The Infrared Phase of QCD and Anderson Localization

Ivan Horváth

Institute of Nuclear Physics, Řež-Prague & University of Kentucky, Lexington, KY

People involved: [various stages/aspects]

Andrei Alexandru (George Washington), Peter Markoš (Comenius), Robert Mendris (Shawnee)

Beijing (Yi-Bo Yang et al), Keh-Fei Liu (Kentucky), Massimo D'Elia (Pisa), Claudio Bonanno (Madrid)

Literature: [w degrees of separation]

0-th	1-st	2-nd	3-rd
1906.08047	1502.07732	1405.2968	1807.03995
	2103.05607	1412.1777	1809.07249
	2110.04833	hep-lat/0607031	2110.11266
	2305.09459	hep-lat/0610121	2205.11520
	2404.12298	hep-lat/0703010	2207.13569
	2310.03621	0803.2744	2212.09806

Technical credits: Dimitrios Petrellis

OUTLINE:

A. IR Phase of SU(3) Gauge Theories

B. Anderson Localization

C. Effective Dimension(s) [Hausdorff/Minkowski dimension for spaces w probabilities]

D. The Role of Anderson-like Localization in IR Phase

Four talks really. Blue parts will be flyovers. Have to proceed deductively.

A. SU(3) Gauge Theories w Fundamental Quarks

$$S = -\frac{1}{2g^2}\,\mathrm{tr}\, F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f (D + m_f)\psi_f$$ [Euclidean formulation]

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad A_\mu \in su(3)$$ [gluons]

$$D\chi \equiv \gamma_\mu(\partial_\mu + A_\mu)\chi \qquad \chi(x) \in C^{12}$$ [fundamental quarks]

Consider them at arbitrary temperature T: 1/T = Euclidean "time" extent

Language:
- $N_f=0$ pure glue QCD of quenched QCD ⇔ infinite quark masses
- $N_f=2$ two flavors, $m_1=m_2$
- $N_f=2+1$ three flavors, $m_1=m_2 \neq m_3$
- …

(1) Real-world QCD : $N_f=1+1+1+1+1+1$ m_f = physical quark-mass parameters

$N_f=2+1$ at physical quark masses its precise representation

(2) Varied behaviors including conformality

(3) For $N_f < 16.5$ we know how to take the continuum limit

A. IR Phase of SU(3) Gauge Theories…

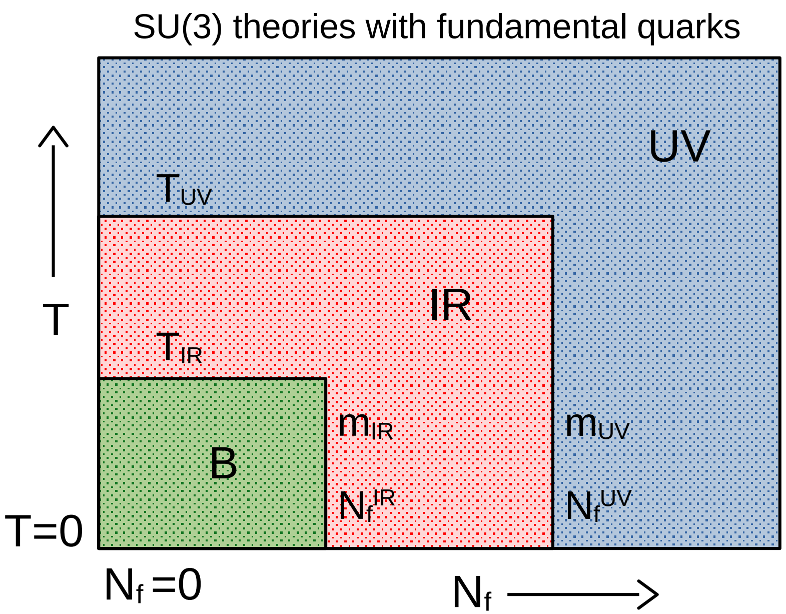
AA & IH 1906.08047

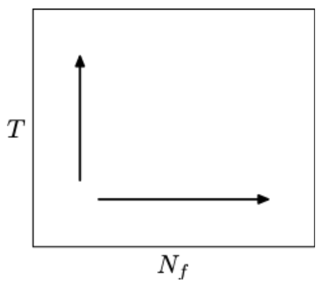
$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \;\; , \;\; \lambda \to 0$$

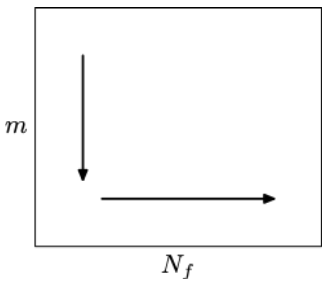
B = IR-broken

IR = IR-symmetric

UV = IR-trivial







Changes consistent with directions of arrows can induce transitions from B➞IR or from IR➞UV.

See also 1502.07732

Most known detail comes from B➞IR thermal case: IR PHASE OF THERMAL QCD

Important also B➞IR light-flavor case (T=0): IR PHASE = STRONGLY COUPLED PART OF CONFORMAL WINDOW

1405.2968, 1412.1777, 1906.08047

A. IR Phase of SU(3) Gauge Theories…

I. IR PHASE OF THERMAL QCD

1906.08047, 2404.12298, 2305.09459

above/different from χ-crossover T_c

$T_c < T_{IR} < T < T_{UV}$

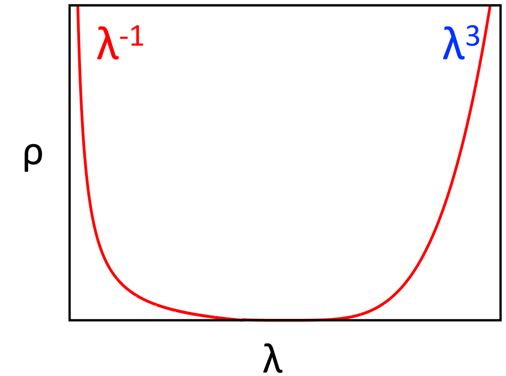
≈155 MeV   200-230 MeV   perturb

II. WHY IR?   Power-law accumulation of DOFs in the IR   AA&IH 1906.08047

Thermal QCD in IR phase:

-highly unusual scales $\Lambda < 1$ MeV

-partial deconfinement 1502.07732



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$?

$a \rightarrow 0$

III. WHY PHASE?

At T_{IR}:

1906.08047

(i) IR BECOMES AN AUTONOMOUS SUBSYSTEM

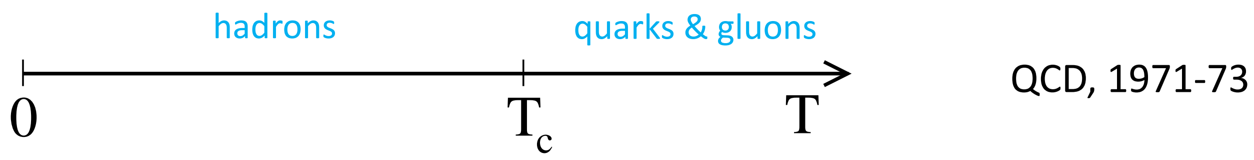
[IR-BULK decoupling, from1-component to 2-component system]

(ii) GLUE OF IR COMPONENT BECOMES SCALE INVARIANT

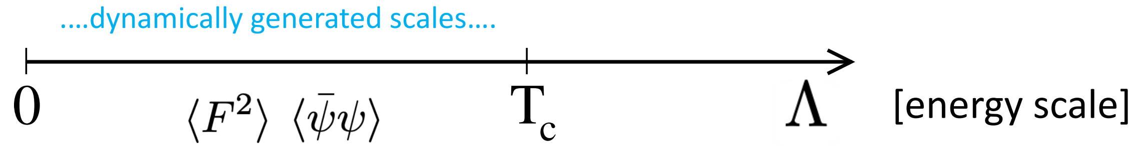
(iii) NON-ANALYTICITIES APPEAR

(iv) INFINITE GLUE SCREENING LENGTHS APPEAR

A. IR Phase: Stories of Temperature & QCD



Effects of Temperature:



Scales Story: thermal agitation erodes condensates and melts them upon T reaching T_c

DOFs Story: thermal agitation reduces IR dof-s and depletes them when T reaches T_c

[DOFs = quark & glue]

A. IR Phase: Stories of Temperature & QCD…

Scales Story: thermal agitation erodes condensates and melts them upon T reaching T_c

DOFs Story: thermal agitation reduces IR DOF-s and depletes them when T reaches T_c

BUT IS THIS TRUE? LATTICE PERFECT FOR UNAMBIGUOUS ANSWER [if lucky w scales].

Need a construct that expresses the distribution of DOFs over scales:

$$D = D[A] \qquad D\psi_\lambda = i\lambda\,\psi_\lambda \qquad \rho(\lambda, V_4) \equiv \frac{\#\,\mathrm{modes\ near}\ \lambda}{V_4\,\mathrm{d}\lambda}$$

Dirac spectral density

1) DISTRIBUTION OF QUARK DOFs ACROSS ENERGY-LIKE SCALES

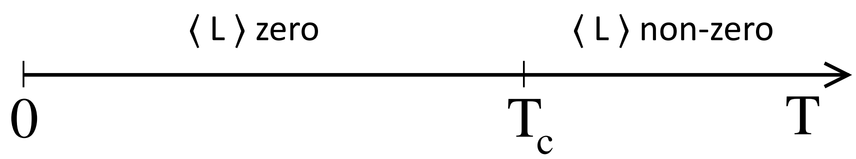
2) GAUGE-INVARIANT SCALE-DEPENDENT GLUE OPERATOR
[Quantifies contributions to F^2 from different energy-like scales.]

This should tell us truth about the stories & it does!

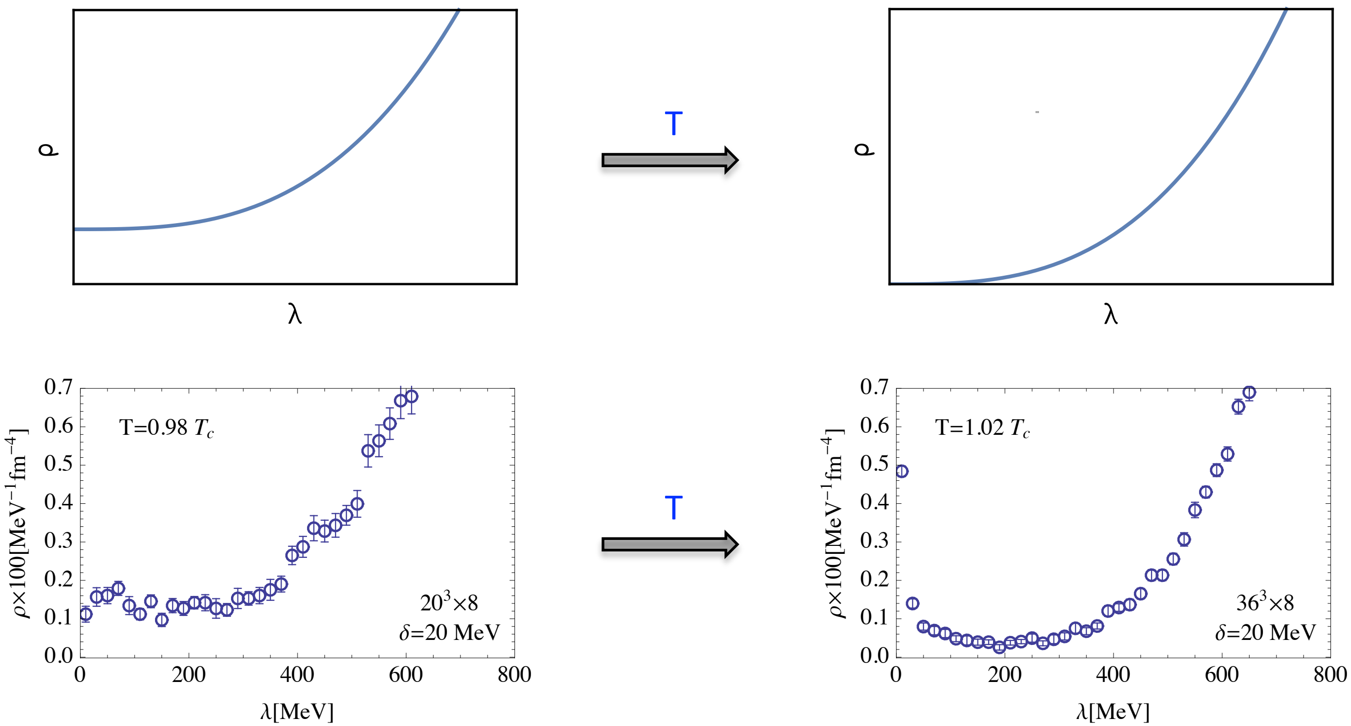
Due to 2) we say since 1502.07732: ``Give us your glue and we will tell you who you are.”

A. IR Phase: Stories of Temperature & QCD…

$N_f=0$ QCD



⟨ L ⟩ = Polyakov loop
1-st order transition

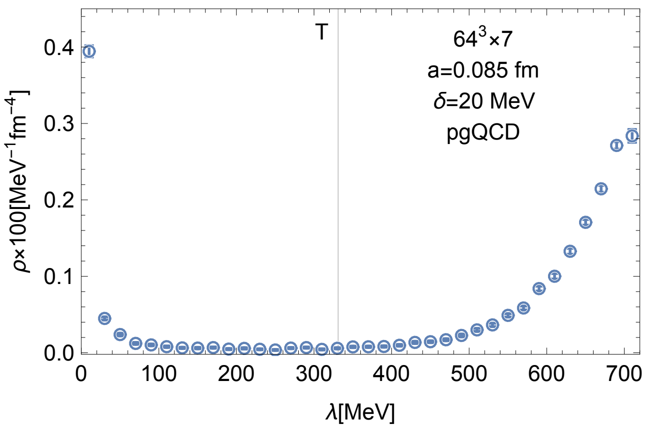


AA & IH 1502.07732

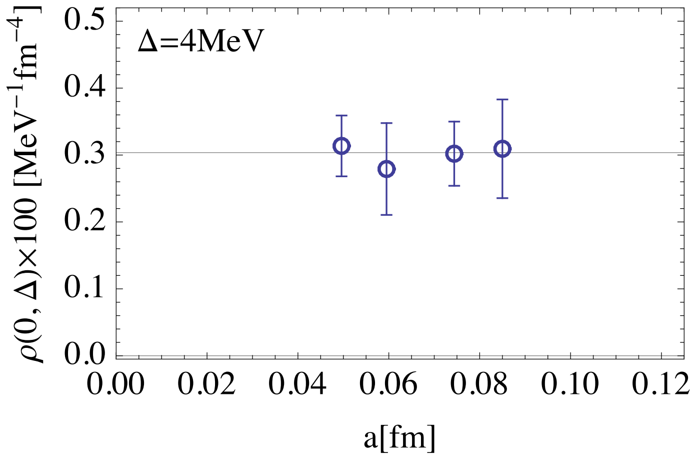
Well, perhaps some sort of an artifact? Edwards et al, hep-lat/9910041

A. IR Phase: Stories of Temperature & QCD…

T=1.12 T_c



L=2 fm, fixed

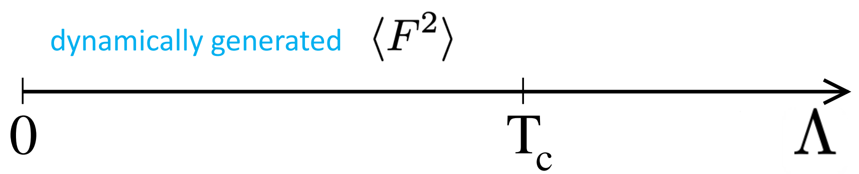


NO ARTIFACT in N_f=0 QCD. Strength of IR peak scales!

AA & IH 1502.07732

Stories: N_f=0 adaptation

Thermal agitation erodes <F^2> and melts it upon T reaching T_c



Thermal agitation reduces IR DOF-s and depletes them upon T reaching T_c. NOT TRUE

Removing scales was supposed to restore IR scale invariance trivially by removing IR DOF-s!

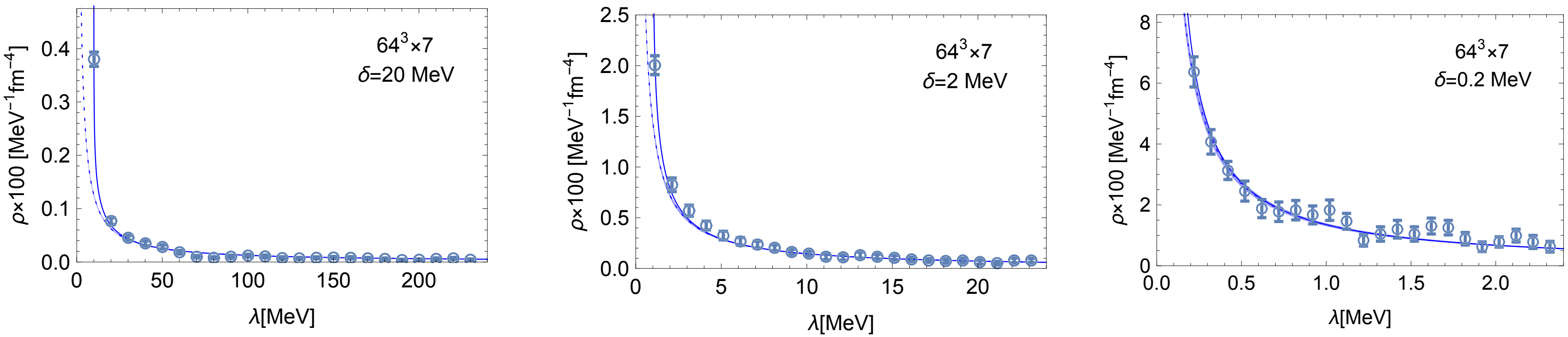
The reality is that IR DOF-s actually proliferate ☺. Thank you lattice!

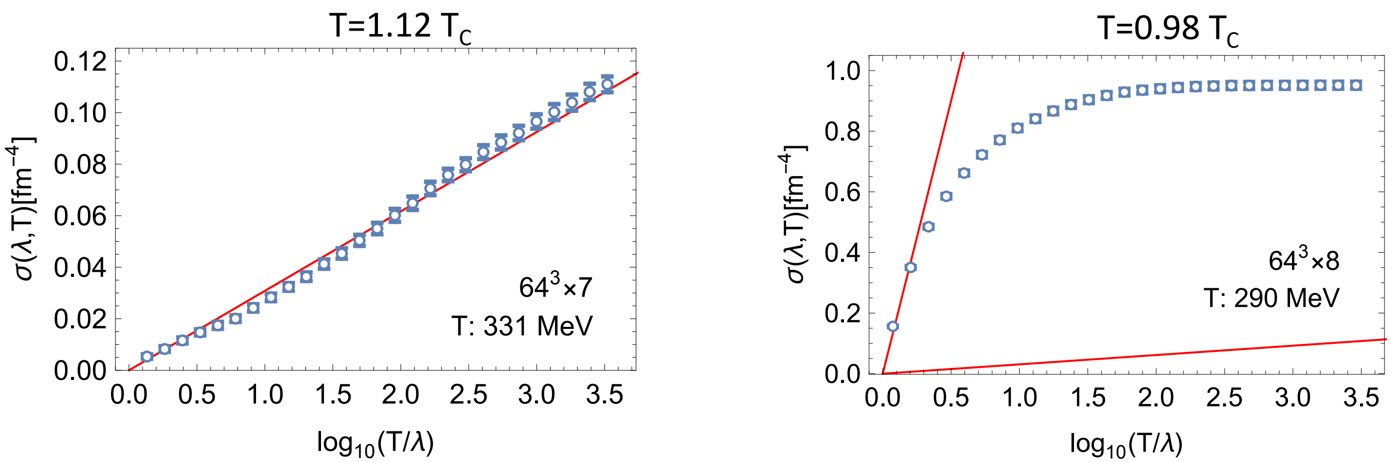
Could it be that IR scale invariance is restored non-trivially???

AA & IH 1906.08047

A. IR Phase: Stories of Temperature & QCD…

Fits to $\rho(\lambda) \propto 1/\lambda$ [N_f=0, T=1.12 T_c]   AA & IH 1906.08047





Data: (1) IR SCALE-INVARIANT DENSITY (λ < T) OVER 3 ORDERS OF MAGNITUDE IN SCALE

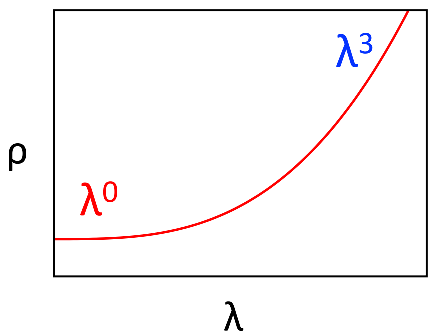
(2) NEGATIVE POWER-LAW ACCUMULATION OF DIRAC MODES IN IR: $\rho(\lambda) \propto \lambda^p$ $p \gtrsim -1$

Proposal: THIS REFLECTS IR SCALE-INVARIANT GLUE: <u>IR PHASE</u>   [p<0]   1906.08047

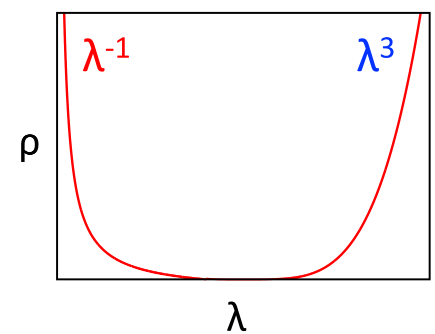
A. IR Phase: Stories of Temperature & QCD…WHAT JUST HAPPENED HERE?

T=0 classically scale invariant theory — quantum fluctuations ⟹ scale anomaly — scales generated world of hadrons

scale-broken T=0 theory — thermal fluctuations ⟹ increasing T — scale-invariant but only for $\Lambda < \Lambda_{IR} < T$



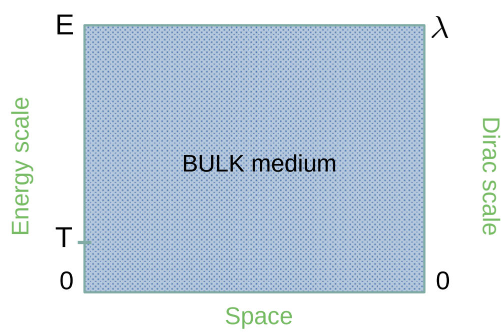
thermal agitation ⟹



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$?

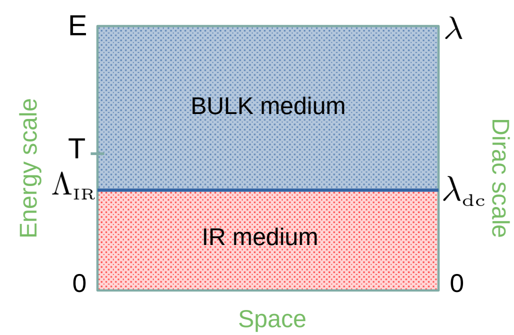
$a \rightarrow 0$



thermal agitation ⟹

IR-BULK SEPARATION

AA & IH 1906.08047



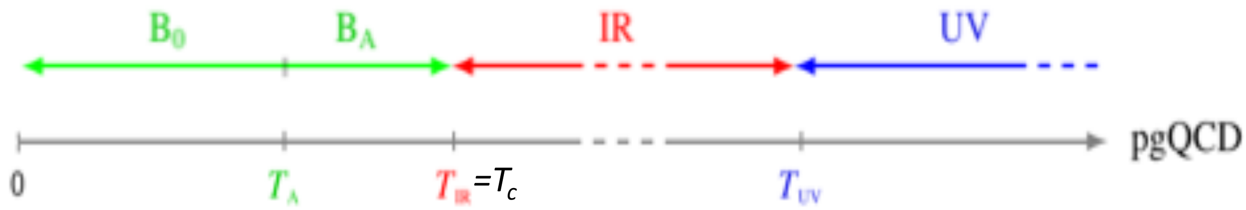
NON-INVARIANT

INVARIANT

AT $T=T_{IR}$ THERMAL QCD BECOMES 2-COMPONENT SYSTEM: IR MEDIUM AN AUTONOMOUS SUBSYSTEM

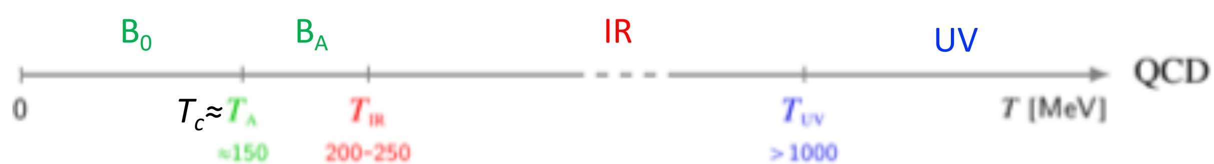
A. Phase Diagrams of Thermal QCD via IR Scale Invariance of Glue AA & IH 1906.08047

■ Thermal phase diagram for N_f=0: Polyakov line transition coincides w IR-phase transition



IR BROKEN p = 0 IR SYMMETRIC p ≈ -1 IR TRIVIAL p ≫ 0

■ Thermal phase diagram of N_f=2+1 ``real-world”: chiral crossover below IR-phase transition



STRONGLY COUPLED p = 0
HADRON GAS

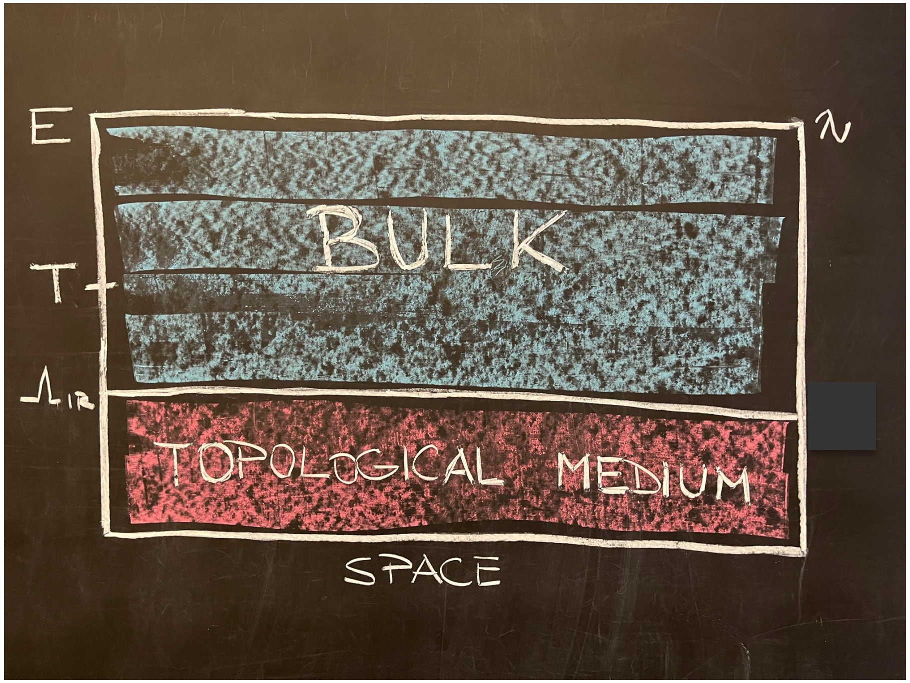
STRONGLY COUPLED p ≈ -1
NEAR-PERFECT FLUID?

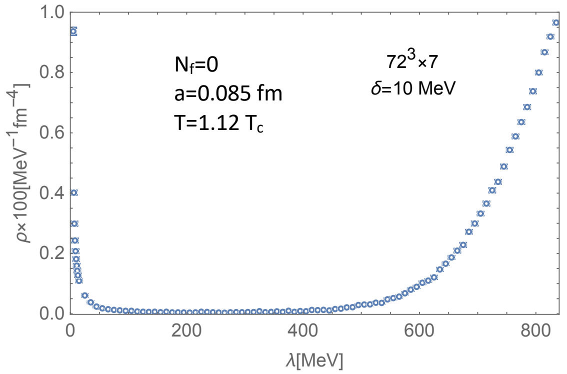
PERTURBATIVE p ≫ 0
WEAKLY-COUPLED QGP

A. IR Phase: Connection to Near-Perfect Fluid? AA & IH 1906.08047

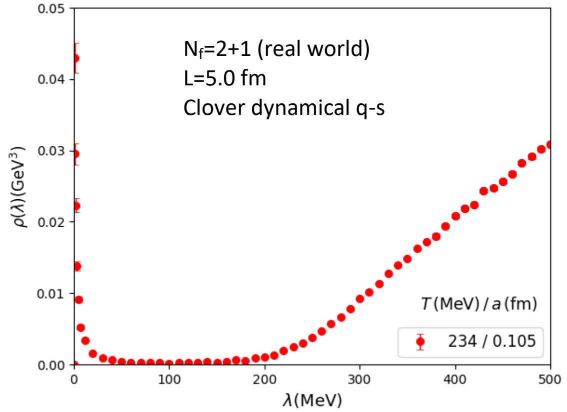
Hypothesis: IR phase describes the near-perfect fluid [RHIC, ALICE] state of matter

Experimental signatures on ALICE3? AA & IH 1906.08047





AA & IH unpublished

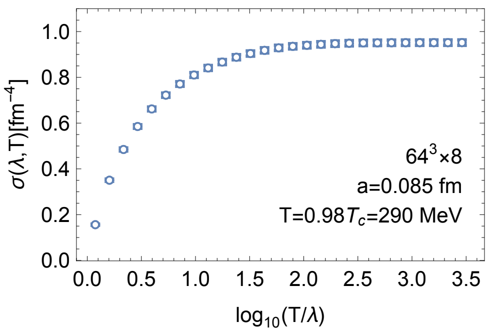


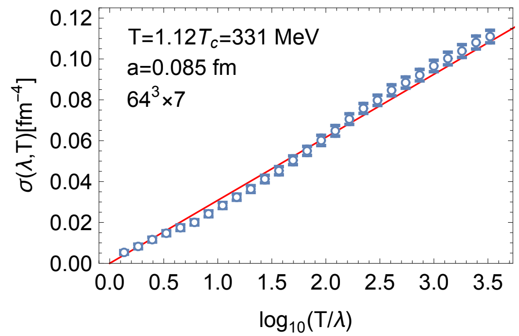
X. Meng et al 2305.09459

- System in IR Phase a 2-component medium: invariant IR and non-invariant bulk
- Tisza/Landau 2-component theory of liquid helium? Maybe in some dual form.

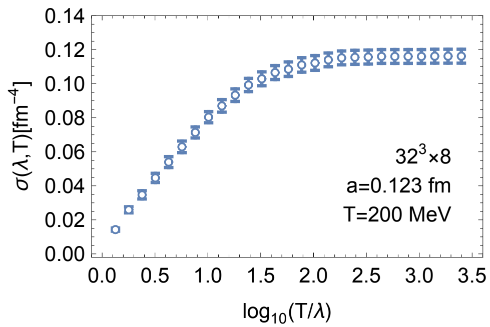
A. IR Phase: Real-World QCD

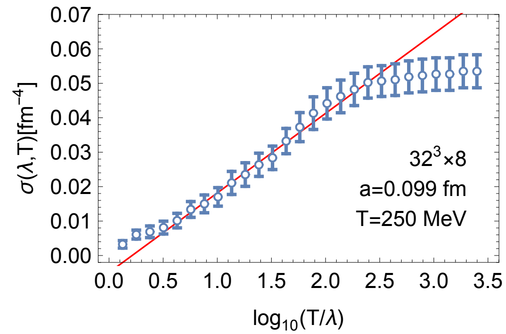
$N_f=0$





$N_f=2+1$





Real-world QCD is $N_f=2+1$ at physical quark masses of stouted staggered quarks (Wuppertal-Budapest) here.

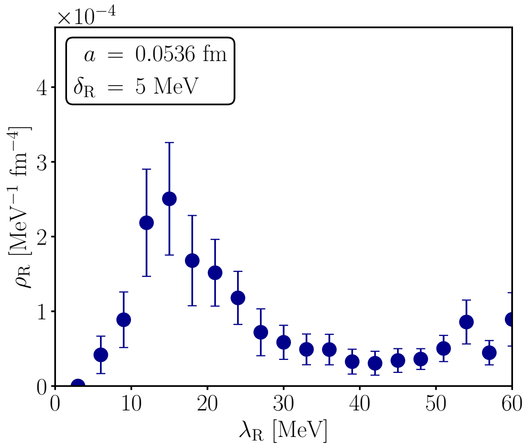
Conjecture: REAL-WORLD QCD HAS IR PHASE WITH p≈-1

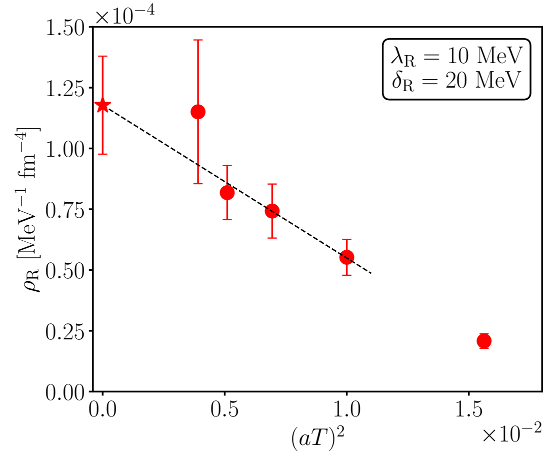
$$200\,\mathrm{MeV} < T_{\mathrm{IR}} < 250\,\mathrm{MeV}$$

AA & IH 1906.08047

A. IR Phase: Real-World QCD…

AA, Bonanno, D`Elia, IH 2404.12298





Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks here.

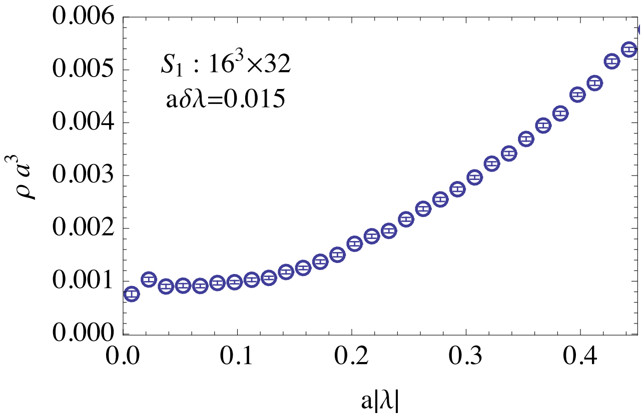
Lattice Dirac operator = stouted staggered [not overlap]

- IR structure exists in Dirac operator describing dynamical quarks
- not a lattice artifact
- IR medium is a quark-glue medium
- green light to study IR phase using overlap: correct & efficient

A. IR Phase: Theories with Many Flavors?

N_f=12, T=0 Configss: A. Hasenfratz et al, 1207.7162 staggered with nHYP AA & IH 1405.2968 1411.1777

am=0.02



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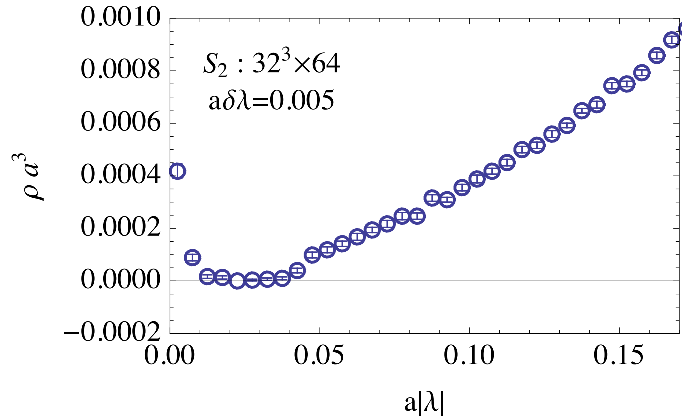
am=0.0025

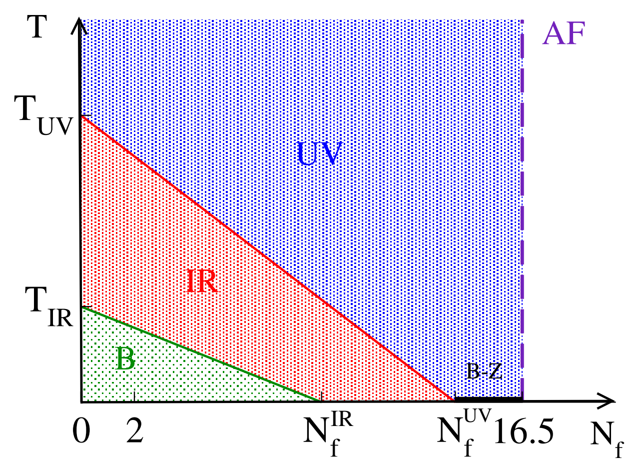


Lowering quark mass at sufficient number of flavors can generate IR phase

Conjecture: AA & IH 1906.08047

Conformal window has a strongly coupled part with p < 0.

$$N_f^c \equiv N_f^{\mathrm{IR}} < N_f < N_f^{\mathrm{UV}} \leq 16.5$$



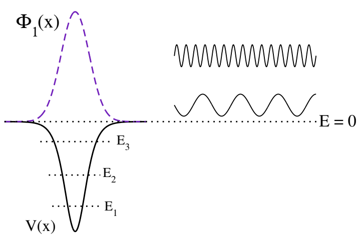
B. Anderson Localization & Transitions

courtesy of P.Markos

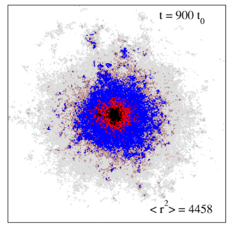
Quantum mechanics: eigenstates of quantum particle could be

bounded extended . . . and localized

[P. W. Anderson 1958]



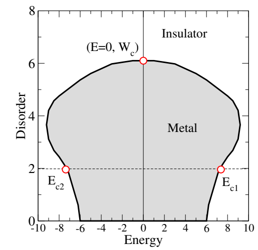
$E < 0$ $E > 0$

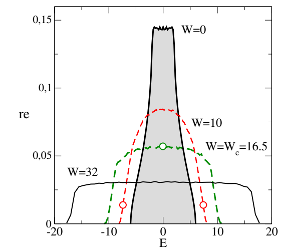


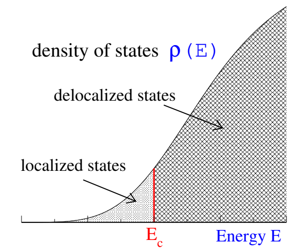
Localization is a consequence of

- disorder
- wave character of particle
- $\Phi(\vec{r}) \propto \exp\left[-r/\lambda\right]$

3D Anderson model: phase diagram, density of state, mobility edge





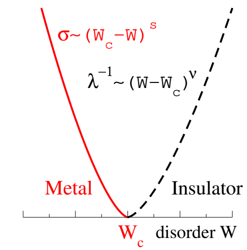


Critical exponents:

conductivity: $\sigma(E) \sim (E_c - E)^s$

localization length: $\lambda(E) \sim (E - E_c)^{-\nu}$

$s = (d-2)\nu$ . . . critical exponents.



λ is a localization length here $\lambda \longrightarrow \ell$

(1) $\ell \propto \xi$ density-density correlation length within the mode

(2) $\langle \psi^2_{loc}(x) \psi^2_{ext}(y) \rangle_c \longrightarrow 0 \quad \text{for} \;\; L \to \infty$

Anderson-like transition in thermal QCD:
Disorder W $\longrightarrow$ Temperature T
Mobility edge $\lambda_A \neq 0$ invoked for understanding chiral phase transition: aka metal-to-insulator picture

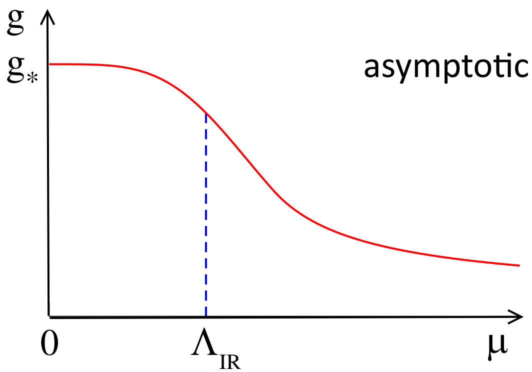
Garcia-Garcia & Osborn hep-lat/0611019

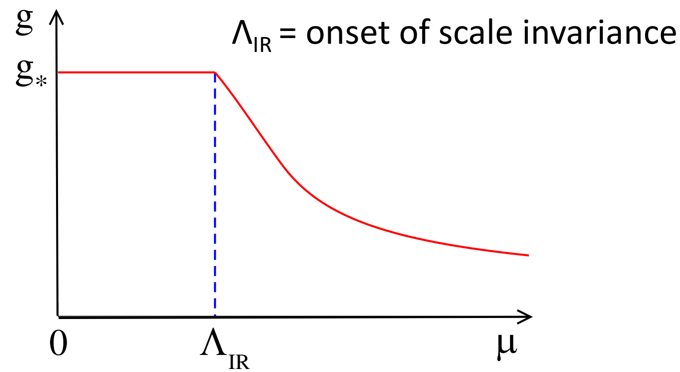
Kovacs & Pitler 1006.1205

Giordano, Kovacs & Pitler 1312.1179

Shortly after AA & IH 1906.0804 we realized that λ_A is a very friendly feature to our claim and accepted it ☺.

B. Why is Anderson-Like λ_A Handy





$g(\mu)$ non-analytic at Λ_{IR}

Q: Do non-analyticities exist and, if so, how do they arise?

Their existence in λ-dependences would also facilitate IR-BULK decoupling!

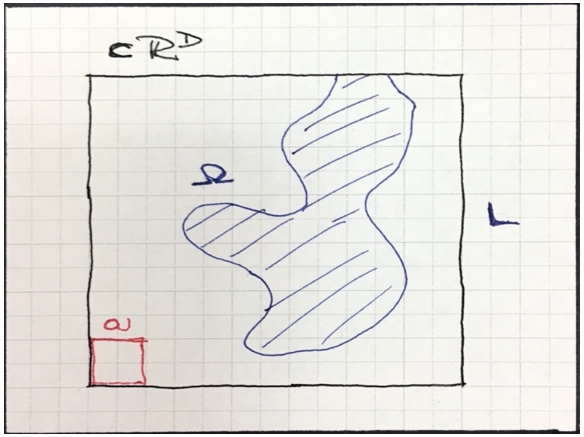
Hint: Given the existence of λ_A and its nature, focus on <u>spatial IR dimensions</u> of modes.

WHAT IS IR DIMENSION OF MODES? Concept didn't exist.

IH & RM 1807.03995 effective-number theory

IH, PM and RM 2205.11520 effective-dimension theory

C. Effective Dimensions



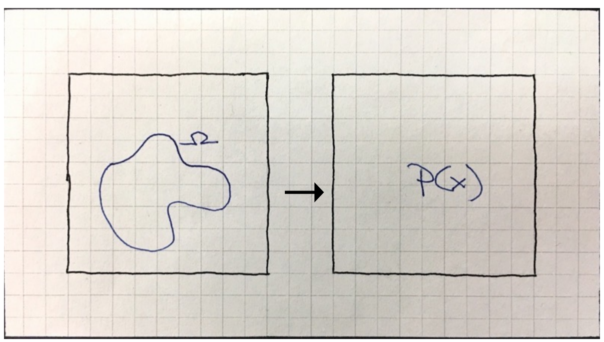
characterize fine [UV] and global [IR] features of fixed sets

all points/elements of regularized space: $N \propto (L/a)^D$

points/elements covering Ω: N_+

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\text{UV}}(L)} \;, \quad a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\text{IR}}(a)} \;, \quad L \to \infty$$



$$P(x) \quad \Longrightarrow \quad \Omega_{\text{eff}}$$

But how to proceed when instead of fixed Ω we have P(x)?

1) Count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P*.

2) Select Ω_{eff} as $\mathcal{N}$ most probable points on the lattice

3) Proceed as Minkowski/box-counting with N_+

Consistent realization of this program leads to unique effective dimensions IH, PM and RM 2205.11520

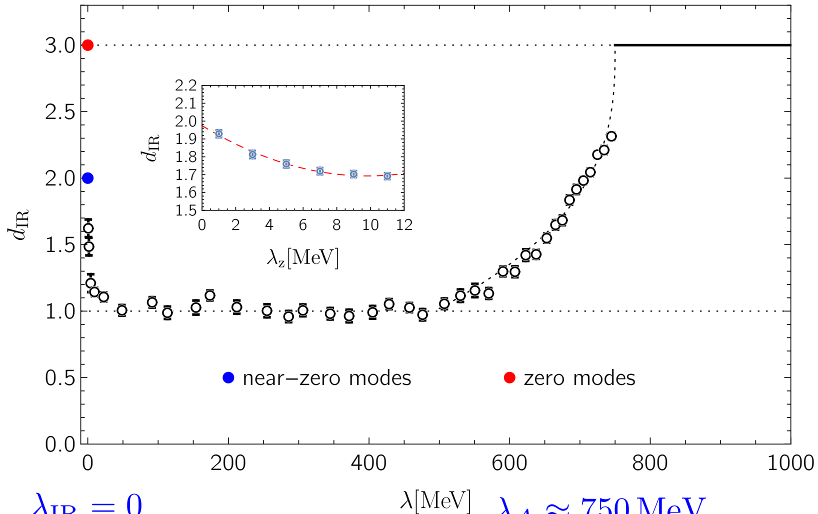
$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$ IH & RM 1807.03995

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

D. Anderson Localization in IR Phase

AA & IH 2103.05607

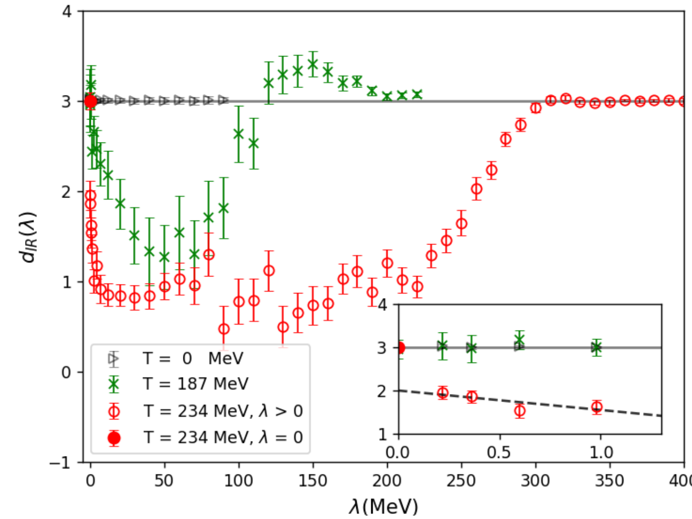
N_f=0, a=0.085 fm, T=1.12T_{IR}

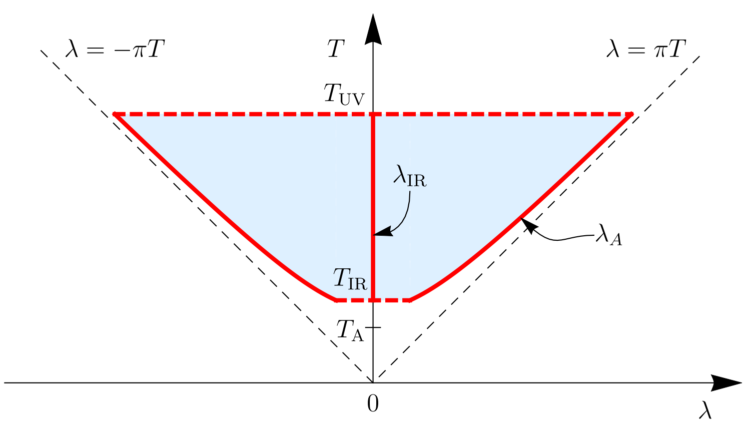


$\lambda_{\rm IR} = 0$ $\lambda_A \approx 750\,\mathrm{MeV}$

X. Meng et al 2305.09459

N_f=2+1 physical point, a=0.105 fm





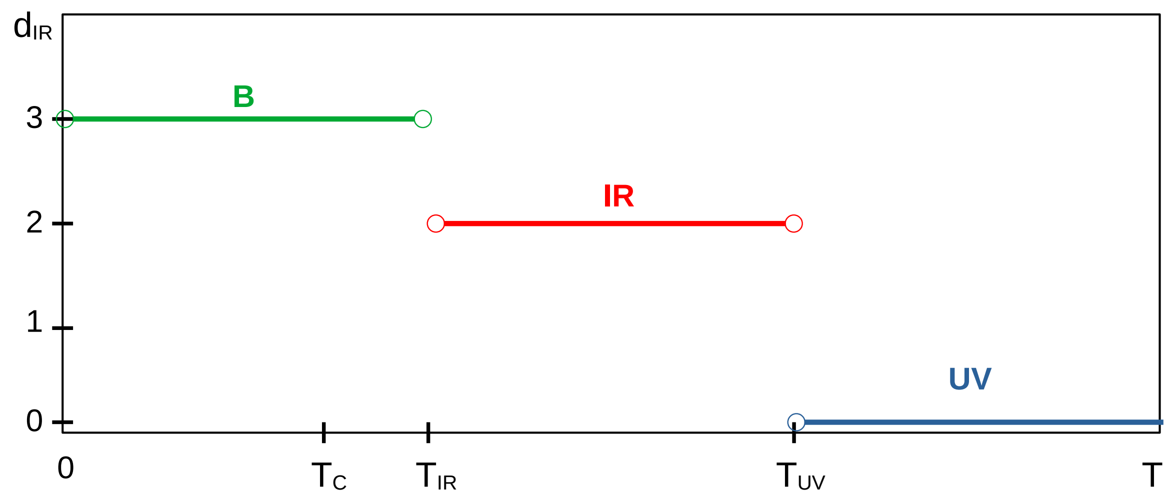
AA & IH 2110.04833

Dirac spectral phase diagram of QCD

- IR phase associated with Dirac non-analyticity
- Explains non-analytic running
- Entails IR-Bulk decoupling
- Second mobility edge λ_{IR}=0 [gives long-range physics]
- Recall that λ_A facilitates decoupling
  [see also model in Kovács 2311.04208 for support of decoupling]
- T=187 MeV is different from T=234 MeV as predicted

D. Anderson Localization in IR Phase…

Clean representation of non-analyticity in IR dimension: represents d_{IR} of near-zero modes and d_{IR} of IR glue field strength



In which aspects of IR phase are Anderson-like localization features relevant?

(i) IR BECOMES AN AUTONOMOUS SUBSYSTEM YES
[IR-BULK decoupling, from1-component to 2-component system]

(ii) GLUE OF IR COMPONENT BECOMES SCALE INVARIANT YES

(iii) NON-ANALYTICITIES APPEAR YES

(iv) INFINITE GLUE SCREENING LENGTHS APPEAR YES

Studies of Dirac Spectra in Other Contexts

$U_A(1)$ problem and other

Dick et al 1502.06190

Kaczmarek et al 2102.06136

Aoki et al 2011.0149

Ding et al 2010.14836

Kehr et al 2304.13617

Glozman et al 2204.05083

Bonanno & Giordano 2312.02857

Kaczmarek et al 2301.11610

Kovacs & Vig 1706.03562

Rohrhofer et al 1902.03191

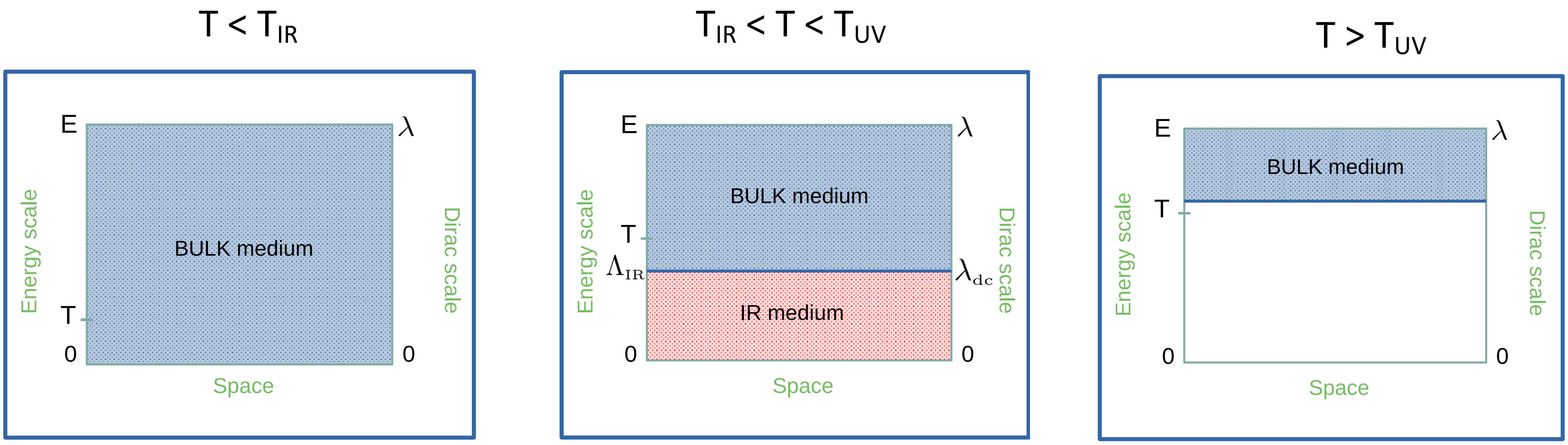
Cardinali et al 2107.02745

Giordano 2404.03546

Pandey et al 2407.09253

and other…

SLIDES FOR QUESTIONS



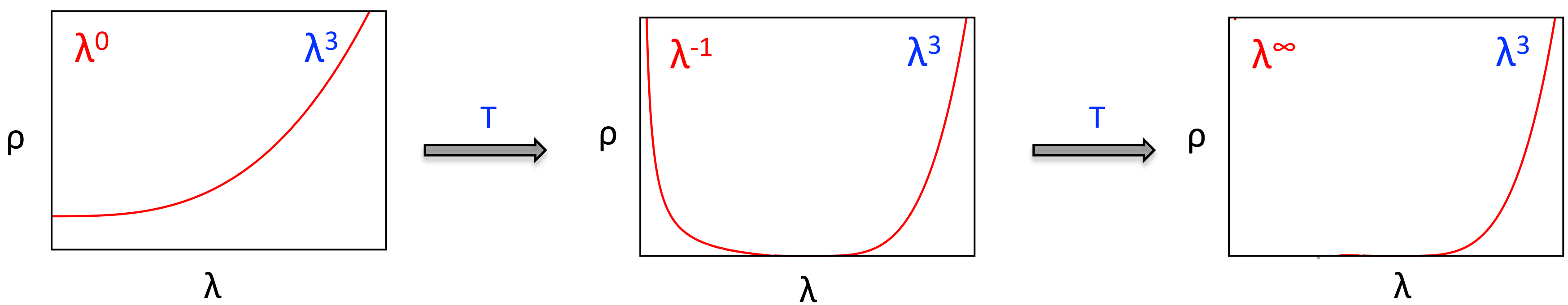
$T < T_{IR}$
E
λ
BULK medium
Energy scale
Dirac scale
T
0
0
Space
$T_{IR} < T < T_{UV}$
E
λ
BULK medium
T
Λ_{IR}
λ_{dc}
IR medium
Energy scale
Dirac scale
0
0
Space
$T > T_{UV}$
E
λ
BULK medium
T
Energy scale
Dirac scale
0
0
Space

WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION

N_F=0 easier to communicate the point [same in N_f=2+1 just more awkward.]

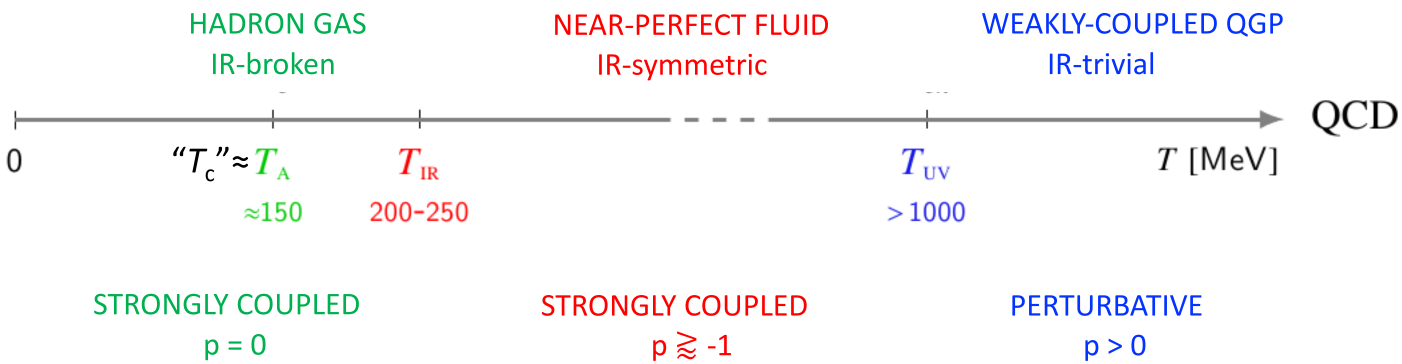
T=0 classically scale invariant theory → (quantum fluctuations / scale anomaly) → scales generated world of hadrons etc

scale-broken T=0 theory → (thermal fluctuations / increasing T) → scale-invariant but only for $\Lambda \leq \Lambda_{\mathrm{IR}} < T$



WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION…

PHASE STRUCTURE OF THERMAL QCD IN TERMS OF GLUE IR SCALE INVARIANCE [AA & IH 1906.08047]



$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \text{with} \qquad \rho(\lambda) \propto \lambda^p \quad \text{for} \quad \lambda \to 0$$

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

Betting on Glue

[1906.08047 , 1502.07732]

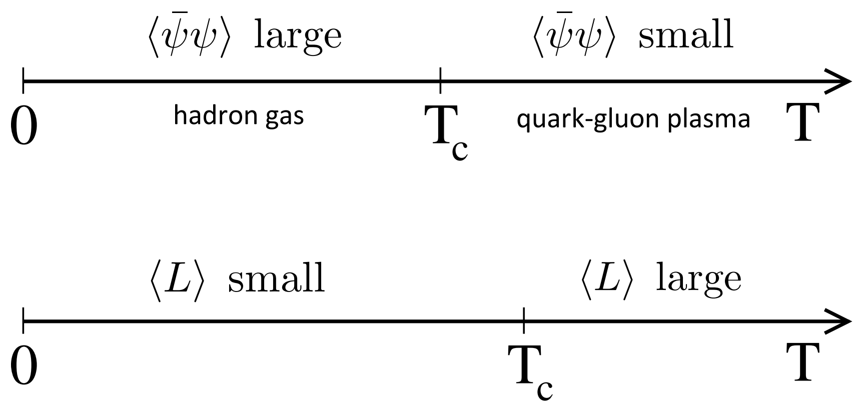
Standard approaches to phases:

quark view:

quark scalar density

glue view:

Polyakov line



Quarks won the popularity contest [$T_c \approx 155$ MeV, crossover , Aoki et al, 2007]

NEED NEW IDEA!

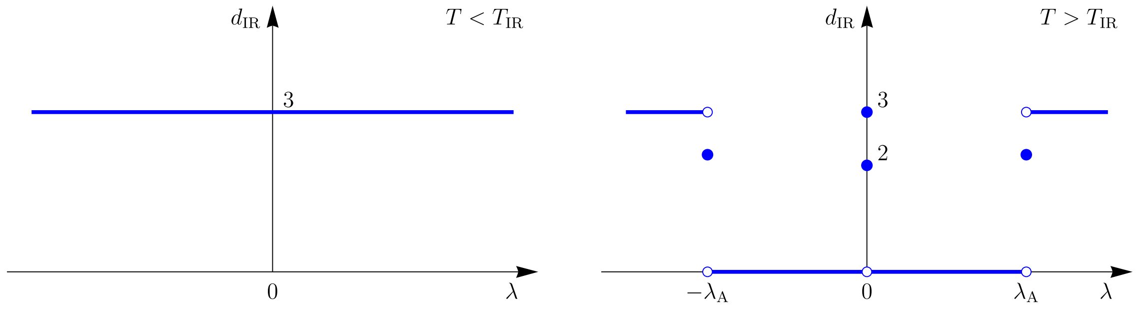
Point 1: $\langle\bar{\psi}\psi\rangle$ is cleaner because it reflects deeper IR of glue

Point 2: both $\langle\bar{\psi}\psi\rangle$ and $\langle L\rangle$ are limited in terms of reflecting glue

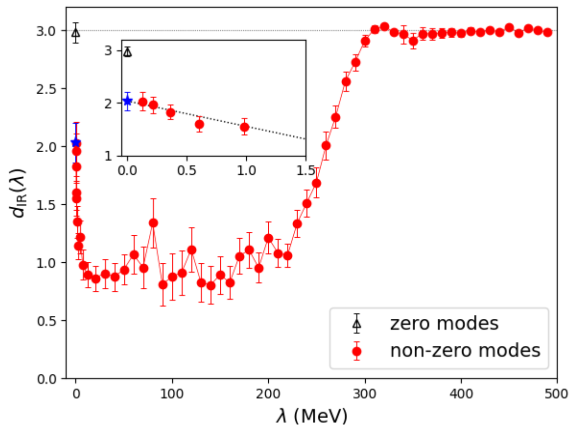
Point 3: need glue probe that is sensitive to any scale by construction
object with explicit scale dependence

TOPOLOGICAL ORIGIN AND NON-ANALYTICITY

A.A. & I.H. 2103.05607, 2110.04833, 2310.03621, 2305.09459



d_{IR}= effective IR dimension of modes

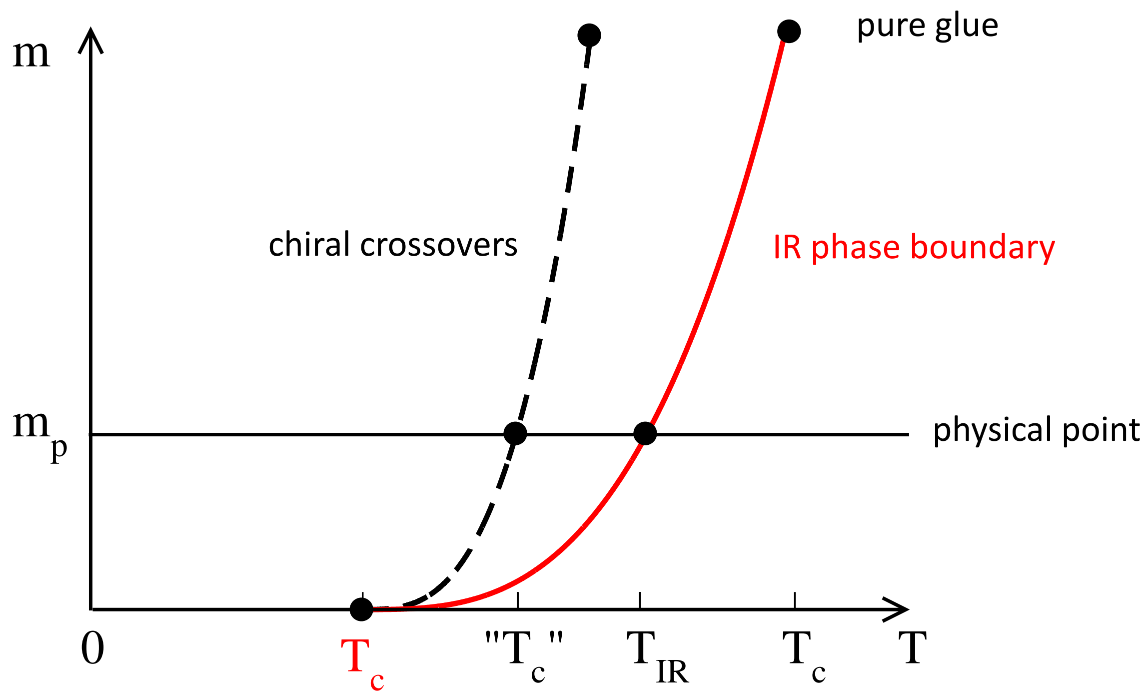


2305.09459

$N_f = 2 + 1$ at physical point T = 234 MeV a=0.105 fm overlap mode-density glue operator

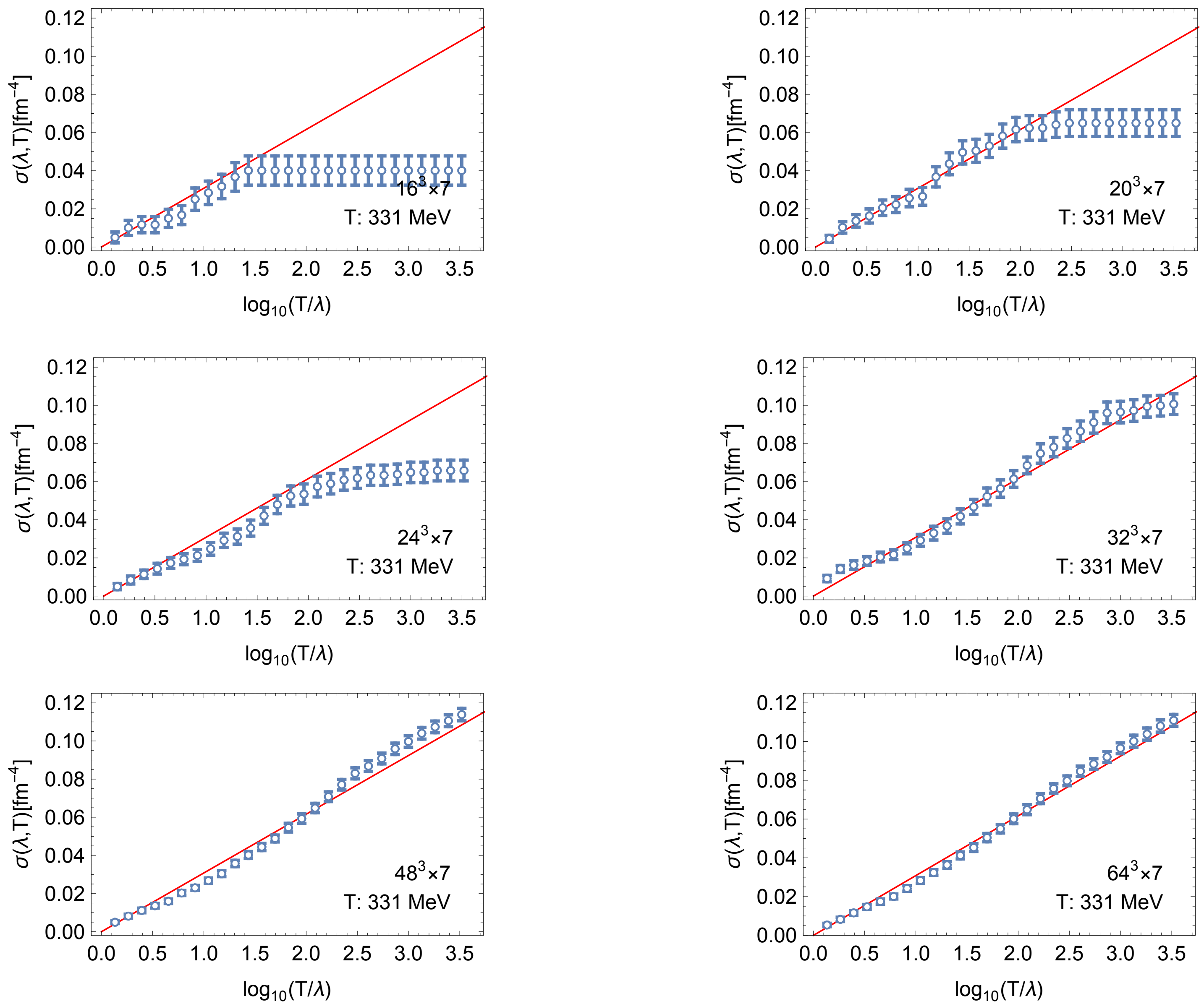
Reconcile IR phase transition with chiral transition:

2-flavor theory: trends, known from A.A & I.H. 1502.07732 and consistent with the following



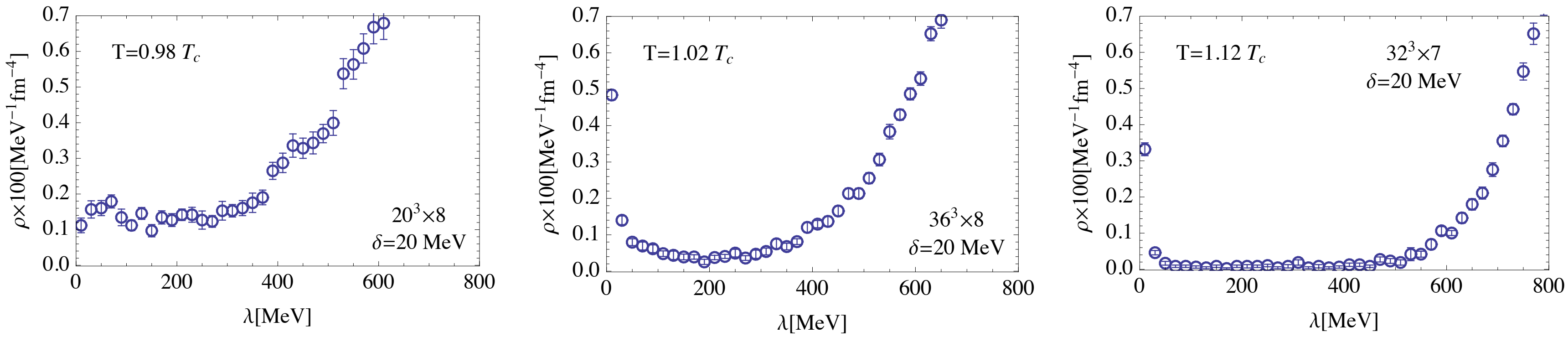
- Note: chiral limit not much closer to physical point than heavy limit
- Secret: story of QCD phase transition is really a T_{IR} story!

Check this further & better... $\sigma(\lambda_1, \lambda_2) \equiv \int_{\lambda_1}^{\lambda_2} d\lambda\, \rho(\lambda)$ $\rho(\lambda) = c/\lambda \longrightarrow \sigma(\lambda, T) = c\ln(T/\lambda)$



- Peak in IR overlap spectrum upon crossing T_c (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]



- knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue
- knee-jerk reaction should be: what on earth is <u>glue</u> doing to produce this? [1502.07732]
- didn't know but went on with it, e.g., around chiral crossover we got this: