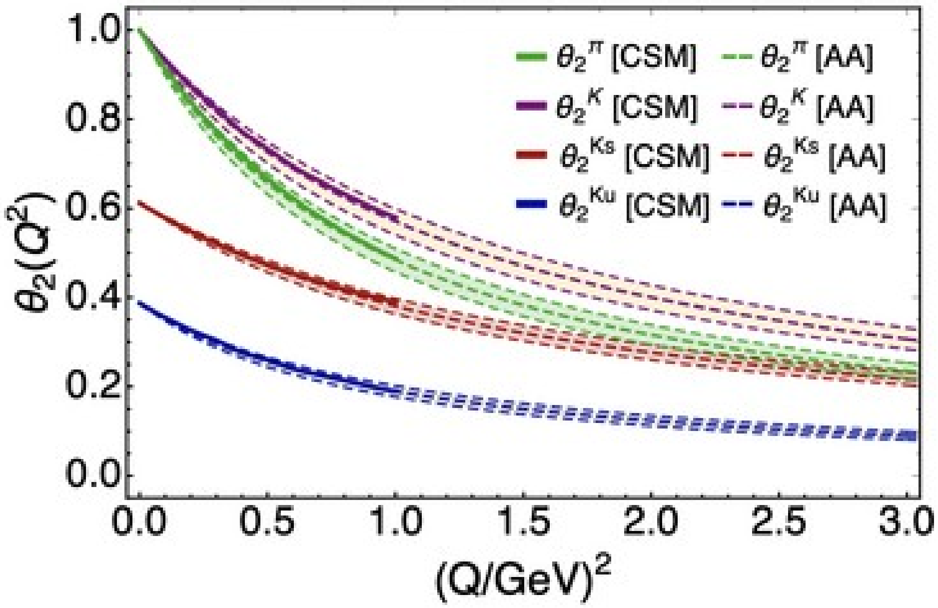

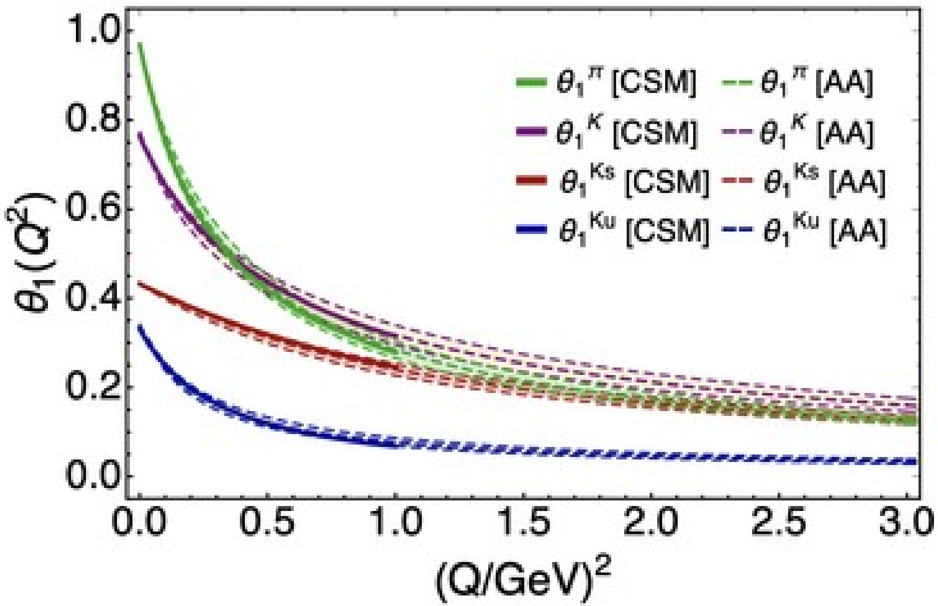

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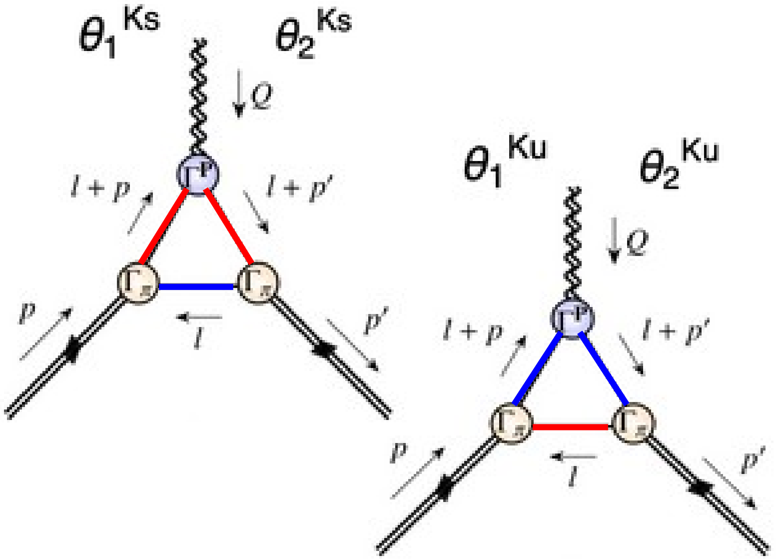

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|---|---|---|---|---|
| $K_{\rm CSM}$ | 0.77 | 0.63 | 0.58 | 0.40 |
| $K_{\rm AA}$ | 0.77 | 0.68(4) | 0.51(3) | 0.41(3) |
| $K^{\bar s}_{\rm CSM}$ | 0.43 | 0.39 | 0.44 | 0.37 |
| $K^{\bar s}_{\rm AA}$ | 0.43 | 0.42(3) | 0.43(3) | 0.36(3) |
| $K^{u}_{\rm CSM}$ | 0.34 | 0.85 | 0.64 | 0.45 |
| $K^{u}_{\rm AA}$ | 0.34 | 0.91(5) | 0.55(4) | 0.48(3) |

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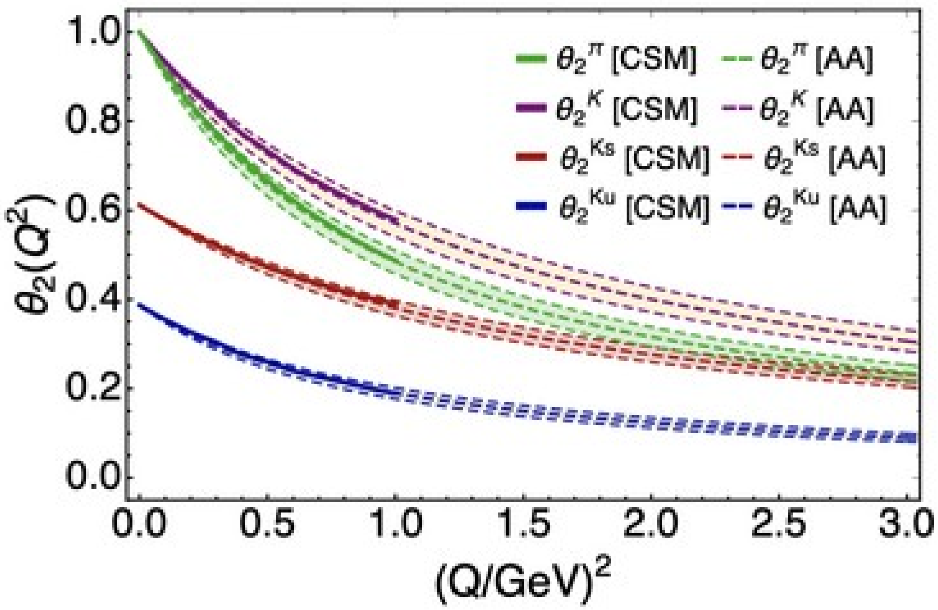
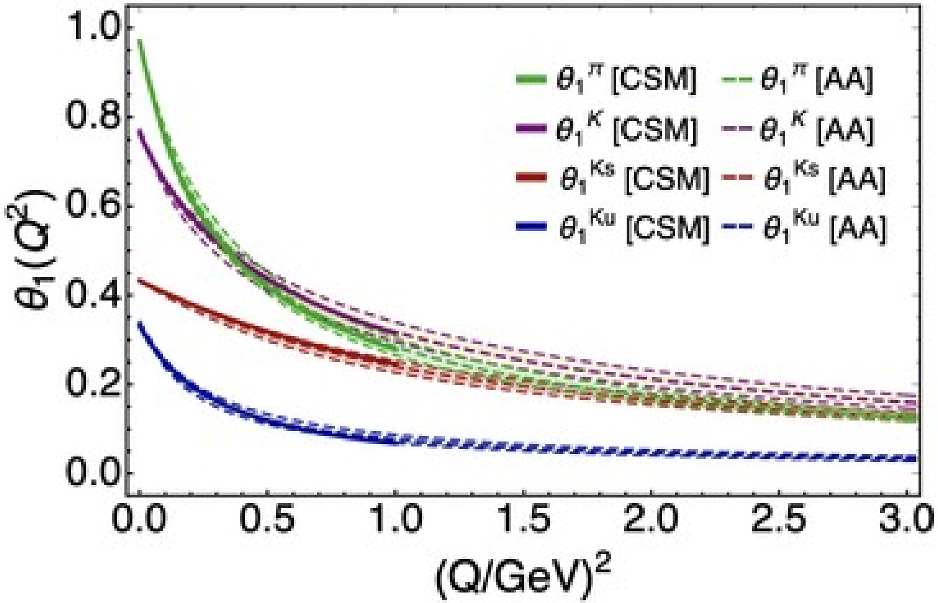
Recent analyses of data yield $r_K^F \approx 0.53$ fm $\theta_1^K(0) \approx 0.77(15)$

Z.-F. Cui et al., Phys. Lett. B822 (2021) 136631.
M. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 18300





# Results: charge, mass and pressure profiles

- **Charge and mas** distributions can be obtained from the FT of FFs

$$\rho_{\mathbf{P}}^{\widetilde{\mathfrak{F}}}(r) = \frac{1}{2\pi}\int_0^{\infty} d\Delta\Delta J_0(\Delta r)\widetilde{\mathfrak{F}}_{\mathbf{P}}(\Delta^2)$$

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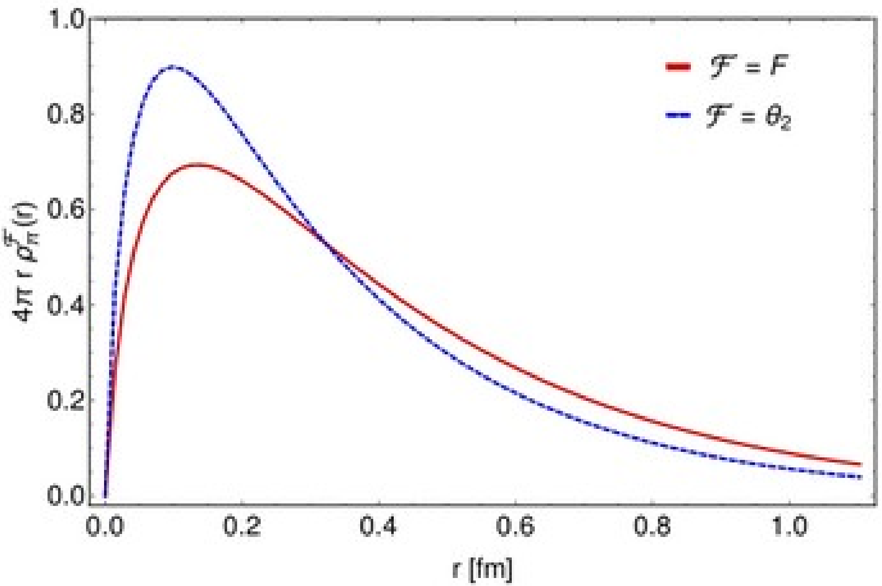

Clearly, as advanced, charge extends over a large domain than mass, the latter’s profile being more compact than the former’s.

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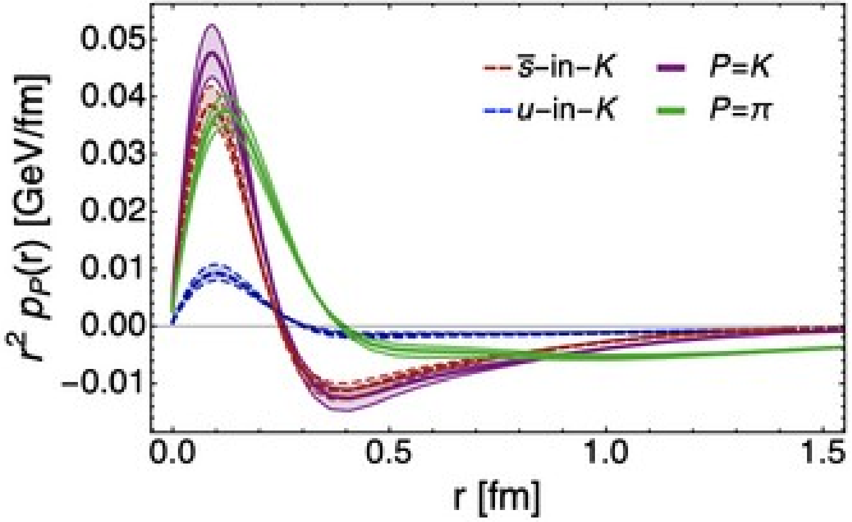
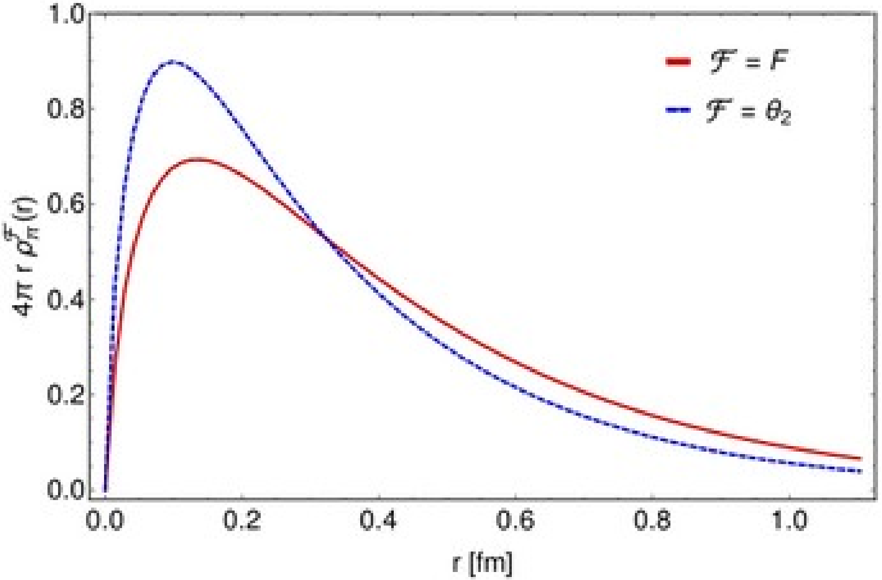
$$\rho_{\mathbf{P}}^{\tilde{\mathfrak{F}}}(r) = \frac{1}{2\pi}\int_0^\infty d\Delta\Delta J_0(\Delta r)\tilde{\mathfrak{F}}_{\mathbf{P}}(\Delta^2)$$

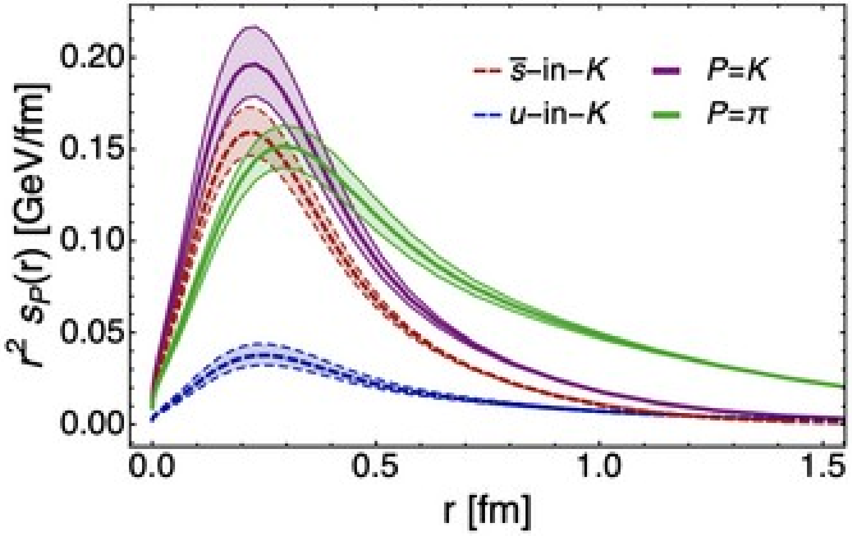
- **Pressure and shear** forces can be calculated as

$$p_{\mathbf{P}}(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta\,\frac{\Delta}{2E(\Delta)}\,\sin(\Delta r)[\Delta^2\theta_1^{\mathbf{P}}(\Delta^2)]$$

$$s_{\mathbf{P}}(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta\,\frac{\Delta^2}{2E(\Delta)} j_2(\Delta r)\,[\Delta^2\theta_1^{\mathbf{P}}(\Delta^2)]$$







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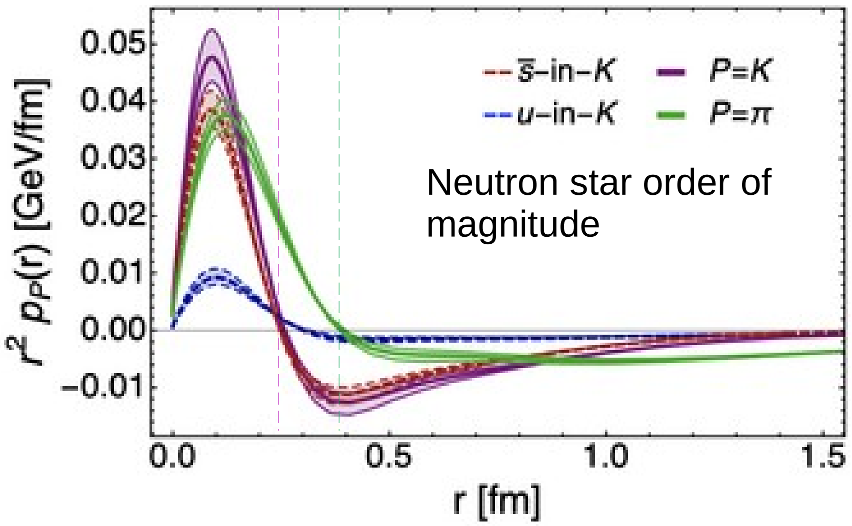
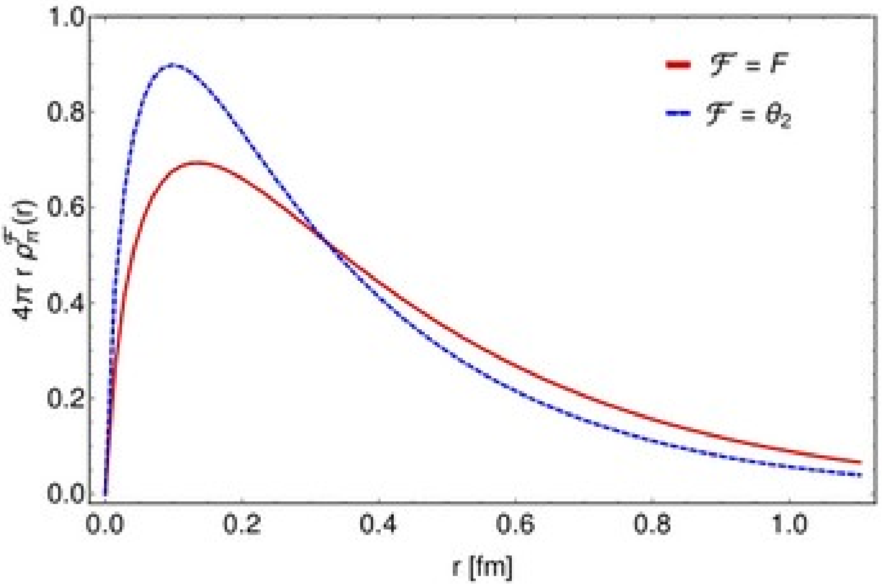
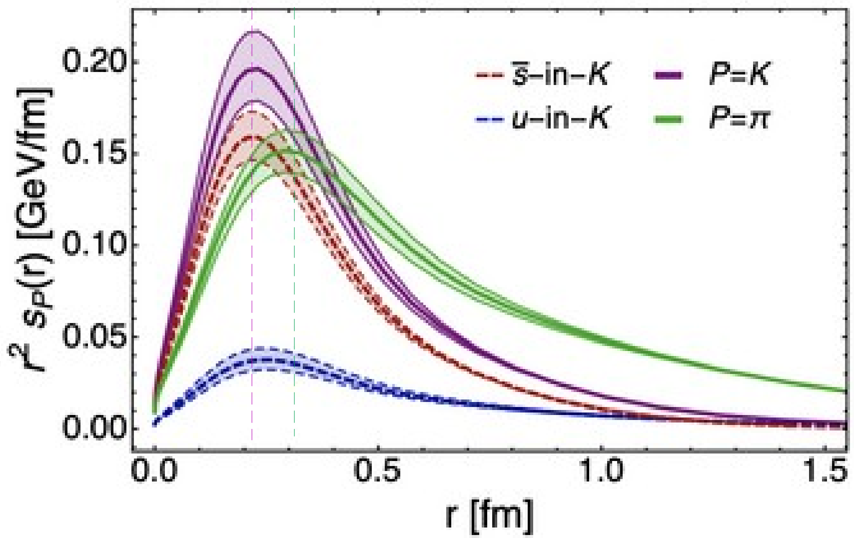


Neutron star order of magnitude

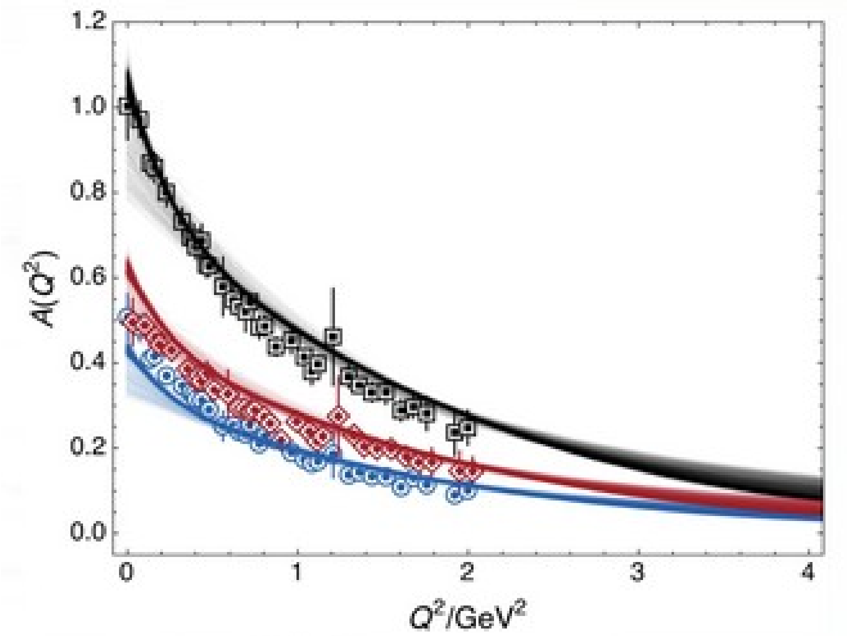


Clearly, as advanced, charge extends over a large domain than mass, the latter’s profile being more compact than the former’s.



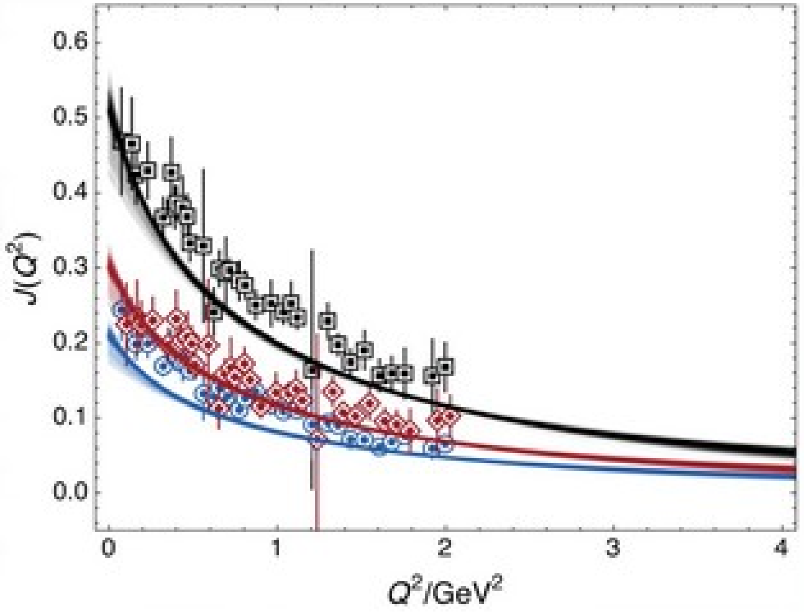
Confining forces become dominant where the pressure density shifts sign (integrated is zero), in the neighborhood of maximal shear forces. Kaon is more compact than pion, and so is s-in-K respect to u-in-K.

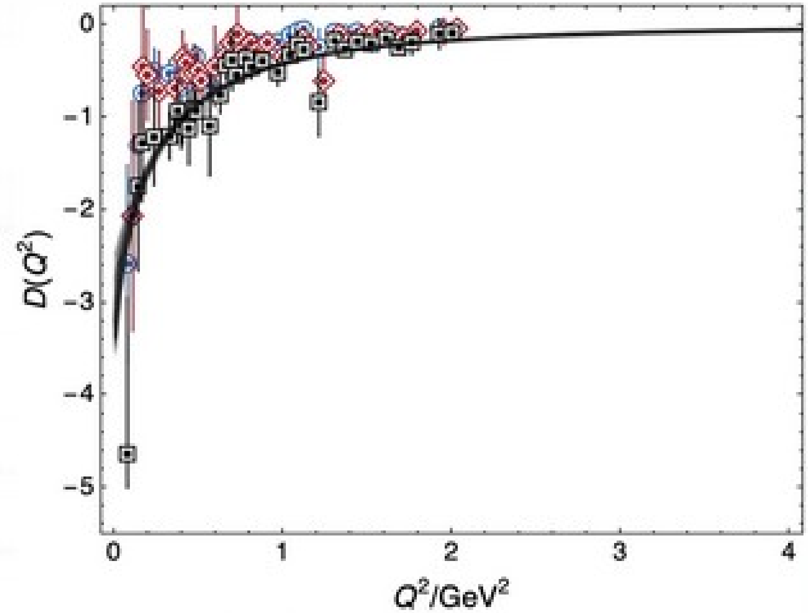
# Results: Preliminary results for the proton



Preliminary results obtained by **Daniele Binosi** and **Zhao-Qian Yao** using our same quark-graviton vertex and realistic Faddeev amplitudes derived from solutions of the three-body bound-state problem!

$$\langle p', \vec{s}\,'|T_a^{\mu\nu}|p, \vec{s}\rangle = \bar{u}(p', \vec{s}\,')\left[A_a(t)\,\frac{P^\mu P^\nu}{M_N} + D_a(t)\,\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + \bar{C}_a(t)\,M_N\,g^{\mu\nu} + J_a(t)\,\frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{M_N} - S_a(t)\,\frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{M_N}\right]u(p, \vec{s})$$





# Summary and scopes

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- We have described a **CSM based** consistent computation of pion’s and kaon’s **EFFs** and **GFFs,** the latter’s brand-new ingredient being the **quark-graviton vertex**.

- The obtained results expose the **robustness** of the framework and the importance of **symmetries**:
  - Both **quark-photon vertex** and **QGV** obey their corresponding **WGTI.**
  - The **QGV** is then completed and used to deliver GFFs, $\theta_1(Q^2)$ taking then zero-momentum values in consistency with the soft-pion theorem.
  - EMT, but not needed for the two other form factors.

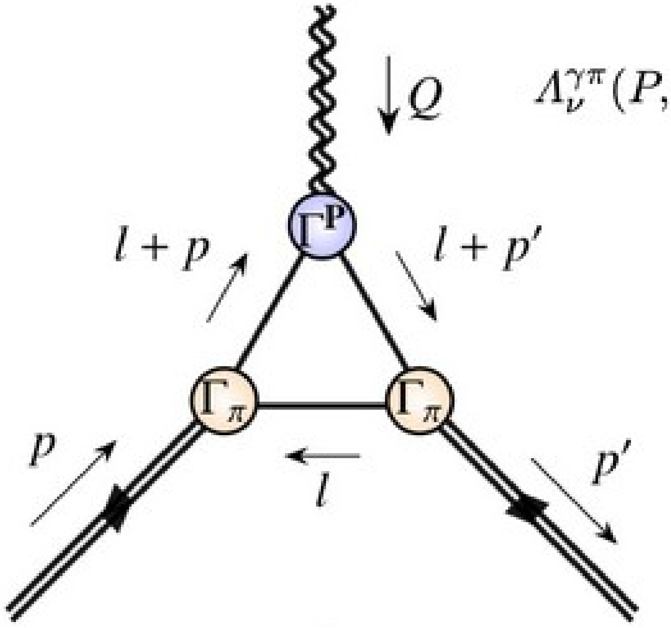
- **Physically** meaningful pictures are drawn:
  - **Charge** effects span over a larger domain than **mass** effects
  - **Shear** forces are maximal where **confinement** forces become dominant
  - Kaon’s profile is more compact than pion’s, and so is s-in-K respect to u-in-K.

- Other hadrons are **within reach:**
  - One can **analogously** proceed with **heavy quarkonia**
  - and, capitalizing on **Faddeev amplitudes**, compute **proton GFFs.** Preliminar results are shown.

**To be continued...**

# Backslides

# EFFs and GFFs: Algebraic Model

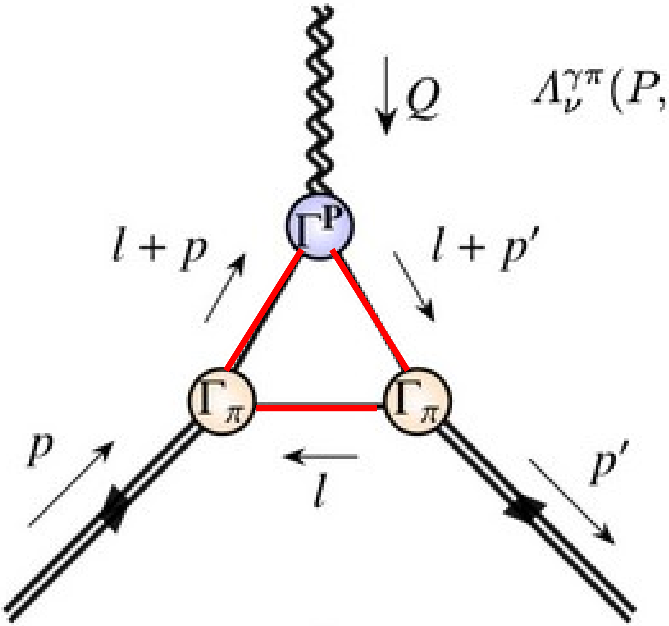


$$\Lambda_\nu^{\gamma\pi}(P,Q) = 2N_c \mathrm{tr}_D \int \frac{d^4 l}{(2\pi)^4} \Gamma_\nu^\gamma(l+p', l+p) L(l,P,Q)$$

$$S(l+p)\Gamma_\pi(l+p/2;p)S(l)\bar{\Gamma}_\pi(l+p'/2;-p')S(l+p')$$

$$(2P = p'+p,\ Q = p'-p,\ p'\cdot p' = -m_\pi^2 = p\cdot p,\ P\cdot Q = 0.)$$

# EFFs and GFFs: Algebraic Model



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$$\boxed{S(l+p)}\Gamma_\pi(l+p/2;p)\boxed{S(l)}\bar{\Gamma}_\pi(l+p'/2;-p')\boxed{S(l+p')}$$

$$(2P = p'+p,\ Q = p'-p,\ p'\cdot p' = -m_\pi^2 = p\cdot p,\ P\cdot Q = 0.)$$

$$S_{q=u,s}(l) = (-i\gamma\cdot l + M_q)/(l^2 + M_q^2)$$

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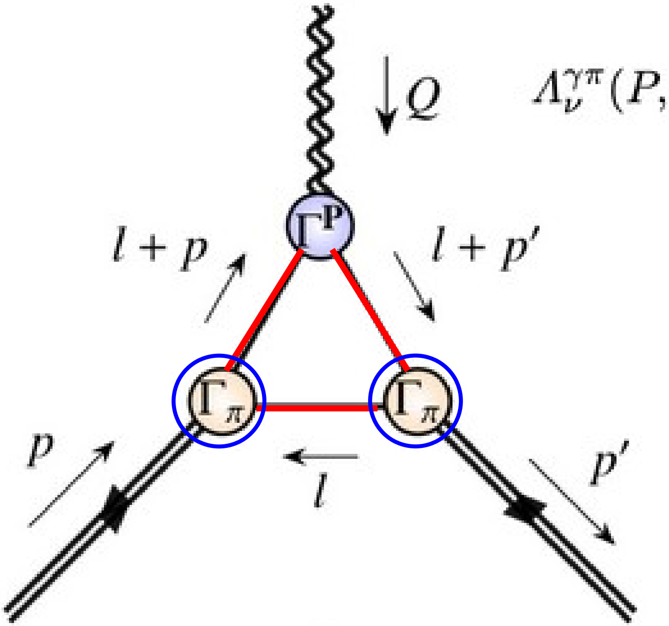

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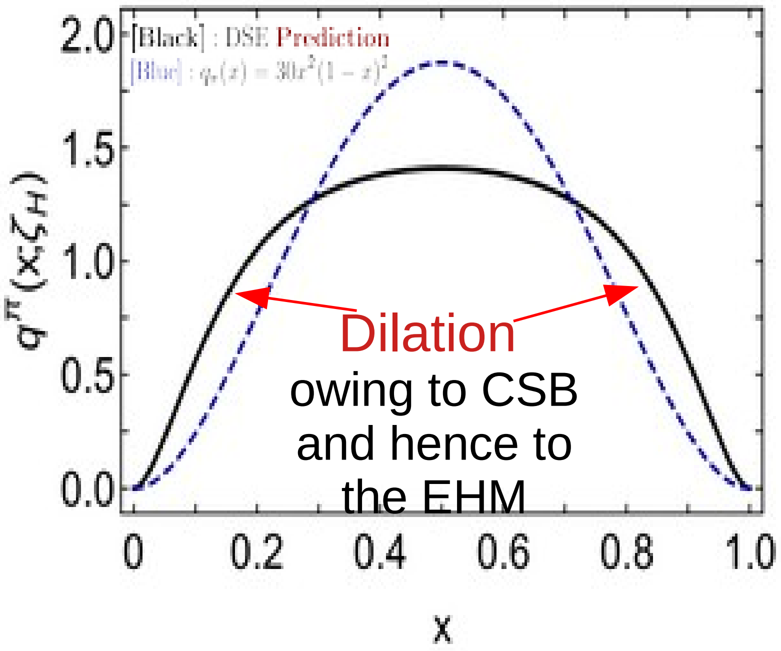
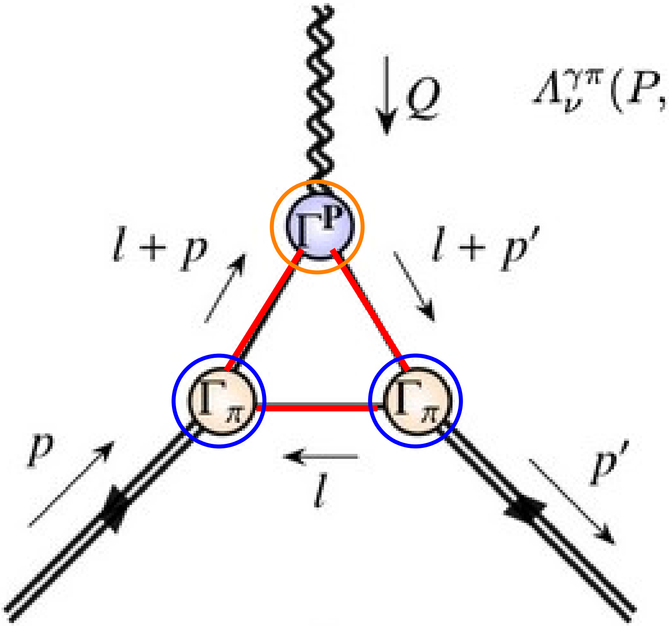
$$\Gamma_{\mathcal{P}=\pi,K}(l;p) = i\gamma_5 \int_{-1}^{1} dz\, \boxed{\rho_{\mathcal{P}}(z)}\, \hat{\Delta}(l_\omega^2, \Lambda_{\mathcal{P}}^2)$$

$$\hat{\Delta}(s,u) = u/[s+u],\ l_z = l + zp/2,\ p^2 = -m_{\mathcal{P}}^2$$



Dilation
owing to CSB
and hence to
the EHM

# EFFs and GFFs: Algebraic Model



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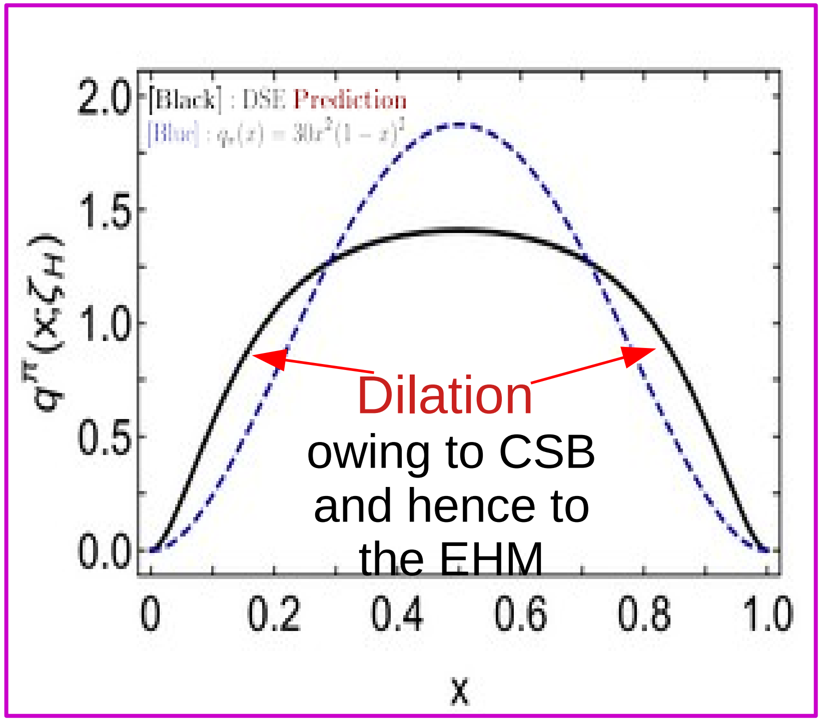
$$S_{q=u,s}(l) = (-i\gamma\cdot l + M_q)/(l^2+M_q^2)$$

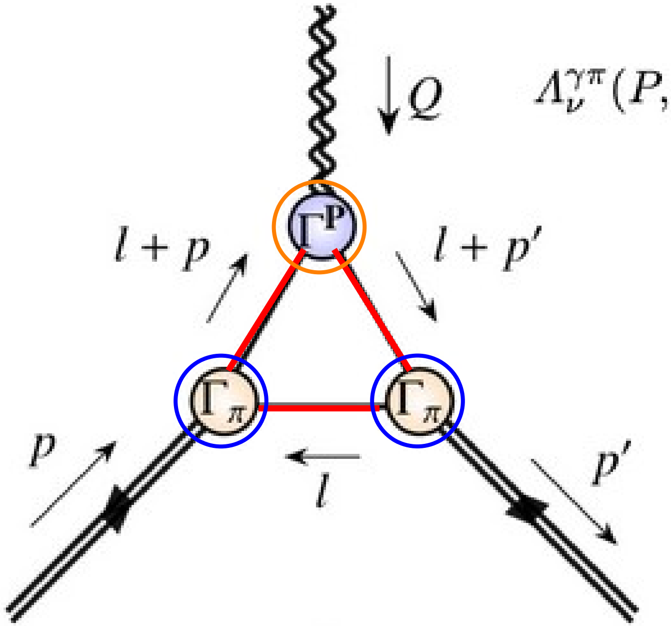
$$\Gamma_{\mathcal{P}=\pi,K}(l;p) = i\gamma_5 \int_{-1}^{1} dz\, \boxed{\rho_{\mathcal{P}}(z)}\, \hat{\Delta}(l_\omega^2, \Lambda_{\mathcal{P}}^2)$$
$$\hat{\Delta}(s,u) = u/[s+u],\, l_z = l + zp/2,\, p^2 = -m_{\mathcal{P}}^2$$

$$\Gamma_{\mu\nu}^{gq}(k;Q) = i\delta_{\mu\nu}[i\gamma\cdot k + M_q] + \gamma_\mu k_\nu + T_{\mu\alpha}(Q)\gamma_\alpha k_\nu 4P_q^{\mathbb{T}}(Q^2) + T_{\mu\nu}(Q)\mathbf{1}P_q^{\mathbb{S}}(Q^2)$$



Dilation
owing to CSB
and hence to
the EHM

# EFFs and GFFs: Algebraic Model



$$\Lambda_\nu^{\gamma\pi}(P,Q) = 2N_c \mathrm{tr}_D \int \frac{d^4 l}{(2\pi)^4} \Gamma_\nu^\gamma(l+p', l+p) L(l,P,Q)$$

$$\boxed{S(l+p)}\boxed{\Gamma_\pi(l+p/2;p)}\boxed{S(l)}\boxed{\bar{\Gamma}_\pi(l+p'/2;-p')}\boxed{S(l+p')}$$

$$(2P = p'+p,\, Q = p'-p,\, p'\cdot p' = -m_\pi^2 = p\cdot p, P\cdot Q = 0.)$$

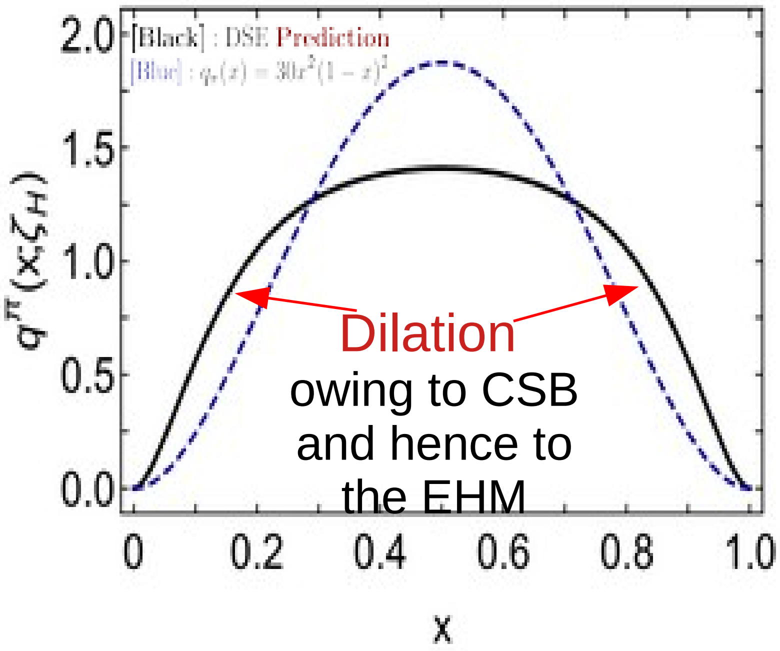
$$S_{q=u,s}(l) = (-i\gamma\cdot l + M_q)/(l^2 + M_q^2)$$

$$\Gamma_{\mathcal{P}=\pi,K}(l;p) = i\gamma_5 \int_{-1}^{1} dz \boxed{\rho_{\mathcal{P}}(z)} \hat{\Delta}(l_\omega^2, \Lambda_{\mathcal{P}}^2)$$

$$\hat{\Delta}(s,u) = u/[s+u],\, l_z = l + zp/2,\, p^2 = -m_{\mathcal{P}}^2$$

$$\Gamma_{\mu\nu}^{gq}(k;Q) = i\delta_{\mu\nu}[i\gamma\cdot k + M_q] + \gamma_\mu k_\nu + T_{\mu\alpha}(Q)\gamma_\alpha k_\nu 4\boxed{P_q^{\mathbb{T}}(Q^2)} + T_{\mu\nu}(Q)\mathbb{1}\boxed{P_q^{\mathbb{S}}(Q^2)}$$

$$\boxed{P_q^{\mathbb{T}}(t) = \frac{-t}{t+m_{\mathbb{T}_q}^2}\frac{m_{\mathbb{T}_q}^2(1-\kappa_q)^2}{t+\kappa_q^2 m_{\mathbb{T}_q}^2}} \qquad \boxed{P_q^{\mathbb{S}}(t) = \frac{-t r_{\mathbb{S}_q}}{t+m_{\mathbb{S}_q}^2}}$$

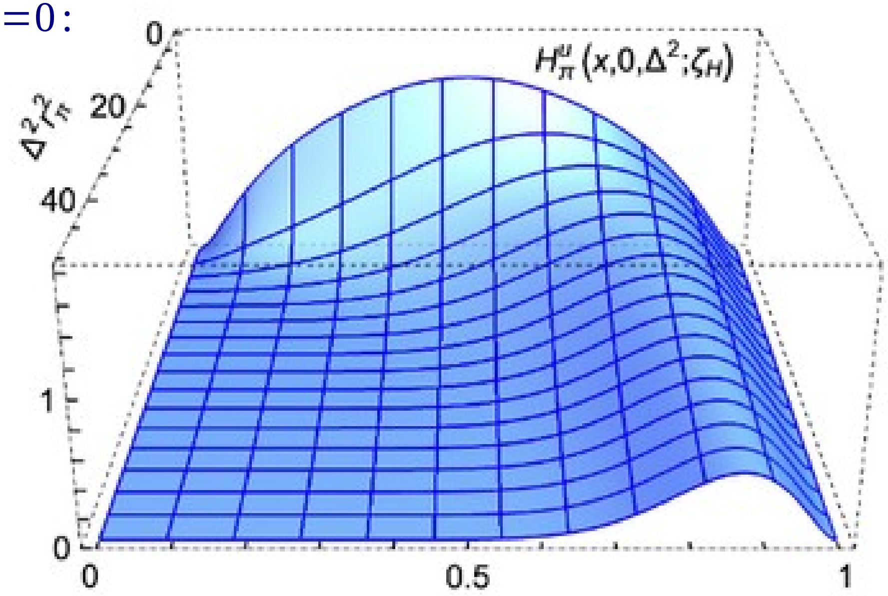


Dilation owing to CSB and hence to the EHM

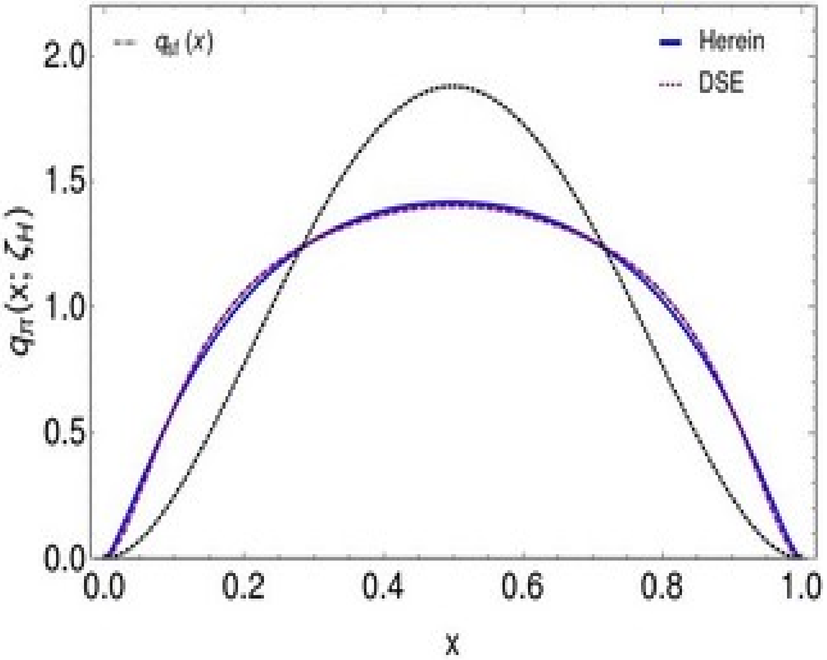
# GPDs from LFWFs

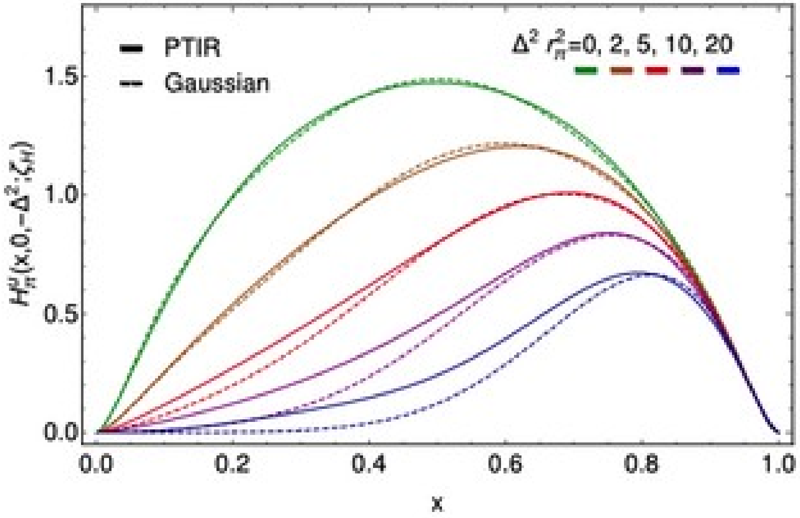
**Pion GPD:** $H_\pi^u\,(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_{\pi\,u}^{\uparrow\downarrow *}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp+\frac{\mathbf{1-x}}{\mathbf{1}-\xi}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi_{\pi\,u}^{\uparrow\downarrow}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp-\frac{\mathbf{1-x}}{\mathbf{1}+\xi}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$

$\xi=0:$



Valence-quark overlap GPD and forward PDF limit





Factorized gaussian ansatz:

$$H_\pi^u(x,\xi,t;\zeta_H) = \theta(x-\xi)\sqrt{u^\pi\left(\frac{x-\xi}{1-\xi}\right)u^\pi\left(\frac{x+\xi}{1+\xi}\right)}\,\exp\left(-\frac{-t\,r_\pi^2(1-x)^2}{6\langle x^2\rangle_u^{\zeta_H}(1-\xi^2)}\right)$$

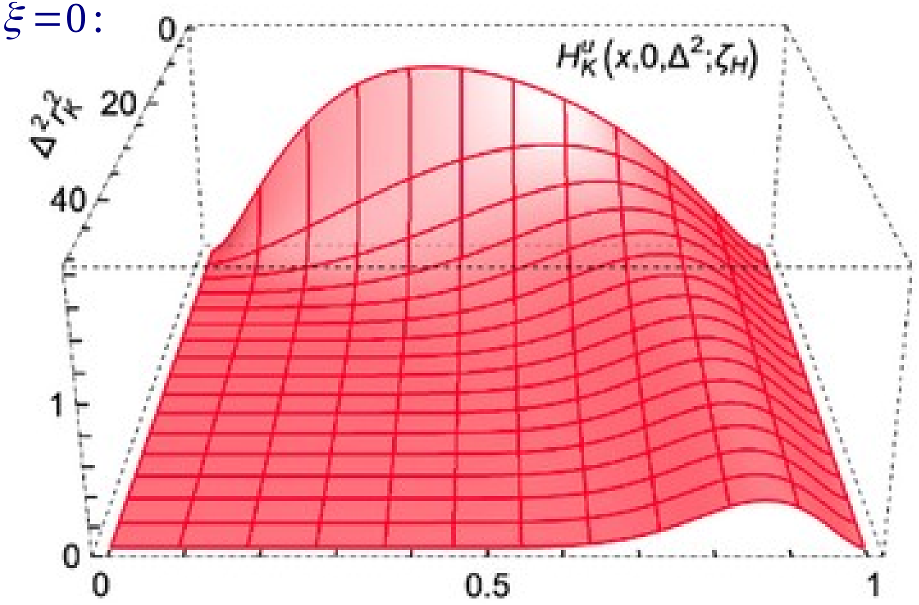
The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG: \; r_\pi = 0.659(8)\,fm \qquad DSE: \; r_\pi = 0.69\,fm\,[PTIR]$

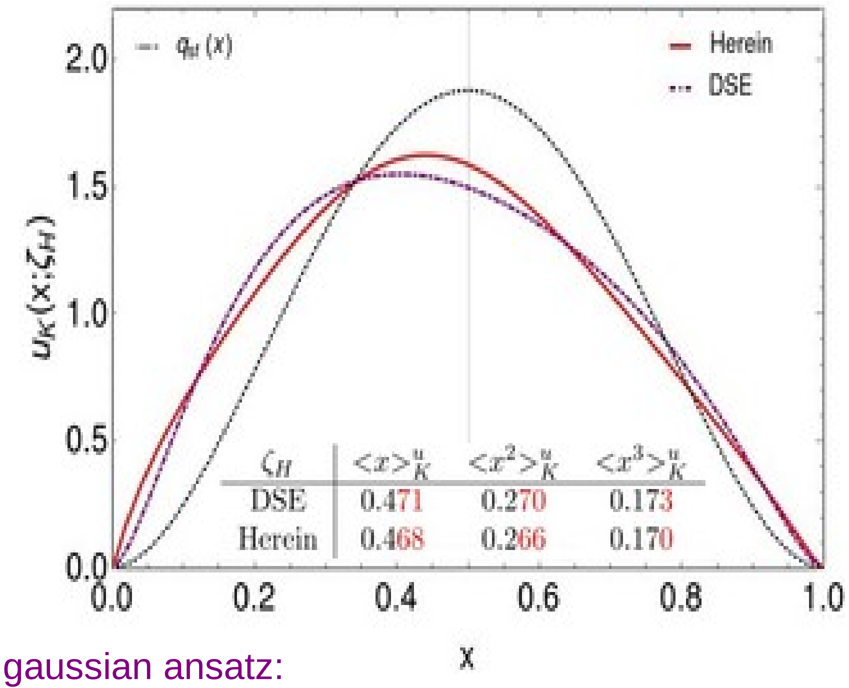
# GPDs from LFWFs

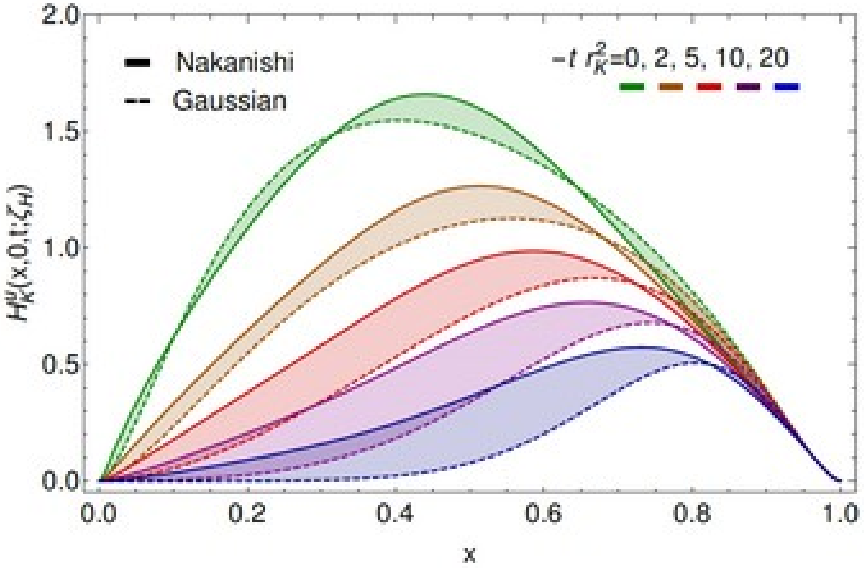
**Kaon GPD:** $H_K^u\left(x,\xi,t;\zeta_H\right)=\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_{K^u}^{\uparrow\downarrow *}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp+\frac{\mathbf{1-x}}{\mathbf{1-\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi_{K^u}^{\uparrow\downarrow}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp-\frac{\mathbf{1-x}}{\mathbf{1+\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$

$\xi=0:$



Valence-quark overlap GPD and forward PDF limit





Factorized gaussian ansatz:

$$H_K^u(x,\xi,t;\zeta_H) = \theta(x-\xi)\sqrt{u^K\left(\frac{x-\xi}{1-\xi}\right)u^K\left(\frac{x+\xi}{1+\xi}\right)} \times\exp\left(-\frac{-t\,r_K^2(1-x)^2}{\left(4\langle x^2\rangle_{\bar{s}}^{\zeta_H}+2(1+\delta)\langle x^2\rangle_u^{\zeta_H}\right)(1-\xi^2)}\right)$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG:\ r_K=0.560(31)\,fm \qquad DSE:\ r_K=0.56\,fm\,[PTIR]$

# Meson gravitational Form Factors

- **Gravitational** form factors connect with Energy-momentum tensor and are obtained from the **t-dependence** of the **GPD's 1-st Mellin moment**:

$$\theta^{\mathrm{M}}_{1,2}(-t) = \theta^{\mathrm{M}_u}_{1,2}(-t) + \theta^{\mathrm{M}_{\bar{h}}}_{1,2}(-t)$$

$$\int_{-1}^{1} dx\, x H^q_{\mathrm{M}}(x,\xi,t;\zeta_H) = \boxed{\theta_2^{\mathrm{M}_q}(-t)} - \xi^2 \boxed{\theta_1^{\mathrm{M}_q}(-t)}$$

Owing to GPD's polynomiality:

mass distribution

pressure distribution

$$\int_{-1}^{1} dx\, x H^q_{\mathrm{M}}(x,0,t;\zeta_H) = \theta_2^{\mathrm{M}_q}(-t)$$

One needs both DGLAP ( $|x| \geq \xi$ ) and ERBL ( $|x| \leq \xi$ ) GPD to derive the pressure distribution.

Radon transform inversion

ERBL completion

J-L. Zhang et al., arXiv:2101.12286