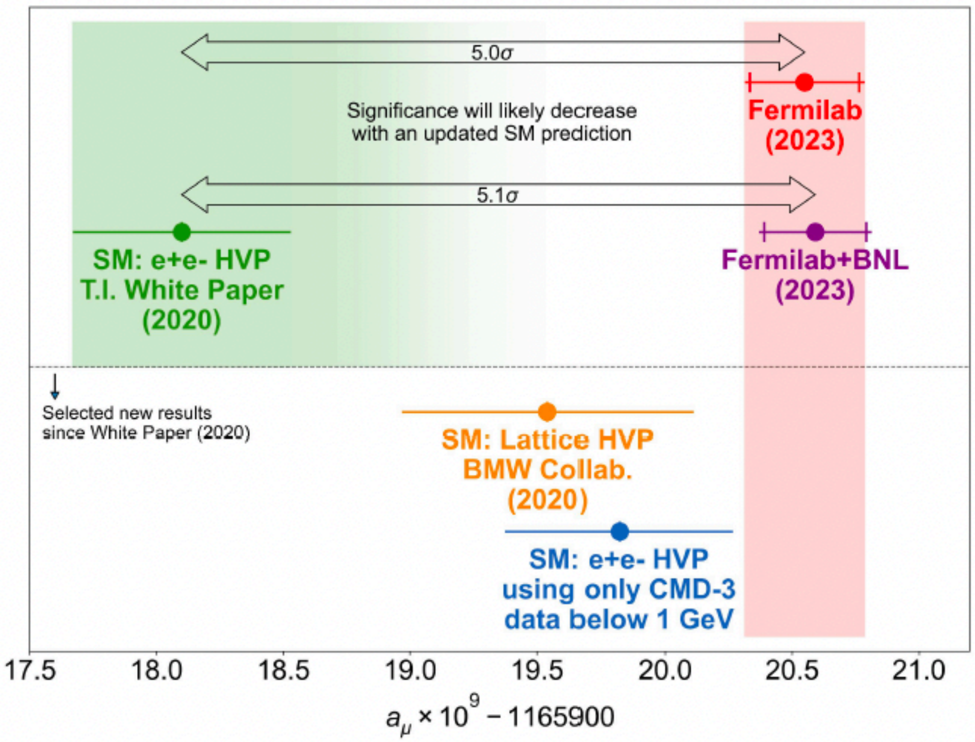
Hadronic Vacuum Polarization and Atomic Binding Effects for the MUonE experiment

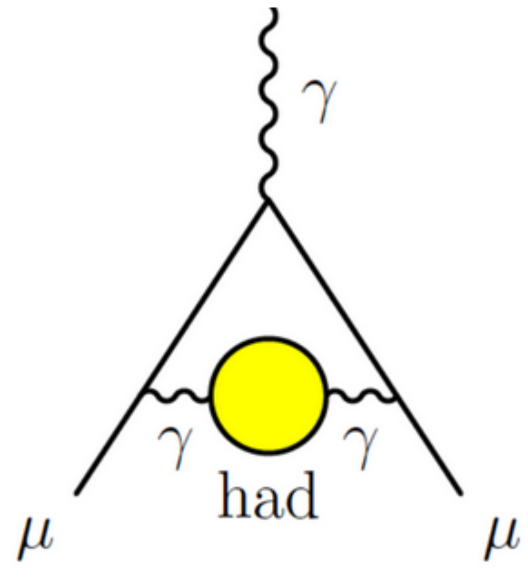
Pleasure to be here in beautiful Cairns Australia.
Talk 8 slides, counting this one (I’m not fast)

Mark Wise August 2024

Anomalous Magnetic Moment of Muon and Hadronic Physics

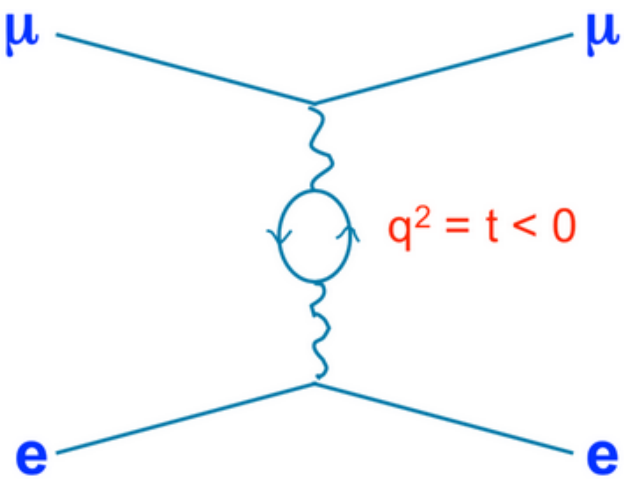


X post from Fermilab



$$a_{\mu-HVP} \sim \int dx f(x) \int \frac{ds}{s} Im\Pi(s) \frac{m_\mu^2}{\frac{x^2}{1-x} m_\mu^2 + s}$$

MUonE Experiment



CERN: 150 GeV muons incident on electrons in carbon

Order 1% correction to muon electron scattering, and want hadronic vacuum polarization contribution to 1% accuracy

Hard experiment aims for accuracy of 10ppm on shape of cross section

Eur. Phys. J. C (2017) 77:139
DOI 10.1140/epjc/s10052-017-4633-z

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Experimental Physics

Measuring the leading hadronic contribution to the muon *g*-2 via *μe* scattering

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Atomic Effects in MUonE (and similar experiments)

For high energy lepton beams scattering off electrons in a target it is an excellent approximation to treat the electron as free and at rest.

1) Corrections from Atomic binding arising both from phase space, and the initial state (many-body) wavefunction of the atom. These corrections stem from the finite three-momentum of electrons in bound atomic orbitals, and the shifted kinematics from binding energies. They account for the non-perturbative bound-state dynamics i.e., from iterated Coulomb exchange in the initial state. [Ryan Plestid and Mark B. Wise, e-Print:2403.12184 (2024)]

$$\mu(\mathbf{k}) + \mathbf{A}(\mathbf{0}) \to \mu(\mathbf{k}') + \mathbf{e}^-(\mathbf{p}') + \mathbf{B}^+(\mathbf{h}') \ ,$$

Momentum Conservation $\mathbf{k}' + \mathbf{p}' = \mathbf{k} - \mathbf{h}'$

Same as muon scattering off free electron not at rest but with momentum $\mathbf{p} = -\mathbf{h}'$

$$0 = E_\mu(\mathbf{k}') + E_e(\mathbf{p}') + E_B(\mathbf{h}') - E_\mu(\mathbf{k}) - m_A \qquad E_B(\mathbf{h}') - m_A = -m_e + \epsilon \qquad \epsilon = \epsilon_A - \epsilon_B$$

Energy conservation becomes $0 = E_\mu(\mathbf{k}') + E_e(\mathbf{p}') - E_\mu(\mathbf{k}) - m_e + \epsilon = E_\mu(\mathbf{k}') + E_e(\mathbf{p}') - E_\mu(\mathbf{k}) - \frac{\mathbf{h}'^2}{2m_e} - m_e + \epsilon + \frac{\mathbf{h}'^2}{2m_e}$

Include effects of order $\mathbf{p}^2/m_e^2$ but neglect terms of order $\mathbf{p}^2/E_{\mu,e}^2$ (phase space)

Atomic Effects in MUonE (continued)

$$\sum_{\text{spins}} |\mathsf{M}|^2 \simeq 32e^4 \frac{(k \cdot p')(k' \cdot p) + (k \cdot p)(k' \cdot p') - m_\mu^2 (p \cdot p')}{[(k-k')^2]^2} \ .$$

Use shifted energy conservation and initial electron 3-momentumand expand matrix element in initial electron momentum and $\epsilon = \epsilon_A - \epsilon_B$

After averaging over directions of initial electron three momentum $cos(\theta_{ee'}) \simeq cos(\theta_{e\nu'}) \simeq cos(\theta_{e\nu})$

Get corrections of order $\mathbf{p}^2/m_e^2$ and ϵ/m_e

$$\langle e^- B^+ | \bar{\psi}_e \gamma_\alpha \psi_e | A \rangle = \int \frac{d^3p}{2E_e(\mathbf{p})} \bar{u}(\mathbf{p}') \gamma_\alpha u(\mathbf{p}) \langle B^+ | \hat{a}_{\mathbf{p}} | A \rangle$$

$$\langle B^+ | \mathbf{a}_{\mathbf{p}} | A \rangle \epsilon = \langle B^+ | [H, \mathbf{a}_{\mathbf{p}}] | A \rangle$$

Square sum over final states $|B^+\rangle$ Should use correct spinor $u(\mathbf{p})$

Atomic Effects in MUonE (continued)

$$\hat{H} = \sum_{\mathbf{p}} \frac{\mathbf{p}^2}{2m_e} + \sum_{\mathbf{p},\mathbf{p}'} V_1(\mathbf{p}, \mathbf{p}') a^\dagger_{\mathbf{p}'} a_{\mathbf{p}} + \sum_{\mathbf{p},\mathbf{p}'\mathbf{q},\mathbf{q}'} V_2(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}') a^\dagger_{\mathbf{p}'} a^\dagger_{\mathbf{q}'} a_{\mathbf{p}'} a_{\mathbf{q}'}$$

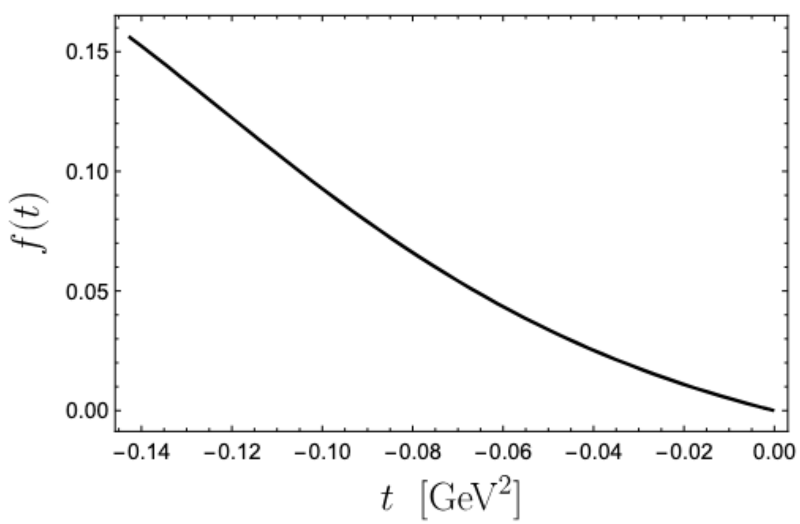
Evaluate commutator use viral theorem $\langle \hat{T} \rangle_A = \epsilon_A$, and $\langle V_1 + V_2 \rangle_A = -2\epsilon_A$

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma_0}\frac{d\sigma_0}{dt}\left(1 - f(t)c\right)$$

$$c = \frac{1}{Z_A m_e}\left[\frac{11}{3}\epsilon_A + \langle \hat{V}_1 \rangle_A\right] \quad c = 45 \times 10^{-5}$$

Used $\langle V_1 \rangle_C = -2.41 keV$

J. B. Mann Tech. Rep. LA-3691 (Los Almos National Lab (1968)



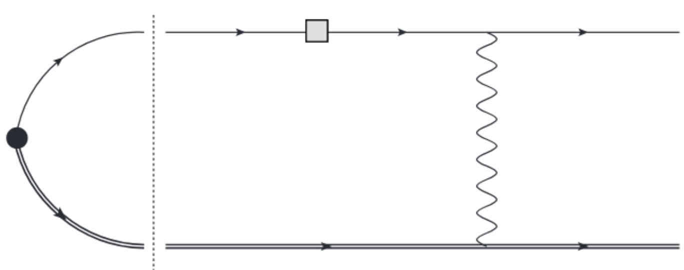
Atomic Effects in MUonE (continued)

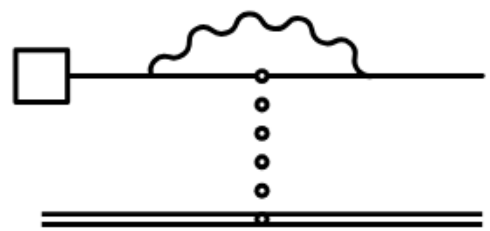
Corrections discussed so far are of order $\frac{\mathbf{p}^2}{m_e^2} \sim \alpha^2$ with some enhancement from the number of electrons in the target atom

Further corrections arise from perturbative photon exchange between ``hard'' leptons, and ``soft'' (i.e., non-relativistic) electrons and nuclei. These effects are also inherently absent from calculations for a free electron at rest.

In a further paper Ryan Plestid and Mark Wise e-Print: 2405.08110 [hep-ph] those were studied

For example below leaving out the muon lines attaching to the square





After **summing over final atomic debris states** no corrections enhanced by number of electrons in atom at order α^2

Conclusions

- Understanding the hadronic contribution to muon g-2 better is important for predicting its value at the level of the present experimental error.
- The proposed experiment MuonE can make a contribution there
- Theory needed at order α^2. This is under control for muon free electron scattering (For a review with references: A. Gurgone [on behalf of MuonE collaboration] e-Print:2401.06491 [hep-ph] 2024.)
- Mostly under control for the order α^2 corrections that care about the fact that the struck electron is in an atom and that final state contains low momentum atomic debris.