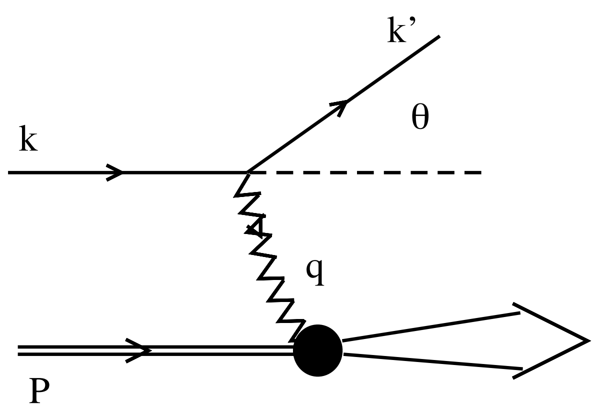
Gluons from the Dressing of Quarks: <x>, J and Mass Distributions



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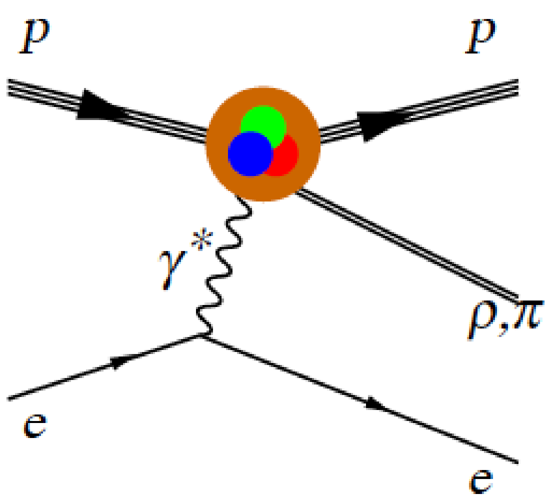
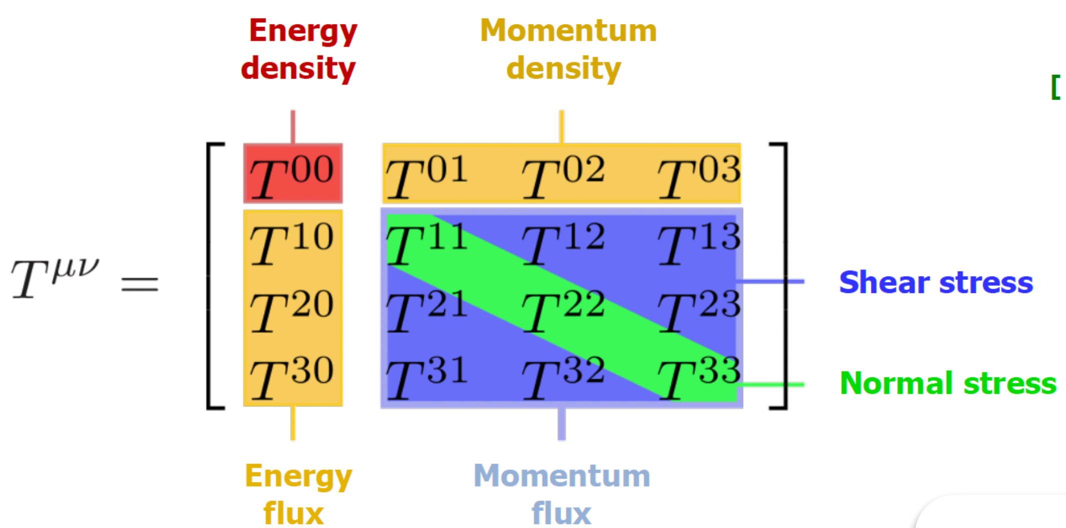
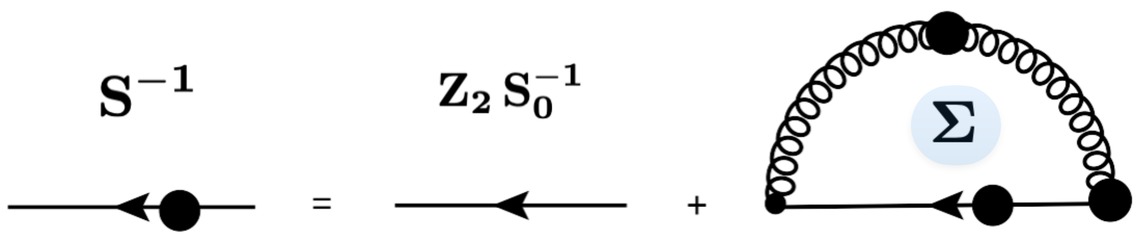


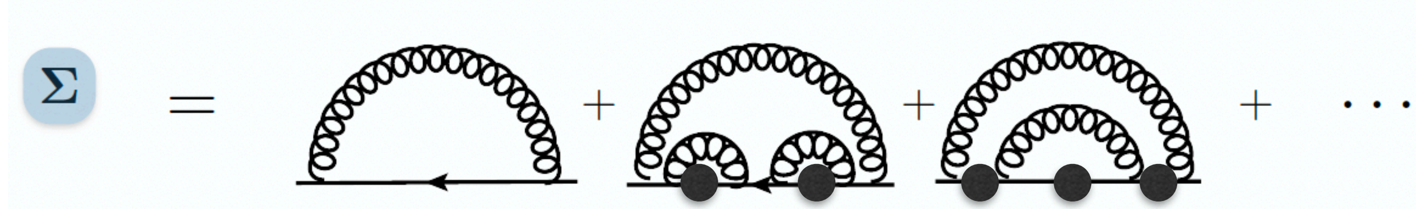


Outline

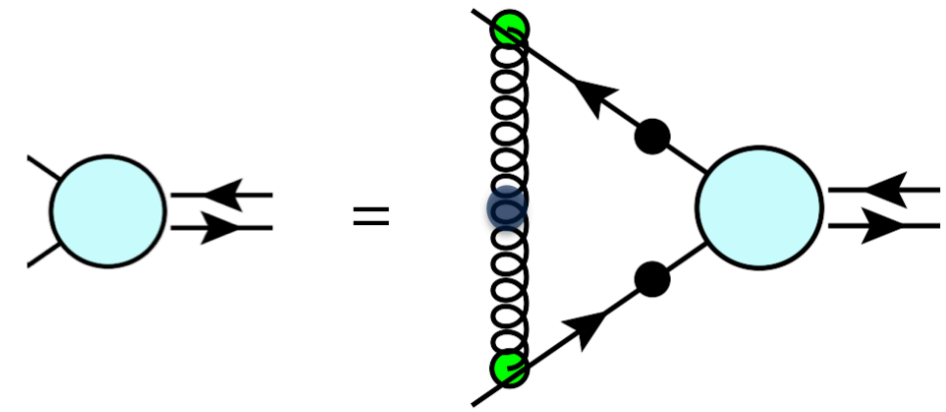
- **Modeling QCD for PDFs $\langle x\rangle_q$, $\langle x\rangle_g$, then $T^{00}_{q/g}(Q)$ & Grav FFs**
- **Employ DSE-RL approach to N and π. Very strong light quark dressing & mass generation.**
- **Obtain contribution of those gluons to Grav FFs, especially energy distribution $\mathcal{E}_{g/q}(Q)$**
- **An initial exploration of capabilities & qualitative features**
- **Compare radii R_g, R_q for $\mathcal{E}_{g/q}(Q)$ & other Grav FFs**
- **Extract $D_{\rm tot}(Q^2)$ and $\bar{C}_{q/g}(0)$**

DSE Ladder-Rainbow Truncation





$$S^{-1}(P) = Z_2\, S_0^{-1}(P) + \frac{4g^2}{3}\int_k^R \gamma_\mu\, \Delta_{\mu\nu}(P-k)\, S(k)\, \gamma_\nu$$



- **DSE-RL has 1-loop UV QCD renorm and 2 IR parameters fitted to** $\langle \bar{q}q \rangle,\ m_\pi,\ f_\pi$
- **Maintains** $\partial_\mu J_\mu = 0, \quad \partial_\mu J_{5\mu} \propto m_q$ **;** $m_\pi^{ch} = 0$**, indep of parameters**
- $M_{\rho,\phi,K^\star},\ f_{\rho,\phi,K^\star}$**, (5%, 10%);** $F_\pi(Q^2),\ F_K(Q^2)$ **P. Maris, P.C. Tandy, PRC60, 055214 (1999)**

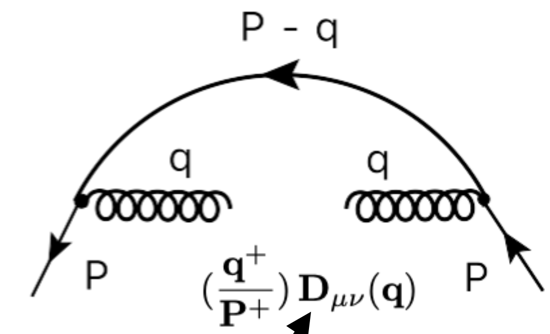
Simple Case Insight: Parton content of "physical" quark, dressed to 1-loop

"gluon-in-quark"

$$\mathbf{g(x) = \int \frac{dz^-}{4\,\pi}\, \frac{e^{\frac{i}{2} xP^+ z^-}}{xP^+} \left\langle P \middle| G^{+\,\alpha}(\frac{-\hat{z}}{2})\; W\; G_{\alpha\,+}(\frac{\hat{z}}{2}) \middle| P \right\rangle_c}$$

$$\mathbf{\hat{z} = z|_{z^+=0=\vec{z}_\perp}}$$

$\longrightarrow$ $<x>_g$ =



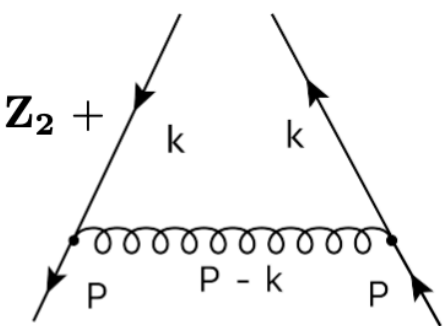
QCD in any gauge :

$$\langle x \rangle_g = \bar{u} \left[\frac{4g^2}{3} \int_q^R \frac{q \cdot n}{P \cdot n}\, \gamma_\mu\, [-n \cdot \partial_q \Delta^{LC}_{\mu\nu}(q)]\, S_0(P-q)\, \gamma_\nu \right] u$$

"quark-in-quark"

$$\mathbf{q(x) = \int \frac{dz^-}{8\,\pi}\, e^{\frac{i}{2} xP^+ z^-} \left\langle P \middle| \bar{q}(\frac{-\hat{z}}{2})\, \gamma^+\, W(\frac{-\hat{z}}{2}, \frac{\hat{z}}{2})\, q(\frac{\hat{z}}{2}) \middle| P \right\rangle_c}$$

$\longrightarrow$ $<x>_q \; = -i\not{p}\, Z_2 +$



LC gauge :

$$\langle x \rangle_q^{LC} = \bar{u} \left\{ -i\not{p}\, Z_2 - \frac{4g^2}{3} \int_k^R \frac{k \cdot n}{P \cdot n}\, \gamma_\mu\, \Delta^{LC}_{\mu\nu}(P-k)\, [n \cdot \partial_k S_0(k)]\, \gamma_\nu \right\} u$$

$$= \; 1 \; - \; \langle x \rangle_g \qquad \Leftarrow \quad \int \text{by parts}$$

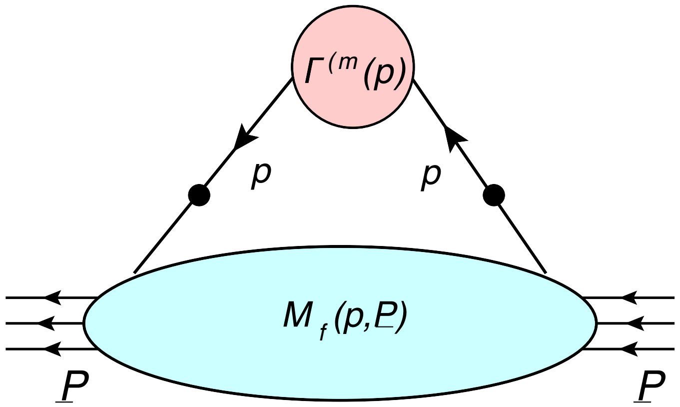
$$\mathbf{n = (1, -1; \vec{0}_T)}$$

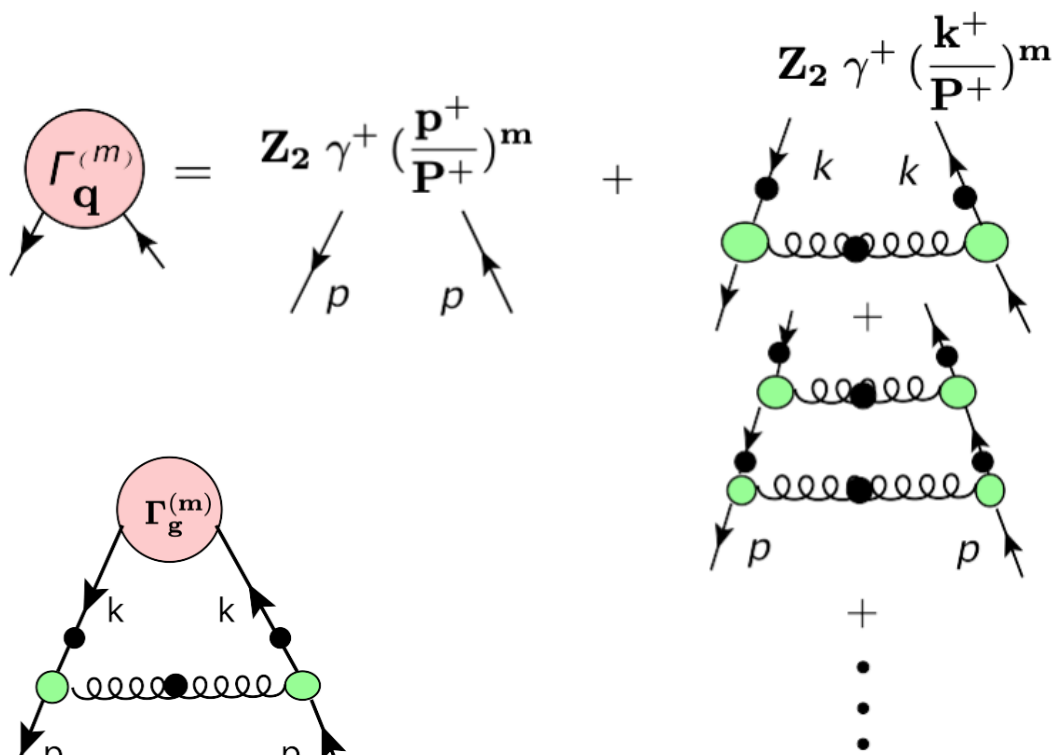
$$\mathbf{k^+ = k \cdot n}$$

$$\langle x \rangle_q^{LG} = 1 - \langle x \rangle_g \; + \langle x \rangle_q^W, \; (\approx 7\% \text{ too big})$$

Dressing $g(x)$ is not 0 at any scale

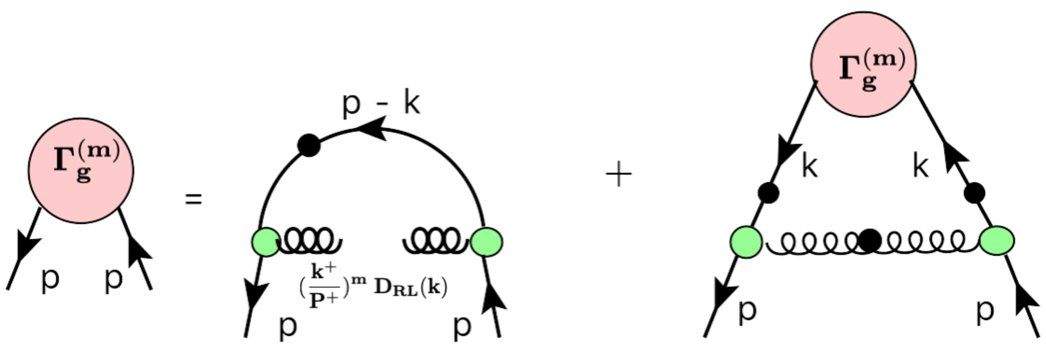
DSE-RL model calculations for hadrons





$\Rightarrow \langle x^m \rangle$ of quark-in-quark

Quark-in-quark PDF:
Bednar, Cloet, PCT, PRL 124, 042002 (2020)



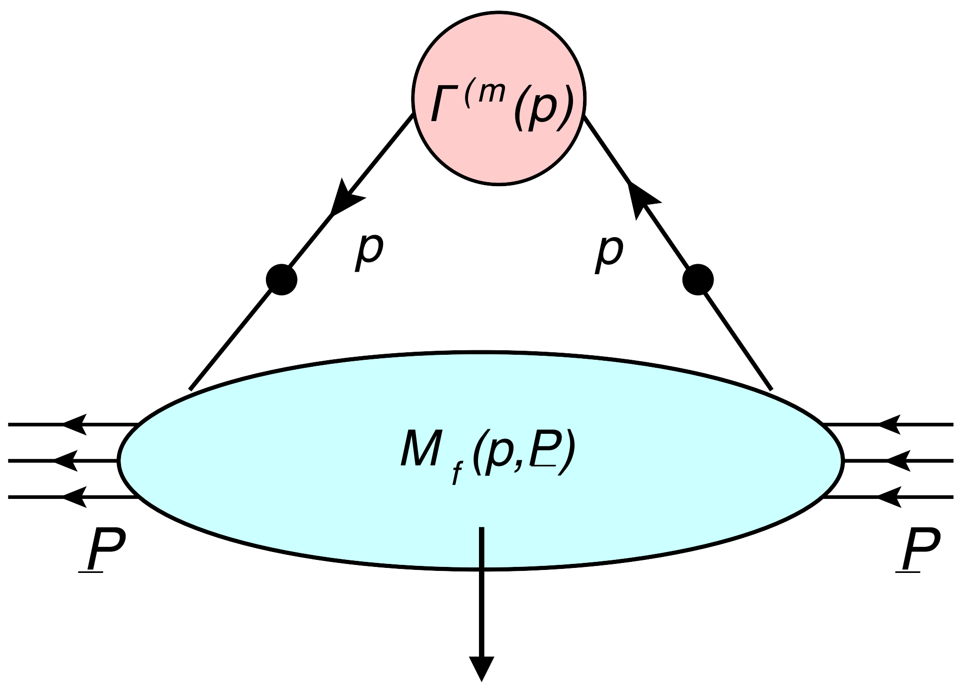
$\Rightarrow \langle x^m \rangle$ of gluon-in-quark

Gluon-in-quark PDF:
Freese, Cloet, PCT, PLB 823, 136719 (2021)

- **"cut-gluon" kernel IR parameter fit to $\langle x \rangle_g^\pi$ @ 1.3 GeV, $\sqrt{10}$ GeV**

Nucleon: First use an exploratory model

Freese, Cloet, PCT, PLB 823, 136719 (2021)



$$\mathcal{M}_\mathbf{f}(\mathbf{p},\mathbf{P}) = \mathbf{D_f}(\mathbf{p_c})\,\mathbf{A}(\mathbf{p},\mathbf{P})$$

$$\mathbf{D_f}(\mathbf{p_c}) = \sum_\mathbf{s} \mathcal{P}_{\mathbf{s},\mathbf{f}}\, \mathbf{u_s}\, \bar{\mathbf{u}}_\mathbf{s} = \frac{-\mathbf{i}\, \not{\mathbf{p}}_\mathbf{c} + \mathbf{M_c}}{\mathbf{2\,M_c}} \left(\frac{\mathbf{n_f} - \mathbf{i}\,\gamma_5\, \not{\mathbf{v}}_\mathbf{f}}{\mathbf{2}}\right)$$

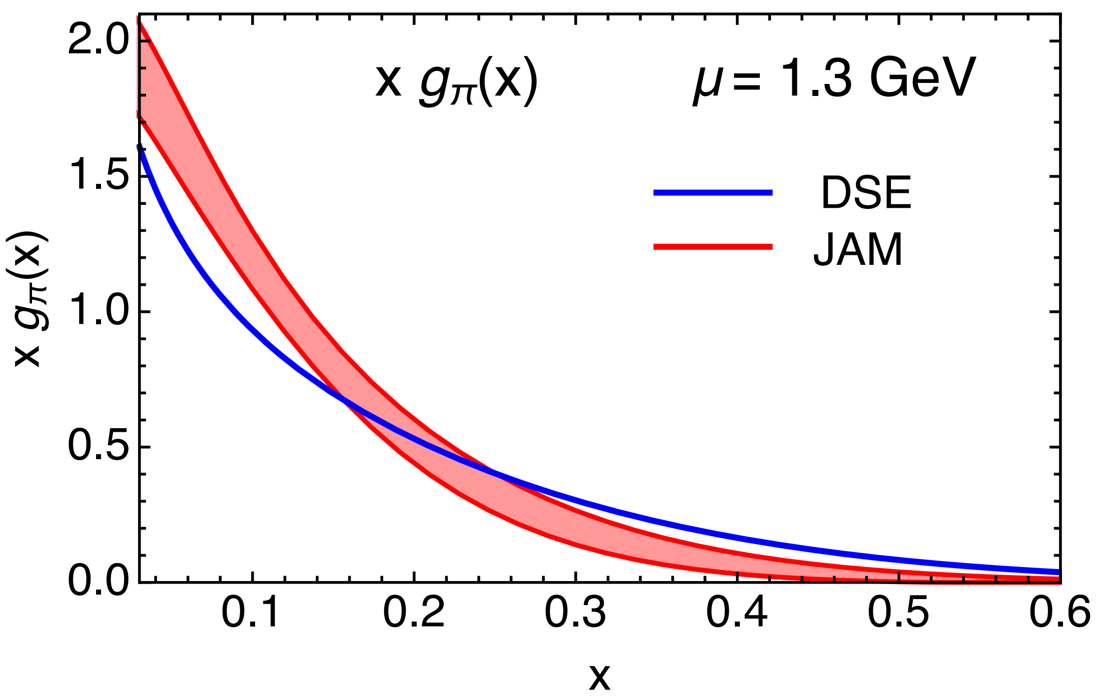
- **Spin-flavor Probs** $\mathcal{P}_{s,f} \Rightarrow n_f = \sum_s \mathcal{P}_{s,f}$**, polarizations** $v_f = (0, \mathcal{P}_f; \vec{0})$ **with** $\mathcal{P}_f = \sum_s s\,\mathcal{P}_{s,f}$
- **Use the** $SU(6)$ **state generalized to allow spatial behavior correlated with the spin-isospin of the spectator pair and fitted to Faddeev** $q - qq$ **calculations (Cloet et al, 2009)**

Pion: Use BSE solution

Pion & Nucleon DSE-RL calculations:

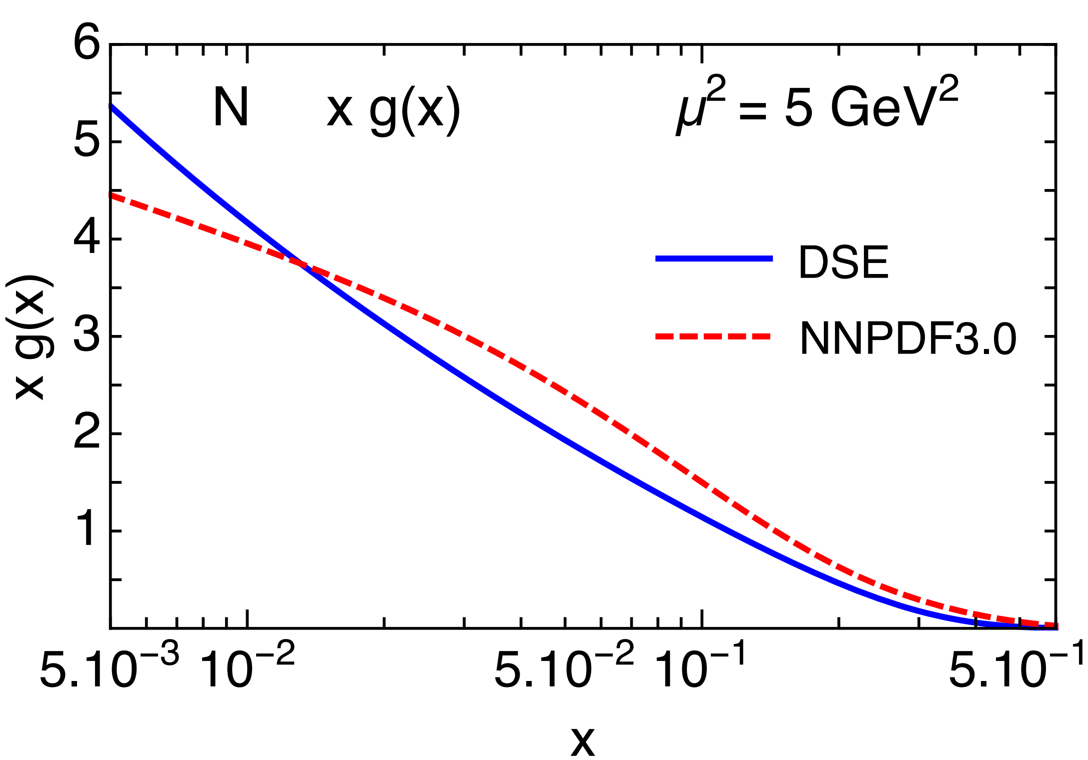
Freese, Cloet, PCT, PLB 823, 136719 (2021)

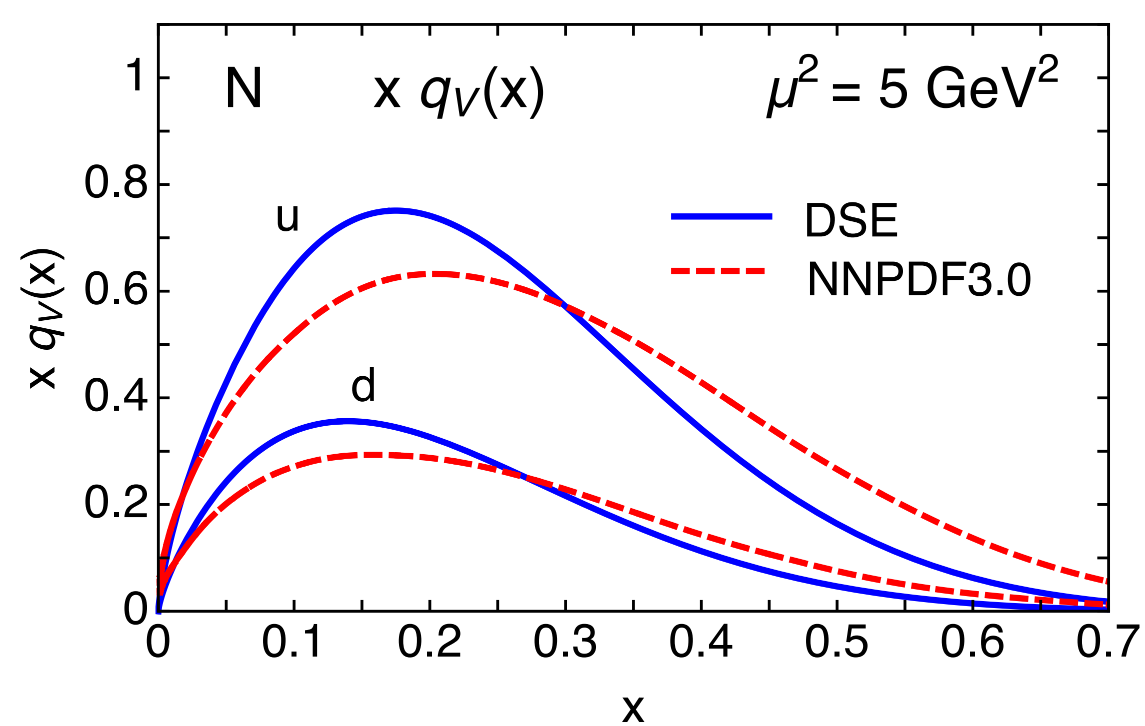
Bednar, Cloet, PCT, PRL 124, 042002 (2020)



	μ (GeV)	$2\langle x\rangle_{u_v}$	$\langle x\rangle_{sea}$	$\langle x\rangle_g$
π: Here	0.9	0.602	0.141	0.257
π: Here	$\sqrt{10}$	0.453	0.152	0.395
JAM [38]	$\sqrt{10}$	0.45 ± 0.01	0.17 ± 0.01	0.37 ± 0.02

	μ (GeV)	$\langle x\rangle_{u_v}$	$\langle x\rangle_{d_v}$	$\langle x\rangle_{sea}$	$\langle x\rangle_g$
N: Here	0.64	0.413	0.175	0.160	0.252
N: Here	$\sqrt{5}$	0.265	0.112	0.198	0.425
NNPDF3.0, Ref. [39]	$\sqrt{5}$	0.273	0.111	0.175	0.441





Parton Content of Nucleon J

GPDs

$$\mathbf{J_{q/g} = \frac{1}{2}\{A_{q/g}(0) + B_{q/g}(0)\} \quad = \frac{1}{2}\int dx\, x\,\{ H^{q/g}(x, \xi = 0, Q^2 = 0) + E^{q/g}(x, \xi = 0, Q^2 = 0)\}}$$

$$\mathbf{A_{q/g}(0) \to \langle x \rangle_{q/g}}$$

$$\xi = -\frac{Q^+}{2K^+} \qquad P',\ P = (K^0,\ 0;\ \pm\frac{Q_\perp}{2})$$

$$(K^0)^2 = E^2(Q_\perp) = M^2 + \frac{Q_\perp^2}{4}$$

$$\mathcal{J}^{q}_{s',s} = \int \frac{d\lambda}{4\pi}\, e^{ixK\cdot n\lambda}\, \Big\langle P', s' \Big| \bar{q}(-\lambda\frac{n}{2})\, \not{n}\, W(\lambda, A)\, q(\lambda\frac{n}{2}) \Big| P, s \Big\rangle_c \ ,$$

$$\mathcal{J}^{g}_{s',s} = \int \frac{d\lambda}{2\pi}\, \frac{e^{ixK\cdot n\lambda}}{xK^+}\, \Big\langle P', s' \Big| G^{+\alpha}(-\lambda\frac{n}{2})\, W\, G_{\alpha +}(\lambda\frac{n}{2}) \Big| P, s \Big\rangle_c$$

general form :

$$\mathcal{J}^{q/g}_{s',s}(x, \xi, Q^2) = \bar{u}_{s'}(P') \left[\not{n}\, H^{q/g} + \frac{i\sigma^{n\alpha} Q_\alpha}{2M}\, E^{q/g} \right] u_s(P)$$

Need $Q > 0$ to extract $H^{q/g}$ and $E^{q/g}$. (**Then extend to** $\langle P' | T^{++}(0) | P \rangle$)

$$\Rightarrow \quad A_{q/g}(Q^2),\ B_{q/g}(Q^2)$$

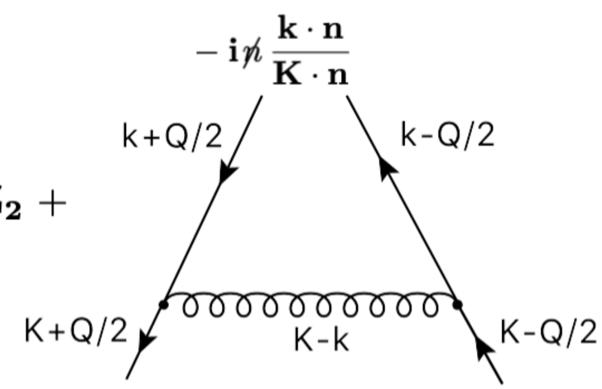
Simple Case Insight: Parton content of J of "physical" q, dressed to 1-loop

Tandy, Phys Lett B842, 137972 (2023)

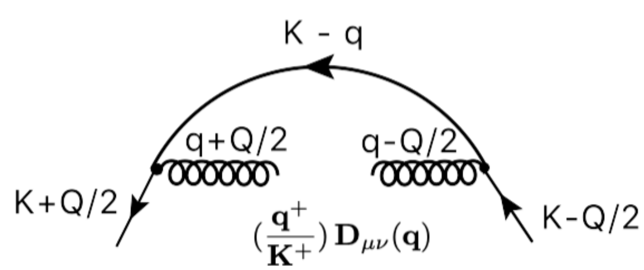
LC gauge :

$$\mathbf{J_q^{LC}} = \lim_{\mathbf{Q}\to 0} \left(\frac{-\mathbf{iM}}{\mathbf{Q}_\perp}\right) \bar{\mathbf{u}}_\uparrow(\mathbf{P}') \left[-\mathbf{i}\not{n}\,\mathbf{Z_2} - \frac{\mathbf{4g^2}}{\mathbf{3}} \int_{\mathbf{k}}^{\mathbf{R}} \left(\frac{\mathbf{k}\cdot\mathbf{n}}{\mathbf{K}\cdot\mathbf{n}}\right) \gamma_\mu\, \mathbf{\Delta}^{\mathbf{LC}}_{\mu\nu}(\mathbf{K}-\mathbf{k})\, [\mathbf{n}\cdot\partial_{\mathbf{k}} \mathbf{S_0}(\mathbf{k})]\, \gamma_\nu \right] \mathbf{u}_\downarrow(\mathbf{P})$$

$\mathbf{J_q} \propto \quad -\mathbf{i}\not{n}\,\mathbf{Z_2} +$



$\mathbf{J_g} \propto$



$\int$ **by parts** $\Rightarrow$

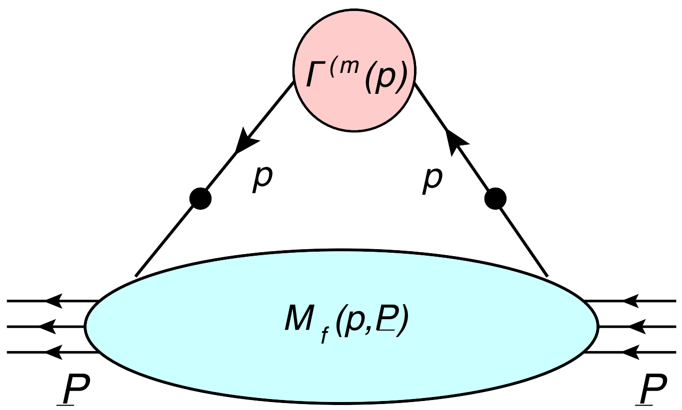
$$\mathbf{J_q^{LC}} = \frac{\mathbf{1}}{\mathbf{2}} - \mathbf{J_g}$$

In any gauge :

$$\mathbf{J_g} = \lim_{\mathbf{Q}\to 0} \left(\frac{-\mathbf{iM}}{\mathbf{Q}_\perp}\right) \bar{\mathbf{u}}_\uparrow(\mathbf{P}') \left[\frac{\mathbf{4g^2}}{\mathbf{3}} \int_{\mathbf{q}}^{\mathbf{R}} \frac{\mathbf{q}\cdot\mathbf{n}}{\mathbf{K}\cdot\mathbf{n}} \gamma_\mu [-\mathbf{n}\cdot\partial_{\mathbf{q}}\, \mathbf{\Delta}^{\mathbf{LC}}_{\mu\nu}(\mathbf{q})]\, \mathbf{S_0}(\mathbf{K}-\mathbf{q})\, \gamma_\nu \right] \mathbf{u}_\downarrow(\mathbf{P})$$

$$\mathbf{J_q^{LG}} = \left(\frac{\mathbf{1}}{\mathbf{2}} - \mathbf{J_g}\right) + \mathbf{J_q^W}, \quad (\text{ 7\% too big})$$

Tandy, Phys Lett B842, 137972 (2023)



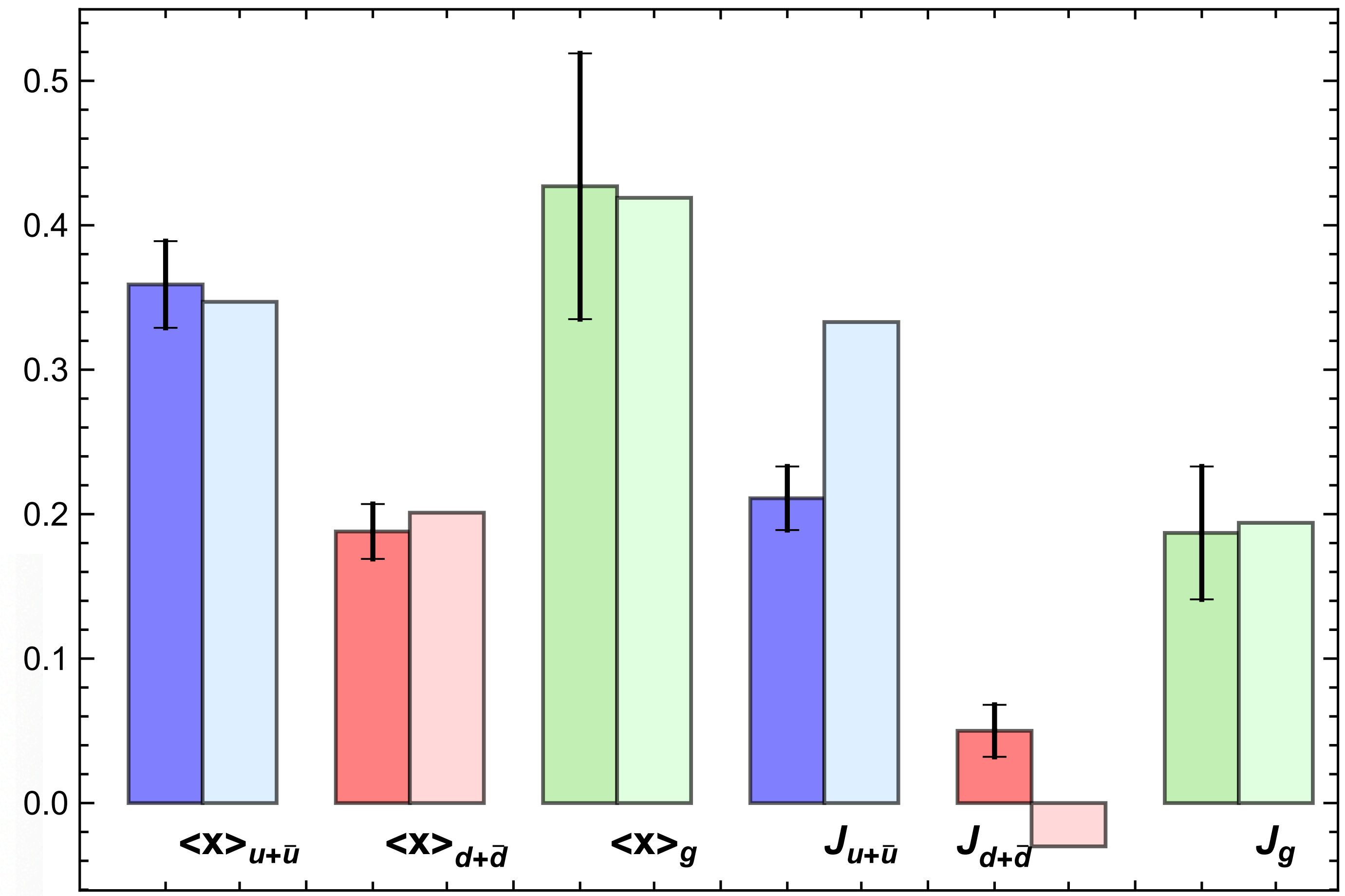
J_q , J_g of Proton at μ_0

	J_{u_v}	J_{d_v}	J_g	J_{tot}	$J_{\bar{u}}$	$J_{\bar{d}}$
v only	0.478	-0.119	0.119	0.478	0	0
	100%	-25%	25%	100%	0	0

	$J_{u+\bar{u}}$	$J_{d+\bar{d}}$	J_g	J_{tot}	$J_{\bar{u}}$	$J_{\bar{d}}$
v + πN	0.439	-0.051	0.119	0.507	0.0104	0.0520
	86.6%	-10.1%	23.5%	100%	2.1%	10.3%

Table III. Parton sharing of the proton J at the model scale μ_0 = 0.64 GeV. The first section at the top is from the previously established DSE-RL model containing only valence quarks and the dynamically involved dressing glue. The next section shows the quark J values from pion cloud dressing of the proton involving πN Fock terms.

Proton: $\langle x \rangle_{q/g}$, $J_{q/g}$ @ $\mu = 2$ GeV



LQCD—-C. Alexandrou, et al, PRD 101, (2020)

H. Pagels (1966) X. Ji (1997) M. Polyakov, C. Weiss (1999)

$$\mathbf{P}', \ \mathbf{P} = (\mathbf{K^0}, \ \mathbf{0}; \ \pm\frac{\mathbf{Q}_\perp}{\mathbf{2}})$$

$$(\mathbf{K^0})^2 = \mathbf{E^2}(\mathbf{Q}_\perp) = \mathbf{M^2} + \frac{\mathbf{Q}_\perp^2}{\mathbf{4}}$$

EMT/Gravitational Form Factors :

N $$\langle \mathbf{P}' | \mathbf{T}^{\mu\nu}_{q/g}(\mathbf{0}) | \mathbf{P} \rangle = \bar{\mathbf{u}}' \{ \mathbf{A}_{q/g}(\mathbf{Q^2}) \, \mathbf{2} \, \mathbf{K}^\mu \mathbf{K}^\nu + \mathbf{J}_{q/g}(\mathbf{Q^2}) \, \mathbf{t}^{\mu\nu}(\mathbf{K}, \mathbf{Q}, \sigma) + \mathbf{D}_{q/g}(\mathbf{Q^2}) \, \frac{\mathbf{Q}^\mu \mathbf{Q}^\nu - \mathbf{g}^{\mu\nu} \mathbf{Q^2}}{\mathbf{2}} + \bar{\mathbf{c}}_{q/g}(\mathbf{Q^2}) \, \mathbf{2} \, \mathbf{M^2} \, \mathbf{g}^{\mu\nu} \} \mathbf{u}$$

π $$\langle \mathbf{P}' | \mathbf{T}^{00}_{q/g}(\mathbf{0}) | \mathbf{P} \rangle = \mathbf{A}_{q/g}(\mathbf{Q^2}) \, \mathbf{2} \, (\mathbf{K^0})^2 + \mathbf{D}_{q/g}(\mathbf{Q^2}) \, \frac{\mathbf{Q}_\perp^2}{\mathbf{2}} + \bar{\mathbf{c}}_{q/g}(\mathbf{Q^2}) \, \mathbf{2} \, \mathbf{M^2}$$

Approach for calculations :

Already have GPDs $\Rightarrow$ $\mathbf{A}_{q/g}(\mathbf{Q^2}), \ \mathbf{B}_{q/g}(\mathbf{Q^2})$

Directly calc QCD matrix elem for $\left\langle \mathbf{P}' \middle| \mathbf{T}^{00}_{q/g}(\mathbf{0}) \middle| \mathbf{P} \right\rangle$

Deduce $\mathcal{E}_{q/g}(\mathbf{Q^2}), \ \mathbf{J}_{q/g}(\mathbf{Q^2}), \ \mathbf{D}_{\rm tot}(\mathbf{Q^2}), \ \bar{\mathbf{C}}_{q/g}(\mathbf{0})$

$\left\langle \mathbf{P} \middle| \mathbf{T}^{++}_{\mathbf{q/g}}(\mathbf{0}) \middle| \mathbf{P} \right\rangle$ and PDF $\langle \mathbf{x} \rangle_{\mathbf{q/g}}$ are same calculation

$$\mathbf{q}(\mathbf{x}) = \int \frac{\mathbf{d}\lambda}{\mathbf{4}\,\pi}\, \mathbf{e}^{\mathbf{ix\,P\cdot n}\,\lambda}\, \langle \mathbf{P}|\, \bar{\mathbf{q}}(-\lambda\frac{\mathbf{n}}{\mathbf{2}})\, \not{n}\, \mathbf{W}(\lambda, \mathbf{A})\, \mathbf{q}(\lambda\frac{\mathbf{n}}{\mathbf{2}})\, |\mathbf{P}\rangle_{\mathbf{c}} \qquad \mathbf{g}(\mathbf{x}) = \int \frac{\mathbf{d}\lambda}{\mathbf{2}\,\pi} \frac{\mathbf{e}^{\mathbf{ix\,P\cdot n}\,\lambda}}{\mathbf{xP^+}}\, \langle \mathbf{P}|\, \mathbf{G}^{+\,\mu}(-\lambda\frac{\mathbf{n}}{\mathbf{2}})\, \mathbf{W}\, \mathbf{G}_{\mu\,+}(\lambda\frac{\mathbf{n}}{\mathbf{2}})\, |\mathbf{P}\rangle_{\mathbf{c}}$$

QCD

$$\Downarrow$$

$$\langle \mathbf{x} \rangle_{\mathbf{q}} + \langle \mathbf{x} \rangle_{\mathbf{g}} = \frac{\mathbf{1}}{\mathbf{2}(\mathbf{P}^+)^{\mathbf{2}}} \left\langle \mathbf{P} \middle| \bar{\mathbf{q}}(\mathbf{0})\, \frac{\mathbf{i}}{\mathbf{2}}\gamma^+ \overleftrightarrow{\mathbf{D}}^+\, \mathbf{q}(\mathbf{0}) + \mathbf{G}^{+\mu}(\mathbf{0})\, \mathbf{G}_{\mu}{}^{+}(\mathbf{0}) \middle| \mathbf{P} \right\rangle_{\mathbf{c}}$$

Collins, Soper, 1982

$$= \frac{\langle \mathbf{P}|\, \mathbf{T}^{++}(\mathbf{0})\, |\mathbf{P}\rangle_{\mathbf{c}}}{\mathbf{2}(\mathbf{P}^+)^{\mathbf{2}}} = \mathbf{1}$$

Momentum Sum Rule (scale invariant)

$$\mathbf{T}^{++} = \mathbf{n}_{\mu}\, \mathbf{T}^{\mu\nu}\, \mathbf{n}_{\nu}$$

$$\mathbf{n} = (\mathbf{1}, -\mathbf{1}; \vec{0}_{\mathbf{T}})$$

$$\left\langle P' \middle| T^{++}_{q/g}(0) \middle| P \right\rangle \text{ extends PDF } \langle x \rangle_{q/g} \text{ to } Q > 0$$

$$P',\ P = (K^0,\ 0;\ \pm\frac{Q_\perp}{2})$$

$$(K^0)^2 = E^2(Q_\perp) = M^2 + \frac{Q_\perp^2}{4}$$

Breit frame

$$\langle P' \,|\, T^{++}(0) \,|\, P \rangle = \left\langle P', s' \middle| \bar{q}(0)\, \frac{i}{2} \gamma^+ \overleftrightarrow{D}^+ q(0) \; + G^{+\mu}(0)\, G_\mu{}^+(0) \middle| P, s \right\rangle_c$$

$$\frac{\langle P' | T^{++}_q(0) | P \rangle_c}{2(K^+)^2} = \int dx\, x \int \frac{dz^-}{8\,\pi}\, e^{\frac{i}{2} x K^+ z^-} \left\langle P' \middle| \bar{q}(\frac{-\hat{z}}{2})\, \gamma^+\, W(\frac{-\hat{z}}{2}, \frac{\hat{z}}{2})\, q(\frac{\hat{z}}{2}) \middle| P \right\rangle_c \quad \hat{z} = z|_{z^+=0=\vec{z}_\perp}$$

$$\frac{\langle P' | T^{++}_q(0) | P \rangle}{2(K^+)^2} = \frac{1}{2K^+} \int \frac{d^4k}{(2\pi)^4} (\frac{k^+}{K^+}) \int d^4z\, e^{ik\cdot z} \left\langle P' \middle| \bar{q}(-\frac{z}{2})\, \gamma^+\, W\, q(\frac{z}{2}) \middle| P \right\rangle_c$$

$\mathrm{k - space\ repn}$

$\mathrm{eg\ DSE - RL\ Feyn\ Diags}$

$$\langle P' \,|\, T^{++}_q(0) \,|\, P \rangle |_{v^+ \to v^0} \quad \to \langle P' \,|\, T^{00}_q(0) \,|\, P \rangle$$

$$\frac{\langle P' | T^{00}_q(0) | P \rangle}{2(K^0)^2} = \frac{1}{2K^0} \int \frac{d^4k}{(2\pi)^4} (\frac{k^0}{K^0}) \int d^4z\, e^{ik\cdot z} \left\langle P' \middle| \bar{q}(-\frac{z}{2})\, \gamma^0\, W(-\frac{z}{2}, \frac{z}{2})\, q(\frac{z}{2}) \middle| P \right\rangle_c$$

$$\left\langle P' \middle| T_g^{++}(0) + \frac{g^{++}}{4} G\, G \middle| P \right\rangle |_{v^+ \to v^0} \quad \to \langle P' \,|\, T_g^{00}(0) \,|\, P \rangle$$

$$\frac{\langle P' | T_g^{++}(0) | P \rangle_c}{2(K^+)^2} = \int dx\, x \int \frac{dz^-}{8\,\pi} \frac{e^{\frac{i}{2} x K^+ z^-}}{x K^+} \left\langle P' \middle| G^{+\,\alpha}(\frac{-\hat{z}}{2})\, G_{\alpha\,+}(\frac{\hat{z}}{2}) \middle| P \right\rangle_c \cdot \hat{z} = z|_{z^+=0=\vec{z}_\perp}$$

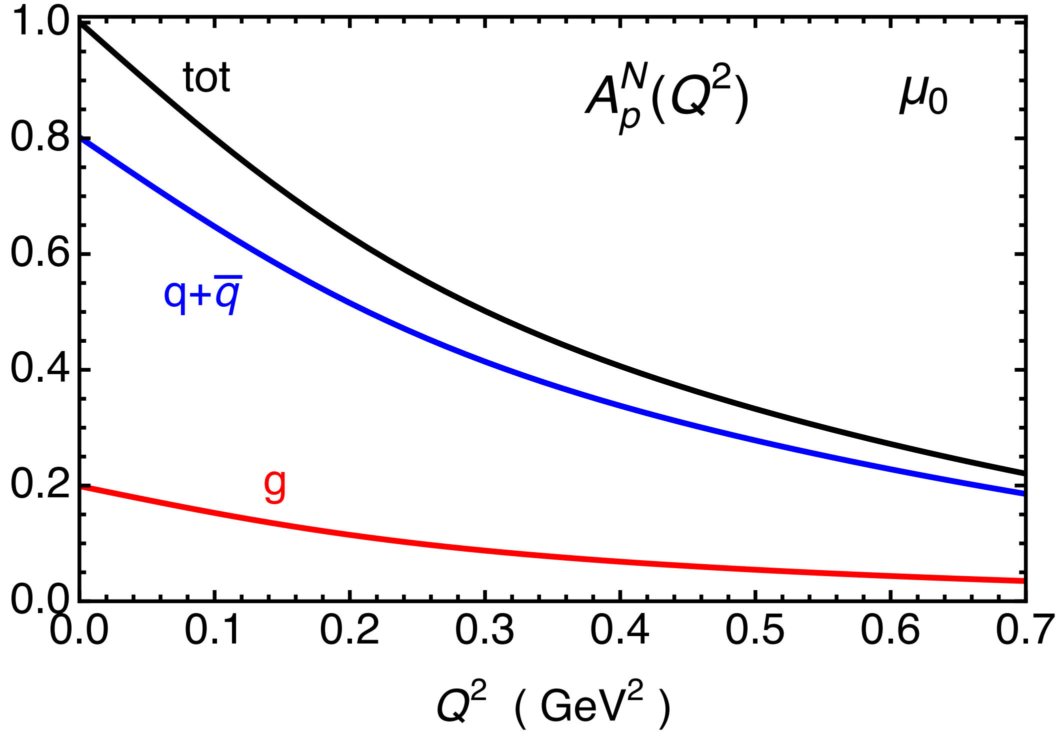
$$\frac{\langle P' | T_g^{++}(0) | P \rangle}{2(K^+)^2} = \frac{1}{2K^+} \int \frac{d^4k}{(2\pi)^4} (\frac{k^+}{K^+}) \int d^4z \, \frac{e^{ik\cdot z}}{k^+} \left\langle P' \,\middle|\, G^{+\alpha}(-\frac{z}{2})\, G_\alpha{}^+(\frac{z}{2}) \,\middle|\, P \right\rangle_c$$

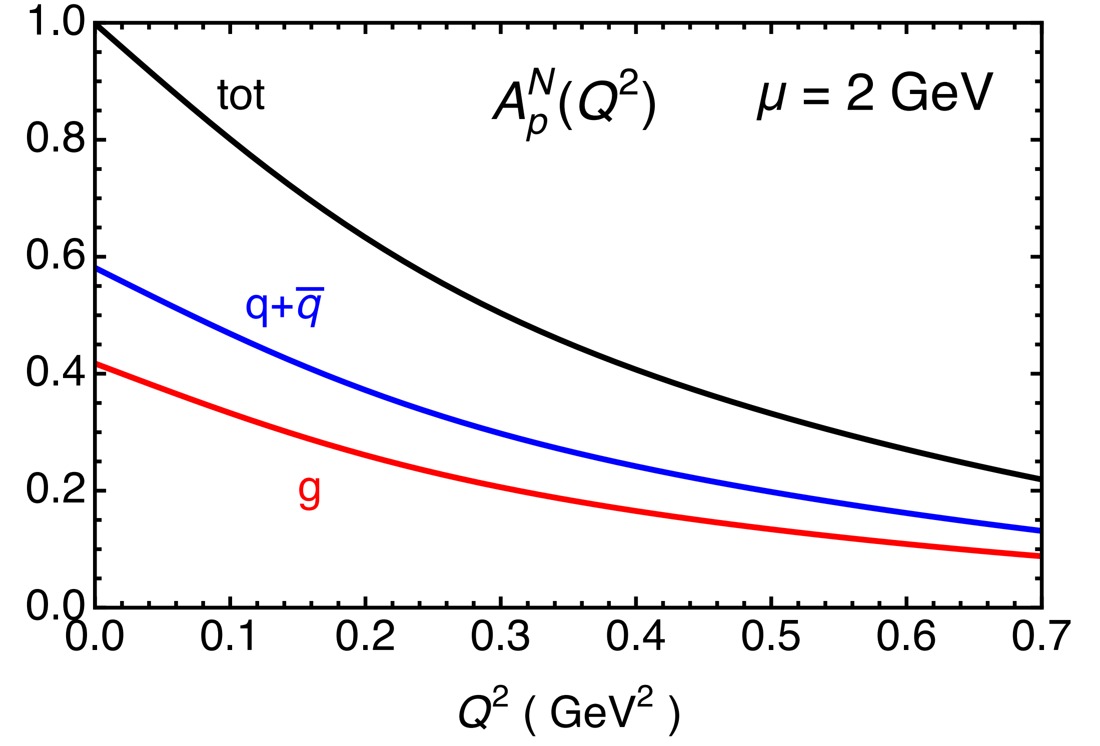
k – space repn

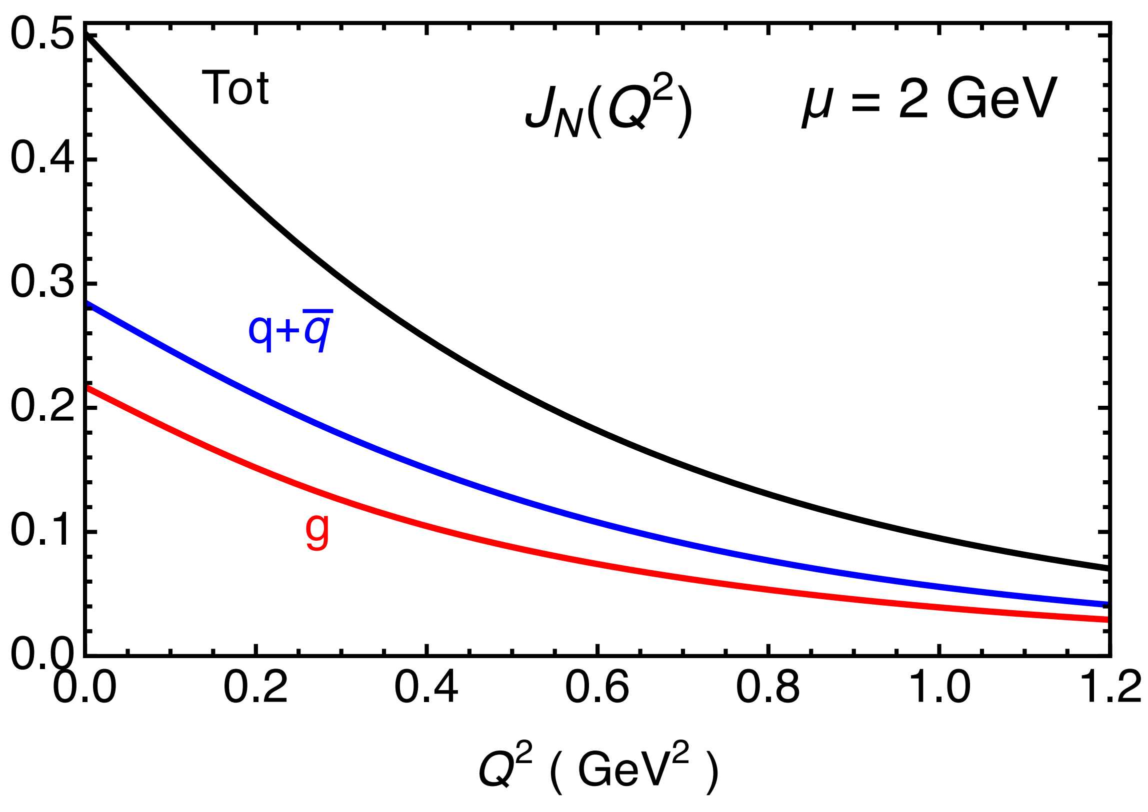
eg DSE – RL Feyn Diags

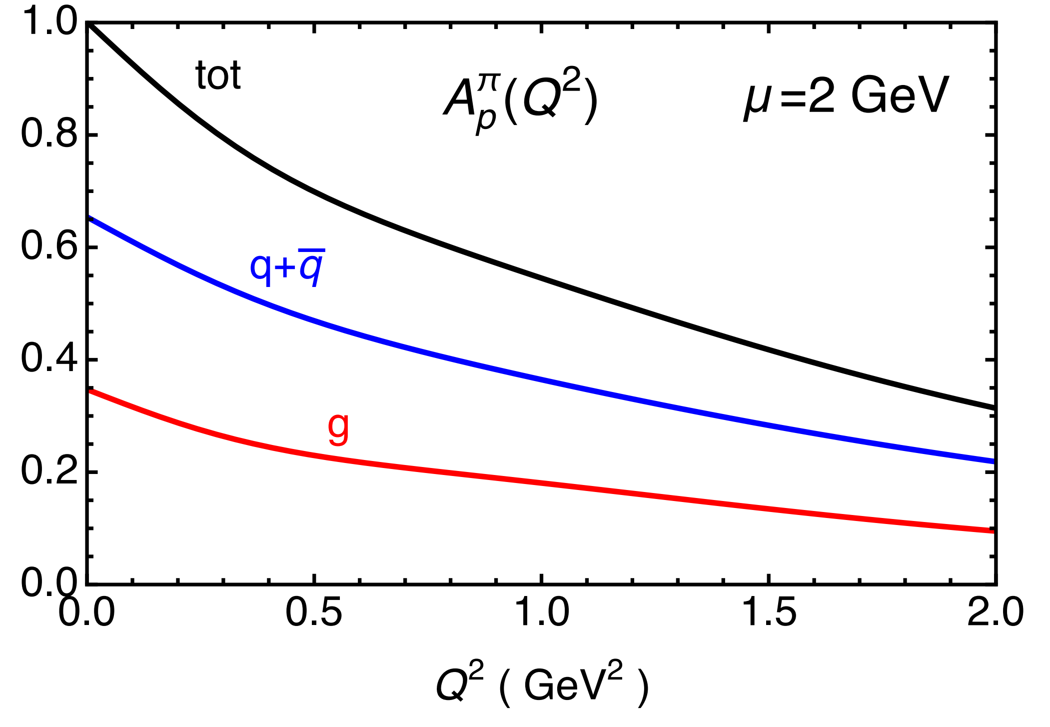
$$\frac{\langle P' | T_g^{00}(0) | P \rangle_c}{2(K^0)^2} = \frac{1}{2K^0} \int \frac{d^4k}{(2\pi)^4} (\frac{k^0}{K^0}) \int d^4z \frac{e^{ik\cdot z}}{k^0} \left\langle P' \,\middle|\, G^{0\,\alpha}(\frac{-z}{2})\, G_{\alpha\,0}(\frac{z}{2}) + \frac{1}{4} G^{\alpha\beta}(\frac{-z}{2})\, G_{\alpha\beta}(\frac{z}{2}) \middle| P \right\rangle_c$$

Present results for $A_{q/g}(Q^2)$ and $J_{q/g}(Q^2)$









Gravitational FF : $\mathcal{E}_{q/g}(Q^2)$ and energy distribution radius

$$\langle P'|P\rangle = 2\,E\,(2\pi)^3\delta^3(P'-P)$$

Eg: Polyakov & Schweitzer, arXiv:1805.06596 (2018)

$$H = \int d^3x\, T^{00}(x) \qquad \mathcal{E}(Q^2) = \frac{\langle P'|T^{00}(0)|P\rangle}{2\,E(Q^2)}$$

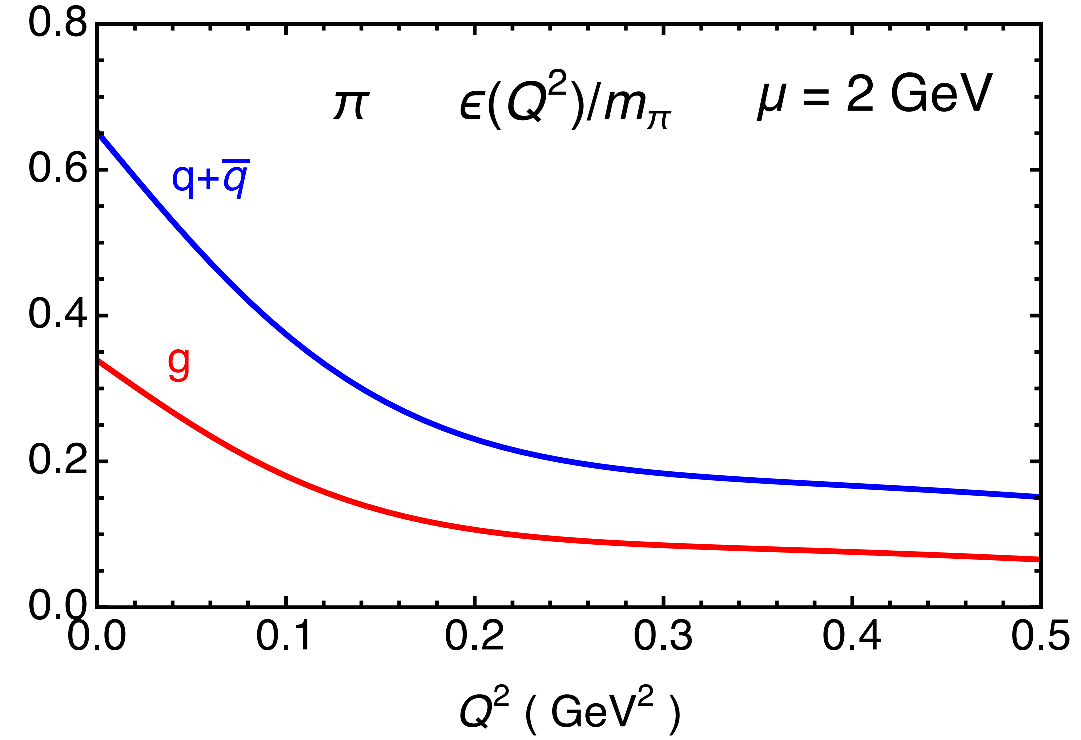
$$\frac{\mathcal{E}(Q^2)}{M} = \frac{\langle P'|T^{00}(0)|P\rangle/2\,M^2}{\sqrt{1+Q^2/4M^2}}$$

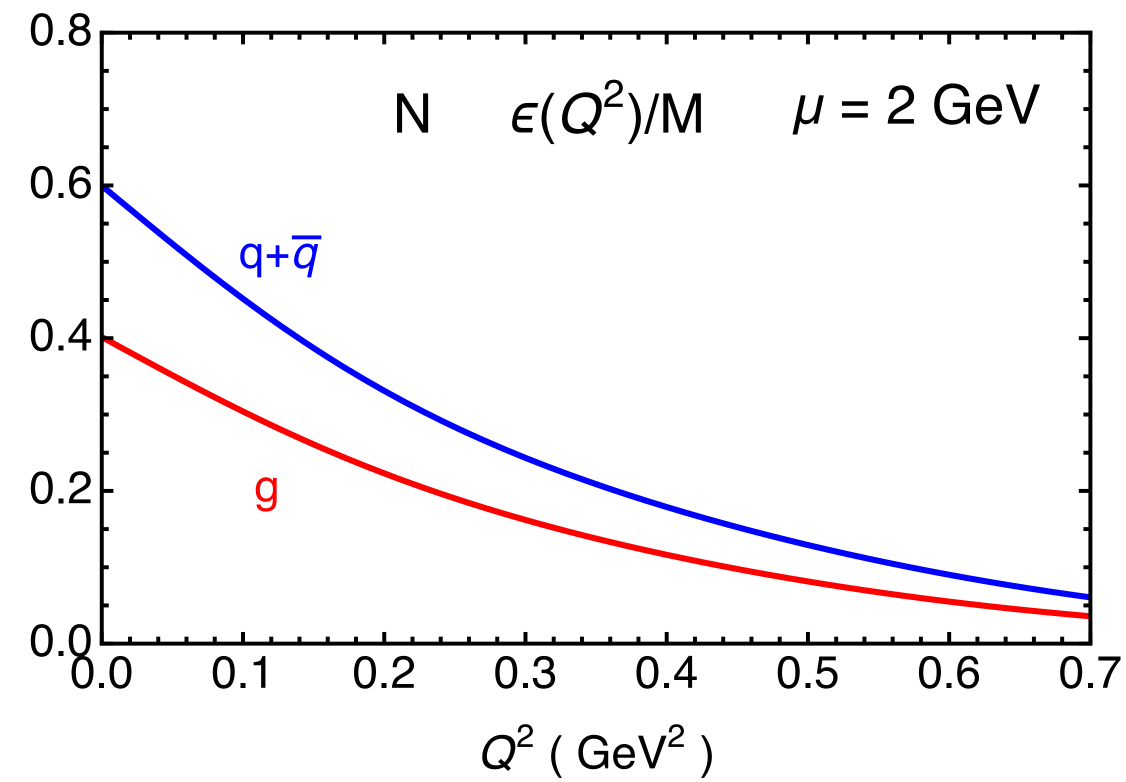
Adopt Breit frame viewpoint: $R^2 = -6\,\frac{d}{dQ^2}\frac{f(Q^2)}{f(0)}\,|_{Q\to 0}$

Energy ("mass") radius $\quad R^2_{\mathcal{E}} = \frac{\int d^3r\, r^2\,\mathcal{E}(r)}{\int d^3r\,\mathcal{E}(r)}$

For any parton : $\quad R^2_{\mathcal{E}} = R^2_T + \frac{3}{4\,M^2}$

$\Rightarrow$ **large effect for** π





Radii (fm) from q, g parts of $A(Q^2)$, $T^{00}(Q)$, $\mathcal{E}(Q^2)$ at $\mu = \mu_0$

	A			T^{00}			$\mathcal{E}$		
	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q
$\pi(\mu_0)$	0.412	0.503	1.22	0.782	1.07	1.37	1.45	1.62	1.12
$N(\mu_0)$	0.711	0.795	1.12	0.951	0.765	0.81	0.966	0.784	0.812

Expected > 1

Agrees $\mathbf{R}_{\mathcal{E}}^2 = \mathbf{R}_{\mathrm{T}}^2 + \frac{3}{4\,\mathrm{M}^2}$

Radii (fm) from $A(Q^2)$, $T^{00}(Q)$, $\mathcal{E}(Q^2)$ at $\mu = 2$ GeV

Agrees $\mathbf{R}^2_{\mathcal{E}} = \mathbf{R}^2_{\mathbf{T}} + \frac{3}{4\,\mathbf{M}^2}$

	A		T^{00}		$\mathcal{E}$	
	R_q	R_g/R_q	R_q	R_g/R_q	R_q	R_g/R_q
$\pi(\mu = 2$ GeV$)$	0.396	1.21	0.681	1.05	1.18	1.01
$N(\mu = 2$ GeV$)$	0.627	1.03	0.823	0.945	0.835	0.897

Expected > 1

cf Hackett, Oare, Pefkou, Shanahan, LQCD (2023) 1.1

cf Raya, Cui, et al (2021) 1.31 AlgA

Rq = 0.64

Here: Rg=0.749
Hackett, Pefkou, Shanahan LQCD (2024) => Rg=0.81
Meziani (2024) => Rg=0.778

Both ignored $\bar{\mathbf{C}}_{\mathbf{q/g}}(\mathbf{Q^2})$

R_q/R_{ch}^{expt} **from** $A_q(Q^2)$, $\mathcal{E}_q(Q^2)$ **at** $\mu = 2$ **GeV**

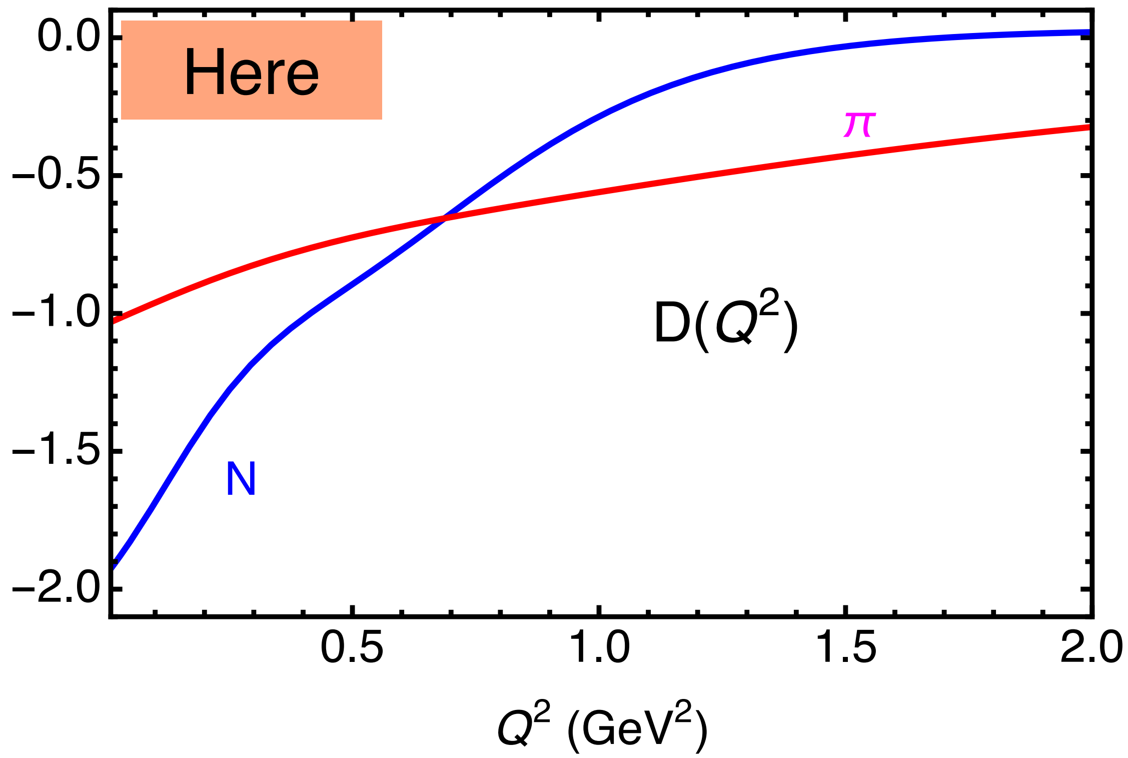
cf Hackett, Oare, Pefkou, Shanahan, LQCD (2023) 0.63

cf Raya, Cui, et al (2021) 0.49 AlgA

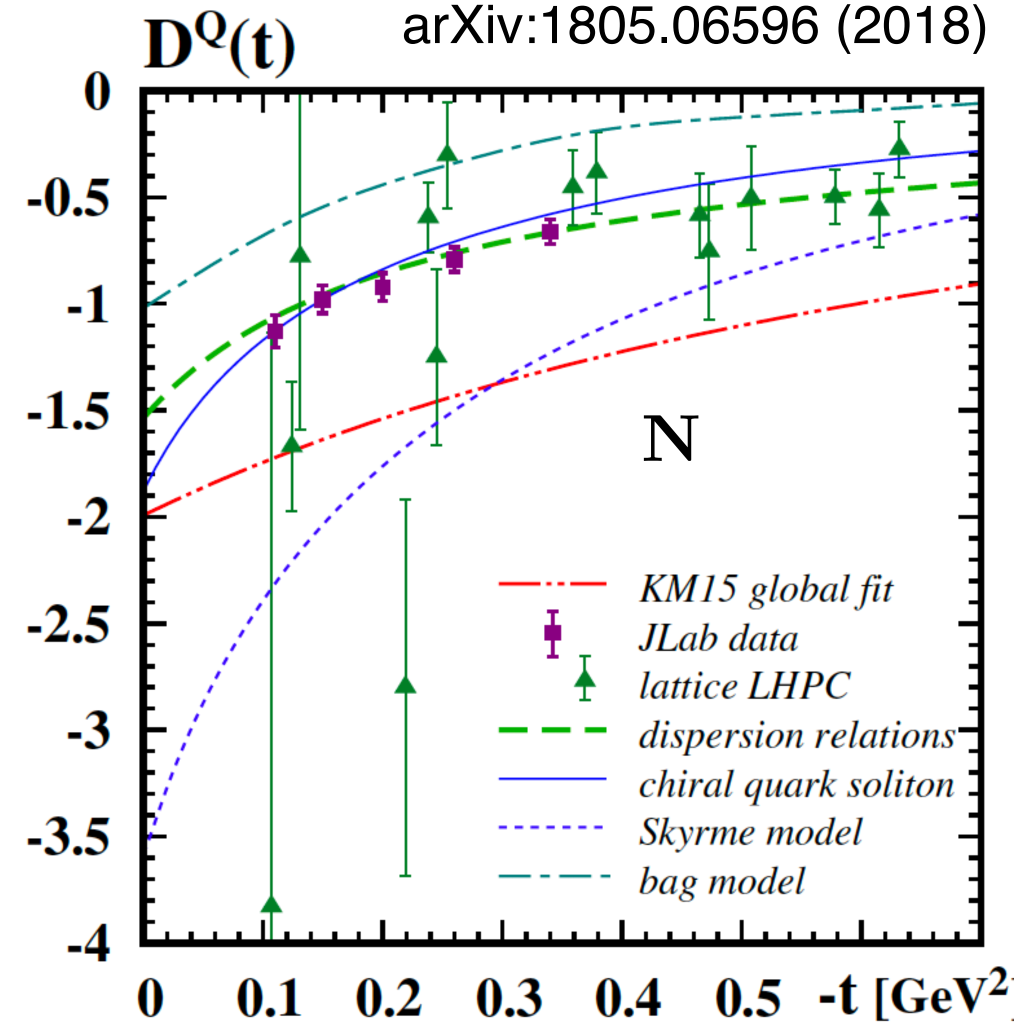
	A	$\mathcal{E}$
	R_q/R_{ch}^{expt}	R_q/R_{ch}^{expt}
$\pi(\mu = 2\,\mathrm{GeV})$	0.60	1.79
$N(\mu = 2\,\mathrm{GeV})$	0.75	0.99

Expected < 1

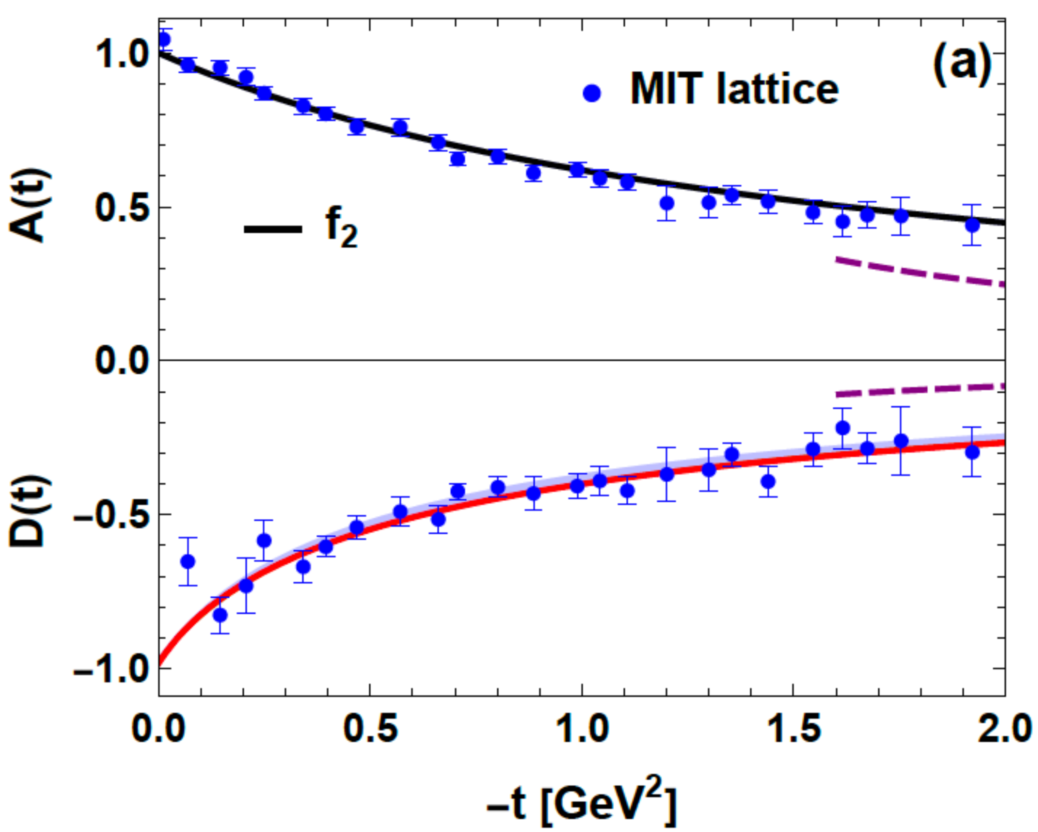
Grav FFs : $\mathbf{D(Q^2),\ \bar{C}_{q/g}(0)}$



From Polyakov & Schweitzer, arXiv:1805.06596 (2018)



π **Grav FFs** From Broniowski & Arriola, 2024



MIT LQCD pts: Hackett, Oare, Pefkou, Shanahan, 2023

Here

$\mu = 2$ GeV	N	π
$\bar{C}_{q_v}(0)$	-0.087	-0.132
$\bar{C}_{q_s}(0)$	+0.106	+0.130
$\bar{C}_{g}(0)$	-0.017	-0.008
	0	0

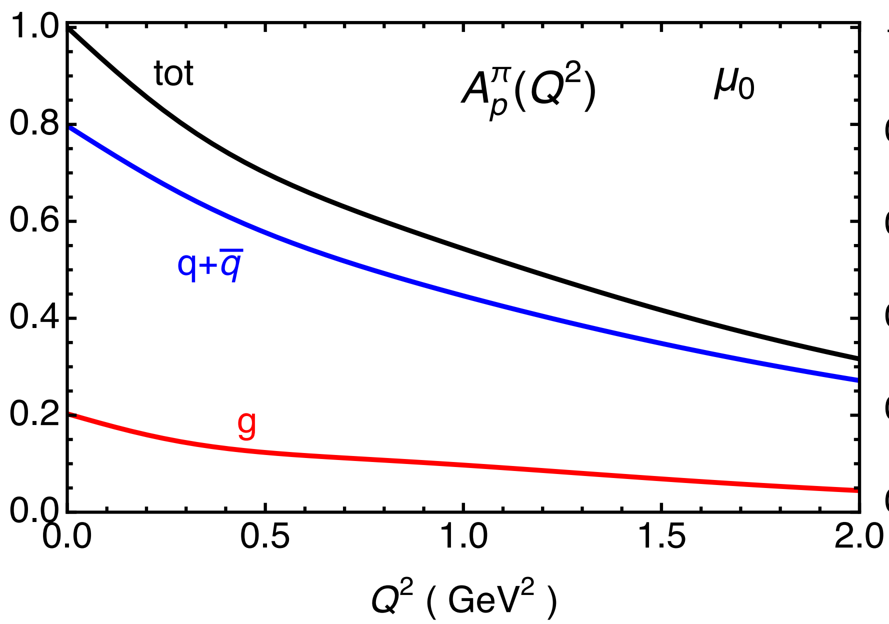
Summary

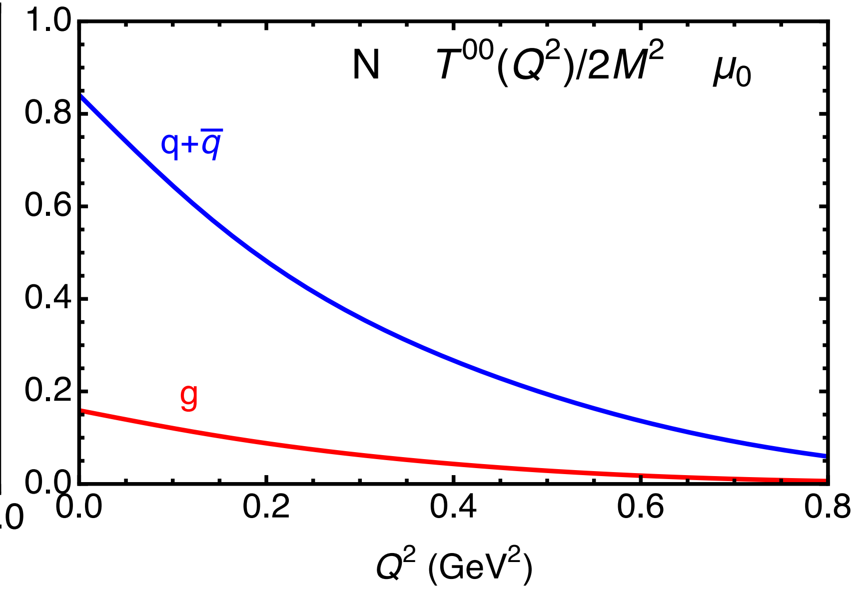
- **DSE-RL modeling of QCD for PDFs $\langle x \rangle_{q/g}$, $J_{q/g}$, extended to $T^{00}_{q/g}(Q)$ & Grav FFs for N & π**
- **DCSB quark dressing is very strong. Those gluons dominate** $T^{00}_g(Q),\ A_g(Q),\ J_g(Q),\ \mathcal{E}_g(Q)$
- **Results for $D(Q)$ are typical, and non-conserving $\bar{C}_{q/g}(0)$ have been identified**
- **$A(Q)$ is not a good estimate of the energy/mass radius** $R_{\mathcal{E}} > R_A$ **Eg:** $\mu = 2$ GeV**:** $(R^{\mathcal{E}}_q/R^A_q)_{\rm N} = 1.33$**,** $(R^{\mathcal{E}}_q/R^A_q)_\pi = 2.98$
- R_g/R_q **less than expected:** $\mu = 2$ GeV**:** $(R^{\mathcal{E}}_g/R^{\mathcal{E}}_q)_{\rm N} = 0.90$**,** $(R^A_g/R^A_q)_\pi = 1.21$
- $\mu = 2$ GeV**:** $(R^{\mathcal{E}}_q/R^{\rm expt}_{\rm ch})_\pi = 1.79$**,** $(R^{\mathcal{E}}_q/R^{\rm expt}_{\rm ch})_{\rm N} = 0.99$

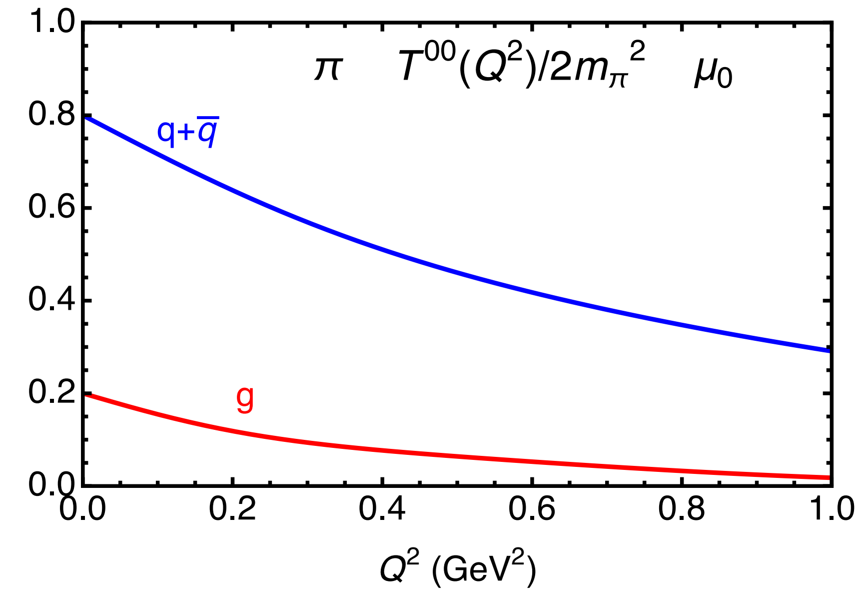
The End

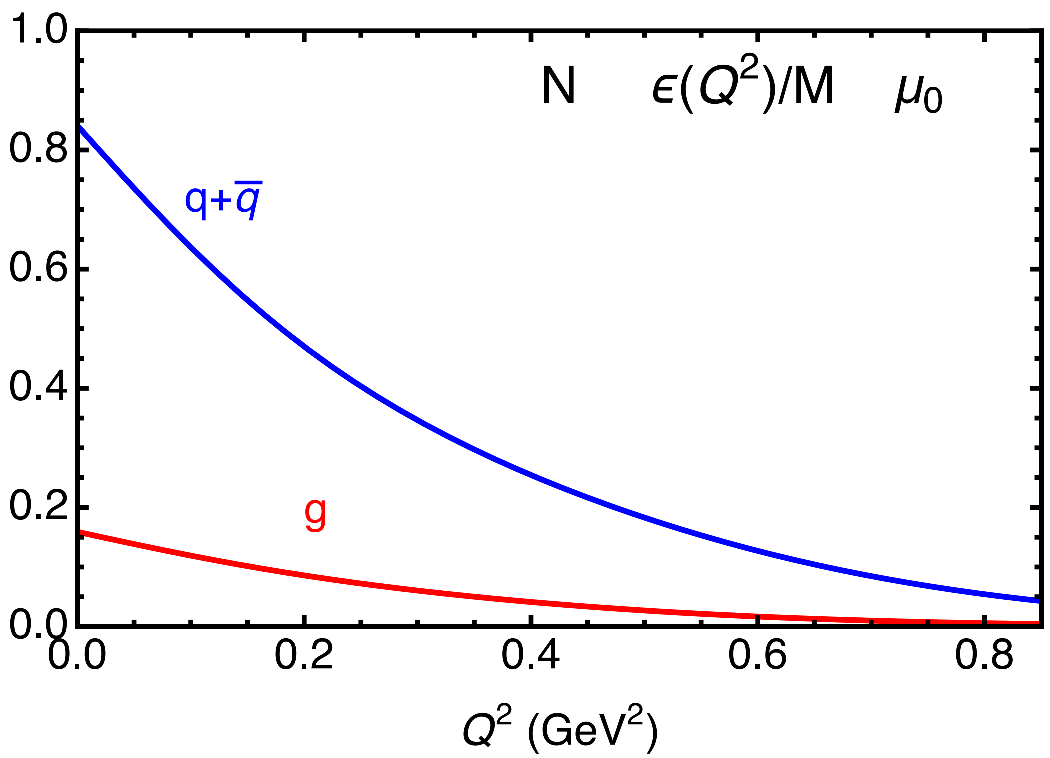
Back-Up Slides

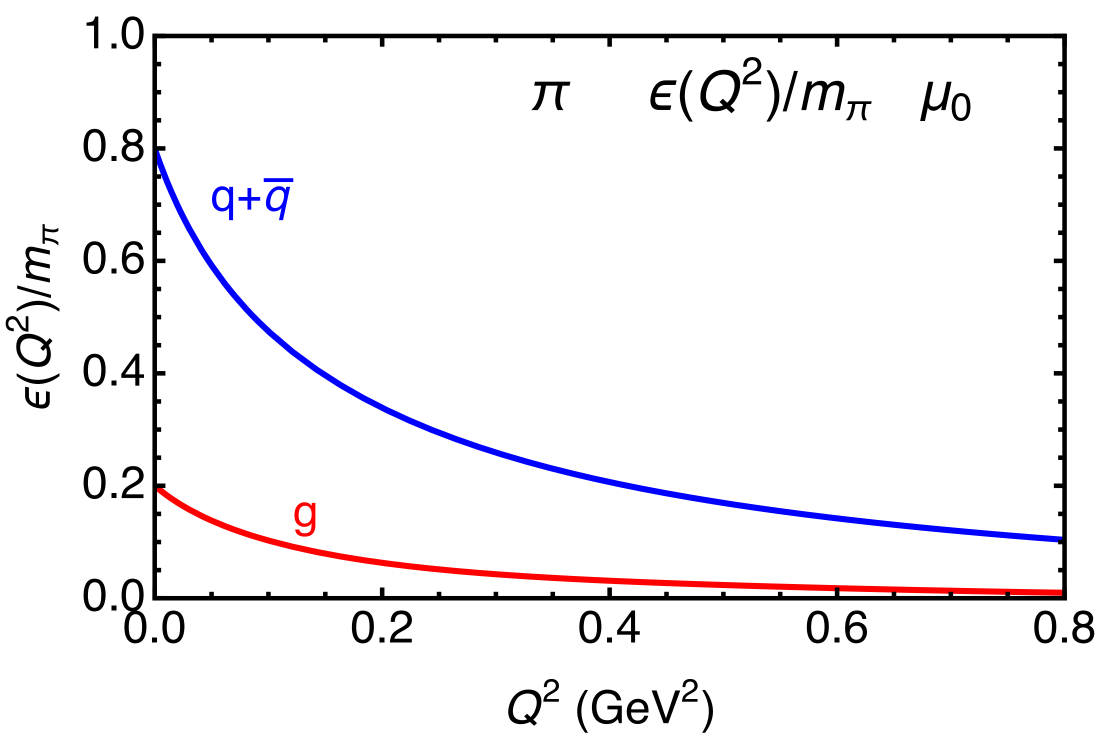
Model scale Grav FFs











Radii (fm) from $A(Q^2)$, $T^{00}(Q)$, $\mathcal{E}(Q^2)$ at $\mu = 2$ GeV

Agrees $\mathbf{R}_{\mathcal{E}}^2 = \mathbf{R}_{\mathrm{T}}^2 + \dfrac{3}{4\,\mathbf{M}^2}$

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Expected > 1

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R_q/R_{ch}^{expt} from $A_q(Q^2)$, $\mathcal{E}_q(Q^2)$ at $\mu = 2$ GeV

cf Hackett, Oare, Pefkou, Shanahan, LQCD (2023) 0.63

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	A	$\mathcal{E}$
	R_q/R_{ch}^{expt}	R_q/R_{ch}^{expt}
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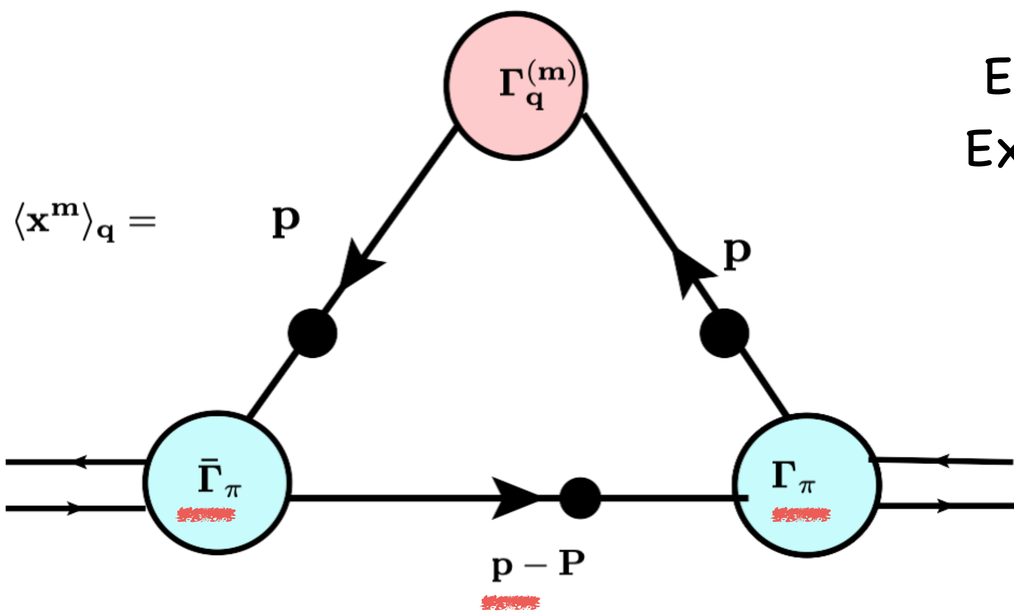
$N(\mu = 2$ GeV$)$	0.75	0.99

Expected < 1

Binding Gluon Contribution to $\langle \mathbf{x} \rangle_{\mathbf{g}}$

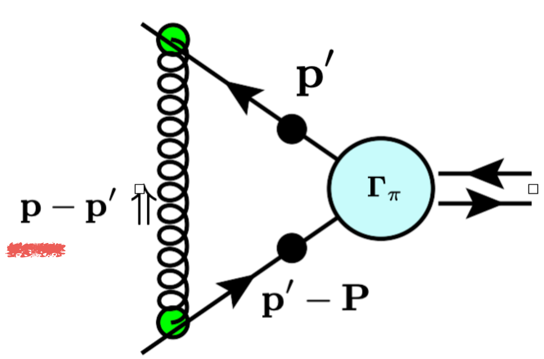
$\langle \mathbf{x^m} \rangle_{\mathbf{q}} =$



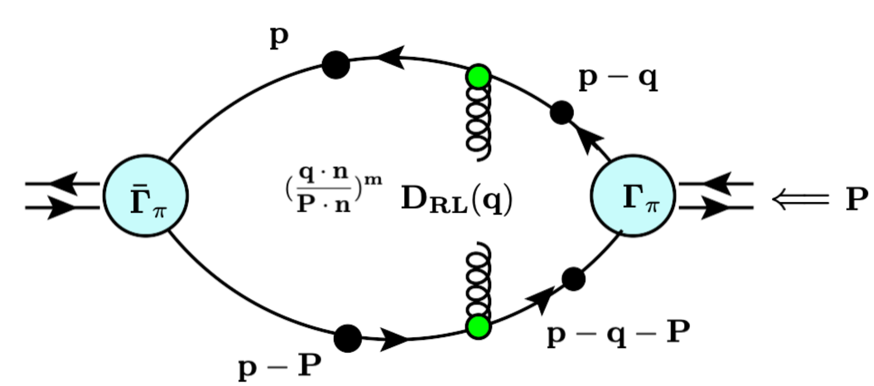
Exact @ 1-loop
Exact @ DSE-RL

$$\left[\Gamma_q^{(m=1)}(p) + \Gamma_g^{(m=1)}(p)\right] = \frac{p \cdot n}{P \cdot n}\left\{ - n \cdot \partial_p S^{-1}(p)\right\}$$

$$\left\{\langle x\rangle_q + \langle x\rangle_{g,\mathrm{Dress}\ q}\right\}_{u+\bar{d}} = 1 - \langle x\rangle_{g,\mathrm{Bind}}$$



$\langle \mathbf{x^m} \rangle_{\mathbf{g,Bind}} =$



~ 0.025

Radii (fm) from q, g parts of $A(Q^2)$, $T^{00}(Q)$, $\mathcal{E}(Q^2)$ at $\mu = \mu_0, 2$ GeV

	A			T^{00}			$\mathcal{E}$		
	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q
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$N(\mu_0)$	0.711	0.795	1.12	0.951	0.765	0.81	0.966	0.784	0.812

	A			T^{00}			$\mathcal{E}$		
	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q	R_q	R_g	R_g/R_q
$\pi(\mu = 2$ GeV$)$	0.396	0.478	1.21	0.681	0.718	1.05	1.18	1.20	1.01
$N(\mu = 2$ GeV$)$	0.627	0.649	1.03	0.823	0.778	0.945	0.835	0.748	0.897

R_q/R_{ch}^{expt} from $A_q(Q^2)$, $\mathcal{E}_q(Q^2)$ at $\mu = \mu_0, 2$ GeV

	A		$\mathcal{E}$	
	R_q	R_q/R_{ch}^{expt}	R_q	R_q/R_{ch}^{expt}
$\pi(\mu_0)$	0.412	0.63	1.45	2.20
$N(\mu_0)$	0.711	0.85	0.966	1.15
$\pi(\mu = 2$ GeV$)$	0.396	0.60	1.18	1.79
$N(\mu = 2$ GeV$)$	0.627	0.75	0.835	0.99

[cf Hackett-Shanahan, LQCD (2023) 63%]

$\mathbf{V.\ Bertone,\ et\ al\ (2022):\ Scale\ evolution\ of}$

$\mathbf{GPDs(x, \xi = 0, Q^2)\ is\ same\ as\ PDFs(x)}$