

Hadron-Hadron Interactions from Lattice QCD

- Theory meets experiments -

Tetsuo Hatsuda (RIKEN iTHEMS)

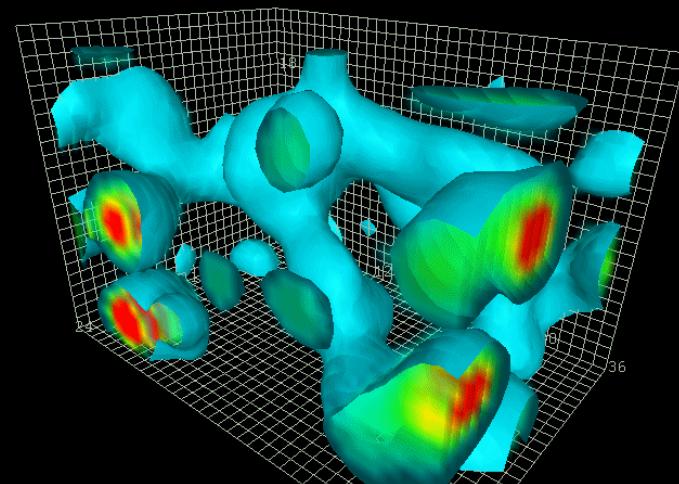
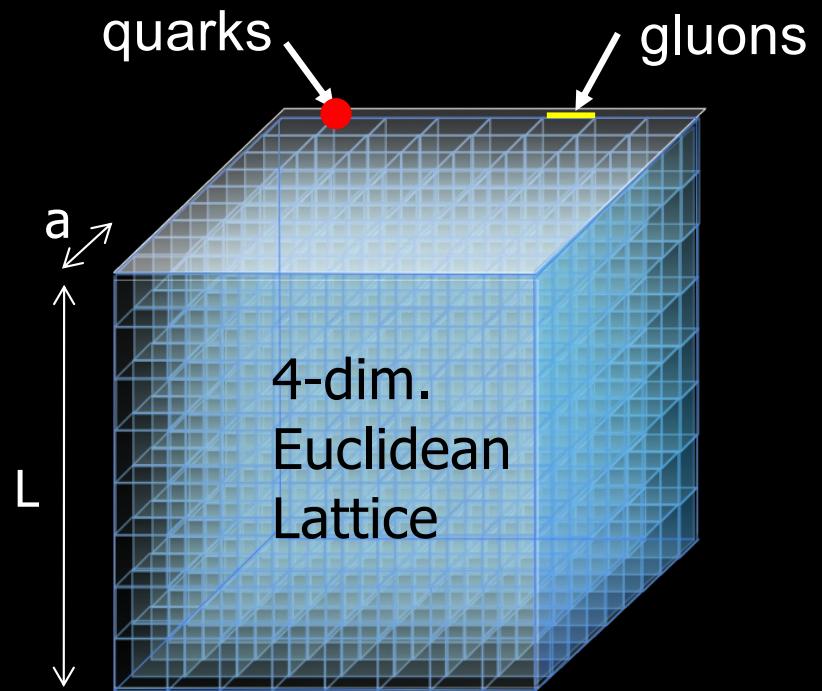


LQCD (since 1974) : Ab initio approach to solve QCD



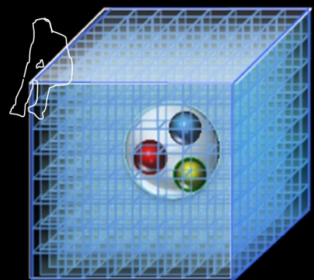
K. Wilson

$$Z_{\text{QCD}} = \int [dU] [dq d\bar{q}] e^{-[S_{\text{glue}}(U) + \bar{q} F(U) q]} = \int [dU] e^{-S_{\text{eff}}(U)}$$

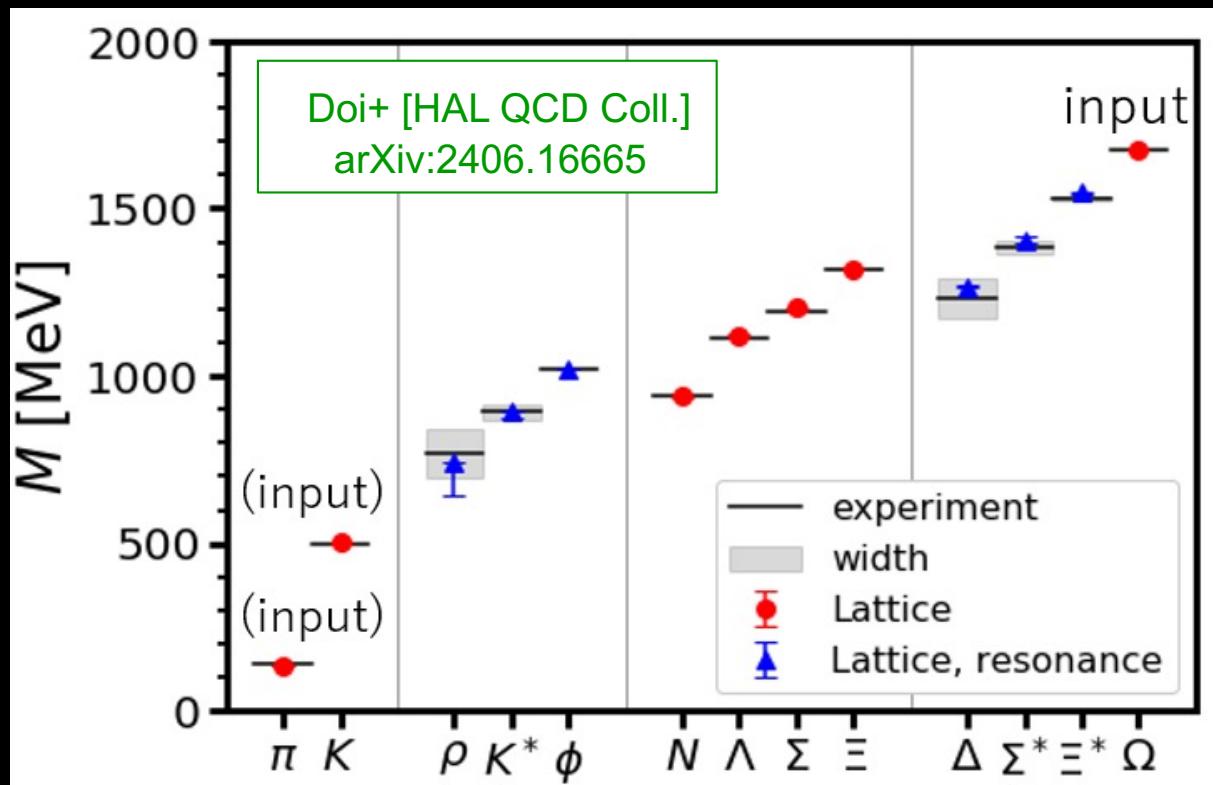


by D.Leinweber

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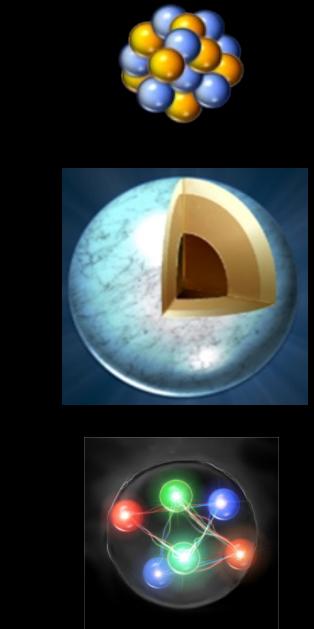
(2+1)-flavor QCD on
FUGAKU@RIKEN (440PFlops)
 $m_\pi=137.1$ MeV, $L=8.1$ fm



Hadron-hadron interactions : Why do we care ?

➤ Nuclear forces & Hyperon forces

↔ Atomic nuclei, hypernuclei, and EoS for neutron stars



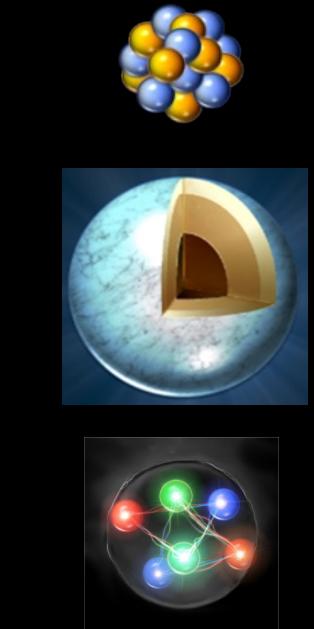
➤ Exotic hadrons & Exotic nuclei

↔ Deeper understanding of “confinement”

Hadron-hadron interactions : Why do we care ?

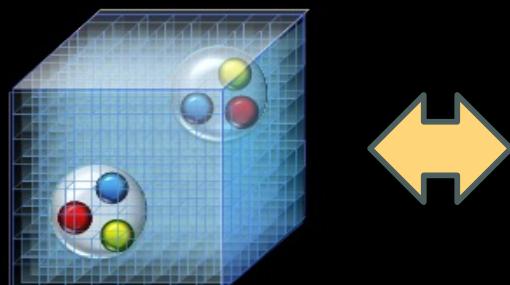
➤ Nuclear forces & Hyperon forces

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➤ Exotic hadrons & Exotic nuclei

↔ Deeper understanding of “confinement”



Era of “co-evolution”
of LQCD and Expt.

BB: $\Lambda\Lambda$ - $N\Xi$, $N\Omega$, $\Omega\Omega$

MM: T_{cc}^+

MB: $s\bar{s}$ - N , $c\bar{c}$ - N

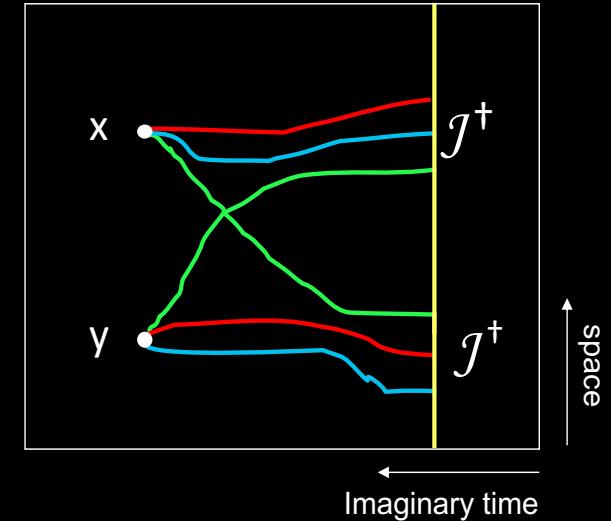


How to extract hadron-hadron int. in LQCD ?

Spacetime correlation of composite particles \rightarrow S-matrix

- Haag–Nishijima–Zimmermann formula (1958)
- Borchers Theorem (1960)

$$R(r, \tau) = \sum_n \Psi_n(r) e^{-E_n \tau}$$

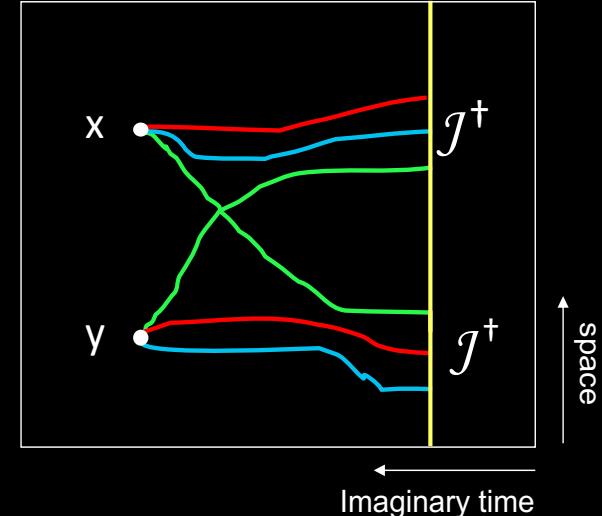


How to extract hadron-hadron int. in LQCD ?

Spacetime correlation of composite particles \rightarrow S-matrix

- Haag–Nishijima–Zimmermann formula (1958)
- Borchers Theorem (1960)

$$R(r, \tau) = \sum_n \Psi_n(r) e^{-E_n \tau}$$



Luscher's Method

$E_n \rightarrow$ relative momentum $k_n \rightarrow$ phase shift, binding energy

Luscher, Nucl. Phys. [B354](#) (1991) 531

HAL QCD Method

$R(r, \tau) \rightarrow$ LS equation ($T = U + GUT$) \rightarrow phase shift, binding energy

Ishii, Aoki & Hatsuda, PRL [99](#) (2007) 022001

Relation between the two:

Iritani+ [HAL QCD Coll.], JHEP [03](#) (2019) 007.

Lyu+ [HAL QCD Coll.], PRD [105](#) (2022) 074512.

How to extract hadron-hadron int. in LQCD ?

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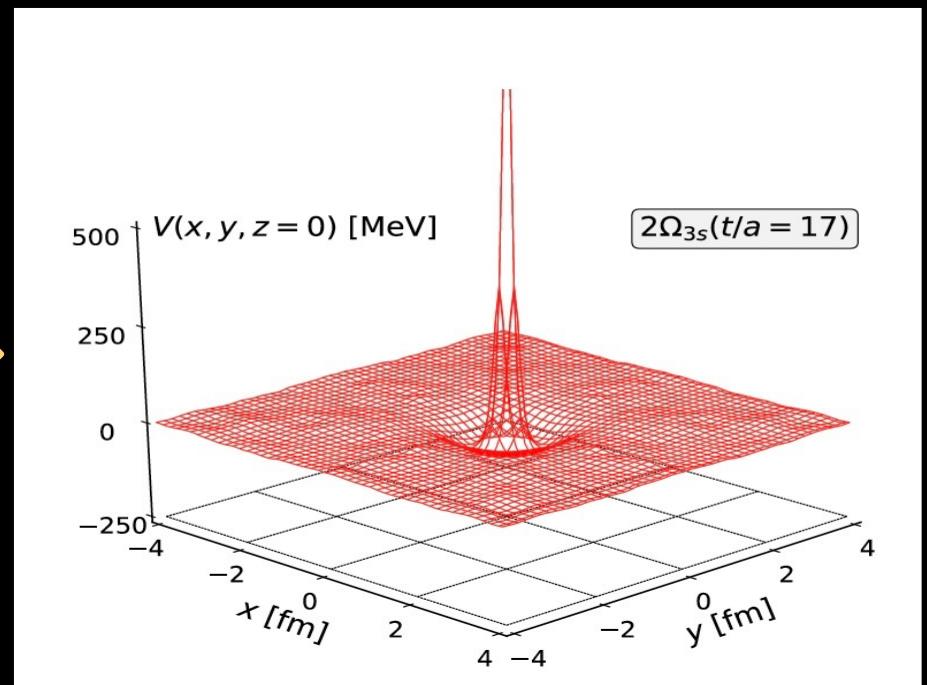
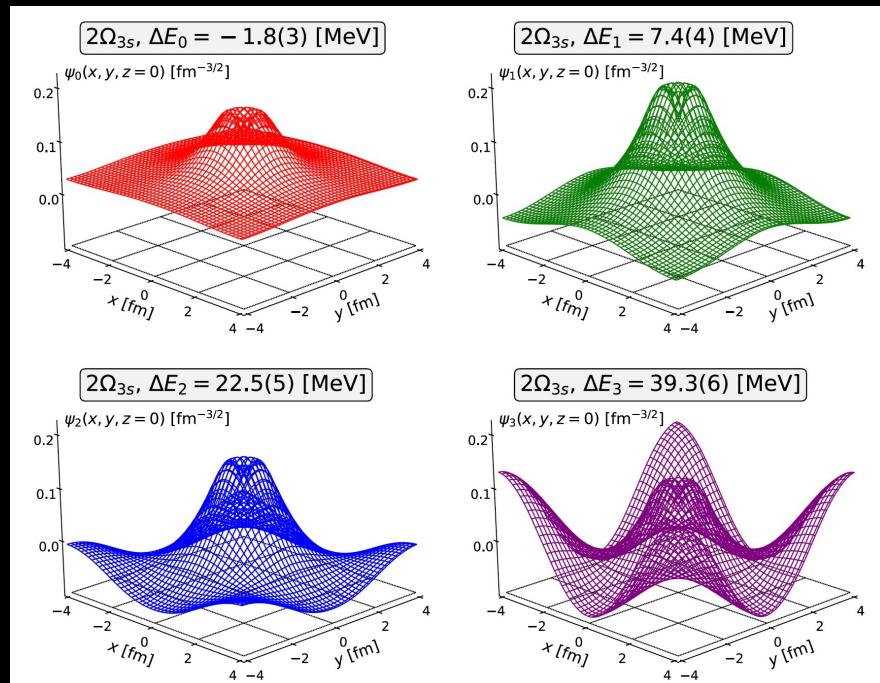
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Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

$R(r, \tau)$

$U(r, r')$



How to extract hadron-hadron int. in LQCD ?

HAL QCD Method

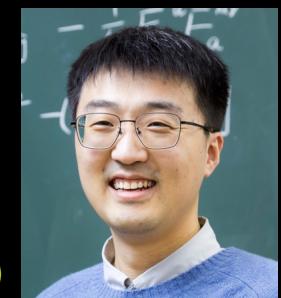
$R(r,\tau) \rightarrow$ LS equation ($T = U + GUT$) \rightarrow phase shift, binding energy

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

$R(r,\tau) \rightarrow U(r, r')$ (Inverse Problem)

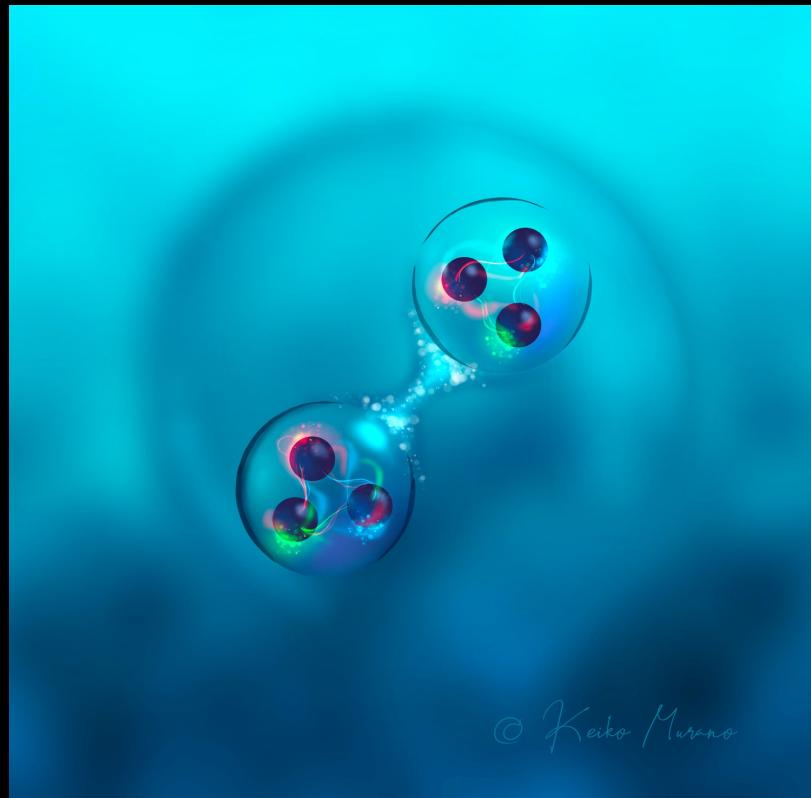
- Derivative expansion: Aoki, Hatsuda, Ishii (2010)
- Separable expansion: L. Meng & Epelbaum (2023)
- Machine learning: L. Wang+ (2024)

Lingxiao Wang
Parallel talk at
H: Statistical
Methods
(18:00-, Aug. 21)



Baryon-Baryon Interactions

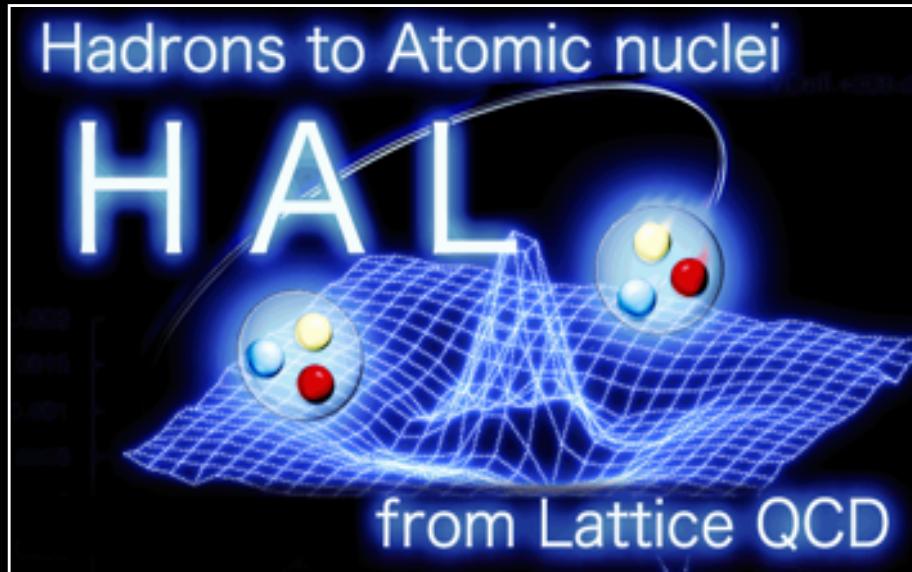
- roles of strangeness and charm -



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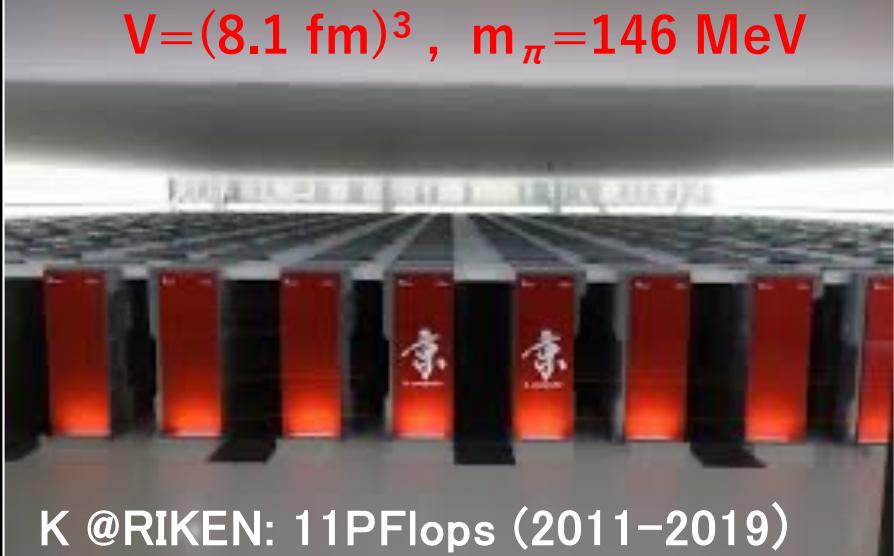
$\Lambda\Lambda$ - $N\Xi$ ($S=2$)
 $N\Omega$ ($S=-3$)
 $\Omega\Omega$ ($S=-6$)

Large Scale LQCD Simulations of Hadron-Hadron Int. (HAL QCD Collaboration since 2010)

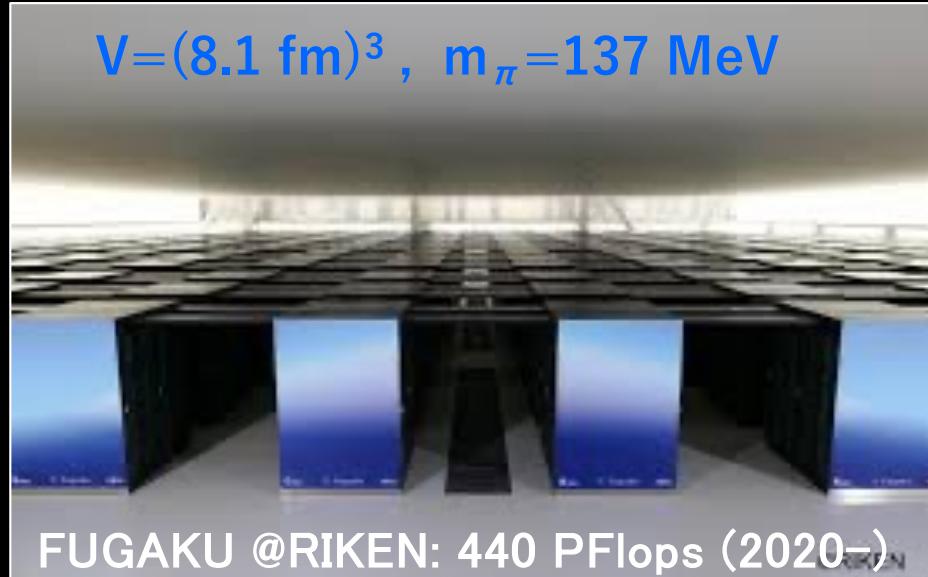


(KEK) T. Aoyama
(RIKEN) T. Doi, T. Hatsuda, Y. Liu, L. Zhang,
R. Yamada, Lingxiao Wang
(Nihon) T. Inoue
(Rissho) T. Sugiura
(TIT) K. Murakami
(TMU) K. Murase
(YITP) S. Aoki, E. Itou
(Kyoto) T. M. Doi
(RCNP) N. Ishii, P. Junnarkar, H. Nemura
(Osaka) Y. Ikeda, K. Sasaki
(Birjand) F. Etminani
(Bonn) H. Tong

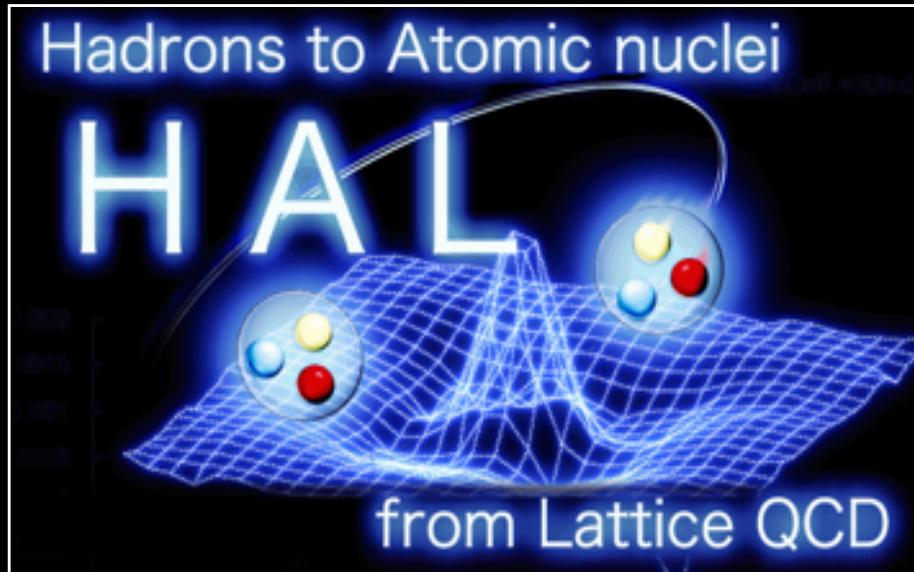
$V=(8.1 \text{ fm})^3$, $m_\pi=146 \text{ MeV}$



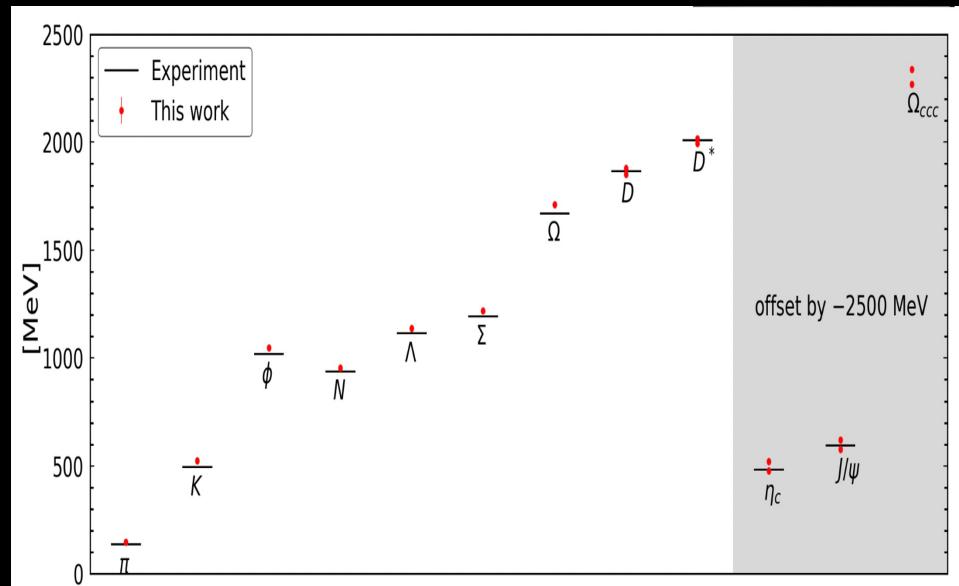
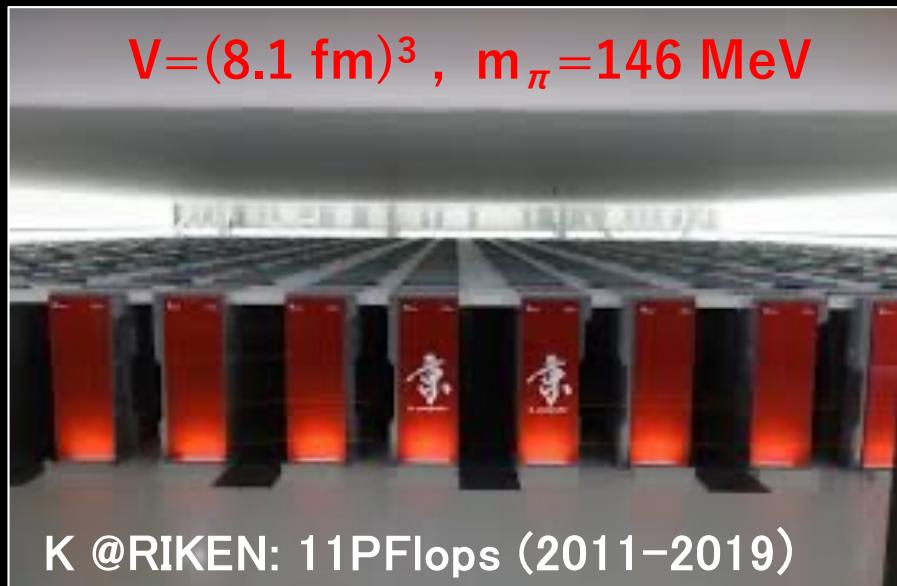
$V=(8.1 \text{ fm})^3$, $m_\pi=137 \text{ MeV}$



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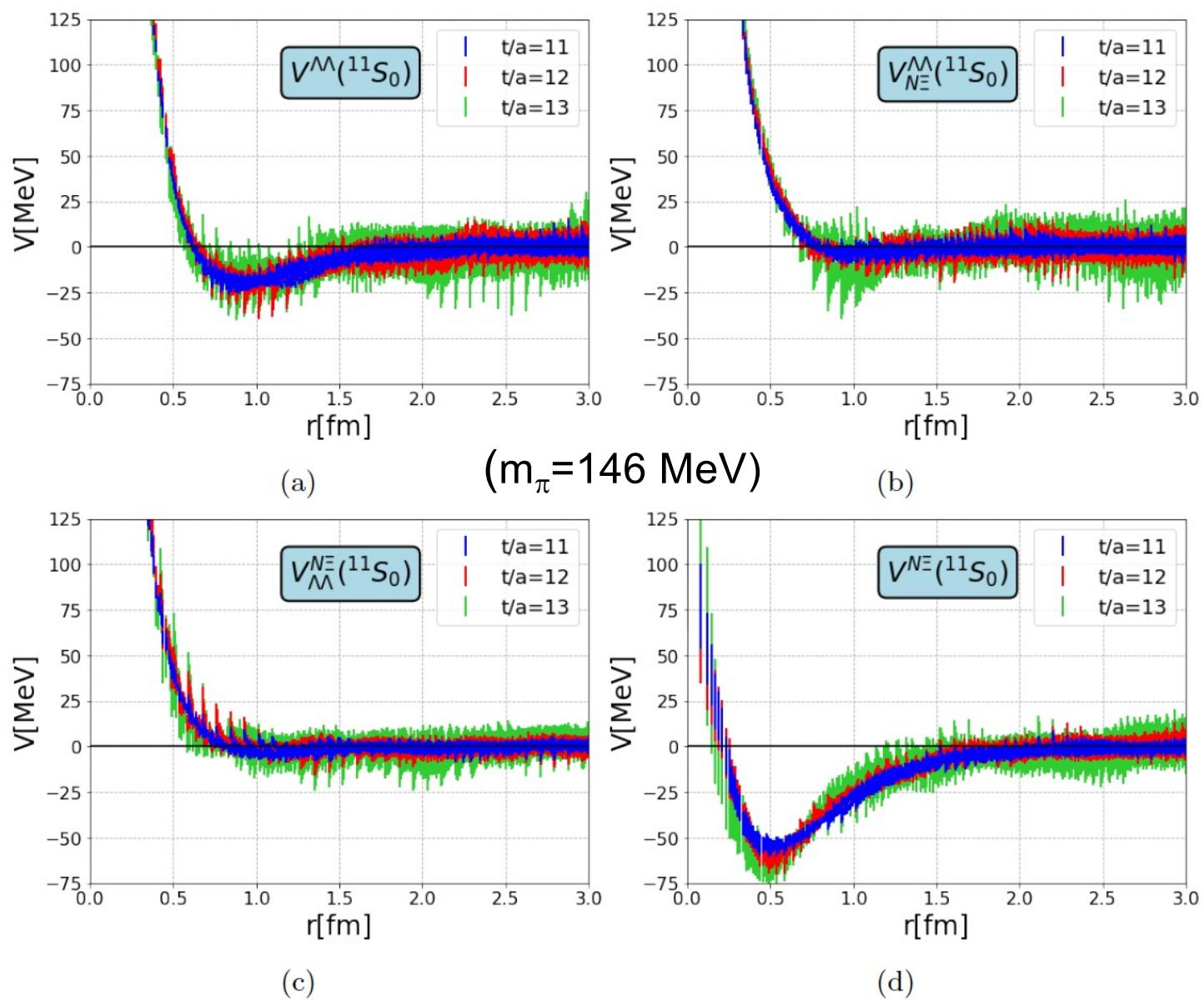
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$\Lambda\Lambda-\bar{N}\Xi$ system (LQCD)

Sasaki+ [HAL QCD Coll.]
NPA (2020)

Small $\Lambda\Lambda$
attraction
Short-range
 $N\bar{\Xi}-\Lambda\Lambda$ coupling

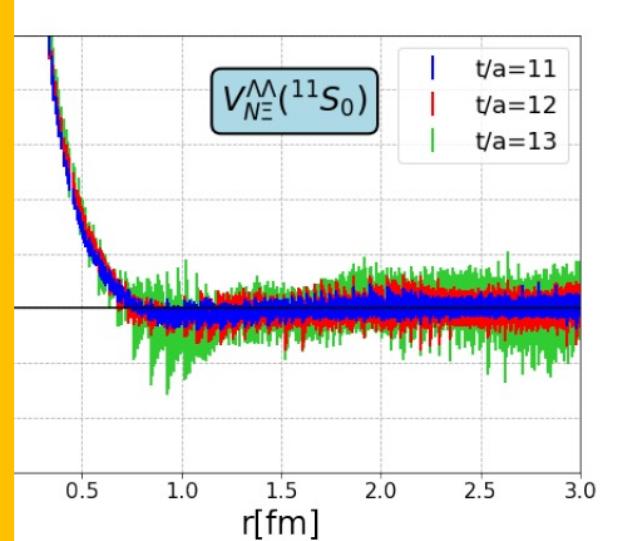
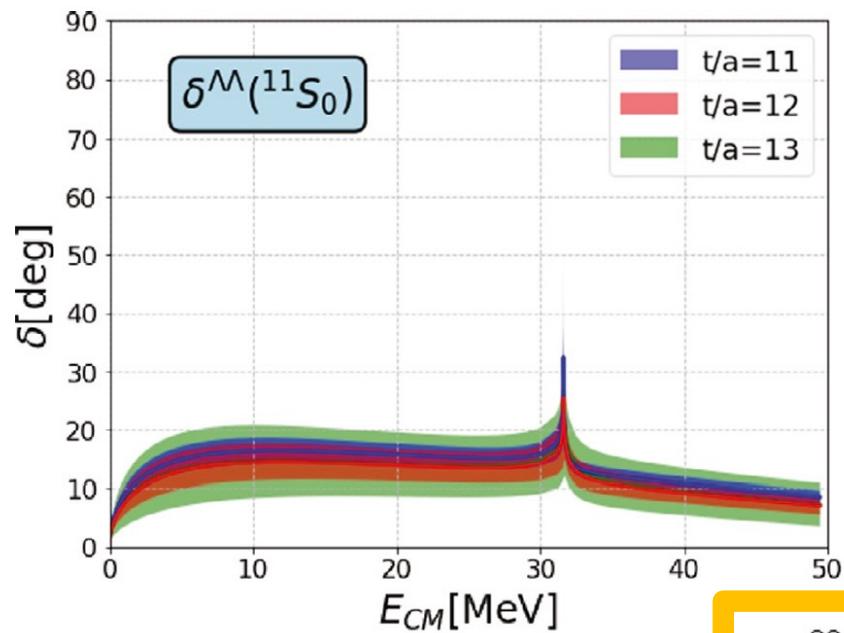


Short-range
 $N\bar{\Xi}-\Lambda\Lambda$ coupling
Large $N\bar{\Xi}$
attraction

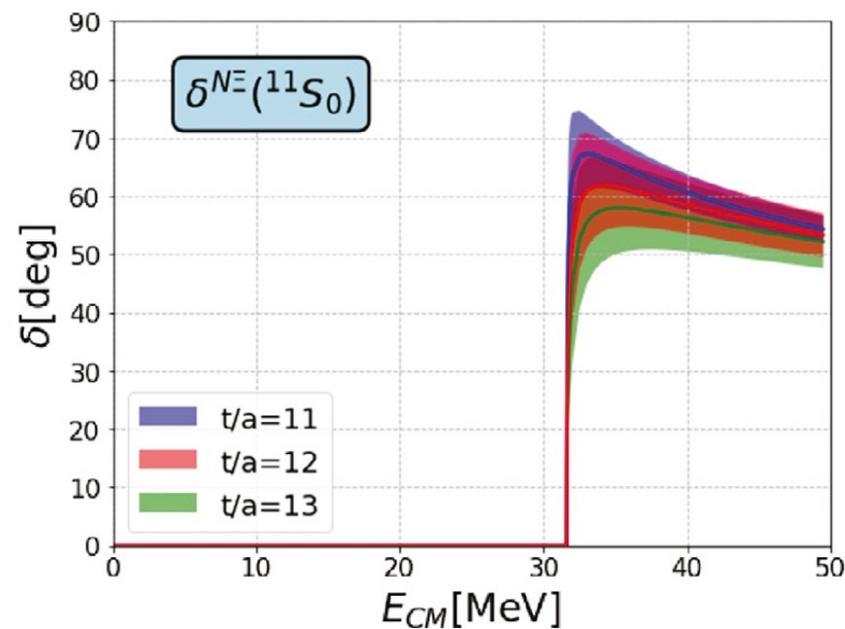
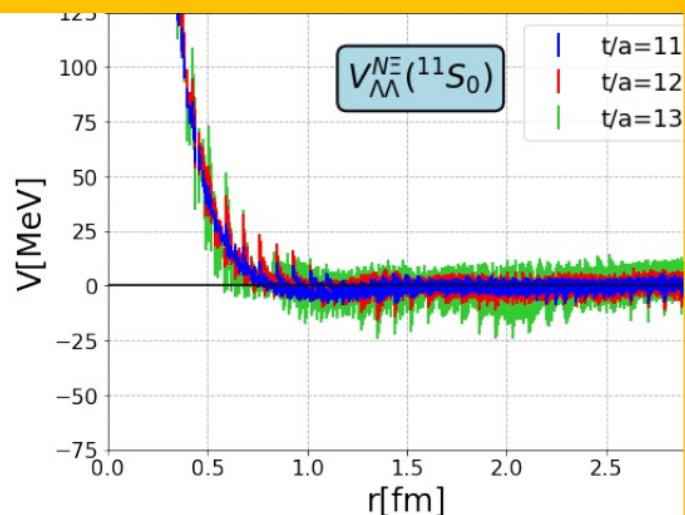
$\Lambda\Lambda-\bar{N}\Xi$ system (LQCD)

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Short-range
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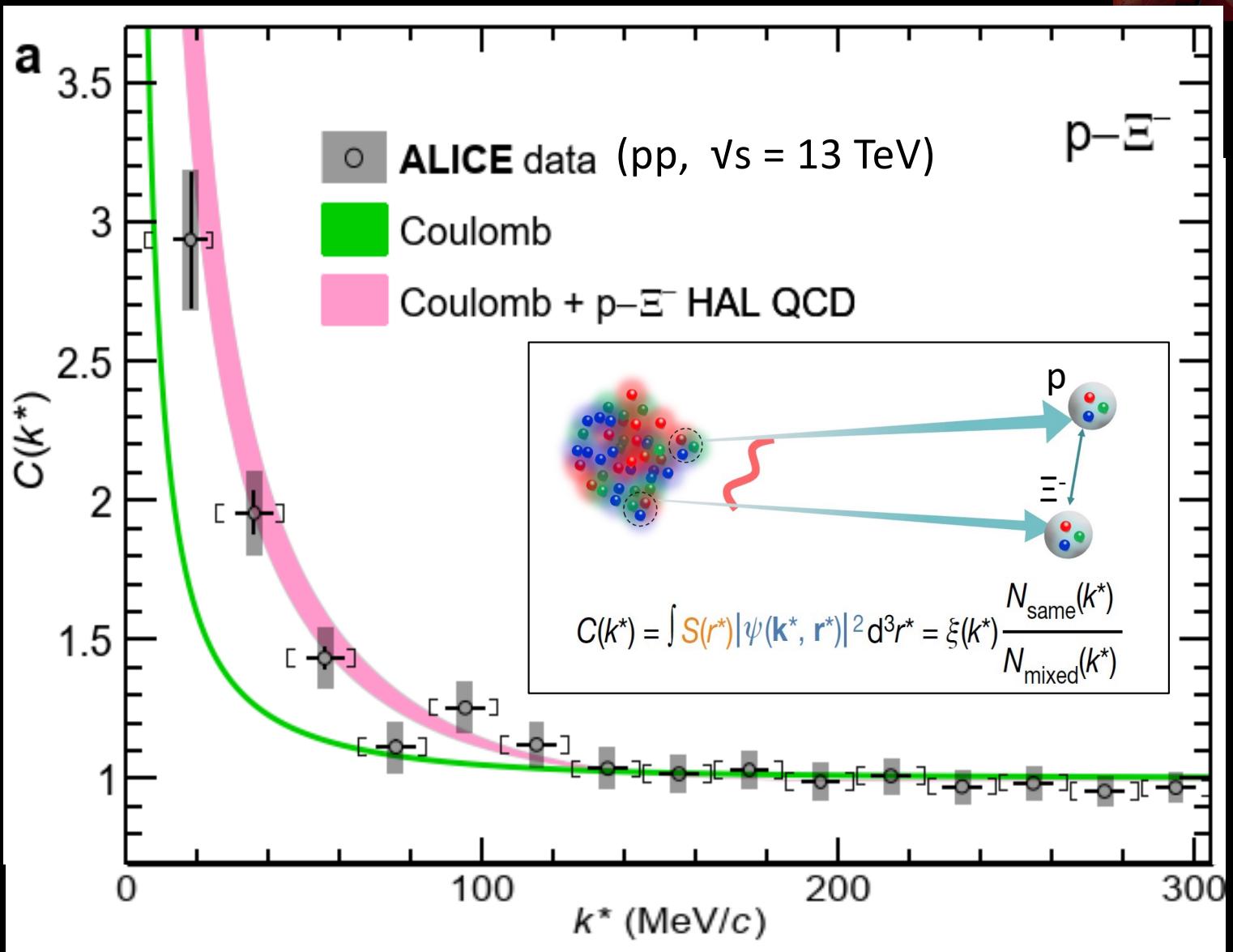
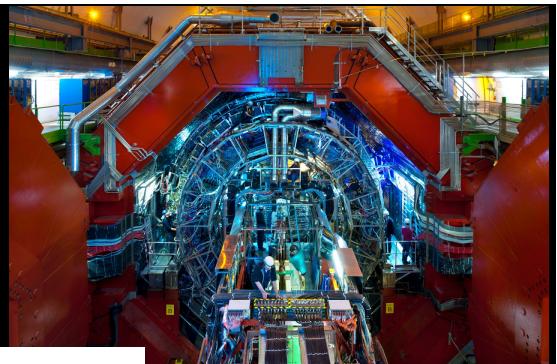
(c)

Short-range
 $N\Xi-\Lambda\Lambda$ coupling

Large $N\Xi$
attraction

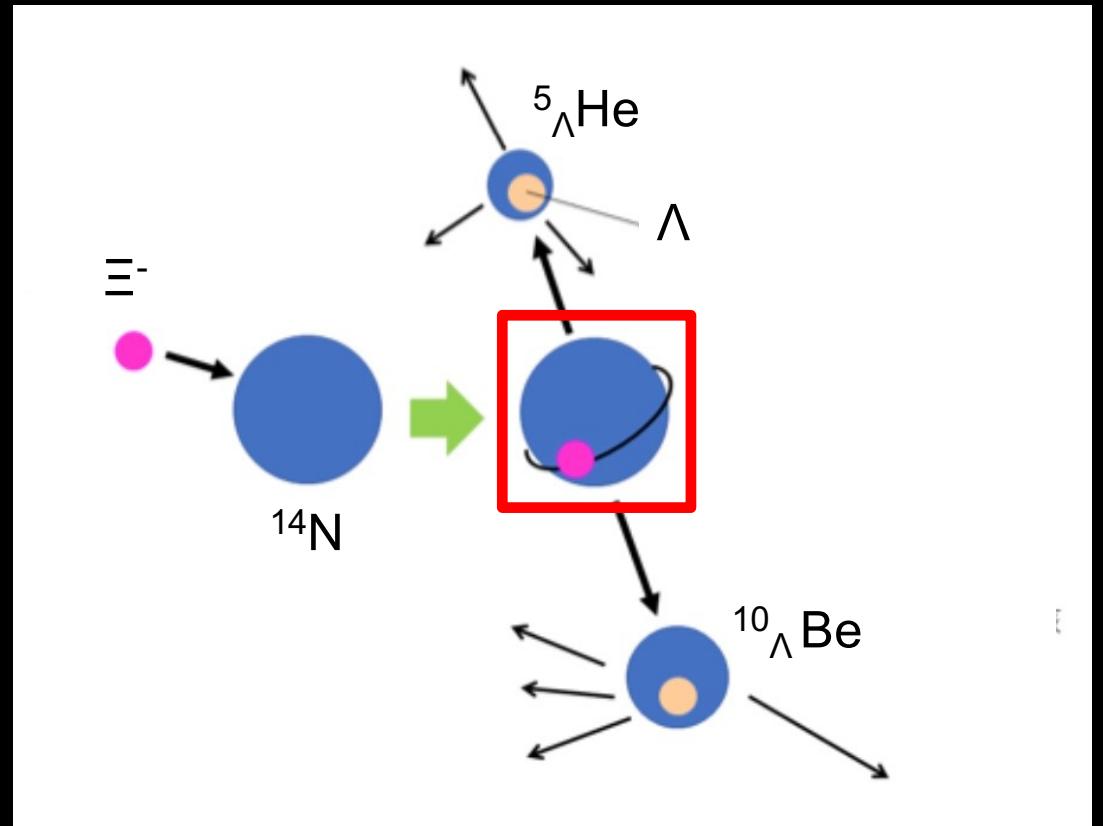
$N\Xi$ femtoscopy (expt.)

ALICE Coll. (LHC), Nature 588 (2020) 232



Ξ hypernuclei at J-PARC (expt.)

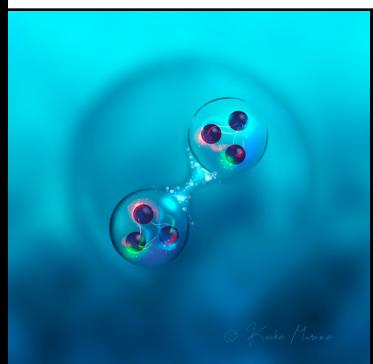
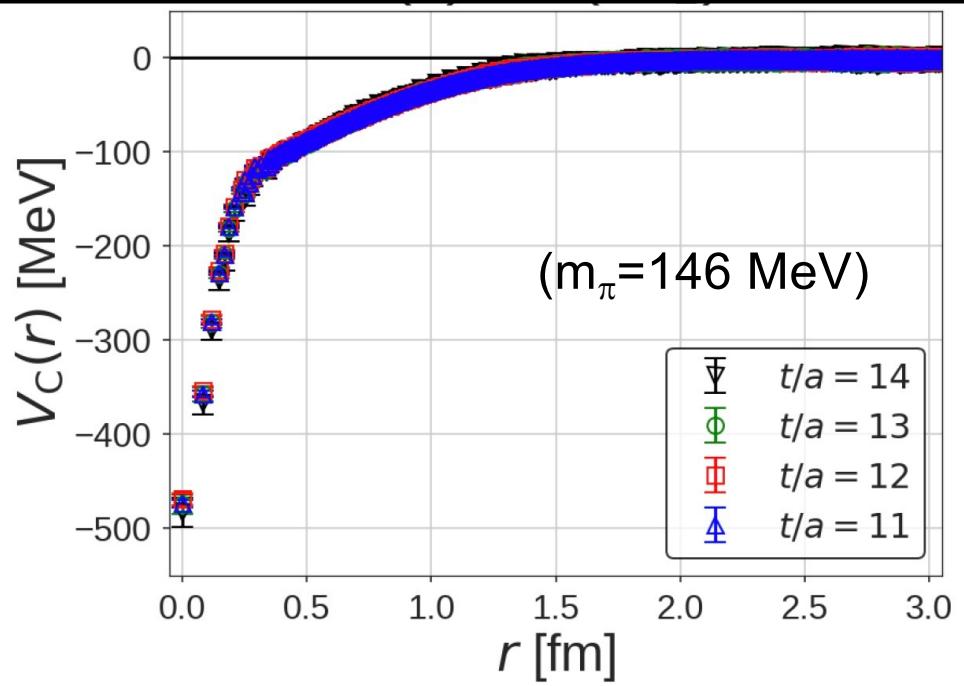
E07 Coll. (J-PARC),
Phys.Rev.Lett. 126 (2021) 062501



LQCD prediction of
• attraction in $\text{N}\Xi$ in $^{11}\text{S}_0$
• weak $\text{N}\Xi - \Lambda\Lambda$ coupling
consistent with the expt. data

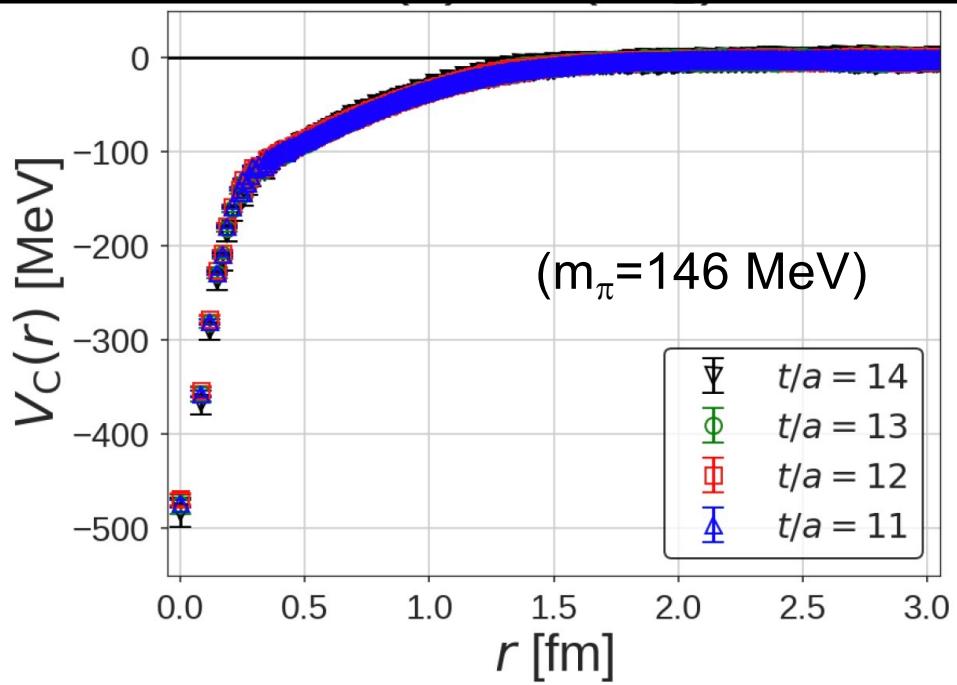
Isaka+, Phys.Rev. C109 (2024) 044317

$N\Omega$ system (LQCD)

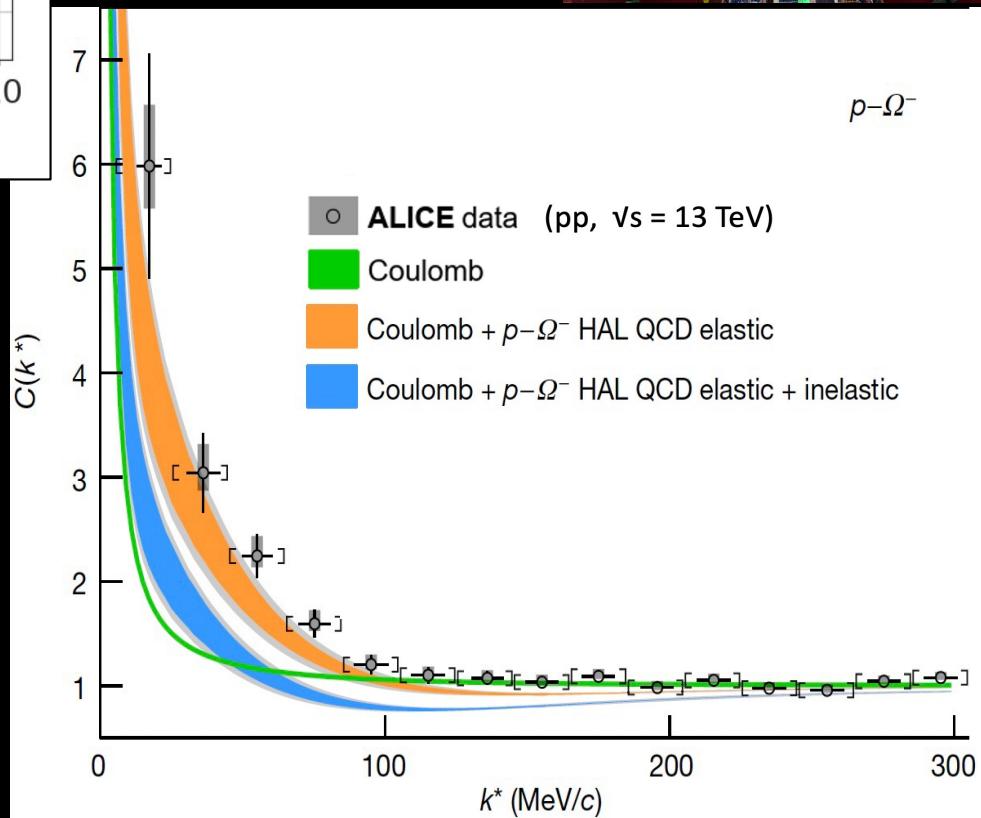
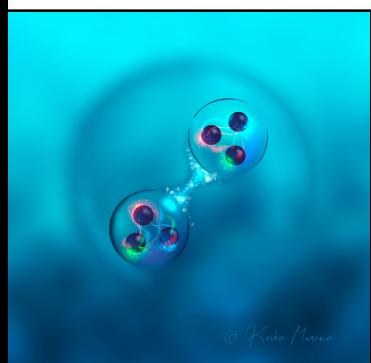
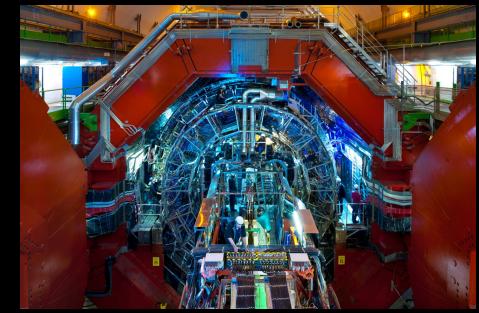


Iritani+ [HAL QCD Coll.],
PLB (2019)

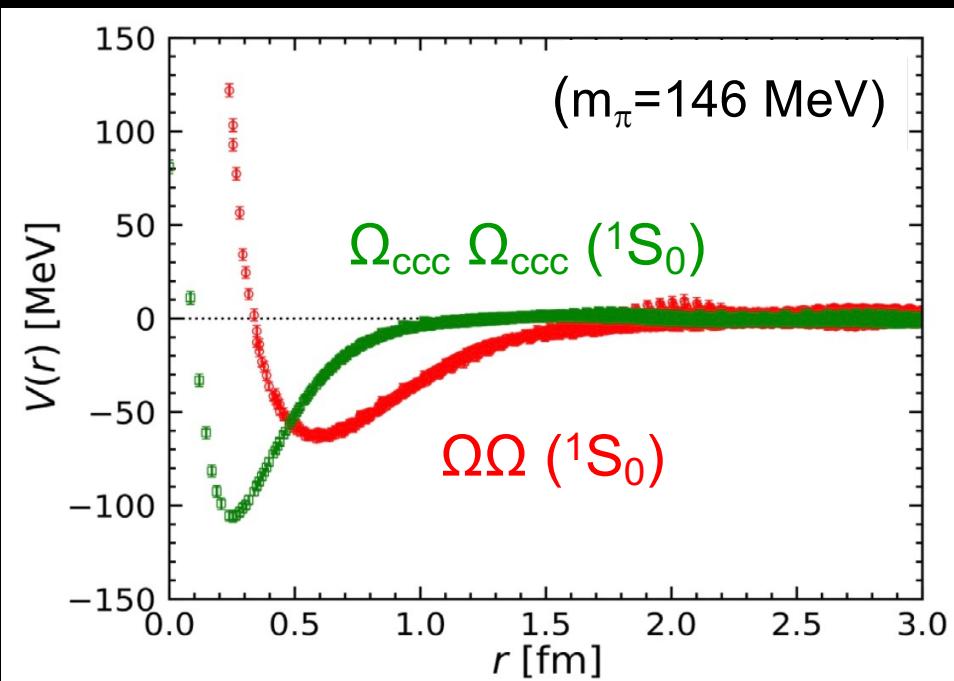
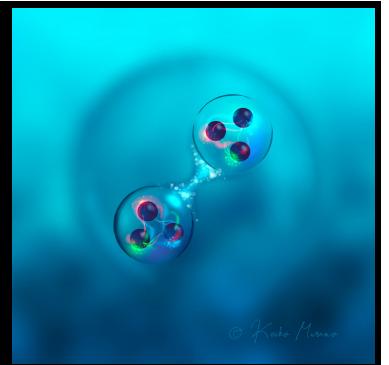
$N\Omega$ system (LQCD)



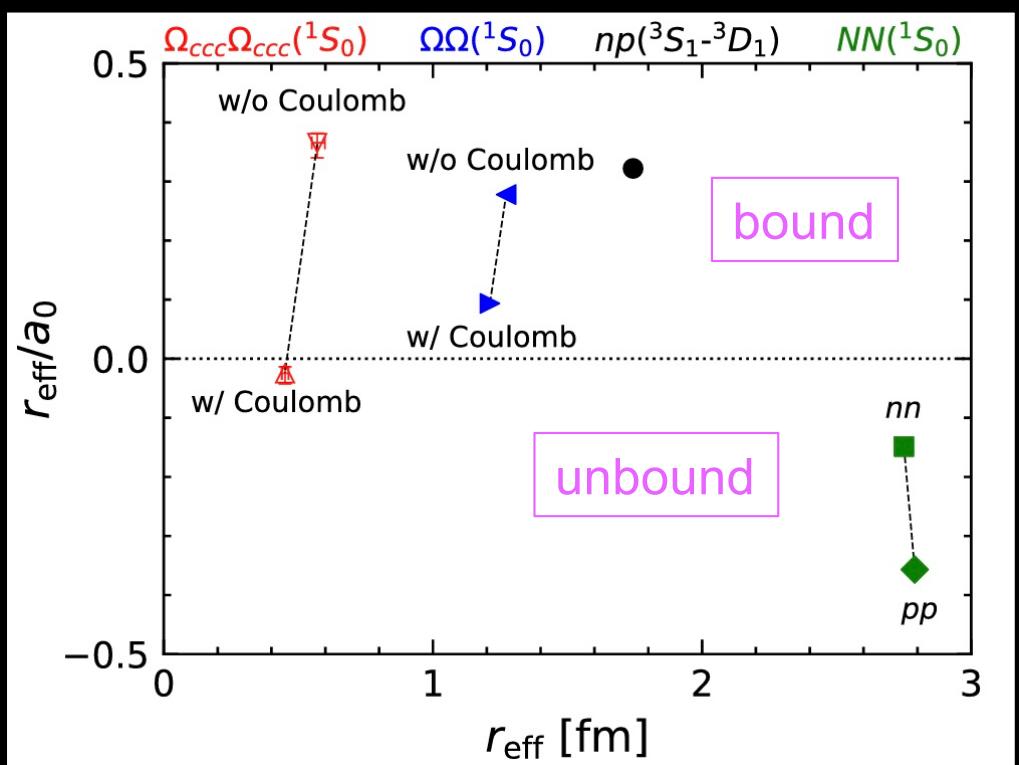
ALICE Coll. (LHC),
Nature (2020)



$\Omega\Omega$ systems (LQCD)

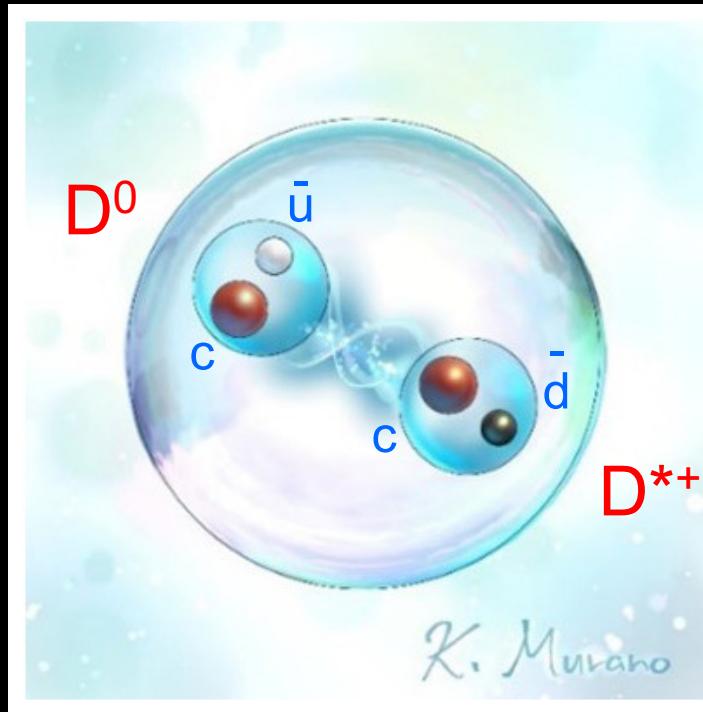


Gongyo+ [HAL QCD Coll.], PRL (2018)
Tong+ [HAL QCD Coll.], PRL (2021)



Meson-Meson Interactions

- genuine tetraquark ?-



$T_{cc}^+(3875)$



OPEN

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*

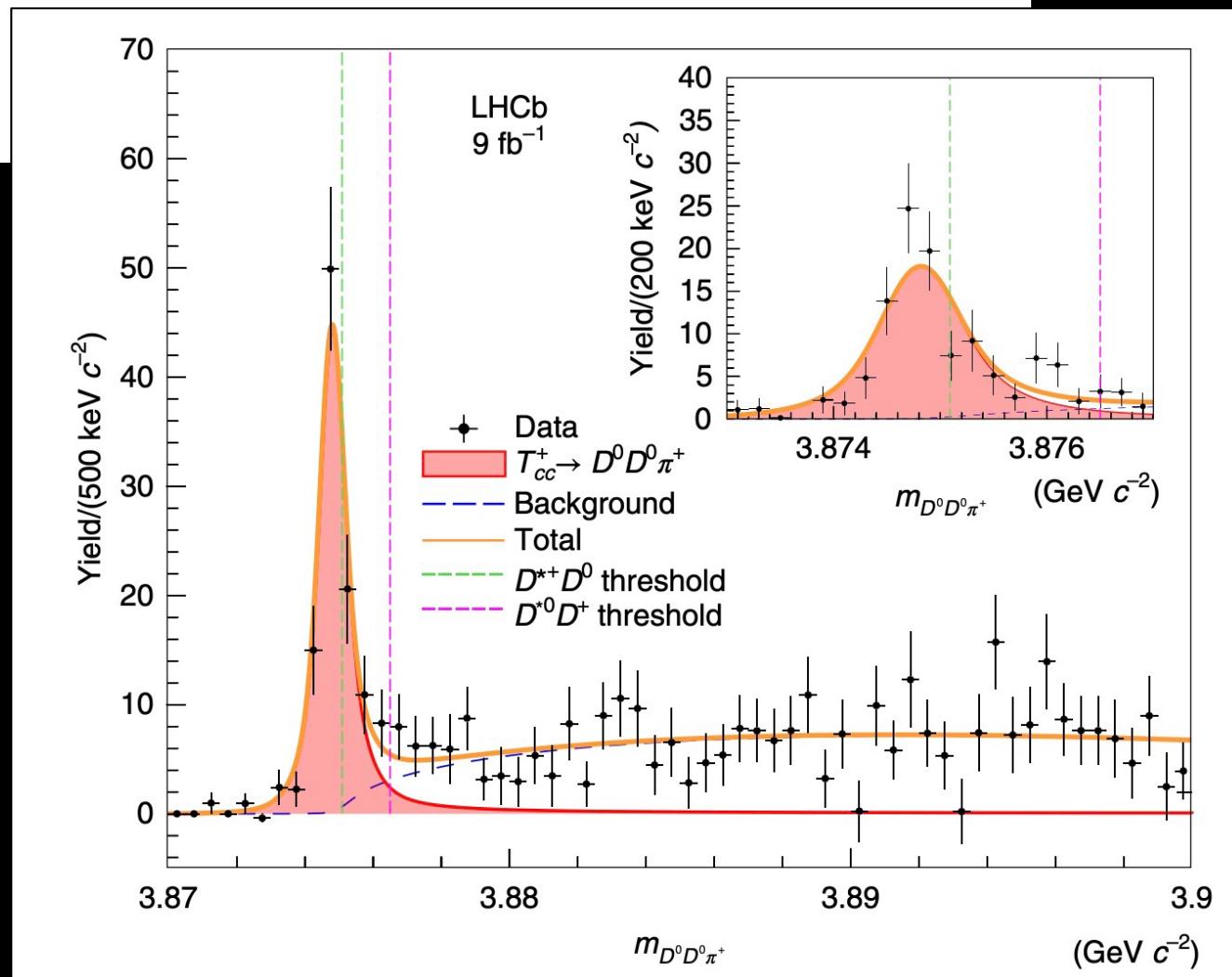
 $T_{cc}^+(3875)$ 

Study of the doubly charmed tetraquark T_{cc}^+

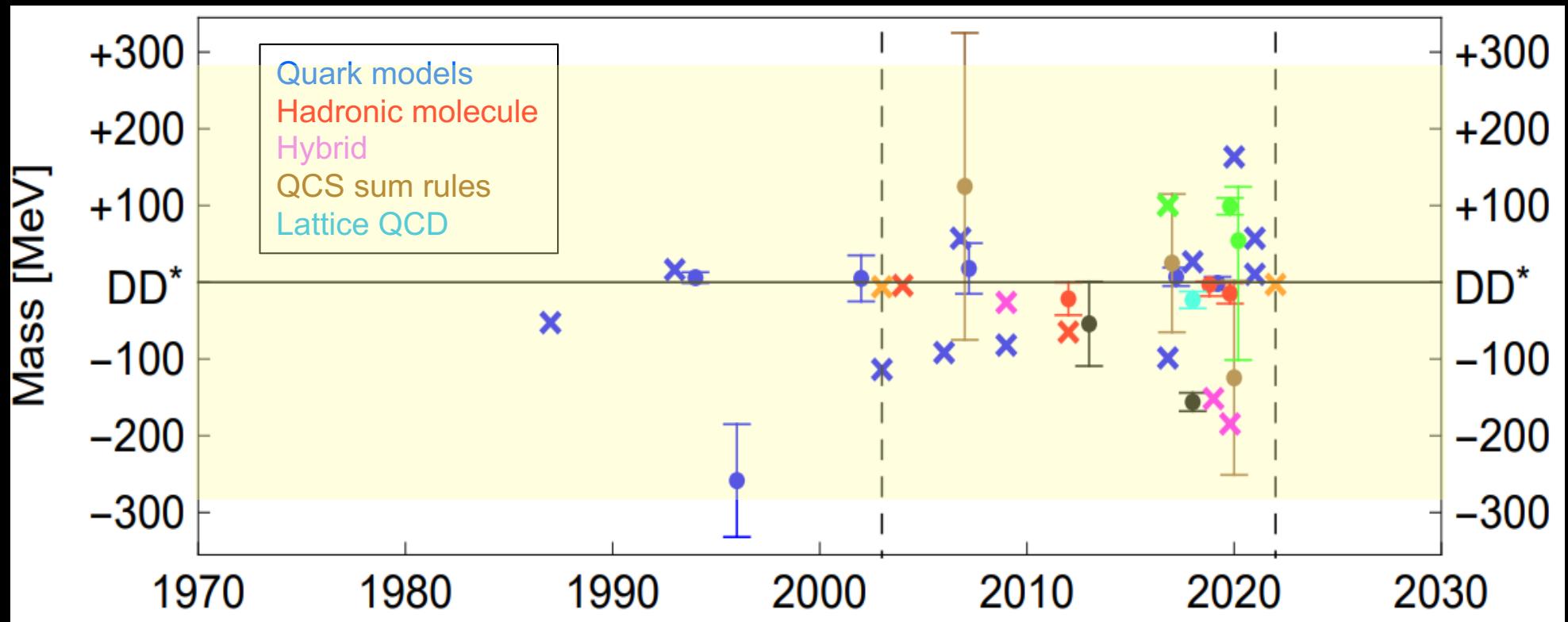
[LHCb collaboration](#)[Nature Communications 13, Article number: 3351 \(2022\) | Cite this article](#)

$I(J^P)$	δm_{pole}	Γ_{pole}
$0(1^+)$	$-360 \pm 40 \text{ keV}$	$48 \pm 2 \text{ keV}$

$\text{Re}(a_0)$	$\text{Im}(a_0)$
$7.16 \pm 0.51 \text{ fm}$	$-1.85 \pm 0.28 \text{ fm}$



Theory history (< 2022) of T_{cc}^+ ($cc\bar{u}\bar{d}$) ($IJ^P = 01^+$)



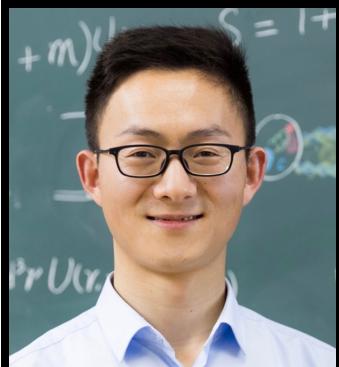
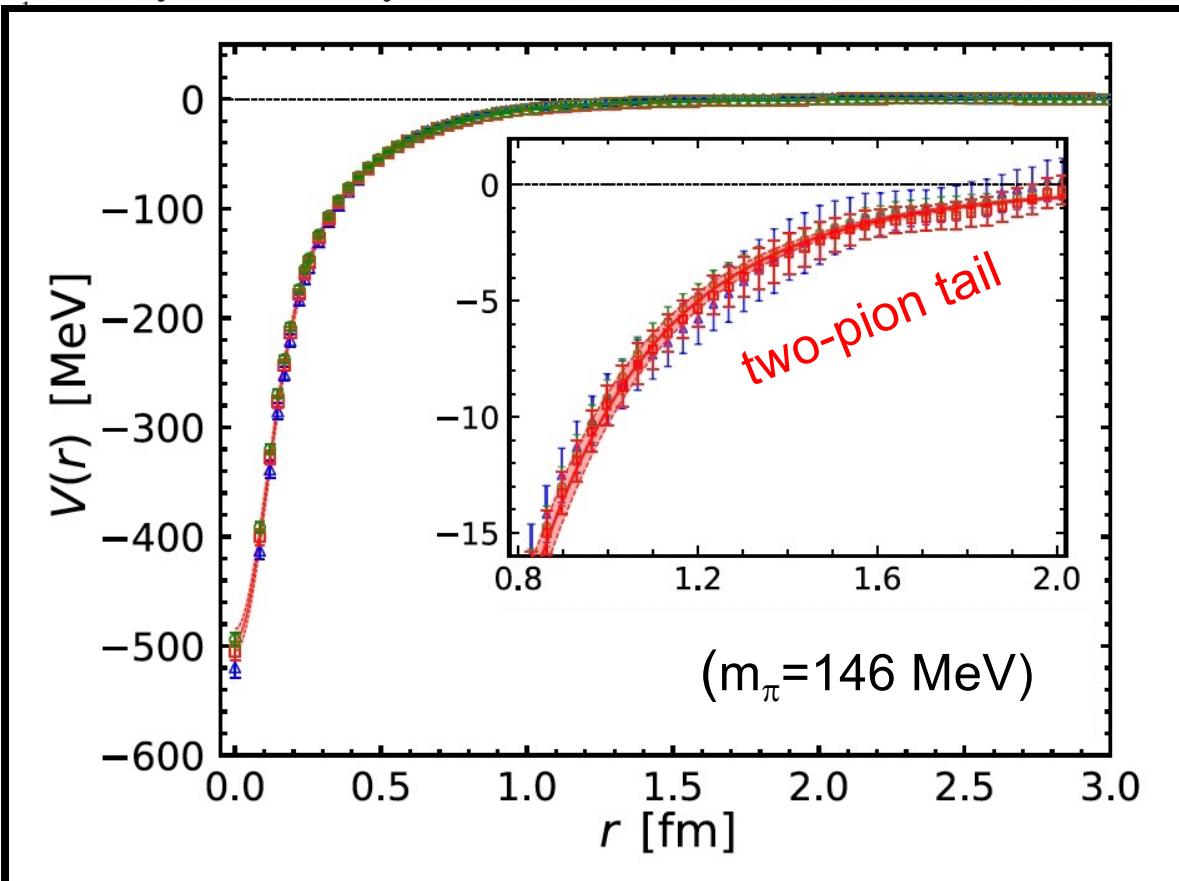
H.-X. Chen *et al.*, Rept. Prog. Phys. 86, 026201 (2023)

±300 MeV uncertainty with respect to DD* threshold

Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point

Yan Lyu^{1,2,*} Sinya Aoki^{3,2,†} Takumi Doi^{1,‡} Tetsuo Hatsuda^{1,§} Yoichi Ikeda^{4,||} and Jie Meng^{1,5,¶}

PHYSICAL REVIEW LETTERS 131, 161901 (2023)



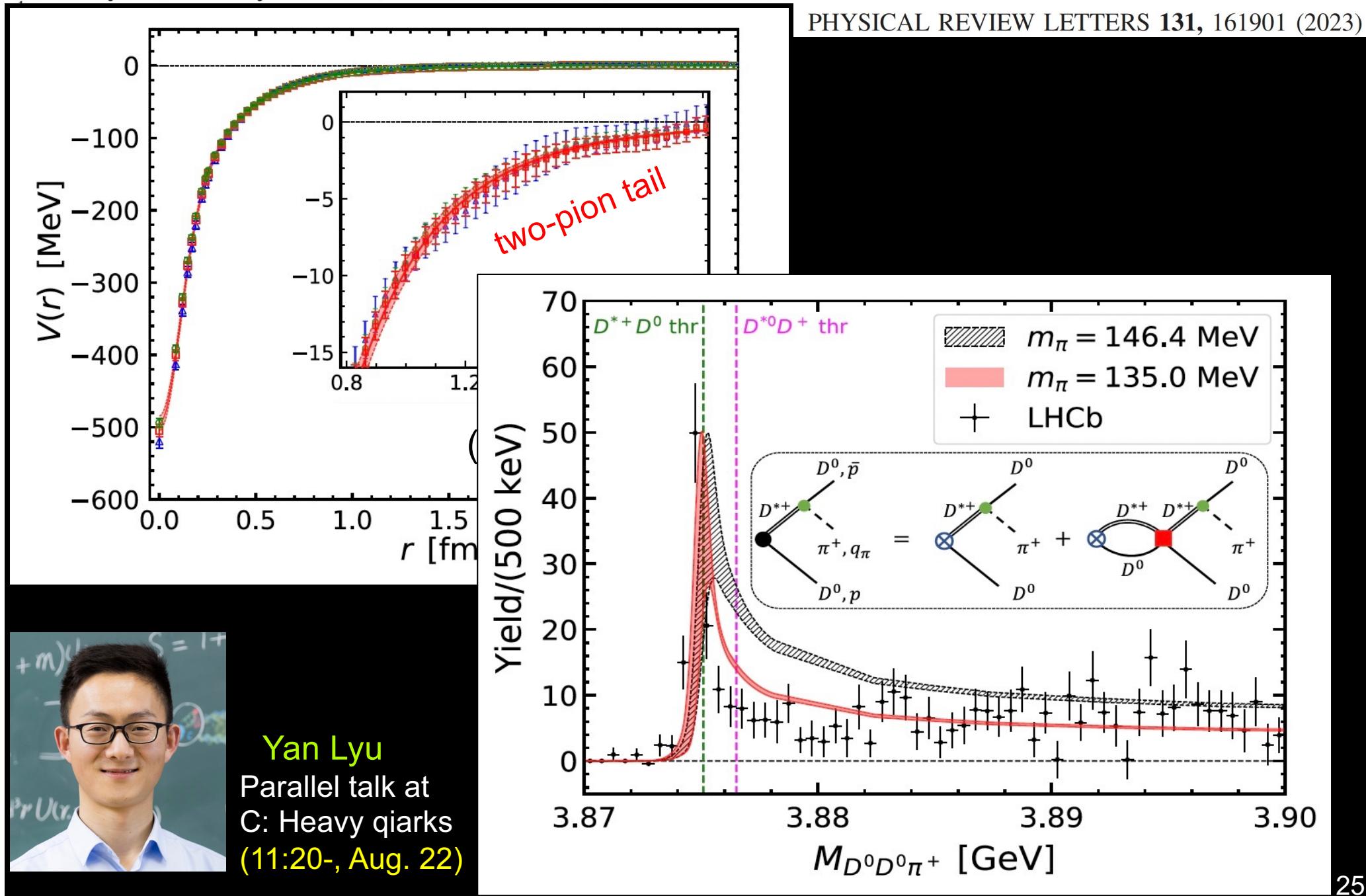
Yan Lyu

Parallel talk at
C: Heavy quarks
(11:20-, Aug. 22)

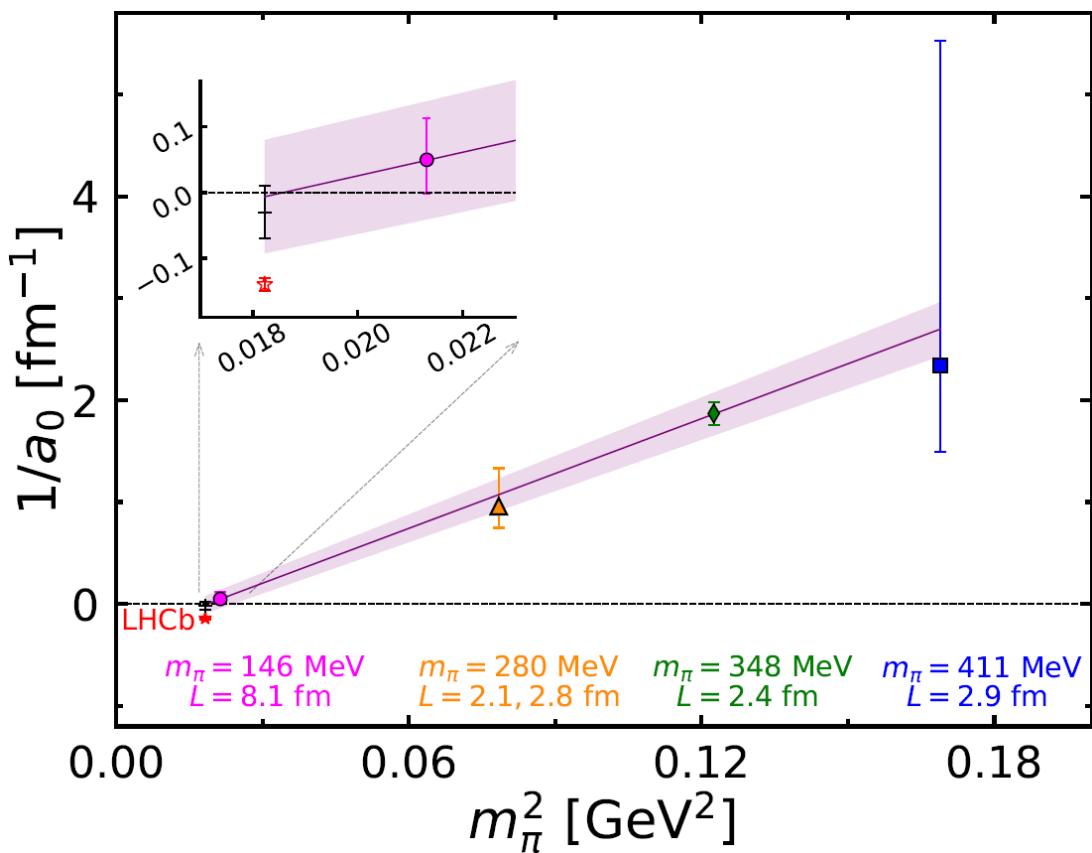
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PHYSICAL REVIEW LETTERS 131, 161901 (2023)



DD* scattering length as a function of the pion mass



HAL QCD: Ikeda+, Phys. Lett. B 729, 85 (2014) HAL QCD

Luscher: Chen et al., Phys. Lett. B 833, 137391 (2022)

Luscher: Padmanath and Prelovsek, Phys. Rev. Lett. 129, 032002 (2022)

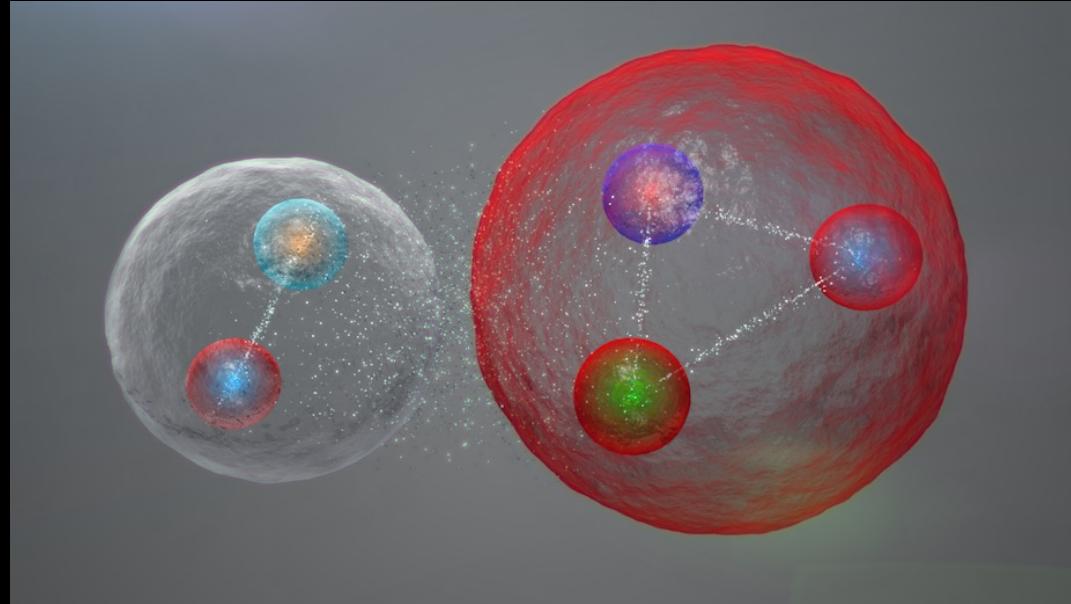
HAL QCD: Yan+, Phys. Rev. Lett. 131, 161901 (2023)

Not the end of the story:

- (i) Isospin breaking (QED and QCD), (ii) coupled channel effect,
- (iii) one-pion exchange and the “left hand cut”, S. Collins et al., PRD 109 (2024) 094509

Meson-Baryon Interactions

- color-dipole and nucleon -

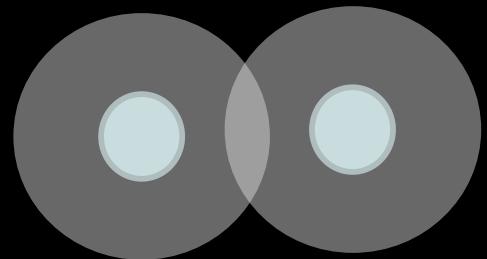


$s\bar{s}$ - N

$c\bar{c}$ - N

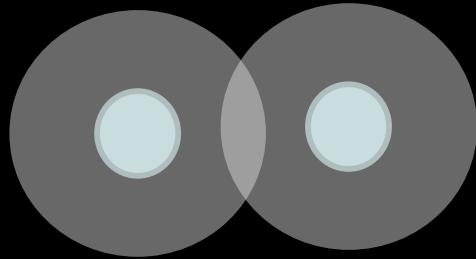
Question:

What is the force between
“neutral particles” at long range ?



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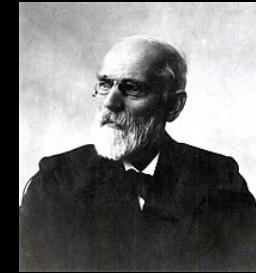


Answer:

QED

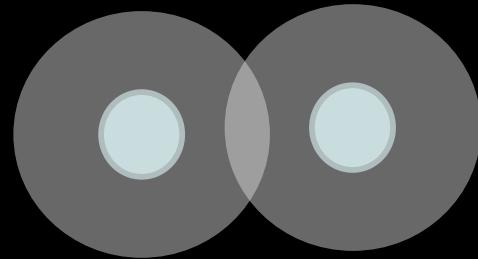
2-photon exchange force

= van der Waals (Casimir-Polder) force $\rightarrow -1/r^7$

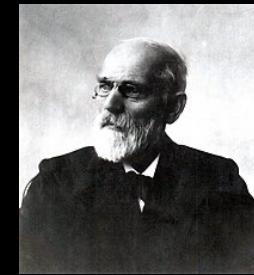


Question:

What is the force between
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Answer:



QED

2-photon exchange force
= van der Waals (Casimir-Polder) force $\rightarrow -1/r^7$

QCD

2-pion exchange force



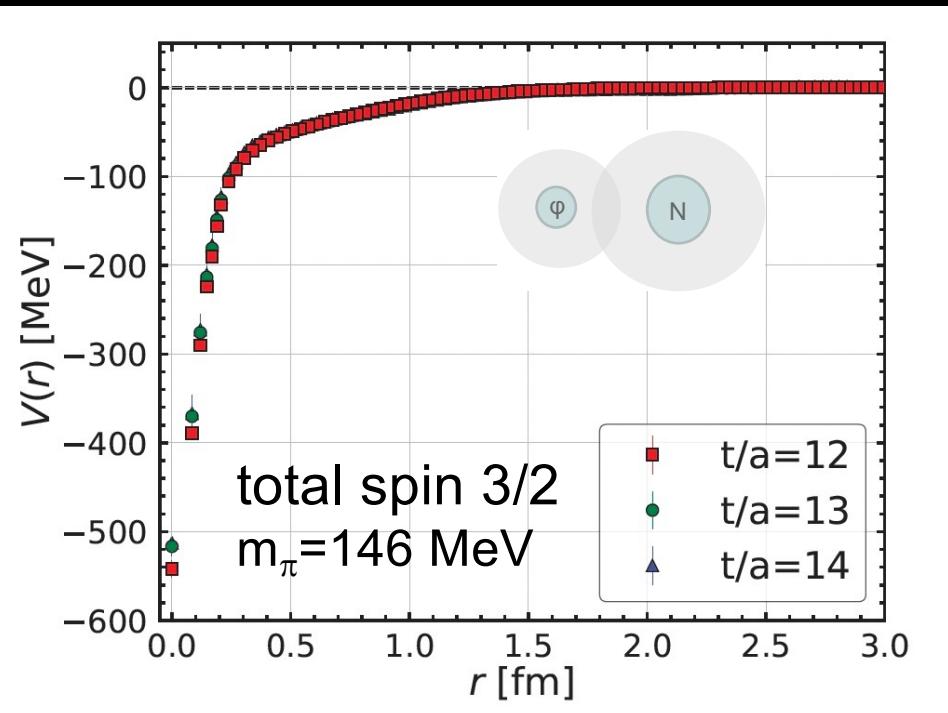
Bhanot and Peskin, Nucl. Phys. B156 (1979) 391

Fujii and Kharzeev, Phys. Rev. D60 (1999) 114039

Brambilla, Klein, Tarrus Castella and Vairo Phys. Rev. D93 (2016) 054002

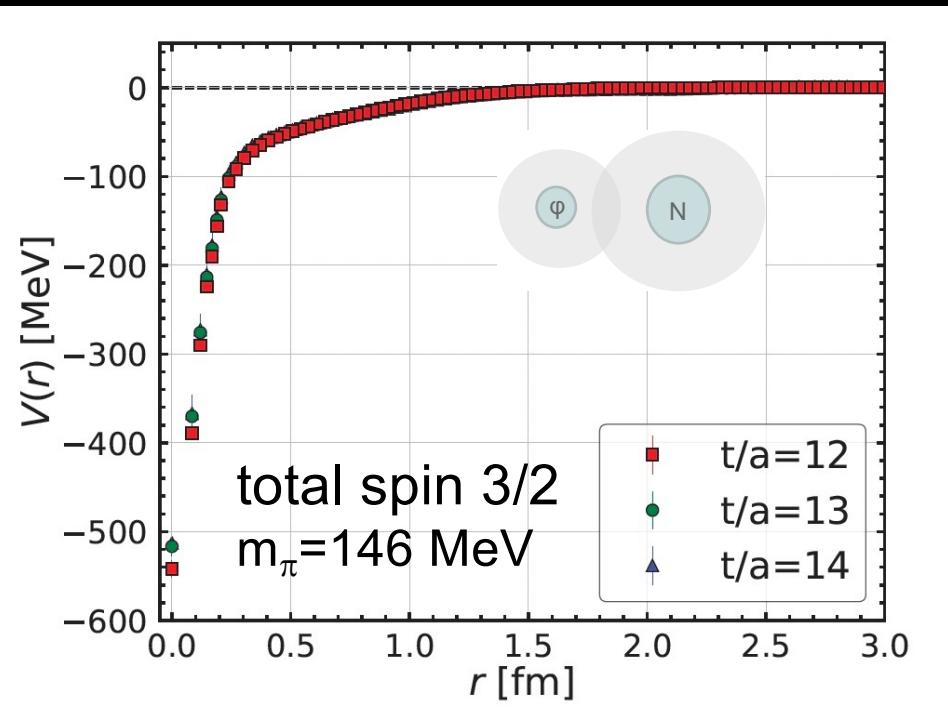
Attractive $N\phi$ interaction and two-pion tail from lattice QCD near physical point

Yan Lyu, Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda, Jie Meng, Kenji Sasaki, and Takuya Sugiura
Phys. Rev. D **106**, 074507 – Published 21 October 2022



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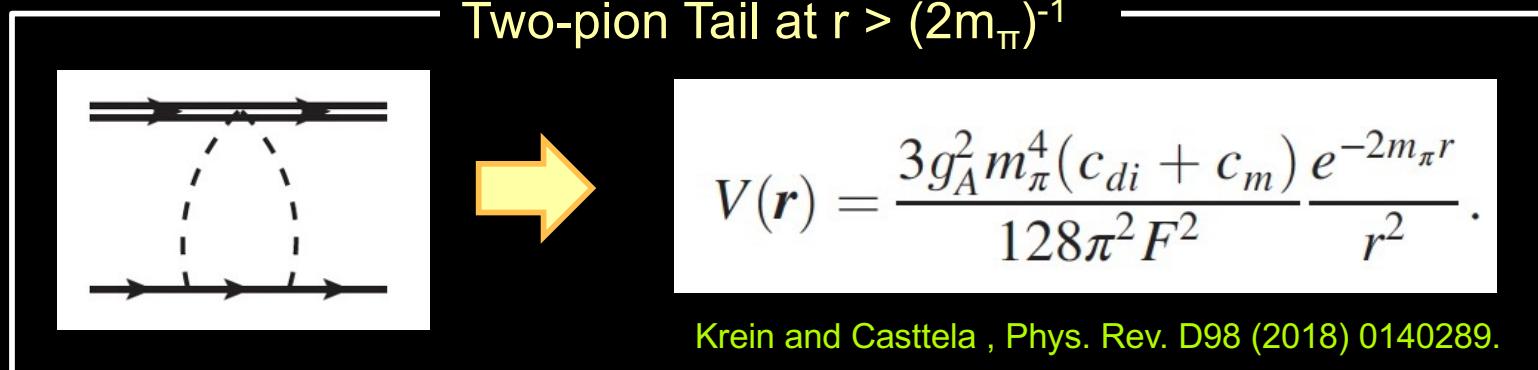
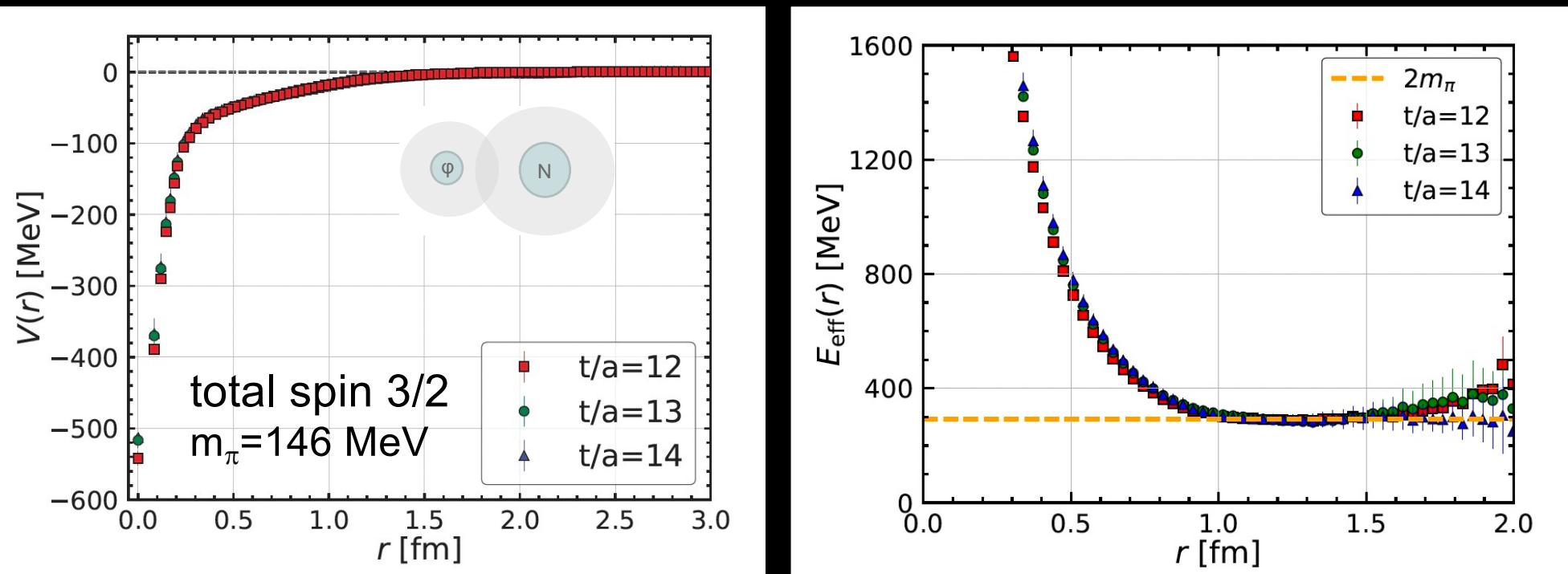
Two-pion Tail at $r > (2m_\pi)^{-1}$

$V(\mathbf{r}) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}.$

Krein and Casttela , Phys. Rev. D98 (2018) 0140289.

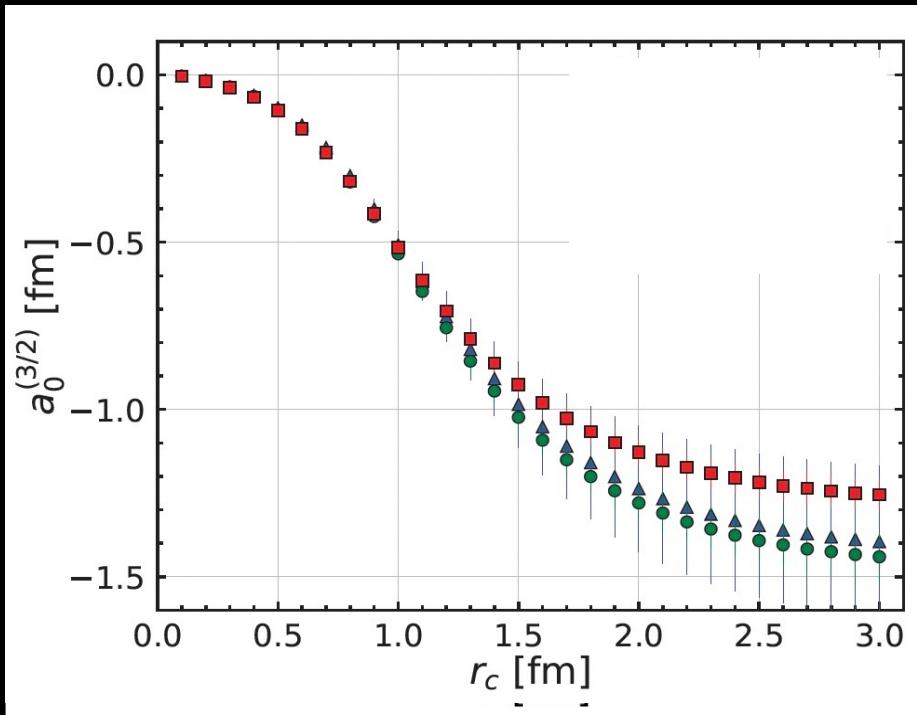
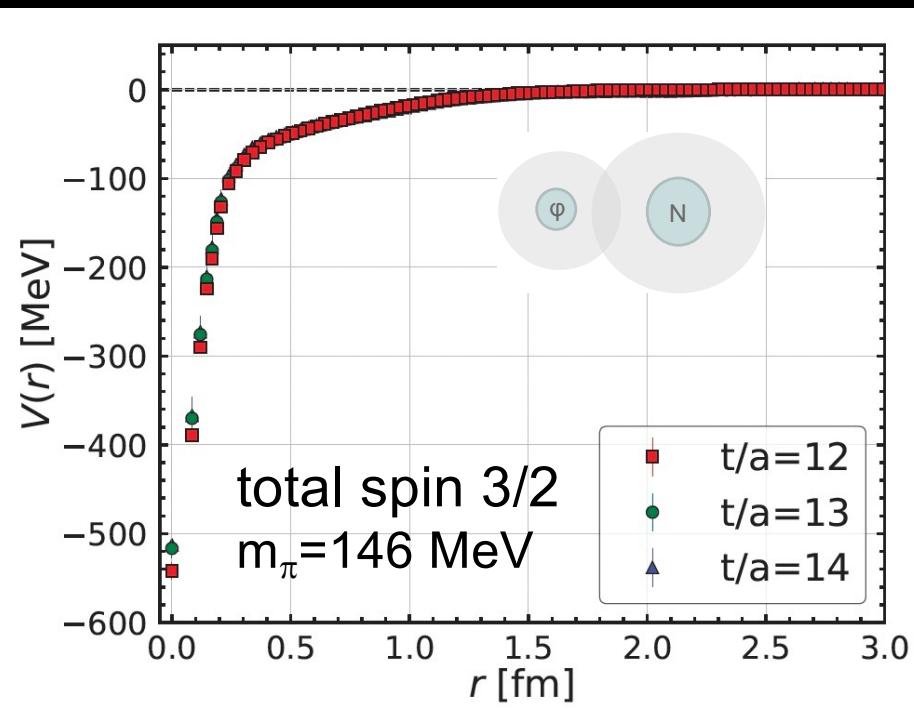
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 Phys. Rev. D **106**, 074507 – Published 21 October 2022



Two-pion Tail at $r > (2m_\pi)^{-1}$

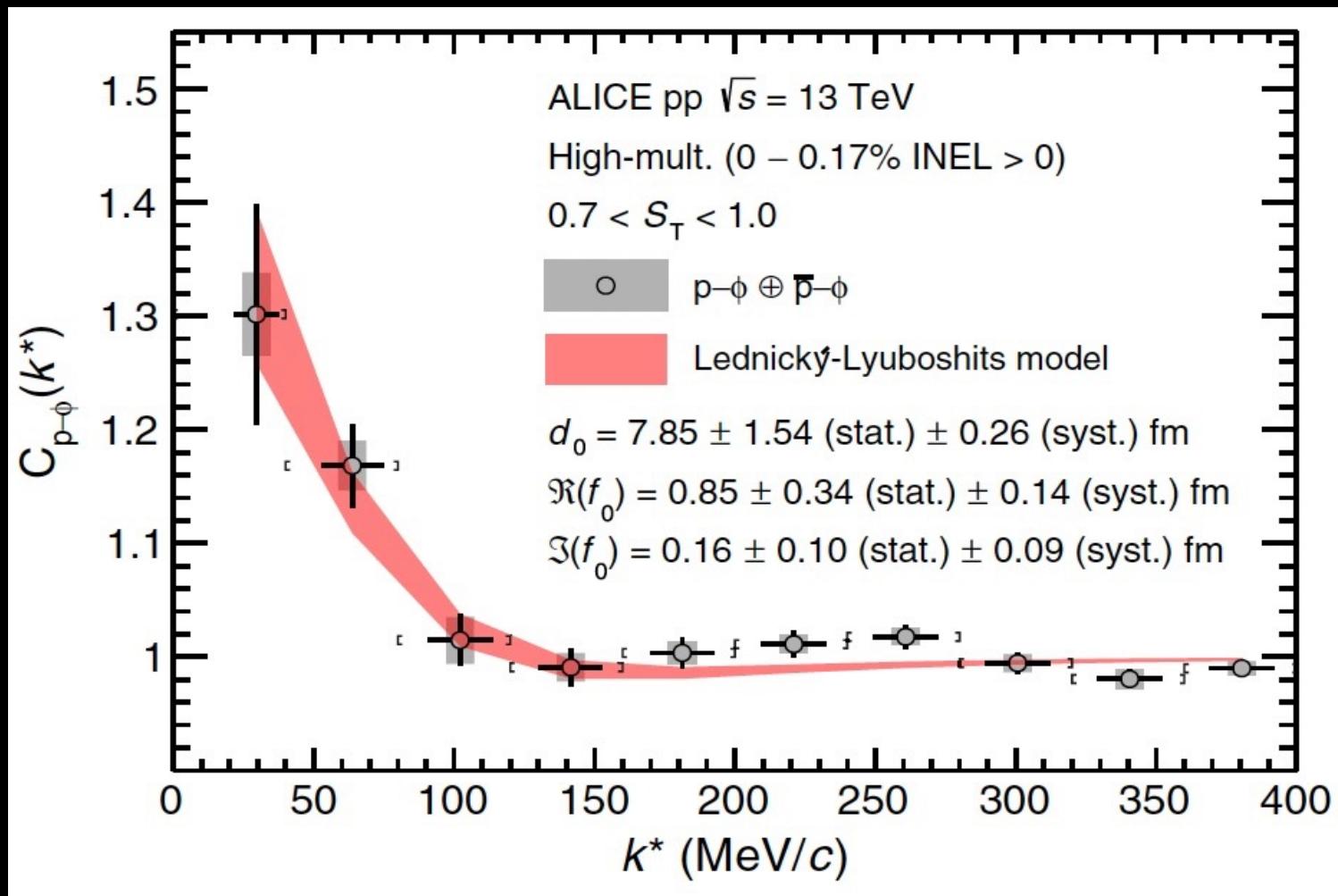
$V(\mathbf{r}) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}.$

Krein and Castella , Phys. Rev. D98 (2018) 0140289.

Experimental Evidence for an Attractive $p\text{-}\phi$ Interaction

S. Acharya *et al.* (ALICE Collaboration)

Phys. Rev. Lett. **127**, 172301 – Published 20 October 2021



→ possible spin $1/2$ bound ϕN system; Chizzali+, Phys. Lett. B848 (2024) 138358

What about $c\bar{c}$ -N interaction ?

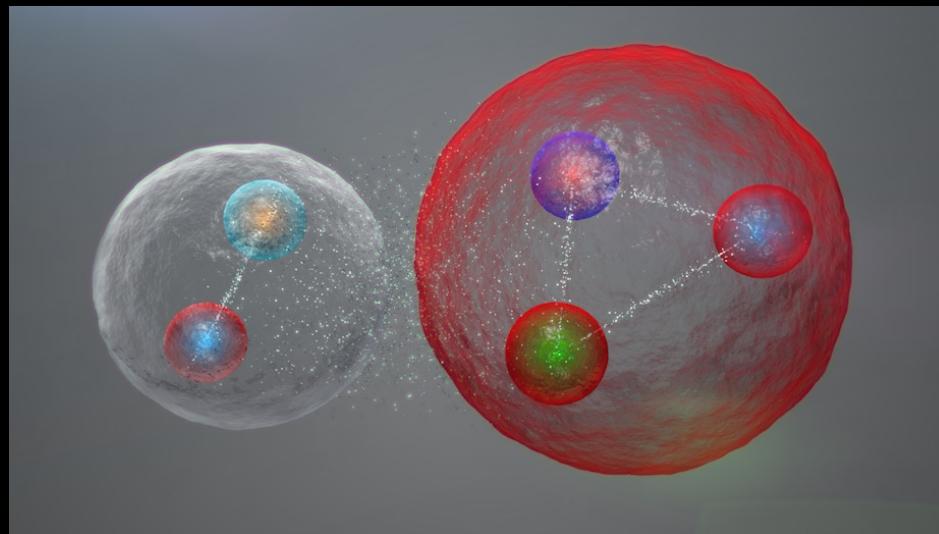
The nucleon-charmonium interactions from lattice QCD

Yan Lyu^a, Takumi Doi^a, Tetsuo Hatsuda^a, Takuya Sugiura^{b,a}

^a*Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, Wako, 351-0198, Japan*

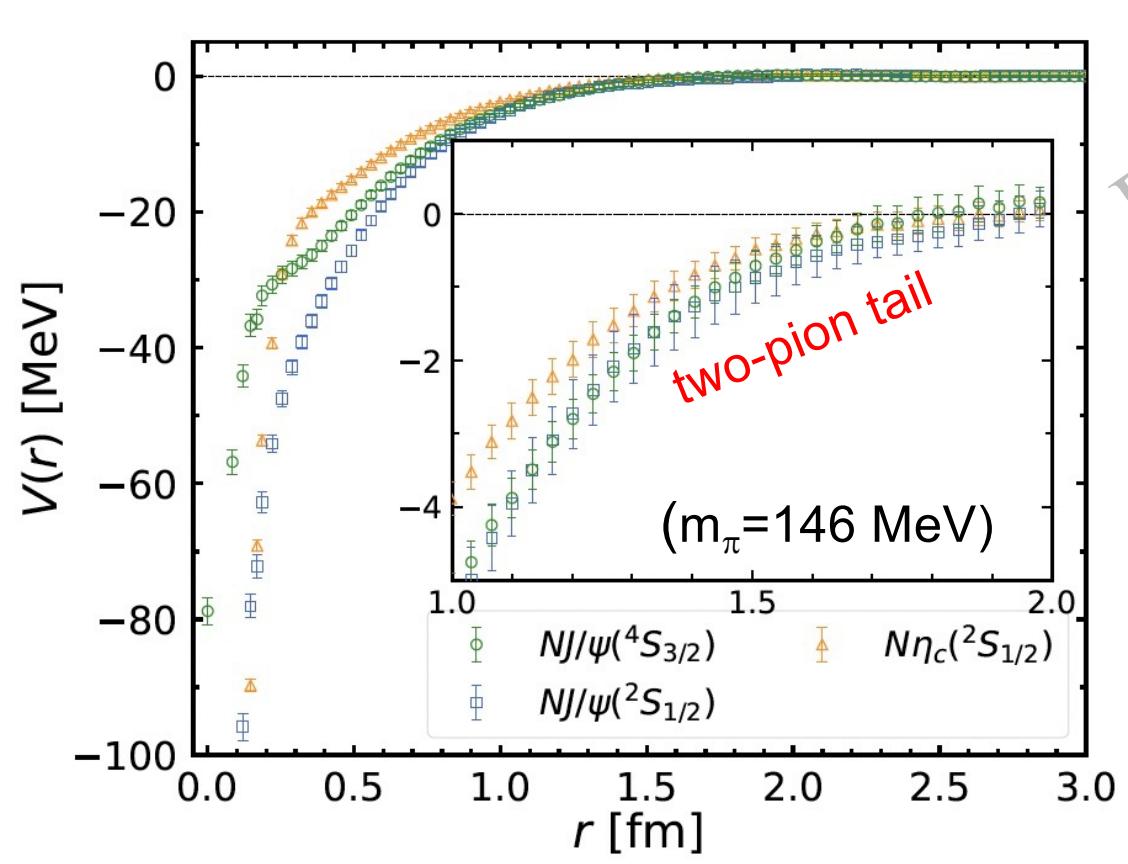
^b*Faculty of Date Science, Rissho University, Kumagaya, 360-0194, Japan*

Preliminary

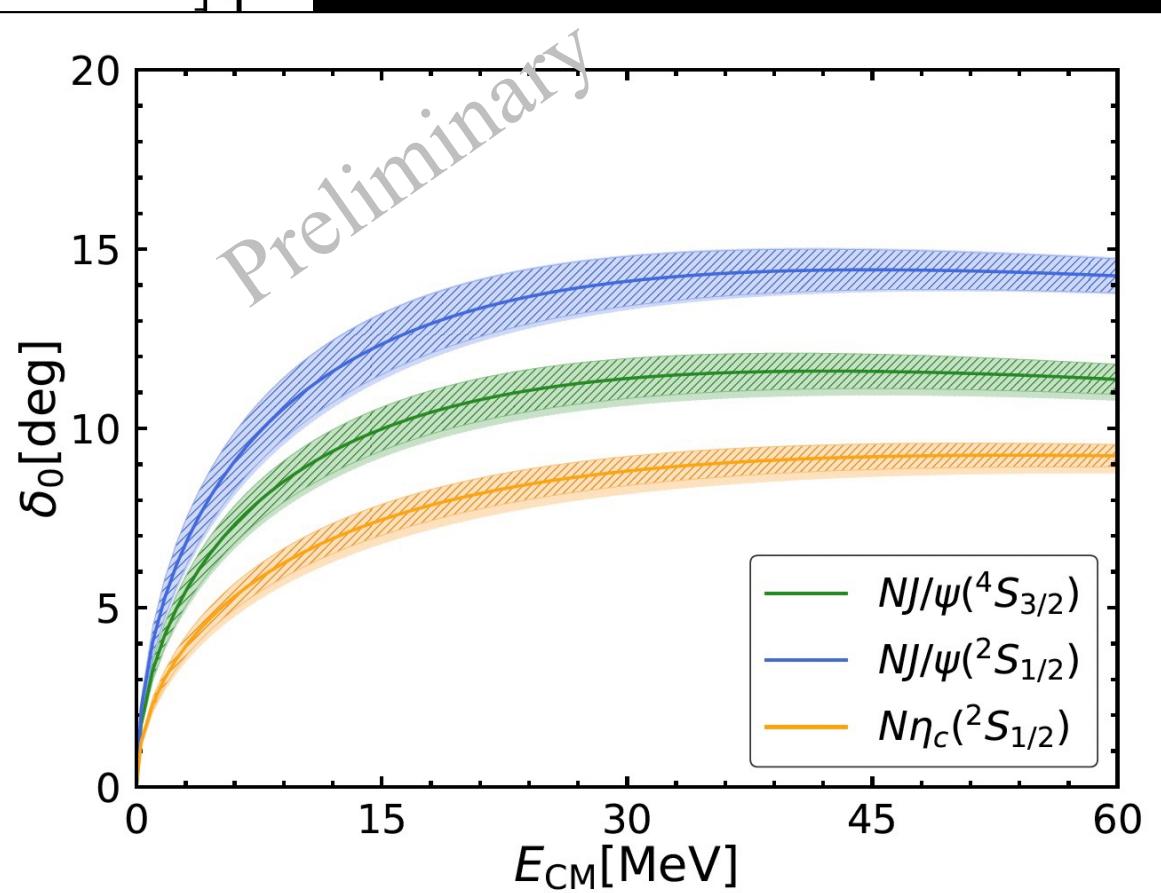
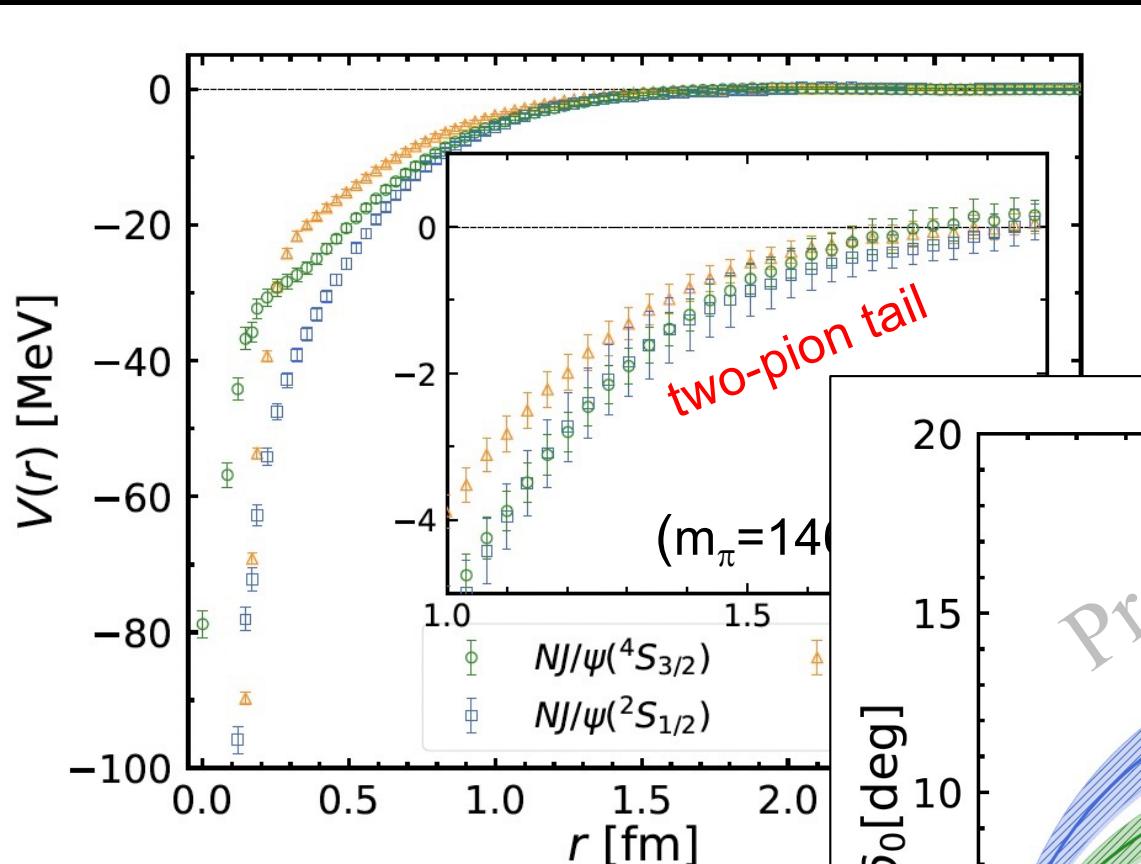


$c\bar{c}$ - N Potentials and phase shifts

Preliminary



$c\bar{c}$ - N Potentials and phase shifts



Scattering length and effective range

channel	a_0 [fm]	r_{eff} [fm]
$NJ/\psi(^4S_{3/2})$	$0.30(2) \begin{pmatrix} +0 \\ -2 \end{pmatrix}$	$3.25(12) \begin{pmatrix} +6 \\ -9 \end{pmatrix}$
$NJ/\psi(^2S_{1/2})$	$0.38(4) \begin{pmatrix} +0 \\ -3 \end{pmatrix}$	$2.66(21) \begin{pmatrix} +0 \\ -10 \end{pmatrix}$
$N\eta_c(^2S_{1/2})$	$0.21(2) \begin{pmatrix} +0 \\ -1 \end{pmatrix}$	$3.65(20) \begin{pmatrix} +0 \\ -6 \end{pmatrix}$

Scattering length and effective range

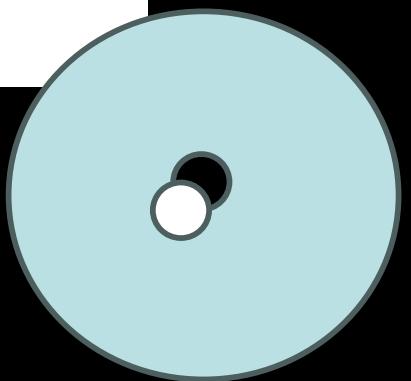
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J/ ψ optical potential in nuclear matter

$$\delta m_{J/\psi} \simeq \frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\text{nm}} = 19(3) \text{ MeV},$$

↔ Krein, Tsushima, Thomas,
Nuclear-bound quarkonia and heavy-flavor hadros
 Prog. in Part. Nucl. Phys. 100 (2018) 161



Summary

Hadron-hadron int. from LQCD at $m_\pi = 146$ MeV opens the door to the “co-evolution” of LQCD and expt.

- examples -

BB: $\Lambda\Lambda$ - $N\Xi$, $N\Omega$, $\Omega\Omega$, MM: T_{cc}^+ , MB: $s\bar{s}$ - N , $c\bar{c}$ - N

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Take-home messages

1. $\Omega\Omega$ could be bound: first stable B=2 system other than the deuteron?
2. Two-pion exchange : universal for long-range part of hadron interactions
3. Heavy quarks: useful probes for the two-pion exchange.

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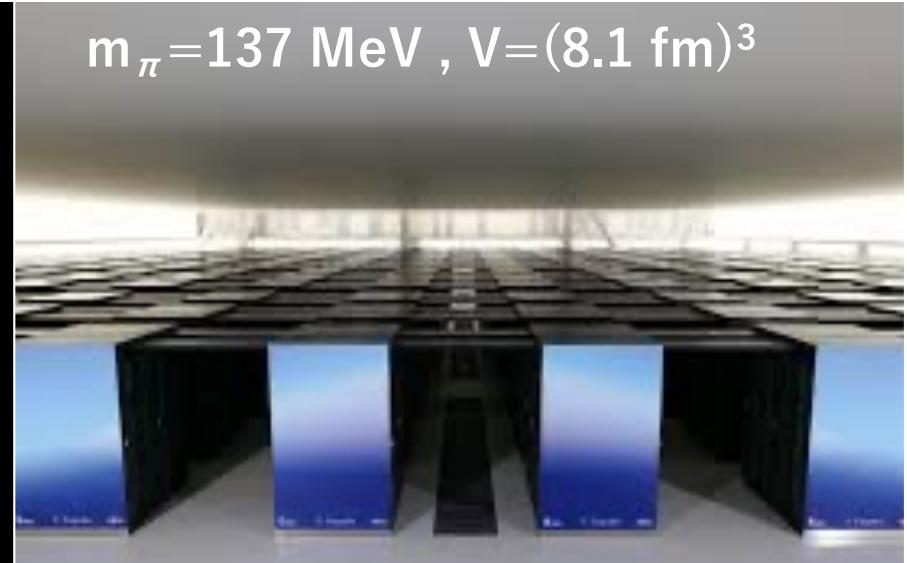
Take-home messages

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2. Two-pion exchange : universal for long-range part of hadron interactions
3. Heavy quarks: useful probes for the two-pion exchange.

- New results at $m_\pi=137$ MeV will come soon.
- Open issues:
 - coupled channel effects
 - isospin breaking (QED & QCD) near threshold.

Stay tuned !

$m_\pi=137$ MeV , $V=(8.1 \text{ fm})^3$

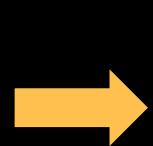
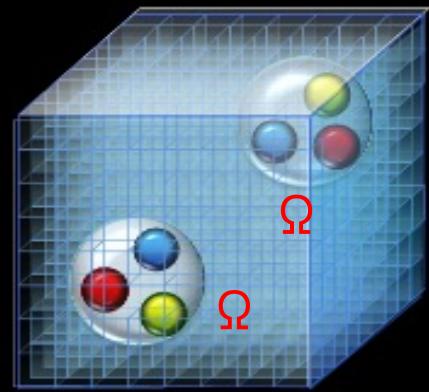


FUGAKU @RIKEN: 440 PFlops (2020-)

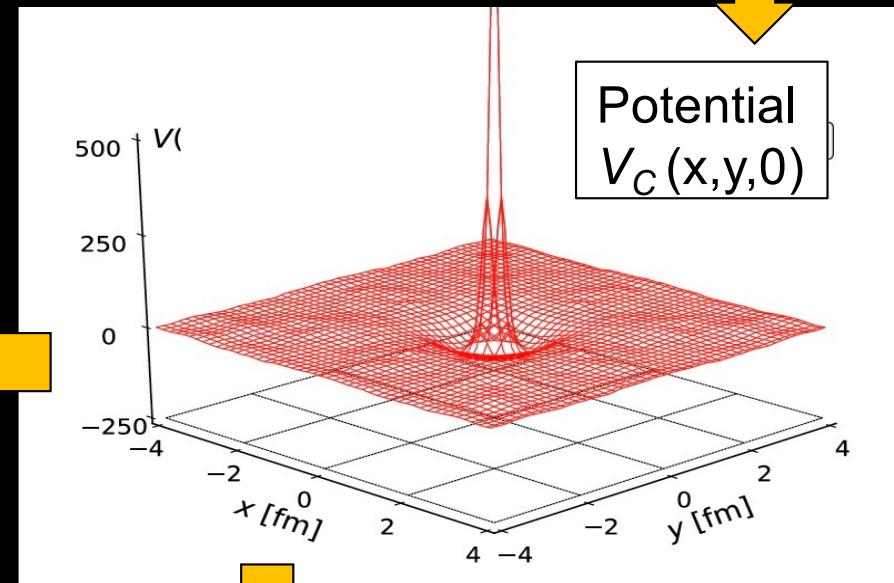
Backup slides

HAL QCD Procedure: an example ($\Omega\Omega$ at $m_\pi=146\text{MeV}$)

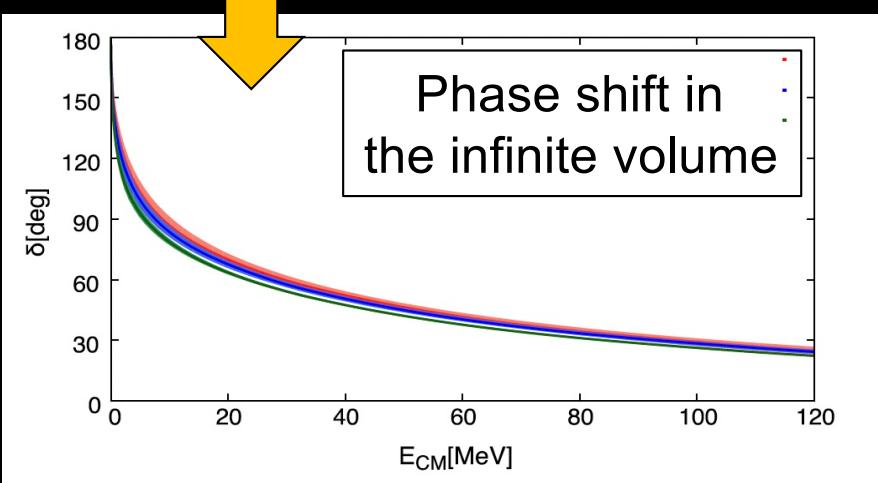
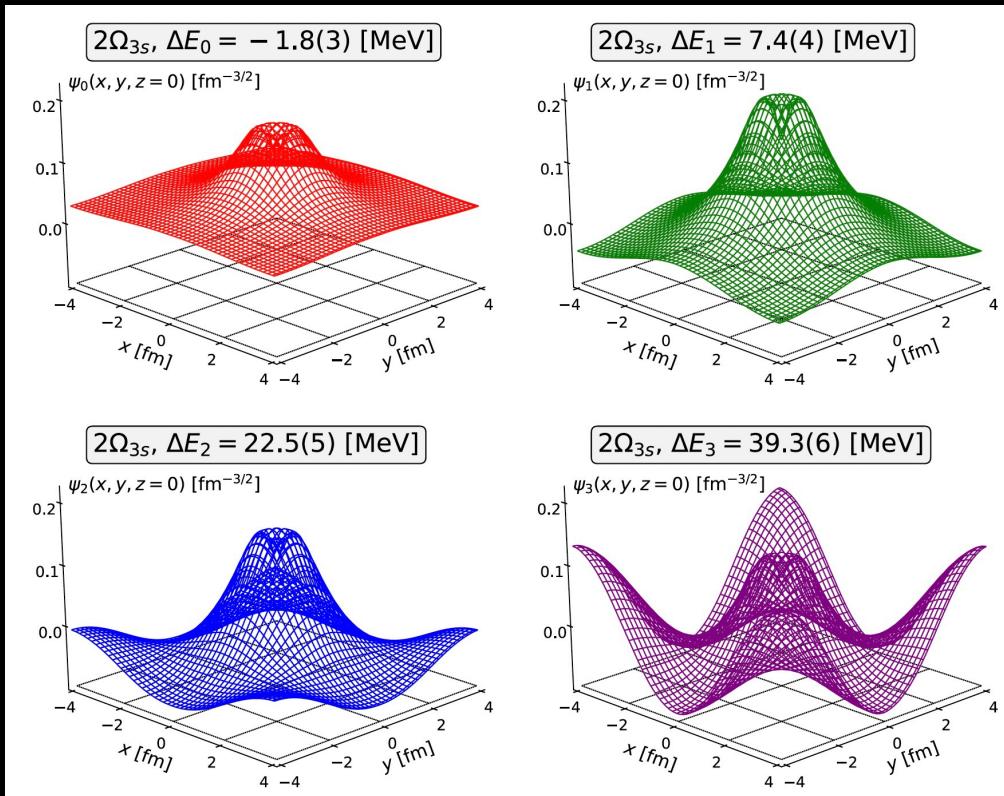
Gongyo+ [HAL QCD], PRL 120 (2018) 212001
Lyu+ [HAL QCD], PRD 105 (2022) 074512



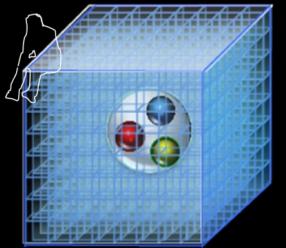
$$F^J(\mathbf{r}, t) = \sum_n a_n^J \psi_n(\mathbf{r}) e^{-E_n t} \rightarrow V_C(r)$$



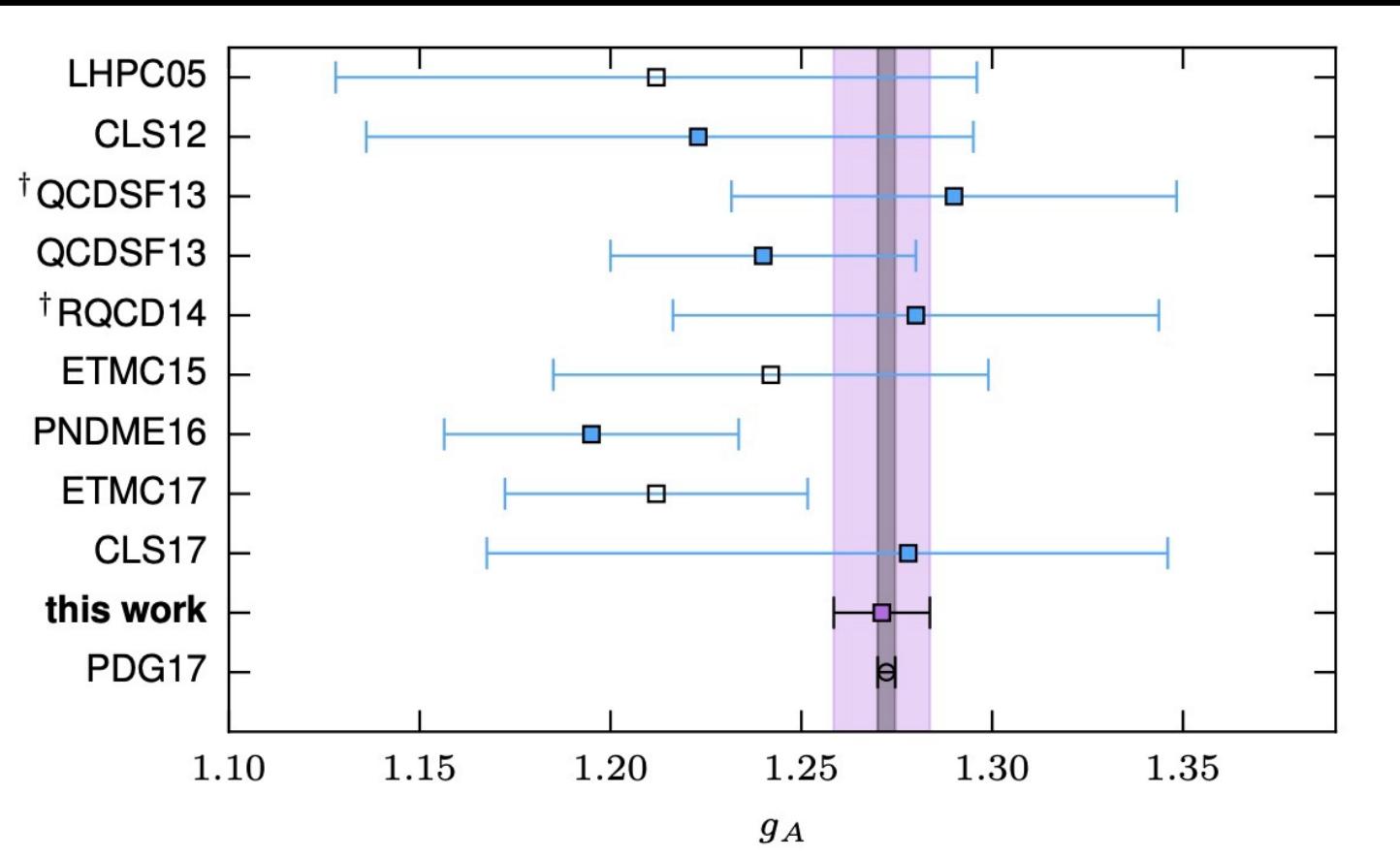
$\Psi_n(\mathbf{r})$ and E_n in a finite volume



Percent-level determination of g_A from LQCD



Chang (LBNL/iTHEMS)+, Nature **558** (2018) 91



$$(g_A)_{\text{LQCD}} = 1.271(13)$$
$$(\tau_n)_{\text{LQCD}} = 884(15) \text{ s}$$

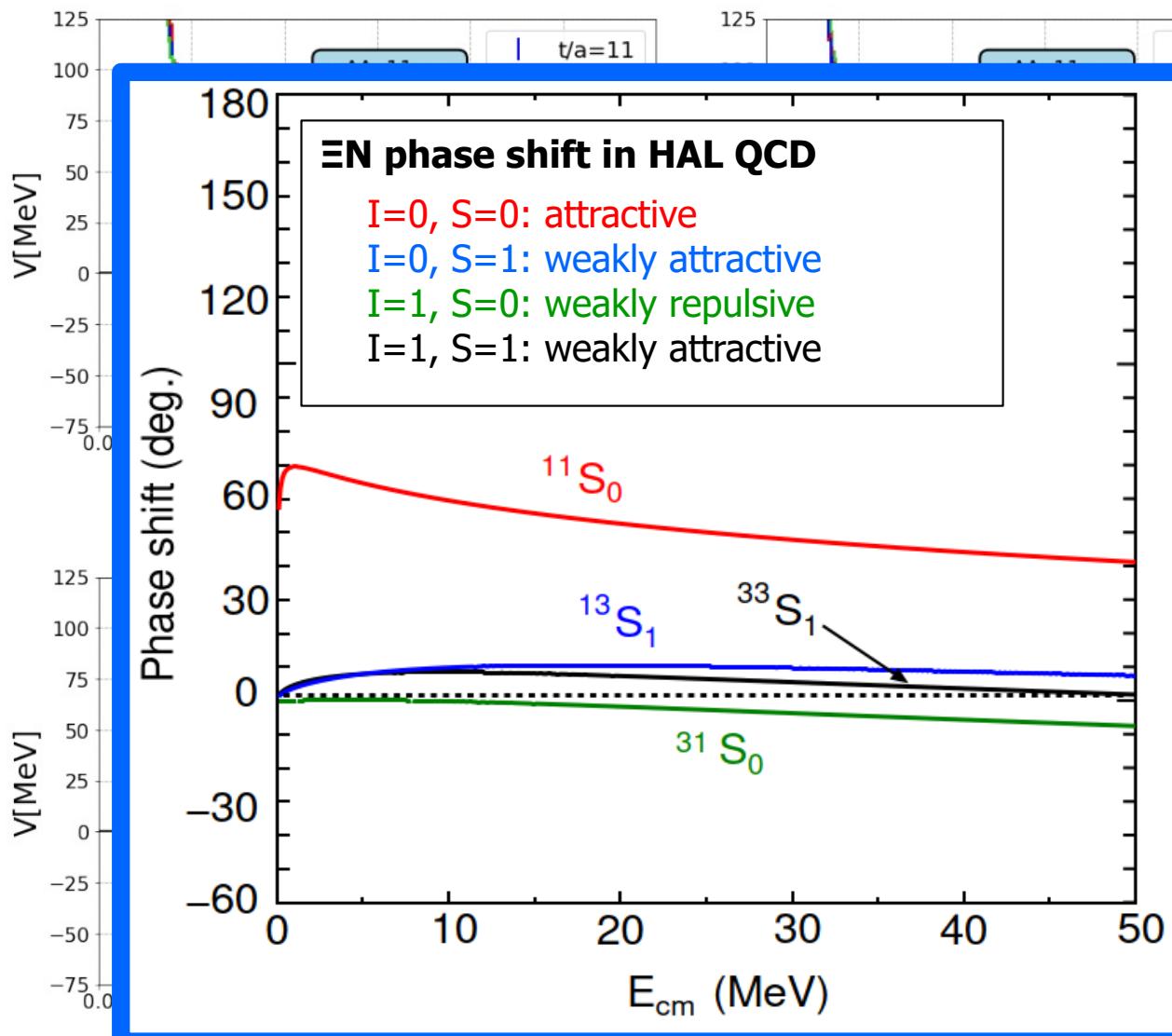


$$(g_A)_{\text{expt}} = 1.272(2)$$
$$(\tau_n)_{\text{PDG}} = 880.2(1.0) \text{ s}$$

Coupled Channel S=-2 system ($^{11}S_0$)

K. Sasaki+ [HAL QCD Coll.]
Nucl. Phys. **A998** (2020)

Small $\Lambda\Lambda$
attraction
Short-range
 $N\bar{N}-\Lambda\Lambda$ coupling

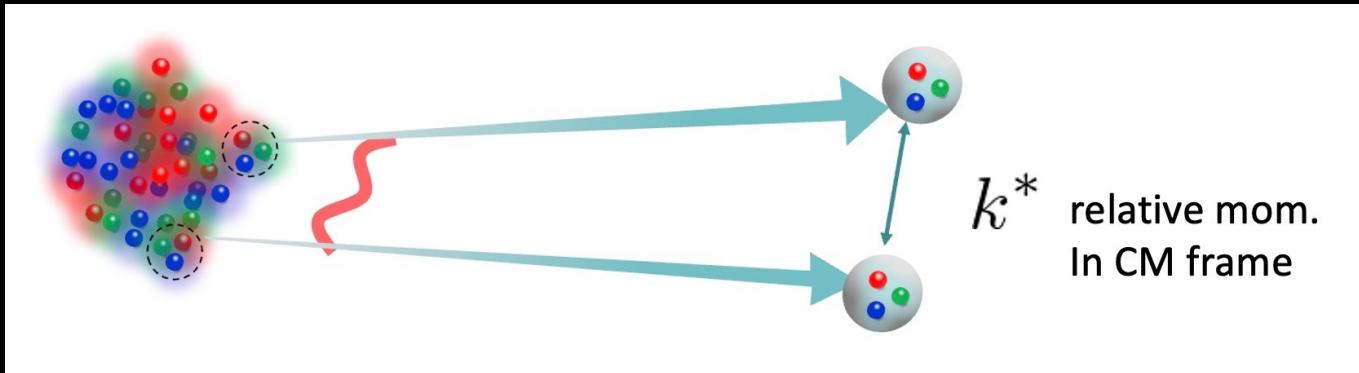


(c)

(d)

Short-range
 $N\bar{N}-\Lambda\Lambda$ coupling
Large $N\bar{N}$
attraction

Femtoscopy : particle correlations in pp, pA and AA



$$C_{\text{expt}}(k^*) = \frac{\xi(k^*) N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

$$\begin{aligned} C_{\text{theo}}(k^*) &= \int d^3r S(r, k^*) |\Psi(r, k^*)|^2 \\ &= 1 + \int d^3r S(r, k^*) [|\Psi_0(r, k^*)|^2 - |j_0(k^* r)|^2] \end{aligned}$$

Koonin, Phys. Lett. B 70, 43 (1977).

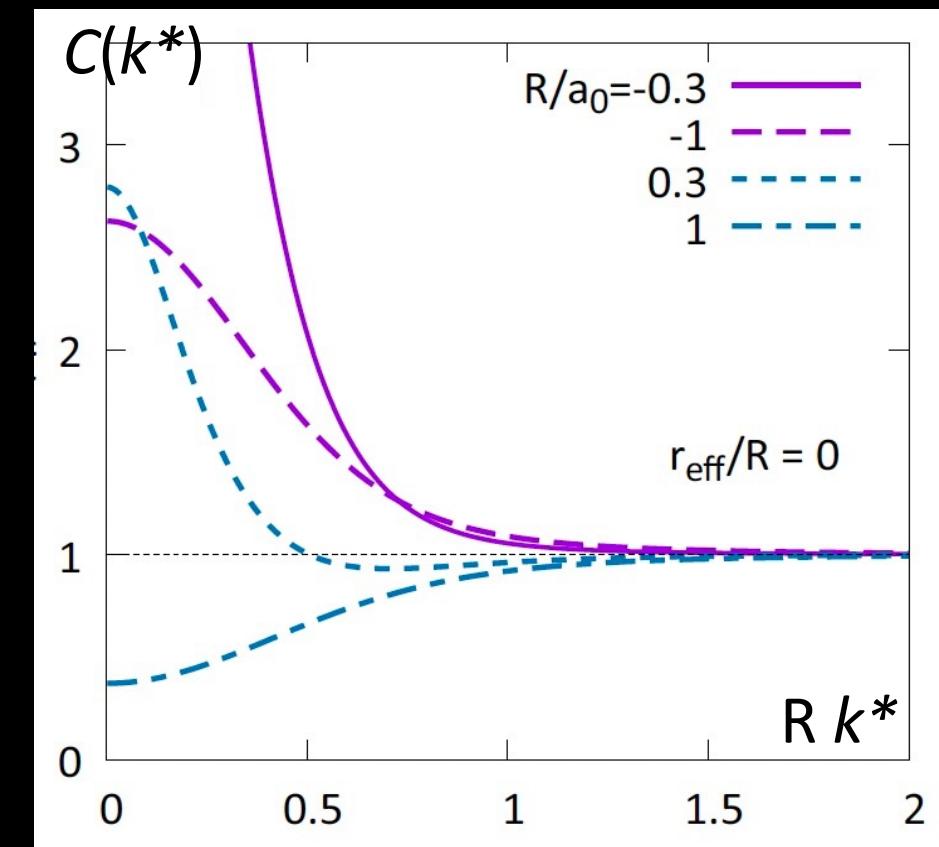
Lednicky and Lyuboshits, Yad. Fiz. 35, 1316 (1981).

Pratt, Phys. Rev. D 33, 1314 (1986).

Anchishkin, Heinz, and Renk, Phys. Rev. C 57, 1428 (1998).

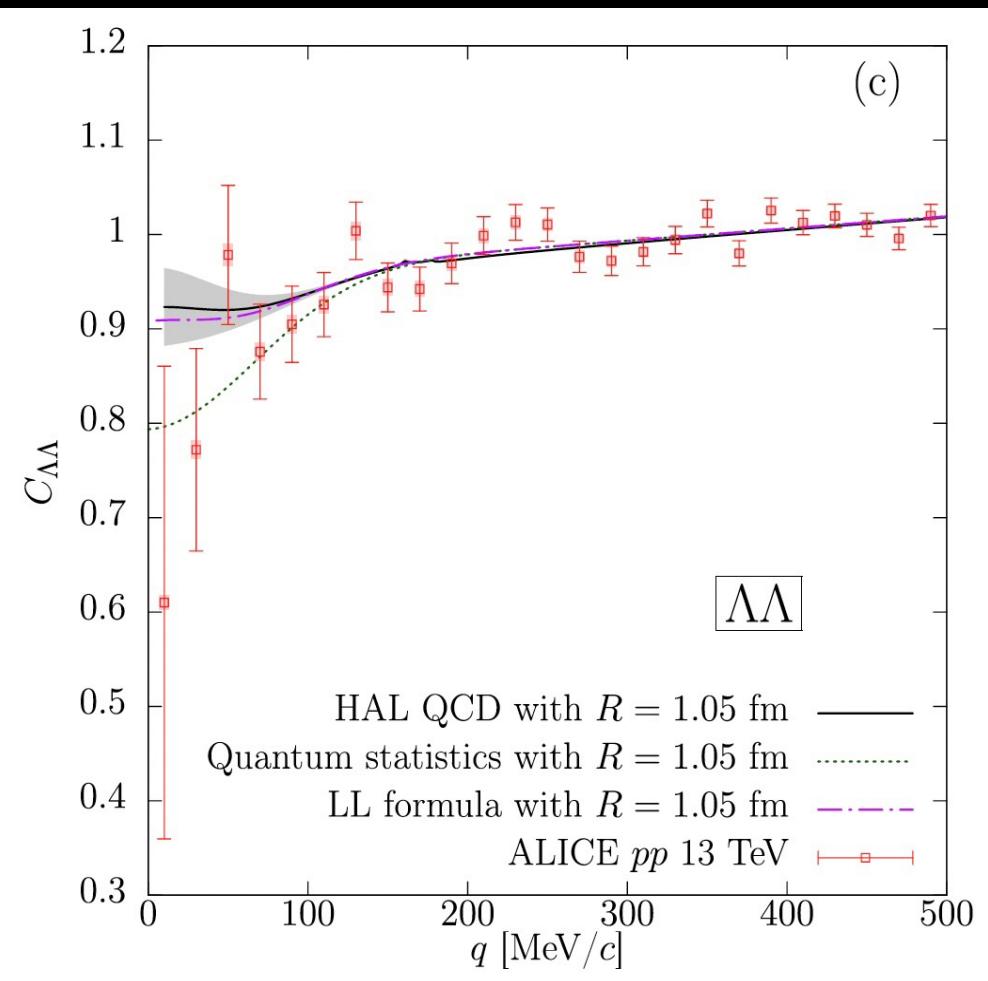
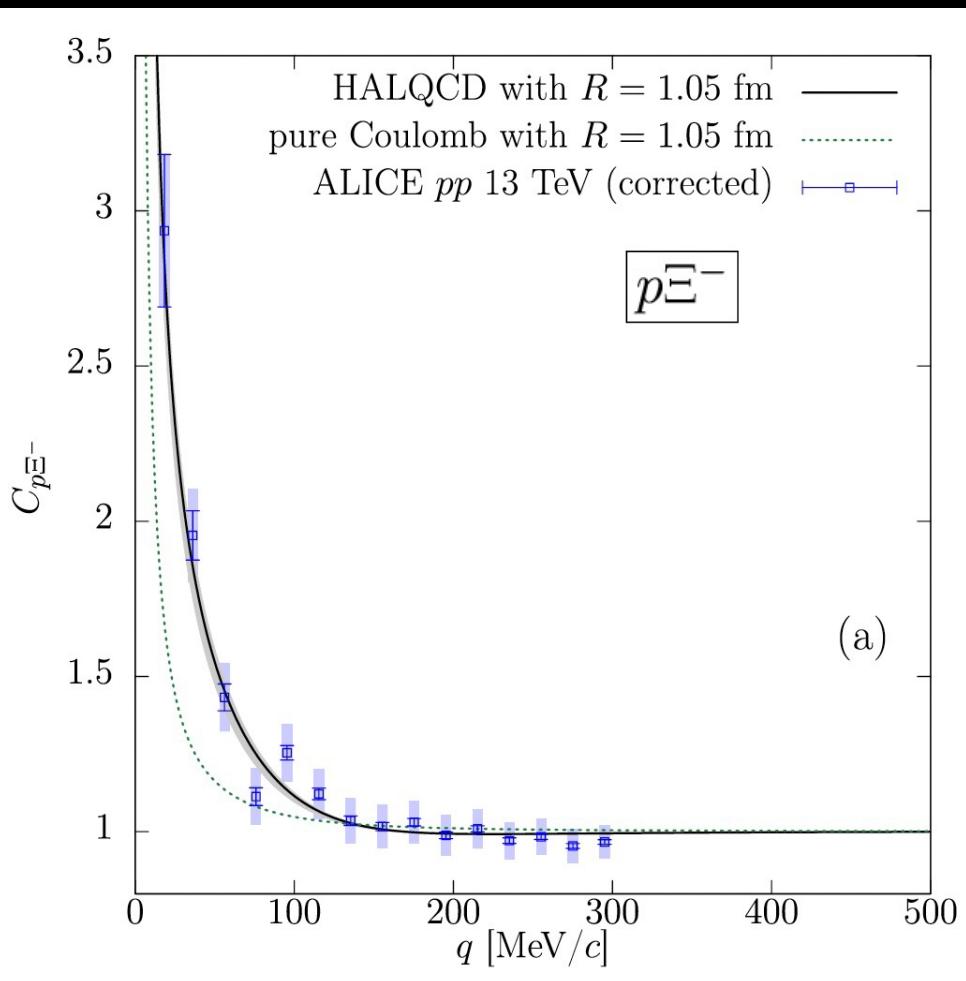
Lednicky, Lyuboshits, and Lyuboshits, Phys. At. Nucl. 61, 2950 (1998).

Haidenbauer, Nucl. Phys. A 981, 1 (2019).

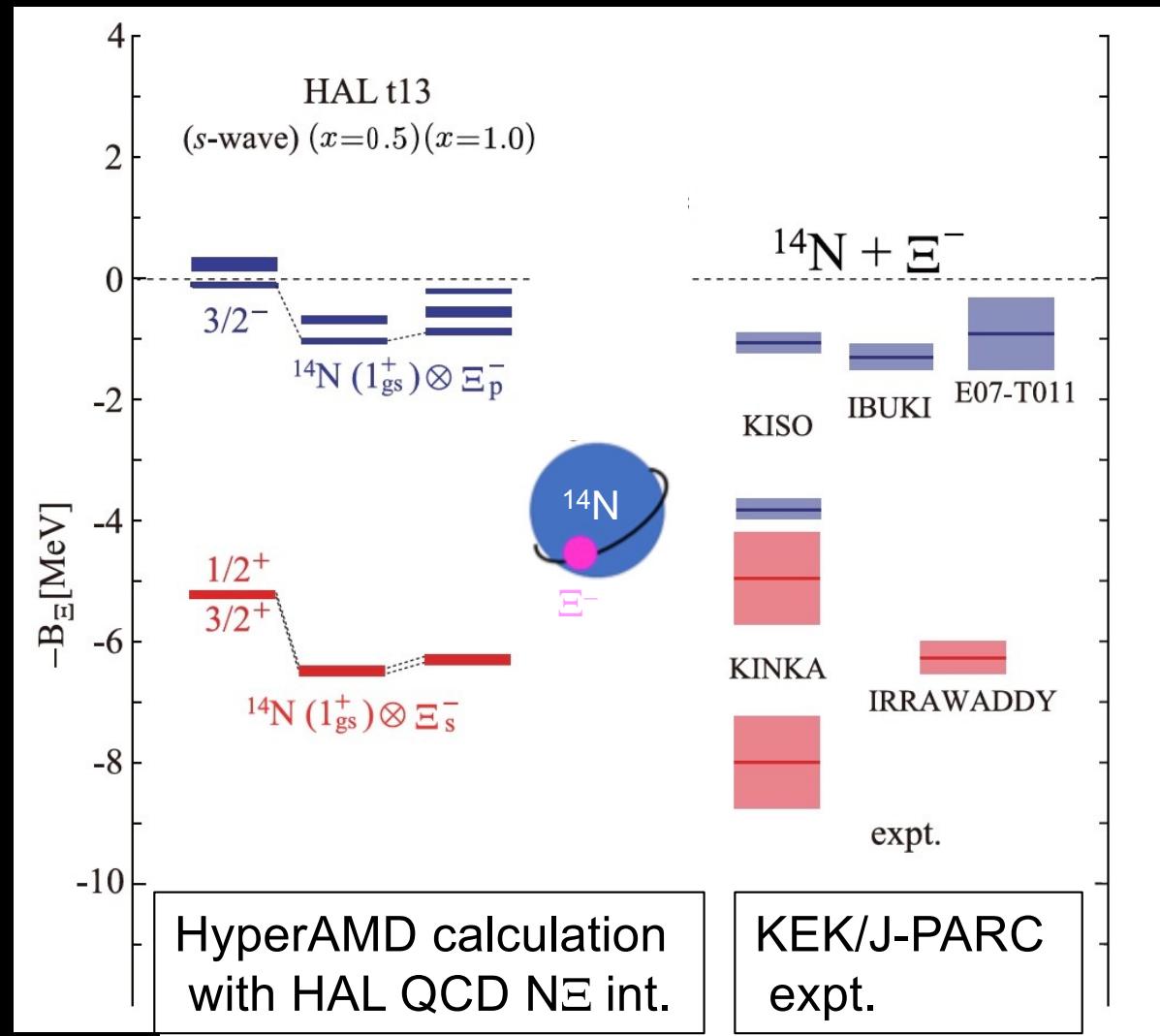


Fully coupled channel analysis

Kamiya+, Phys. Rev. C 105 (2022) 014915



Excitation spectra of $^{15}_{\Xi}$ C and $^{12}_{\Xi}$ Be calculated with a ΞN interaction from lattice QCD



Isaka+,
Phys.Rev. C109 (2024) 044317

p-state

s-state



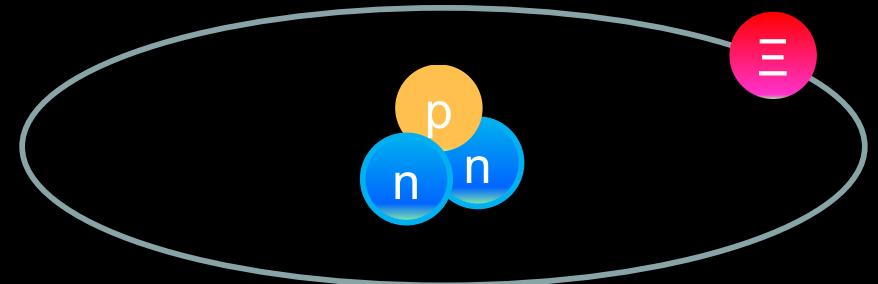
LQCD prediction of

- attraction in $N\Xi$ in 1S_0
- weak $N\Xi - \Lambda\Lambda$ coupling
- cosisistent with the expt. data

Attempts towards lighter Ξ -nuclei such as ${}^3\text{He}-\Xi$ and $\alpha\alpha\Xi$:
Hiyama+, PRL 124 (2020); PRC 106 (2022)

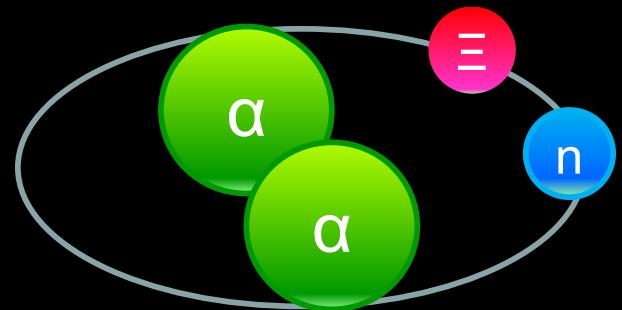
What is the lightest Ξ hypernuclei?

Hiyama, Sasaki, Miyamoto, Doi,
Hatsuda, Yamamoto, Riken,
Phys.Rev.Lett.124 (2020) 092501



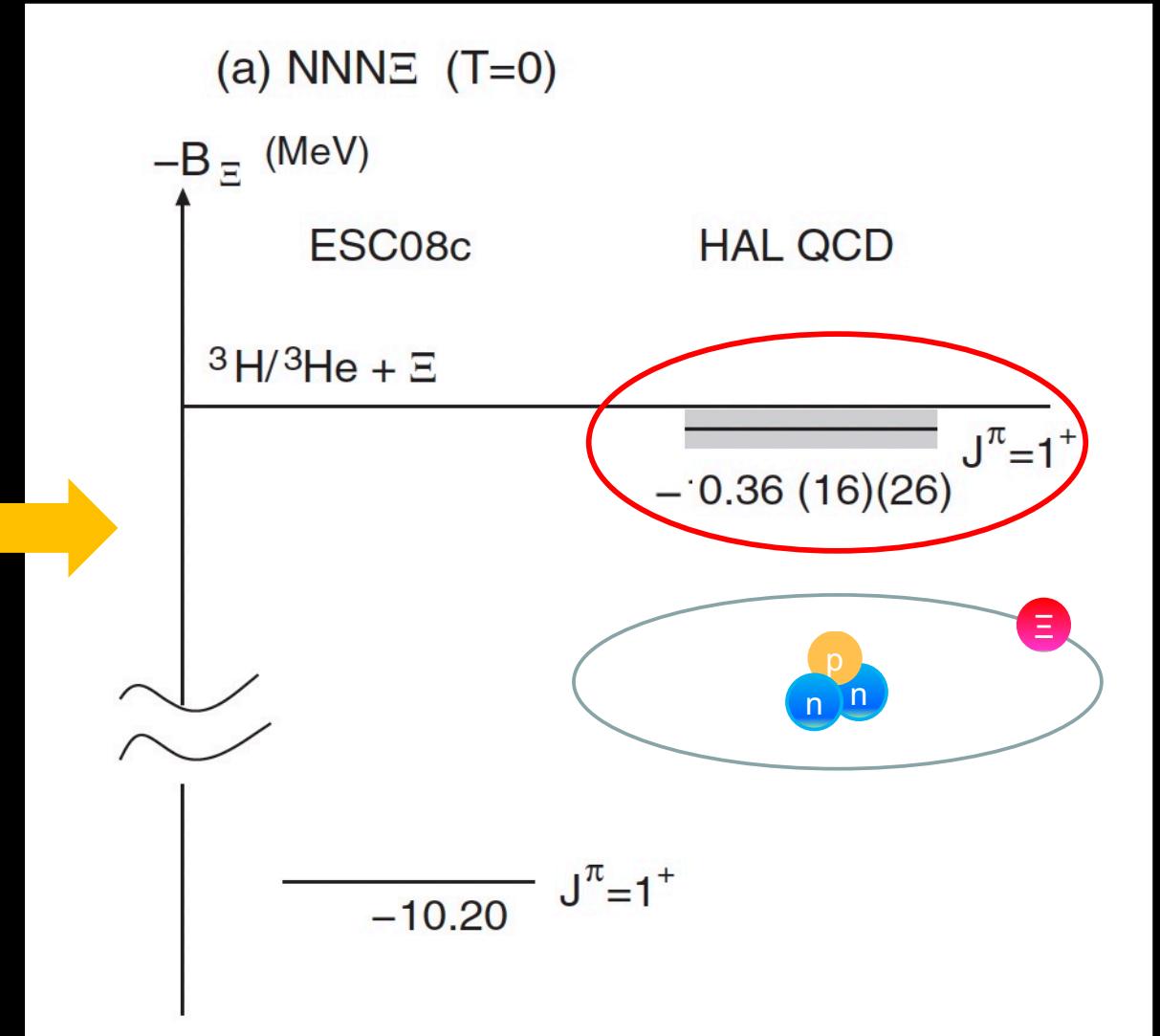
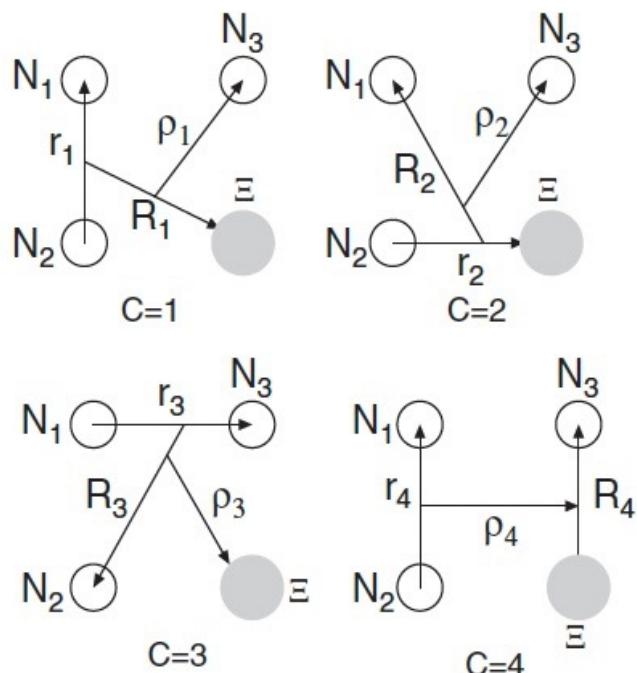
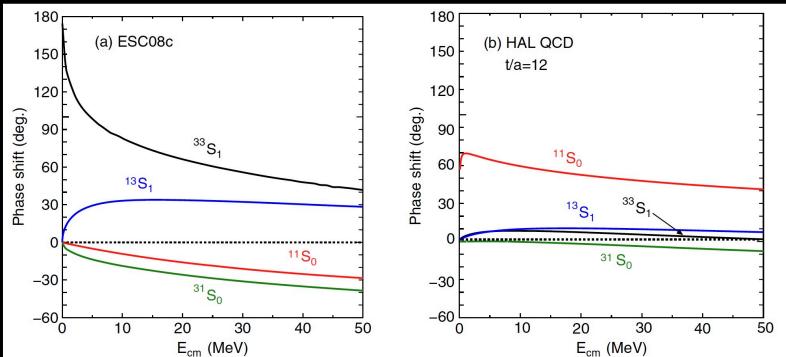
Can we test the spin-isospin dependence of ΞN int.?

Hiyama, Isaka, Doi, Hatsuda,
Phys. Rev. C 106 (2022) 064318



Q1. What is the lightest Ξ hypernuclei?

Hiyama, Sasaki, Miyamoto, Doi, Hatsuda, Yamamoto, Riken,
 Phys.Rev.Lett.124 (2020) 092501

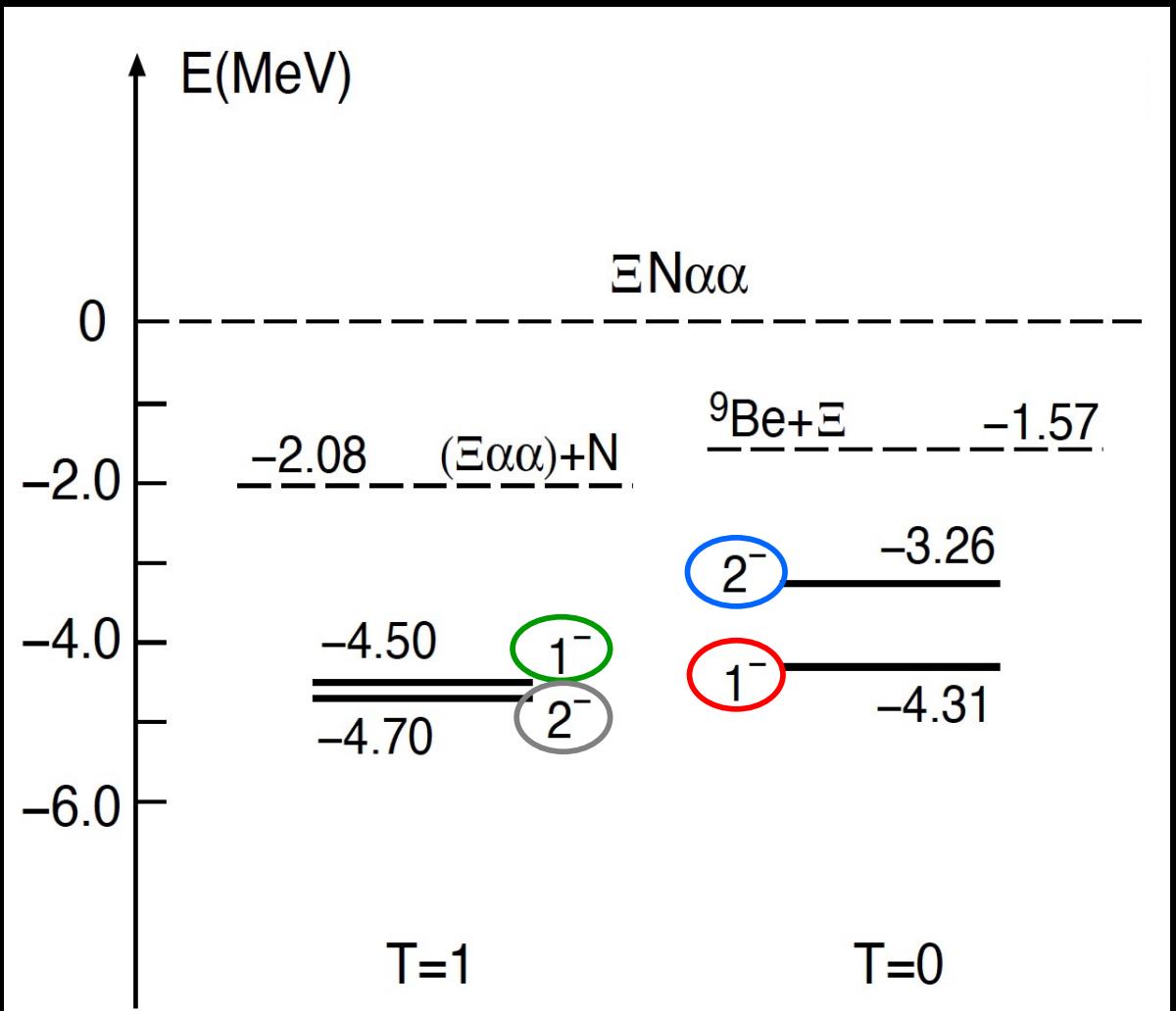
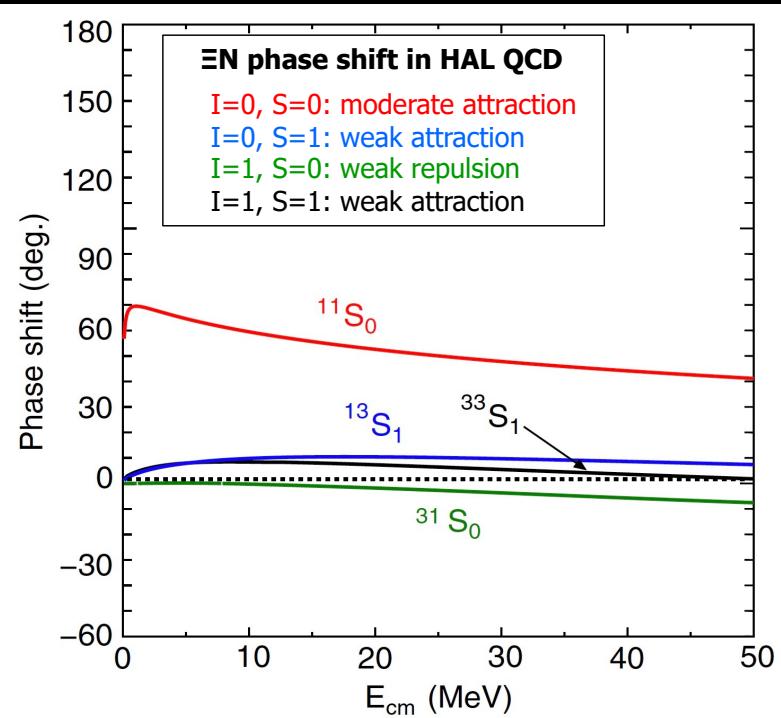
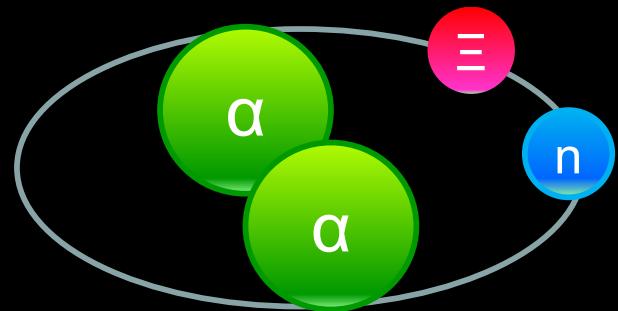


Gaussian expansion method
 (Hiyama et al., 2003)

$\Gamma \sim 60$ keV

Inversion of spin-doublets in ΞNaa system

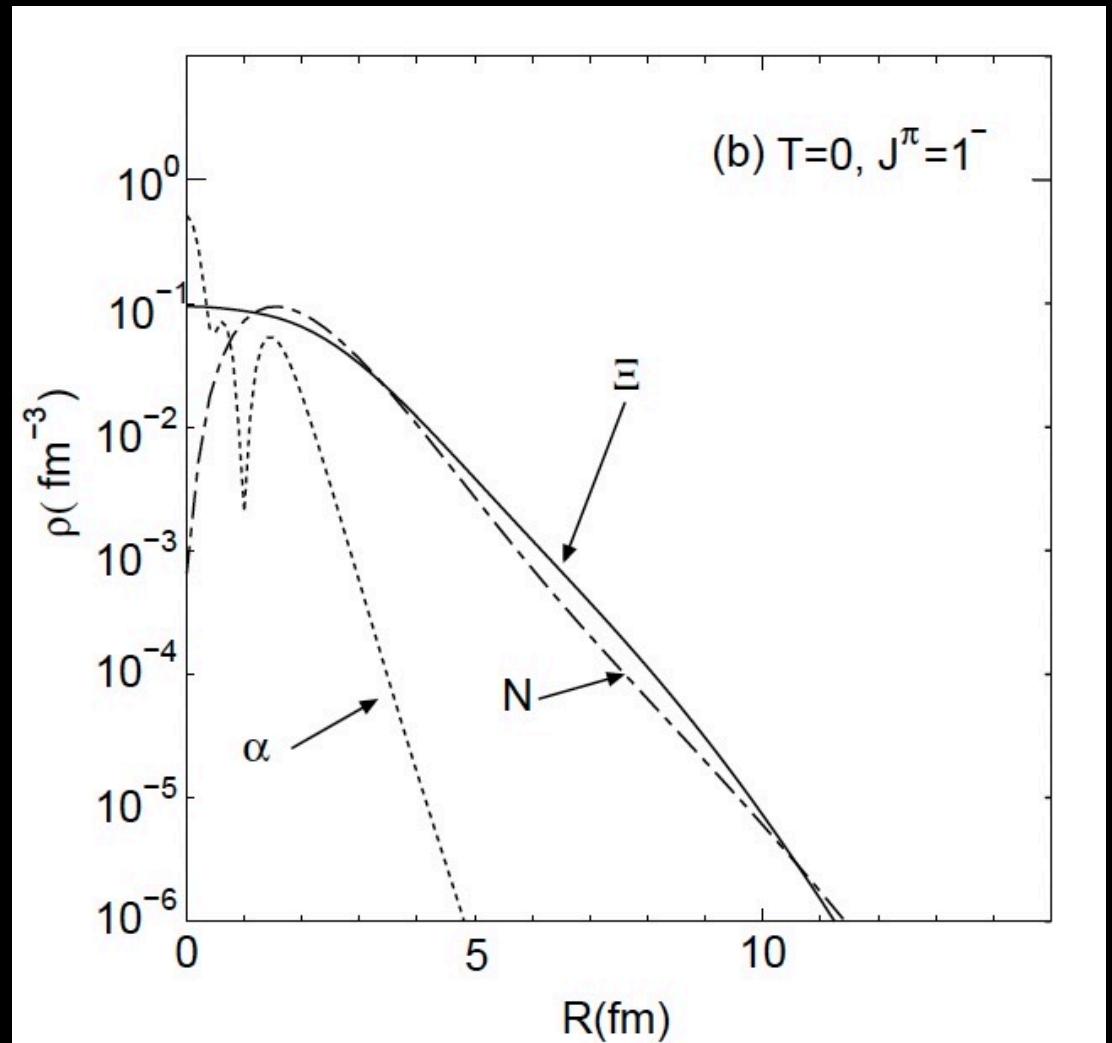
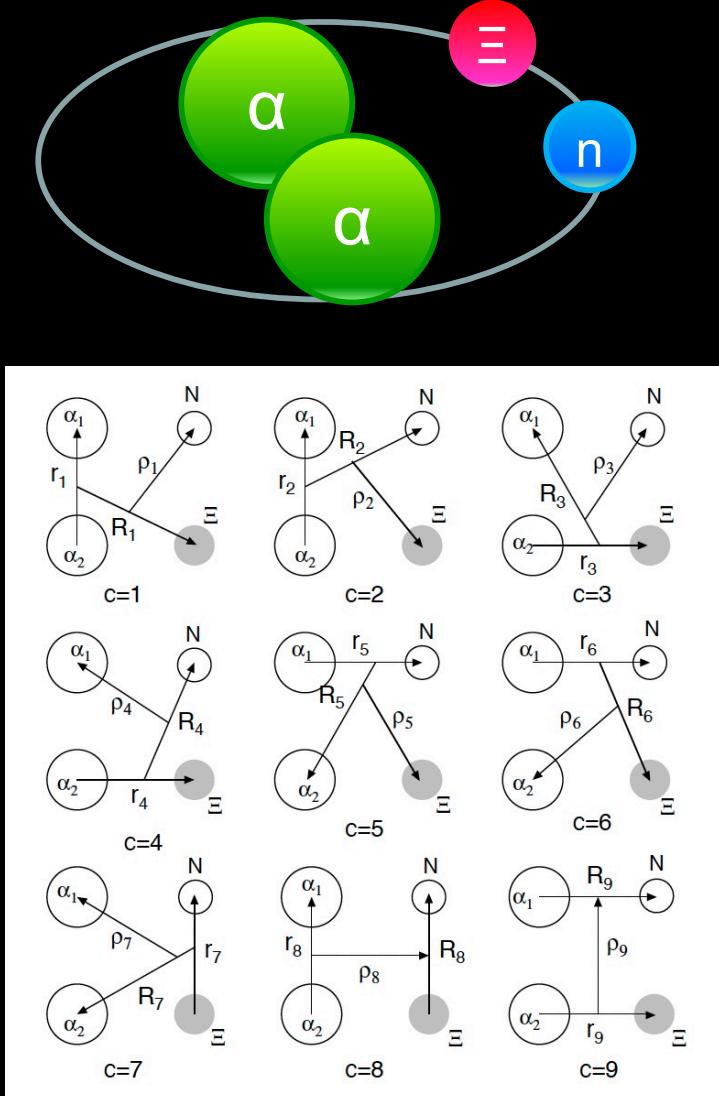
Hiyama, Isaka, Doi, Hatsuda,
arXiv:2209.06711 [nucl-th]



$\Gamma=20-40 \text{ keV}$

Q2. Can we test the spin-isospin dependence of ΞN int.?

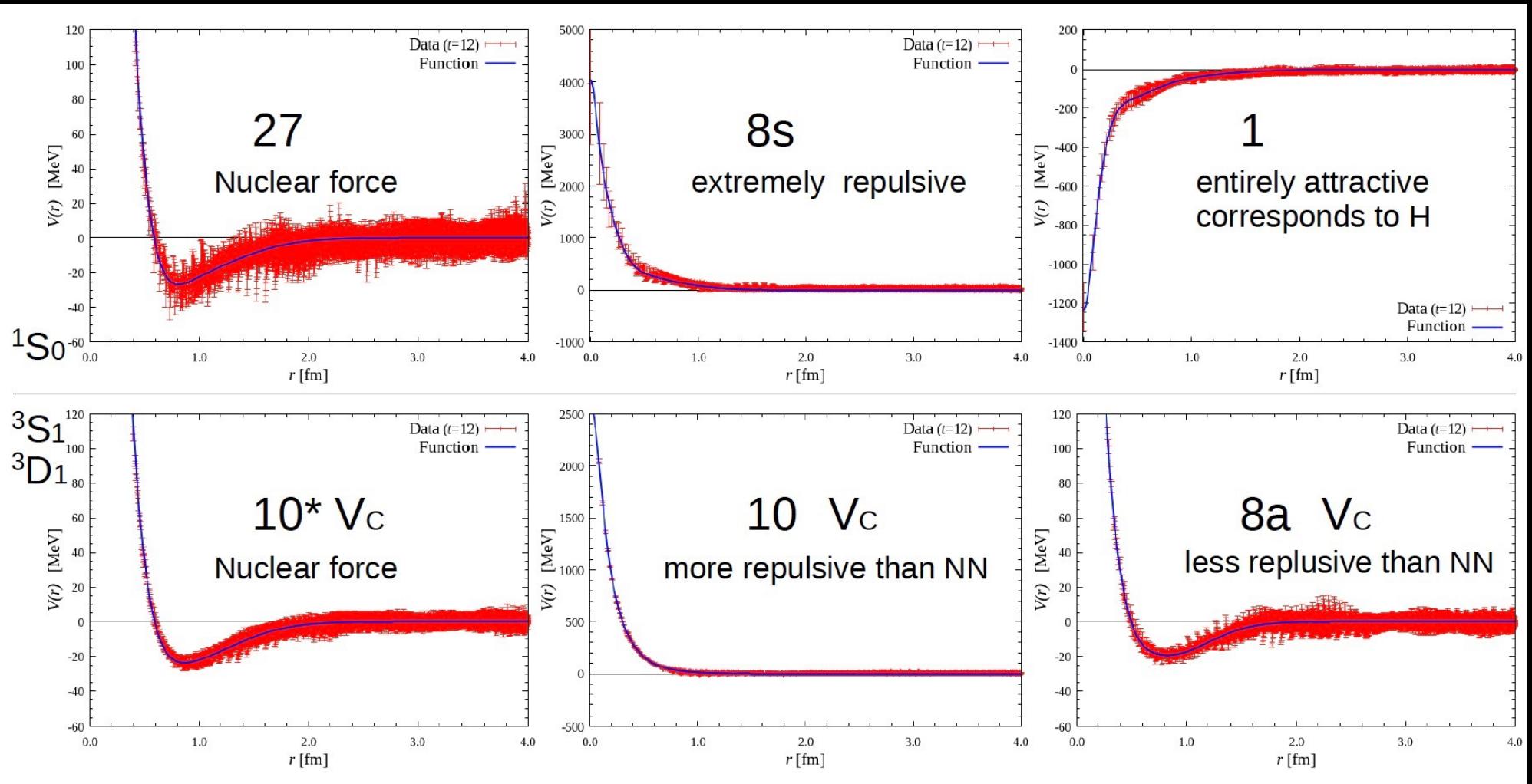
Hiyama, Isaka, Doi, Hatsuda, arXiv:2209.06711 [nucl-th]



Gaussian expansion method (Hiyama et al., 2003)

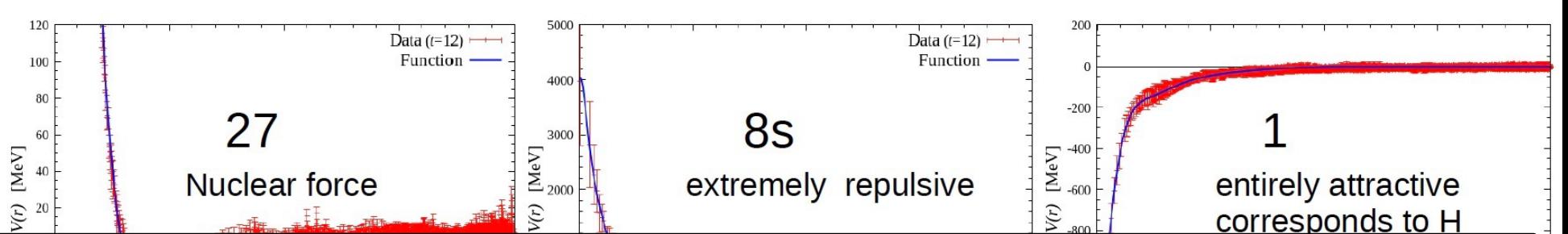
Central BB interactions in the flavor basis

$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$



Central BB interactions in the flavor basis

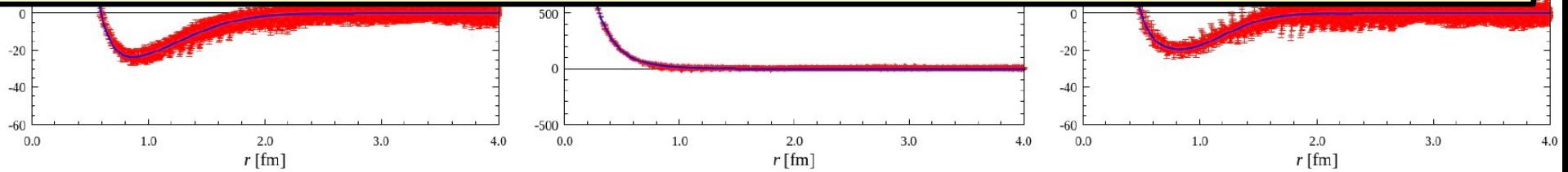
$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$



BB interactions at short distances:

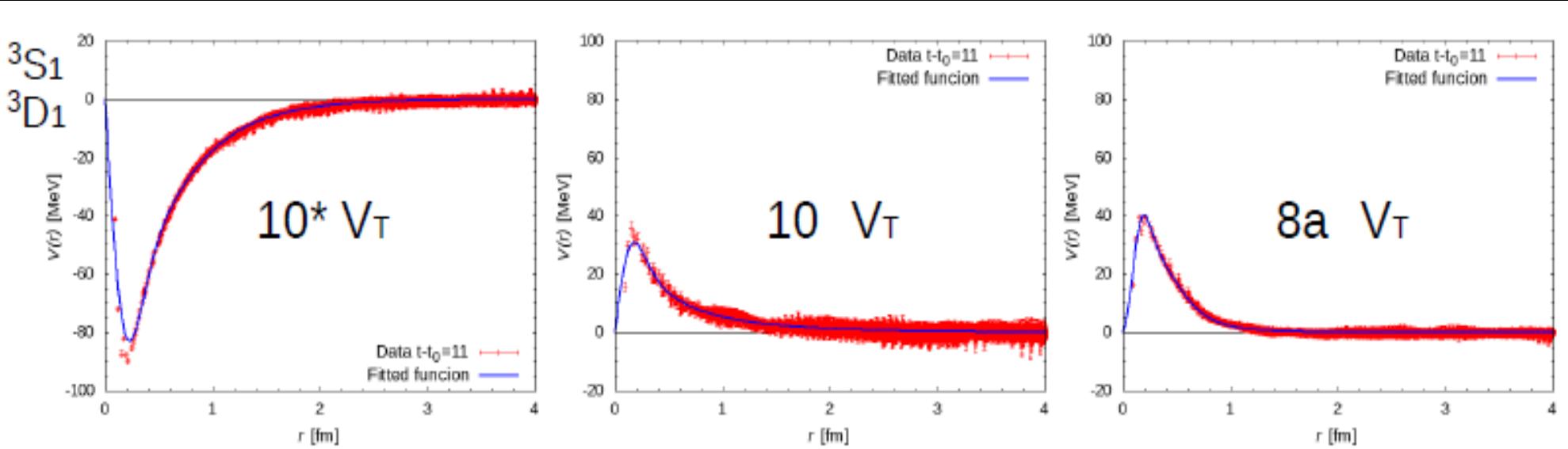
- (i) flavor dependent
- (ii) consistent with Pauli + OGE

Park, Lee, Inoue, Hatsuda, Eur. Phys. J., **A56** (2020)



Tensor BB interactions in the flavor basis

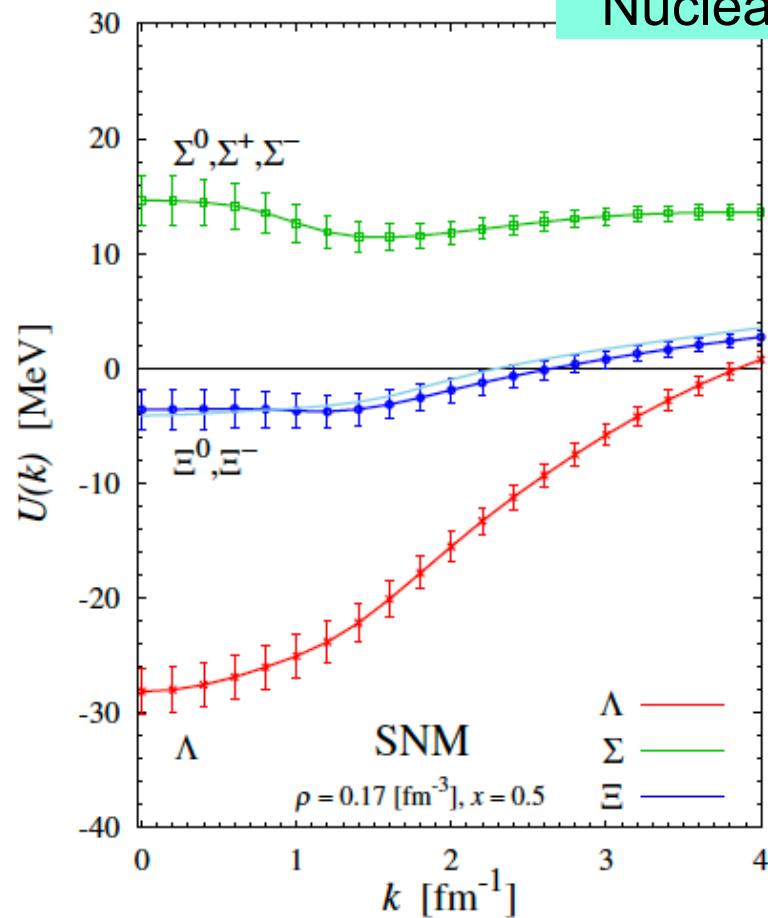
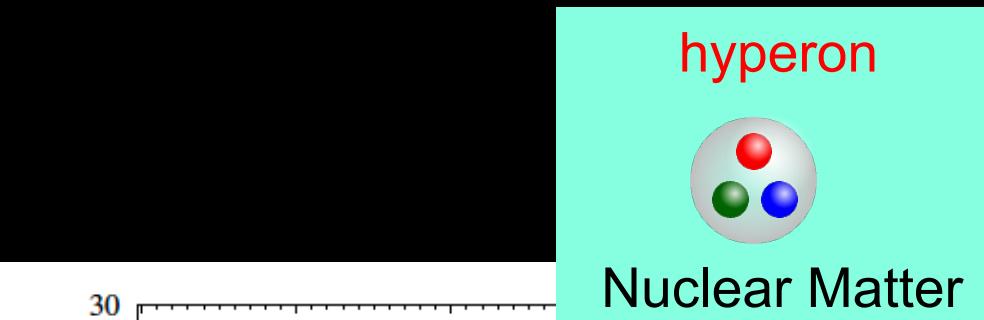
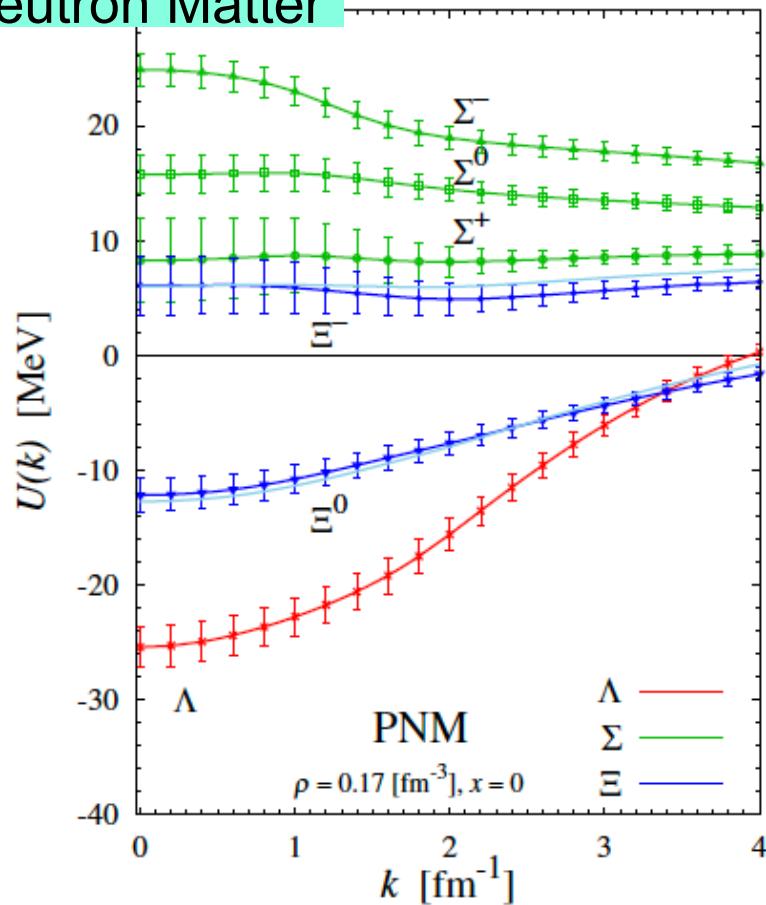
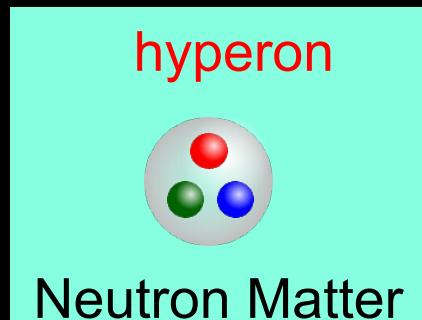
$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$



$$U(\mathbf{r}, \mathbf{r}') = V(\mathbf{r}, \mathbf{v})\delta(\mathbf{r} - \mathbf{r}'),$$

$$V(\mathbf{r}, \mathbf{v}) = \underbrace{V_C(r)}_{\text{LO}} + \underbrace{V_T(r)S_{12}}_{\text{NLO}} + \underbrace{V_{LS}(r)\mathbf{L} \cdot \mathbf{S}}_{\text{NLO}} + \underbrace{O(\mathbf{v}^2)}_{\text{N}^2\text{LO}} + \dots$$

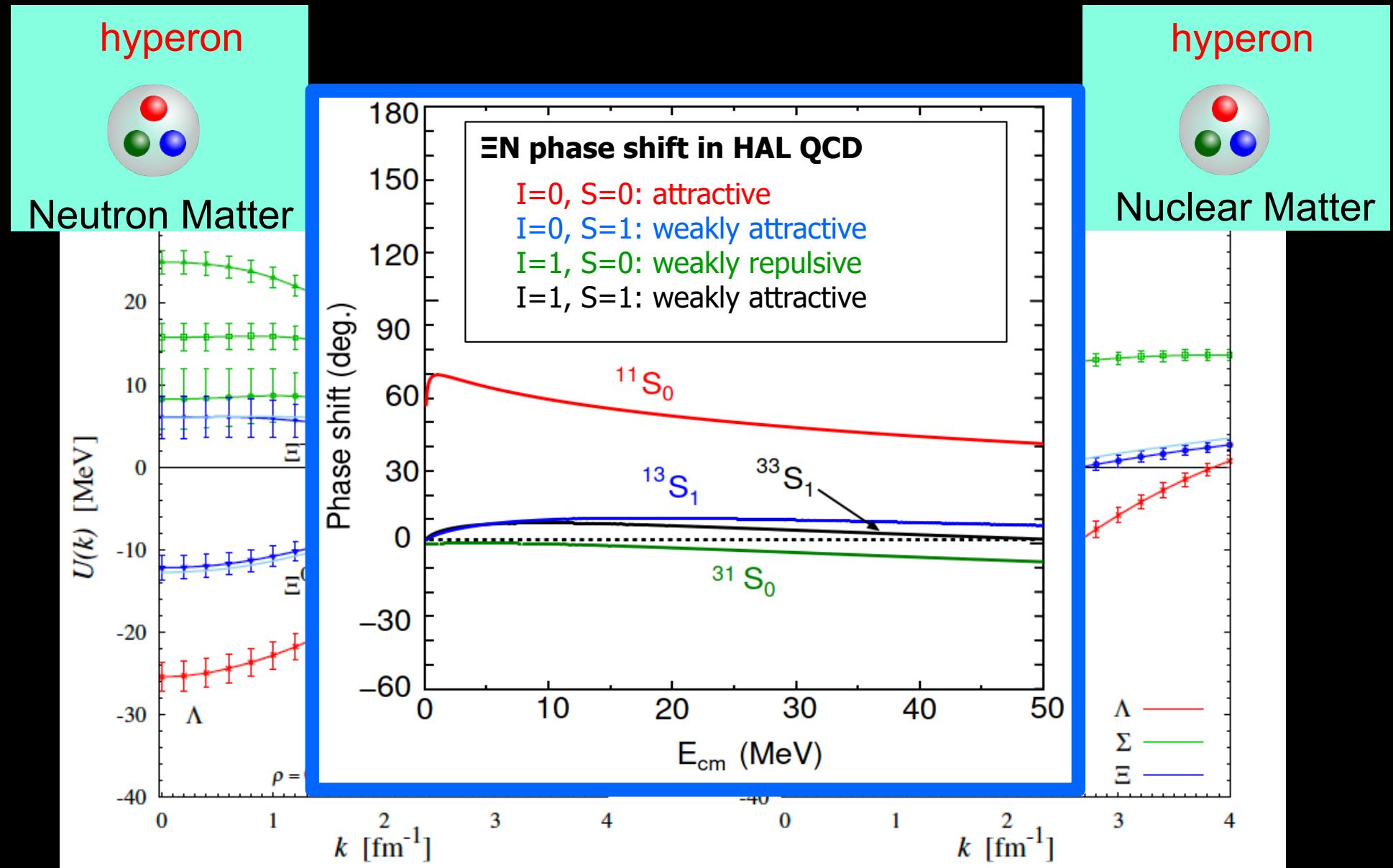
Hyperon embedded in cold nuclear matter (HAL QCD + BHF)



Inoue [HAL QCD Coll.], Few-body Syst. 62 (2021) 106

Hiyama, Sasaki, Miyamoto, Doi, Hatsuda, Yamamoto, and Rijken, PRL 124 (2020) 092501 58

Hyperon embedded in cold nuclear matter (HAL QCD + BHF)

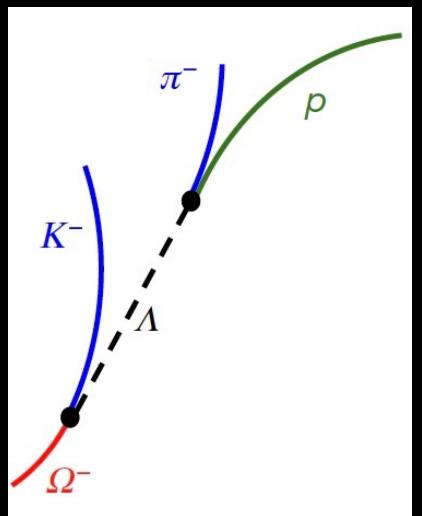
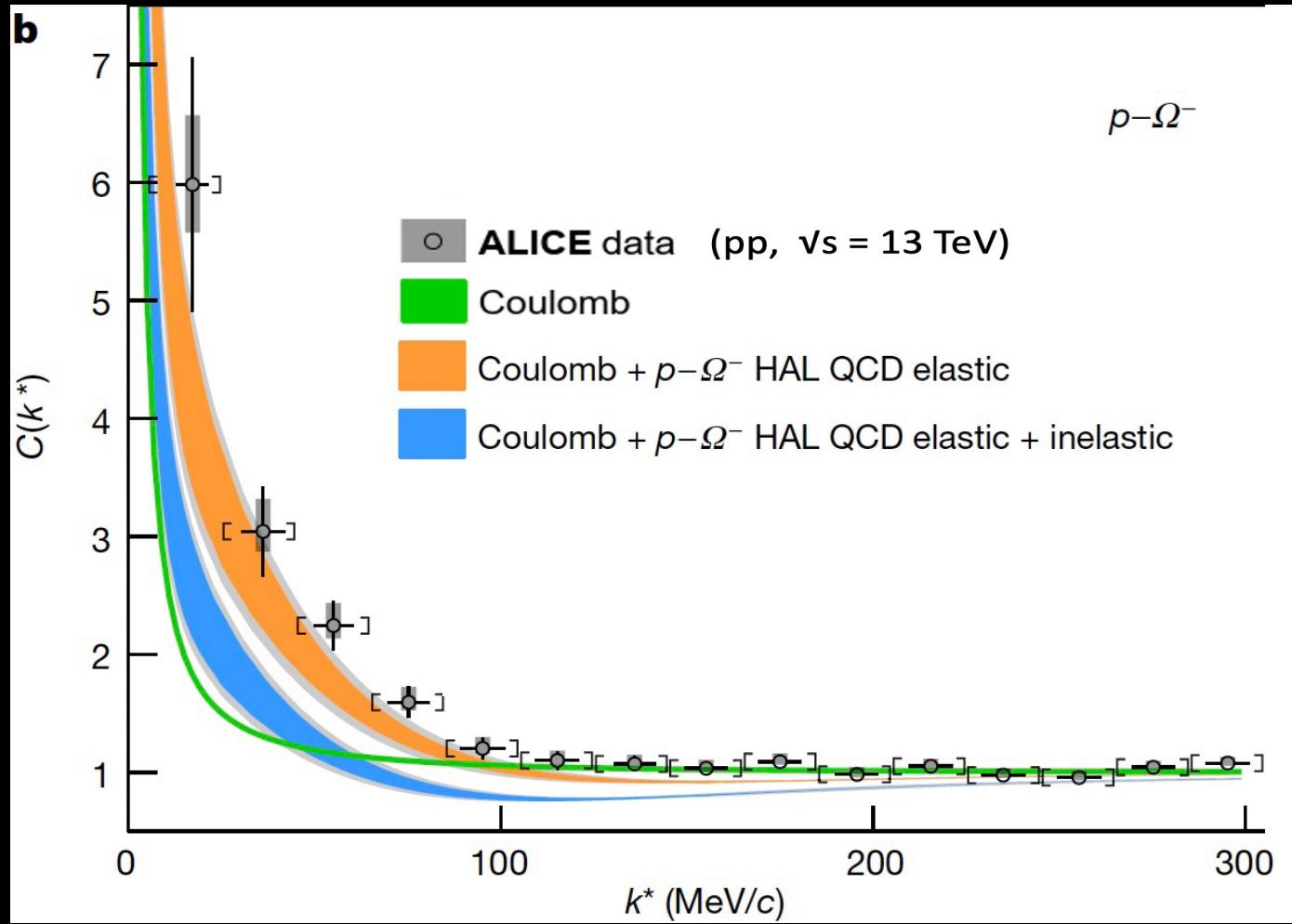
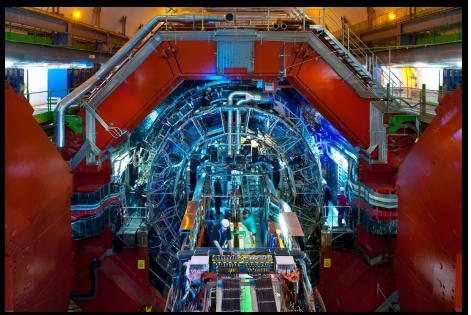


Inoue [HAL QCD Coll.], Few-body Syst. 62 (2021) 106

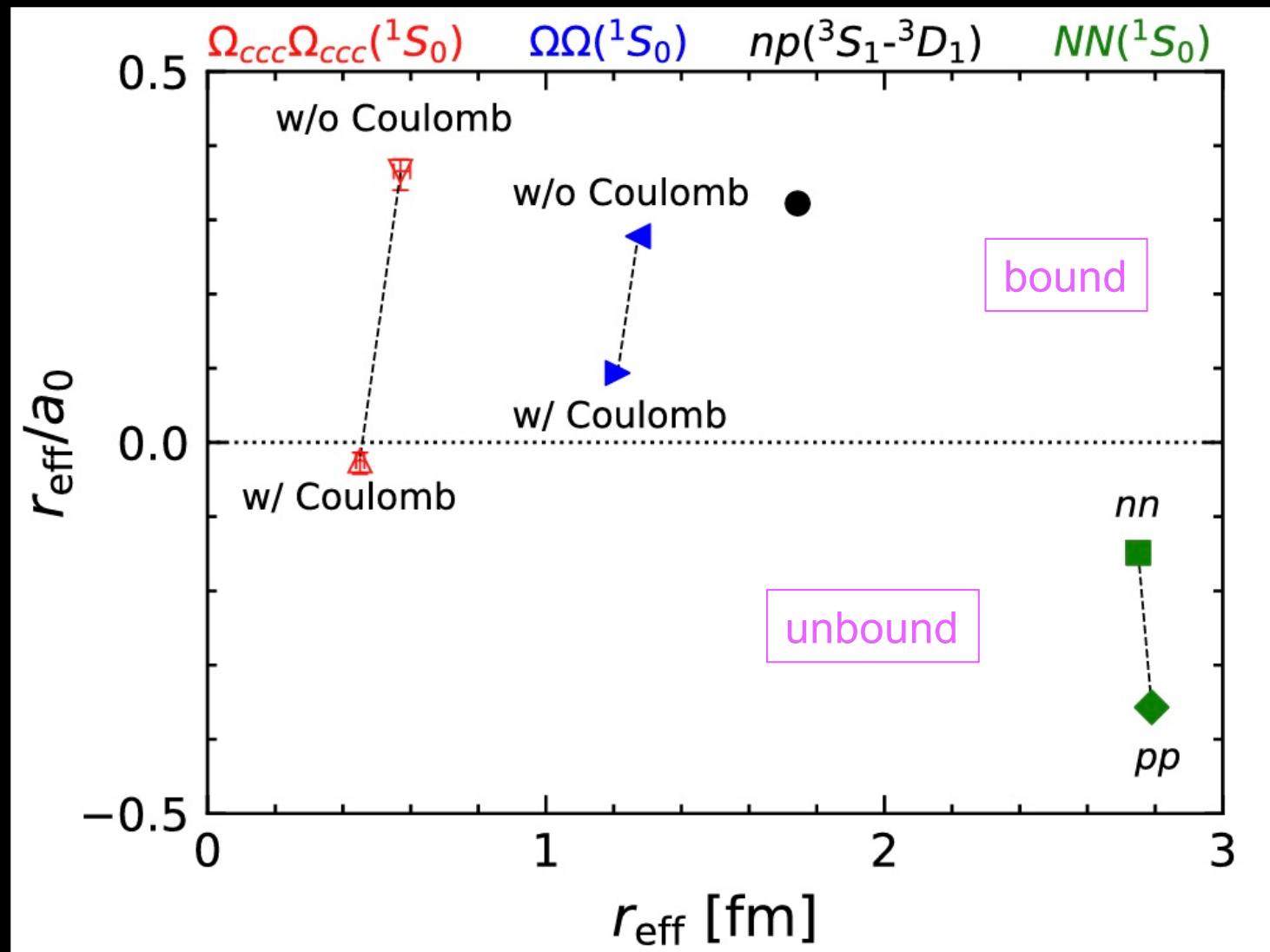
Hiyama, Sasaki, Miyamoto, Doi, Hatsuda, Yamamoto, and Rijken, PRL 124 (2020) 092501 59

$N\Omega$ correlation in pp

LHC ALICE Coll., Nature 588 (2020) 232



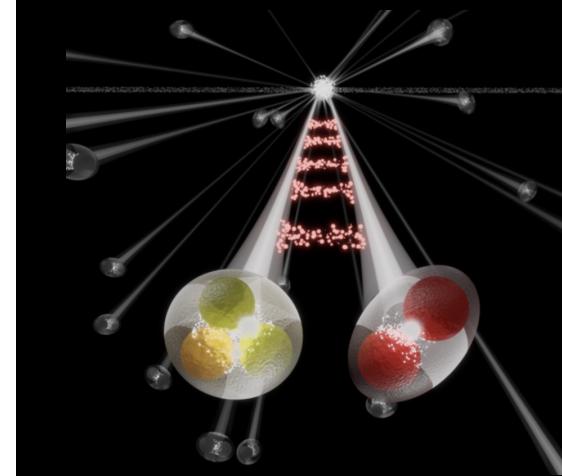
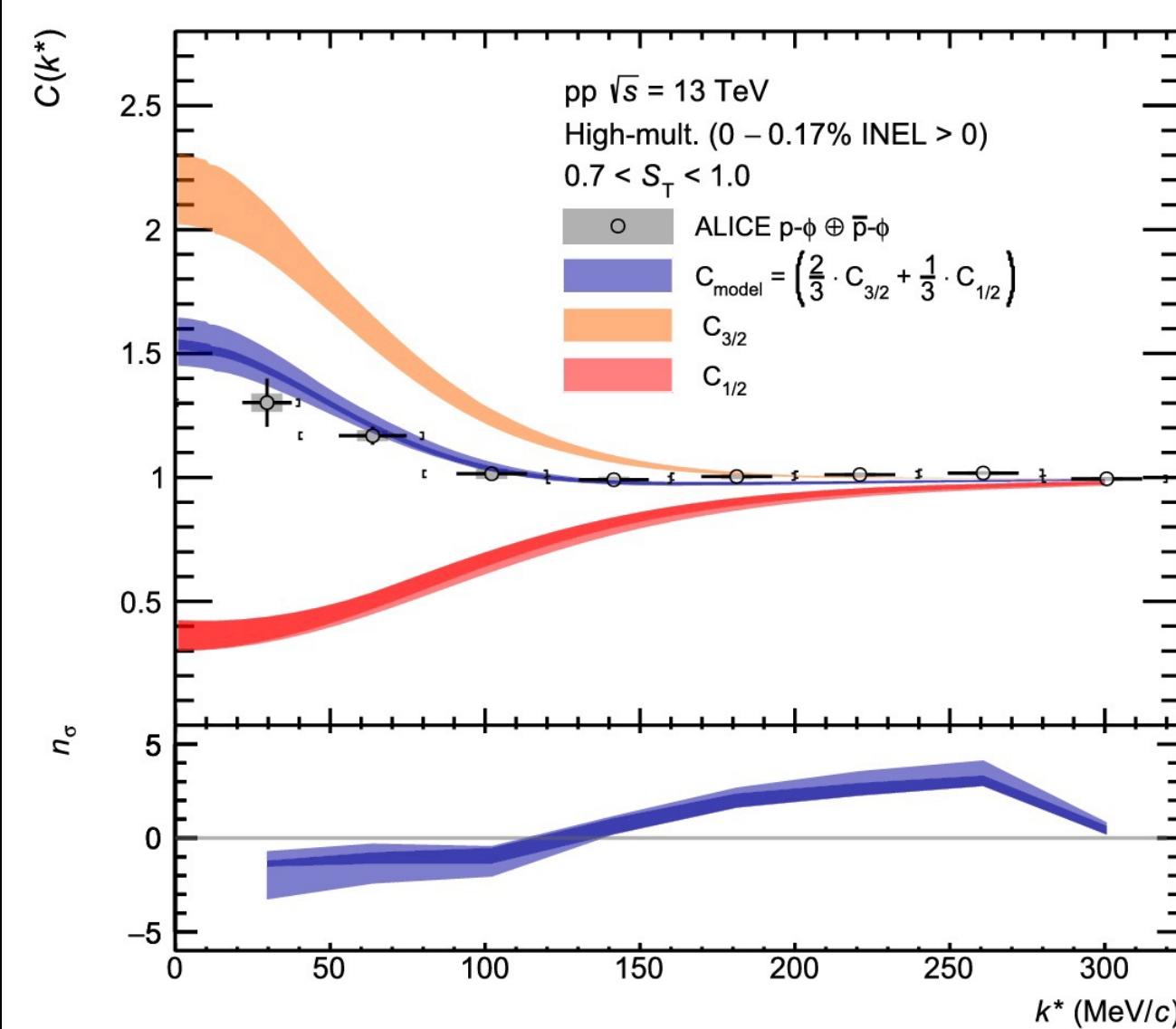
$N\Omega$ and $\Omega\Omega$ systems in LQCD



Indication of a p- ϕ bound state from a correlation function analysis

Emma Chizzali ^{a,b,^{id},*}, Yuki Kamiya ^{c,d,*}, Raffaele Del Grande ^b, Takumi Doi ^d, Laura Fabbietti ^b, Tetsuo Hatsuda ^d, Yan Lyu ^{e,d}

Phys. Lett. B 848 (2024) 138358



ϕp bound state
in spin $1/2$ channel?
(B = 12.8- 56.1 MeV)

Energy levels

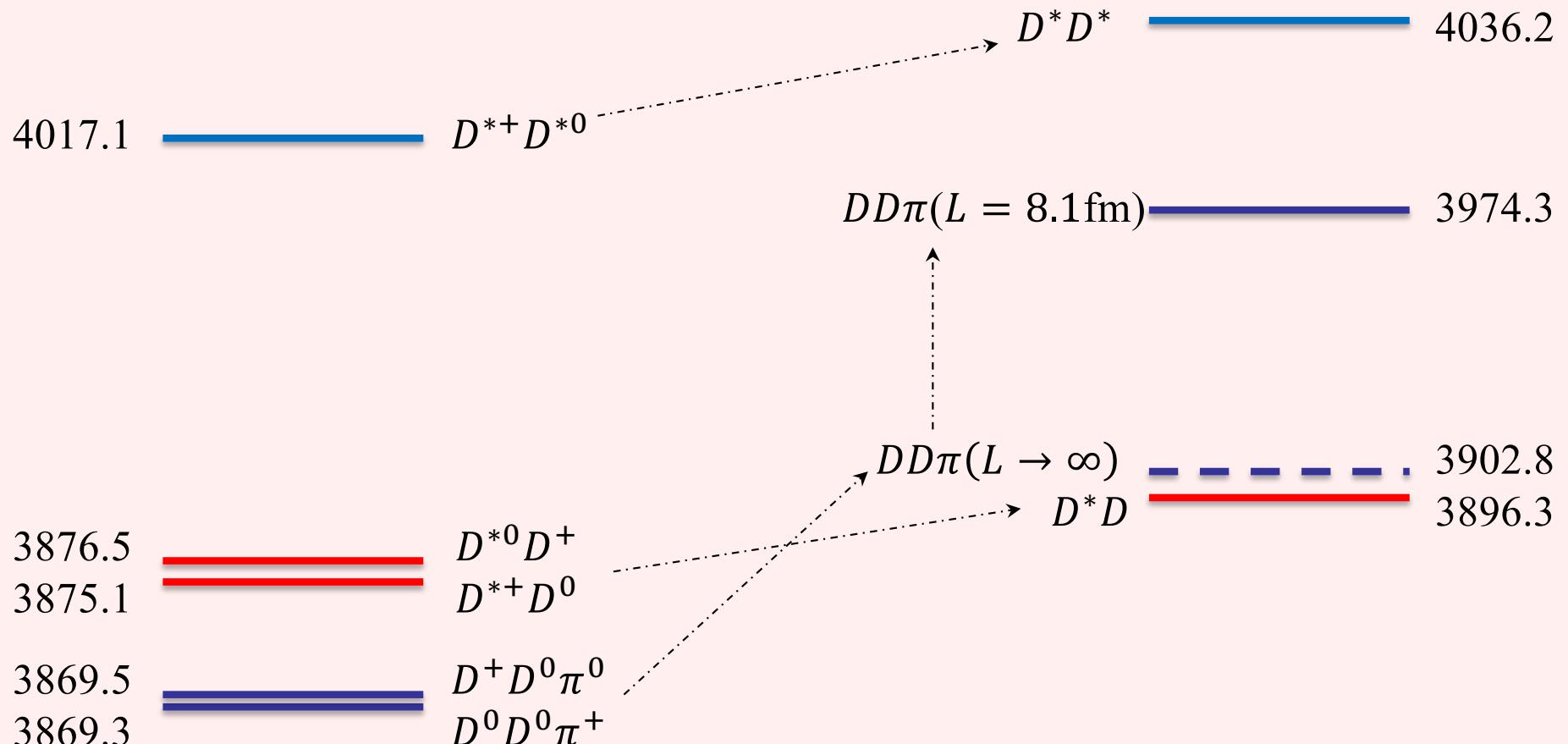
Nature

$\pi^0(134.98)$	$\pi^+(139.57)$
$D^0(1864.84)$	$D^+(1869.66)$
$D^{*0}(2006.85)$	$D^{*+}(2010.26)$

Lattice

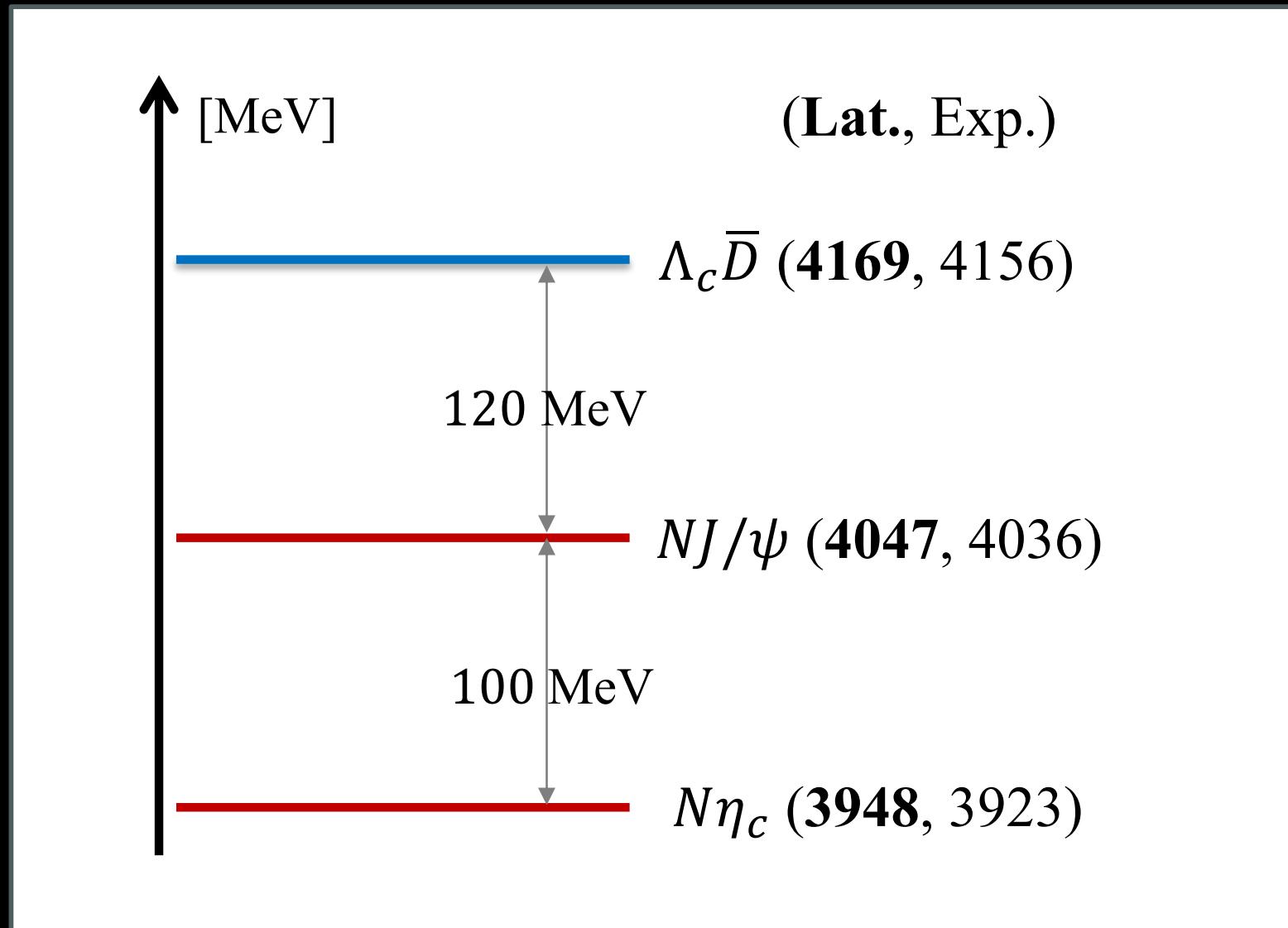
$\pi(146.4)$
$D(1878.2)$
$D^*(2018.1)$

[MeV]



- The lowest energy level of $DD\pi$ (D^*D^*) is around 78 (140) MeV above on the lattice

Thresholds



Hadron masses with “K-configuration”

$L^3 \times T$	a [fm]	La [fm]	m_π [MeV]	m_K [MeV]
$96^3 \times 96$	0.0846	8.1	146	525

