Three-hadron scattering from QCD

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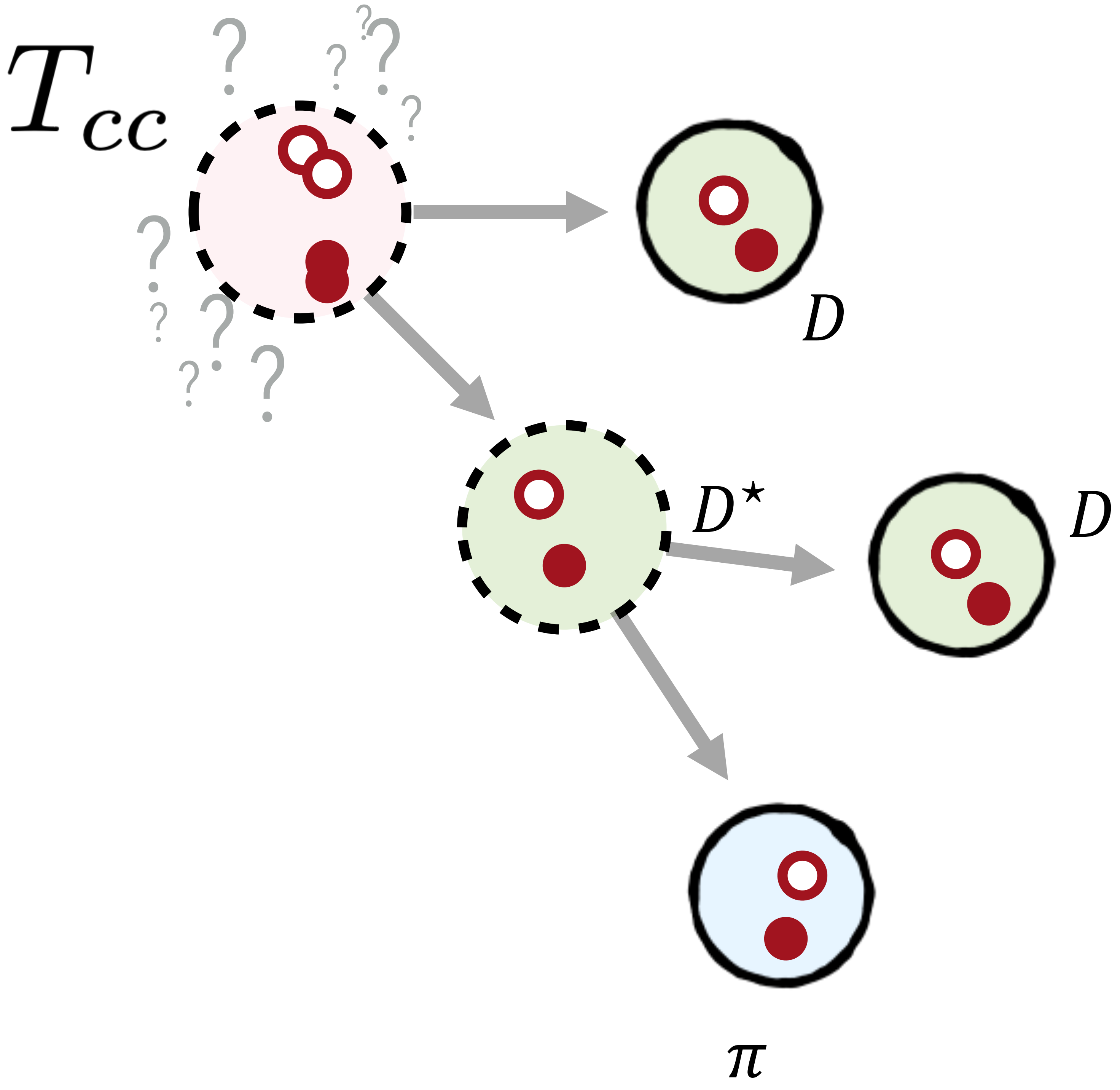
hadspec

ExoHad
Exotic Hadrons Topical Collaboration

why three-body systems?

- hadron spectroscopy



why three-body systems?

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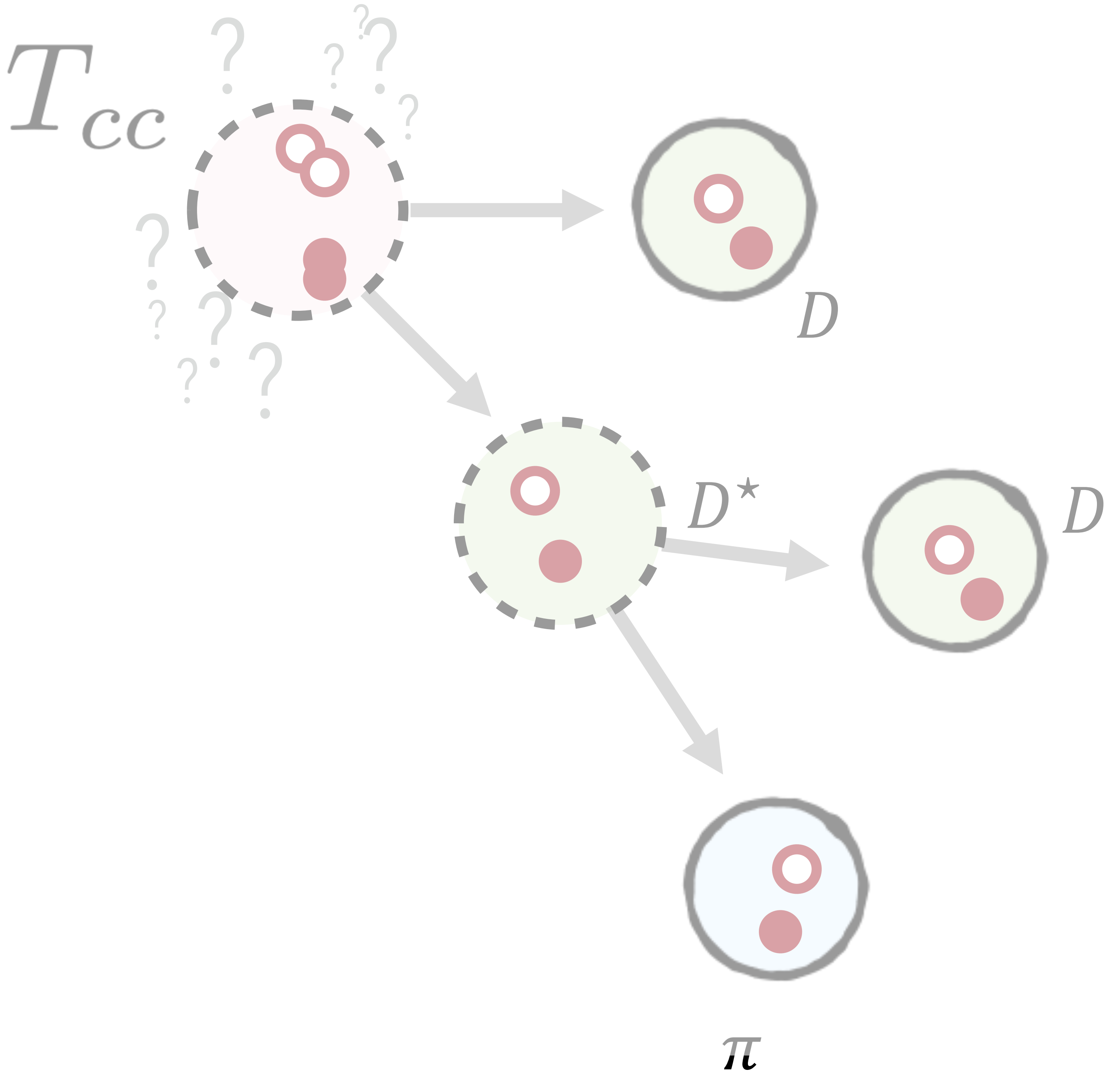
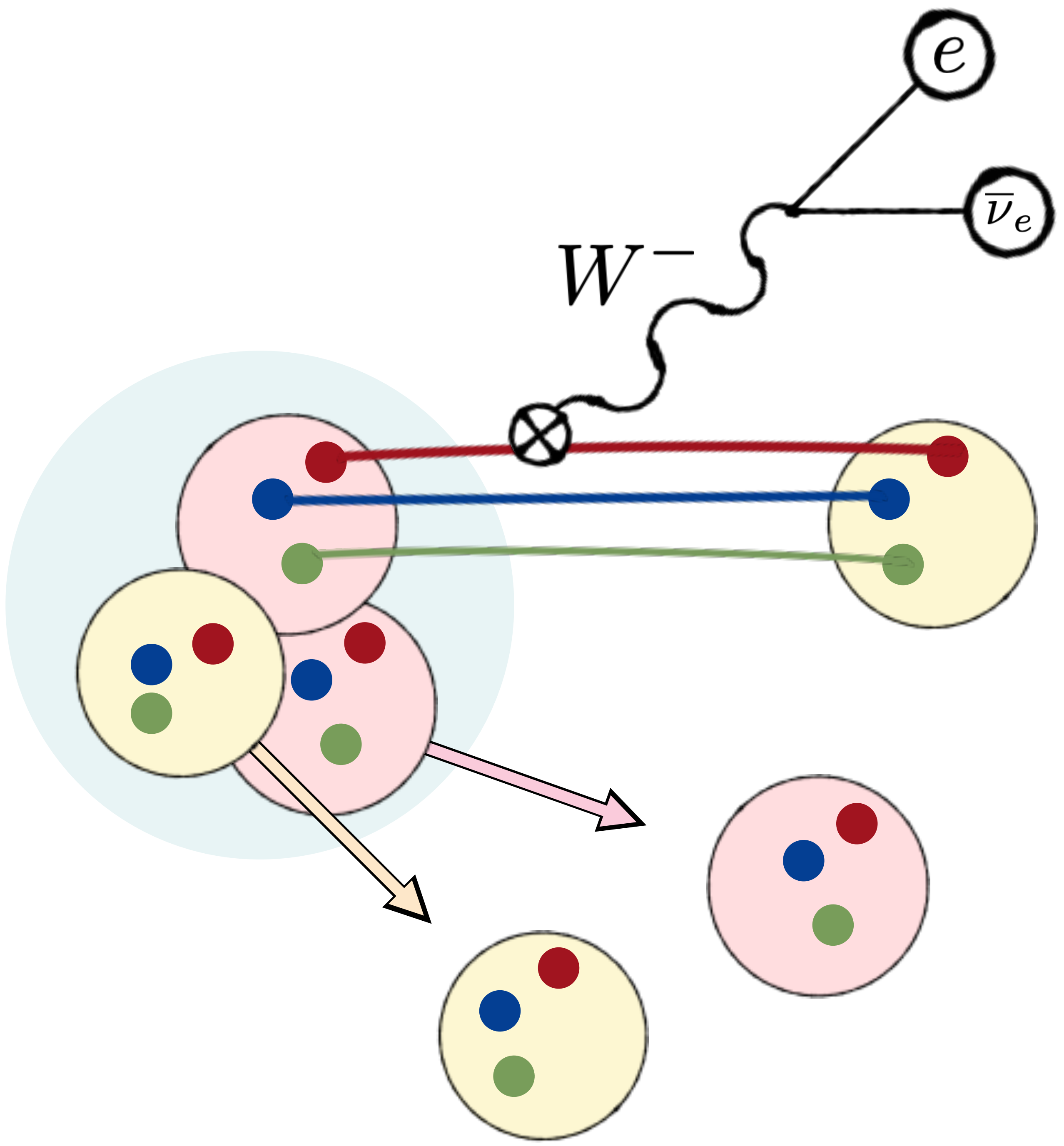
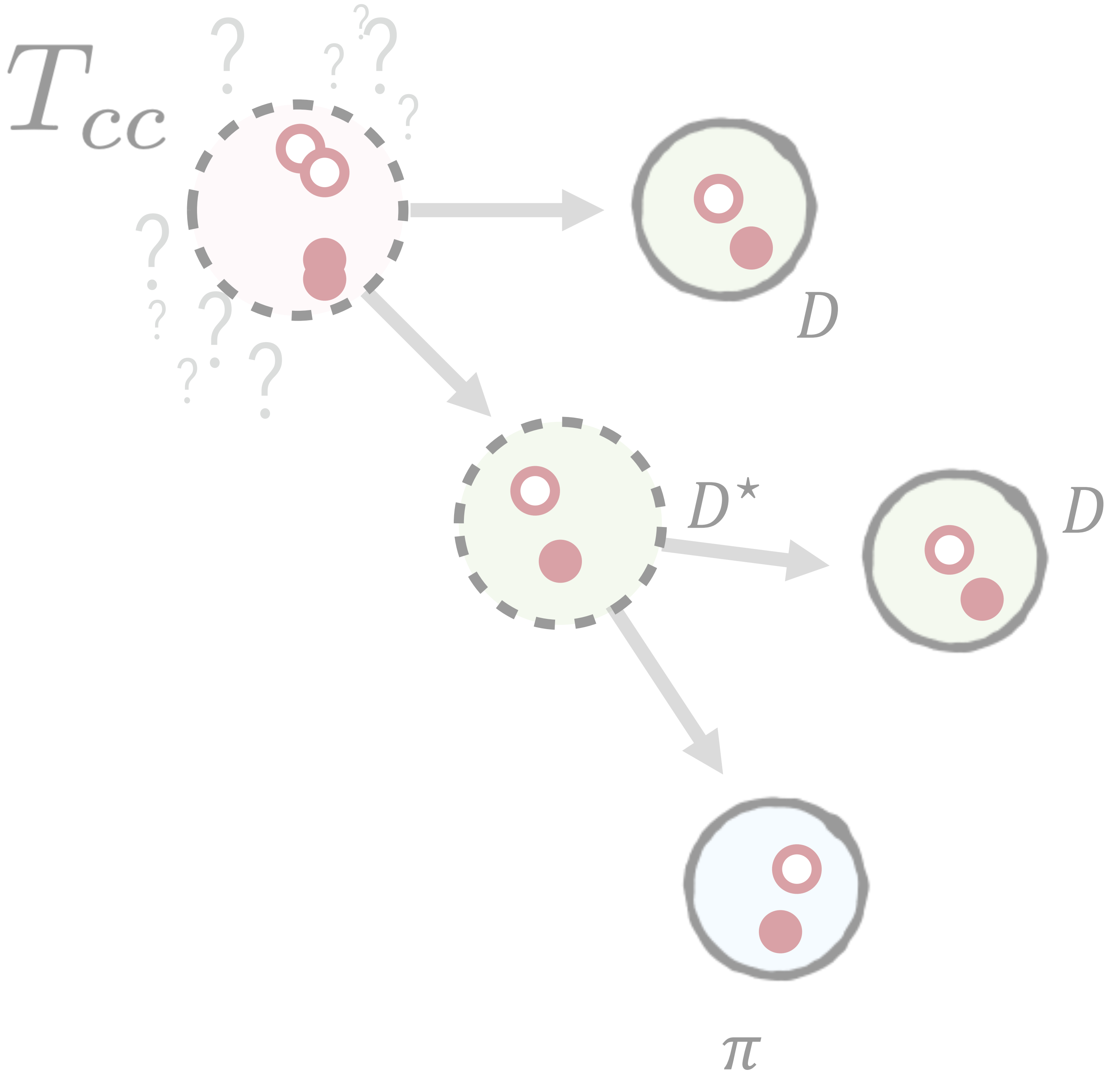


- nuclear structure / neutrino physics



why three-body systems?

❑ hadron spectroscopy



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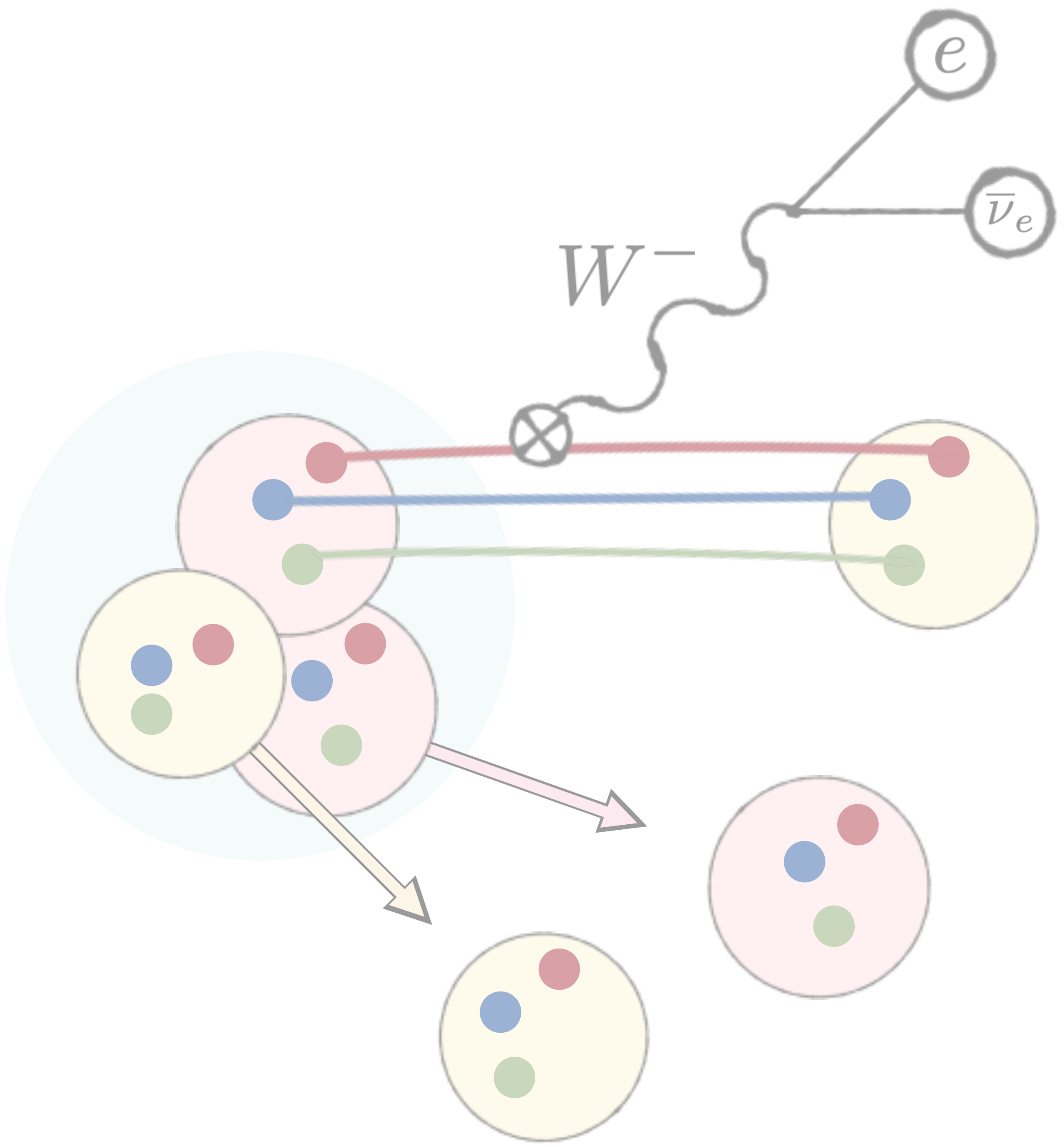
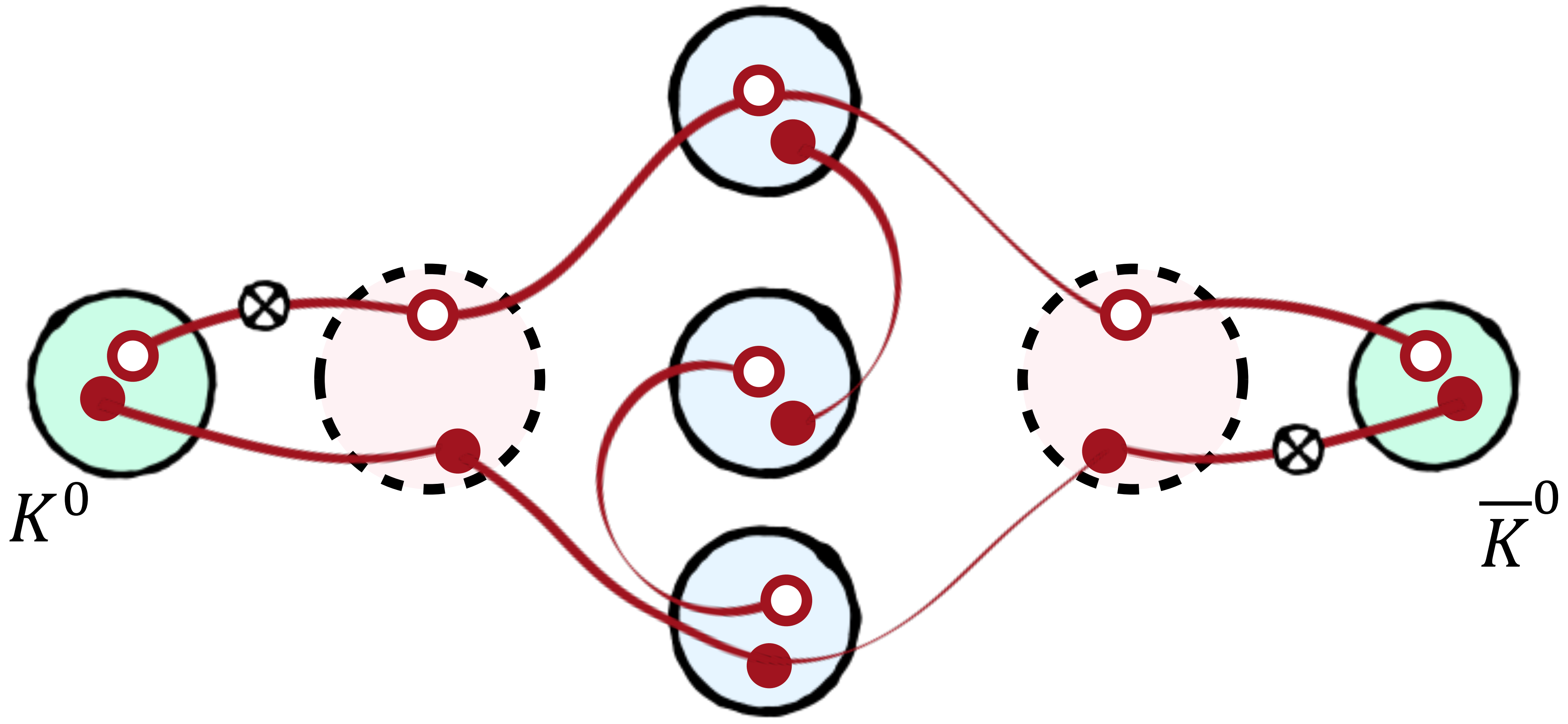
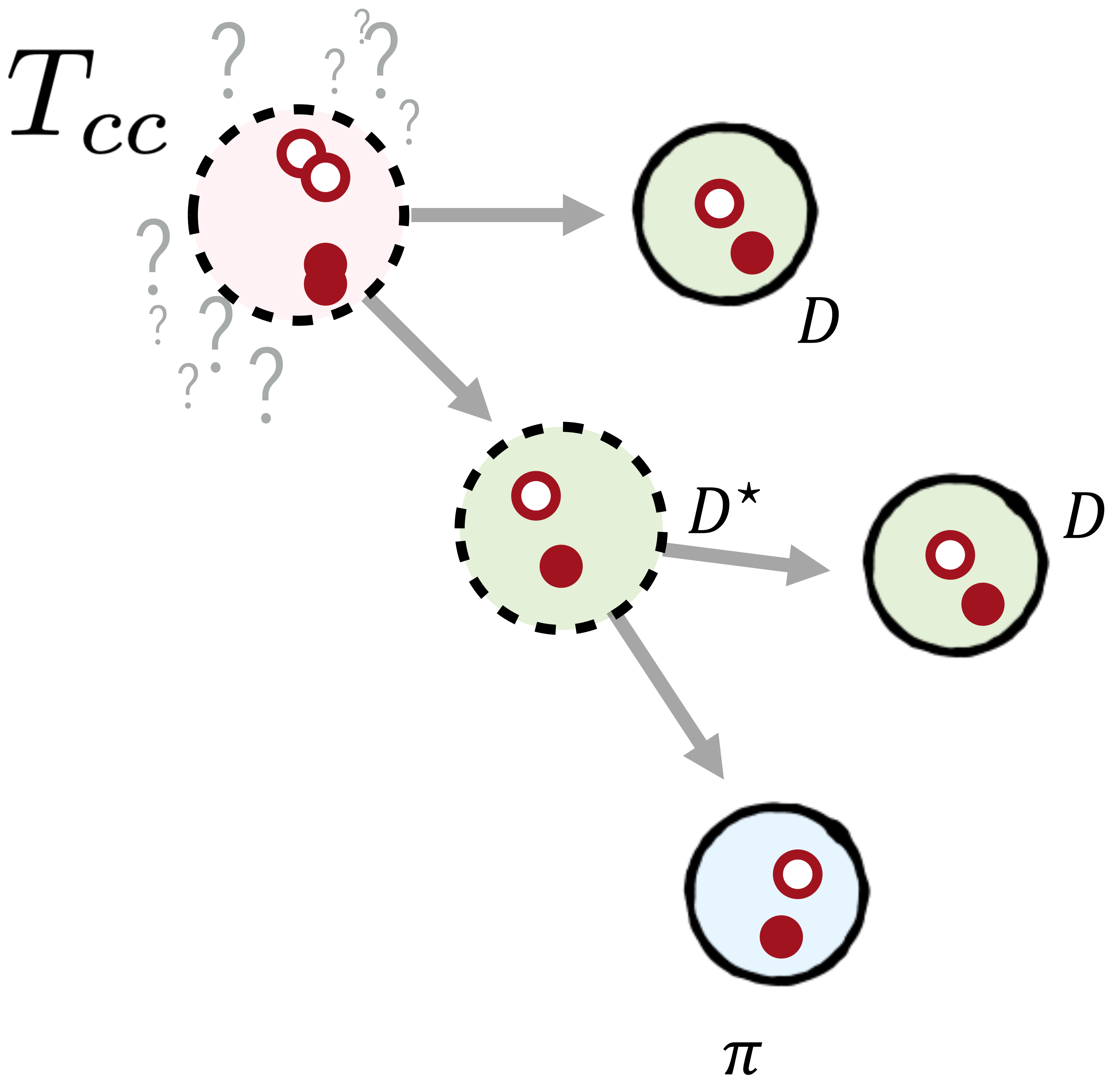
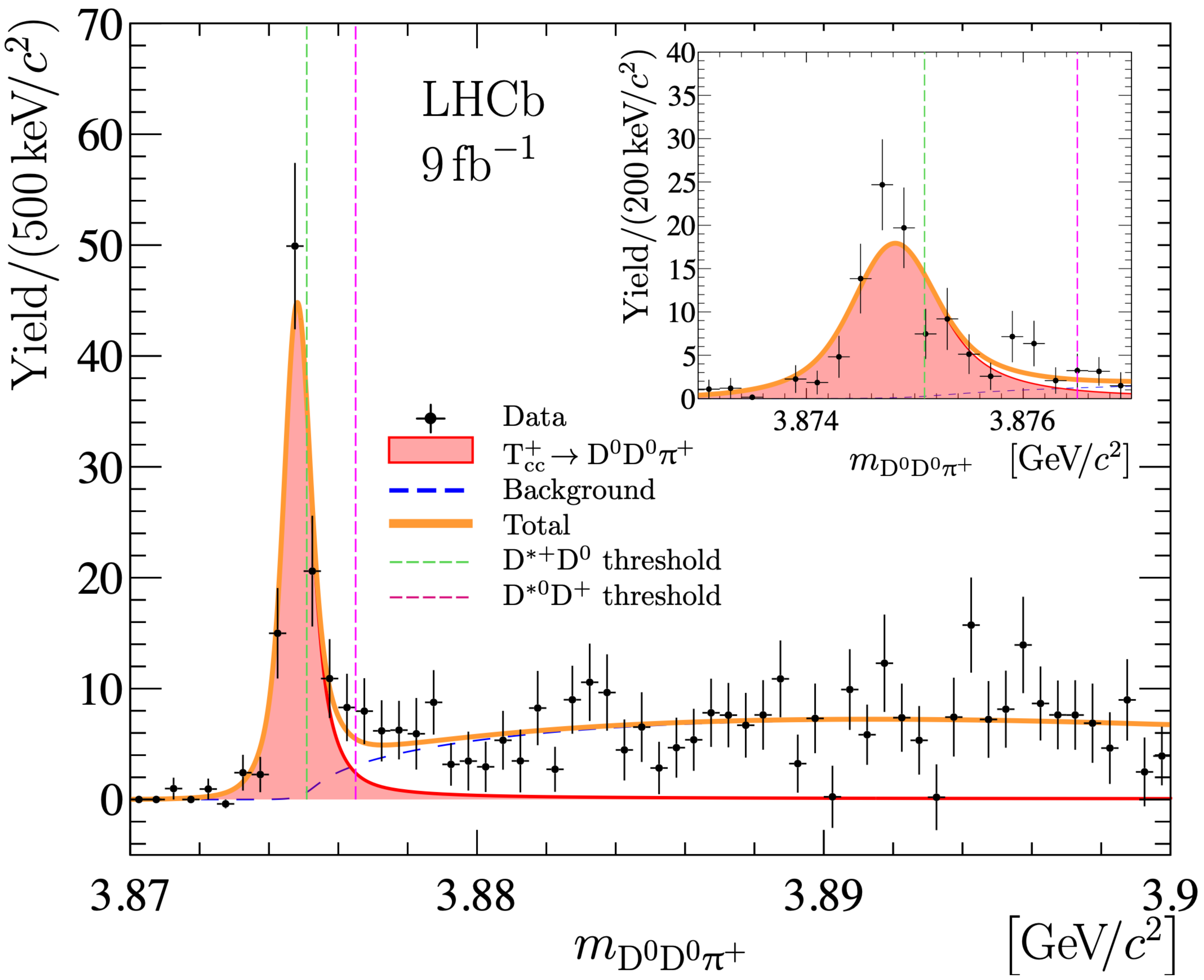


❑ precision tests



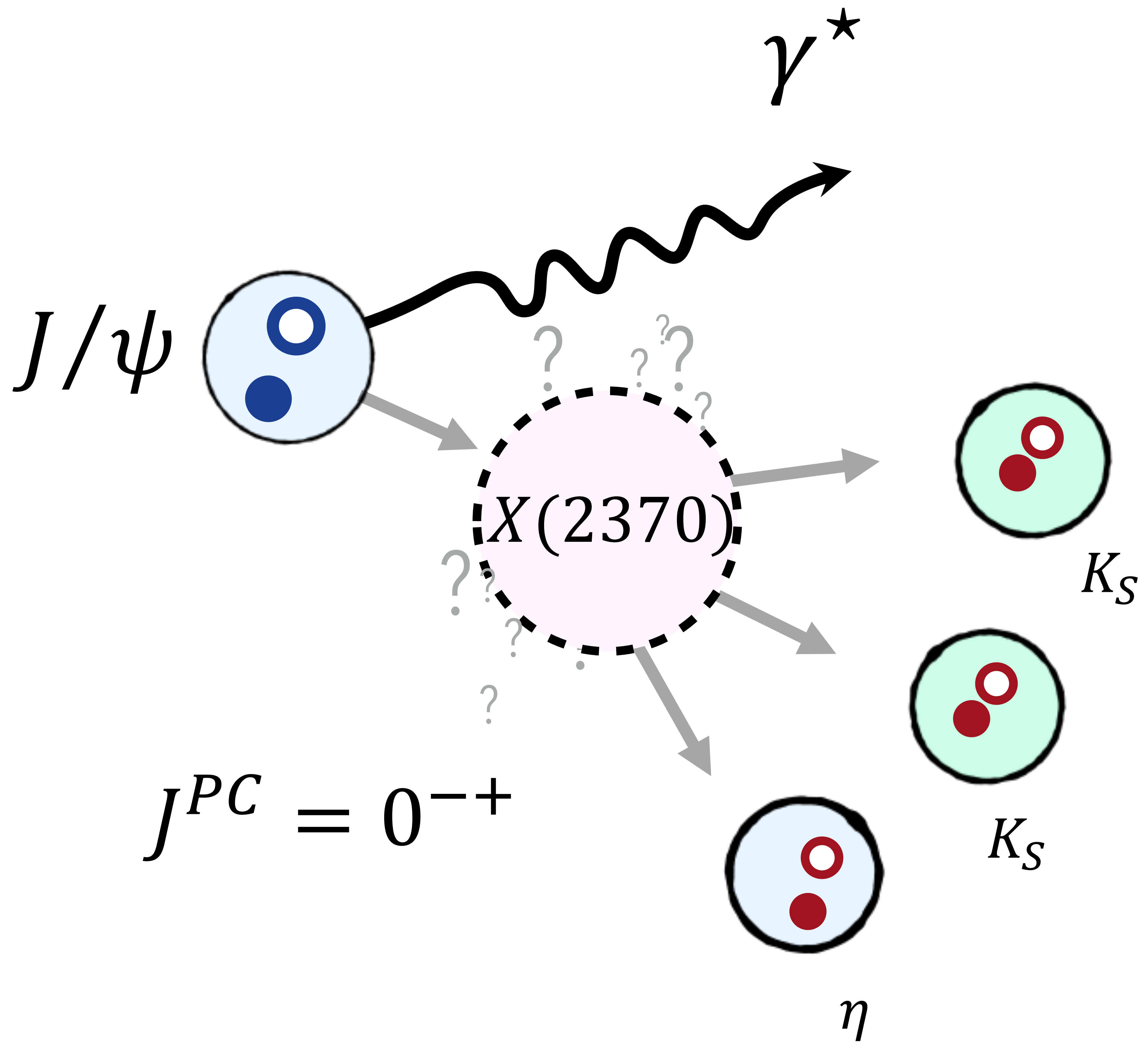
Tetraquarks?

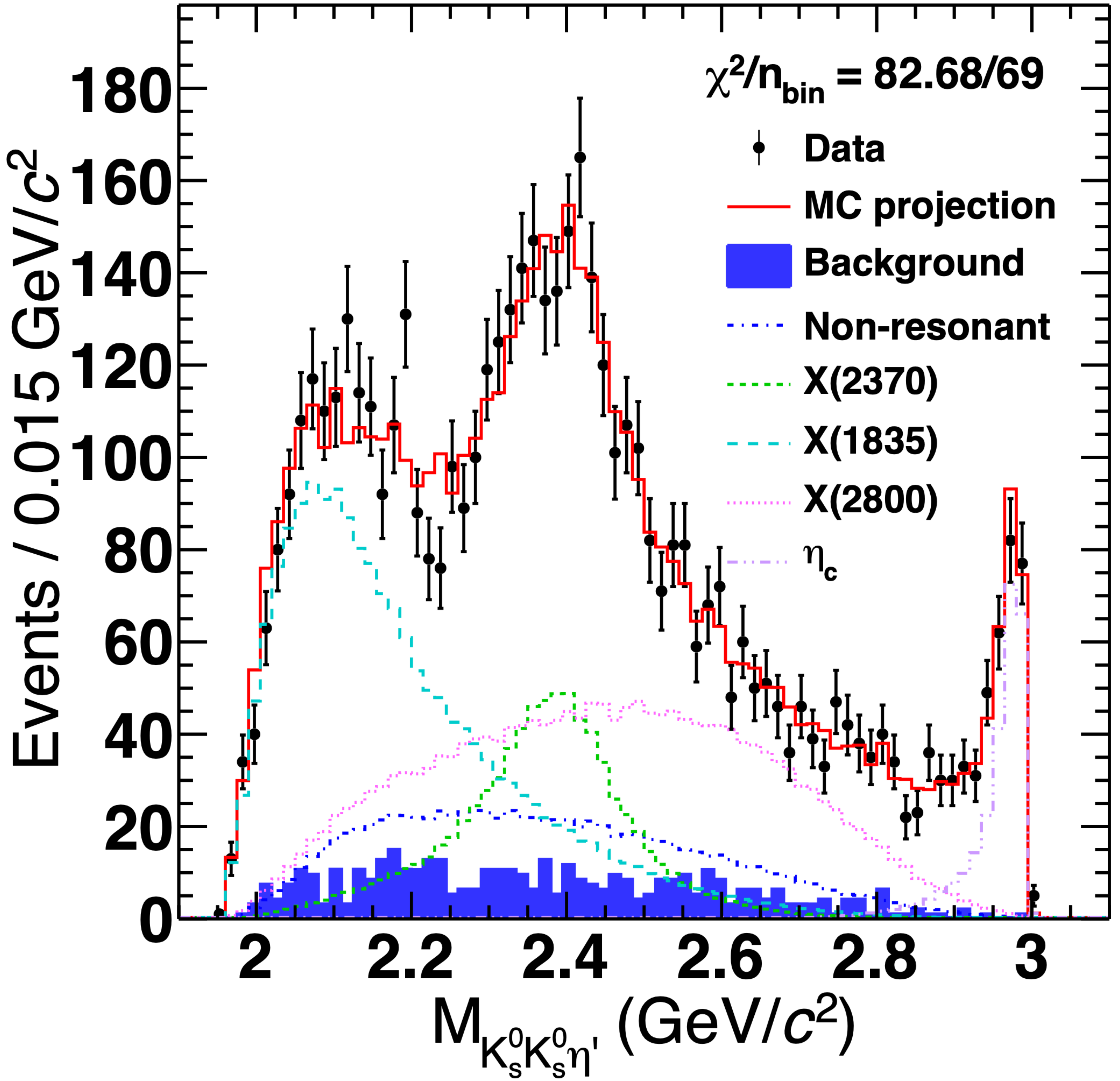




(2021)

Glueballs?





BESIII

The Roper?

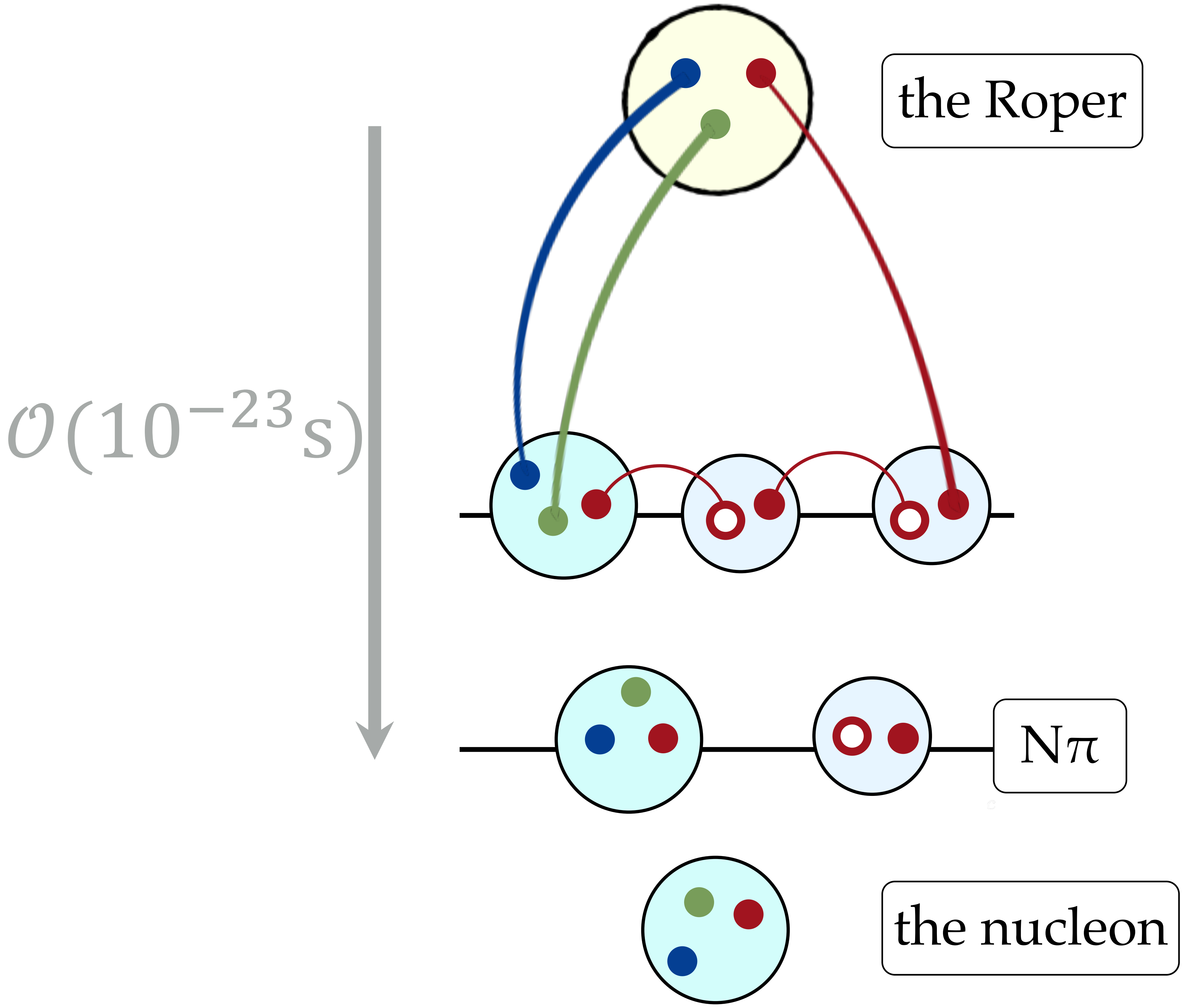
- Not so simple…

$N(1440)\ 1/2^+$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗∗

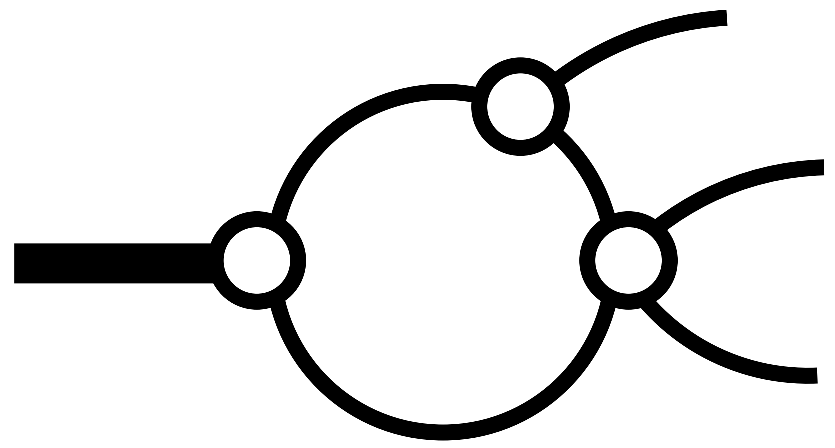
Older and obsolete values are listed and referenced in the 2014 edition, Chinese Physics **C38** 070001 (2014).

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	55–75 %
Γ_2	$N\eta$	<1 %
Γ_3	$N\pi\pi$	17–50 %
Γ_4	$\Delta(1232)\pi$, P-wave	6–27 %
Γ_5	$N\sigma$	11–23 %
Γ_6	$p\gamma$, helicity=1/2	0.035–0.048 %
Γ_7	$n\gamma$, helicity=1/2	0.02–0.04 %



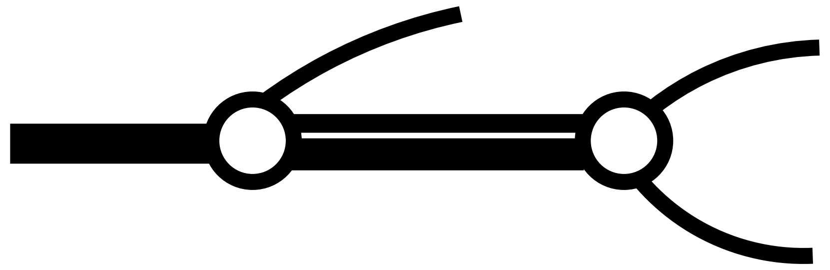
Key questions to answer

- [ ] Which enhancements in cross sections are actual resonances?



$\sim \log(s - s_t)$

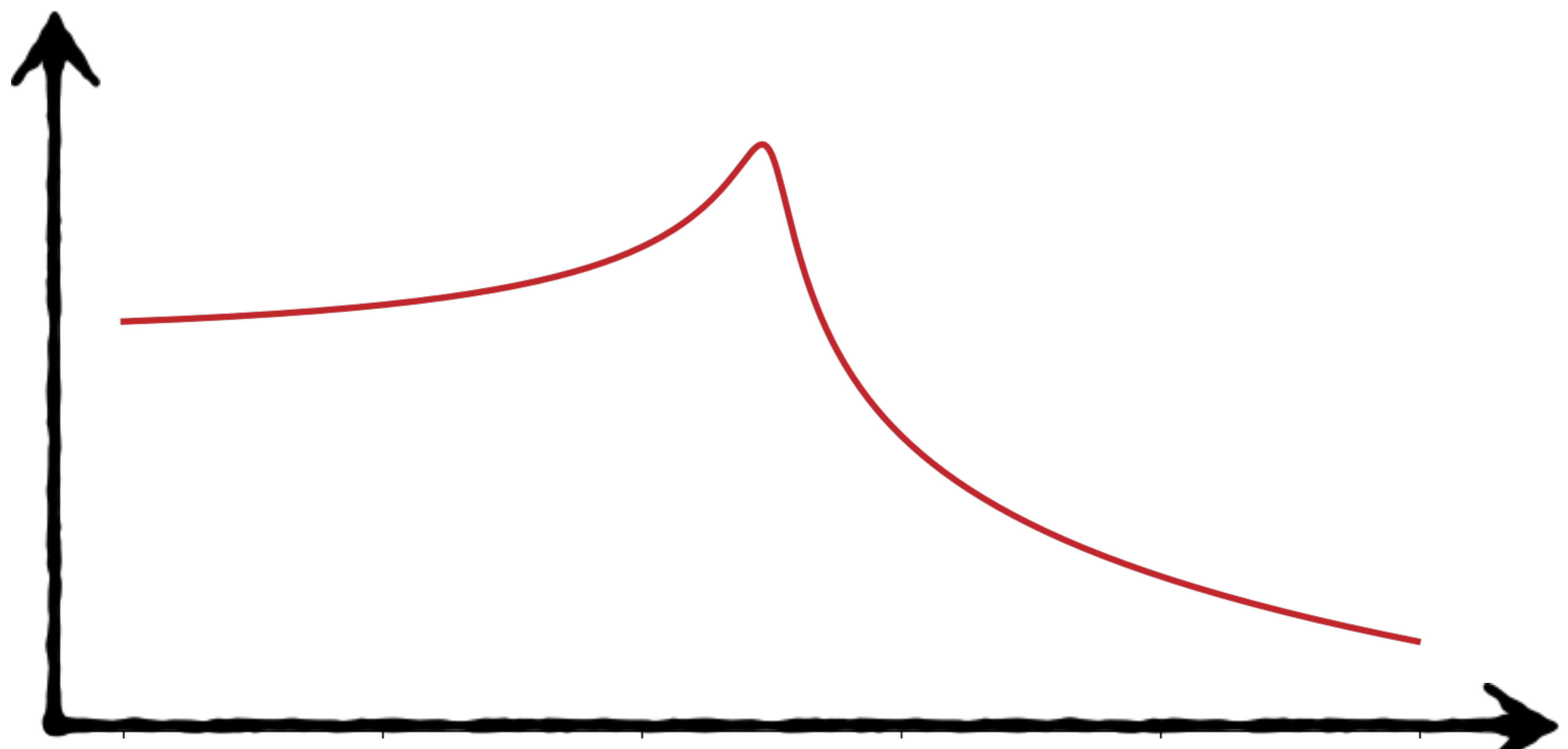
vs.

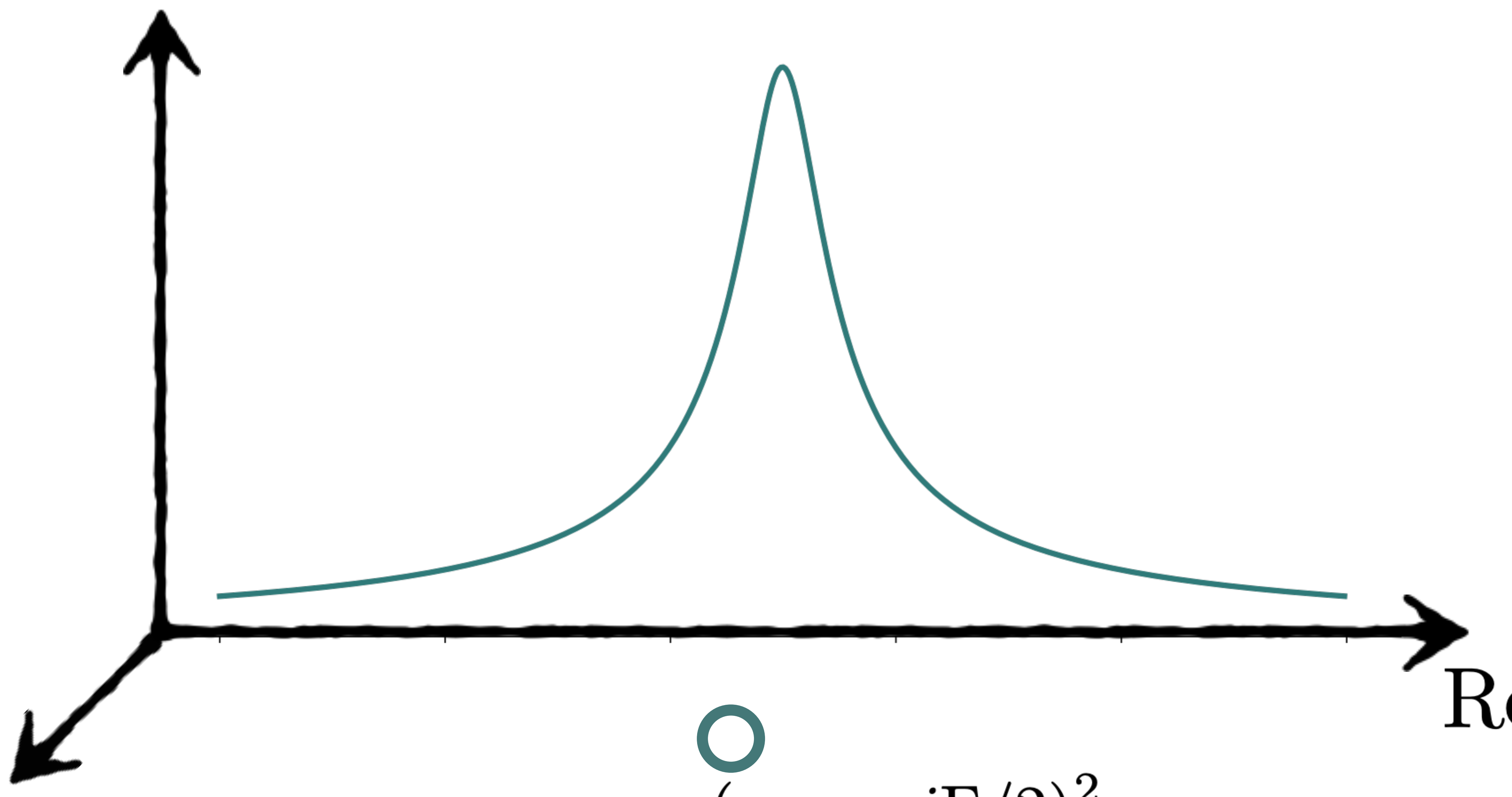


$$\sim \frac{1}{s - (m_R - \frac{i}{2}\Gamma)^2}$$

"just a poser"

"the real deal"





$s = E_{\rm cm}^2$

Key questions to answer

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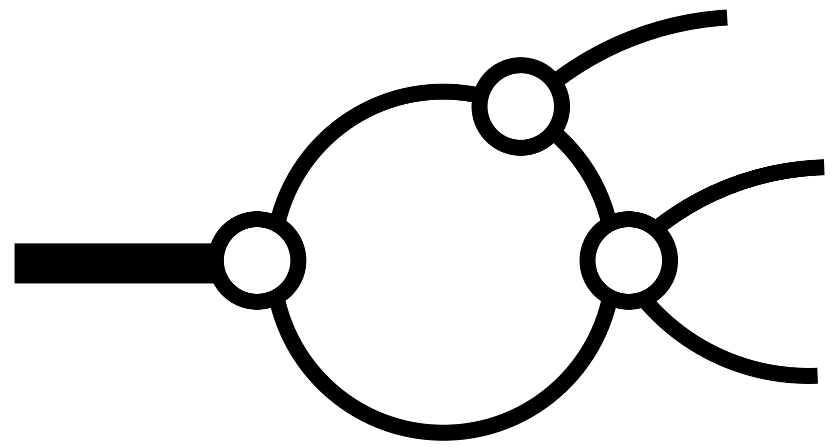

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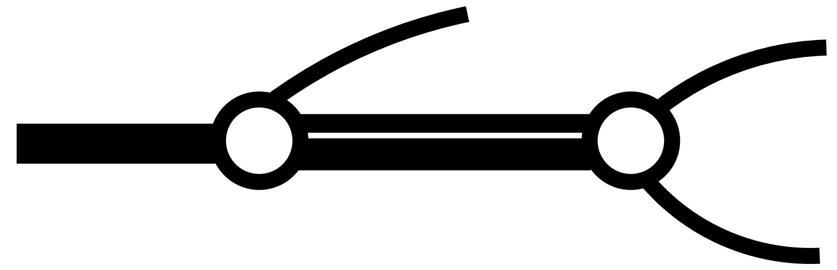

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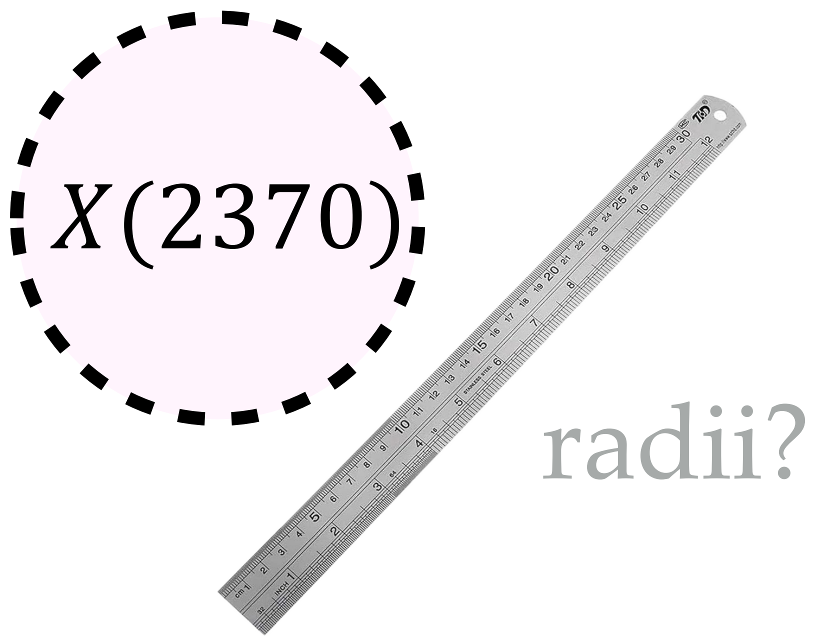
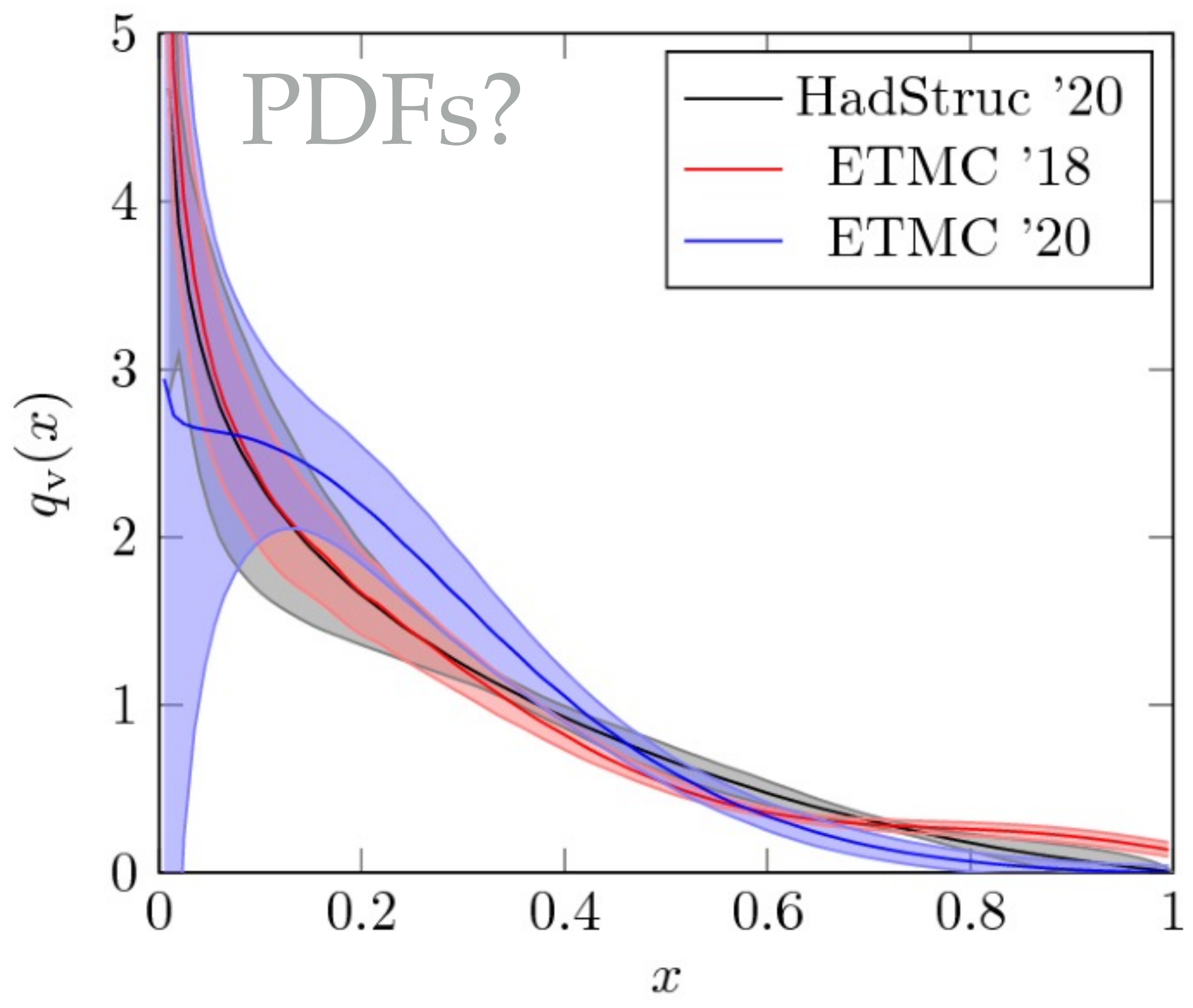
- ☐ If real, what is its inner structure?





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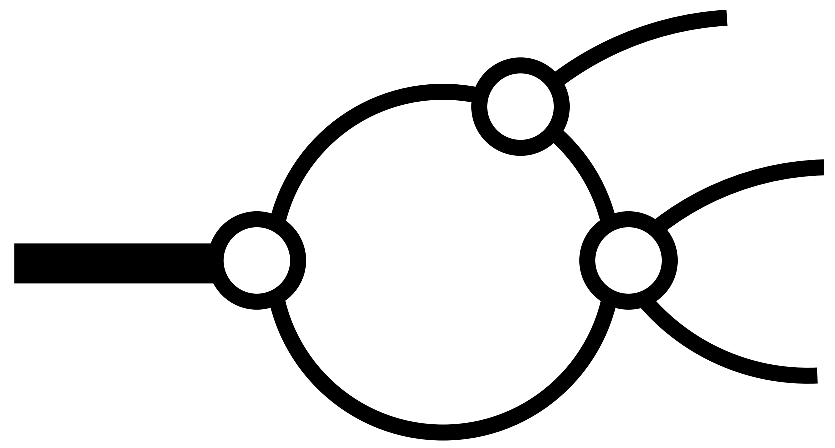

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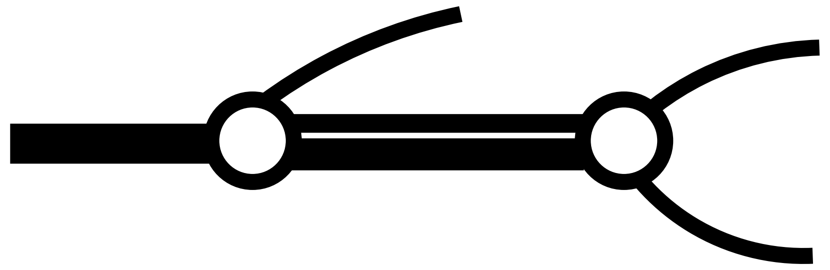

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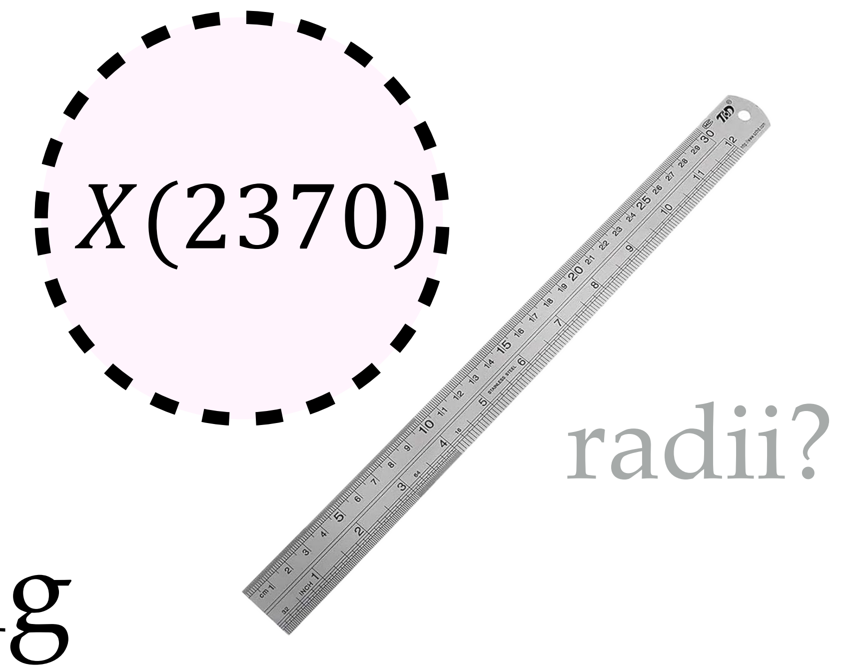
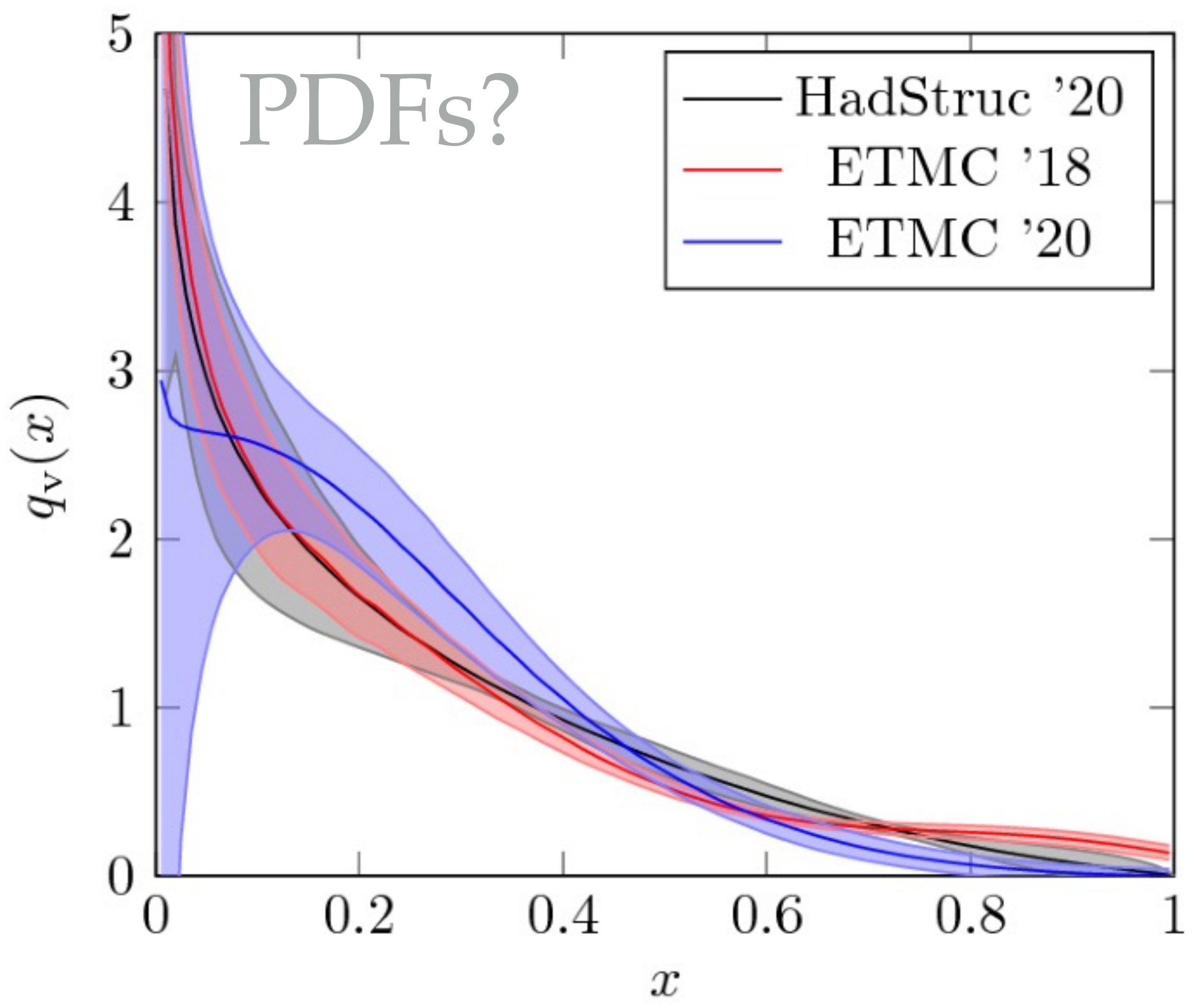
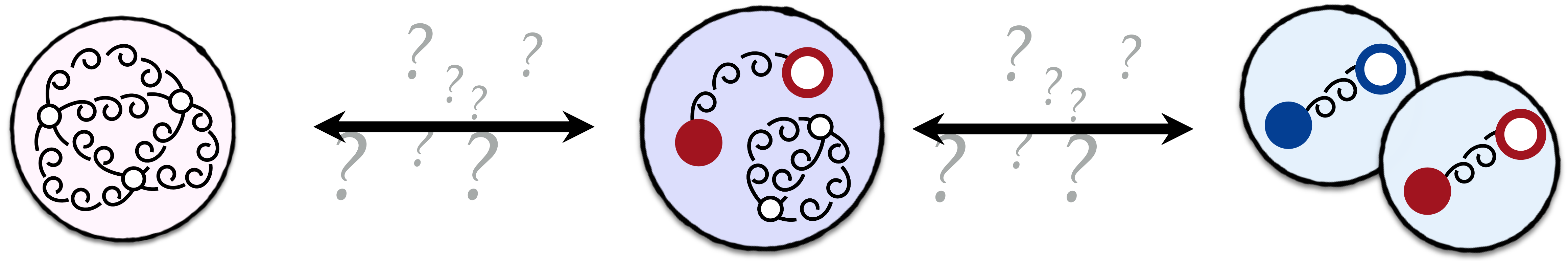




- Given structural information, can we say anything about the nature?



- Can we deduce general principles from the QCD spectrum?

Overarching goal

non-perturbatively constrain two- and three-hadron scattering amplitudes directly from the standard model (including electroweak & BSM probes)

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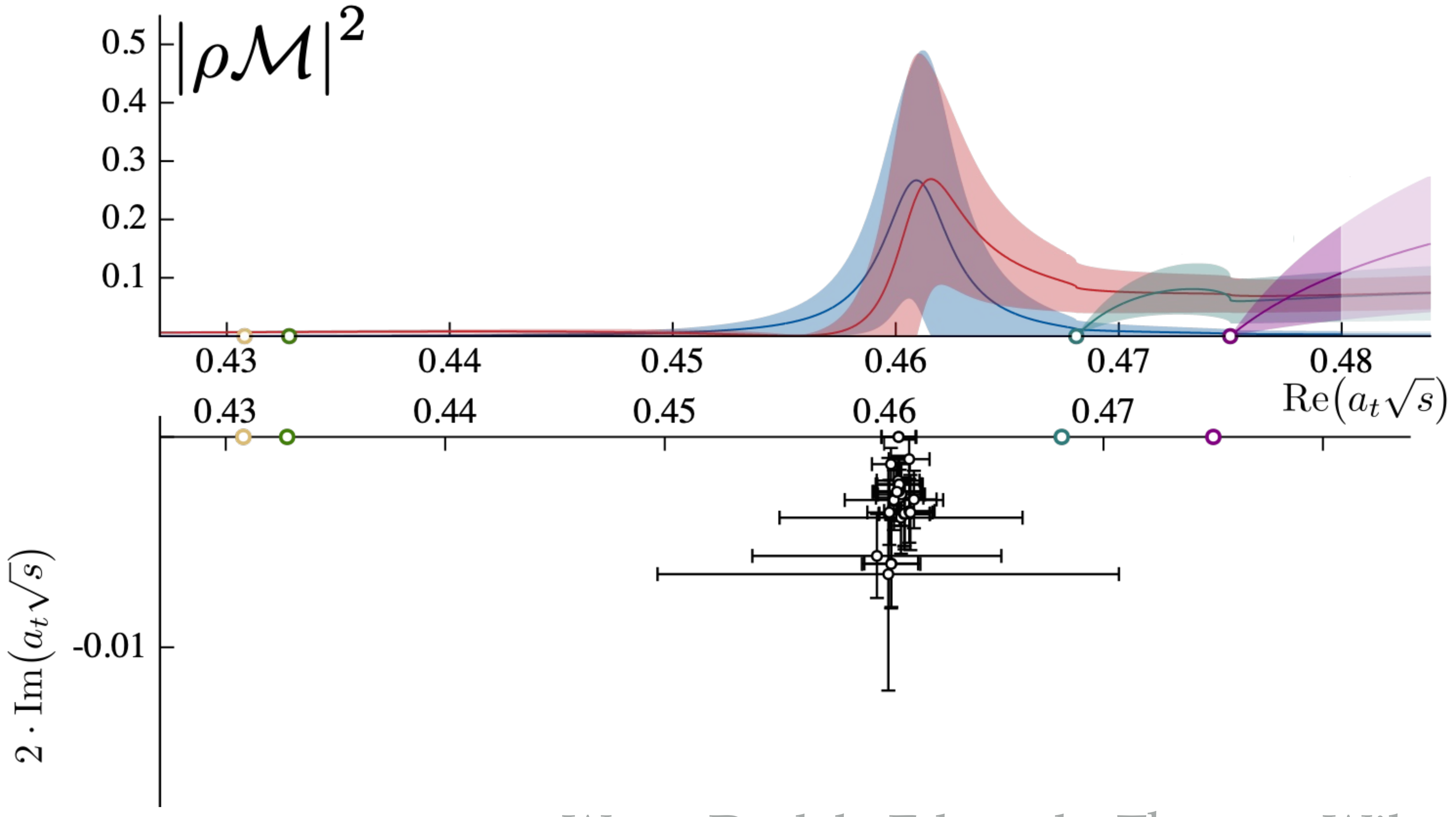
non-perturbatively constrain two- and three-hadron scattering amplitudes directly from the standard model (including electroweak & BSM probes)

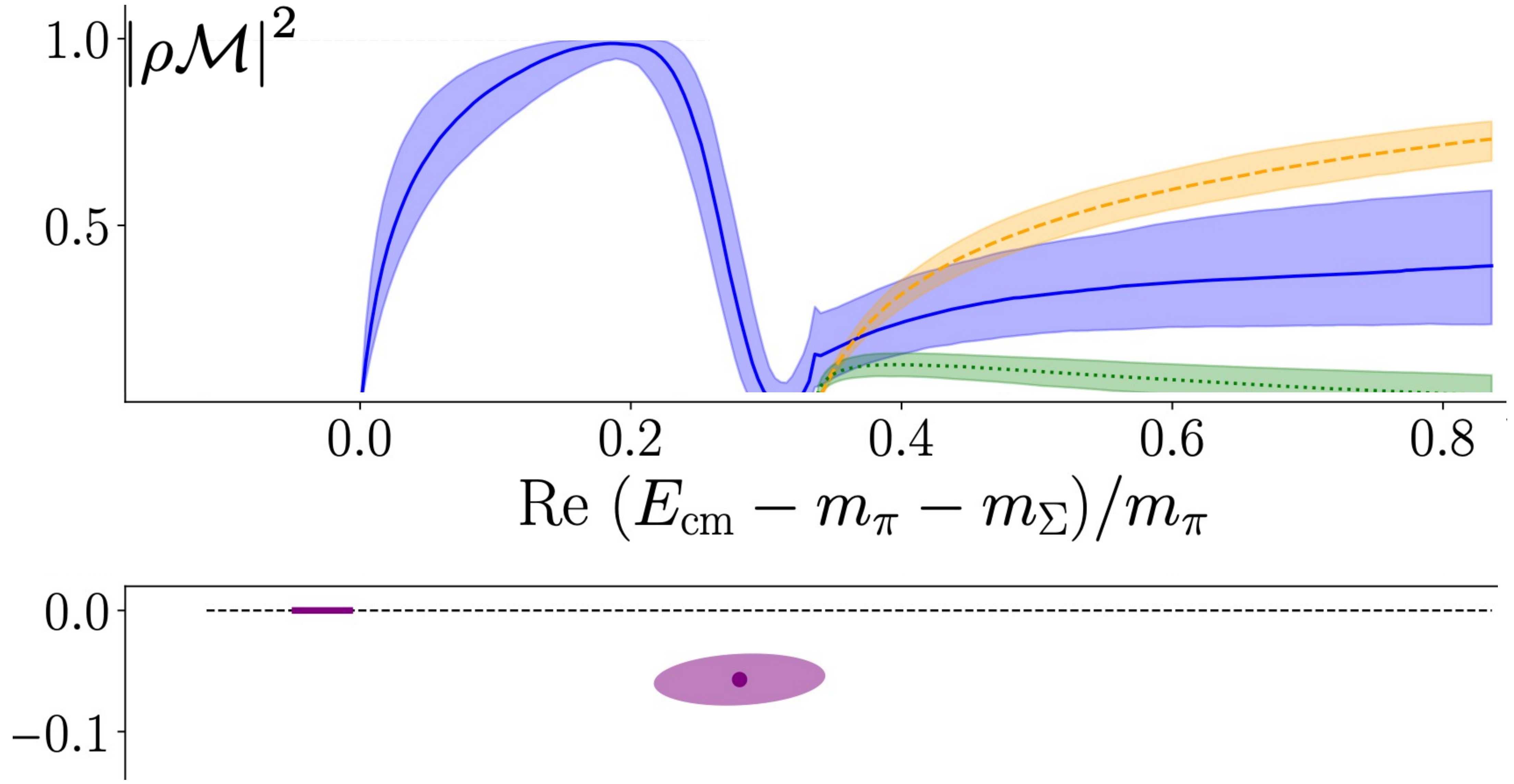
Two-body systems are well studied via lattice QCD

π_1 *channel*



Woss, Dudek, Edwards, Thomas, Wilson (2020)

$\Lambda(1405)$ *channel*



Basc Collaboration (2023)

Overarching goal

non-perturbatively constrain two- and three-hadron scattering amplitudes directly from the standard model (including electroweak & BSM probes)

three questions to answer

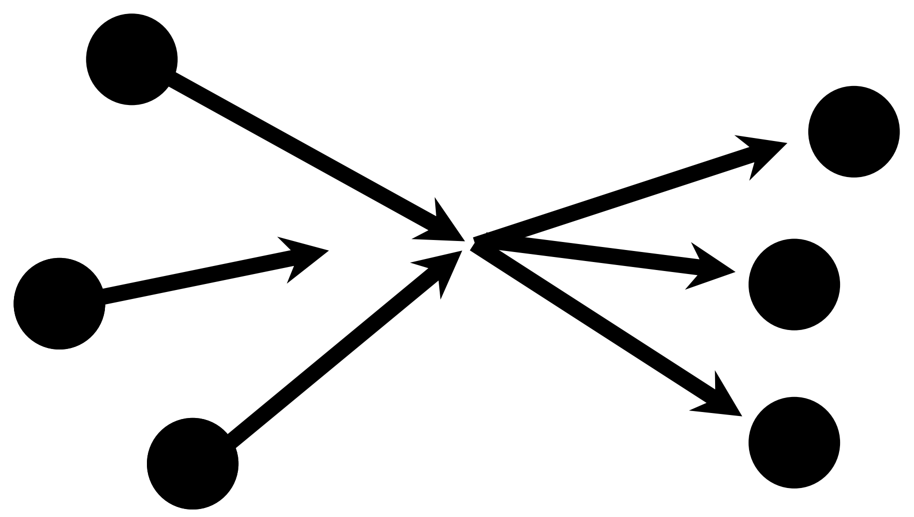
- [ ] why are three-body so much harder? 😬

- [ ] what has been done? 🤓

- [ ] what can we expect to be done? 🤠

Arsenal of non-perturbative tools

Scattering theory

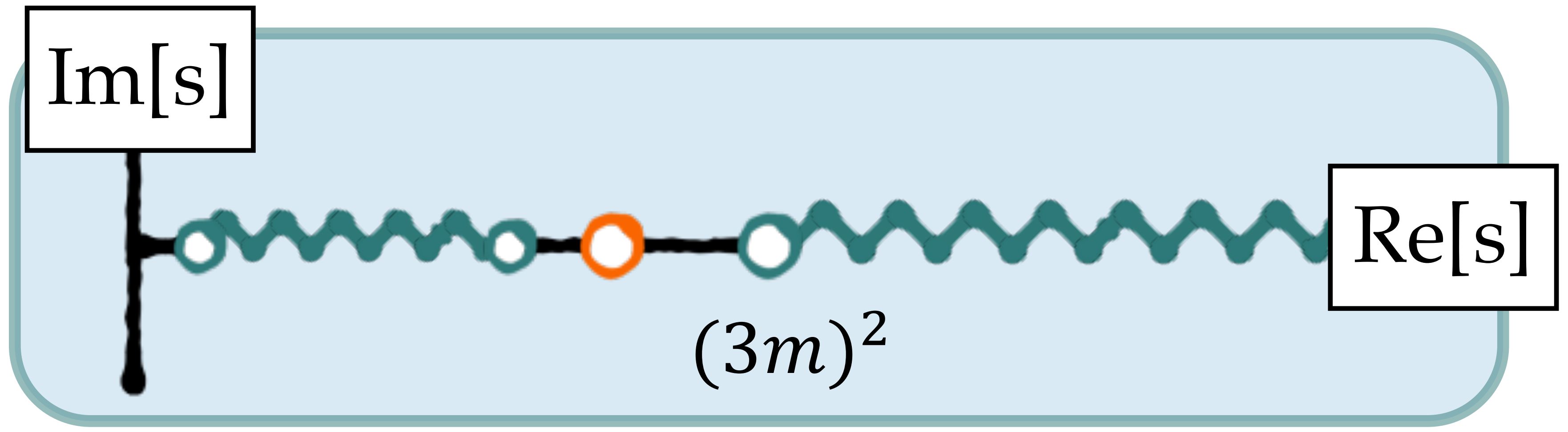


Benefits

- ☑ **analytic description,**
- ☑ **correct singular behavior,**
- ☑ **infinite-volume Minkowski observables**

Limitations

- ☐ **unknown real functions**



EFTs can be understood as a subset of this

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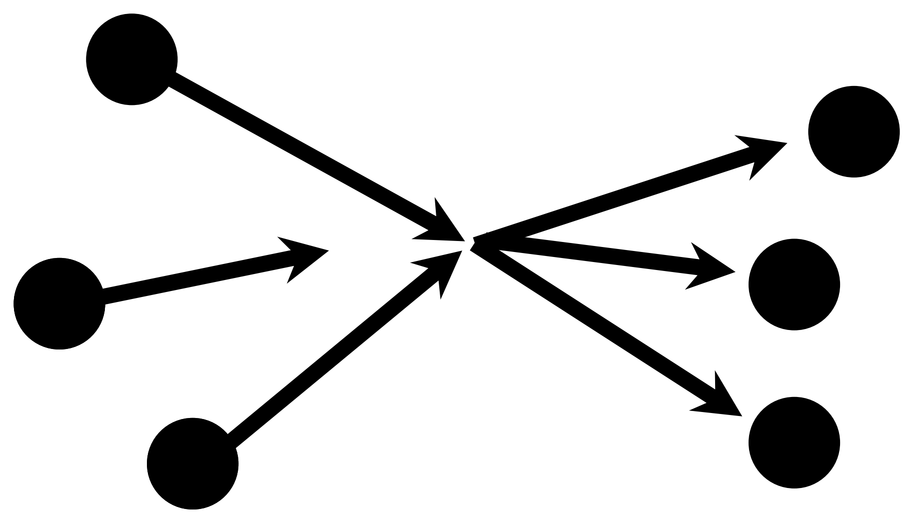

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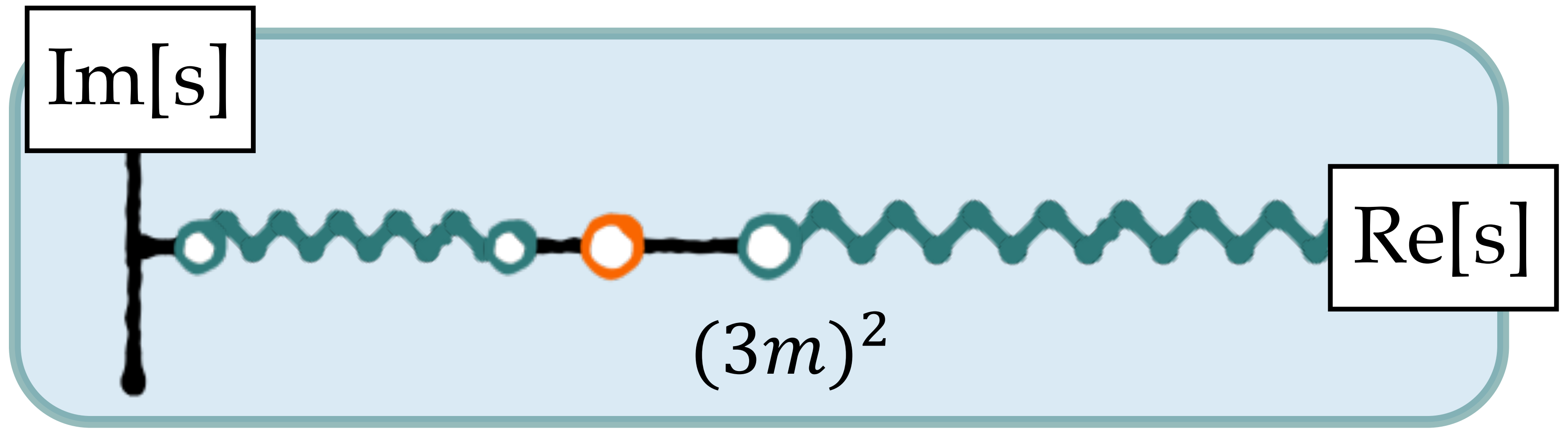
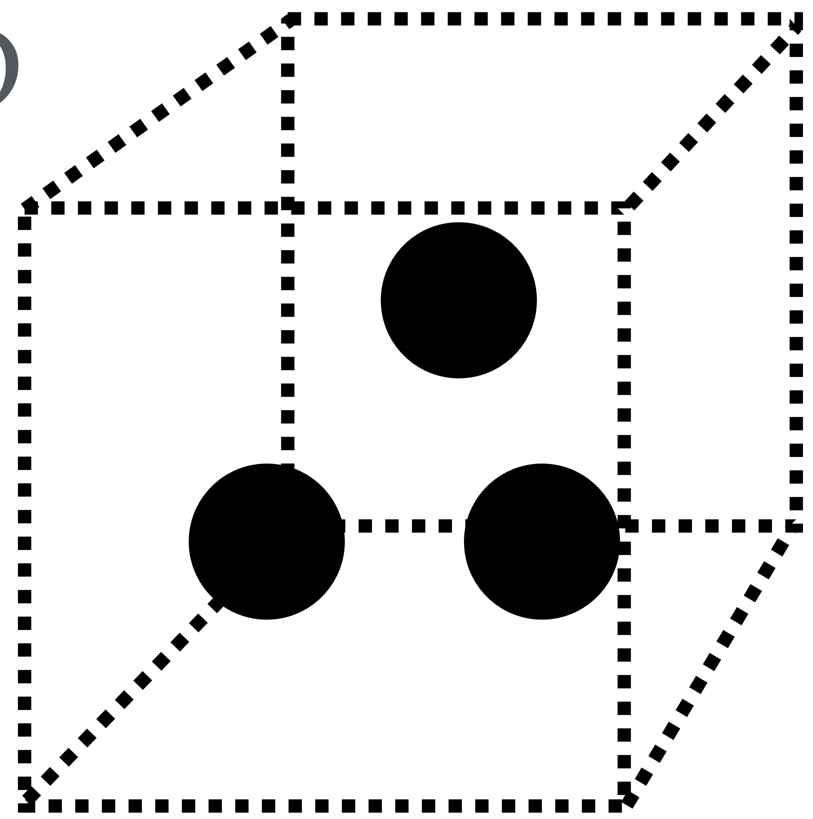


Lattice QCD

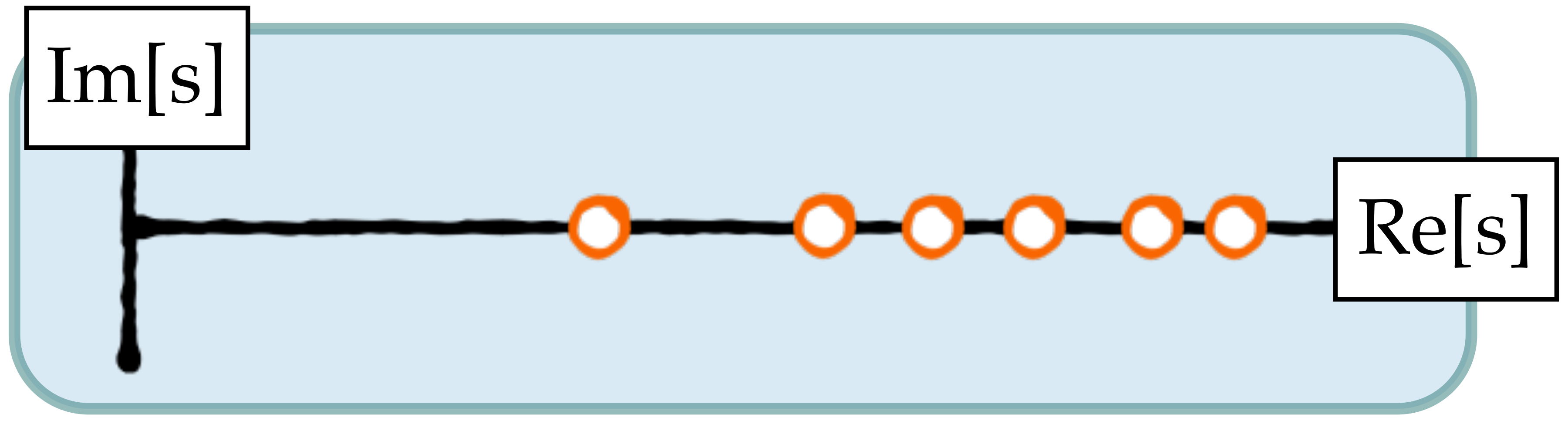


Benefits

- ☑ treats dynamics exactly,

Limitations

- ☐ computationally costly
- ☐ finite Euclidean spacetime
- ☐ no asymptotic states



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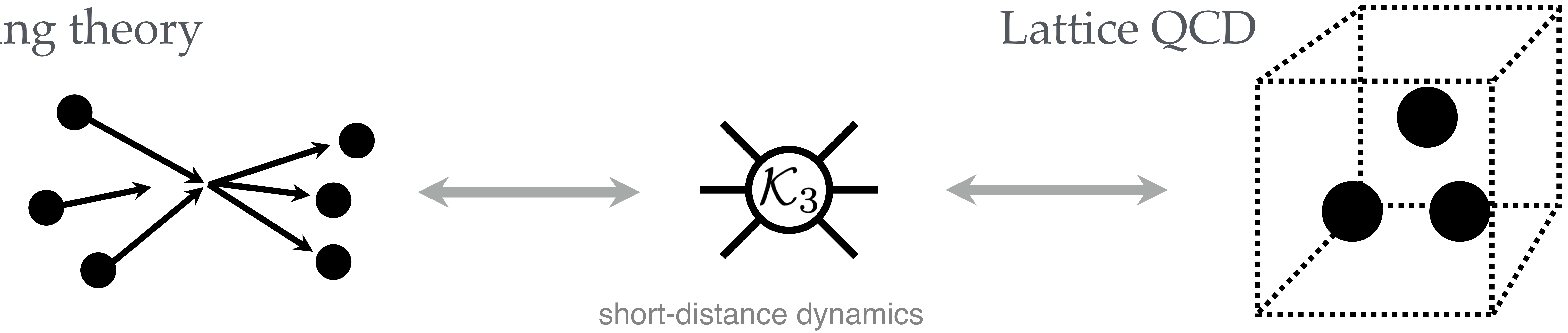

short-distance dynamics

nearly a continuum of references:

Rusetsky & Polejaeva(2012)

RB & Davoudi (2012)

Hansen & Sharpe (2014+)

RB, Hansen, Sharpe, …(2017+)

Mai & Doring (2017)

…

Jackura & RB (2023)

RB, Jackura & Costa (to appear)

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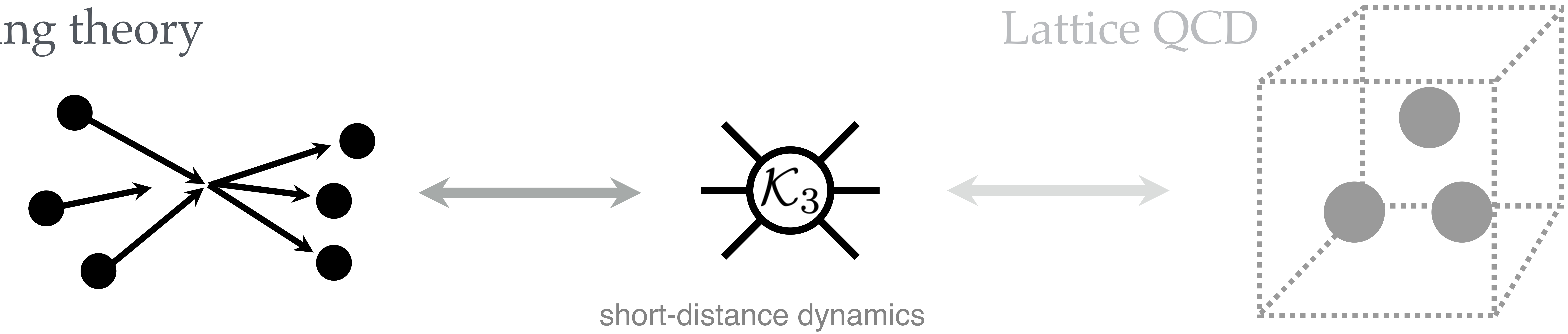
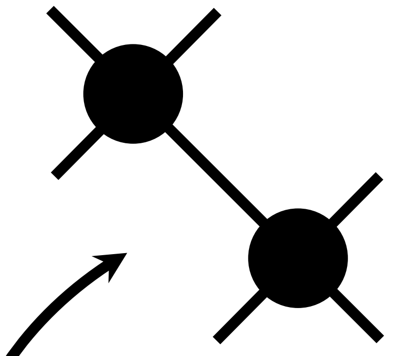
Scattering theory



$i\mathcal{M}_3 =$

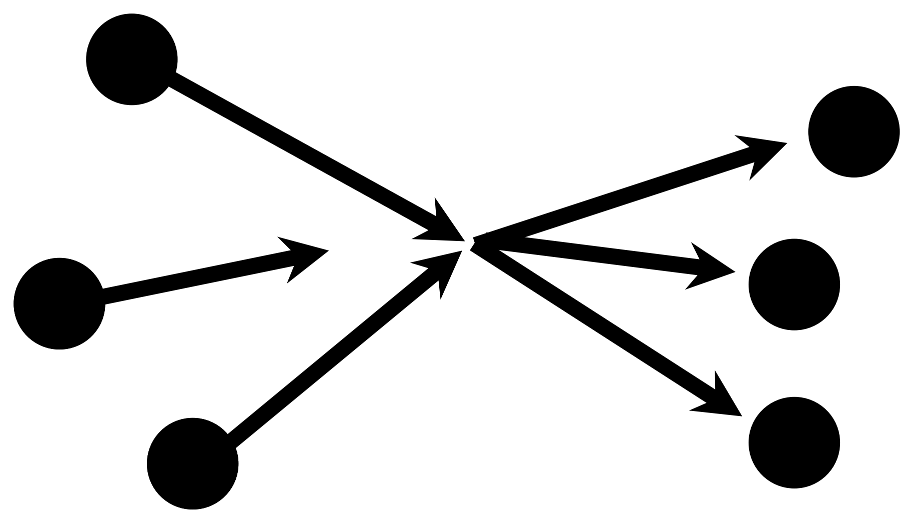


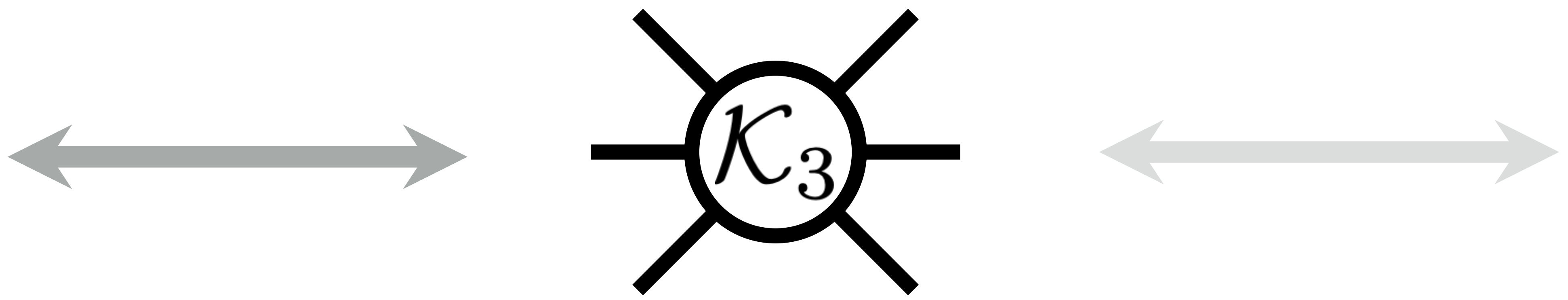
$+ \cdots$

$$G \sim \frac{1}{(P-p-k)^2 - m^2}$$

Arsenal of non-perturbative tools

Scattering theory





short-distance dynamics

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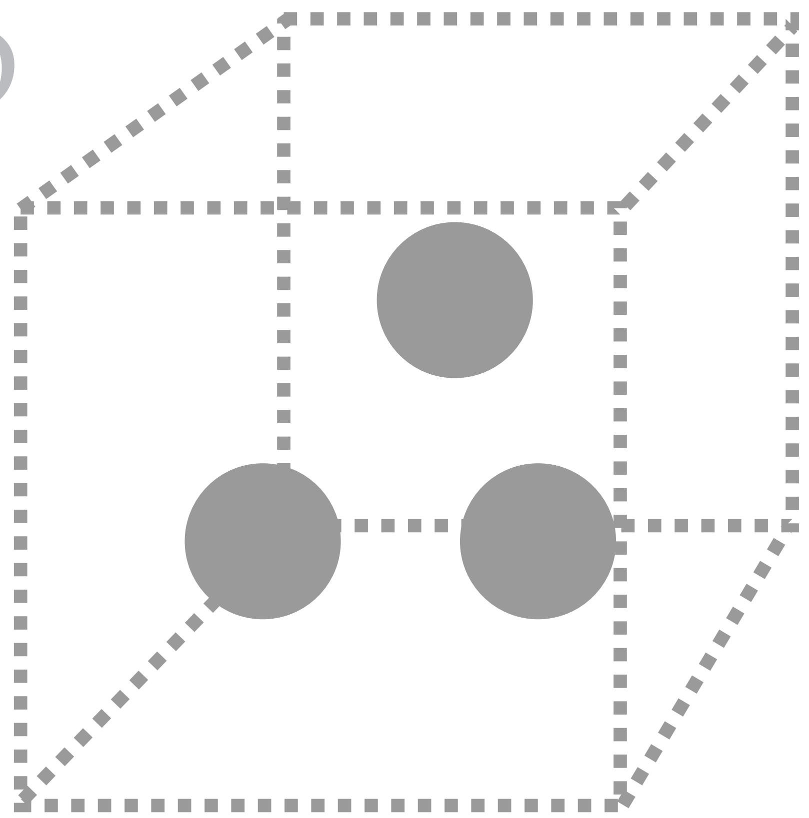

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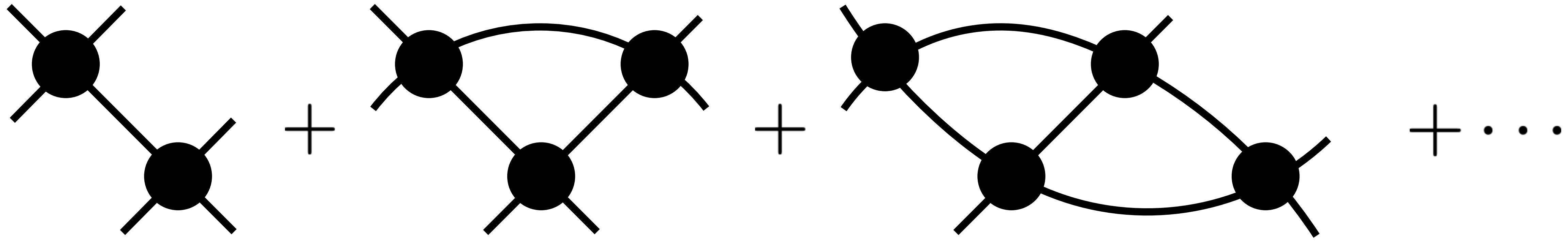
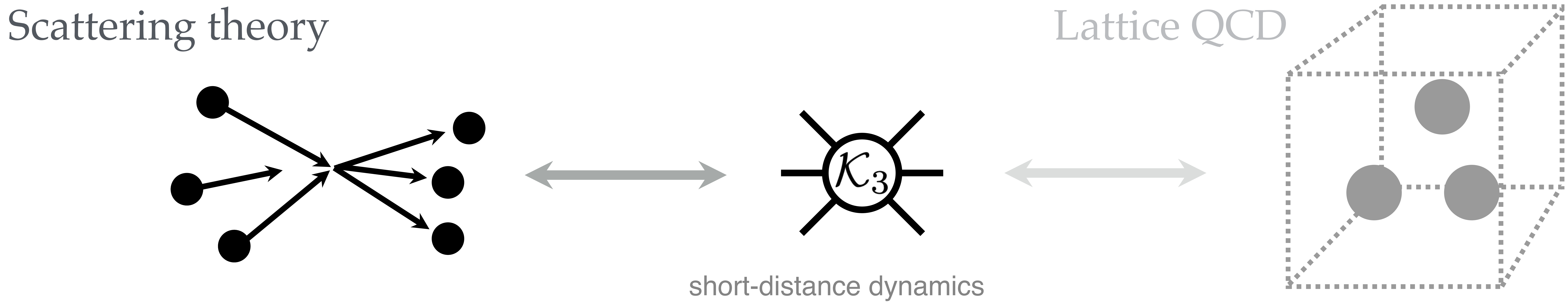


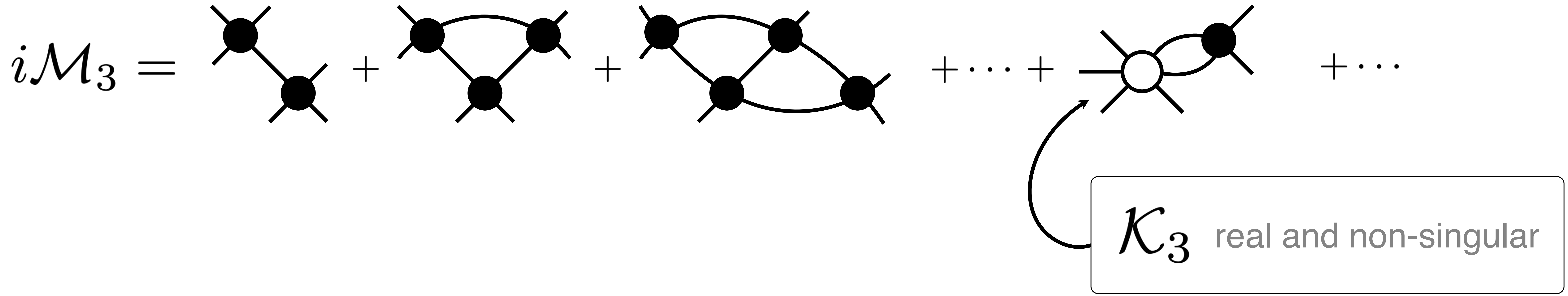
satisfies an integral equation

Where $\mathcal{D} = \mathcal{M}_2\, d\, \mathcal{M}_2$ and

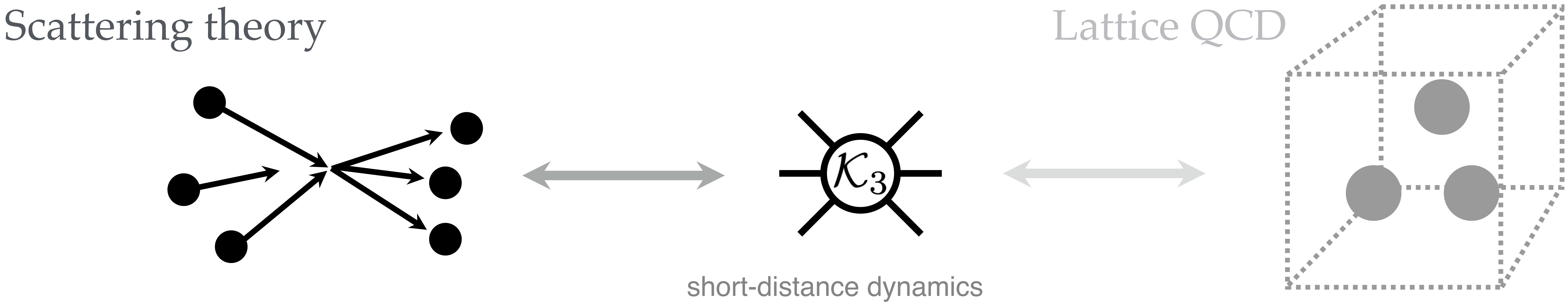
$$d = -G - \int G\, \mathcal{M}_2 d$$

Arsenal of non-perturbative tools





Arsenal of non-perturbative tools



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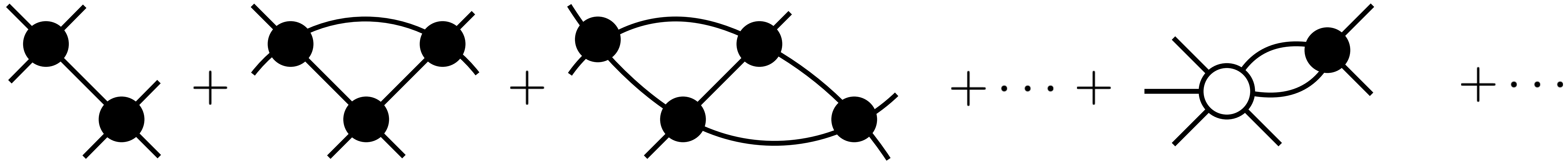

$$= i\mathcal{D} + i\mathcal{L}[\mathcal{D}] \cdot \mathcal{F}[\mathcal{D}, \mathcal{K}_3] \cdot \mathcal{R}[\mathcal{D}]$$

Integral equations

We need to solve:

$$d(\mathbf{p}', s, \mathbf{p}) = -G(\mathbf{p}', s, \mathbf{p}) - \int_0^{q_{\max}} \frac{d^3\mathbf{q}}{(2\pi)^3 2\omega_q} G(\mathbf{p}', s, \mathbf{q}) \mathcal{M}_2(q, s)\, d(\mathbf{q}, s, \mathbf{p})$$

Need to resort to numerical solutions.

"integration kernel"

Integral equations

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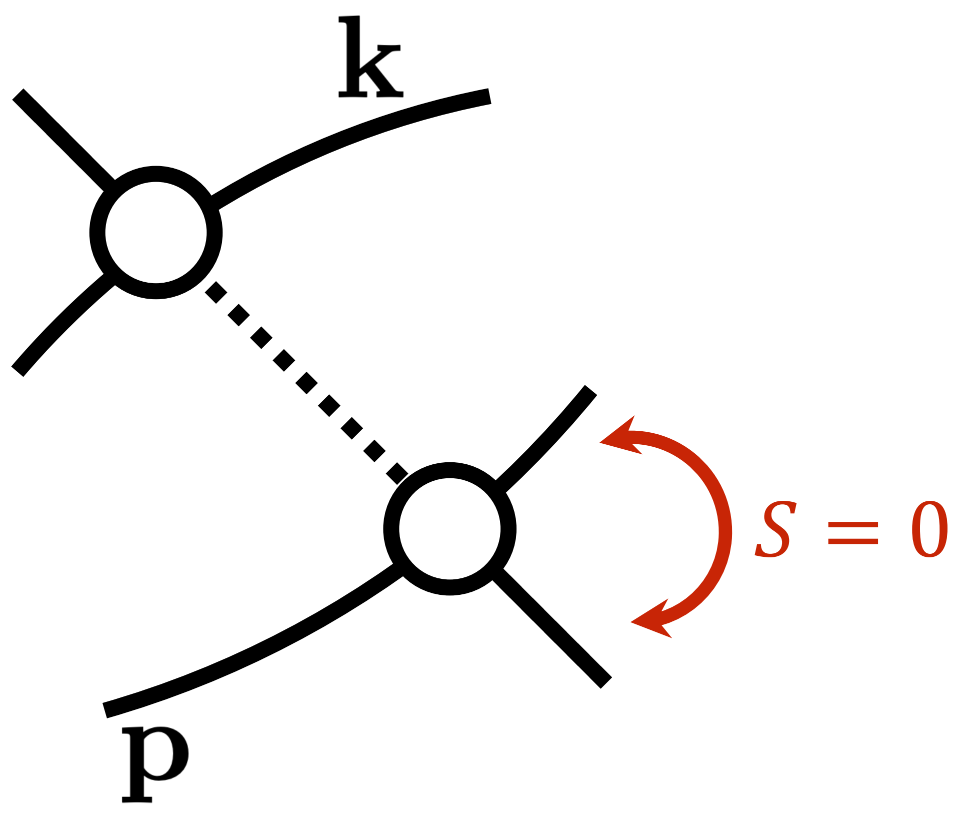
Need to resort to numerical solutions.

Three correlated challenges:

- ☐ 3D integral equation,
- ☐ need to project to **angular momentum and parity**,
- ☐ integration kernel is generally singular.

Partial wave projections

The one-particle exchange is one of the main sources of singularities.
Let us consider the case where $S = 0$:



$$\sim G(\boldsymbol{p}, \boldsymbol{k}) = \frac{1}{(E - \omega_k - \omega_p) - (\boldsymbol{p} + \boldsymbol{k})^2 - m^2 + i\epsilon}$$

$$= \frac{1}{(E - \omega_k - \omega_p) - k^2 - p^2 - m^2 - 2pk\cos\theta + i\epsilon}$$

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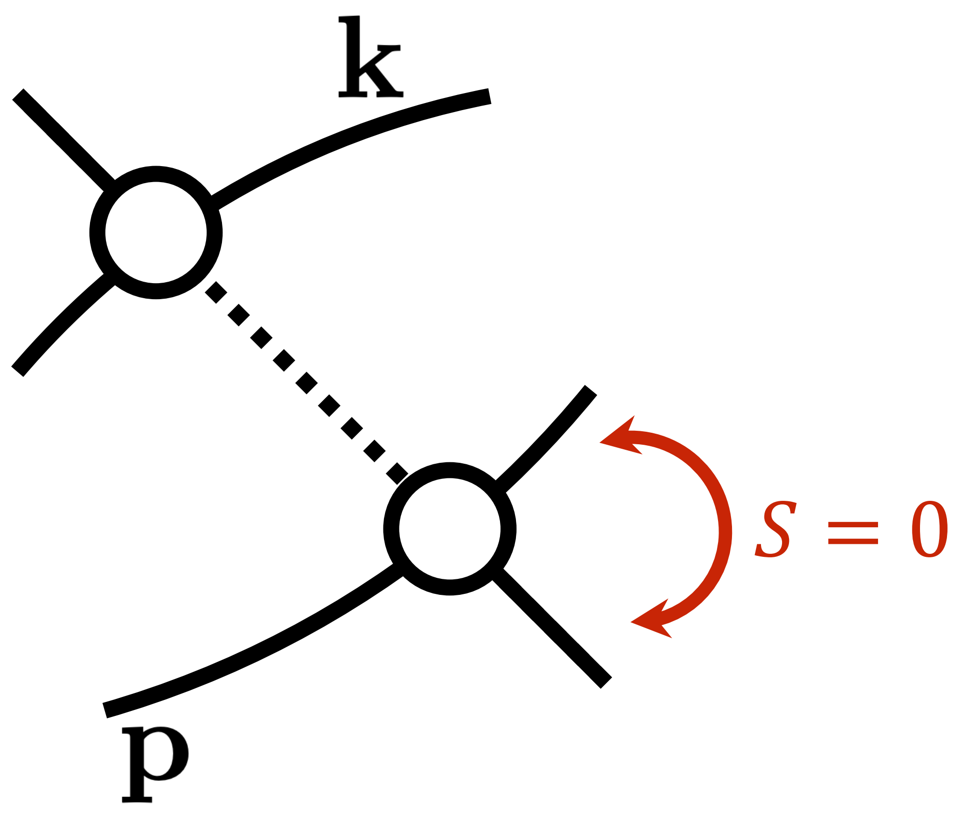

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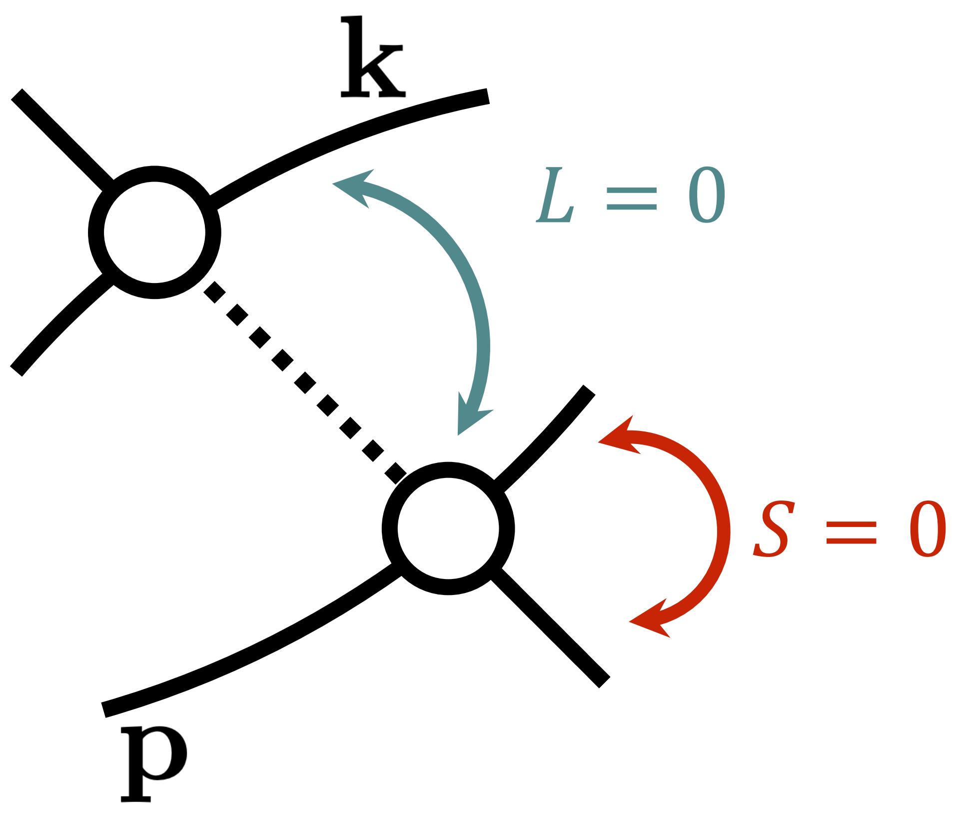
Projecting to total $J = 0$ amounts to integrating over all angles:

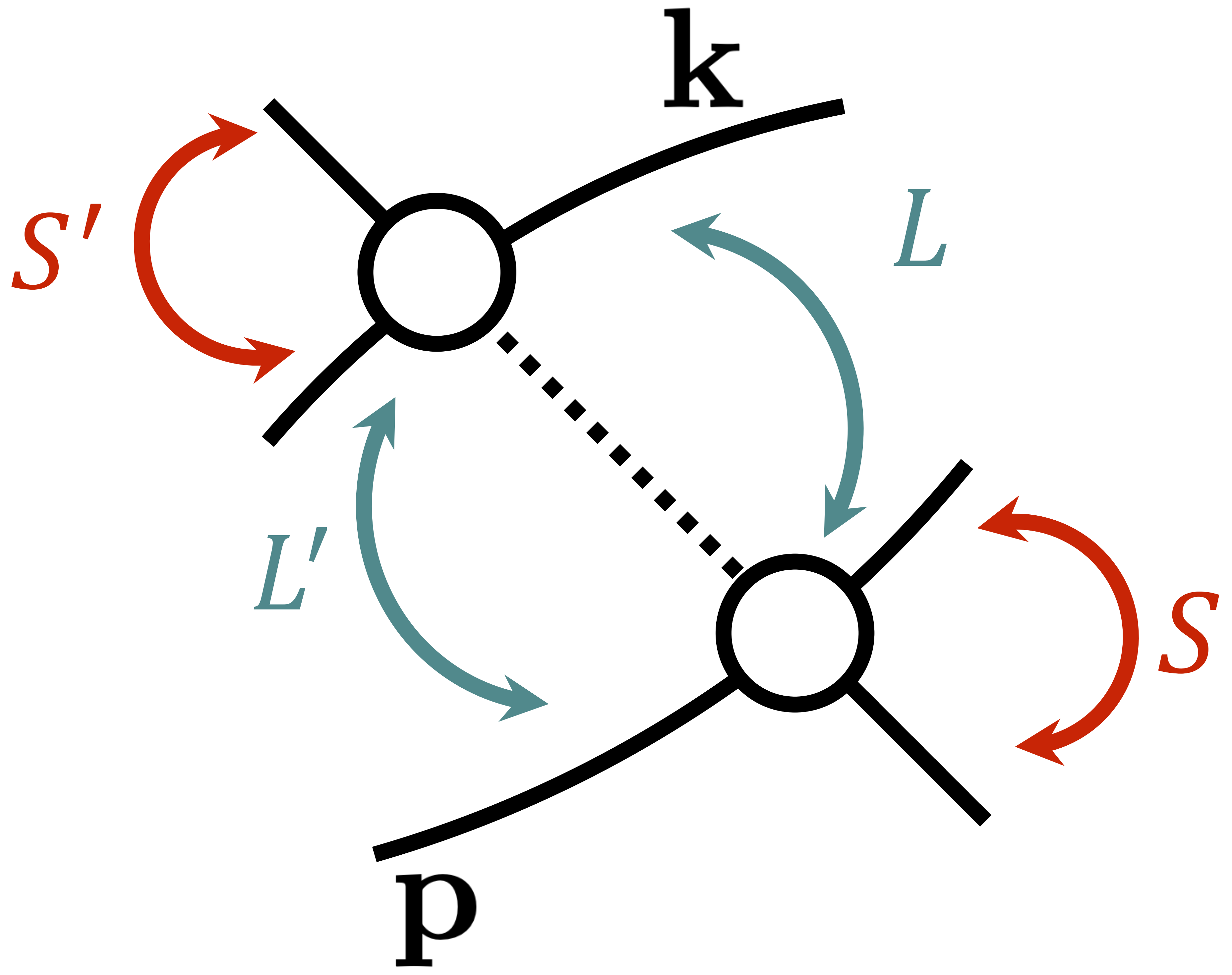


$$\sim G(p, k) = \frac{1}{2}\int_{-1}^{1} d\cos\theta\, G(\boldsymbol{p}, \boldsymbol{k}) = -\frac{1}{4pk}\log\frac{z_{pk} - 1}{z_{pk} + 1}$$

$$z(p, k) = \frac{(E - \omega_k - \omega_p)^2 - k^2 - p^2 - m^2}{2pk}$$

Partial wave projections

In general…



$$\left[\mathcal{G}^{J^P}\right]_{L'S',LS} = \left[\mathcal{K}_{\mathcal{G}}^{J^P}\right]_{L'S',LS} + \left[\mathcal{T}^{J^P}\right]_{L'S',LS} \, Q_0(\zeta_{pk})$$

known kinematic functions

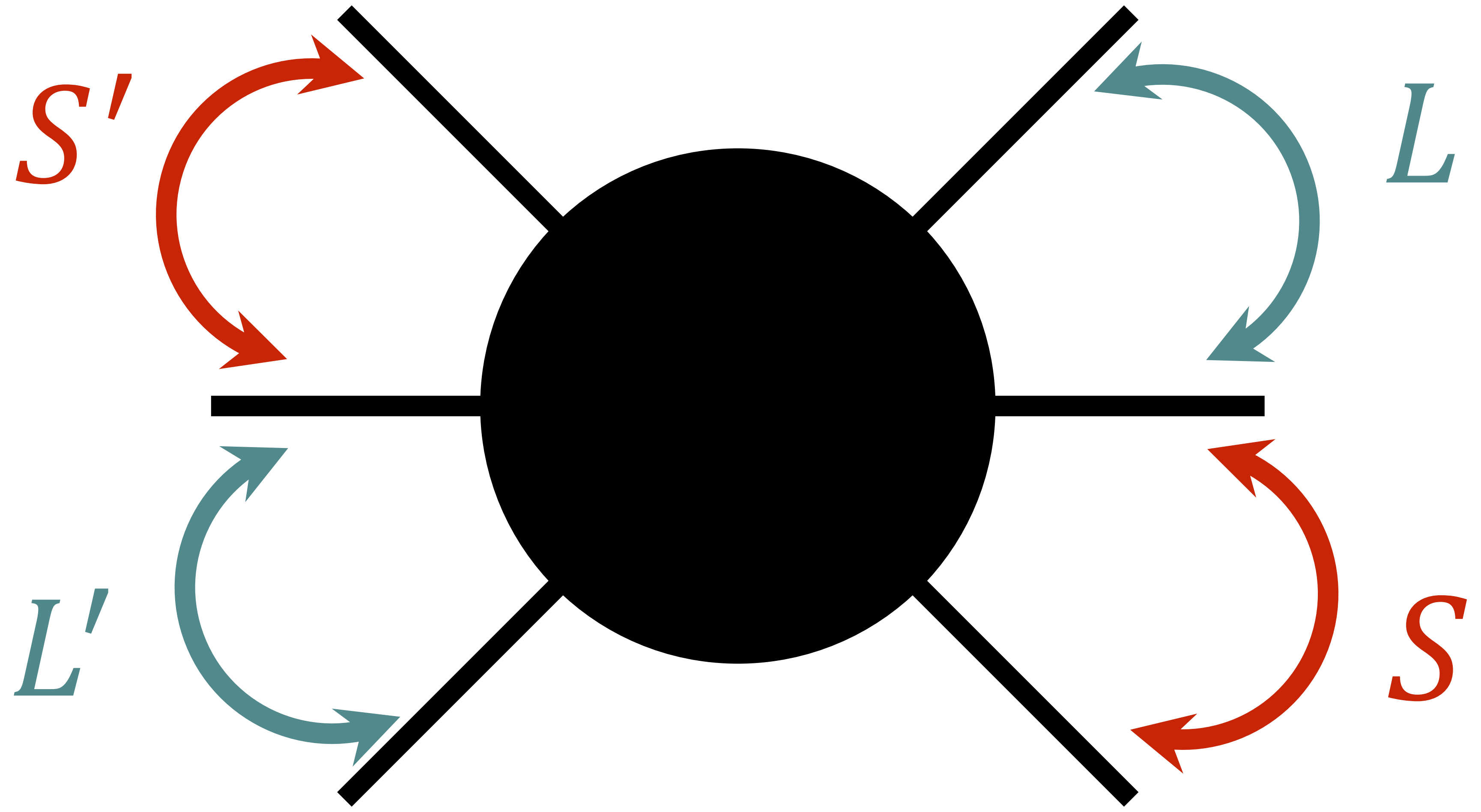
Legendre functions

Jackura, RB (2023)

$$Q_0(\zeta) = \frac{1}{2}\log\left(\frac{\zeta+1}{\zeta-1}\right)$$

Partial wave projections

In general…



$$= i\left[\mathcal{M}_3^{J^P}\right]_{L'S',LS}$$

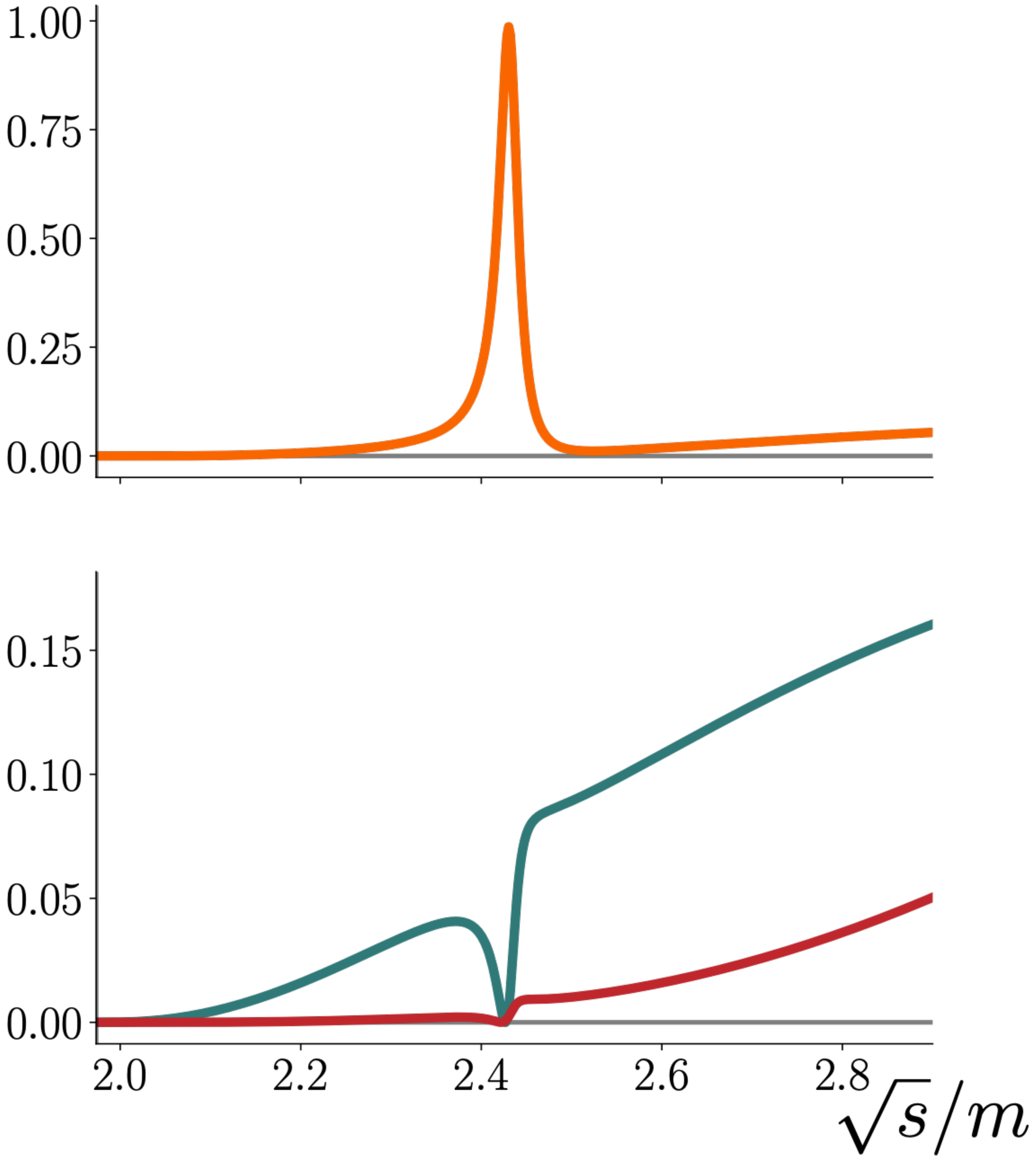
S. R. Costa

Jackura

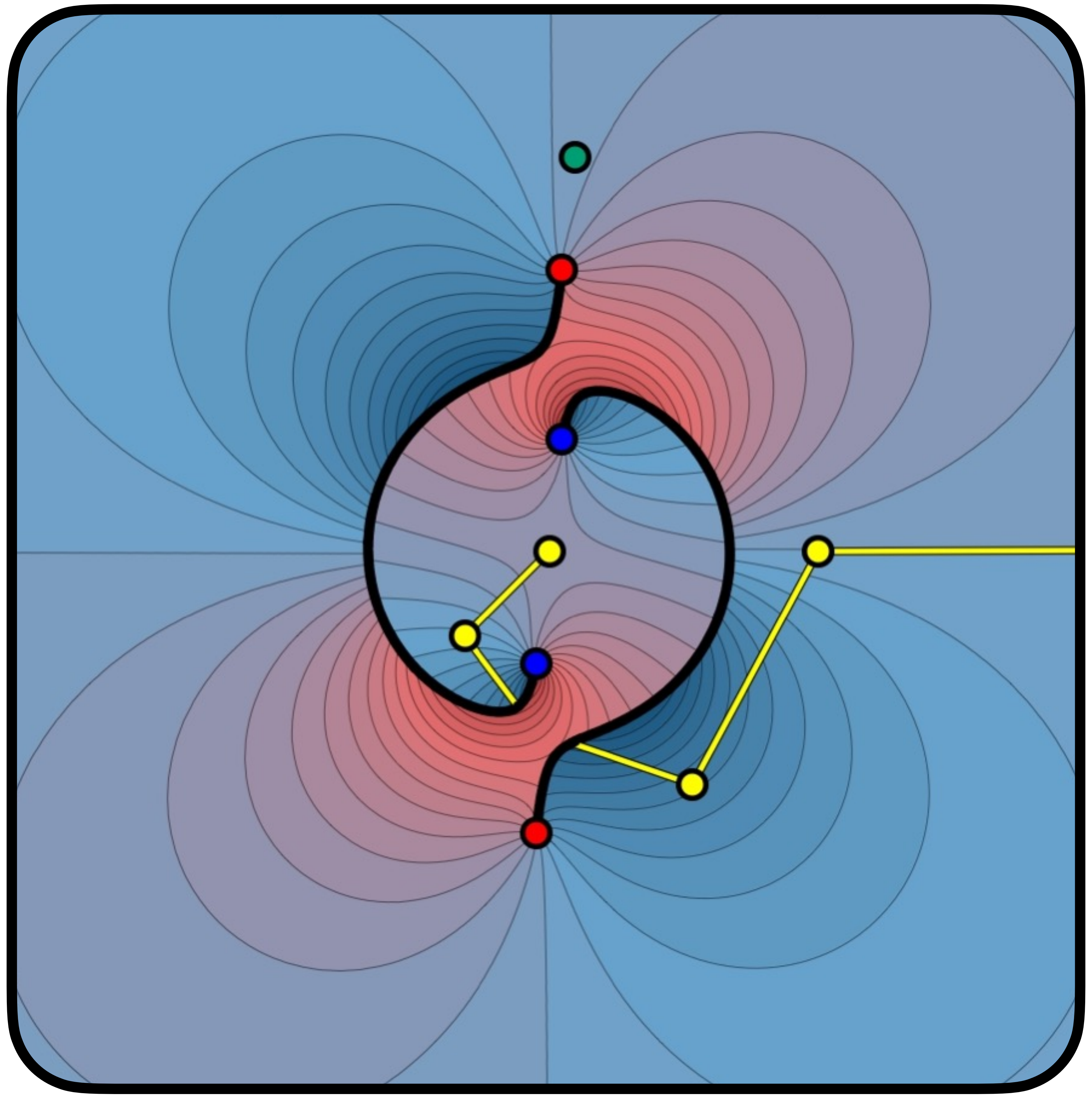
Costa, Jackura, RB (to appear)

Numerical tests

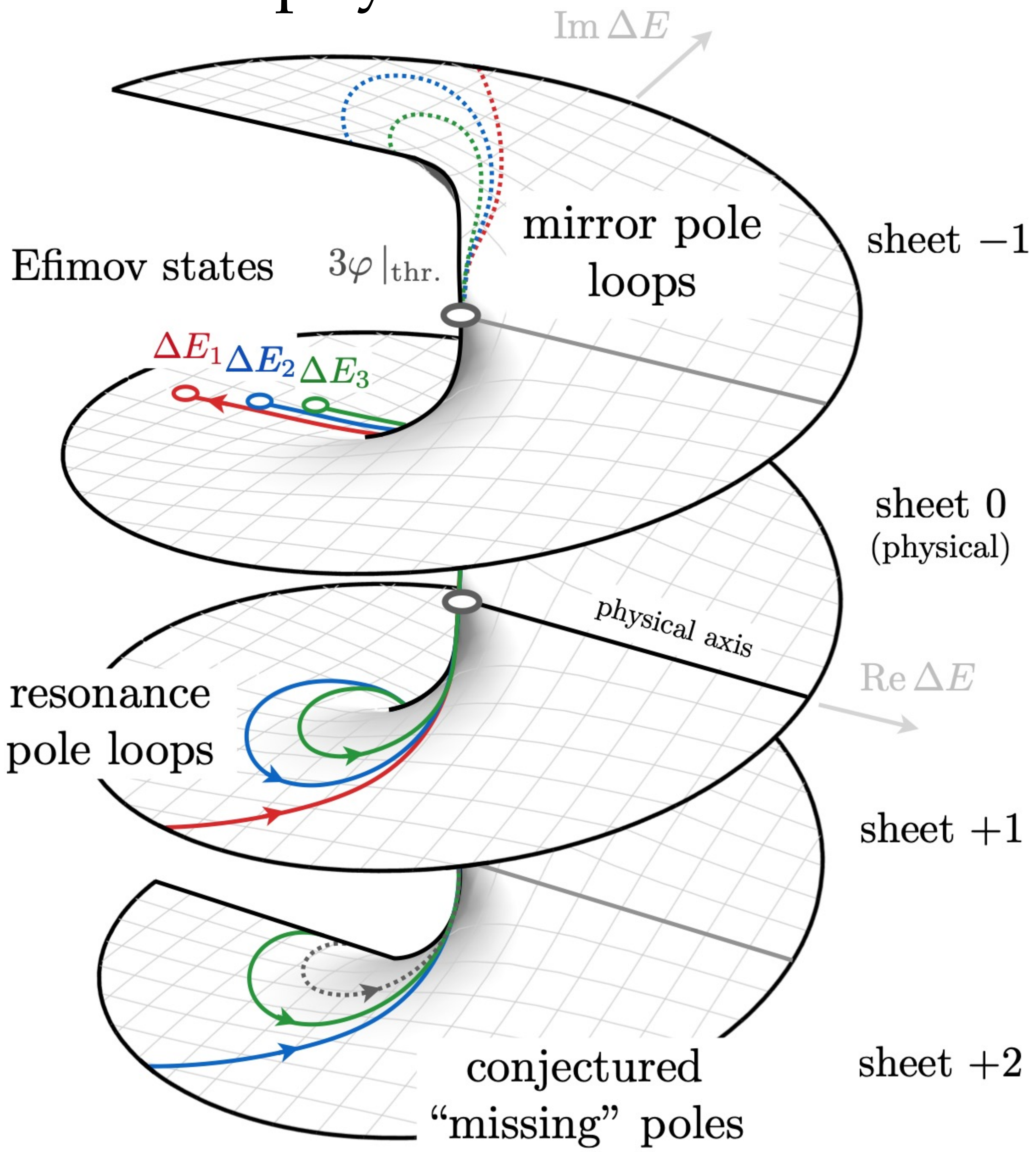
☑ unitarity



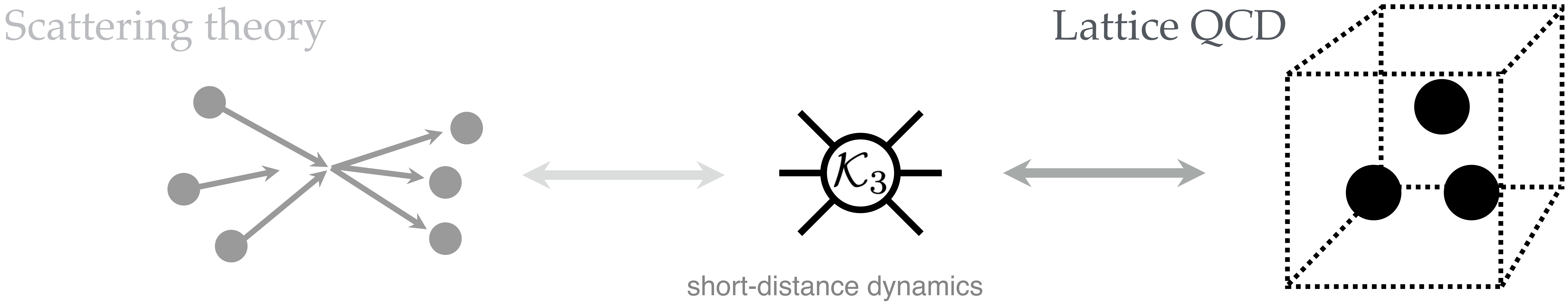
☑ analyticity



☑ Efimov physics



Arsenal of non-perturbative tools



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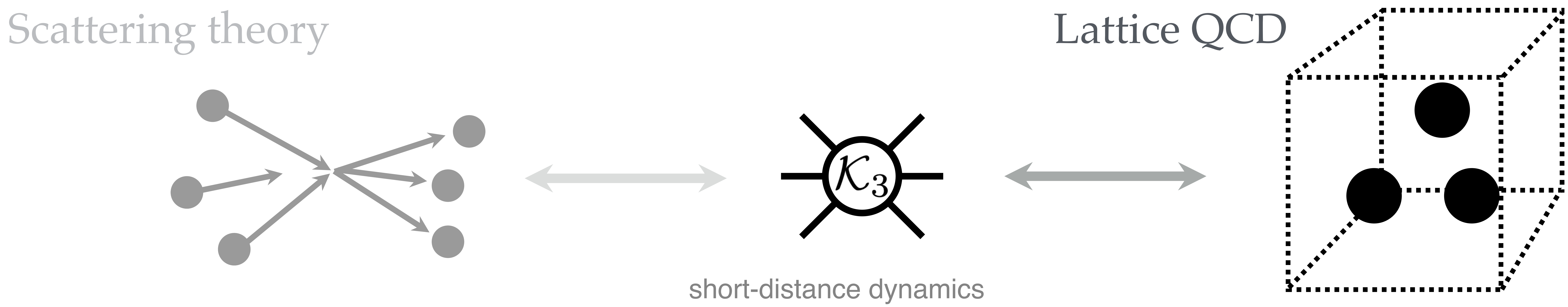
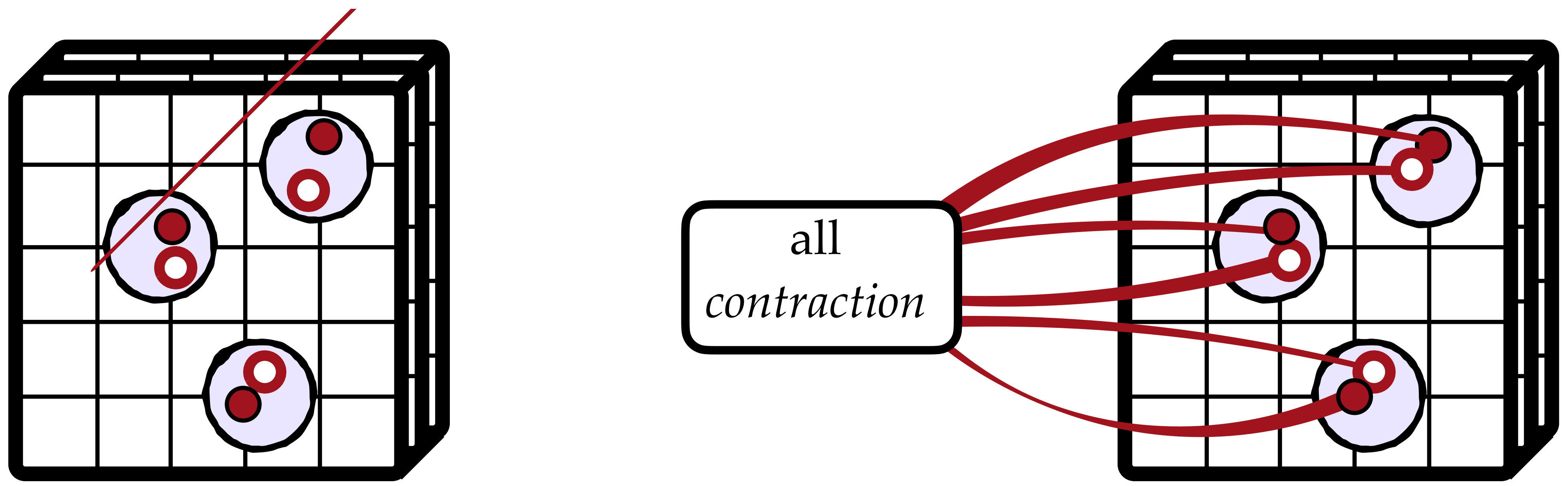


☑ Two point correlation functions:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t} =$$



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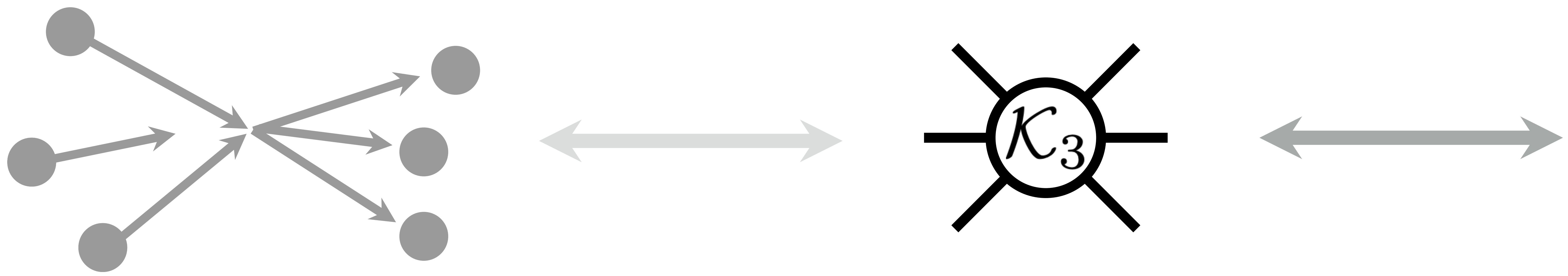
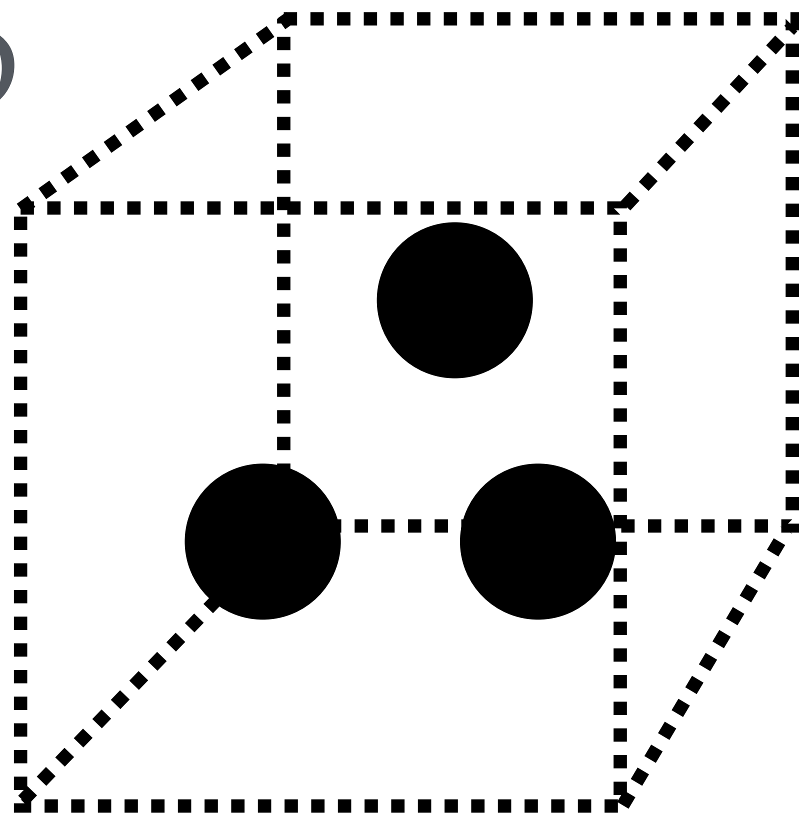
Lattice QCD

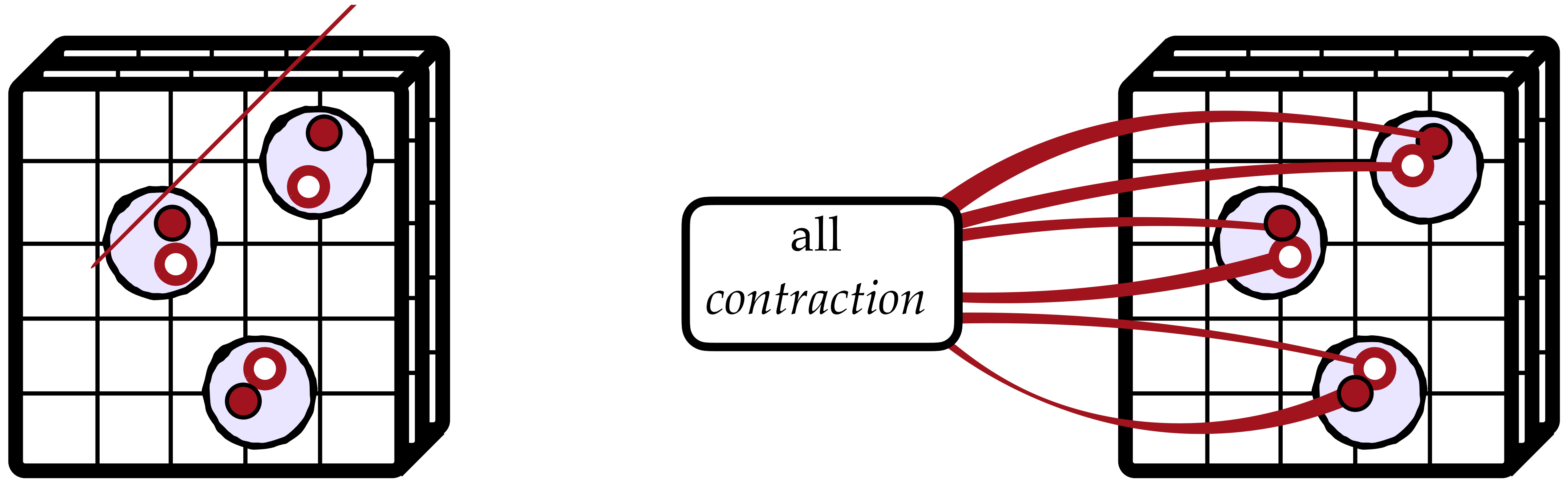




short-distance dynamics

☑ Two point correlation functions:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t} =$$



☑ The energy of three *identical spinless bosons* in a box satisfies:

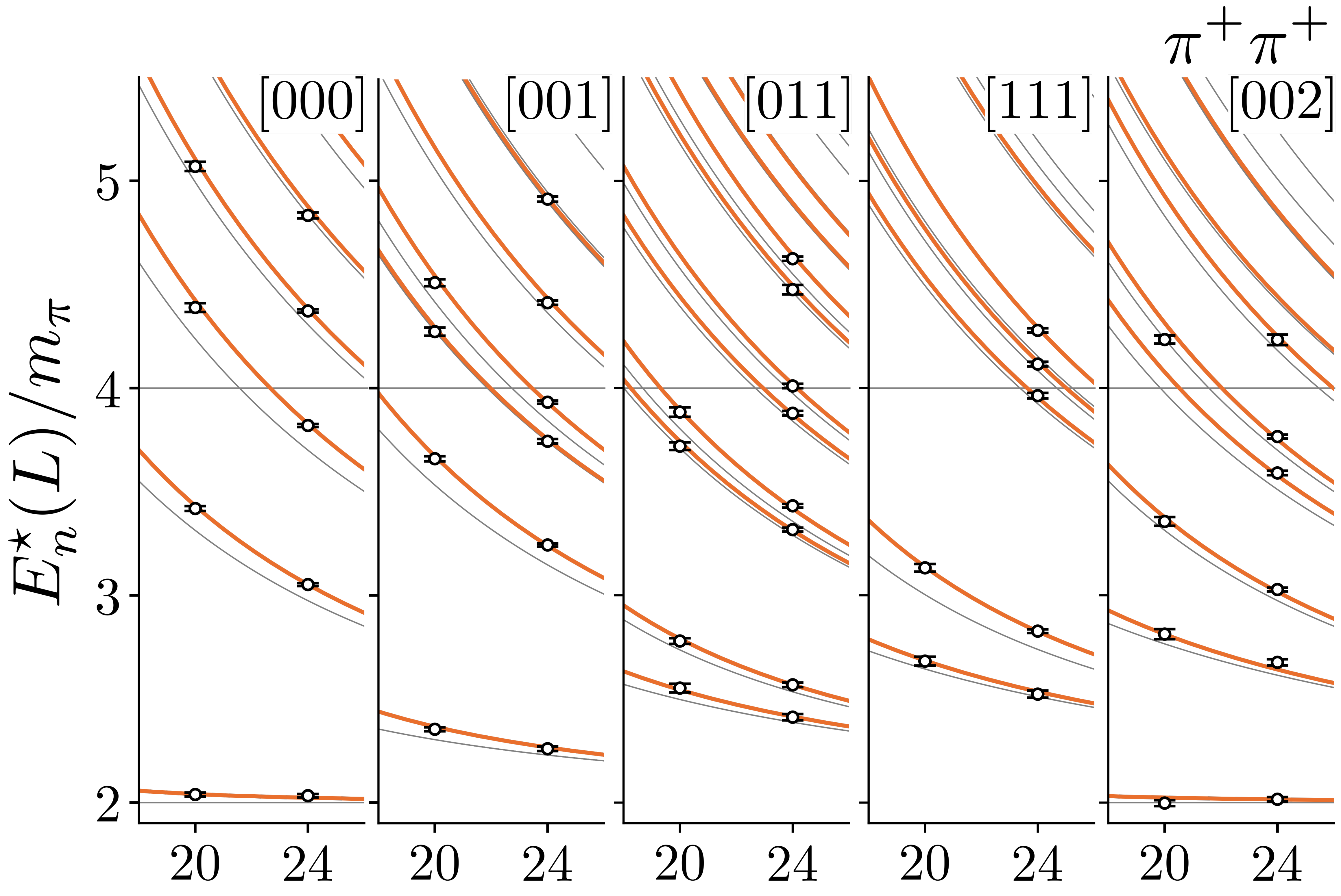
$$F_3^{-1}(P_n, L) + \mathcal{K}_3(P_n^2) = 0 \quad +\mathcal{O}(e^{-mL})$$

[up to details I won't go into 🤓]

Hansen & Sharpe (2014+)

$\pi\pi$ scattering

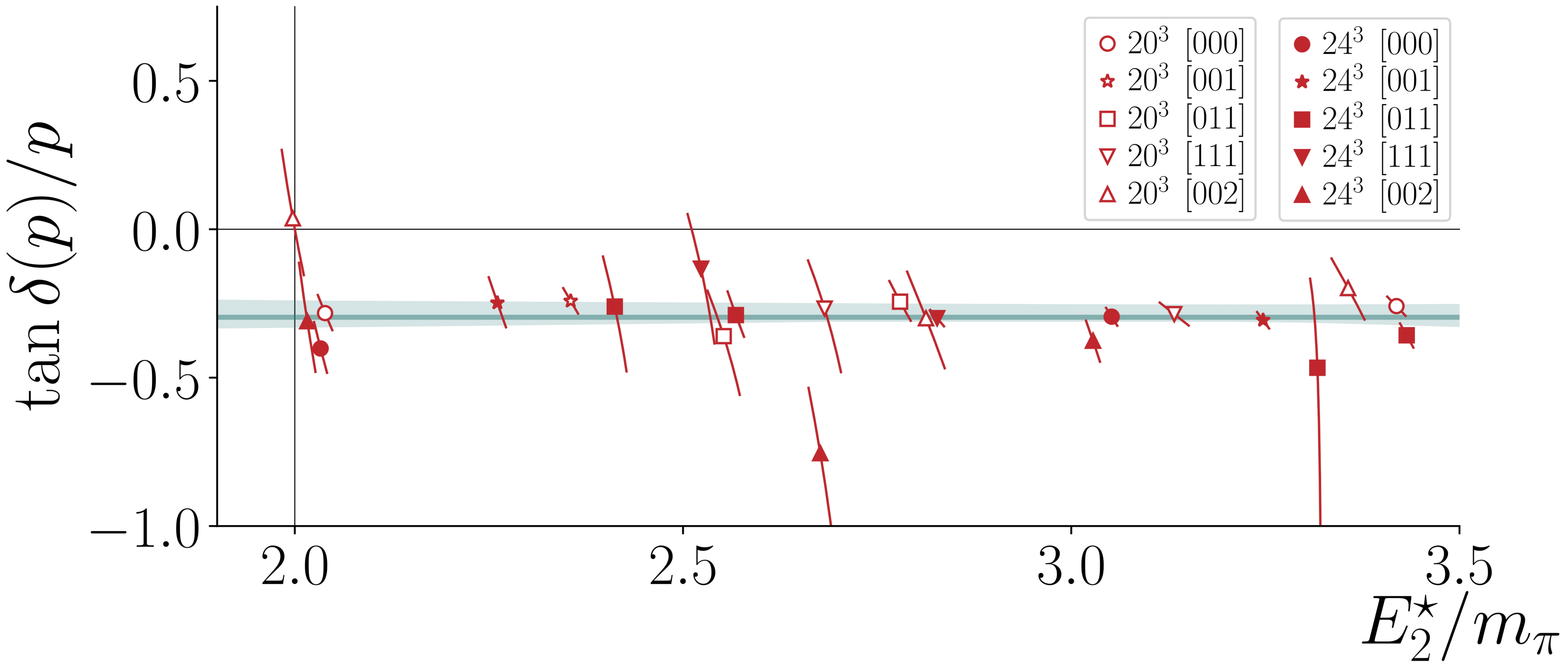
(I=2 channel, $m_\pi \sim 390\text{MeV}$)



"Vanila" Lüscher

$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$



Hansen, RB, Edwards, Thomas, & Wilson (2020)

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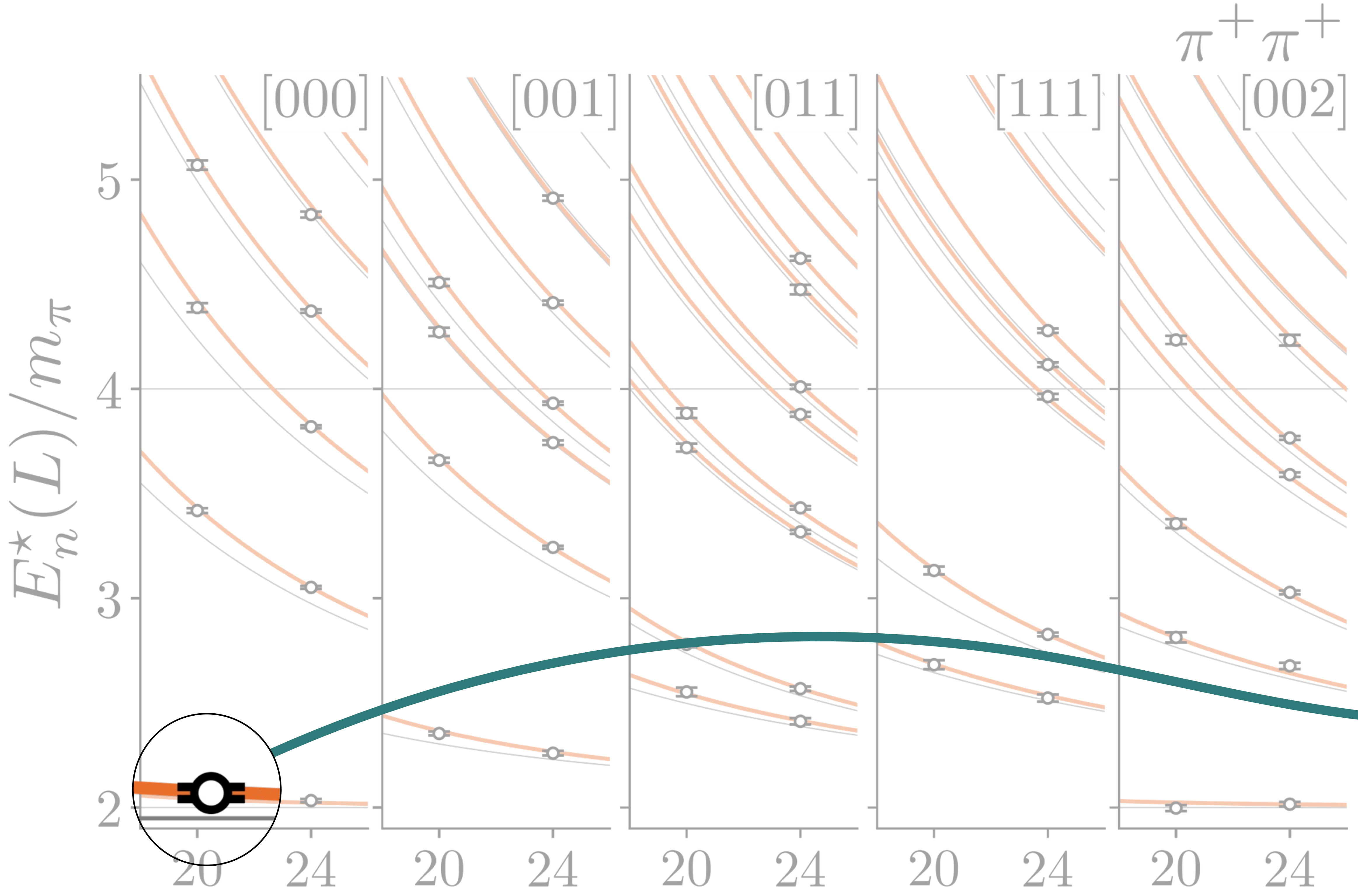

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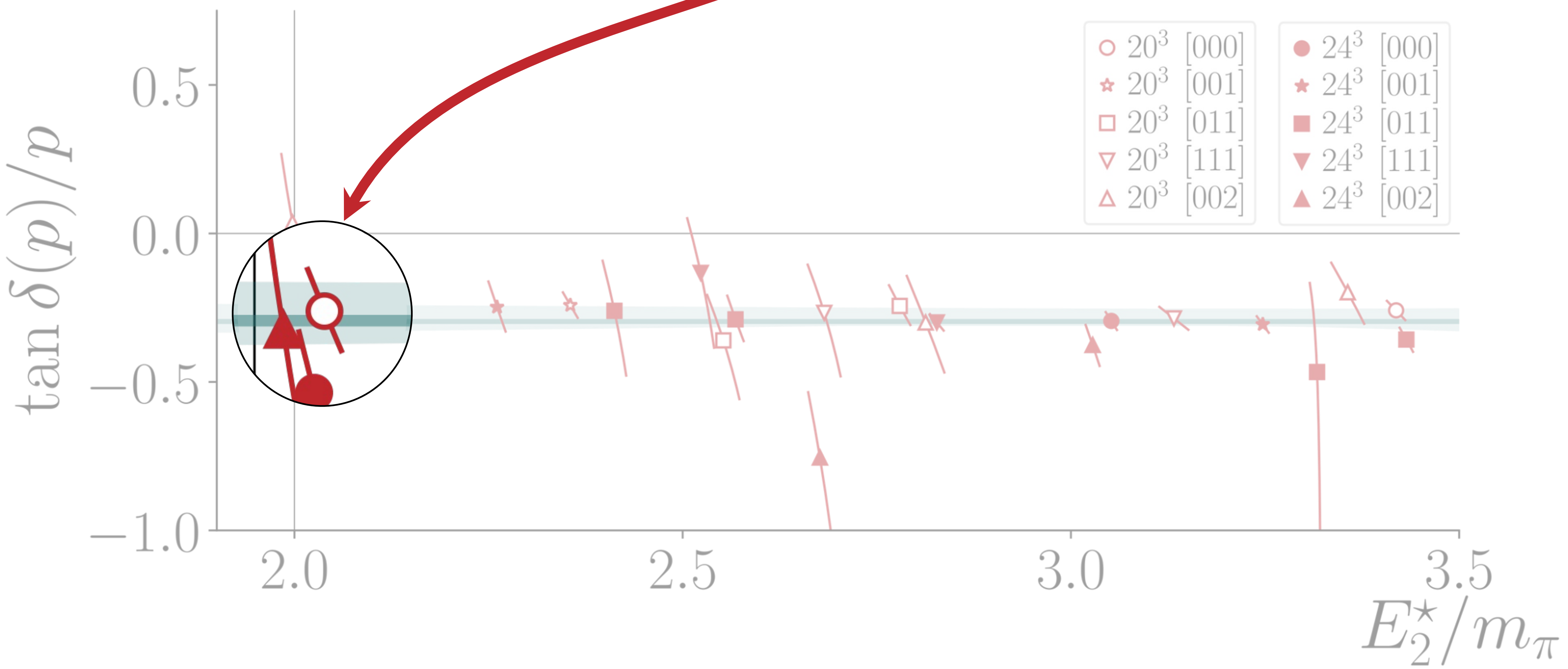
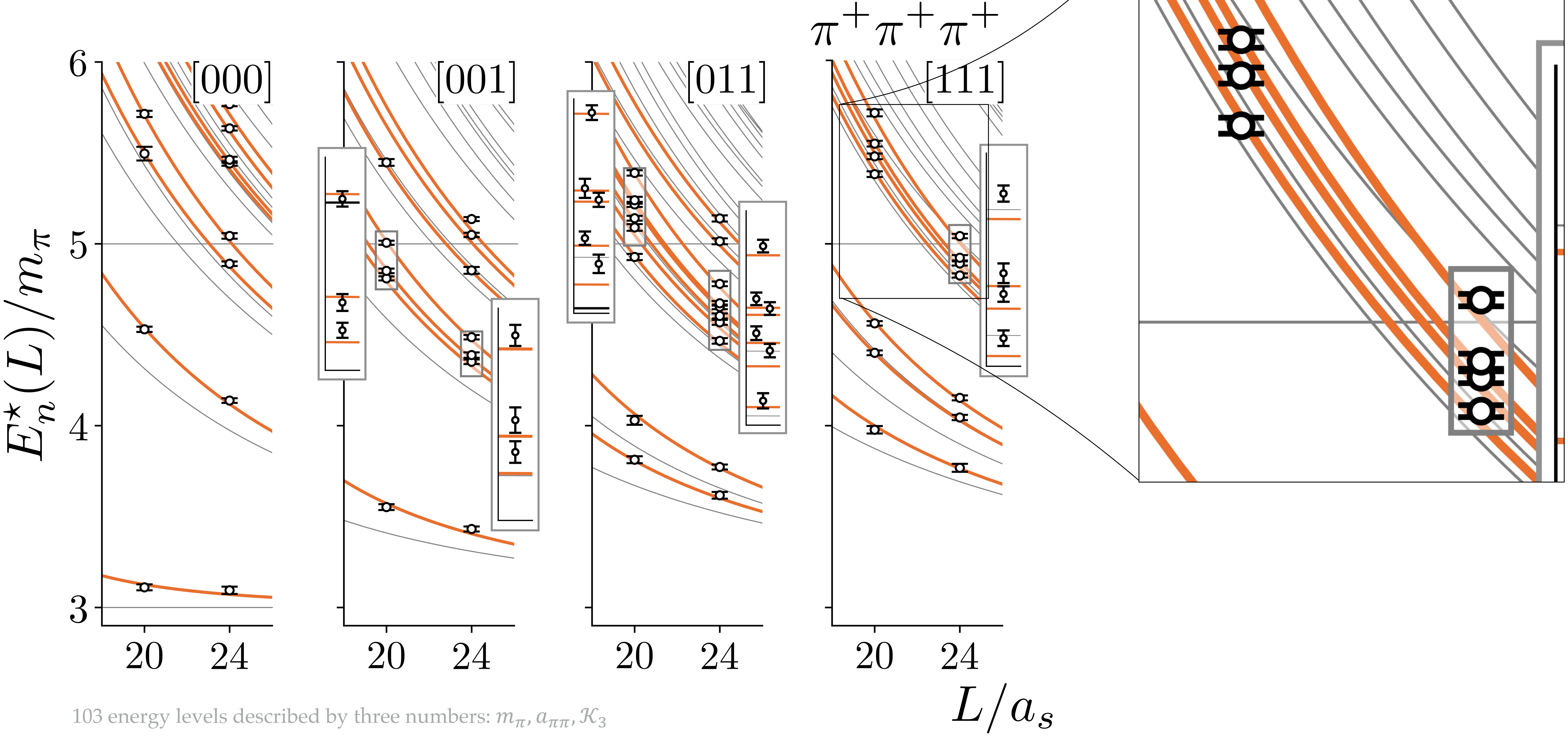


hadspec

Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi\pi$

(I=3 channel, $m_\pi \sim 390\text{MeV}$)



103 energy levels described by three numbers: $m_\pi, a_{\pi\pi}, \mathcal{K}_3$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

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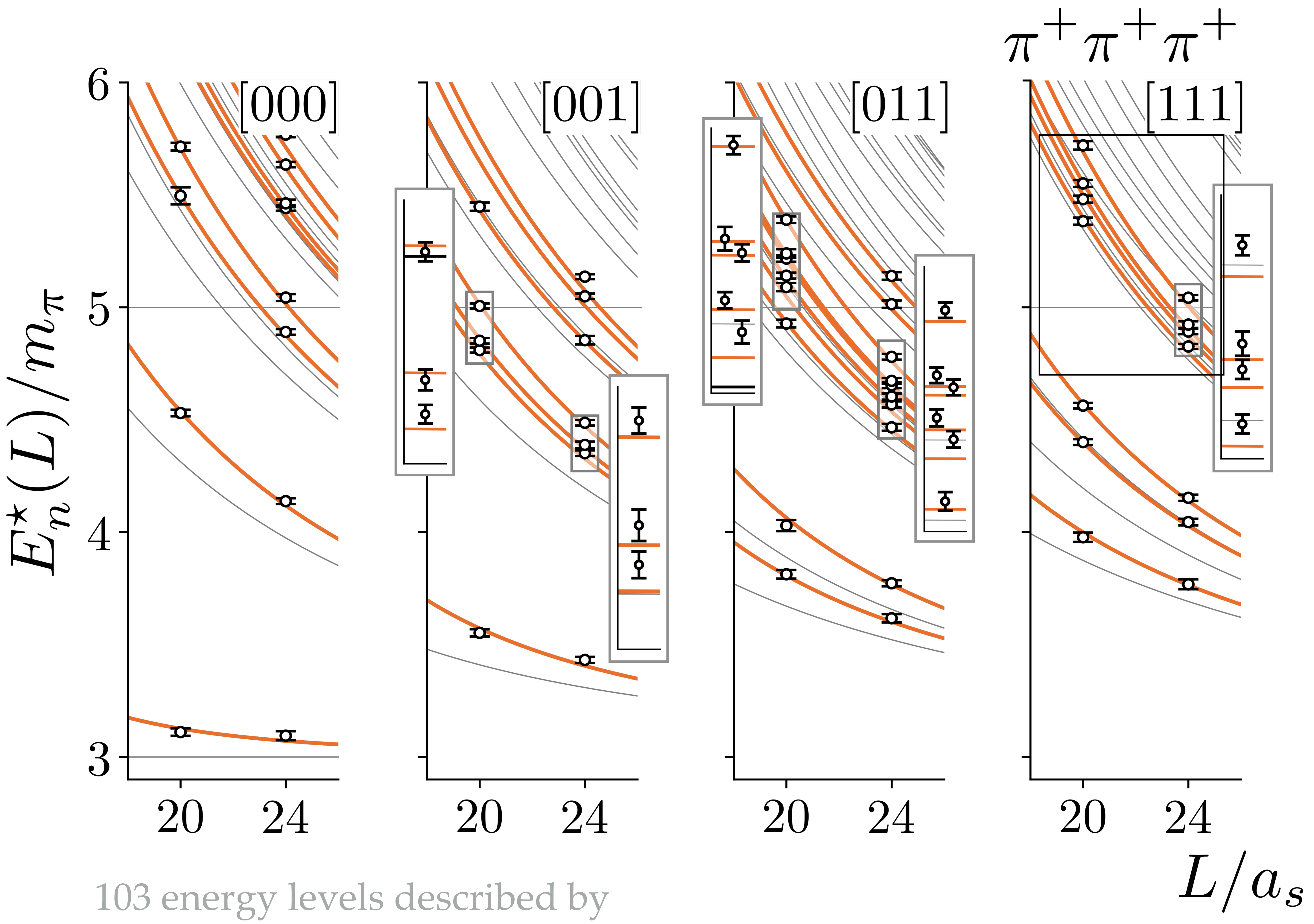

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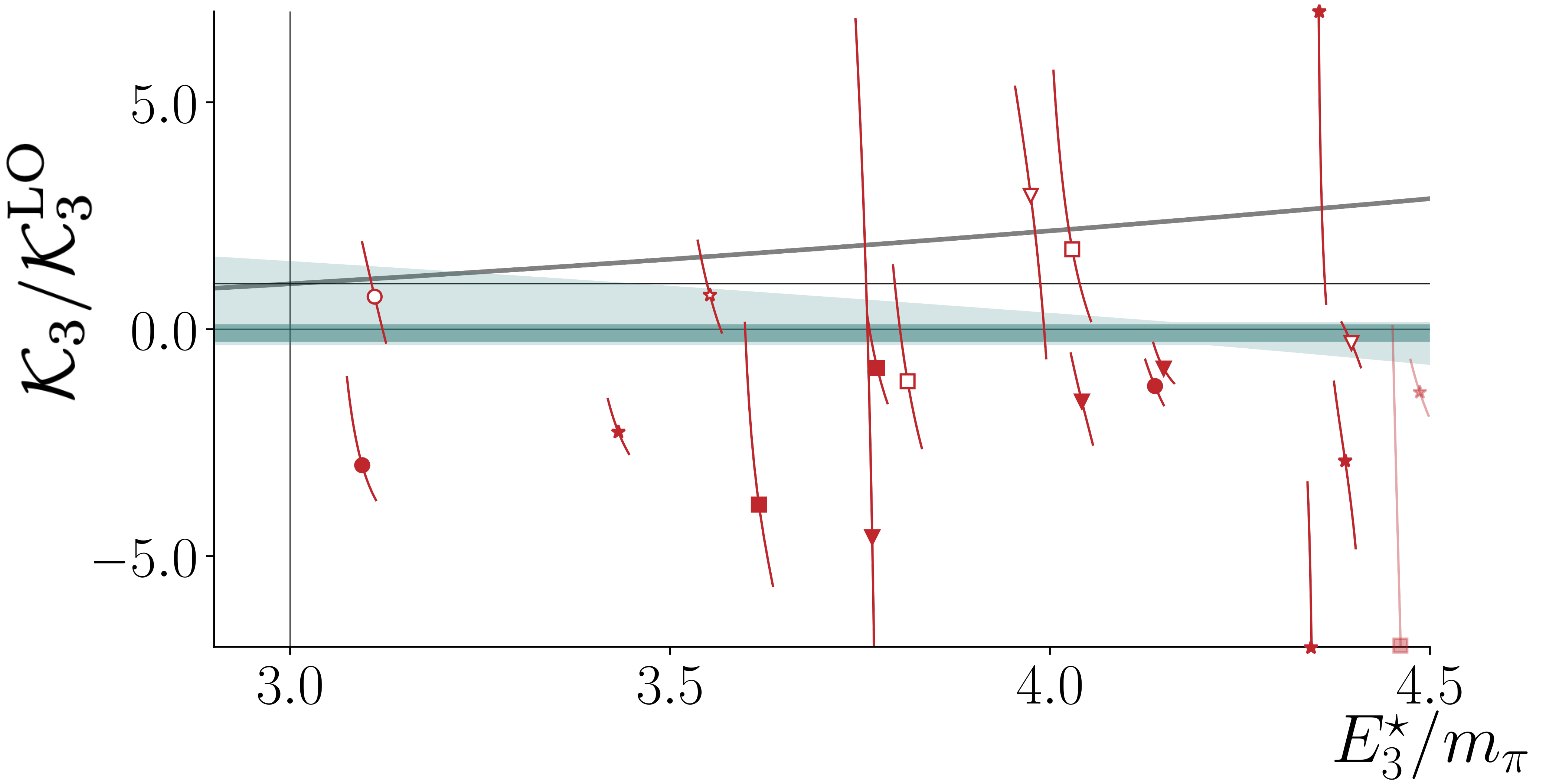

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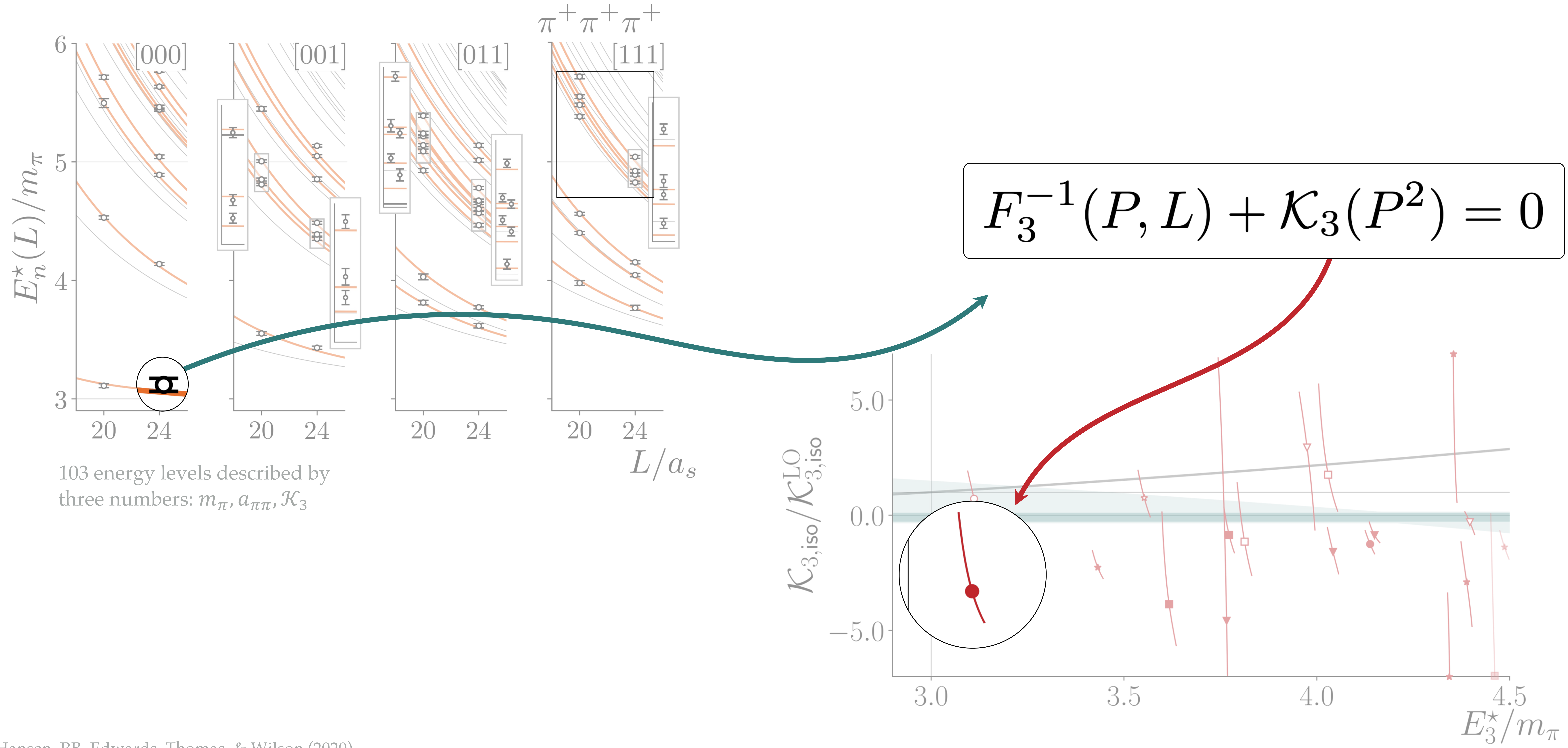

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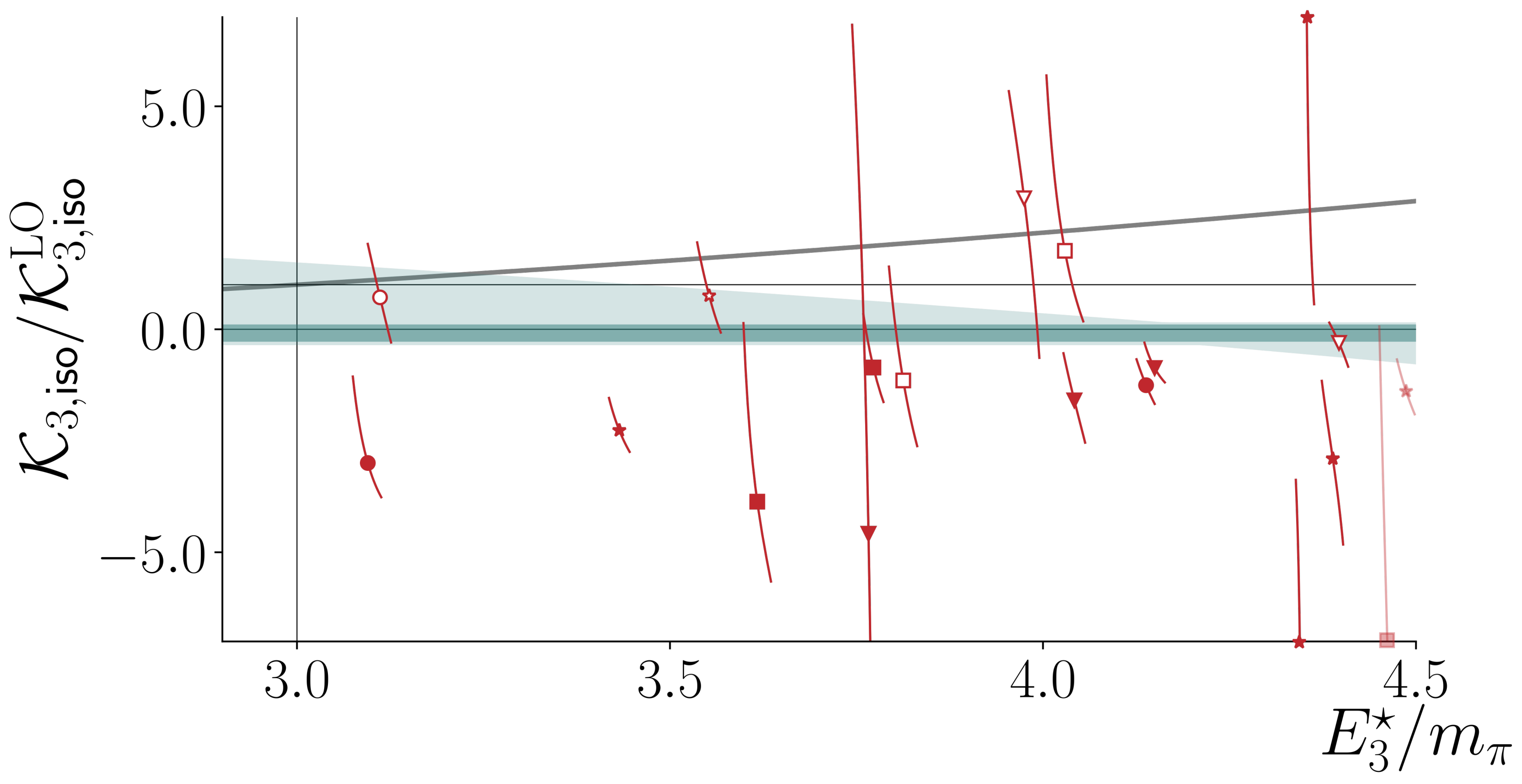
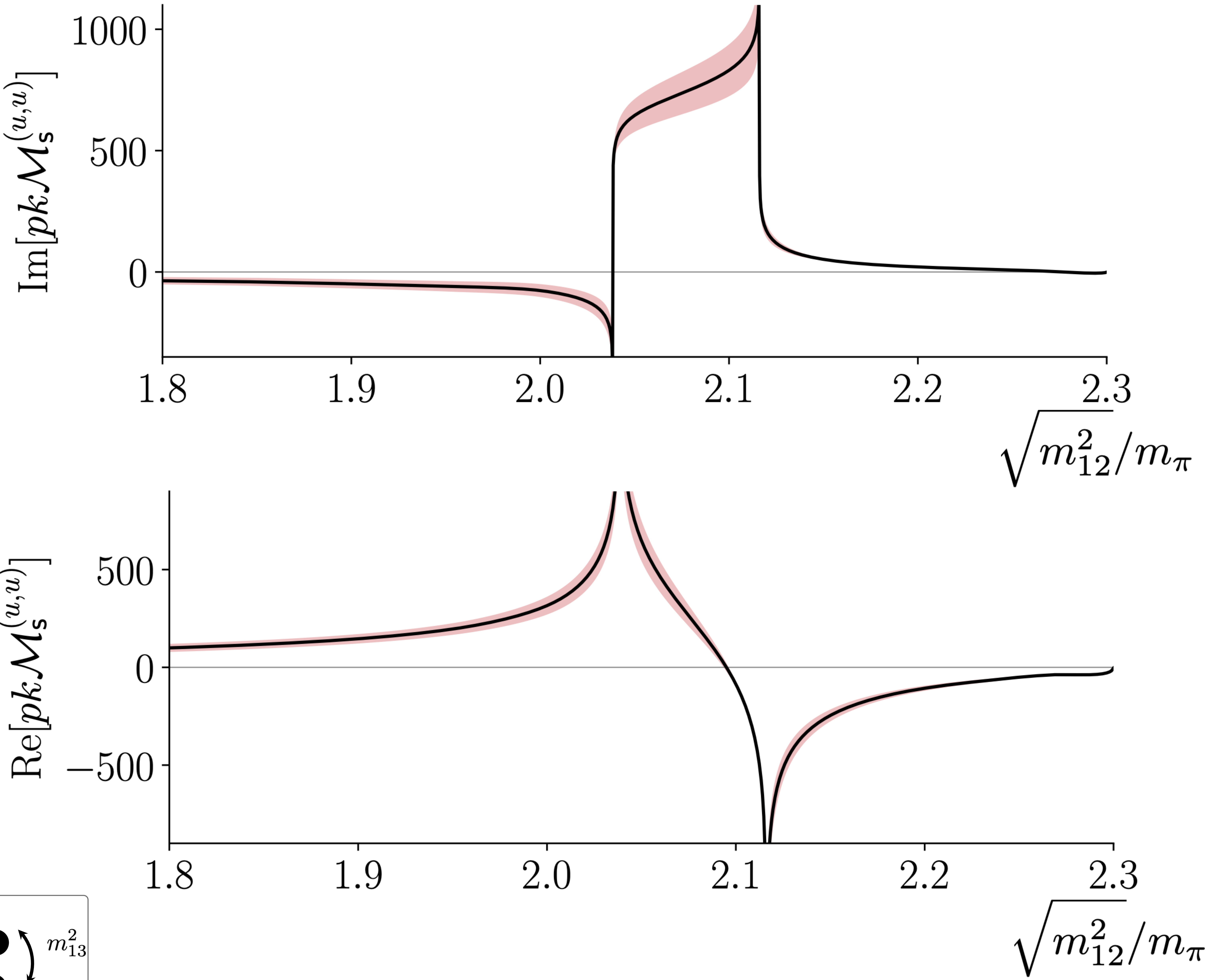
first 3body scattering amplitude from the lattice QCD!



$$i\mathcal{D} + i\mathcal{L}[\mathcal{D}] \cdot \mathcal{F}[\mathcal{D}, \mathcal{K}_3] \cdot \mathcal{R}[\mathcal{D}]$$



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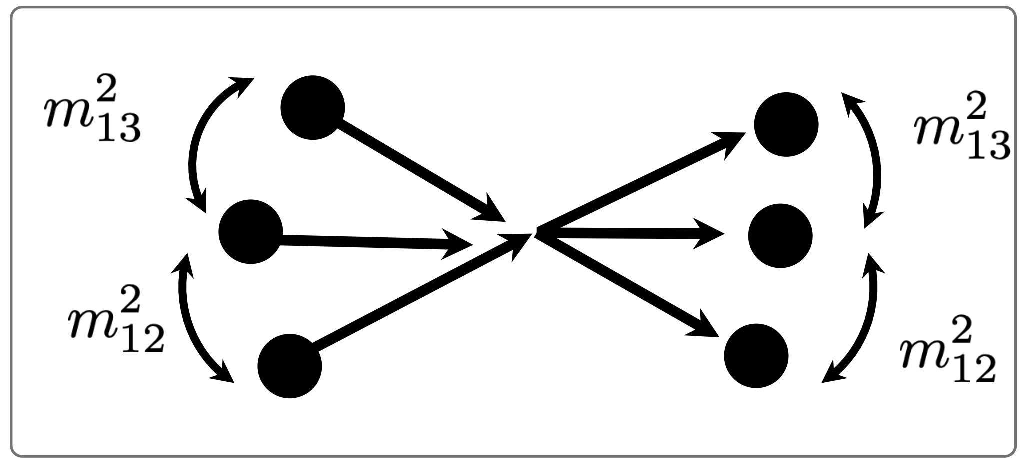

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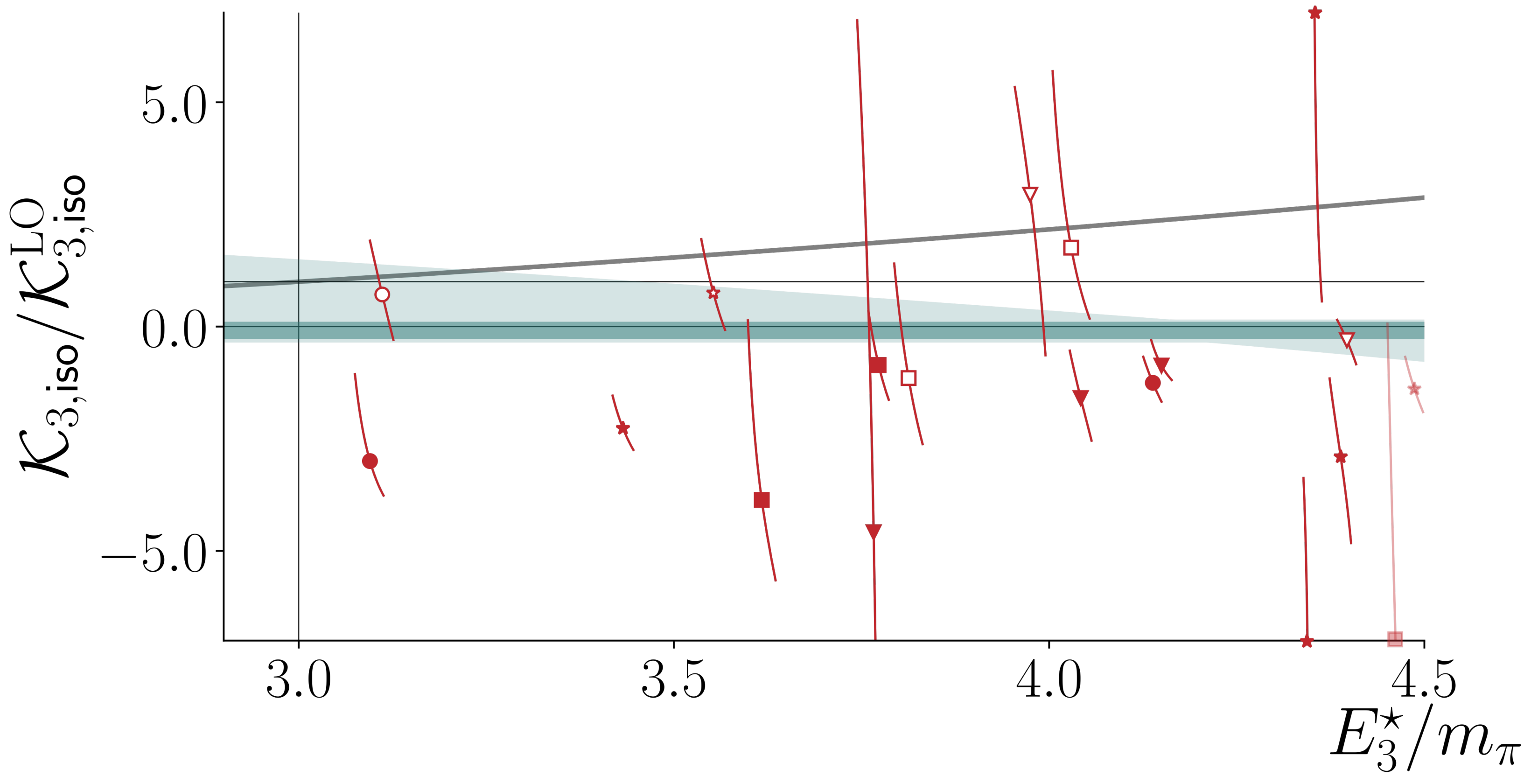

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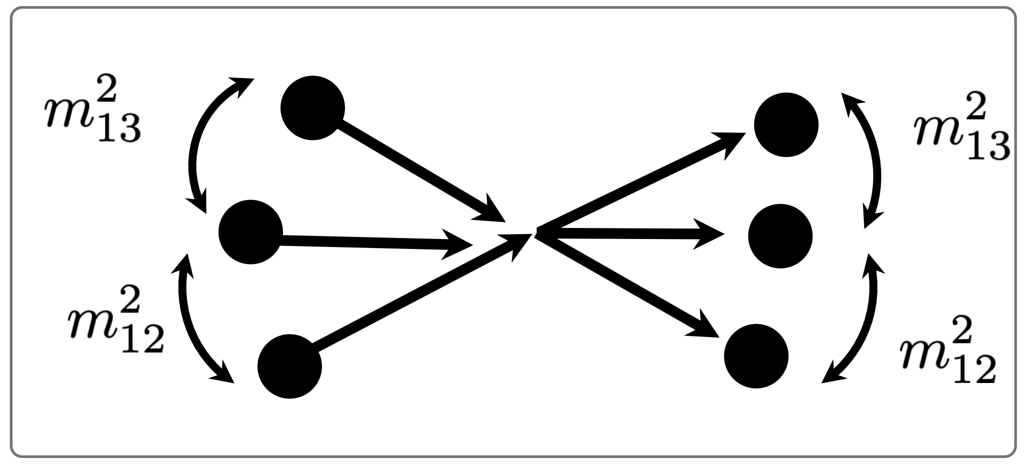
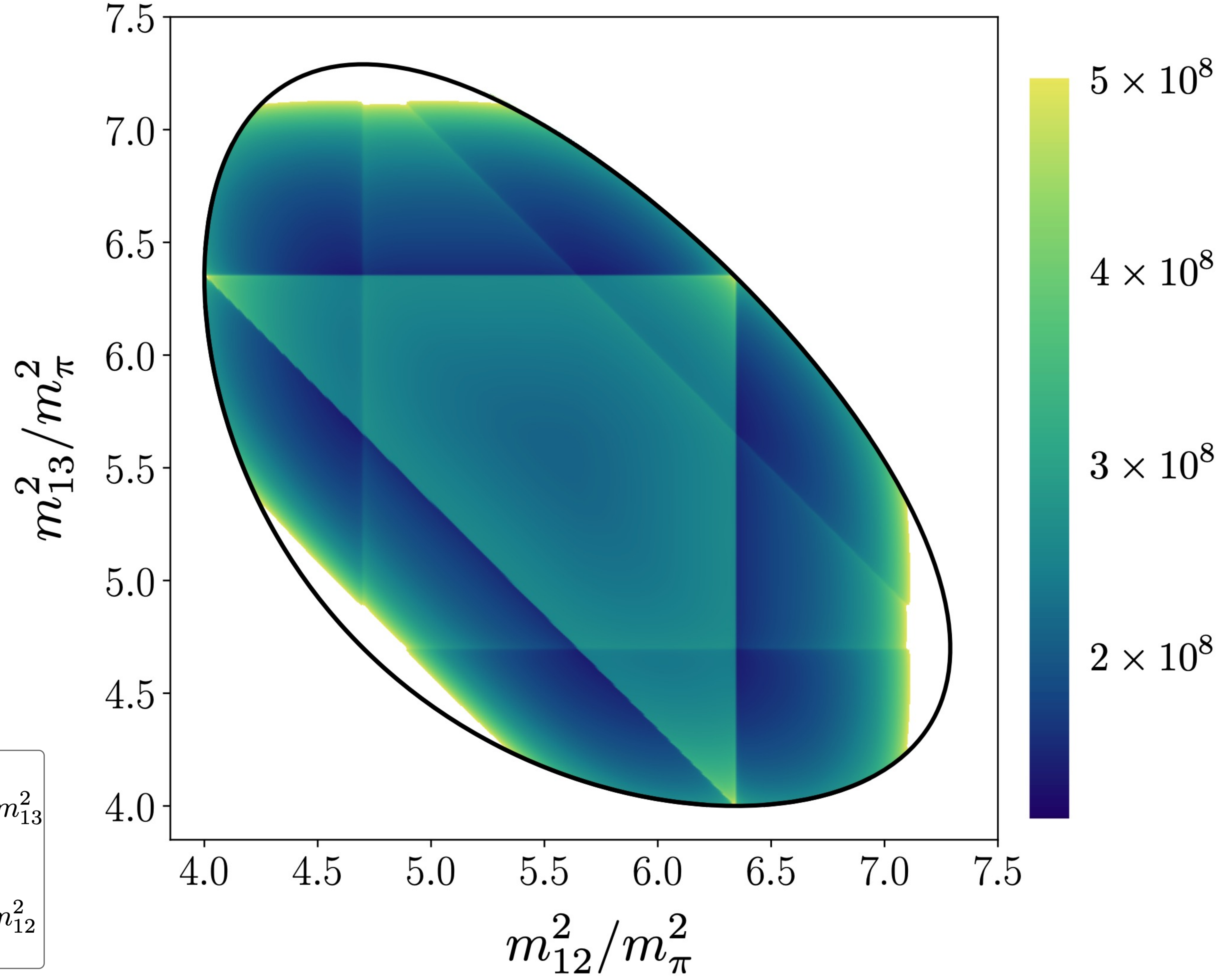
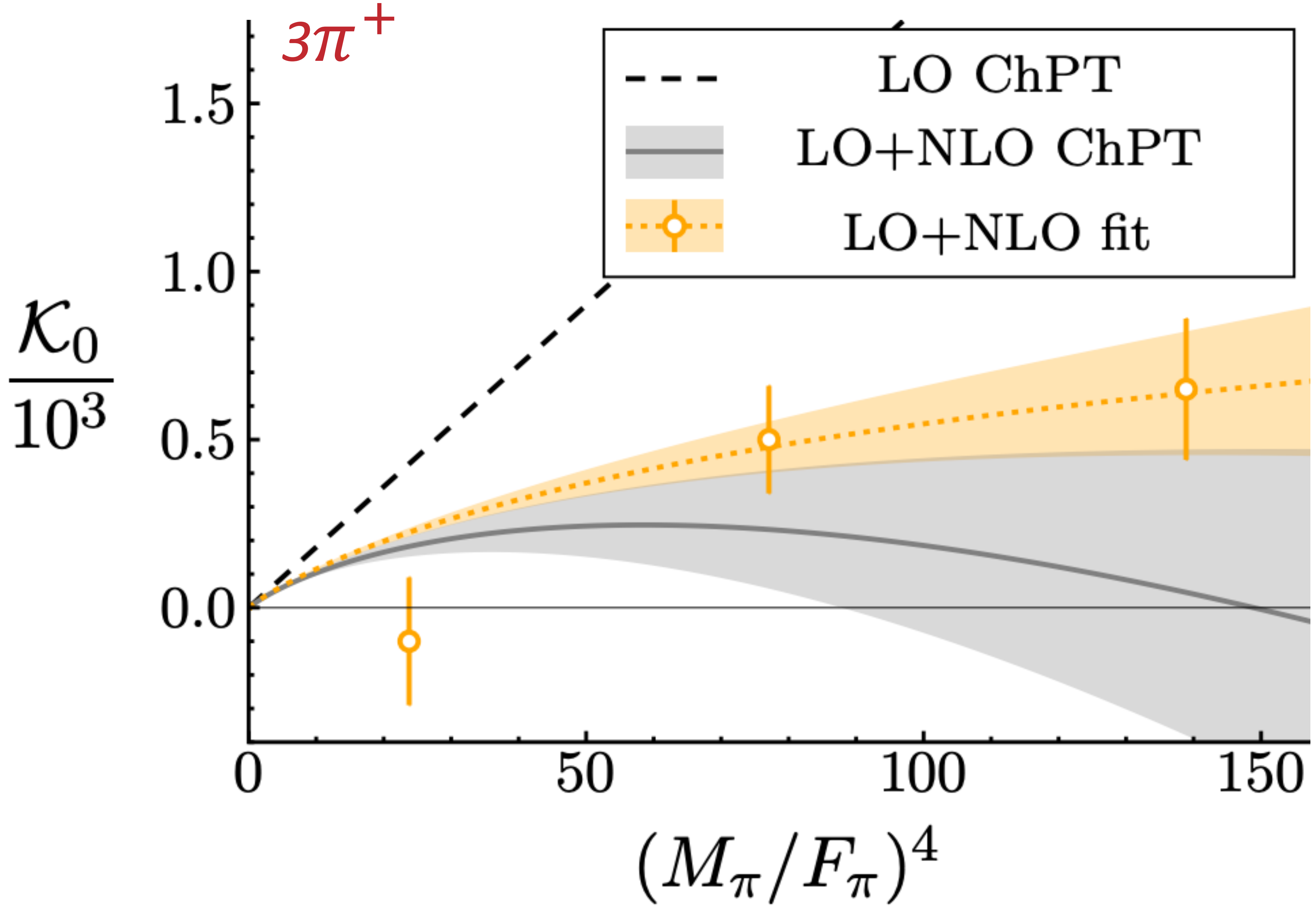


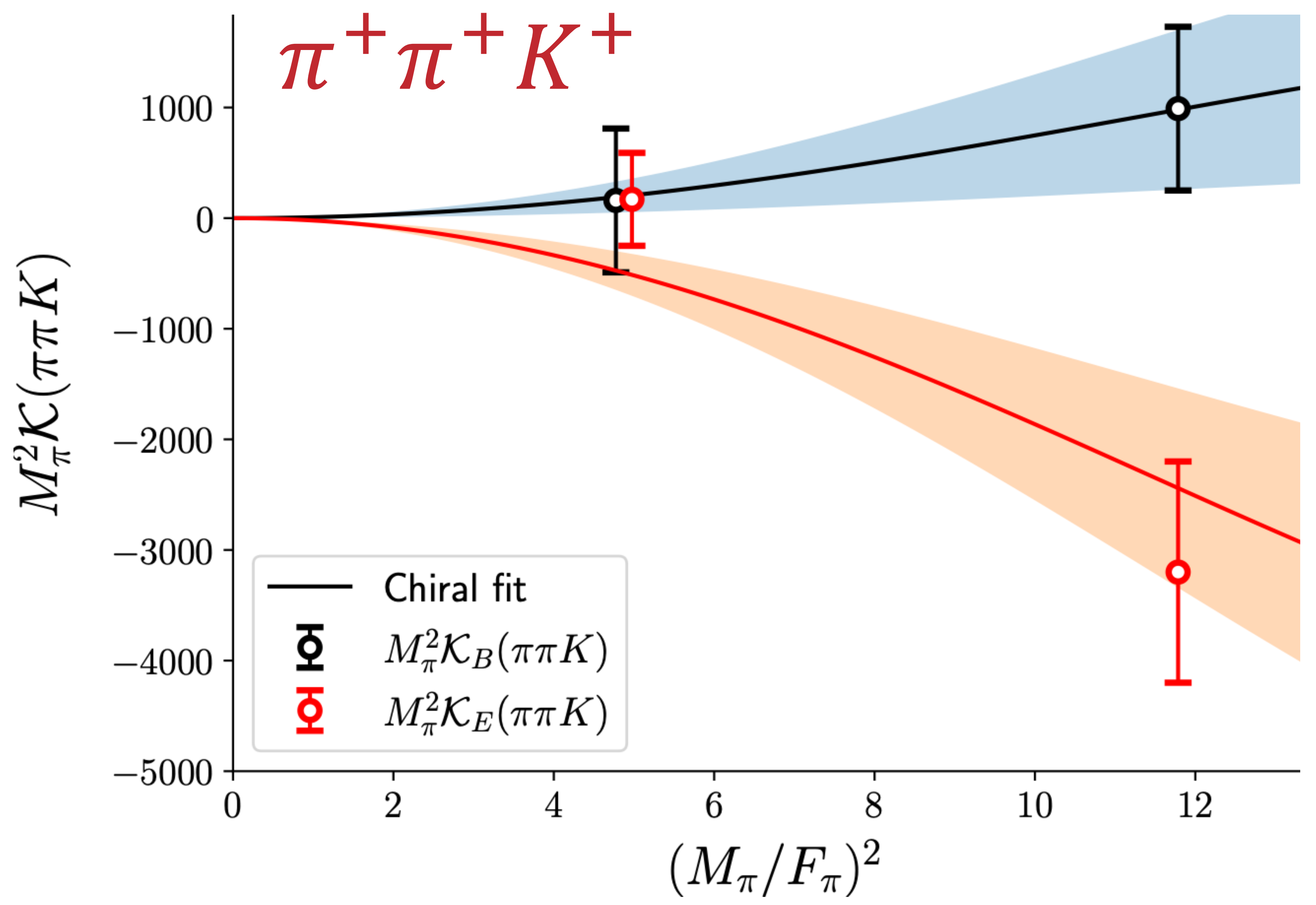


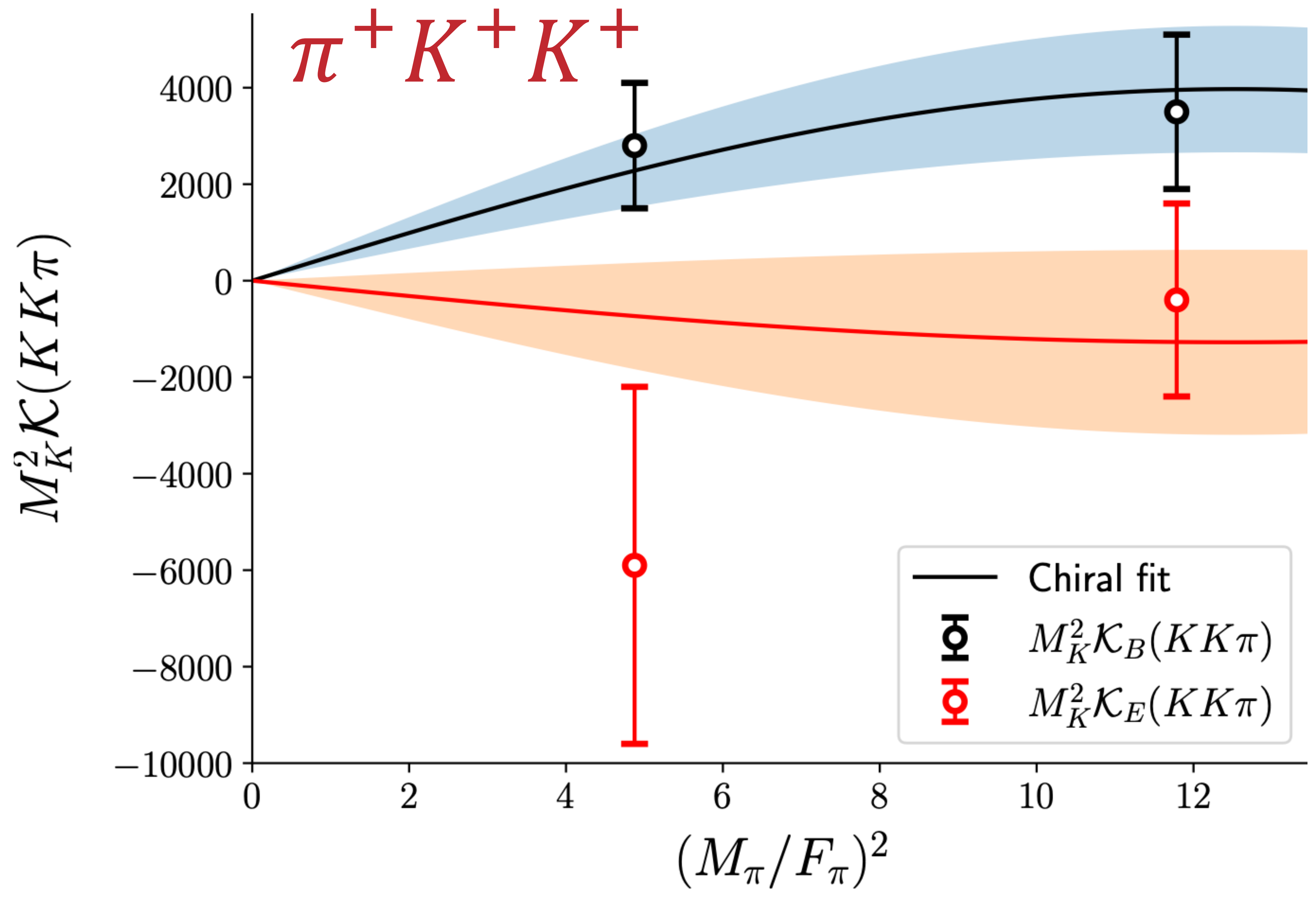
Hansen, RB, Edwards, Thomas, & Wilson (2020)

Quark-mass dependence

exploratory studies of the three-body K matrices







Blanton, et al. (2021); Draper, et al. (2023); Baeza-Ballesteros, et al. (2023)

Overarching goal

non-perturbatively constrain two- and three-hadron scattering amplitudes directly from the standard model (including electroweak & BSM probes)

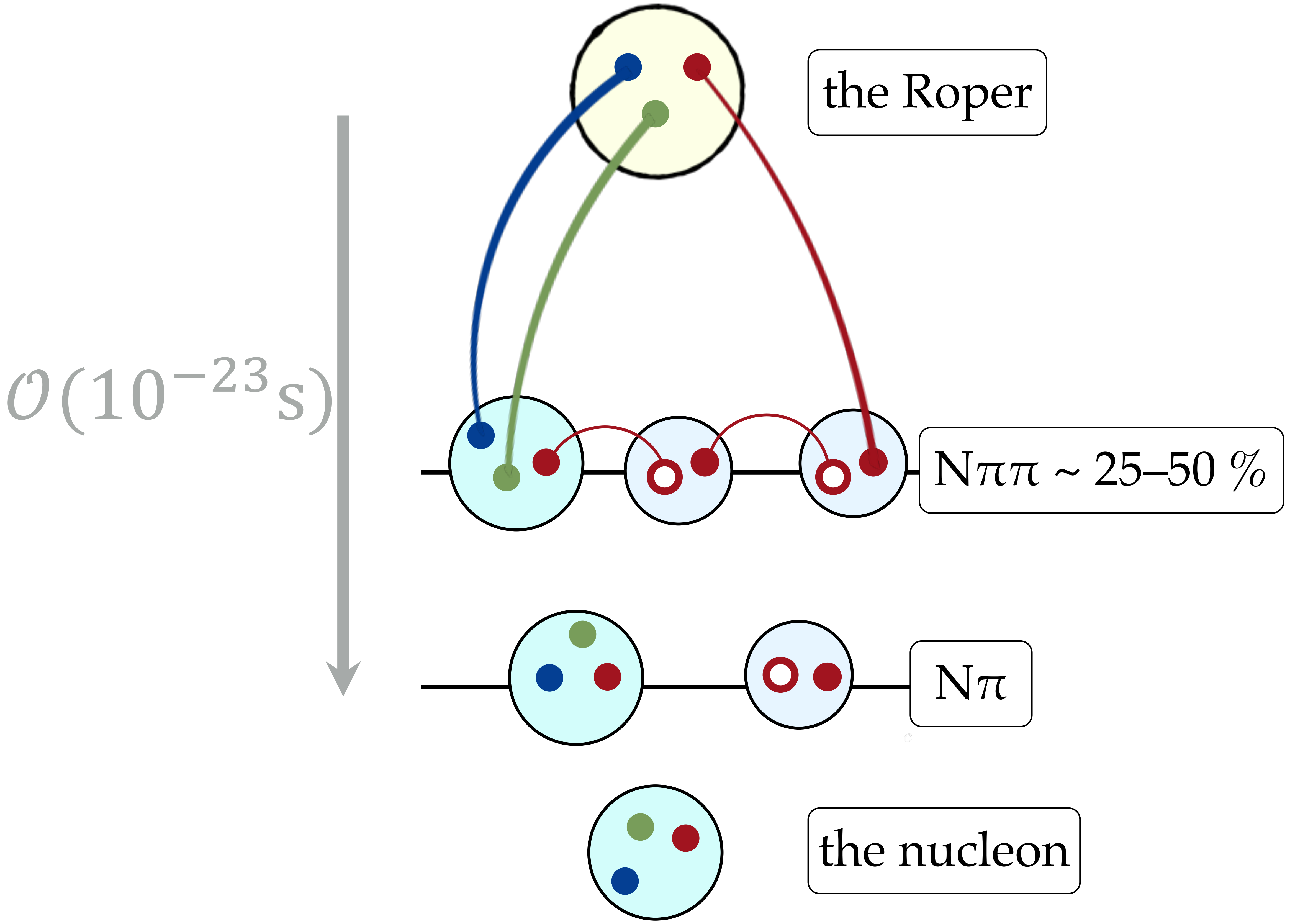
three questions to answer

- ☑ why are three-body so much harder? 😬
- ☑ what has been done? 🤓
- ☐ what can we expect to be done? 🤠

what can we expect to be done? in the next 5yrs

Formal issues:

- ☐ coupled 2-3 bodies,
- ☐ non-identical particles,
- ☐ electroweak production,
- ☐ non-zero intrinsic spin,
- ☐ electroweak probes,
- ☐ …



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Formal issues:

- ☐ coupled 2-3 bodies,
- ☐ non-identical particles,
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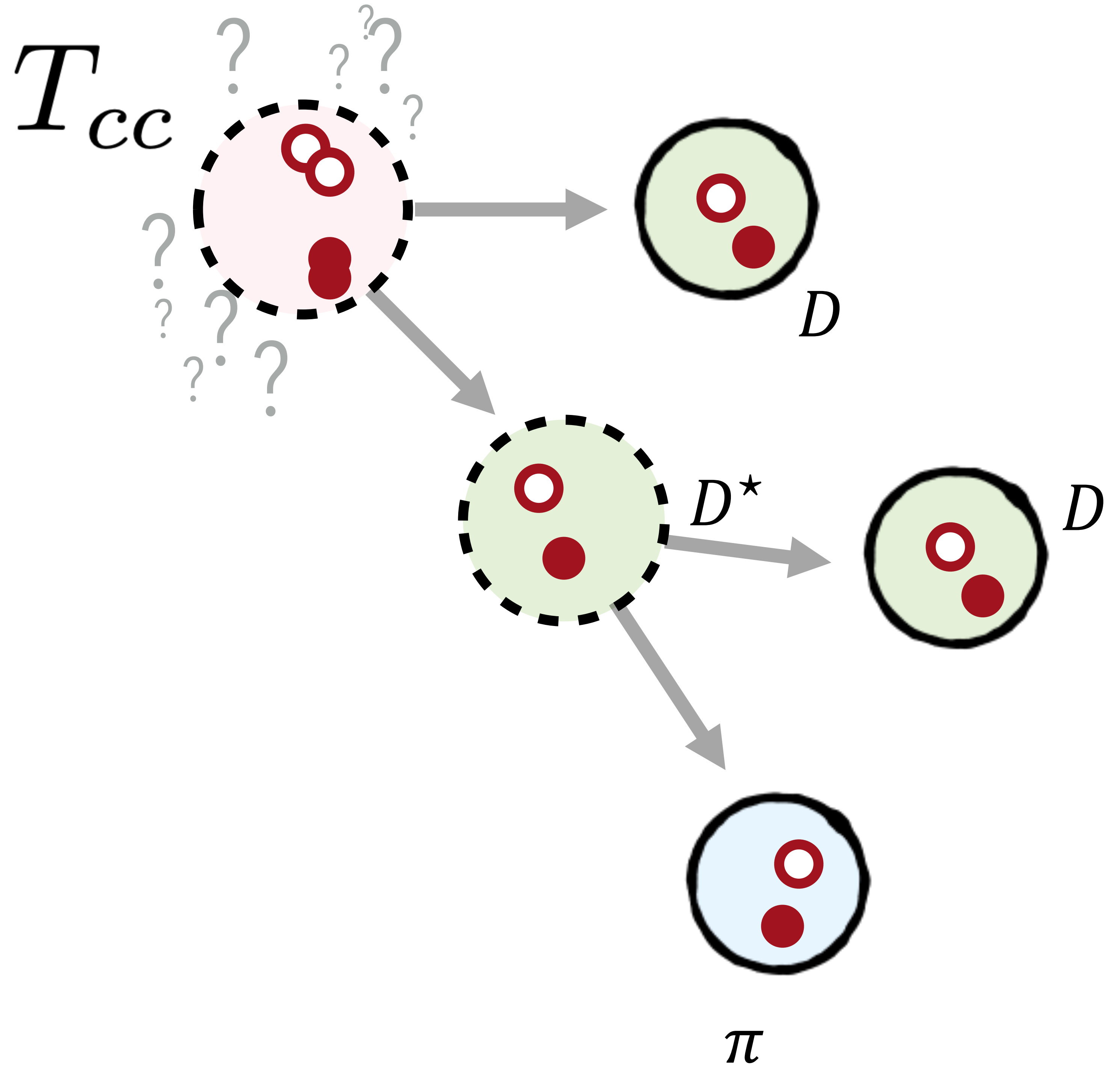
- ☐ non-zero intrinsic spin,
- ☐ electroweak probes,
- ☐ …

Exploratory lattice QCD:

- ☐ resonant / strongly interacting mesonic systems
  - ☐ 3π *channels*
- ☐ $T_{cc} \leftrightarrow DD^{\star} \leftrightarrow DD\pi$
- ☐ … $N\pi - N\pi\pi$…?

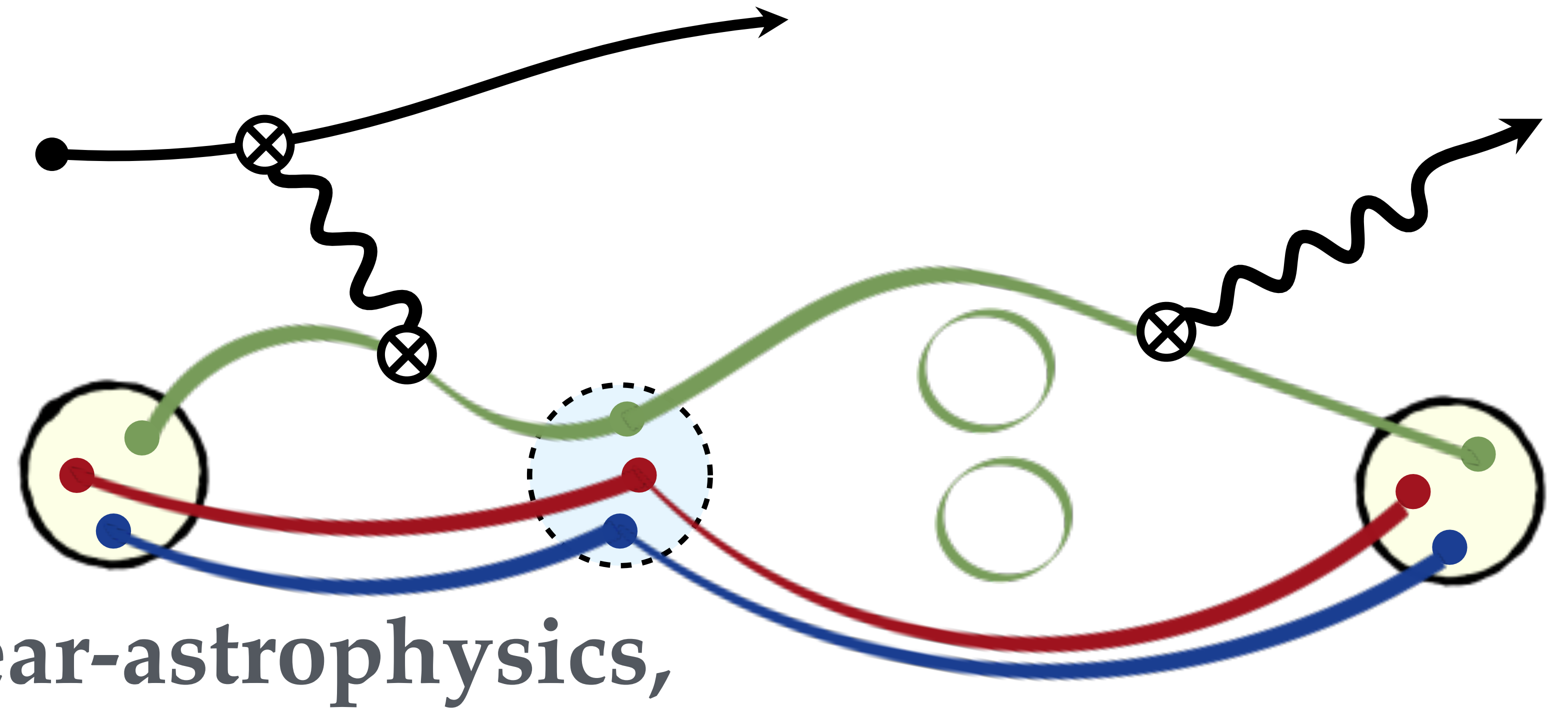


Symbiotic byproducts

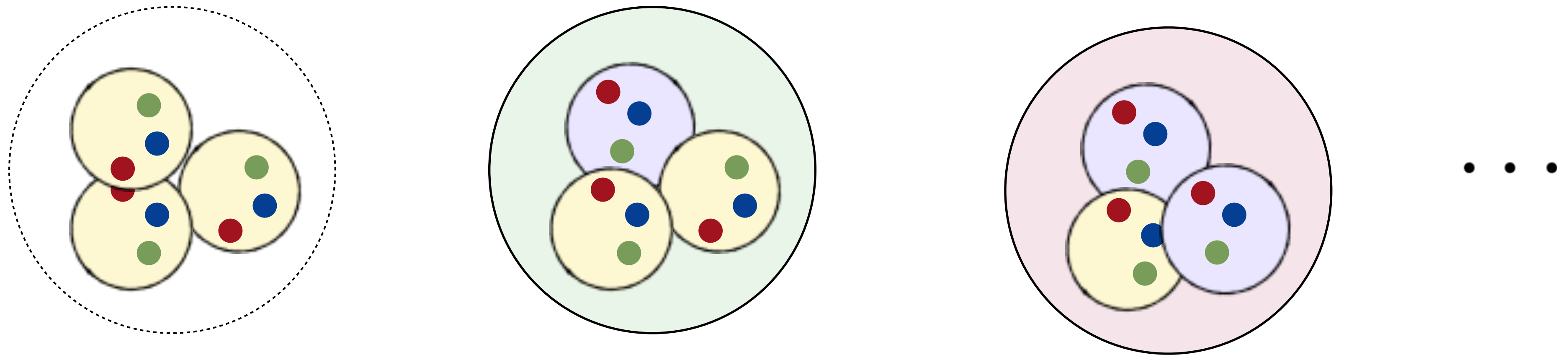
Formal & numerical tools being developed are universal.

These will impact studies in

☑ **hadron structure,**

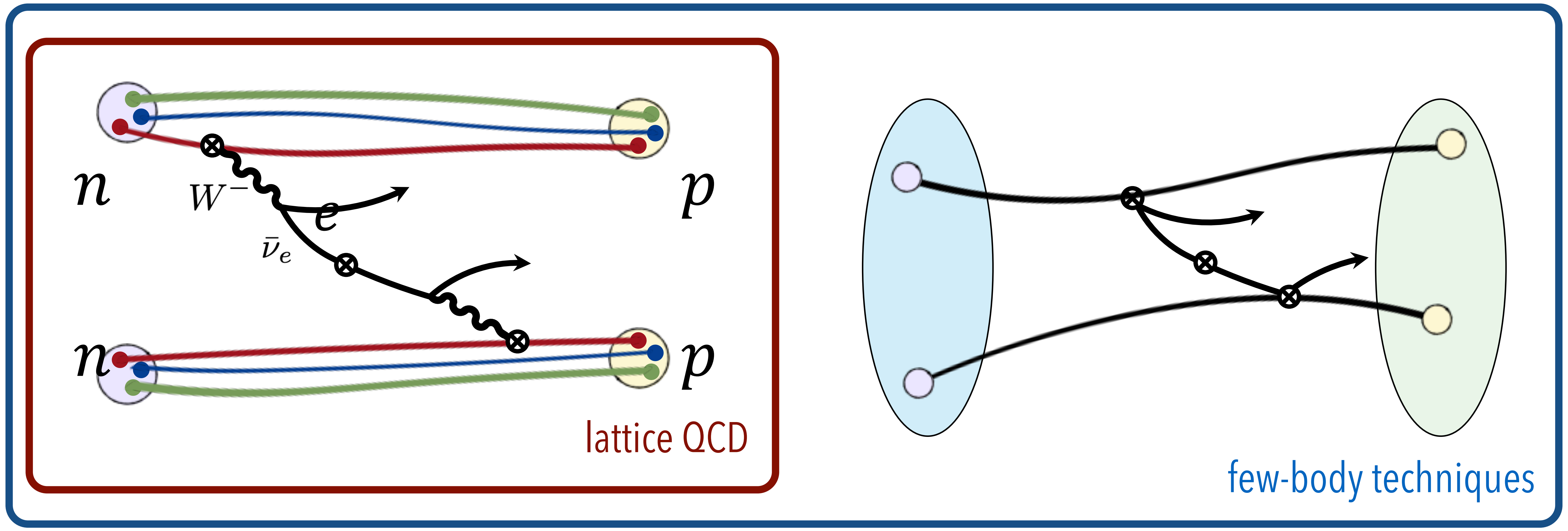


☑ **nuclear structure / nuclear-astrophysics,**

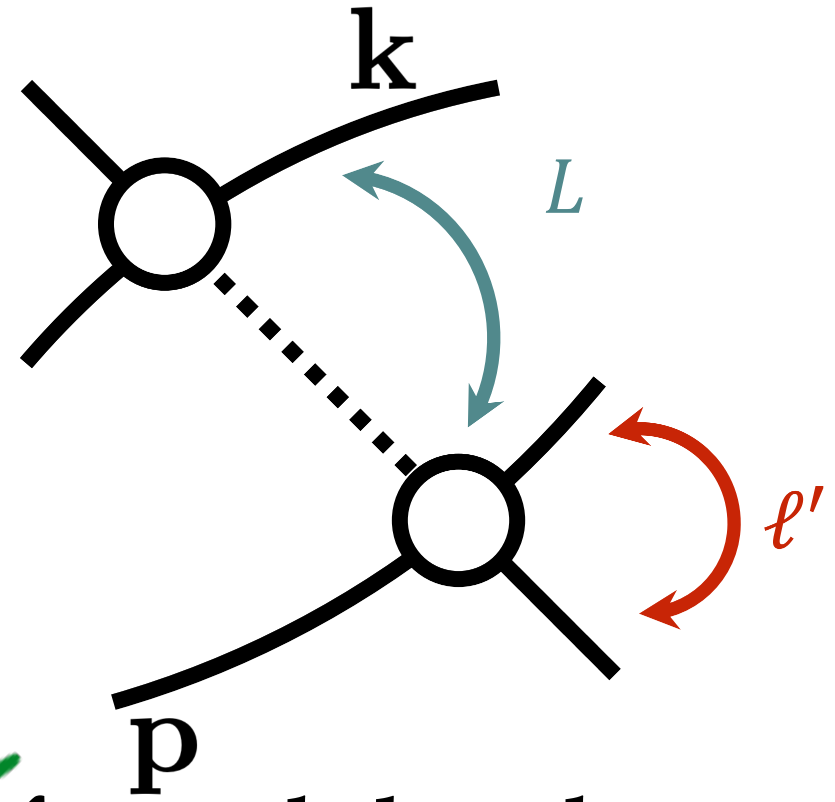


☑ **fundamental symmetries,**

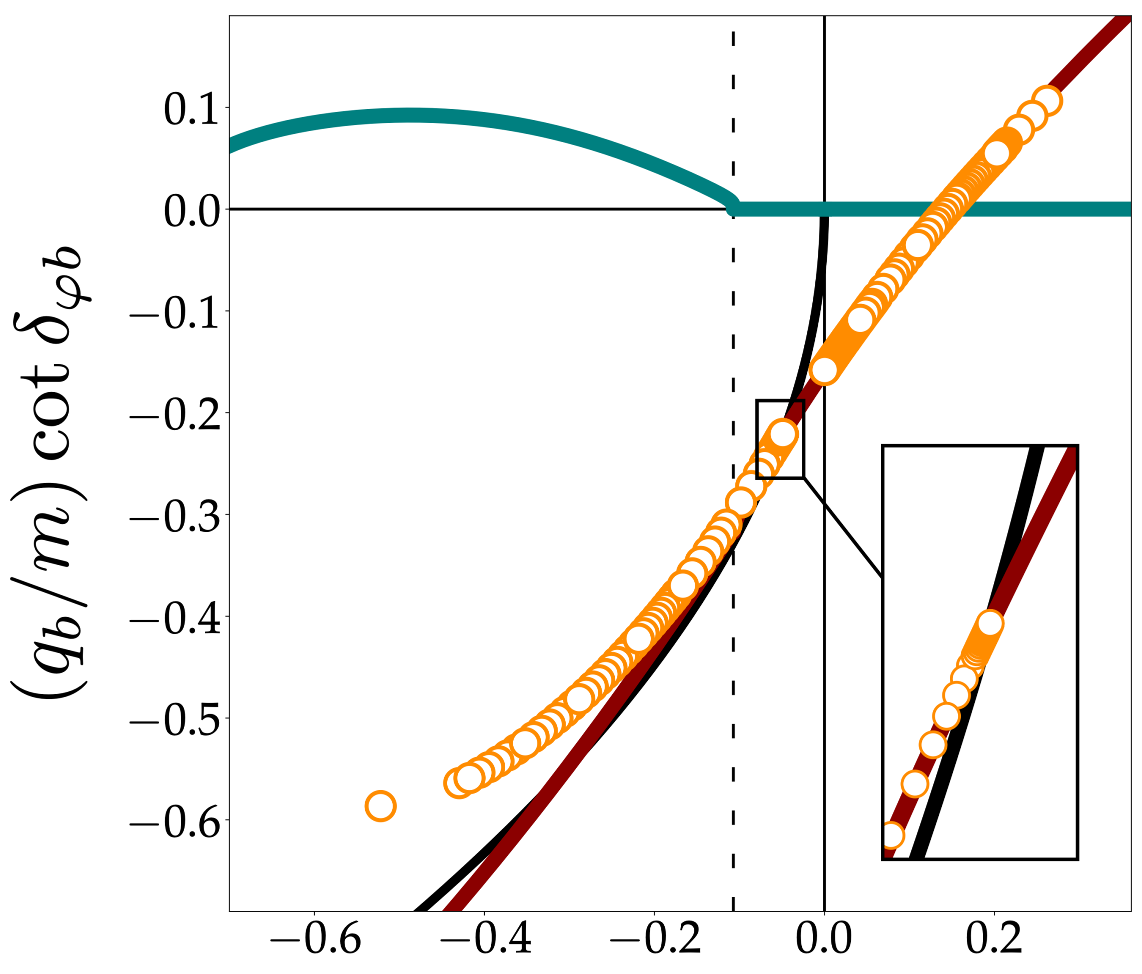
☑ **universal phenomena,….**



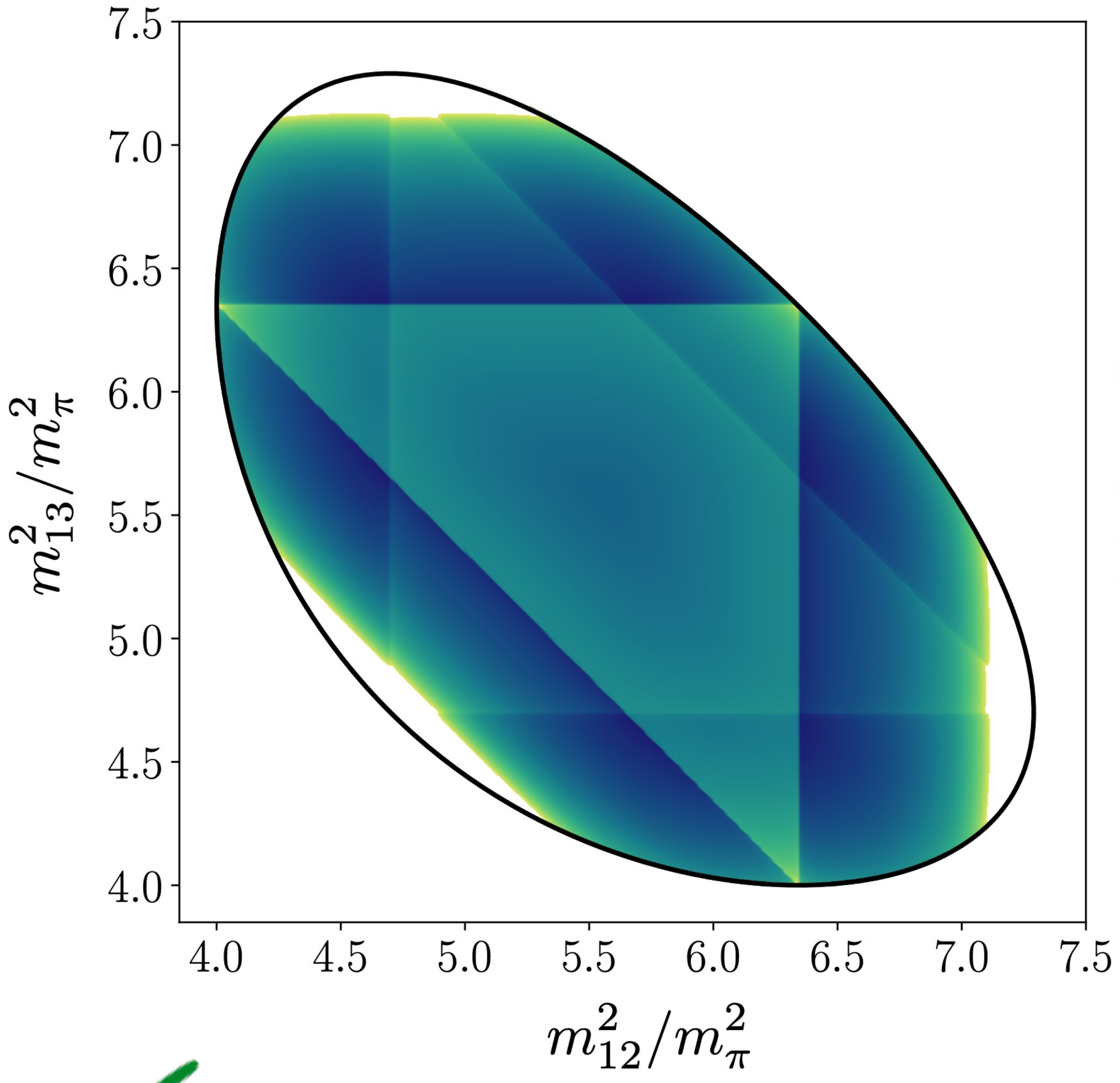
rapidly developing field!



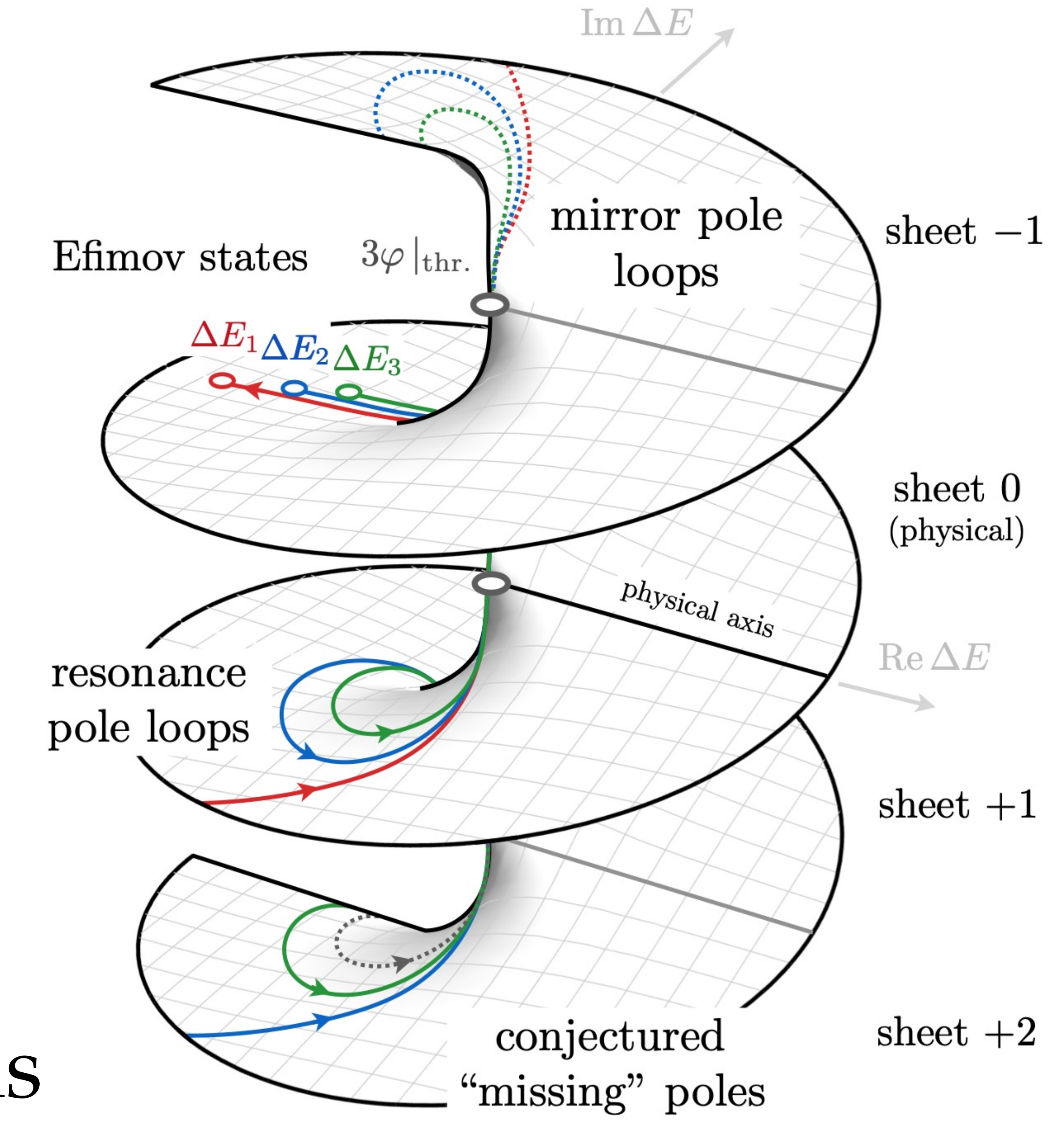
formal developments



checks of the formalism



actual lattice calculations



further explorations

ExoHad/Berkely
2025 School and Workshop

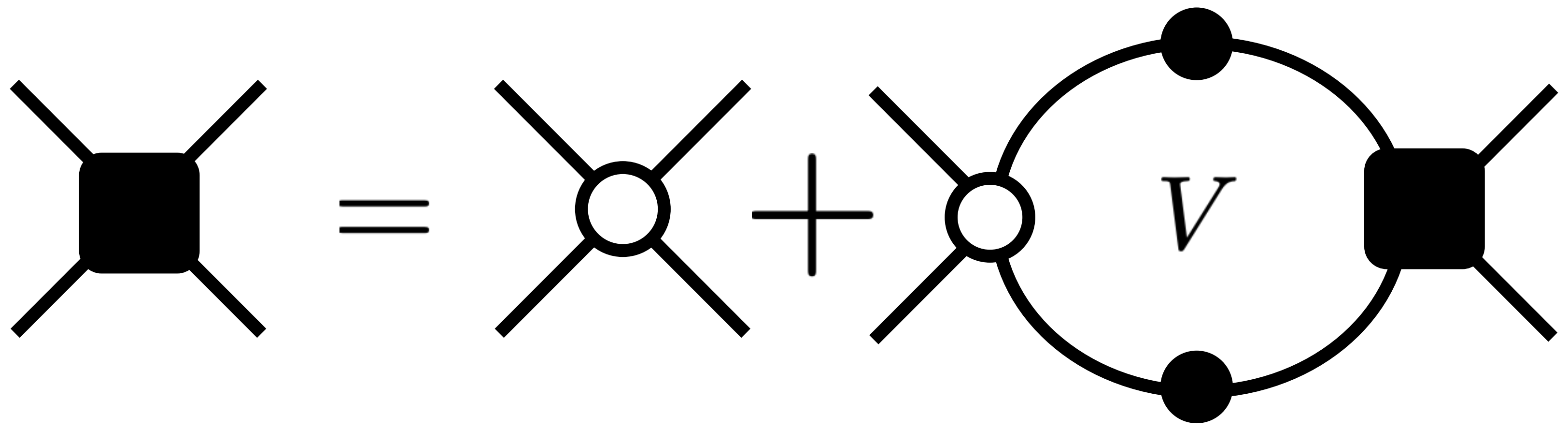
EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

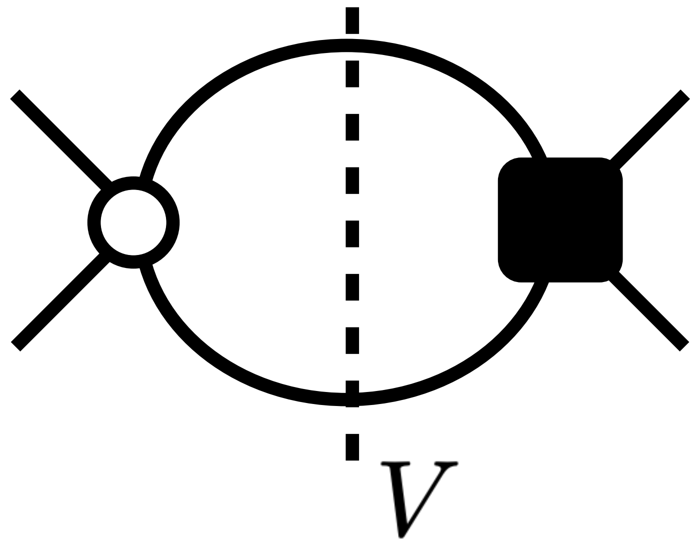
BERKELEY LAB

Berkeley
UNIVERSITY OF CALIFORNIA

Two particle in finite volume

Similar story as before…except momenta are discrete $\vec{k} = 2\pi \vec{n}/L$

$i\mathcal{M}_L =$ 



$$= [iB]_{\ell' m'} \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{2\omega_k)} \frac{i\mathcal{Y}_{\ell' m'}(\hat{k})\, \mathcal{Y}^*_{\ell m}(\hat{k})}{(P-k)^2 - m^2 + i\epsilon} \right) [i\mathcal{M}_L]_{\ell m}$$

$$\equiv [iB]\, iF\, [iB]$$

$$F = \begin{pmatrix} F_{00;00} & F_{00;11} & F_{00;10} & & \\ F_{11;00} & F_{11;11} & F_{11;10} & & \\ F_{10;00} & F_{10;11} & F_{10;10} & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}$$

non-diagonal matrix over partial waves…because angular momentum is not a good quantum number

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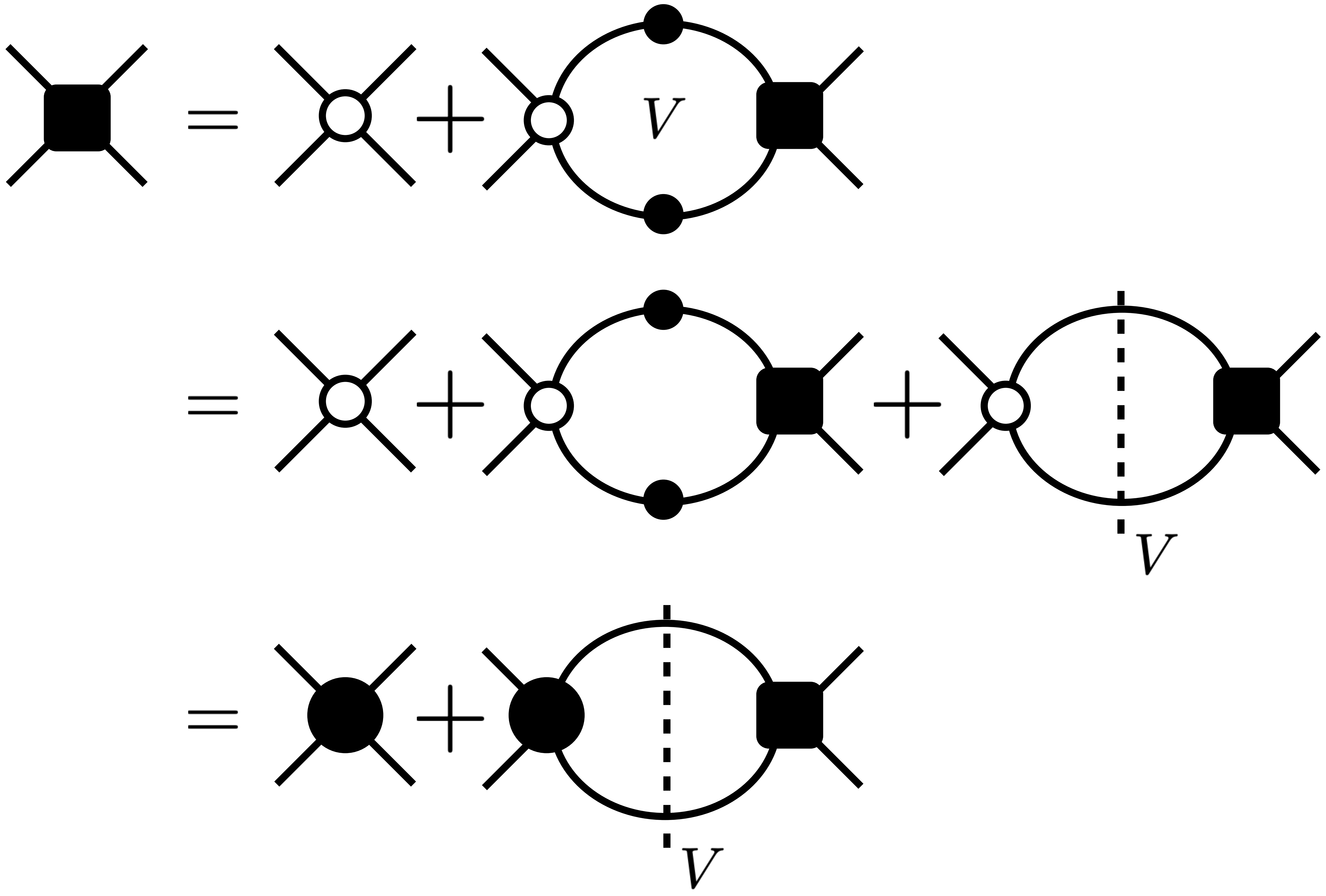

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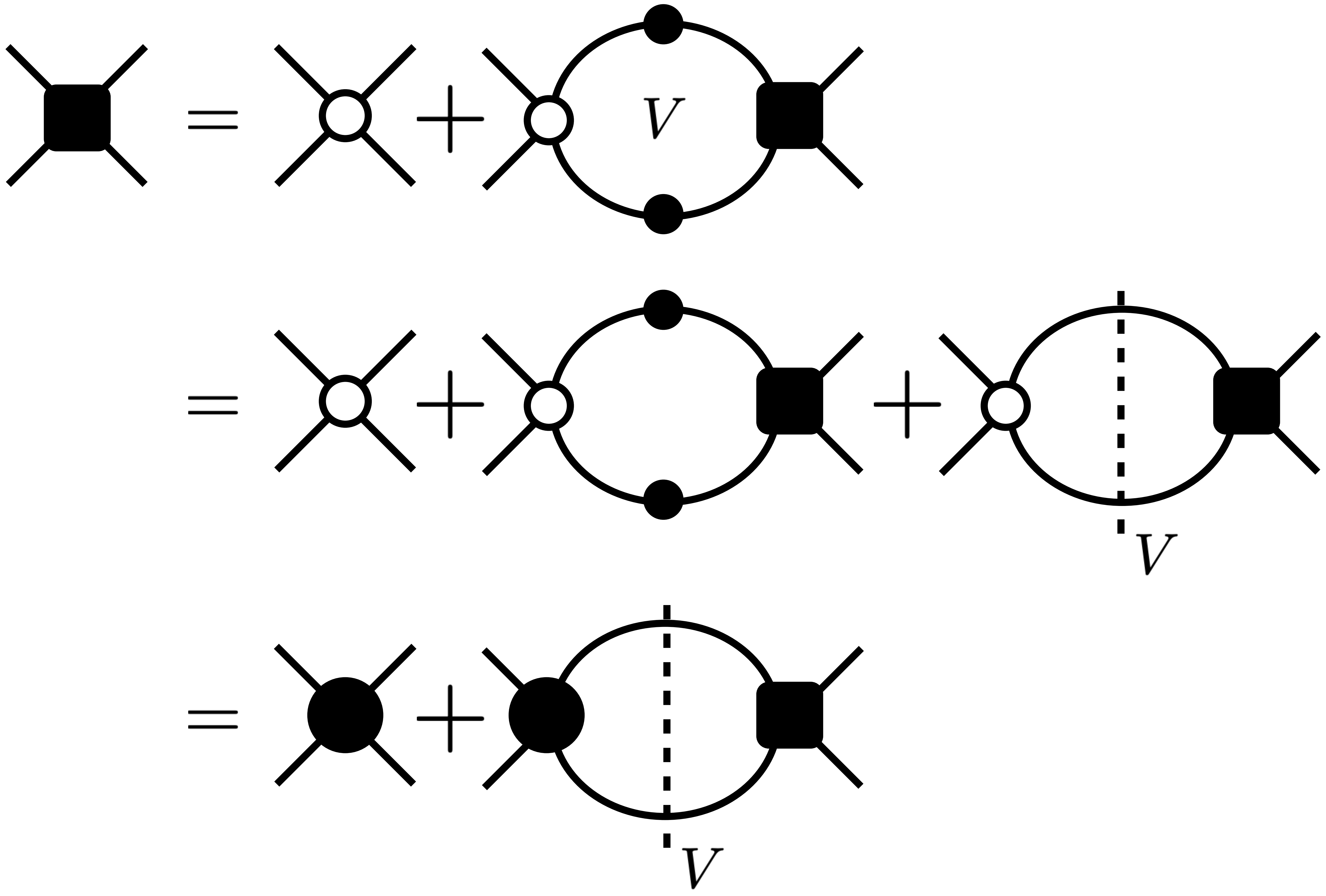


placing all legs on-shell
& partial-wave projecting

$$\longrightarrow \; i\mathcal{M}\frac{1}{1 + F\mathcal{M}}$$

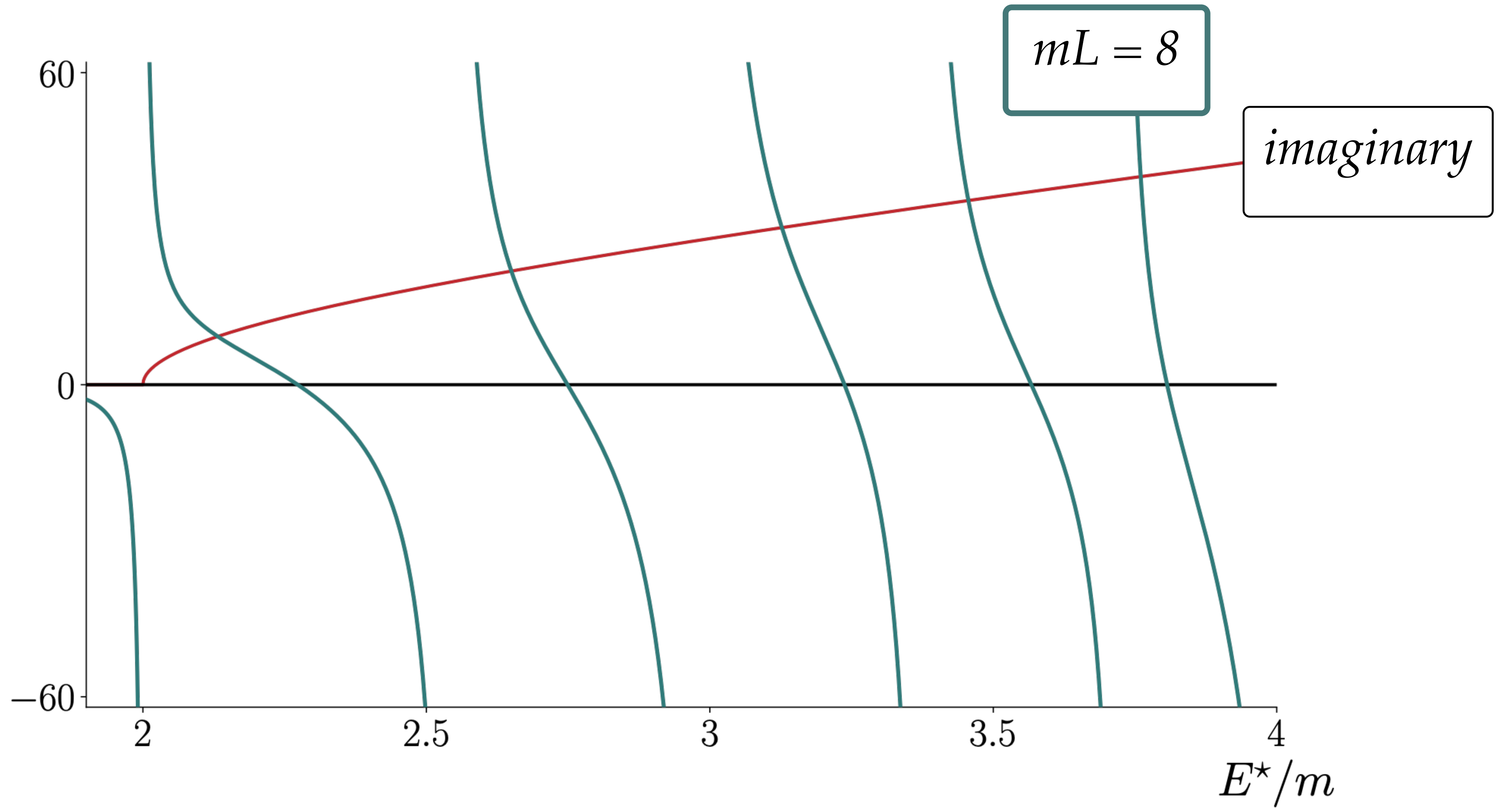
$$\det[F^{-1} + \mathcal{M}] = 0$$

poles satisfy…

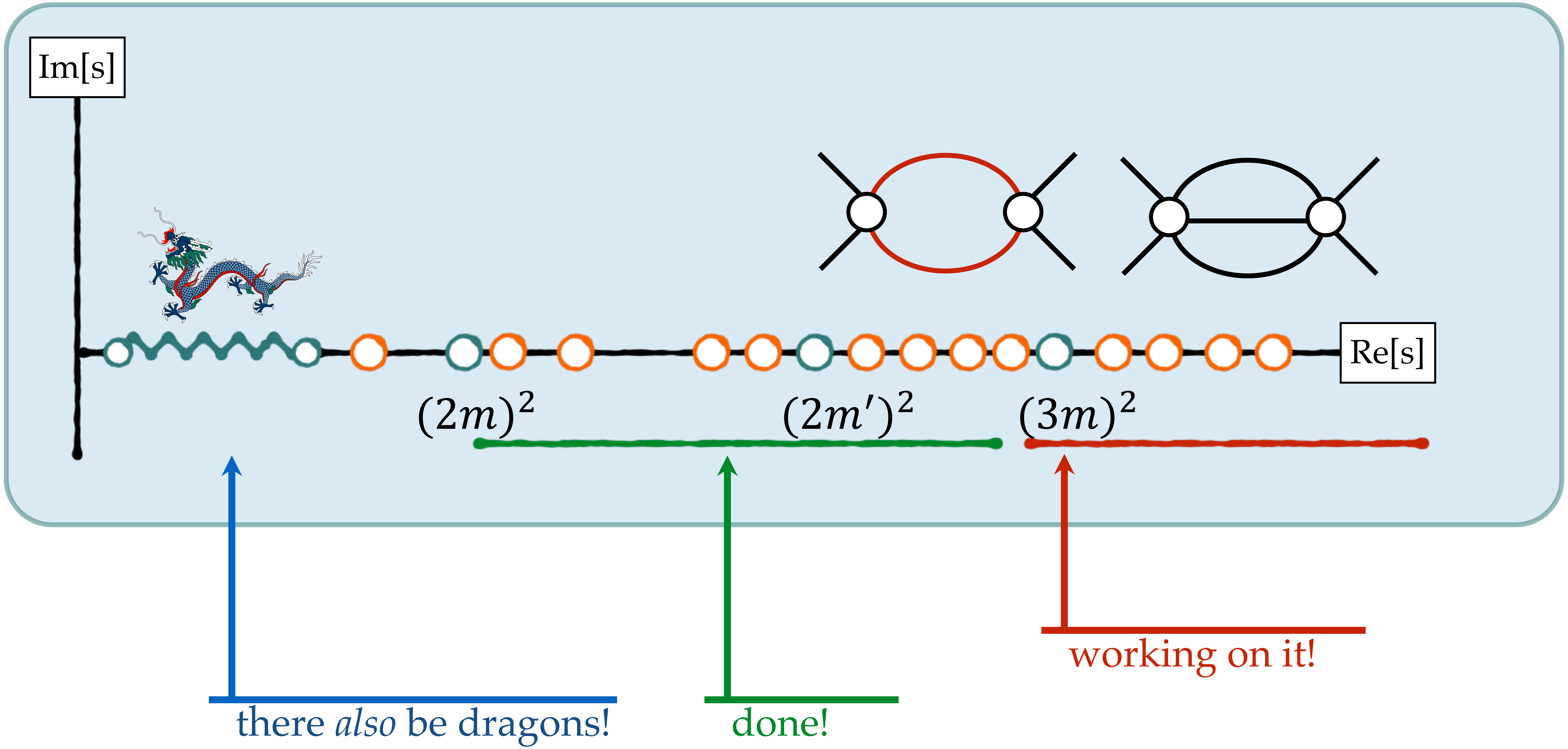
Some comments

$$\det[F^{-1}(P,L) + \mathcal{M}(P^2)] = 0$$

- ☑ exact up to $\mathcal{O}(e^{-m_\pi L})$,
- ☑ Mapping, not an extrapolationg,
- ☑ Not one-to-one [no asymptotic states & angular momentum is not a good quantum number],
- ☑ For moderate energies, low partial waves saturate the amplitude,
- ☑ We know F arbitrary boost, so we can further constraint the amplitude by considered boosted systems.

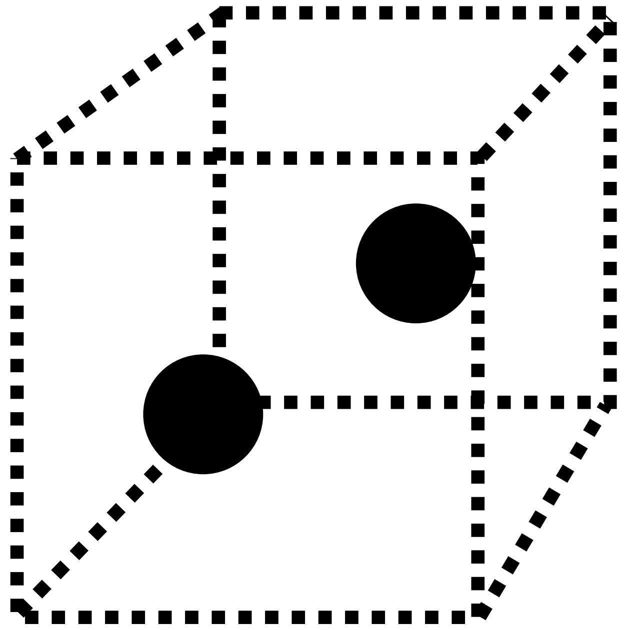


Going to higher energies

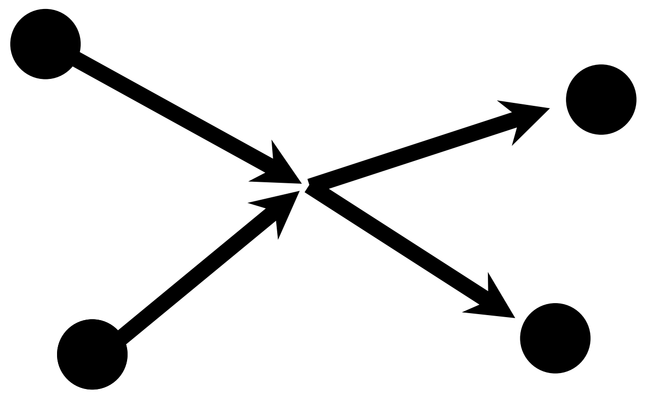


Outline

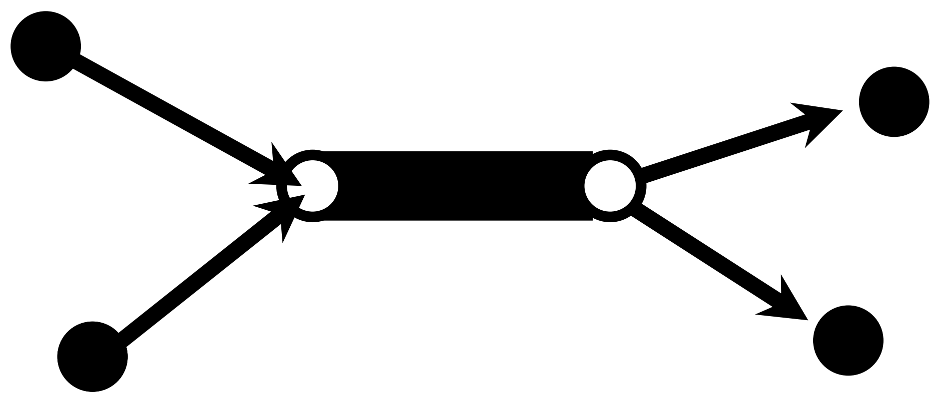
- ☑ Formalism



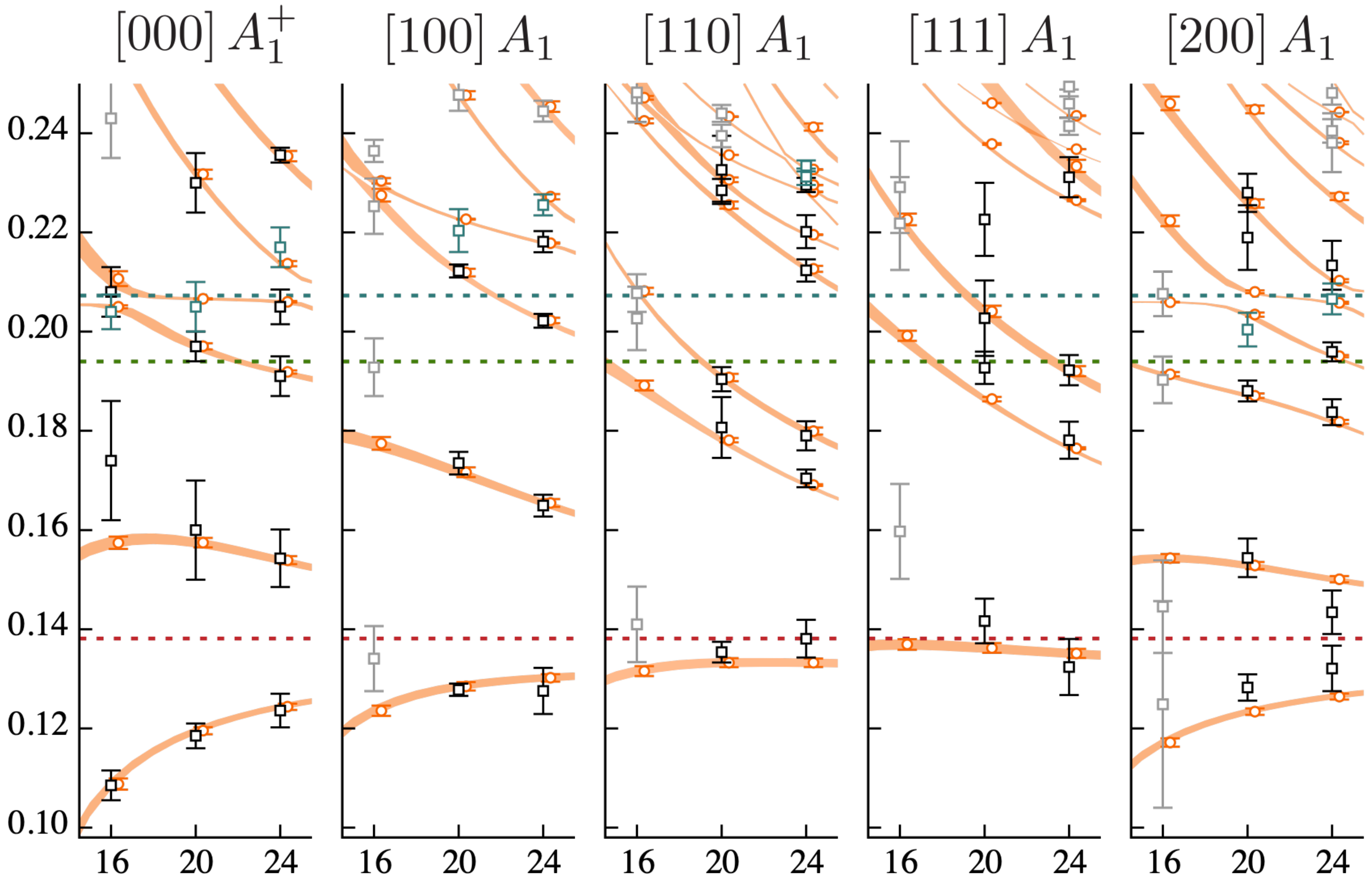
$\det[F^{-1} + \mathcal{M}] = 0$

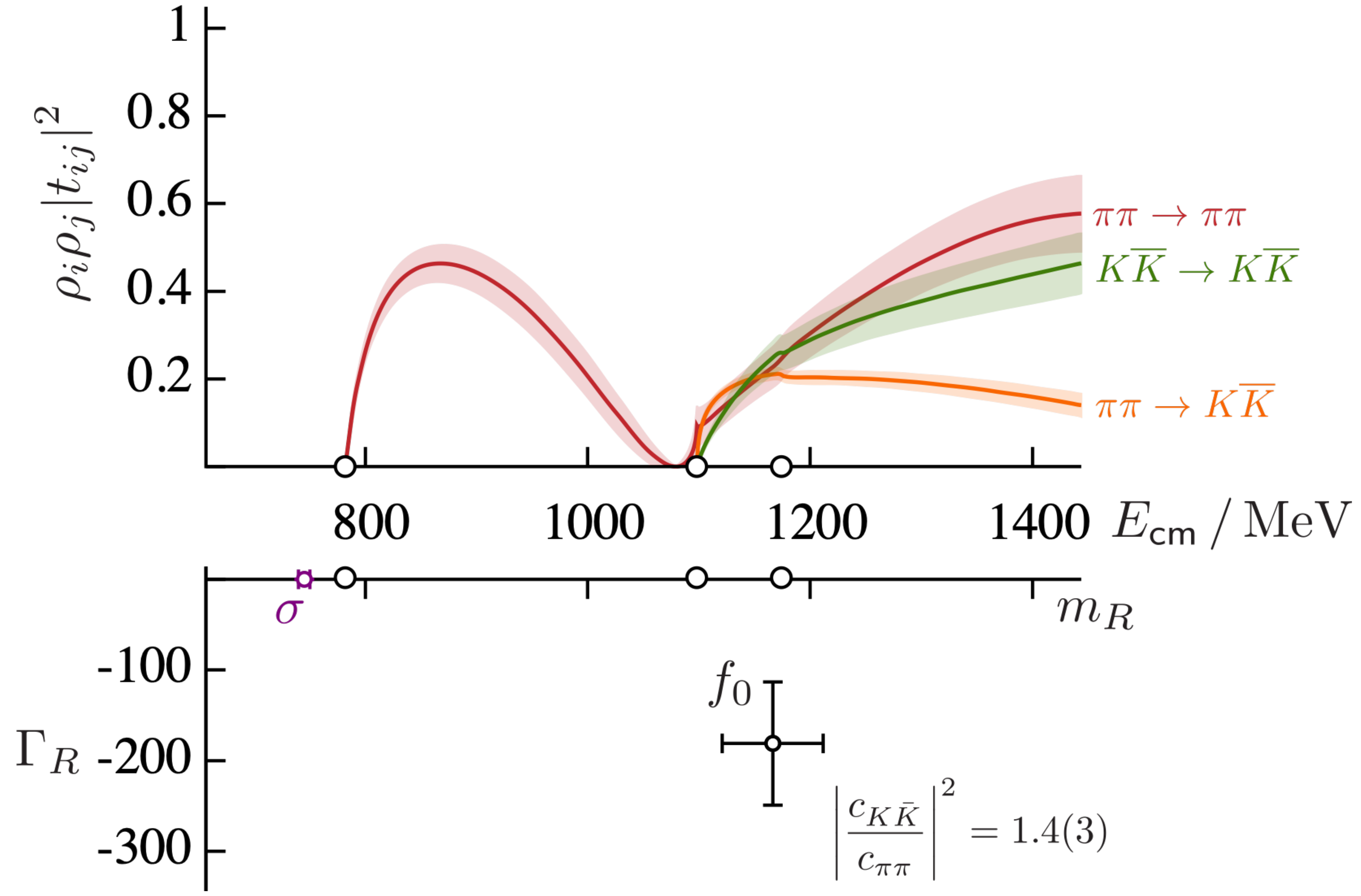


$\mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$



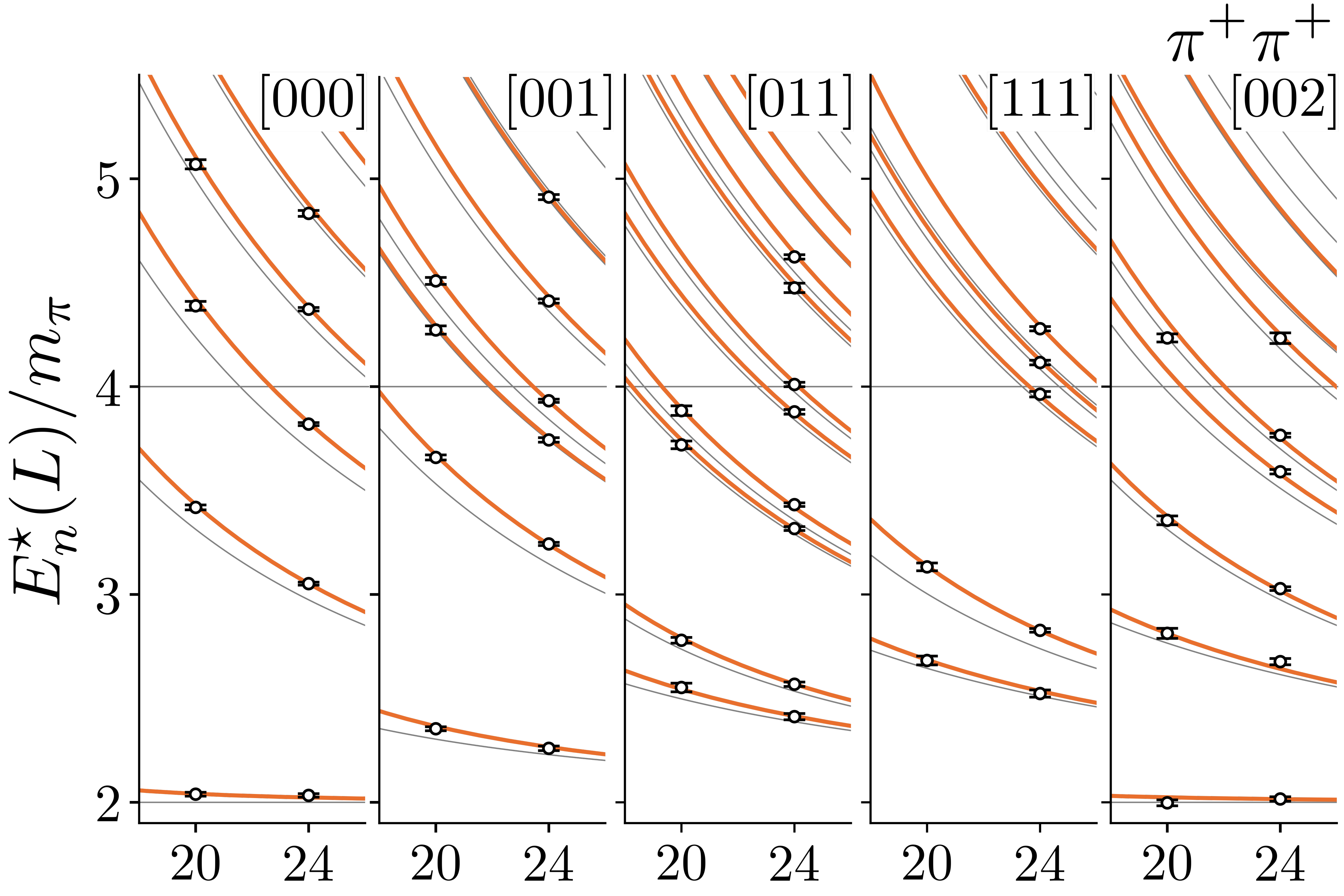
- ☐ Lattice QCD calculations





$\pi\pi$ scattering

(I=2 channel, $m_\pi \sim 390\text{MeV}$)



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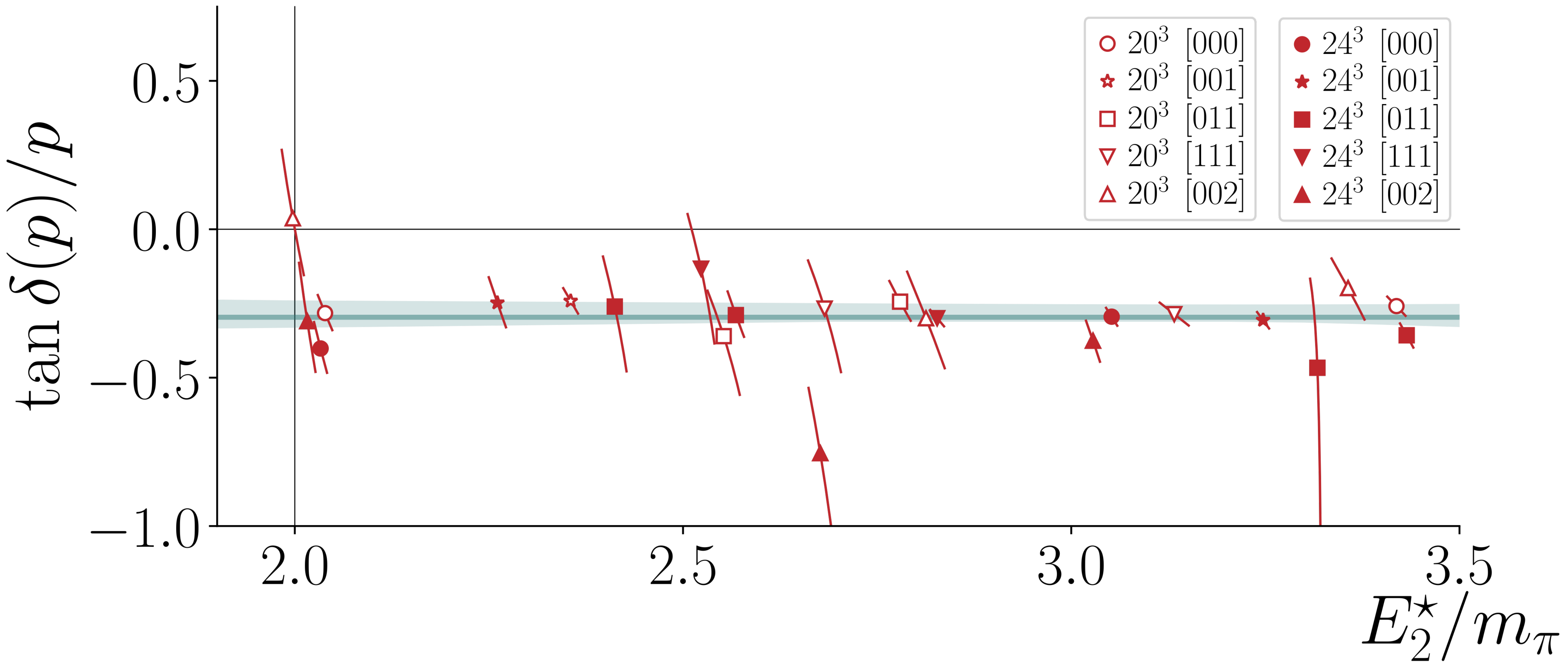

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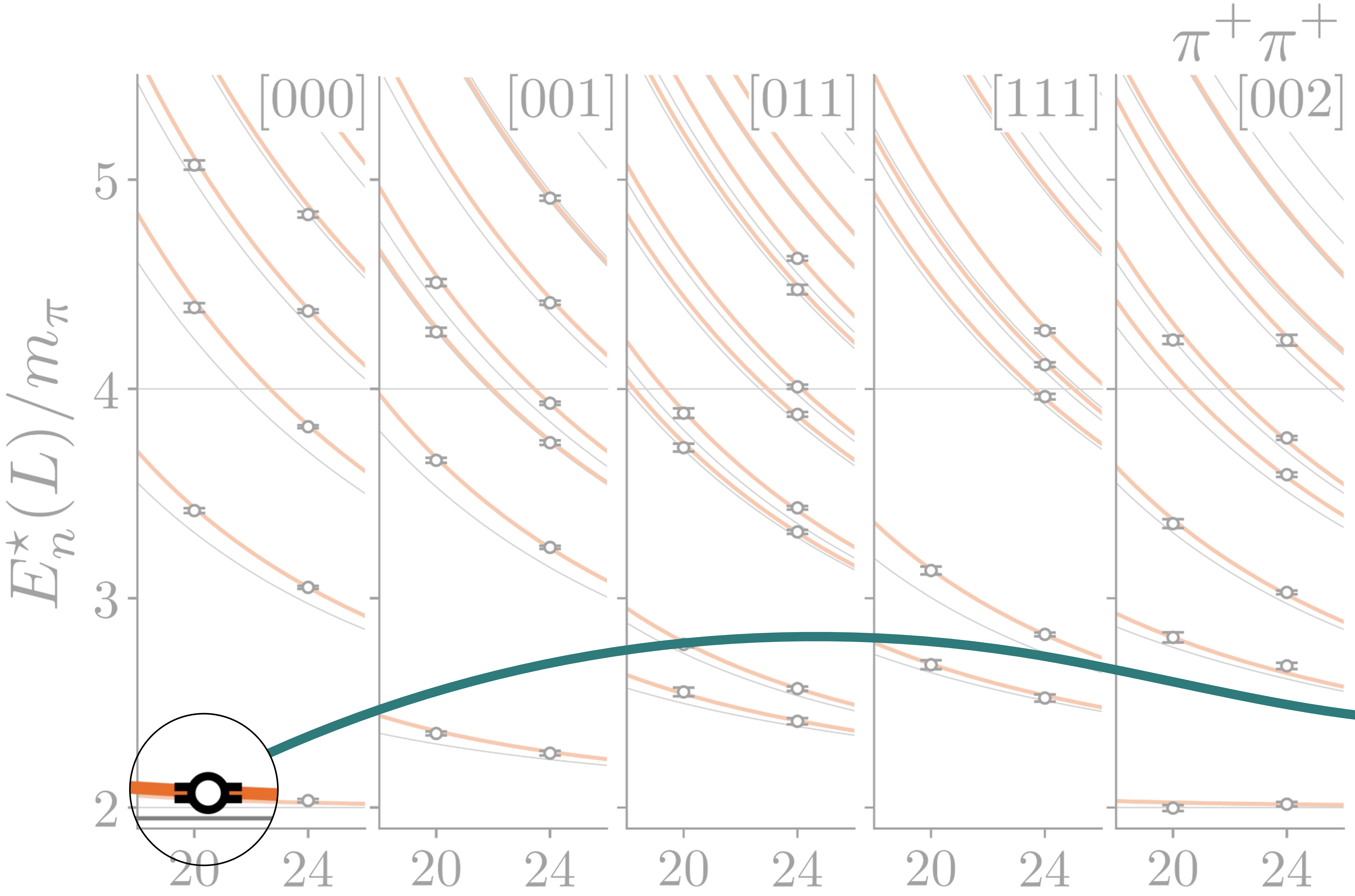

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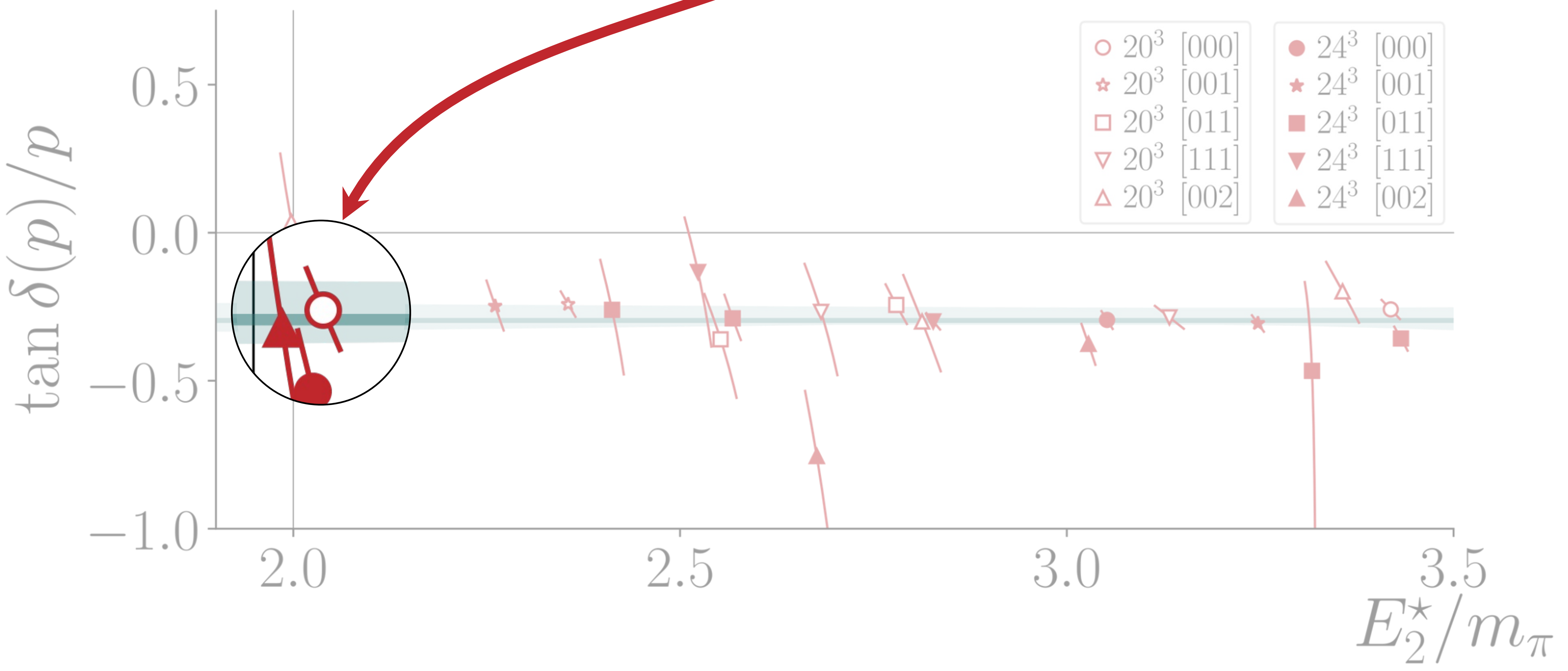
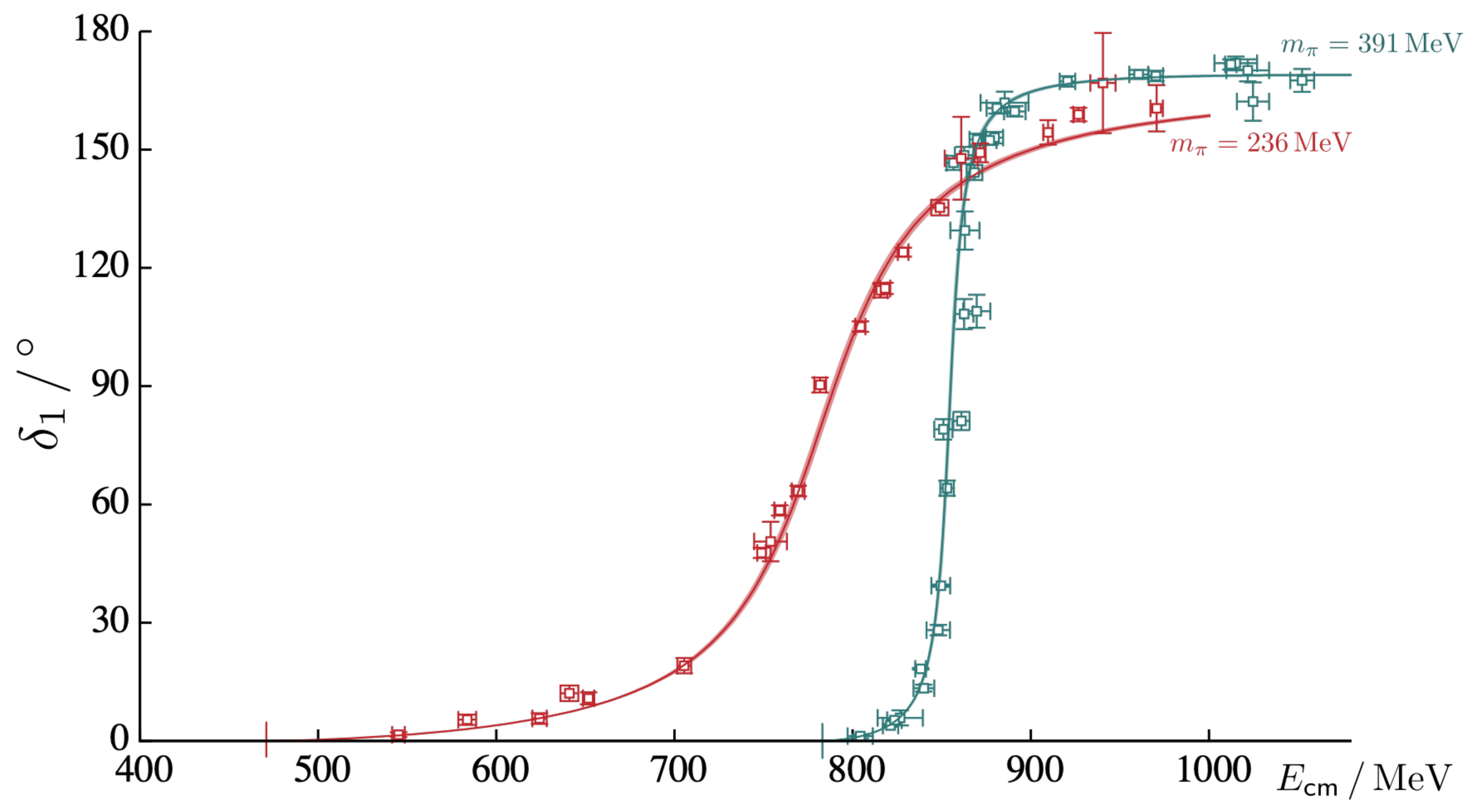
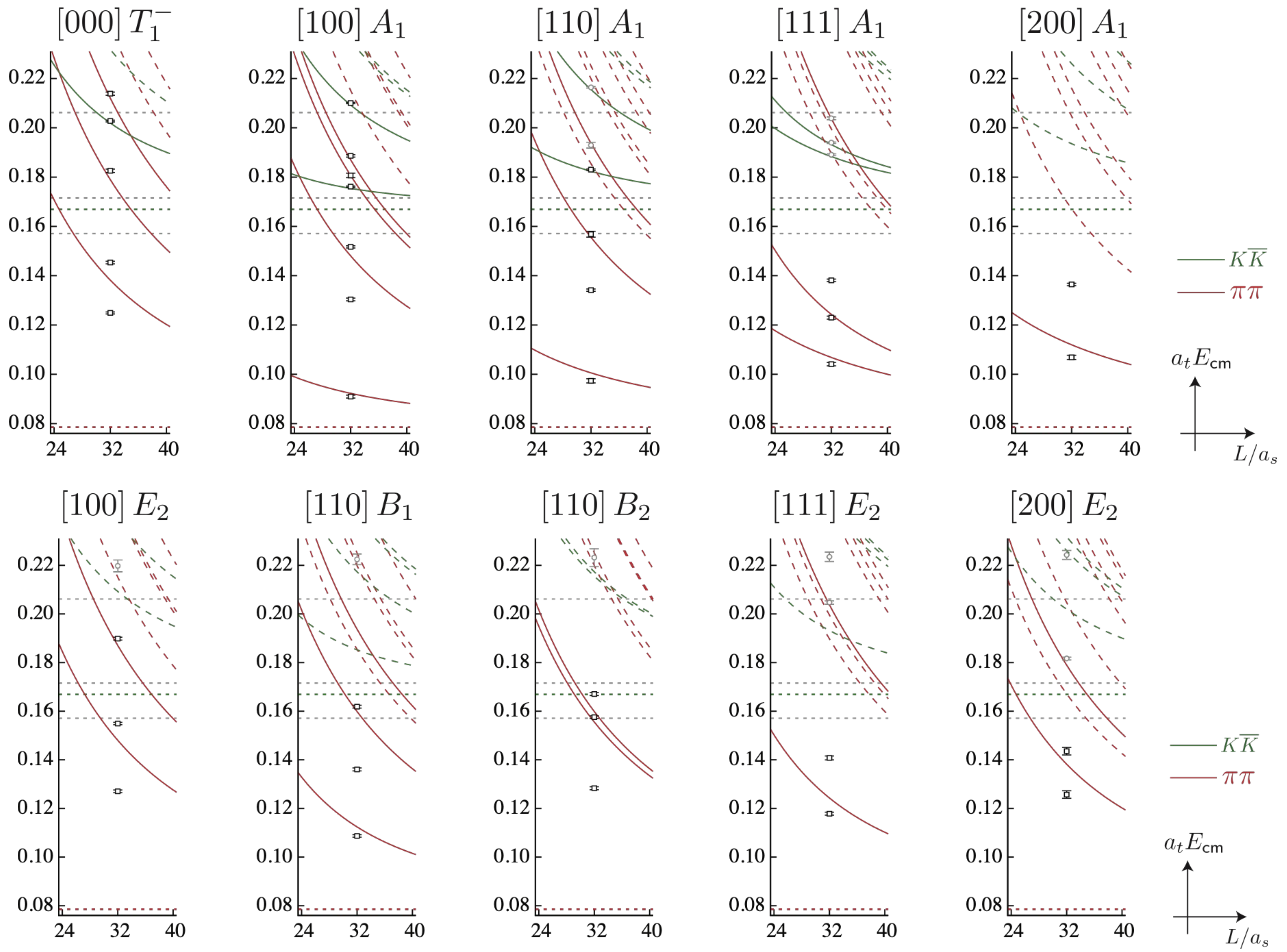


Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi$ scattering

(I=1 channel)





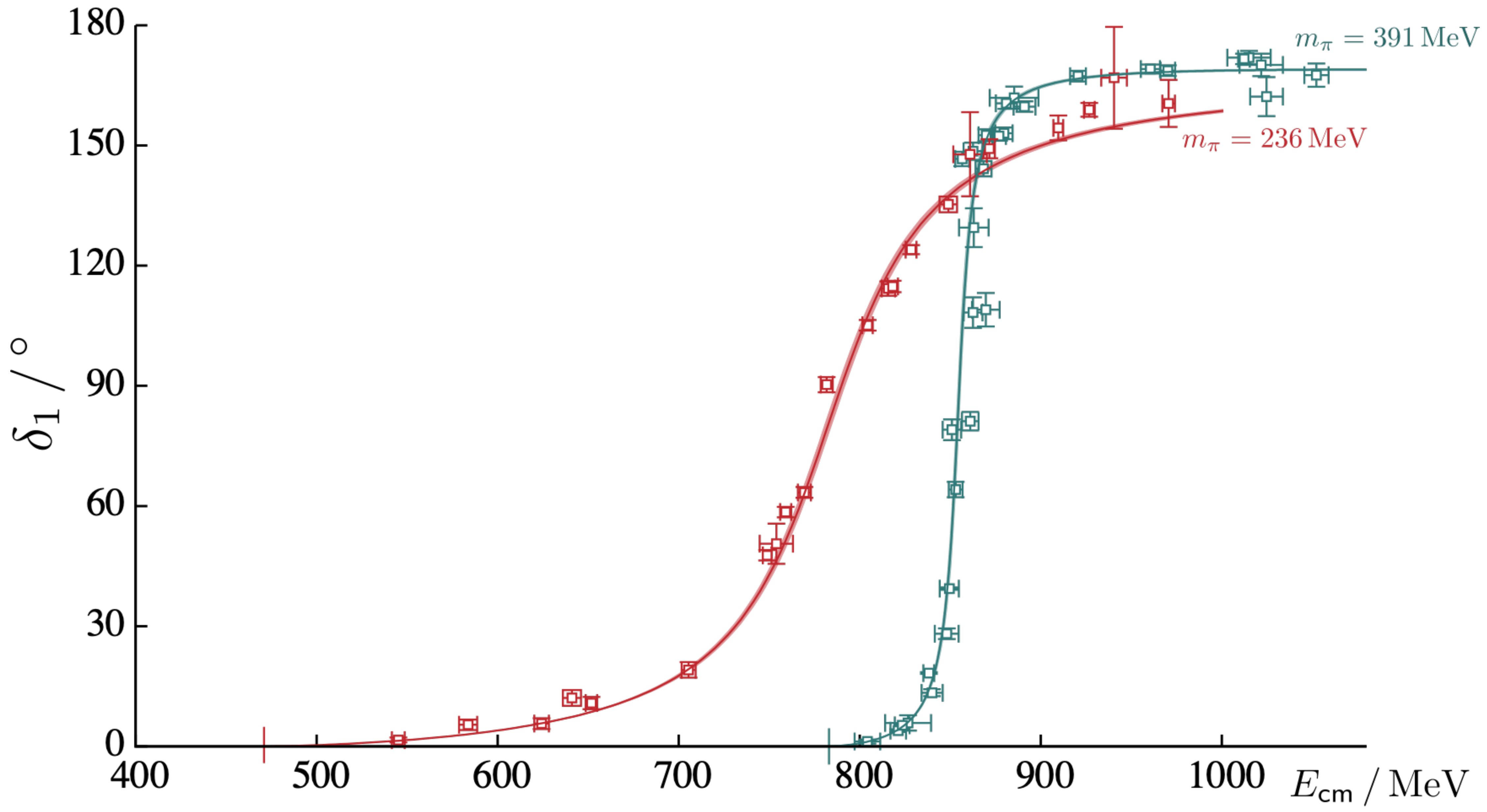
$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$

Dudek, Edwards, & Thomas (2012)

Wilson, RB, Dudek, Edwards, & Thomas (2015)

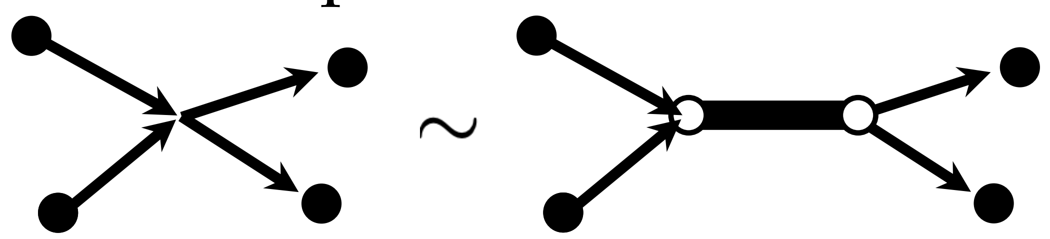
$\pi\pi$ scattering

(I=1 channel)



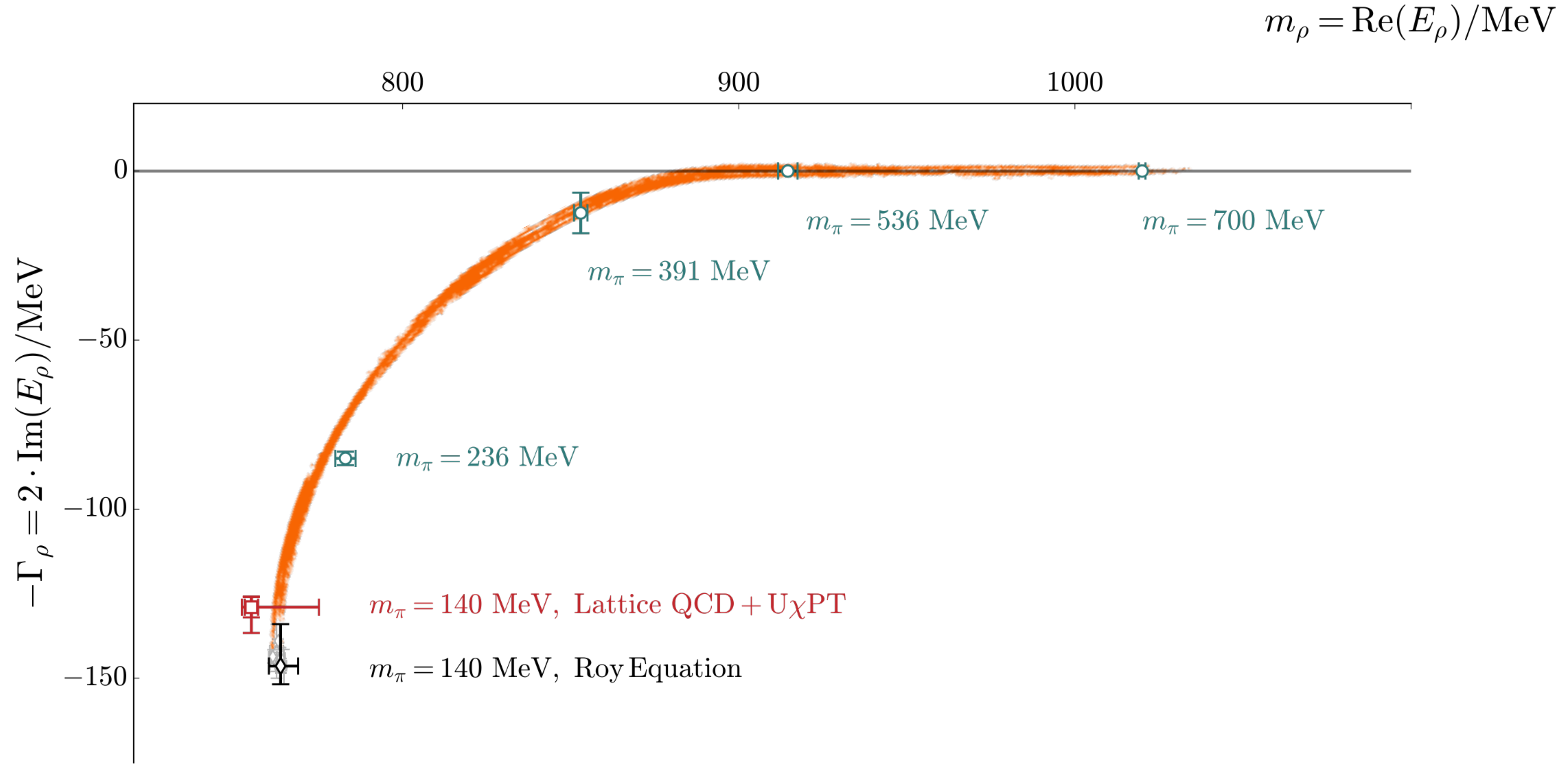
$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

resonance pole



$$\sim \qquad \sim \frac{1}{s - s_0}$$

$$s_0 = (m_R - i\Gamma_R/2)^2$$



Dudek, Edwards, & Thomas (2012)

Wilson, RB, Dudek, Edwards, & Thomas (2015)

Coupled $\pi\pi, K\overline{K}$ and the f_0's

- ☑ Above $K\overline{K}$-threshold, spectrum satisfies:
- ☑ No one-to-one correspondence,
- ☑ Parameterize amplitude and perform global fit.

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$