Femtoscopy for exotic hadrons and nuclei

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Tokyo Metropolitan Univ.

2024, Aug. 20th

Contents

- Introduction — Femtoscopy primer
- Femtoscopy for exotic hadrons
  - $K^- p$ correlations for $\Lambda(1405)$

    Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)
- Femtoscopy for hypernuclei
  - $\Lambda\alpha$ correlations for Λ in medium

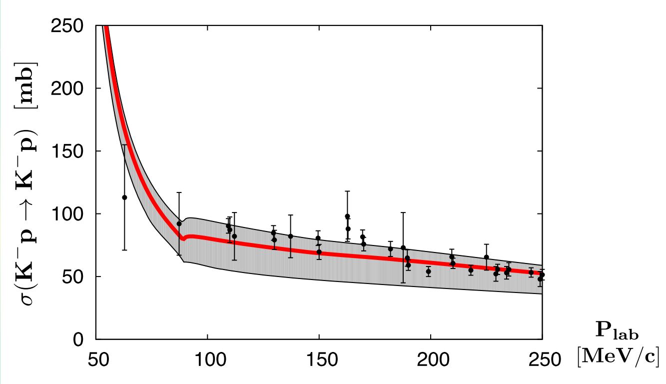
    A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC110, 014001 (2024)
- Summary

Contents

Scattering experiments and femtoscopy

Traditional methods: scattering experiments

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)



- Limited channels: $NN, YN, \pi N, KN, \bar{K}N, \cdots$
- Heavy (c, b) hadrons: impossible
- Limited statistics (low-energy)

Scattering experiments and femtoscopy

Traditional methods: scattering experiments

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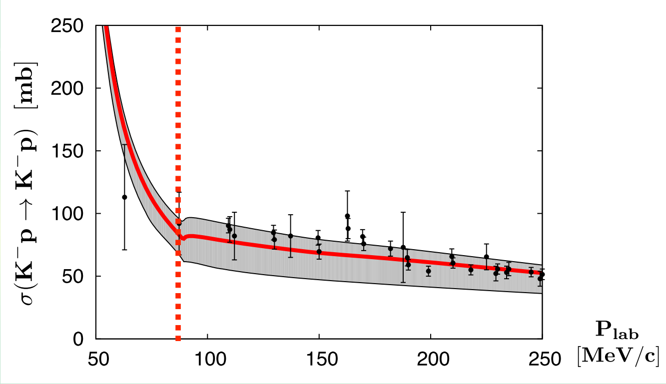

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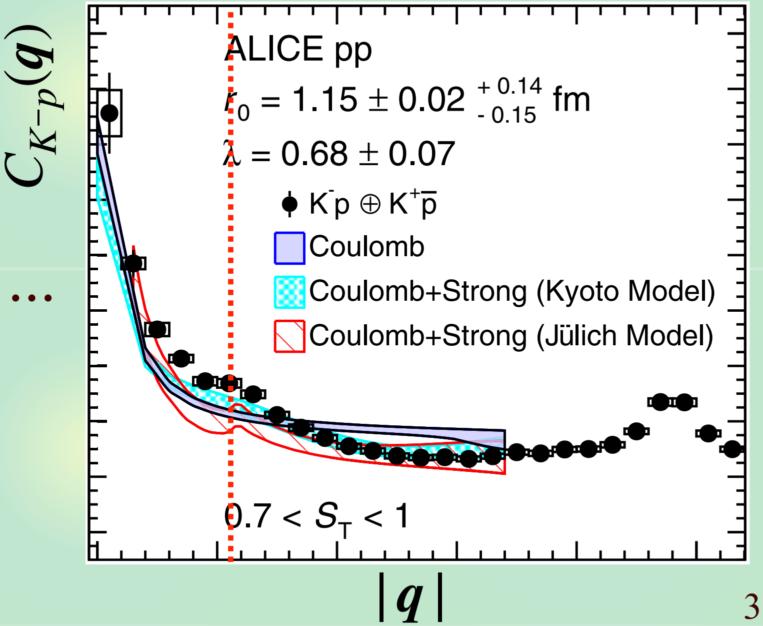
- Heavy (c, b) hadrons: impossible
- Limited statistics (low-energy)

Femtoscopy: correlation function

ALICE collaboration, PRL 124, 092301 (2020)



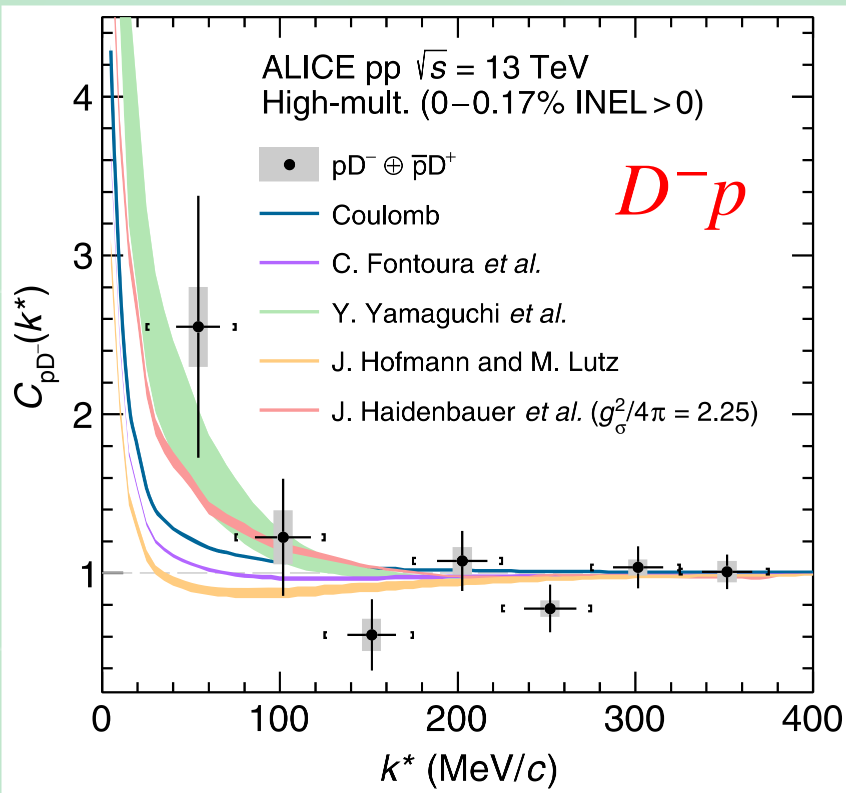
- Various systems: $\Lambda\Lambda, N\Omega, \phi N, \bar{K}\Lambda, DN, \cdots$
- Heavy hadrons: possible!
- Excellent precision ($\bar{K}^0 n$ cusp)

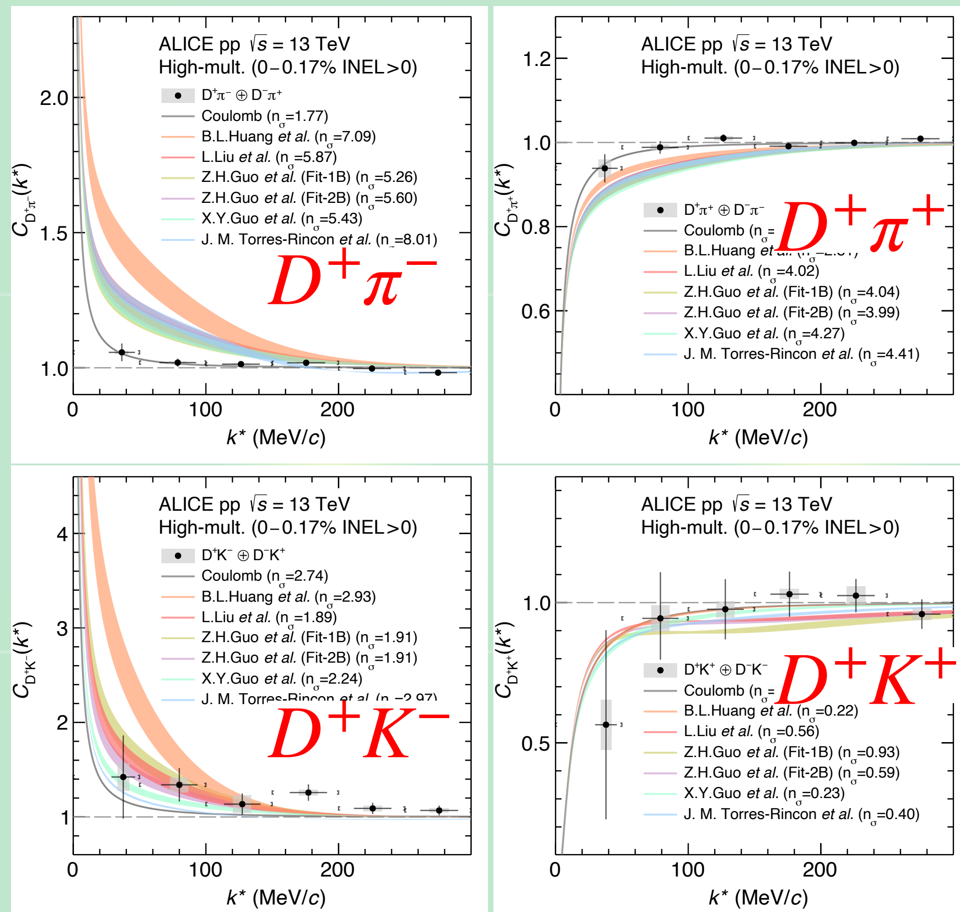
Experimental data in charm sector

Observed correlation functions with charm: $DN, D\pi, DK$

ALICE collaboration, PRD 106, 052010 (2022);

ALICE collaboration, PRD 110, 032004 (2024)



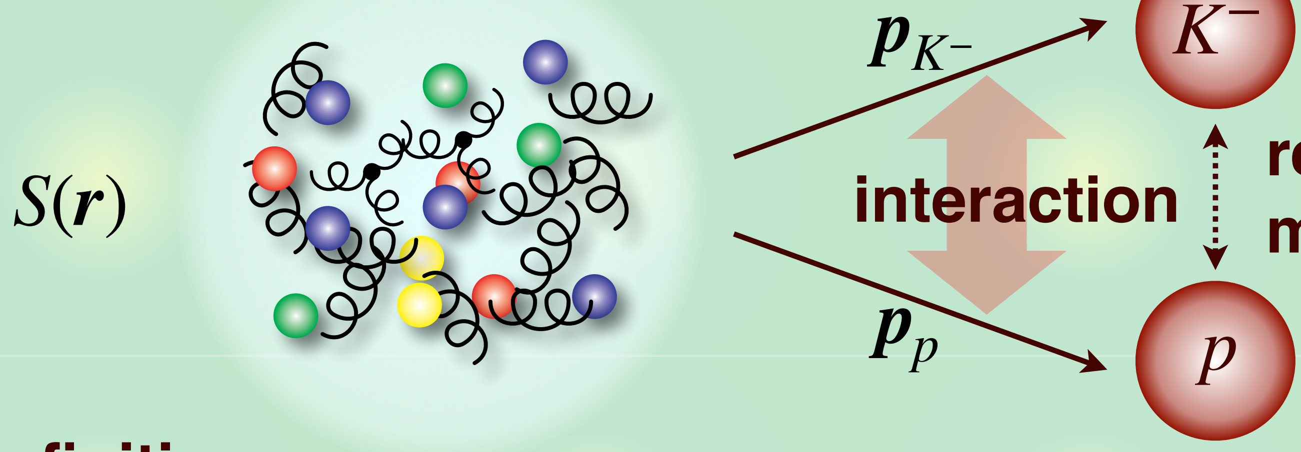


Unique way to obtain data in charm sector (yet low statistics)

Correlation function and KP formula

High-energy collision: chaotic source $S(\boldsymbol{r})$ of hadron emission

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)



- Definition

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)}$$

(= 1 in the absence of FSI/QS)

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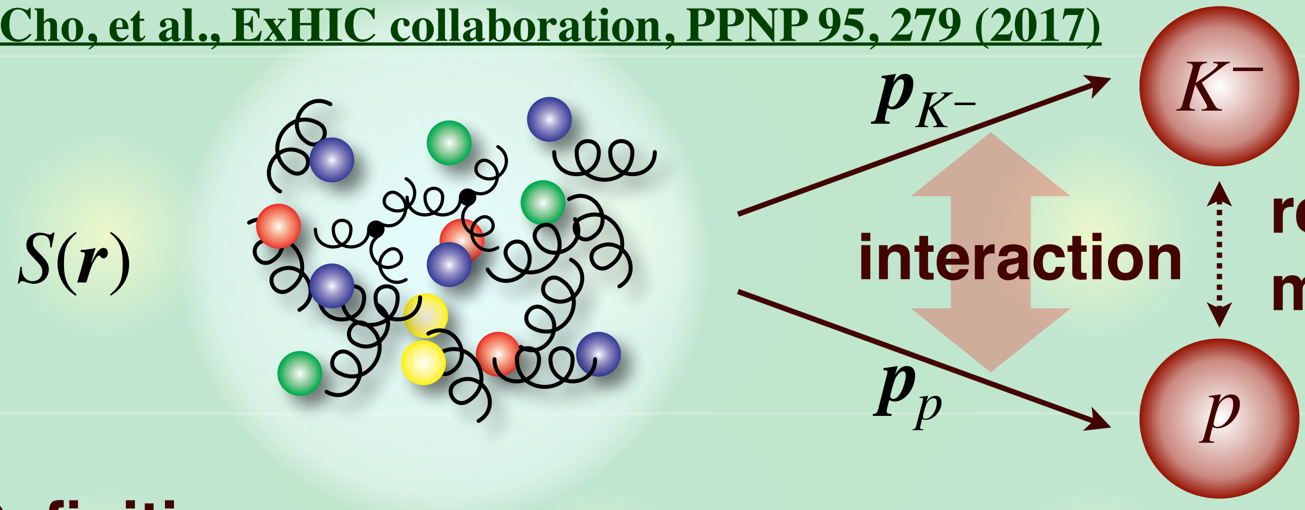


- Definition

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)}$$ (= 1 in the absence of FSI/QS)

- Theory (Koonin-Pratt formula)

incoming + outgoing

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

$$C(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2, \quad \Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r}) \propto S^\dagger e^{-iqr} - e^{+iqr} \quad (r \to \infty)$$

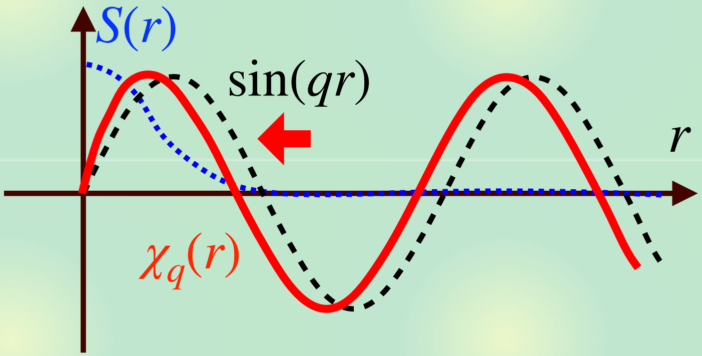
Source function $S(\boldsymbol{r})$ <—> wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$ (interaction)

Wave functions and correlations

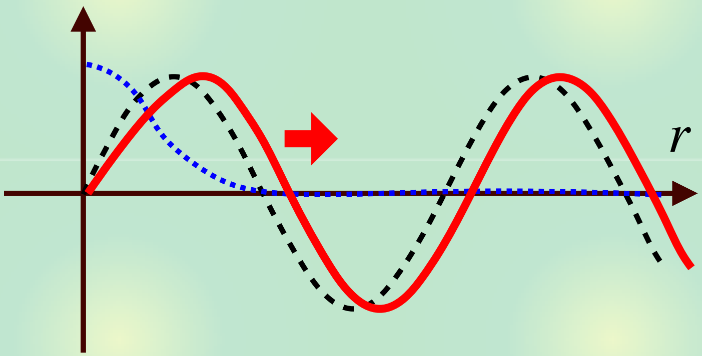
Spherical source with s-wave interaction dominance

$$C(q) \simeq 1 + \int_0^\infty dr\, S(r)\{\,|\chi_q(r)|^2 - \sin^2(qr)\}$$

attraction



repulsion



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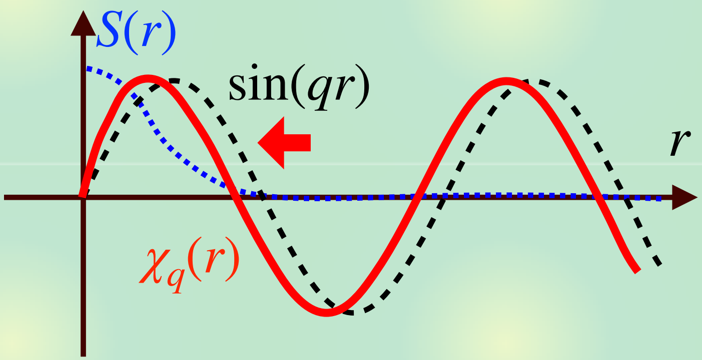

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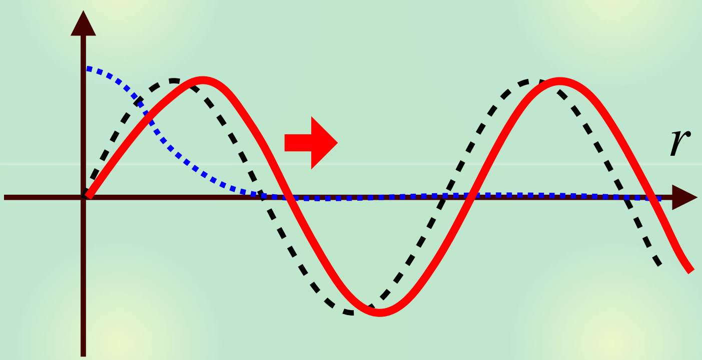

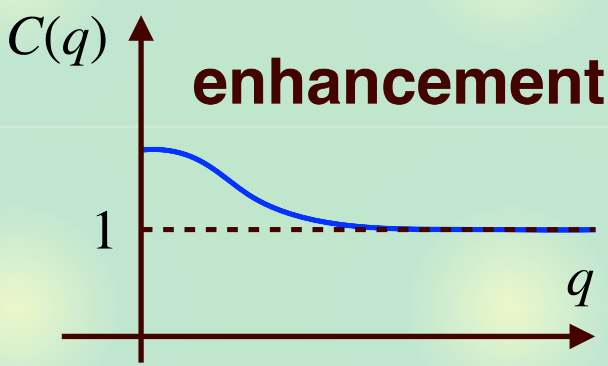

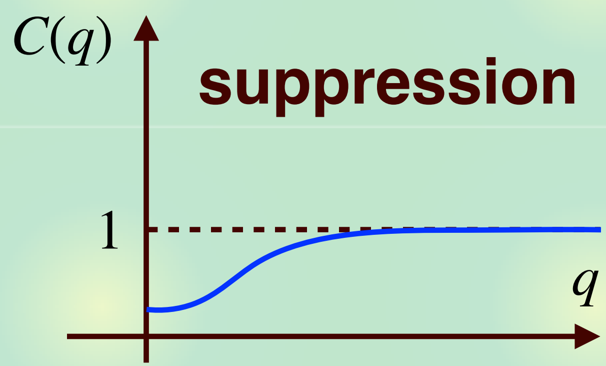

Correlation function <—> nature of interaction

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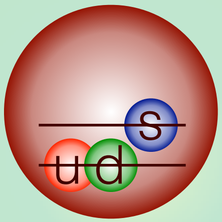
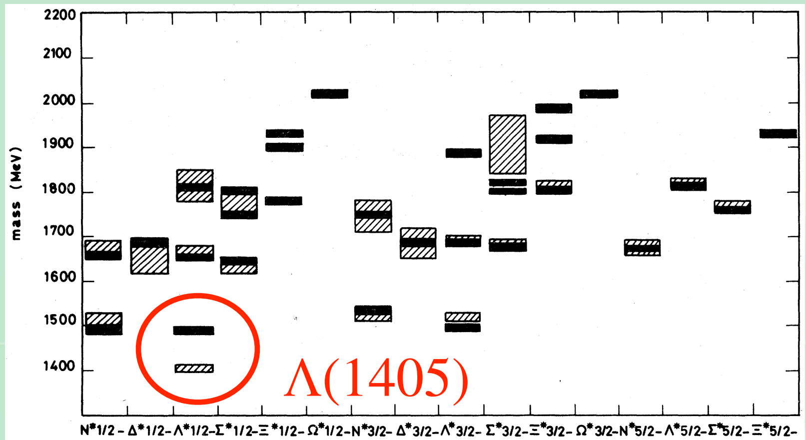
$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)





: theory

: experiment

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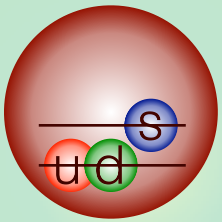
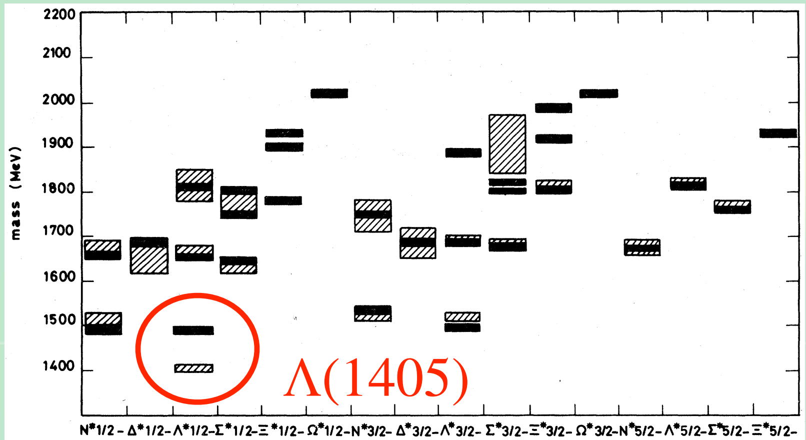


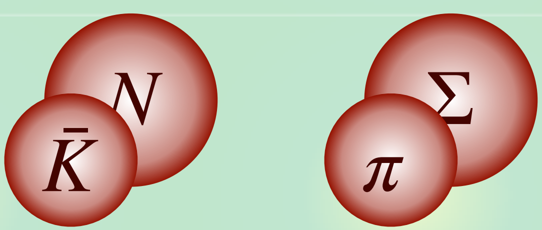


: theory

: experiment

Resonance in coupled-channel scattering

- Coupling to MB: chiral SU(3) dynamics



T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th] (Handbook of Nuclear Physics)

Coupled-channel effects

Schrödinger equation (s-wave)

$$\begin{pmatrix} \frac{-1}{2\mu_1}\frac{d^2}{dr^2} + V_{11}(r) + V_{\mathrm{C}}(r) & V_{12}(r) & \cdots \\ V_{21}(r) & \frac{-1}{2\mu_2}\frac{d^2}{dr^2} + V_{22}(r) + \Delta_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0 n}(r) \\ \vdots \end{pmatrix} = E \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0 n}(r) \\ \vdots \end{pmatrix}$$

Coulomb — threshold energy difference

Coupled-channel effects

Schrödinger equation (s-wave)

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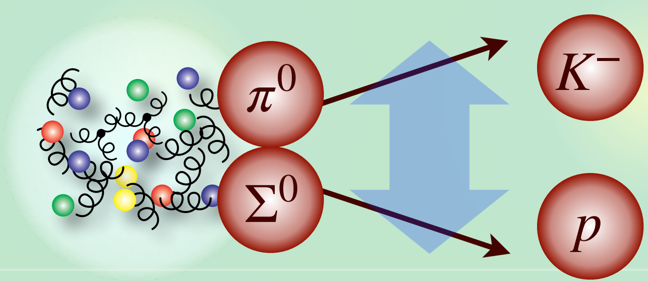
Coulomb — threshold energy difference

Asymptotic ($r \to \infty$) wave function (incoming + outgoing)

$$\begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0n}(r) \\ \vdots \end{pmatrix} \propto \begin{pmatrix} S^\dagger_{11}e^{-iqr} - e^{iqr} \\ S^\dagger_{12}e^{-iq_2r} - 0 \times e^{iq_2r} \\ \vdots \end{pmatrix} \quad (r \to \infty)$$

- **Transition** from $\bar{K}^0n, \pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+, \pi^0\Lambda$ is in $\psi_i(r)$ with $i \neq K^-p$



Coupled-channel correlation function

Coupled-channel Koonin-Pratt formula

R. Lednicky, V.V. Lyuboshitz, V.L. Lyuboshitz, Phys. Atom. Nucl. 61, 2950 (1998);
J. Haidenbauer, NPA 981, 1 (2019);
Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

$$C_{K^-p}(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\; S_{K^-p}(\boldsymbol{r})\,|\,\Psi^{(-)}_{K^-p,\boldsymbol{q}}(\boldsymbol{r})\,|^2$$

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$$C_{K^-p}(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\ S_{K^-p}(\boldsymbol{r})\,|\,\Psi^{(-)}_{K^-p,\boldsymbol{q}}(\boldsymbol{r})\,|^2 + \sum_{i\neq K^-p} \omega_i \int d^3\boldsymbol{r}\ S_i(\boldsymbol{r})\,|\,\Psi^{(-)}_{i,\boldsymbol{q}}(\boldsymbol{r})\,|^2$$

- Transition from $\bar{K}^0 n, \pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+, \pi^0\Lambda$
- ω_i: weight of channel i source relative to K^-p

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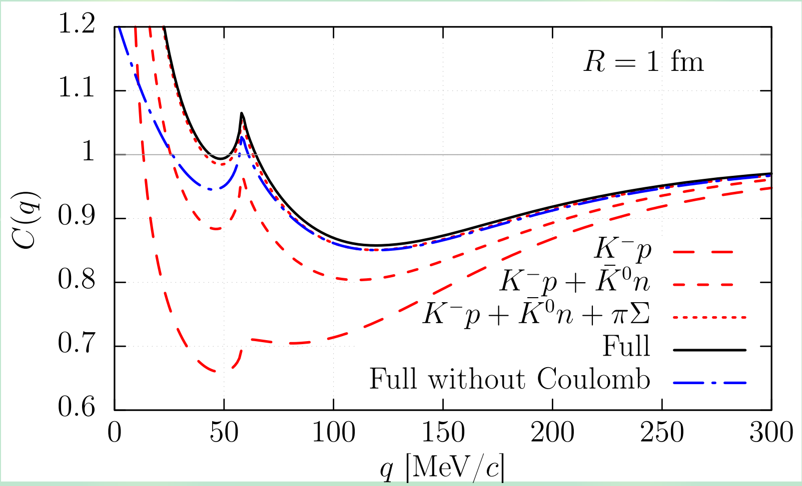
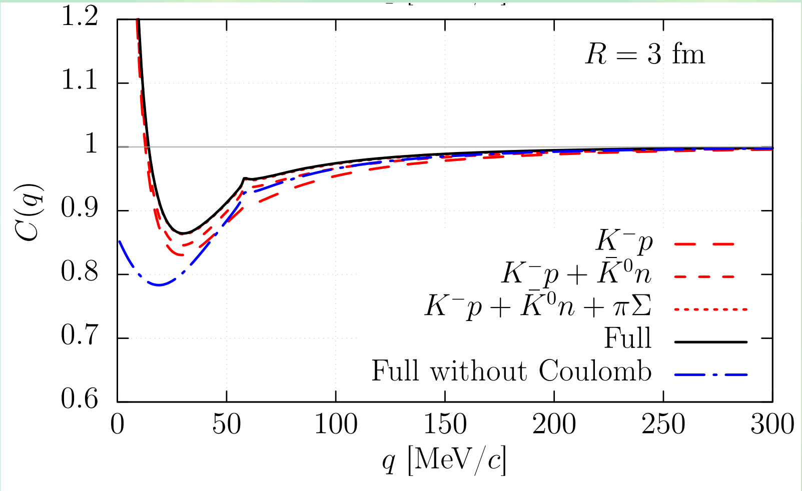




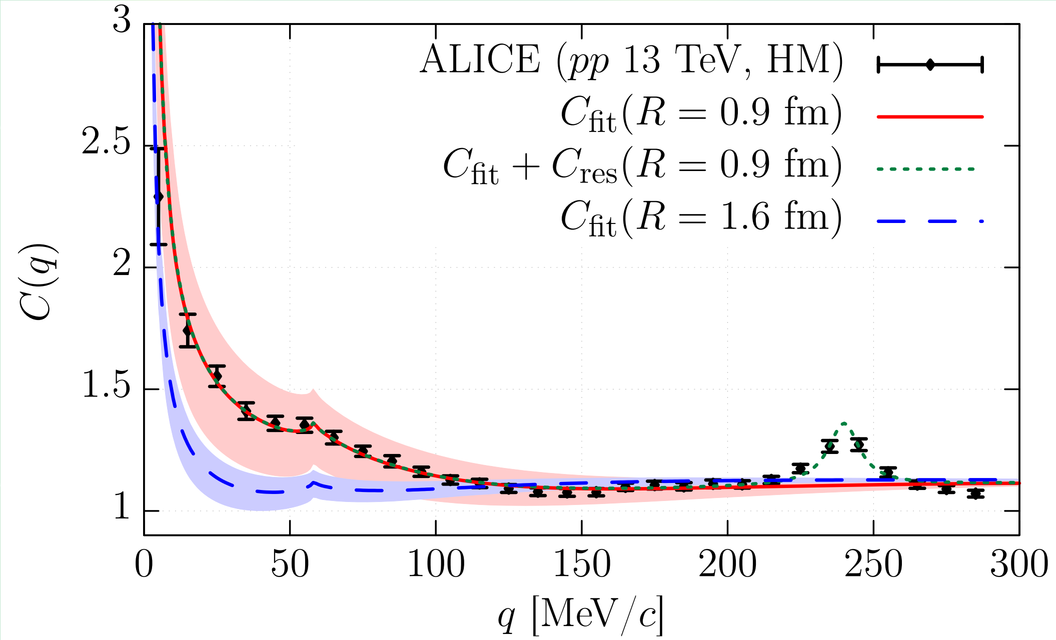
Coupled-channel effect is enhanced for small sources

Correlation from chiral SU(3) dynamics

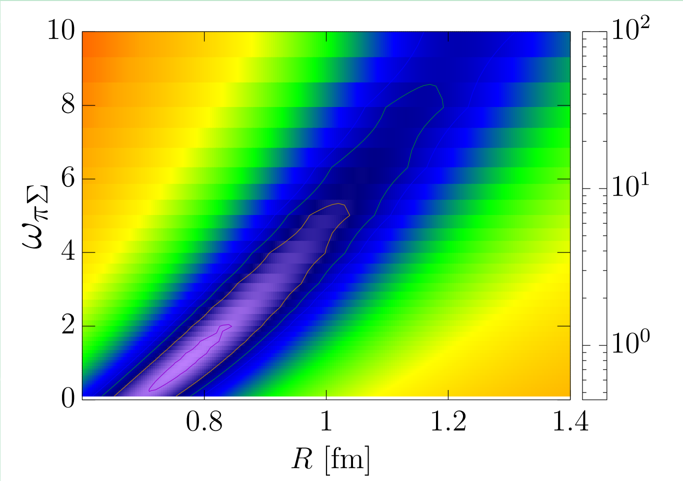
Wave function $\Psi^{(-)}_{i,\boldsymbol{q}}(\boldsymbol{r})$: Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential

K. Miyahara, T. Hyodo, W. Weise, PRC98, 025201 (2018)

- **Source function $S(\boldsymbol{r})$: gaussian, $R \sim 1$ fm from K^+p data**
- **Source weight $\omega_{\pi\Sigma} \sim 2$ by simple statistical model estimate**



reduced χ^2



Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

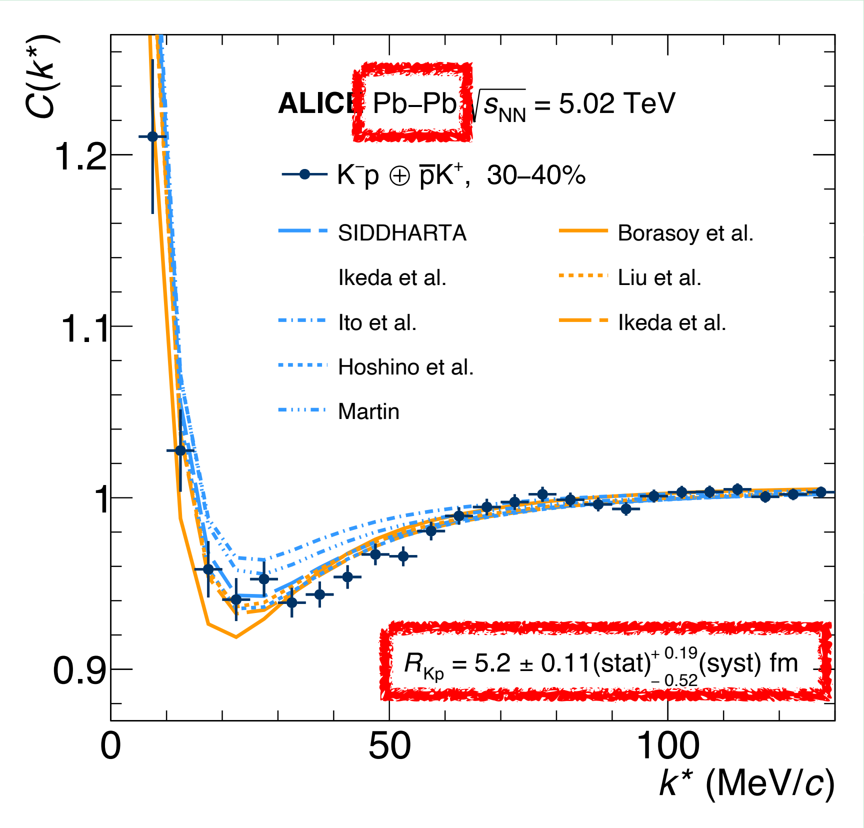
Correlation is well reproduced by chiral SU(3) potential

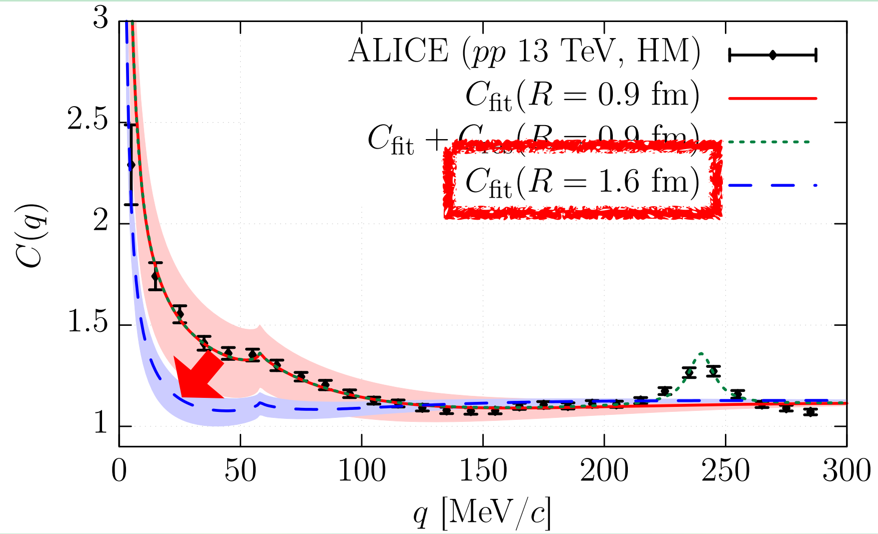
Large source case

New data with Pb-Pb collisions at 5.02 TeV

ALICE collaboration, PLB 822, 136708 (2021)

- Scattering length $a_{K^-p} = -0.91 + 0.92i$ fm





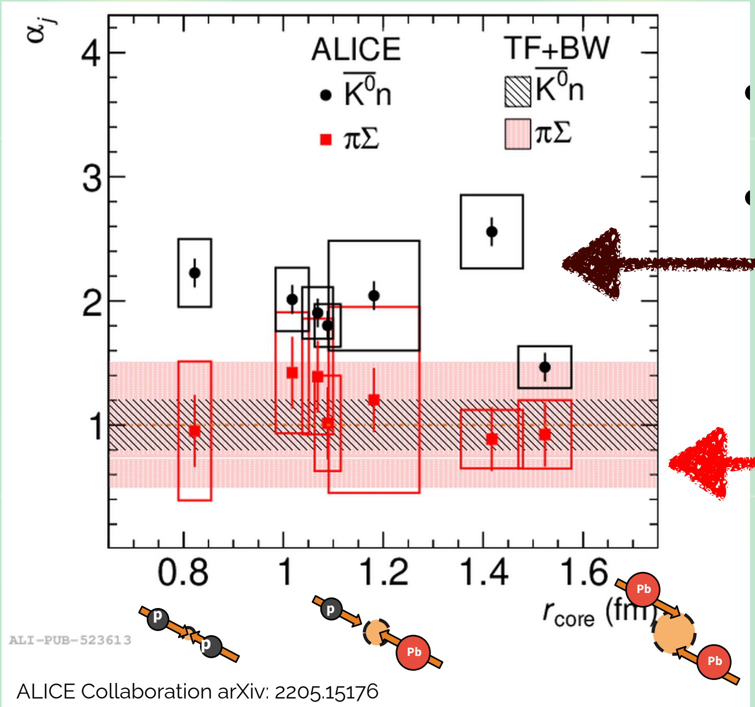
Correlation is suppressed at larger R, as predicted

Systematic study of source size dependence

Correlations in pp, p-Pb, Pb-Pb by Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential

ALICE collaboration, EPJC 83, 340 (2023)

$$C_{K^-p}(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S_{K^-p}(\boldsymbol{r})\,|\Psi^{(-)}_{K^-p,\boldsymbol{q}}(\boldsymbol{r})|^2 + \sum_{i\neq K^-p} \alpha_i\, \omega_i \int d^3\boldsymbol{r}\, S_i(\boldsymbol{r})\,|\Psi^{(-)}_{i,\boldsymbol{q}}(\boldsymbol{r})|^2$$



ALICE Collaboration arXiv: 2205.15176

ω_i: expected weight by Thermal Fist + Blast Wave

enhancement needed to explain data

expected weight is OK

More strength is needed in the $\bar{K}^0 n$ channel

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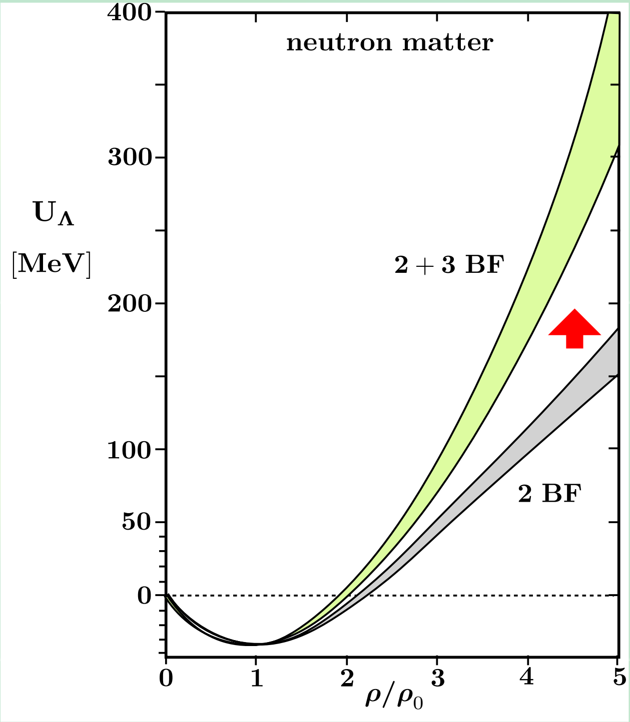
Motivation

A solution to hyperon puzzle in neutron stars

- ΛNN **three-body force** for repulsion at high density

D. Gerstung, N. Kaiser, W. Weise, EPJA 55, 175 (2020)



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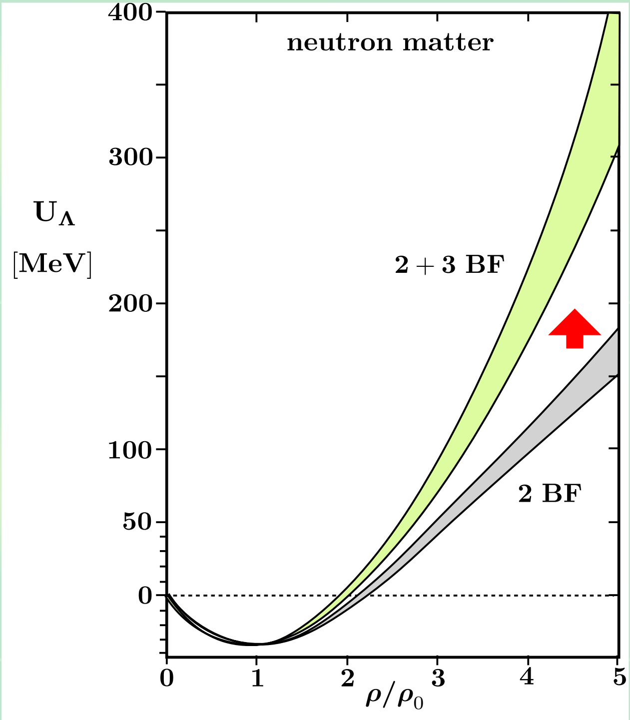

How to verify this in experiments?

- Λ directed flow in heavy ion collisions

Y. Nara, A. Jinno, K. Murase, A. Ohnishi, PRC 106, 044902 (2022)

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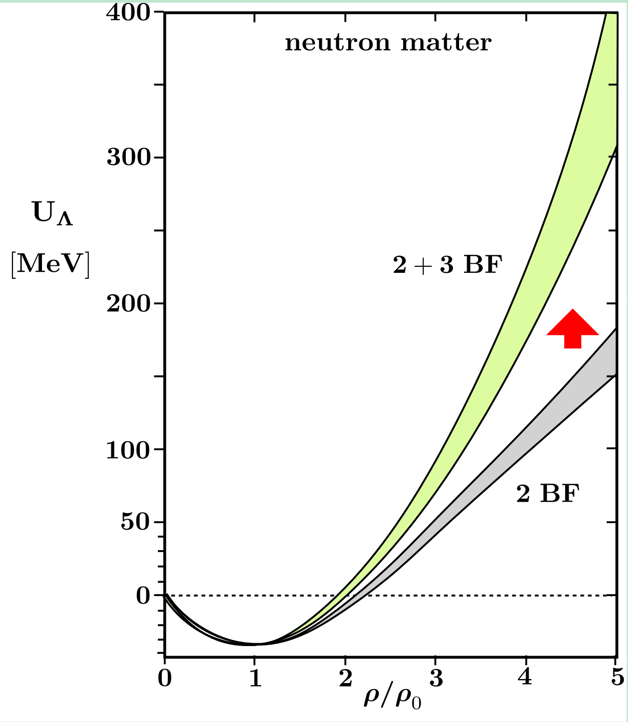


How to verify this in experiments?

- Λ directed flow in heavy ion collisions

Y. Nara, A. Jinno, K. Murase, A. Ohnishi, PRC 106, 044902 (2022)

Λ-nucleus correlation function?

- Heavy nuclei are difficult to produce
- Strong binding of α: two-body treatment justified

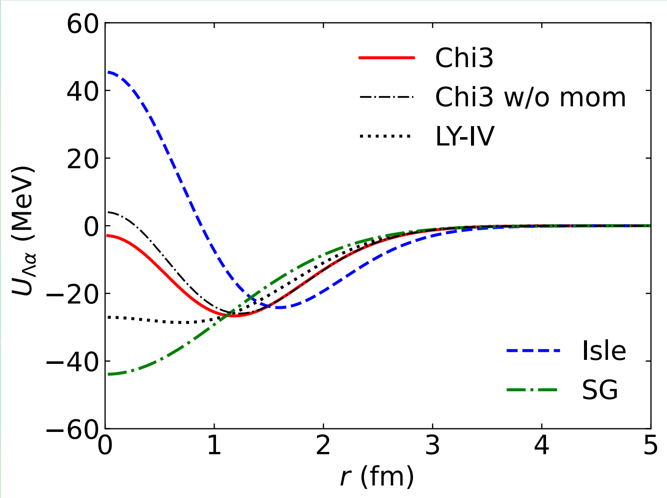
$\Lambda\alpha$ correlation function —> nature of $\Lambda\alpha$ potential?

$\Lambda\alpha$ potentials

Phenomenological $\Lambda\alpha$ potentials ($^{5}_{\Lambda}\mathrm{He}$ binding energy)

I. Kumagai-Fuse, S. Okabe, Y. Akaishi, PLB 345, 386 (1997)

- SG: single gaussian
- Isle: two gaussians (with core)



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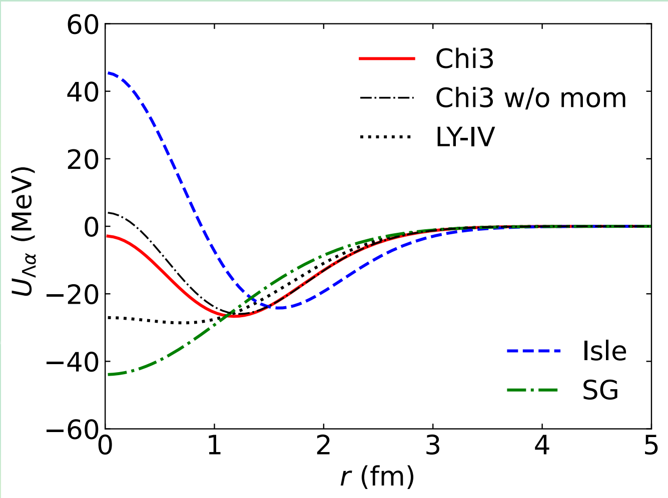


Skyrme-Hartree Fock methods

- LY4: phenomenorogical

D.E. Lanskoy, Y. Yamamoto, PRC 55, 2330 (1997)

- Chi3: based on chiral EFT with ΛNN force

A. Jinno, K. Murase, Y. Nara, A. Ohnishi, PRC 108, 065803 (2023)

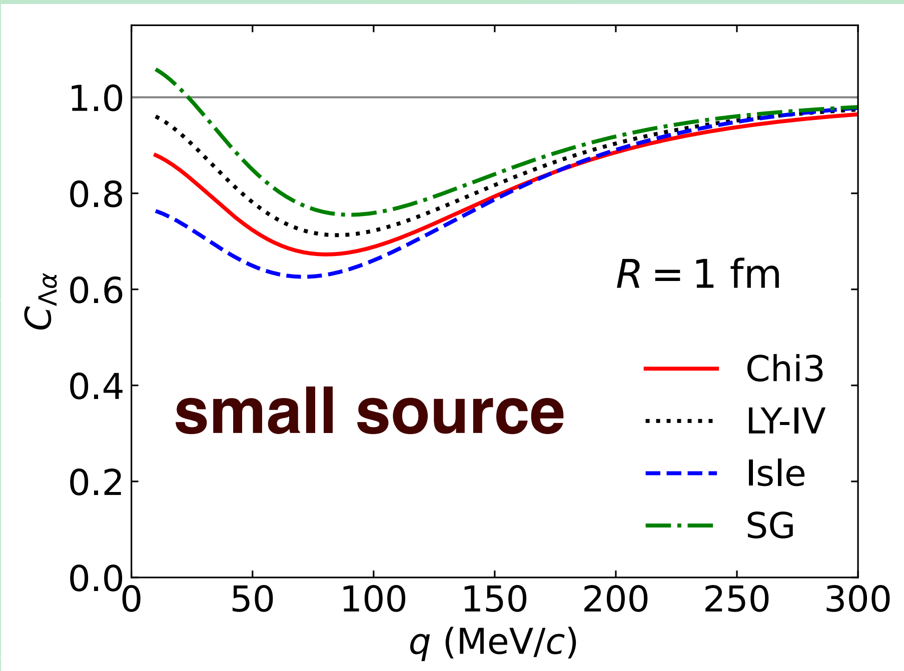
- Both potentials reproduce hypernuclear data from C to Pb
- α density distribution —> $\Lambda\alpha$ potentials

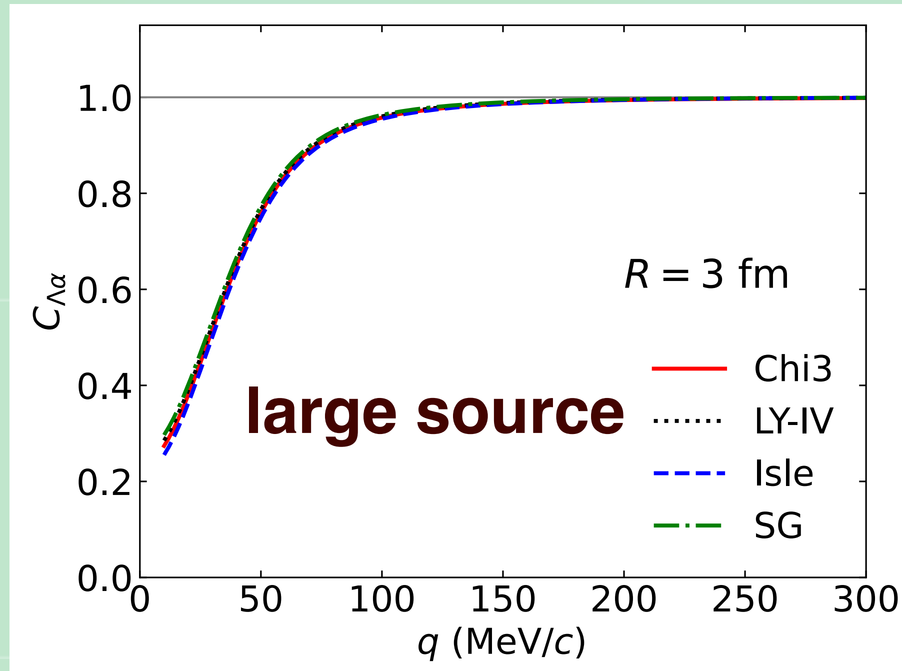
Effect of repulsive core —> correlation function?

$\Lambda\alpha$ correlation functions: source size dependence

Correlation functions from small and large sources

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC110, 014001 (2024)



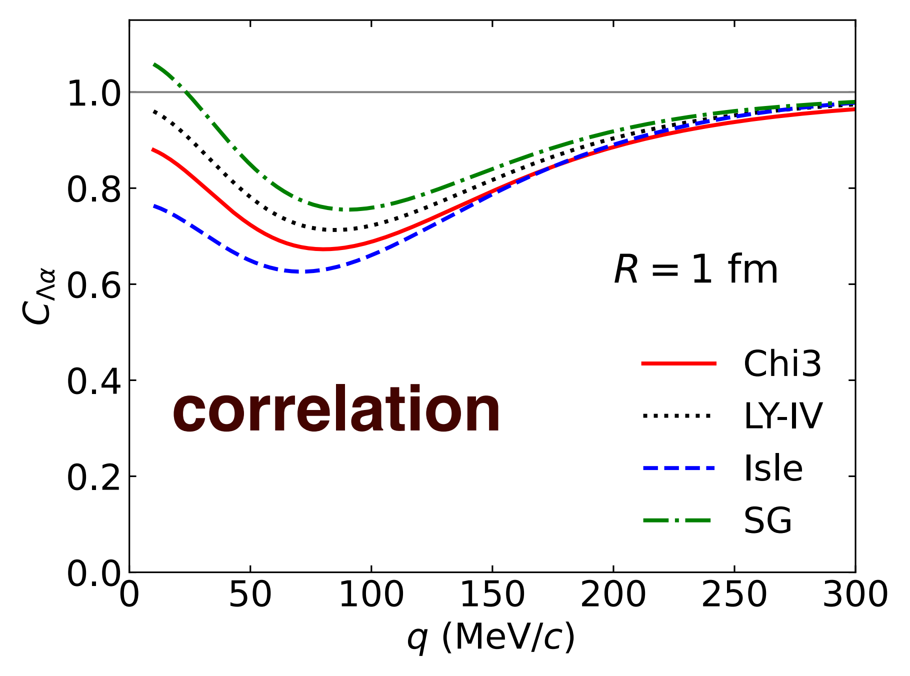


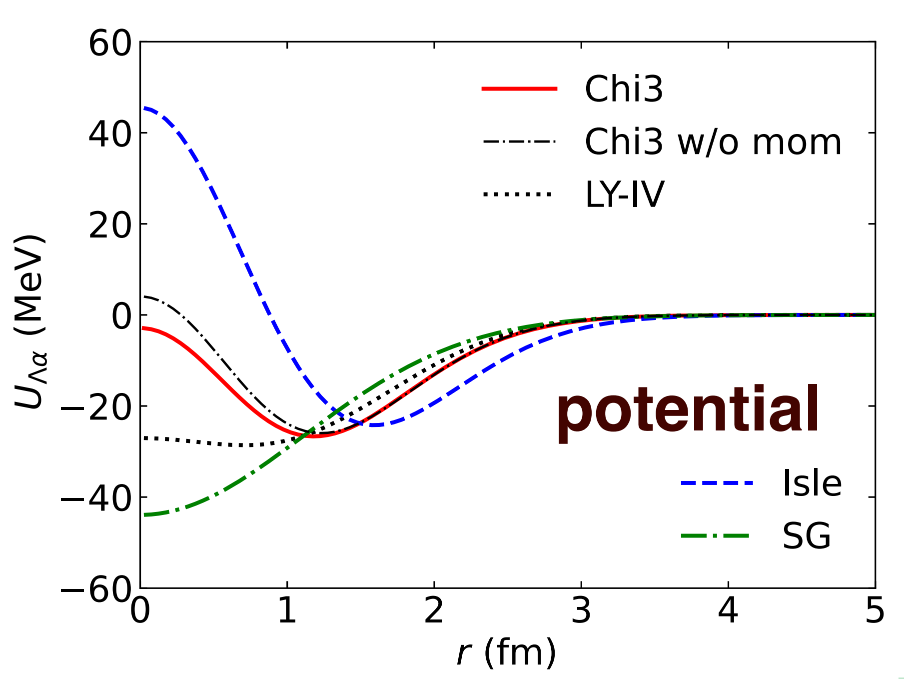
- Bound state signature (dip at low q in small source)
- No difference in large source ($R \sim 3$ fm)
- Potential dependence in small source ($R \sim 1$ fm)

$\Lambda\alpha$ correlation functions: potential dependence

Correlation functions and $\Lambda\alpha$ potentials

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC110, 014001 (2024)





- $U_{\Lambda\alpha}(r=0)$: Isle > LY-IV > Chi3 > SG
- $C_{\Lambda\alpha}(q=0)$: Isle < LY-IV < Chi3 < SG
- Central repulsion suppresses correlation at low q

Summary

- **Femtoscopy: novel and useful method to study interactions of exotic hadrons and nuclei**
  - **- unique tool to study charm sector**
- $K^- p$ **correlations**
  - **- precise test for $\Lambda(1405)$ and $\bar{K}N$ interactions**

  Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)
- $\Lambda\alpha$ **correlations**
  - **- hint for repulsive core in $\Lambda\alpha$ interaction**

  A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC110, 014001 (2024)

Hadron2025

Hadron 2025 conference will be held in Osaka

- **March 27-31, 2025**
- **Registration will be open soon**

https://hadron2025.rcnp.osaka-u.ac.jp/

LL formula

Correlation function <—> observables ($a_0, r_e, f(q)$)

R. Lednicky, V.L. Lyuboshits, Yad. Fiz. 35, 1316 (1981)

- **Gaussian (relative) source** $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- R **: source size (gaussian width is** $\sqrt{2}R$**)**

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- Gaussian (relative) source $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- R : source size (gaussian width is $\sqrt{2}R$)
- s-wave interaction only
- zero-range interaction : $R \gg R_{\rm int}$ (use asymptotic w.f.)

$$C(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR)$$

- $F_i(x)$: known functions, $f(q)$: s-wave scattering amplitude

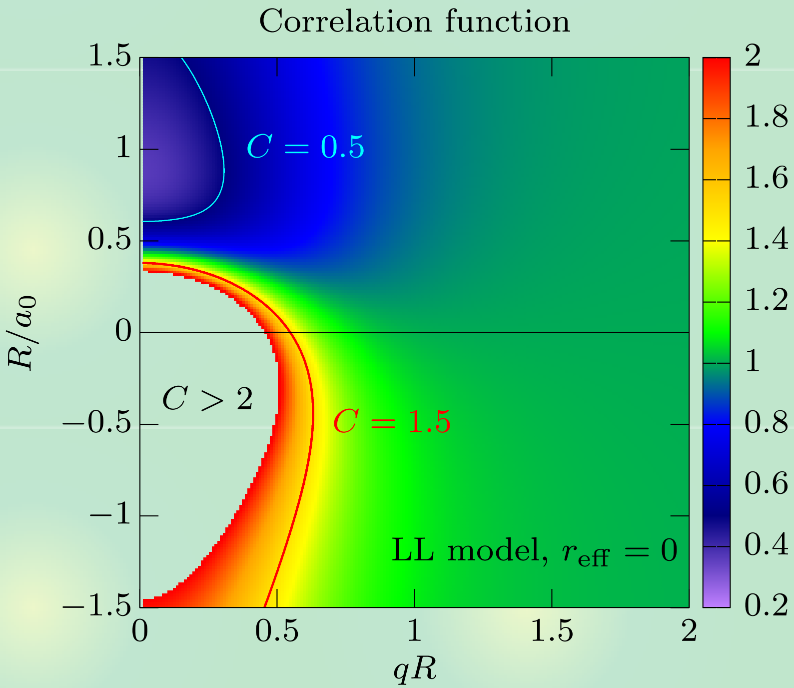
S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)

$$f(q) = \frac{1}{q\cot\delta - iq} \simeq \frac{1}{-\frac{1}{a_0} + \frac{r_e}{2}q^2 - iq}$$

LL formula and correlations

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)



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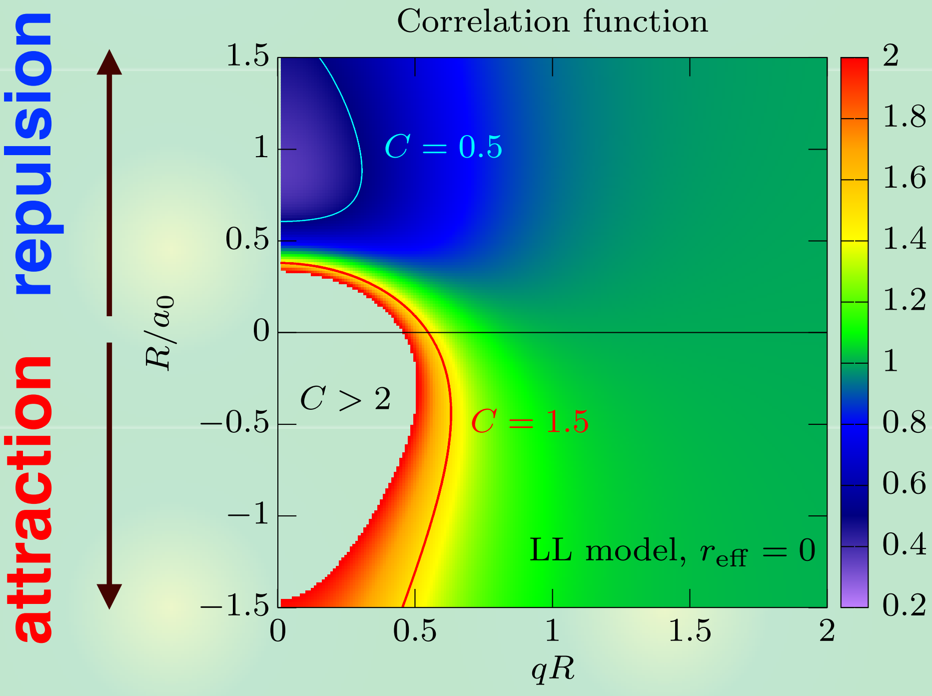
- fixed $R > 0$
- **repulsion**: $R/a_0 > 0$, **attraction**: $R/a_0 < 0$



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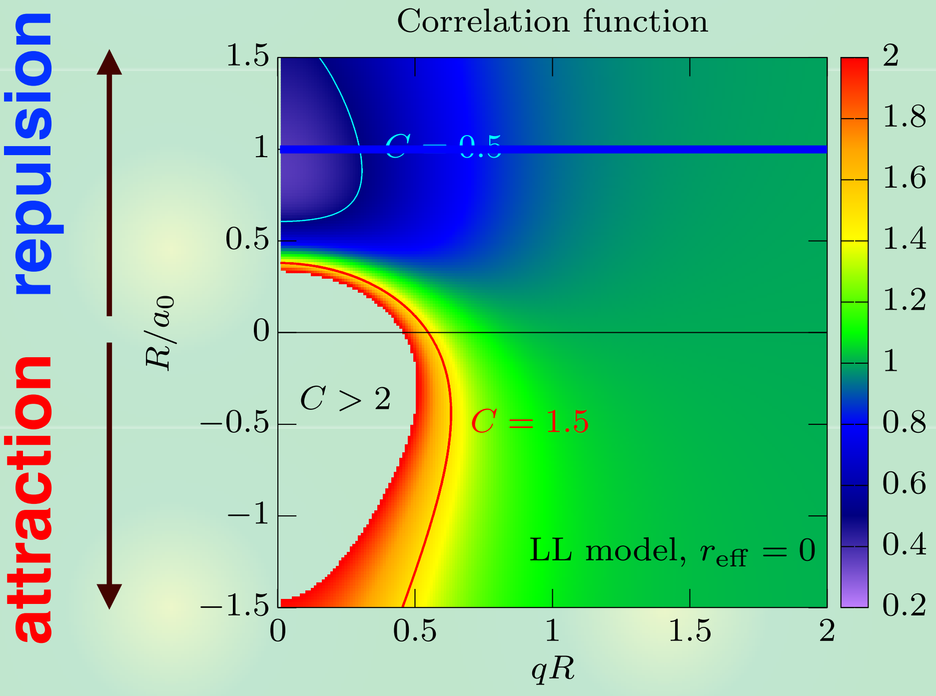

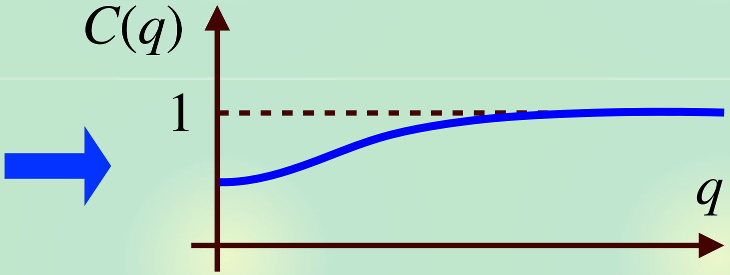

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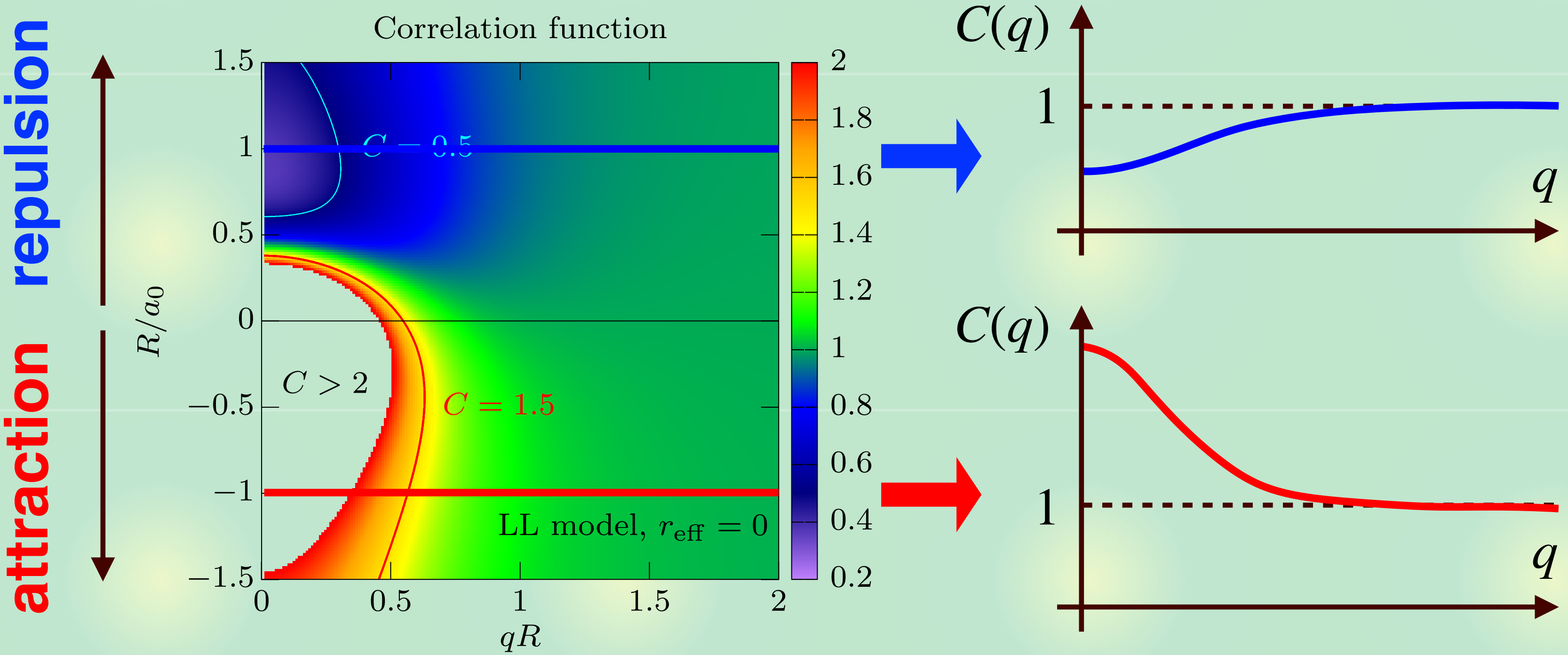
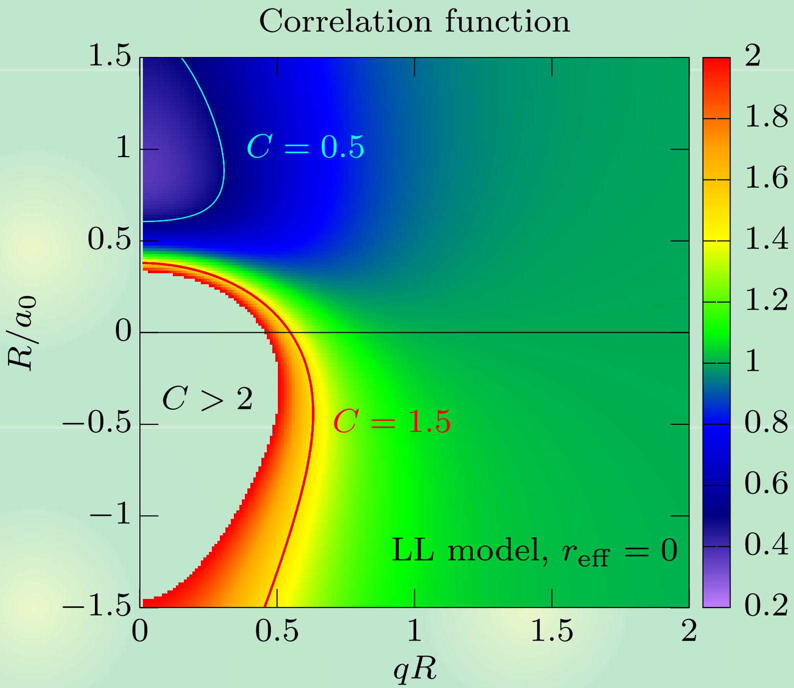


Consistent with KP formula

Shallow bound state case

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)



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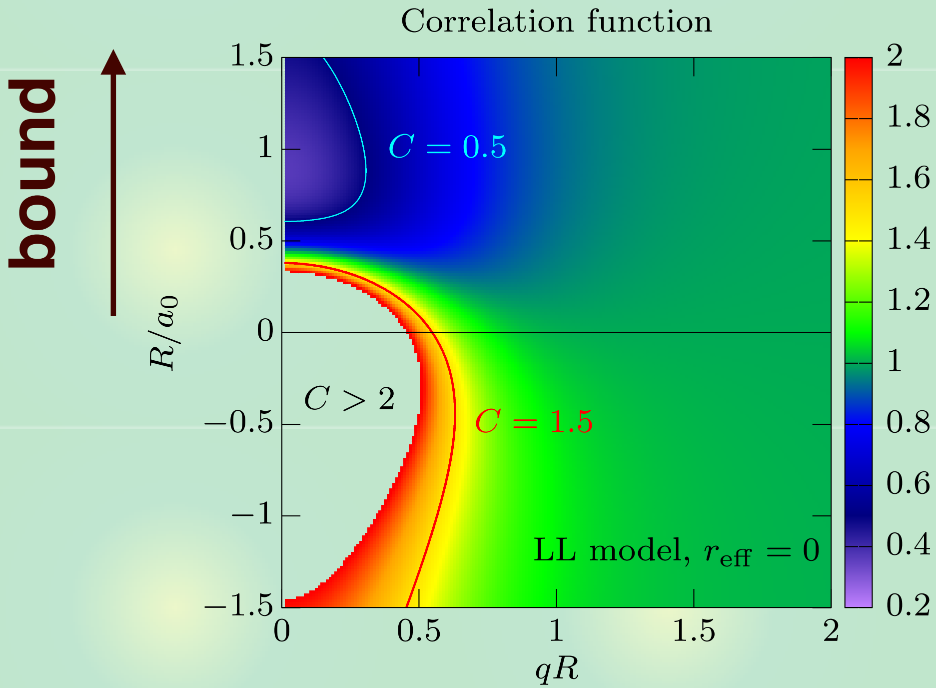
- large source: $R/a_0 \sim 1$, small source: $R/a_0 \ll 1$



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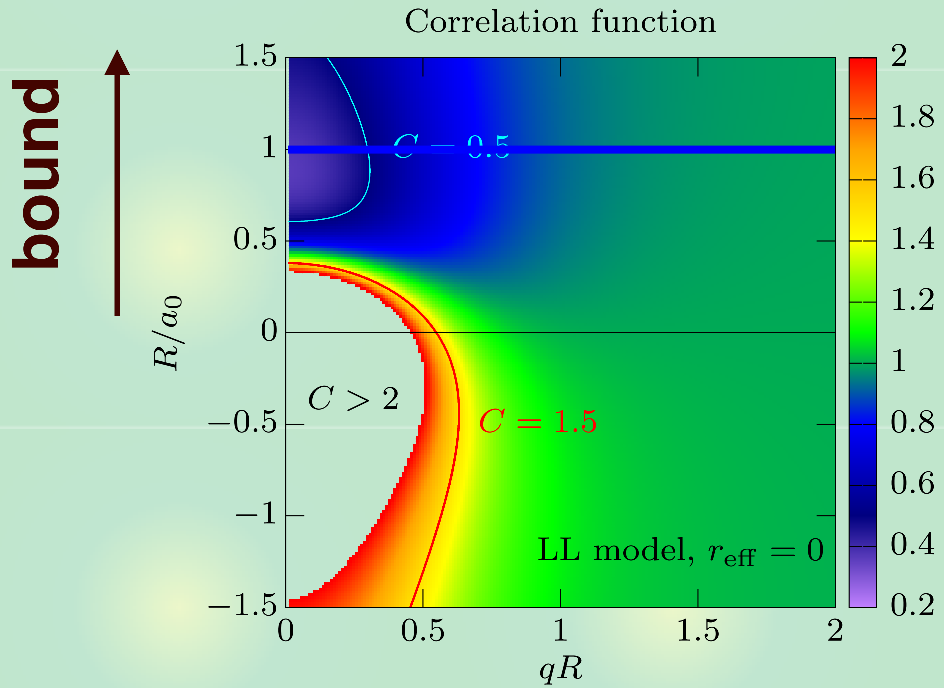

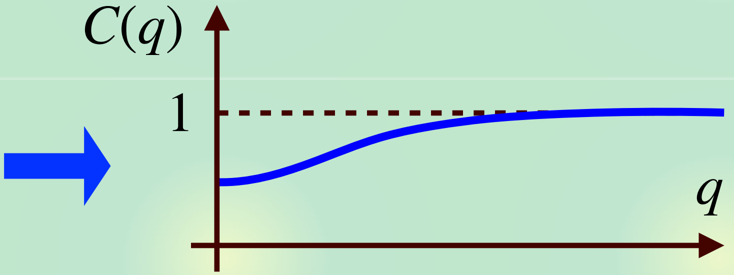

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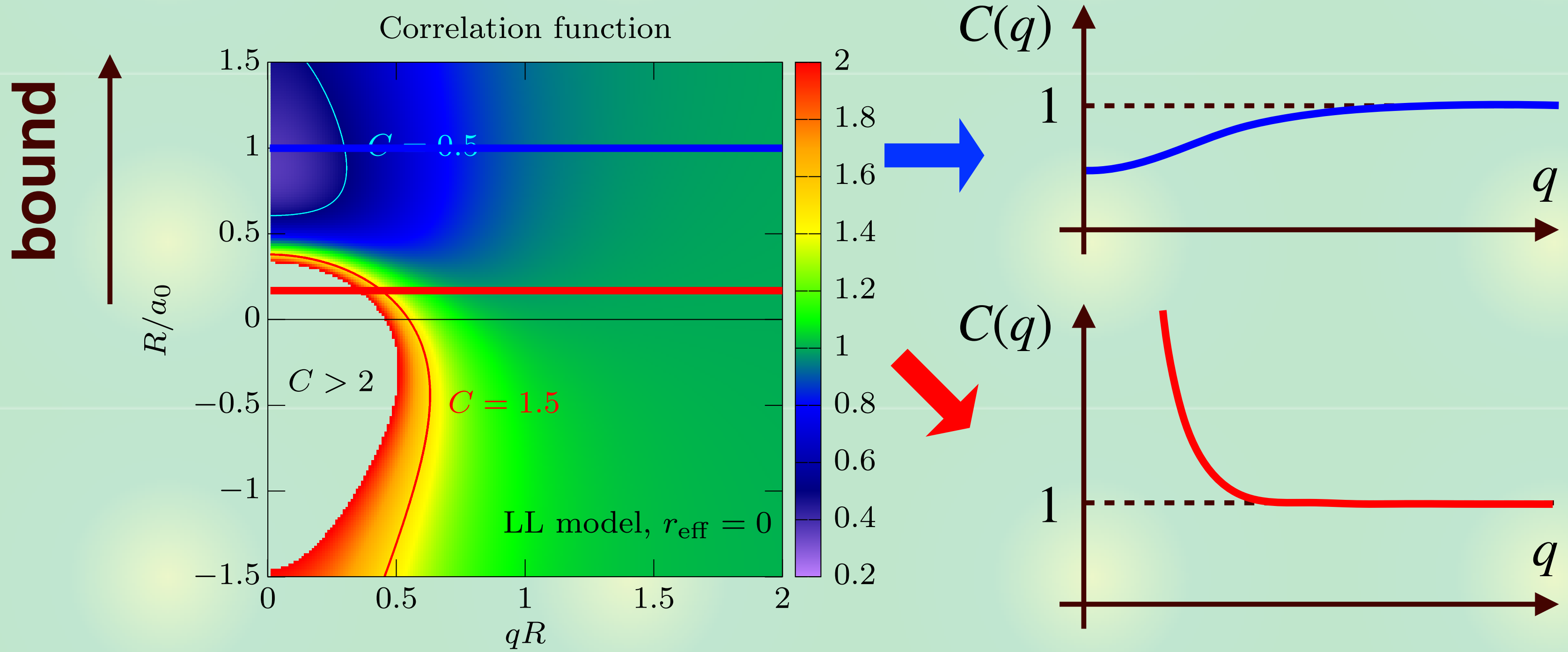
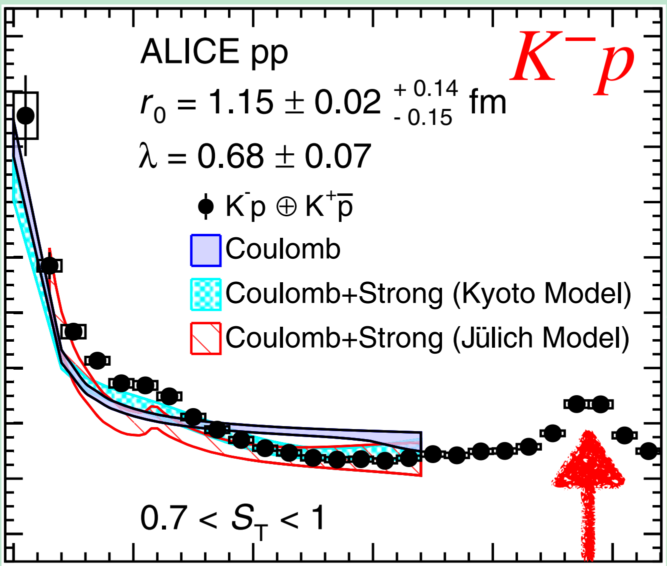


—> qualitative difference in large/small source

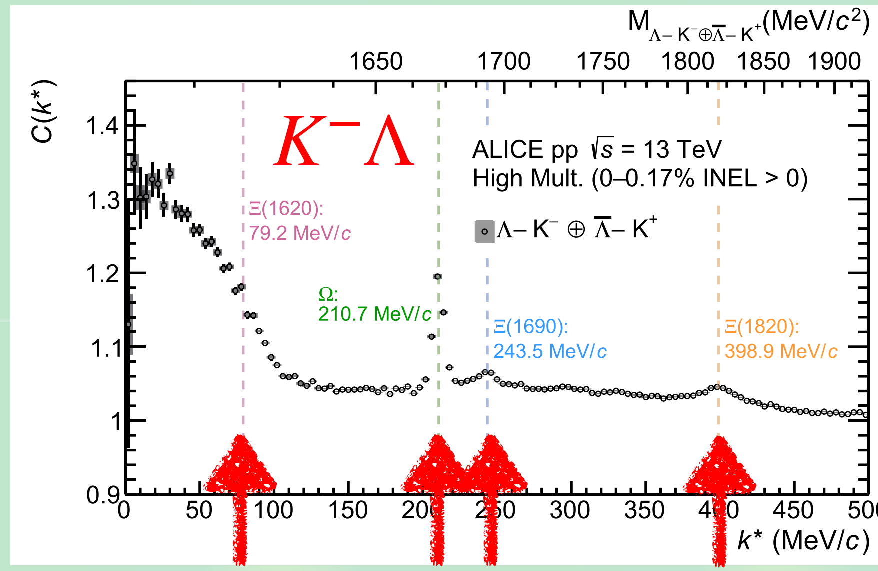
Higher partial waves and resonance contributions

Resonances in $\ell = 0$ and in $\ell \neq 0$ are seen

- Simple Breit-Wigner function has been used



$\Lambda(1520)$: d-wave



$\Xi(1620)$, $\Xi(1690)$: s-wave
Ω : p-wave (weak decay)
$\Xi(1820)$: d-wave

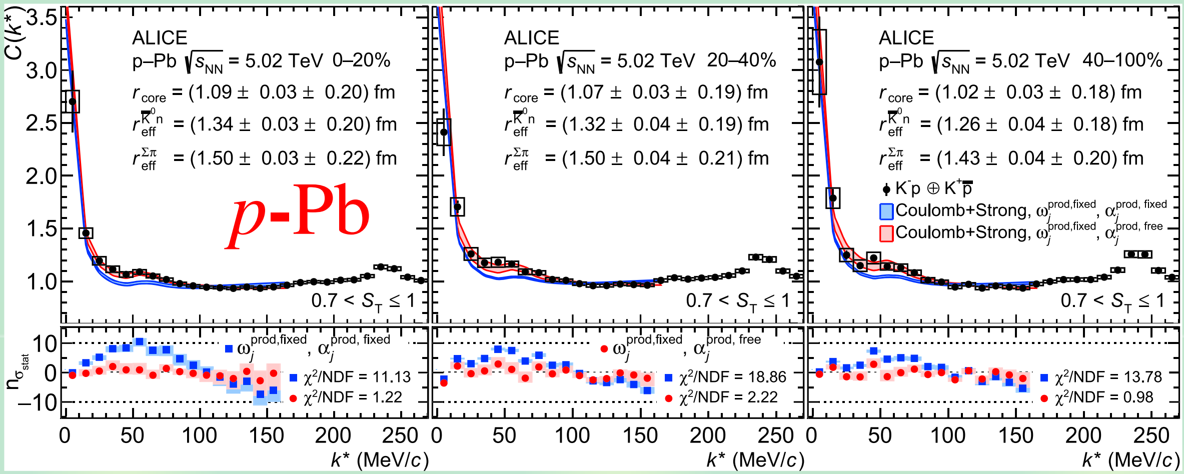
Questions

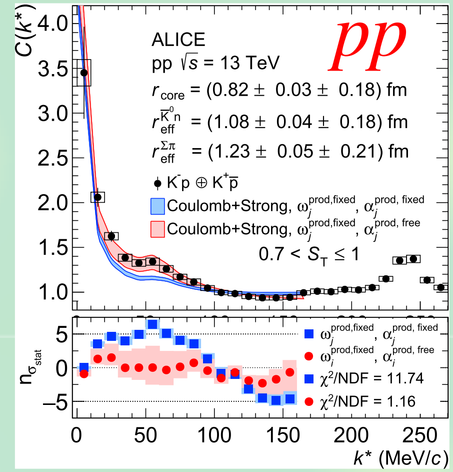
- Contribution from higher partial waves?
- Is Breit-Wigner function fine for resonance?

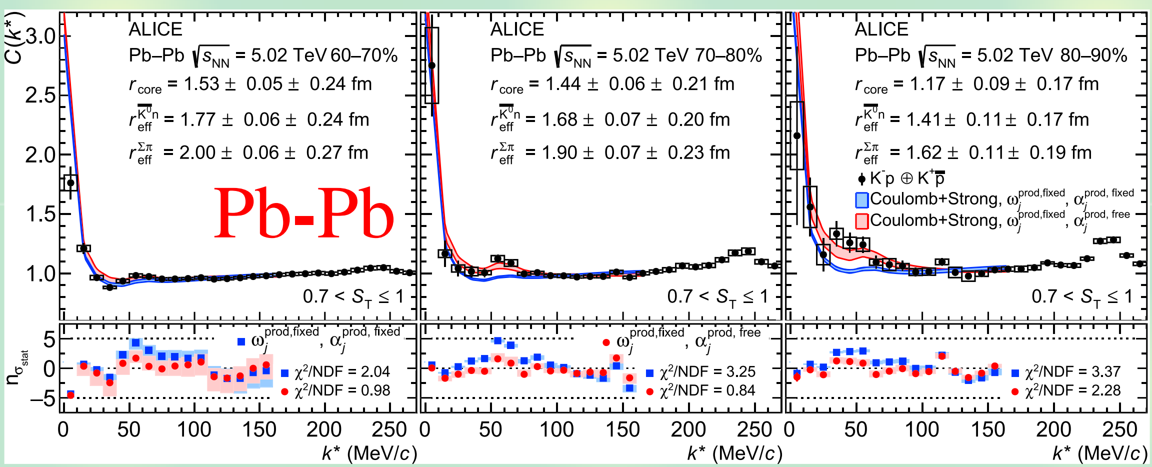
Systematic study of source size dependence

Correlations in pp, p-Pb, Pb-Pb by Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential

ALICE collaboration, EPJC 83, 340 (2023)







Correlations with different source size