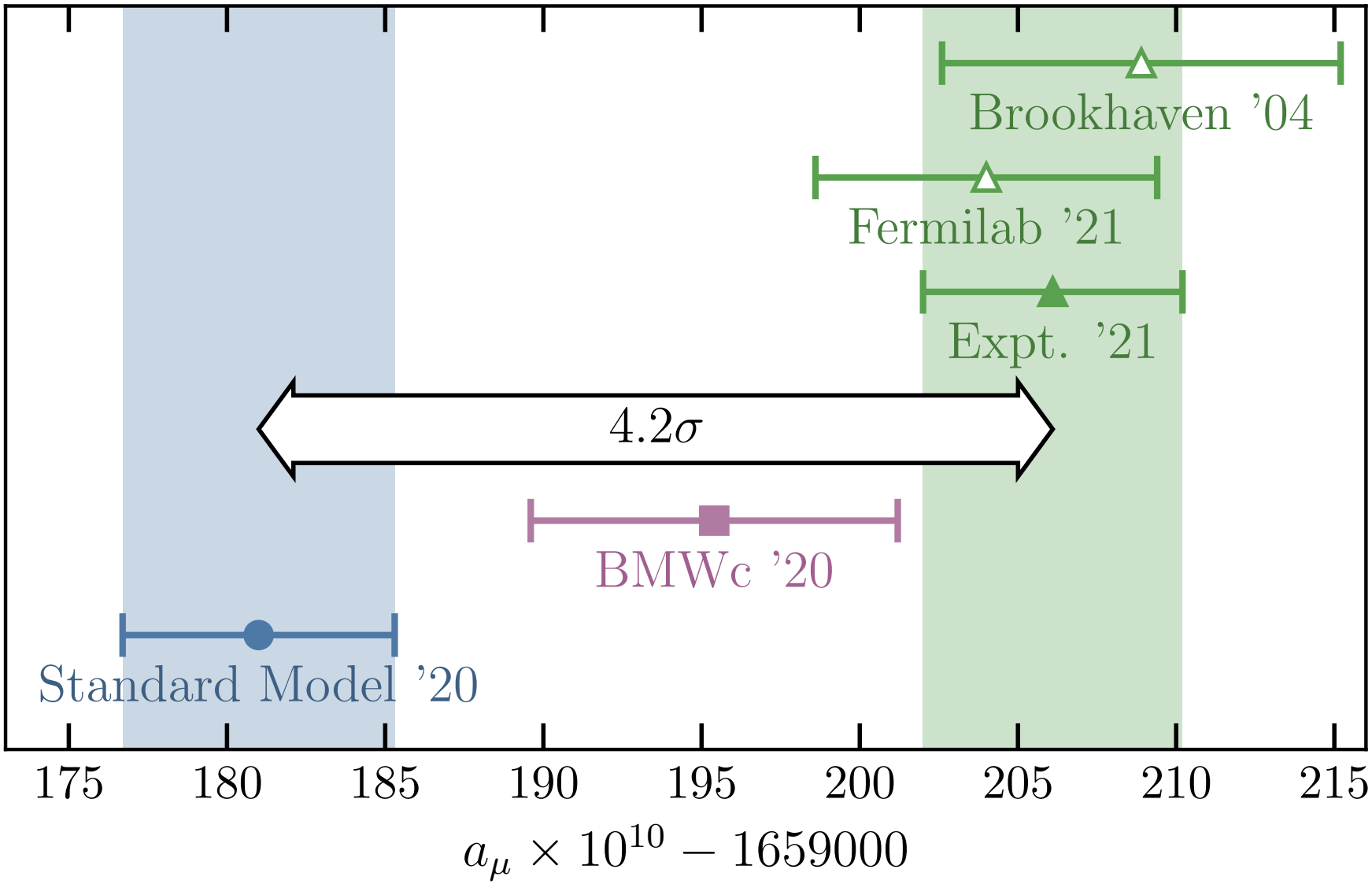
Review of muon g-2: lattice, dispersive, and data driven results

Finn M. Stokes (they/them)

Budapest-Marseille-Wuppertal Collaboration
CSSM, Adelaide, Australia

Overview


Brookhaven '04
Fermilab '21
Expt. '21
4.2σ
BMWc '20
Standard Model '20
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$a_\mu \times 10^{10} - 1659000$

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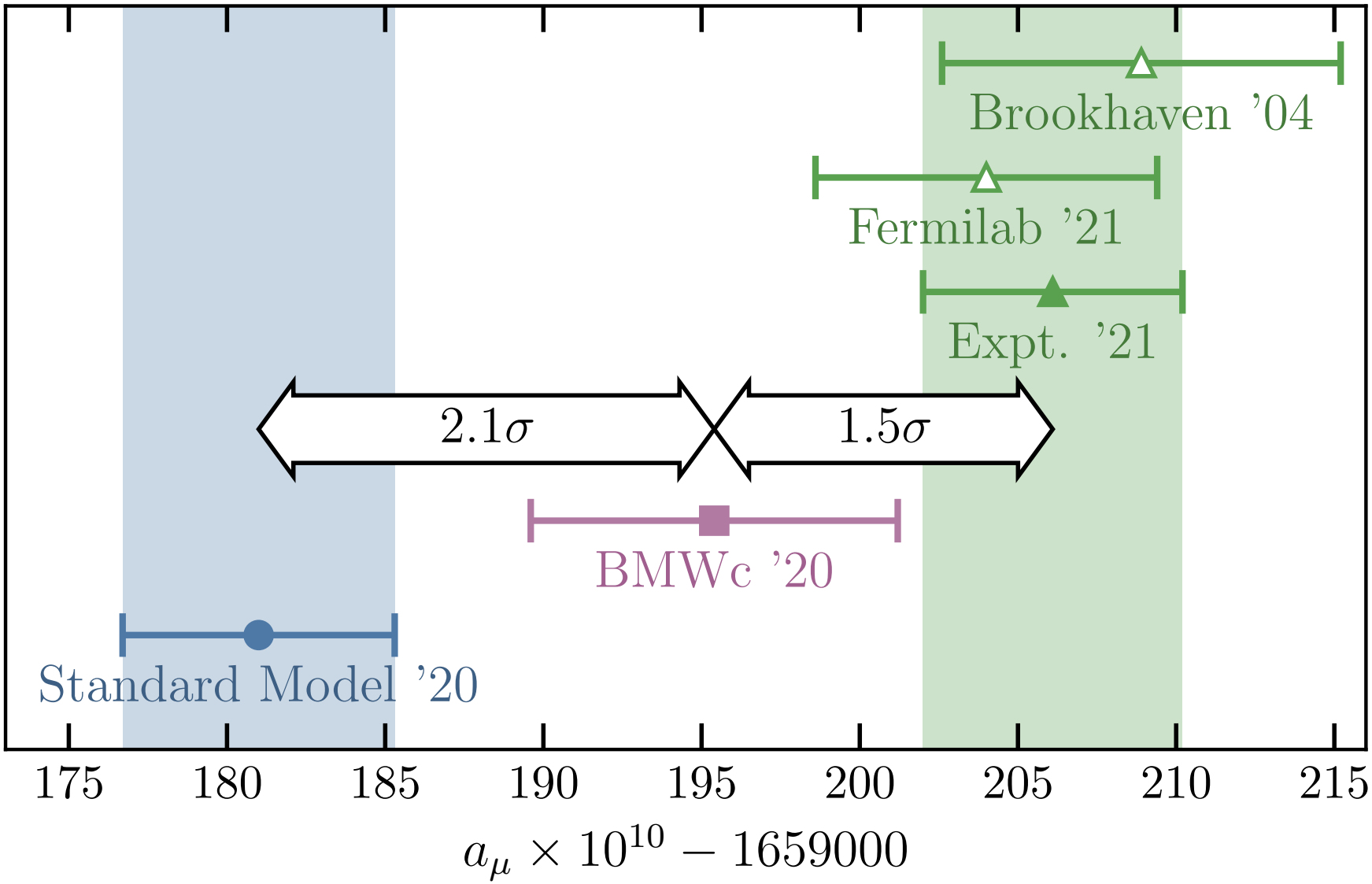
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1.5σ
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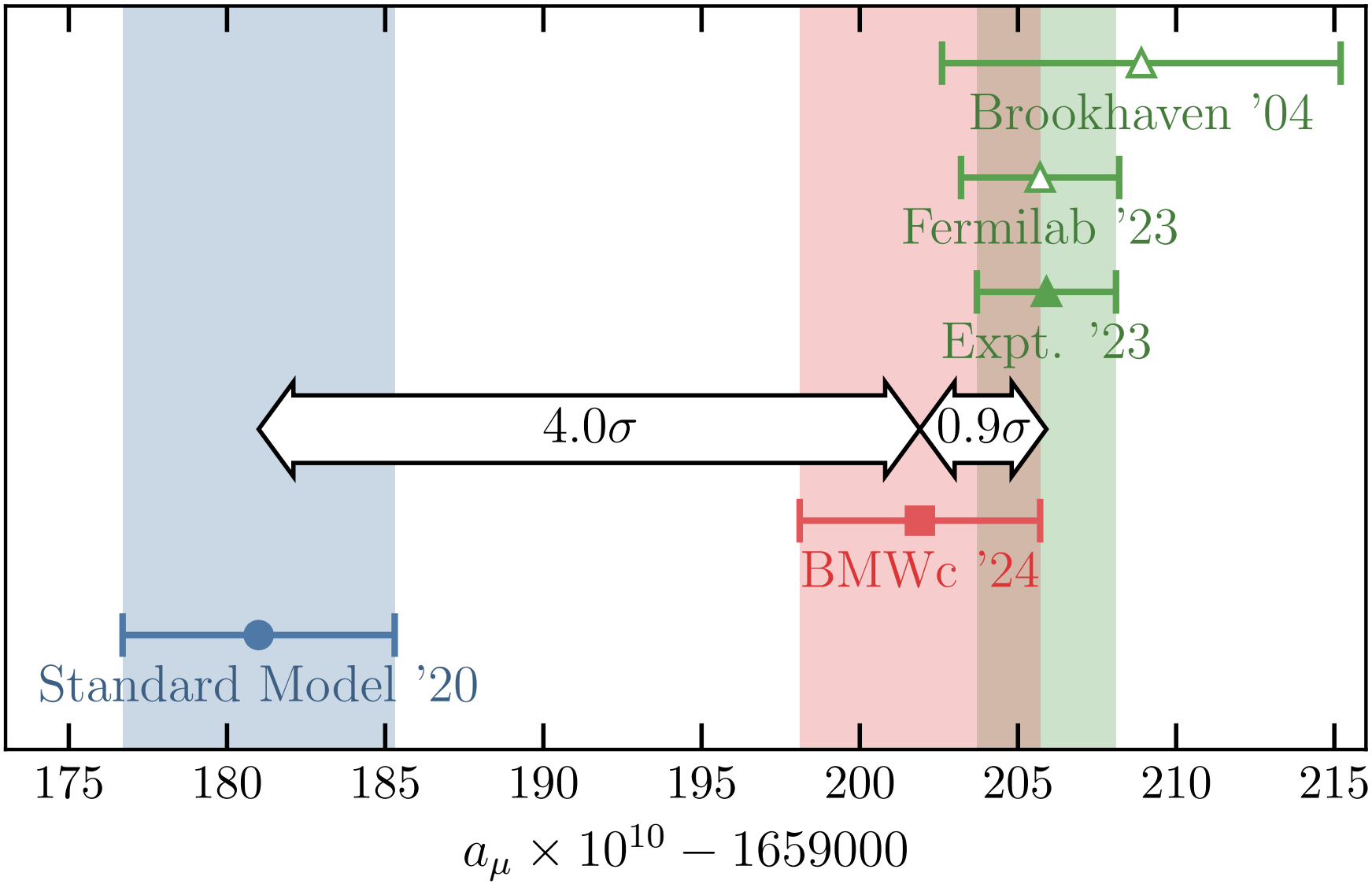
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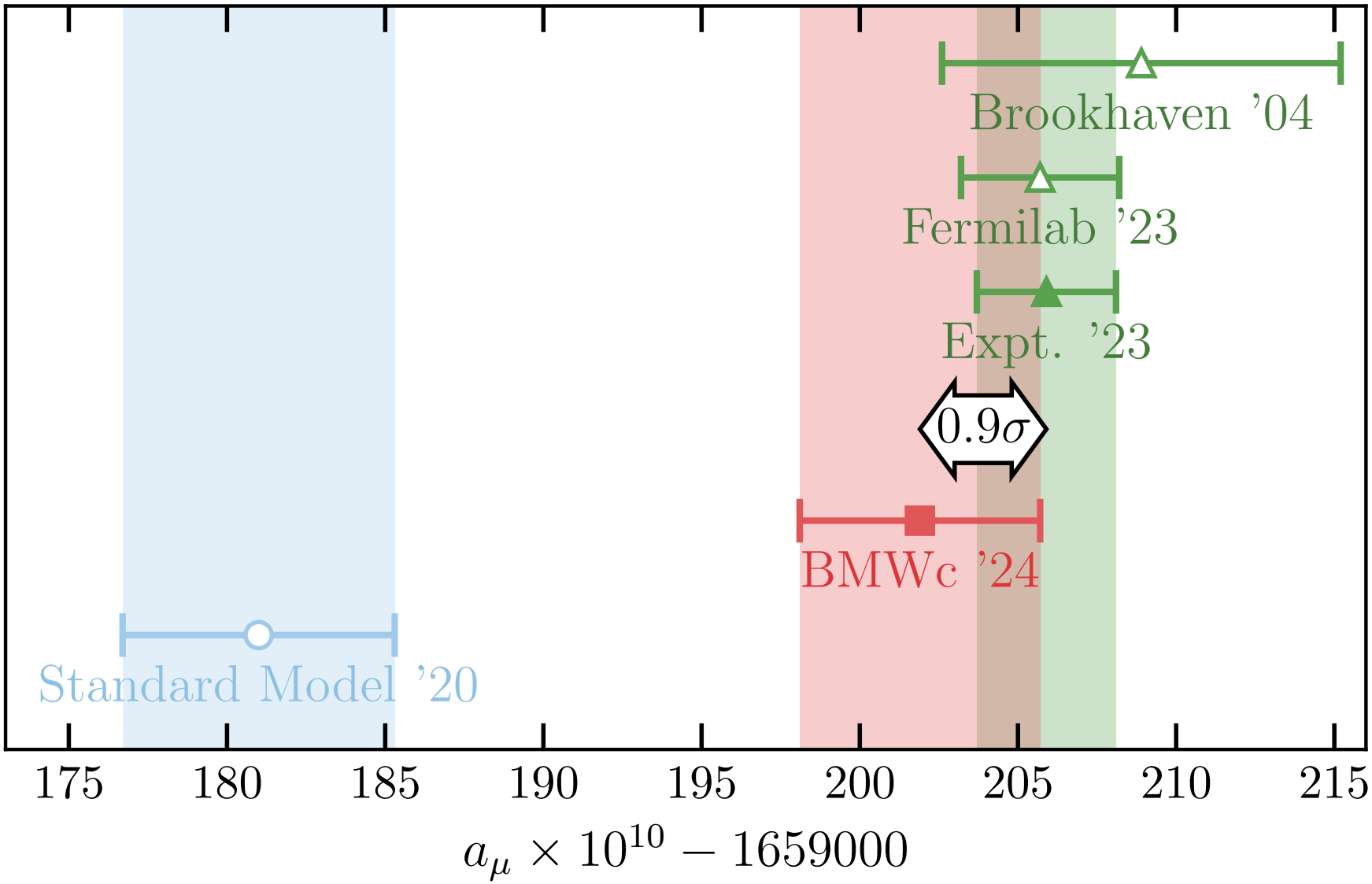
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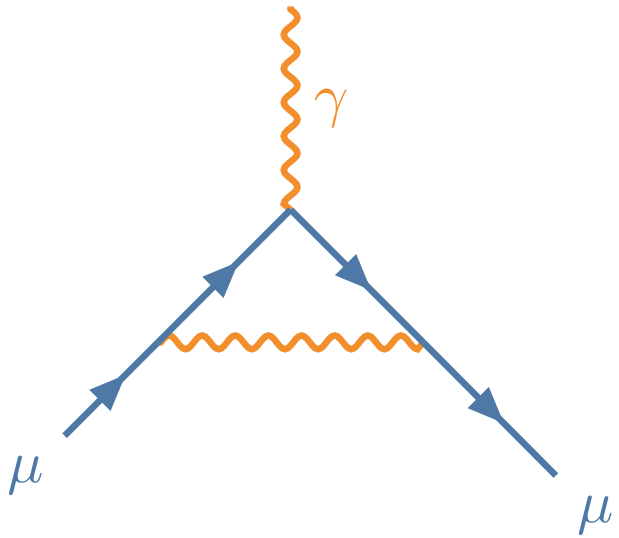
Anomalous magnetic moment

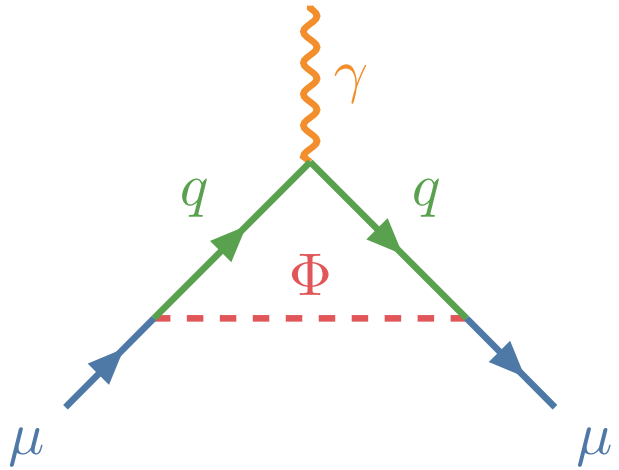
- Anomalous magnetic moment

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{S}$$

$$a = (g-2)/2$$

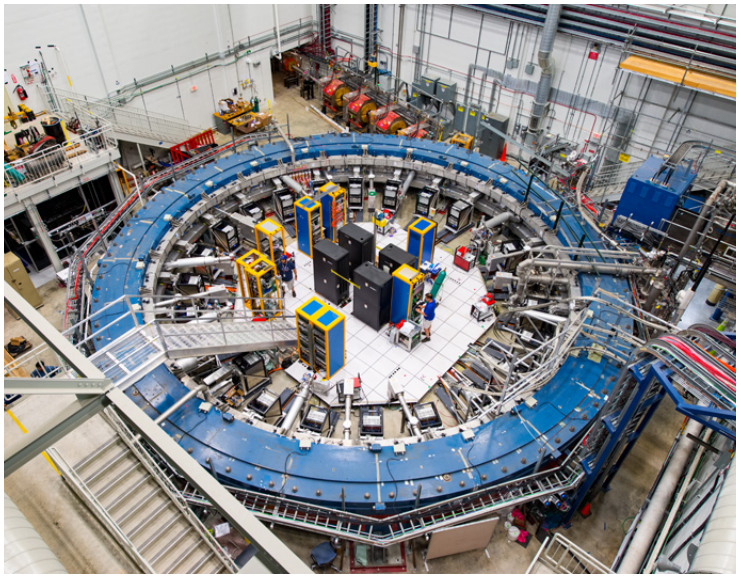
- Quantum corrections to magnetic moment
- New physics probe:
  - Loop in a is sensitive to many types of BSM physics



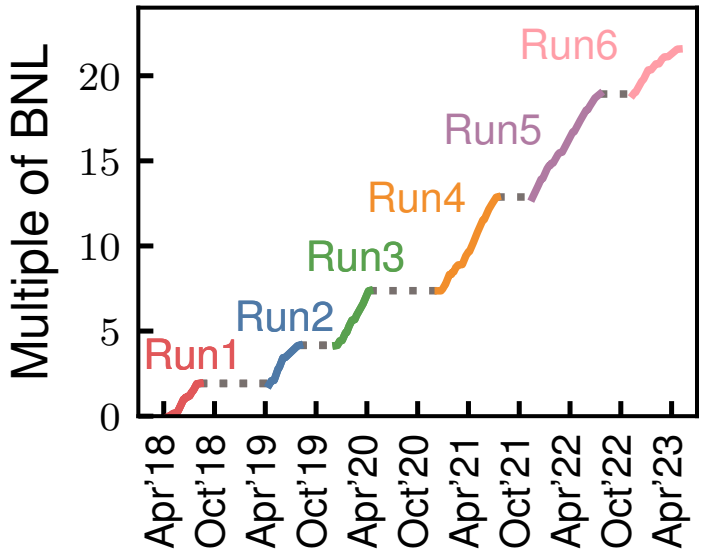


Experimental measurement

- Direct polarised beam of muons into large storage ring
- Precisely measure rate of precession of muon spins
- Latest update from Fermilab in August 2023: 0.20 ppm
- $4\times$ statistics on the way



[Fermi National Accelerator Laboratory 2017]



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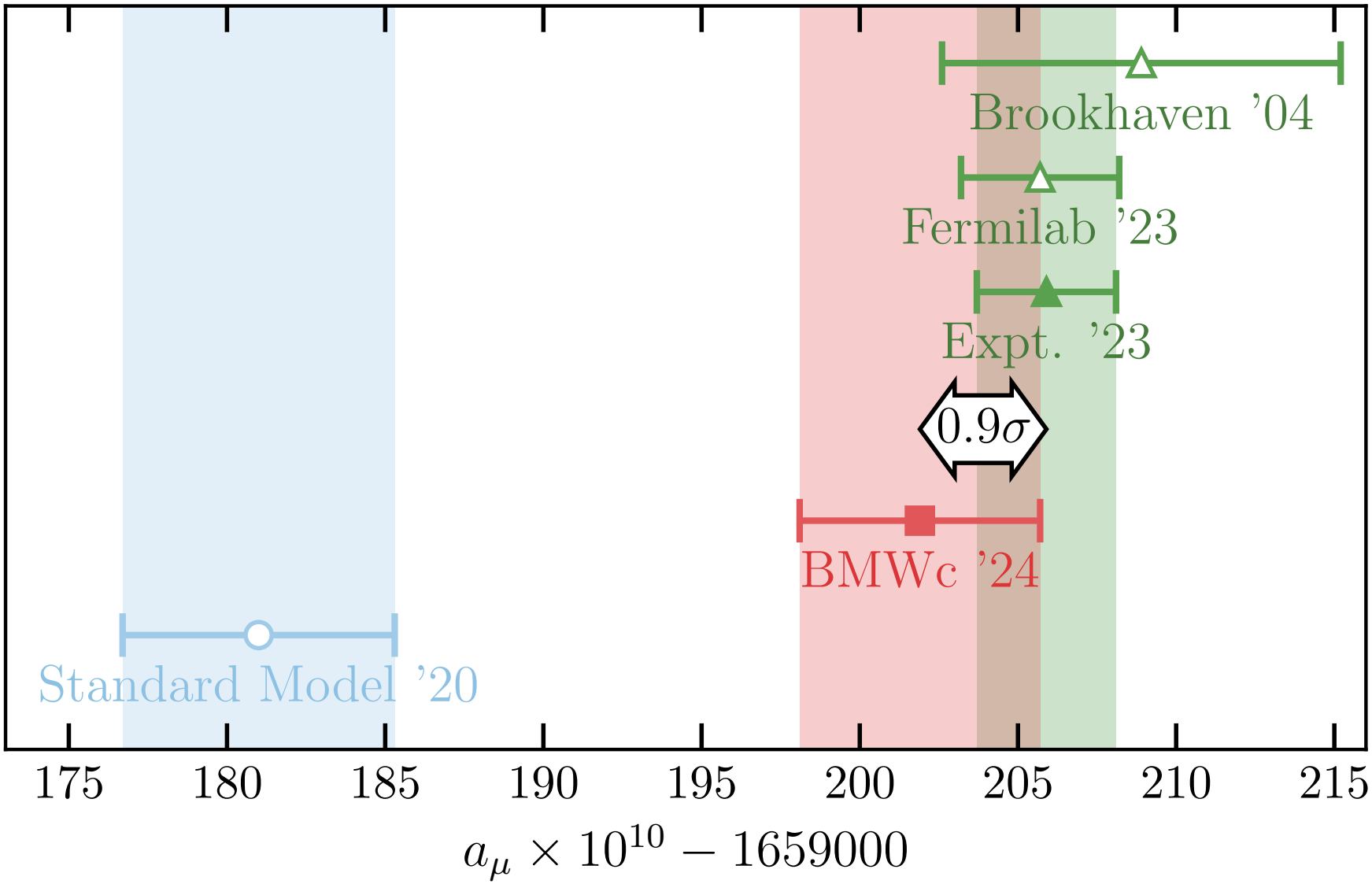
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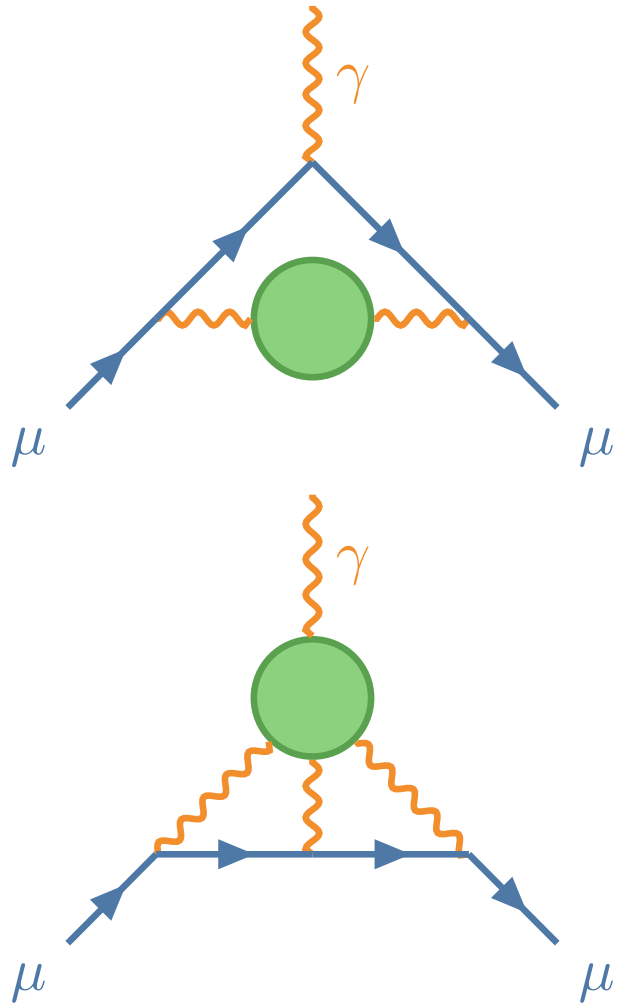
Standard Model contributions

$a_\mu^{QED} \times 10^{10}$	$11\,658\,471.8931$	$\pm$	0.0104
$a_\mu^{EW} \times 10^{10}$	15.36	$\pm$	0.10
$a_\mu^{HVP} \times 10^{10}$	684.5	$\pm$	4.0
$a_\mu^{HLbL} \times 10^{10}$	9.2	$\pm$	1.8
$a_\mu \times 10^{10}$	$11\,659\,208.9$	$\pm$	6.3

[White Paper 2006.04822]

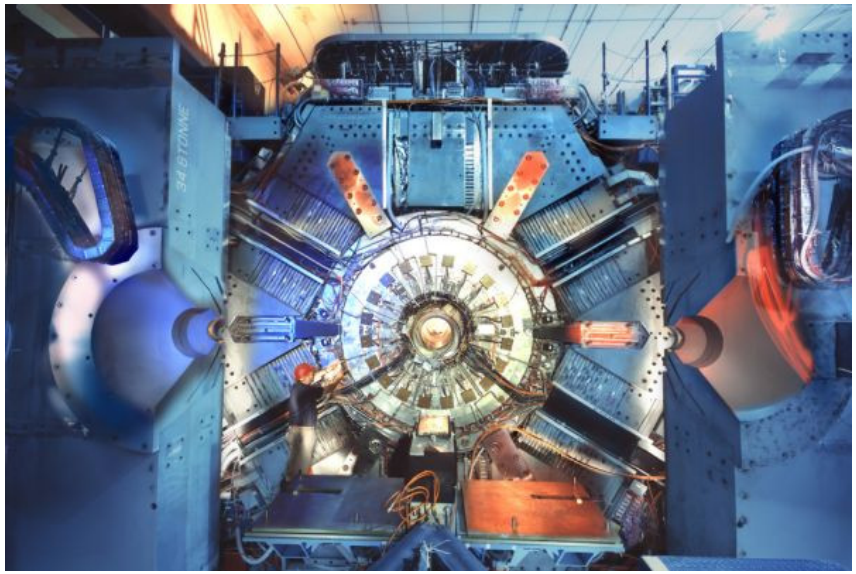


- Value is dominated by electromagnetic part (QED)
- Errors dominated by strong part (QCD)
  - Hadronic vacuum polarisation
  - Hadronic light-by-light scattering

QCD contributions

Data driven

- Input: Experimental cross-sections, form factors
- Optical theorem and dispersion relations



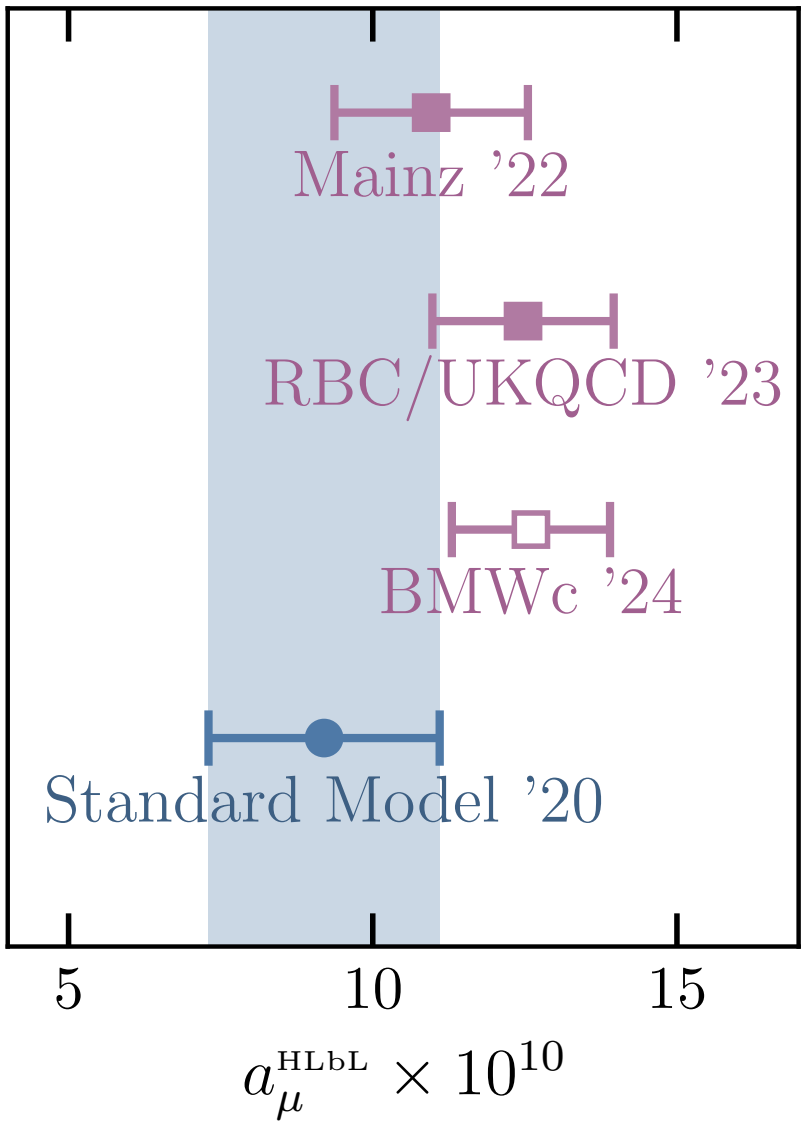
[BaBar at SLAC National Accelerator Laboratory]

Lattice QCD

- Input: QCD+QED Lagrangian with no additional parameters
- Non-perturbative numerical simulation

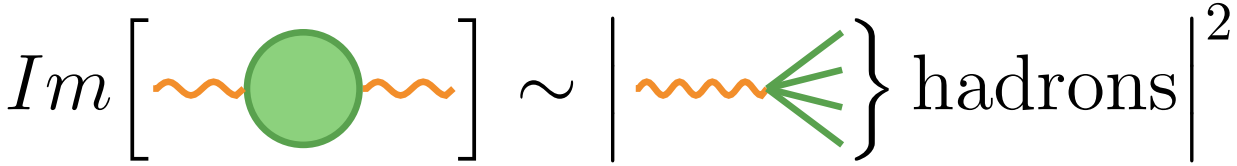
[Forschungszentrum Jülich]

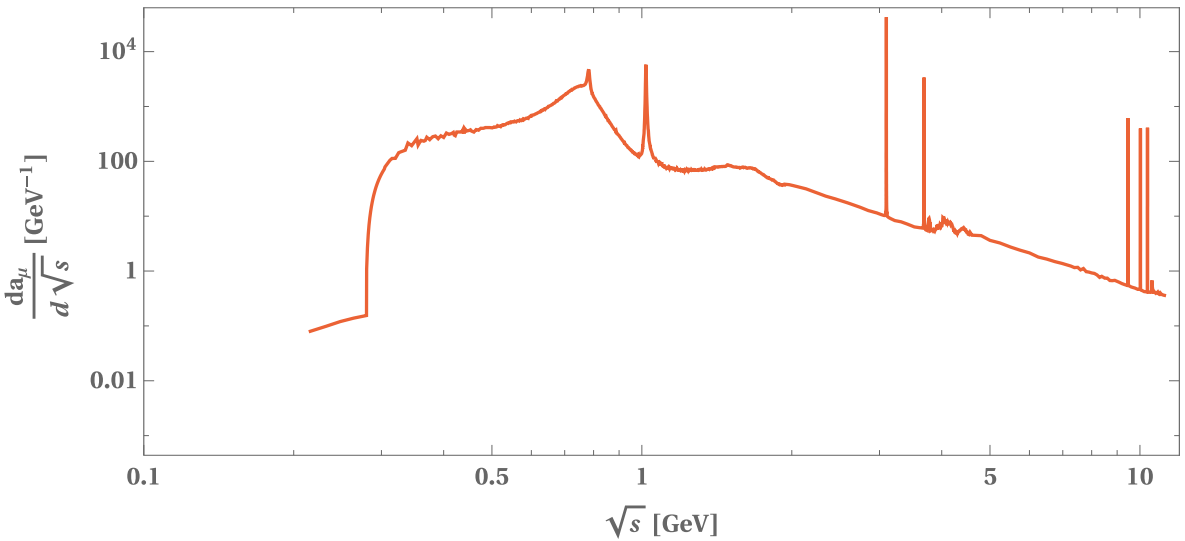
Hadronic Light-by-Light



- Good agreement between data-driven [WP 2006.04822] and lattice [2204.08844, 2304.04423]
- Ongoing efforts to improve precision
- Takeaway: HLbL is in good shape

LO-HVP: data-driven





- $e^+e^- \to$ hadrons integrated with appropriate kernel
- Major contributions from BaBar and KLOE experiments
- Benchmark SM result from [WP 2006.04822]: KNT [1802.02995], DHMZ [1706.09436] & CHHKS [1810.00007, 1907.01556]
- 5σ tension with latest experimental avg.

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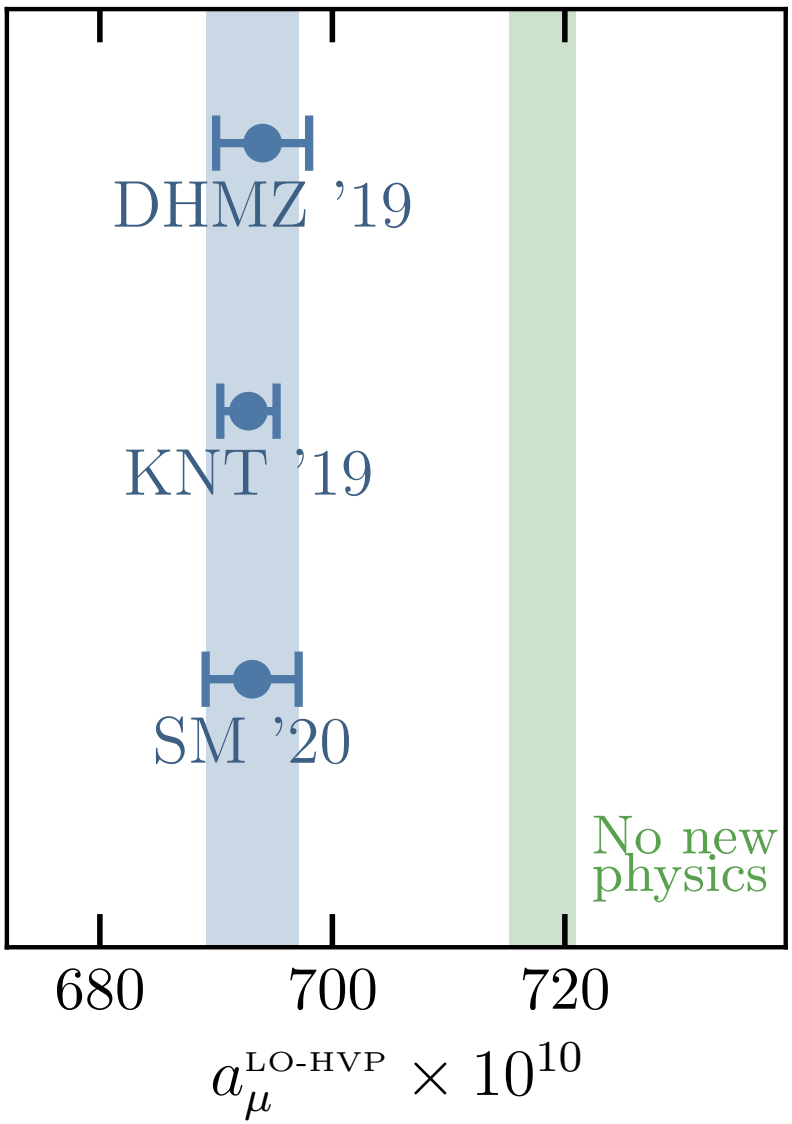

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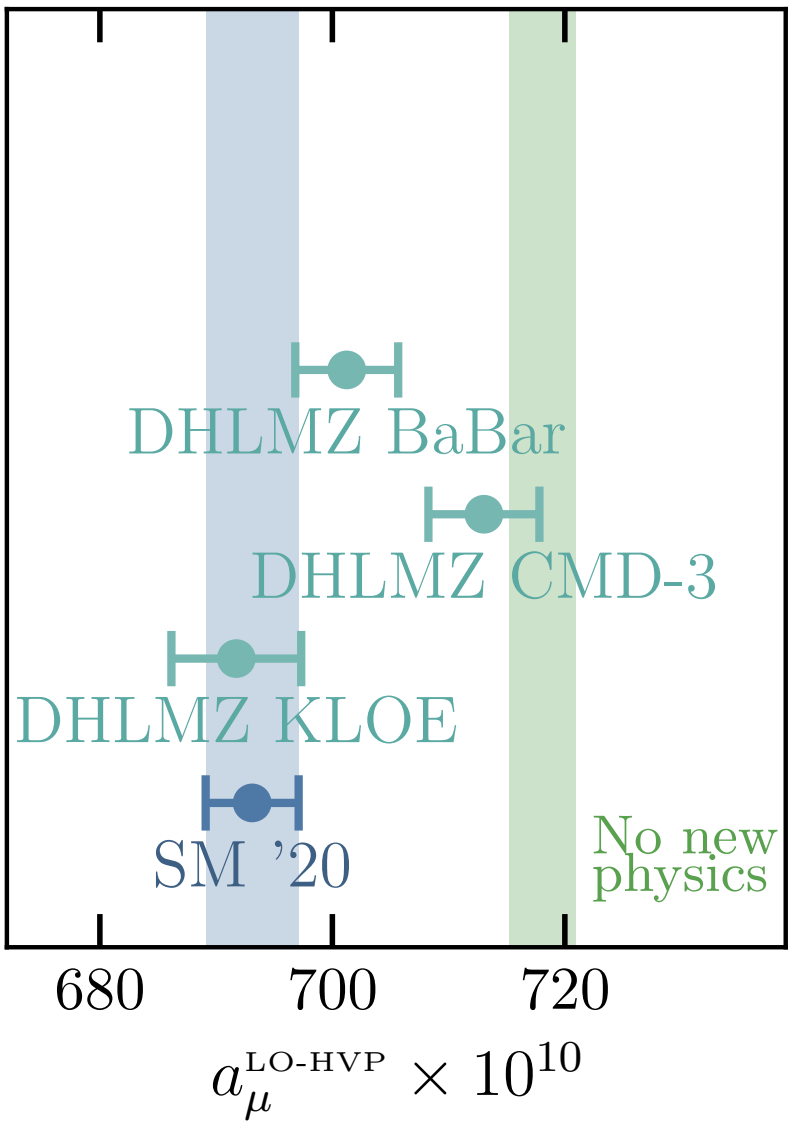
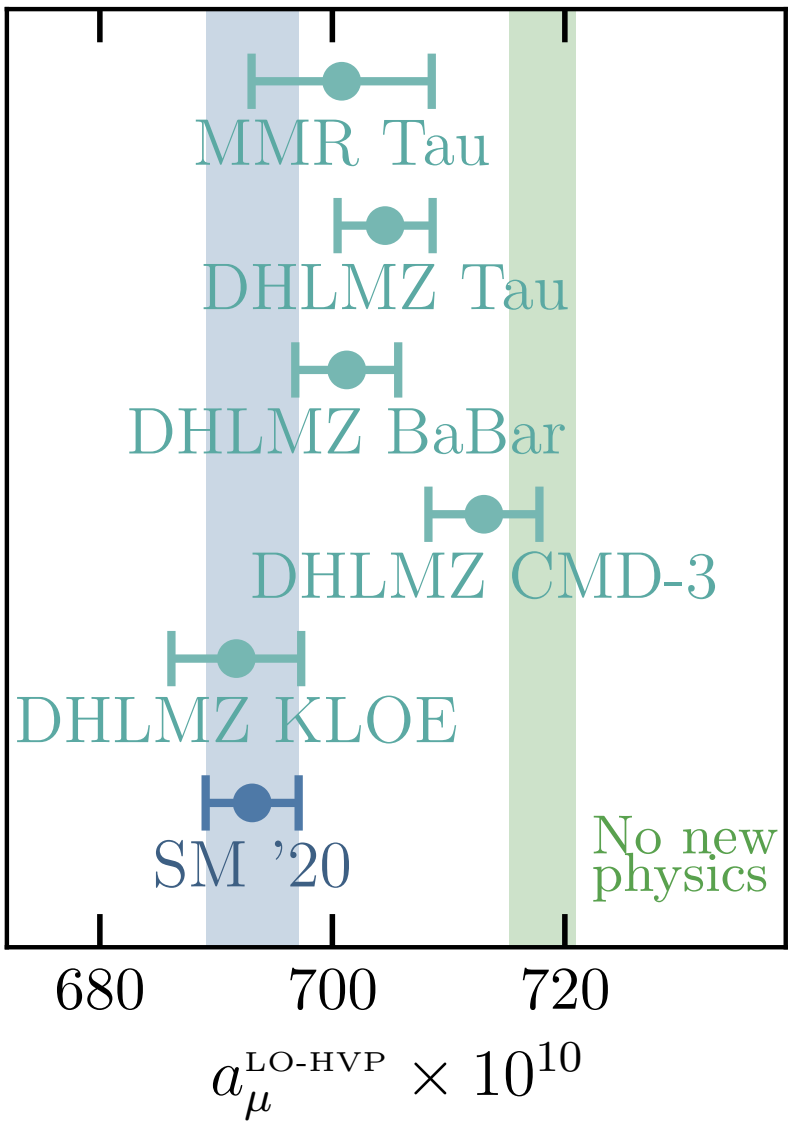
LO-HVP: data-driven discrepancies



- Tensions between experiments near ρ peak
- Since WP, new result from CMD-3 [2302.08834] shows even greater tension
- BaBar: issues in PHOKHARA Monte Carlo may affect KLOE and BES [2308.05233]
- Motivate reevaluation of dispersive calculations and how experiments combined [DHLMZ 2312.02053]
- New efforts to include determinations from τ decays [MMR 2305.20005]

LO-HVP: data-driven discrepancies



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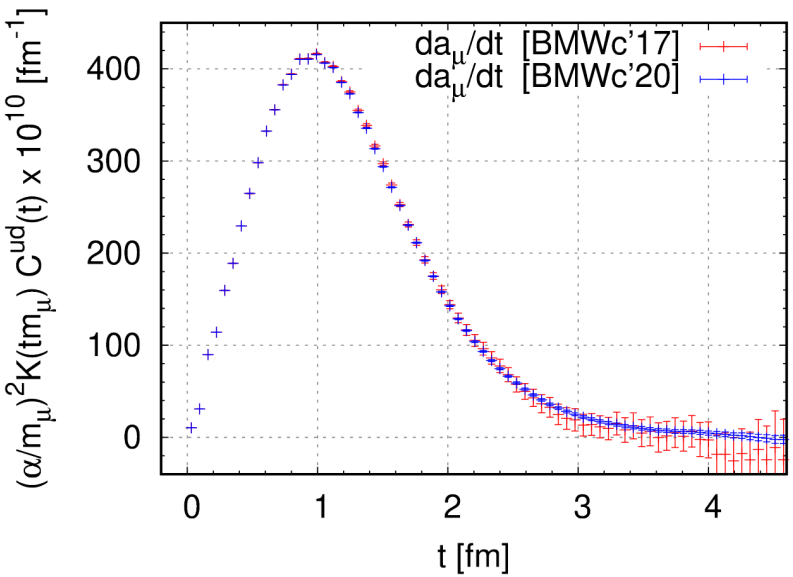
LO-HVP: lattice QCD

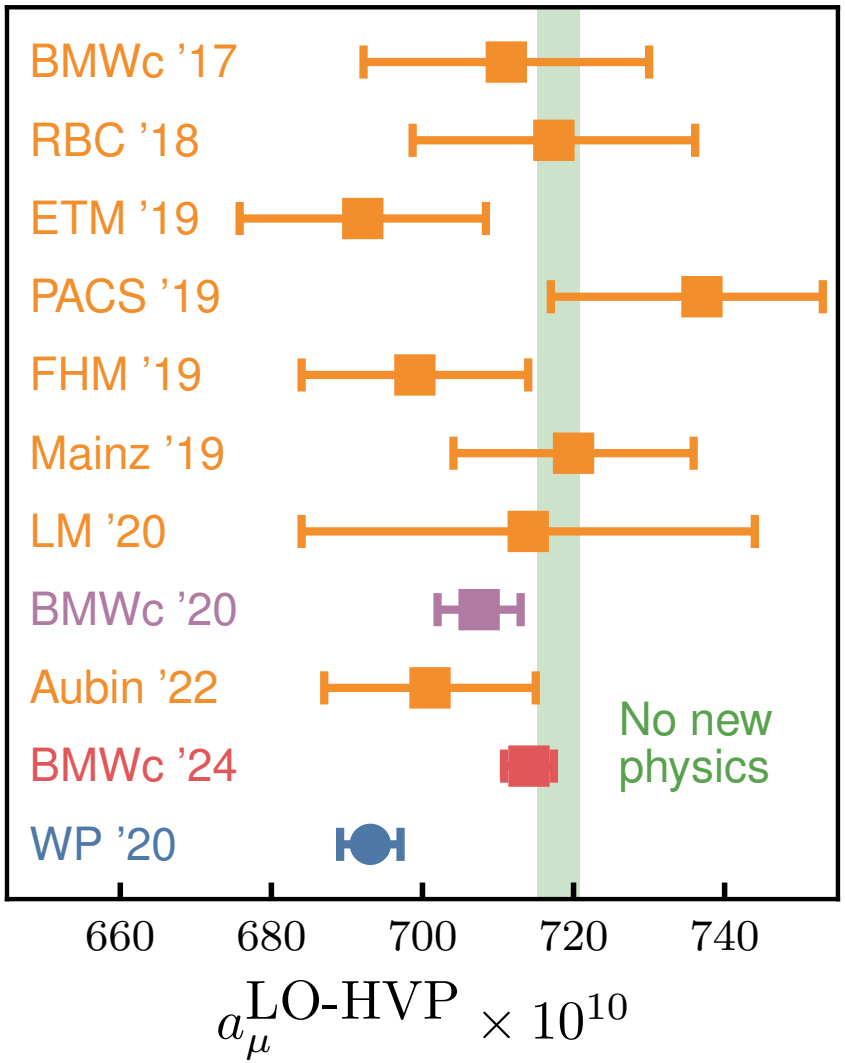
- Formulate QCD+QED Lagrangian on discrete four-dimensional grid
- Fix physical scale and parameters of theory using physical input
  - Scale $\leftarrow M_\Omega$
  - $(m_u + m_d)/2 \leftarrow M^2_{\pi_0}$
  - $m_u - m_d \leftarrow M^2_{K_0} - M^2_{K_+}$
  - $m_s \leftarrow M^2_{K_0} + M^2_{K_+} - M^2_{\pi_+}$
  - $m_c \leftarrow M^2_{D_s}$
  - QED $\leftarrow \alpha$
- a_μ^{LO-HVP} obtained from integral in Euclidean time

[High-Performance Computing Center Stuttgart]



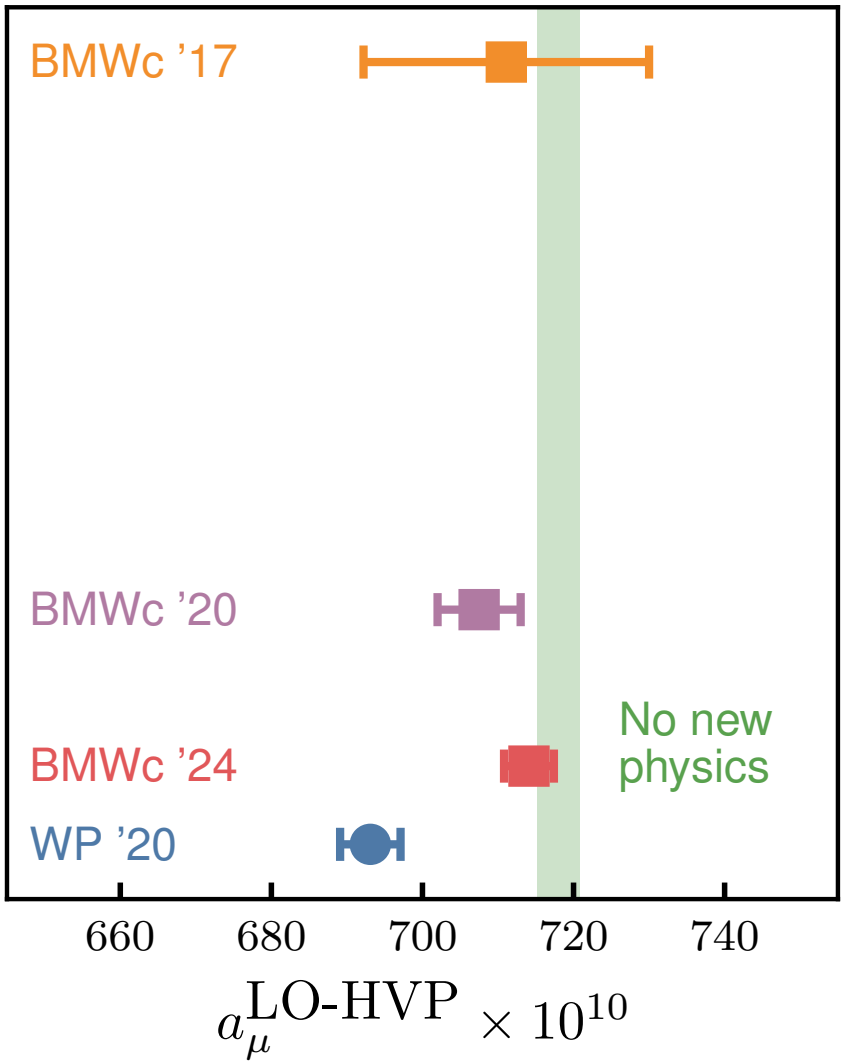
LO-HVP: lattice QCD

- Until 2020, lattice uncertainty larger than data-driven
- Lattice results mostly consistent with both data-driven and experiment
- Sub-percent lattice determination [2002.12347]: First lattice calculation with errors comparable to data-driven determinations
- New BMWc result [2407.10913]: More precise than data-driven determinations

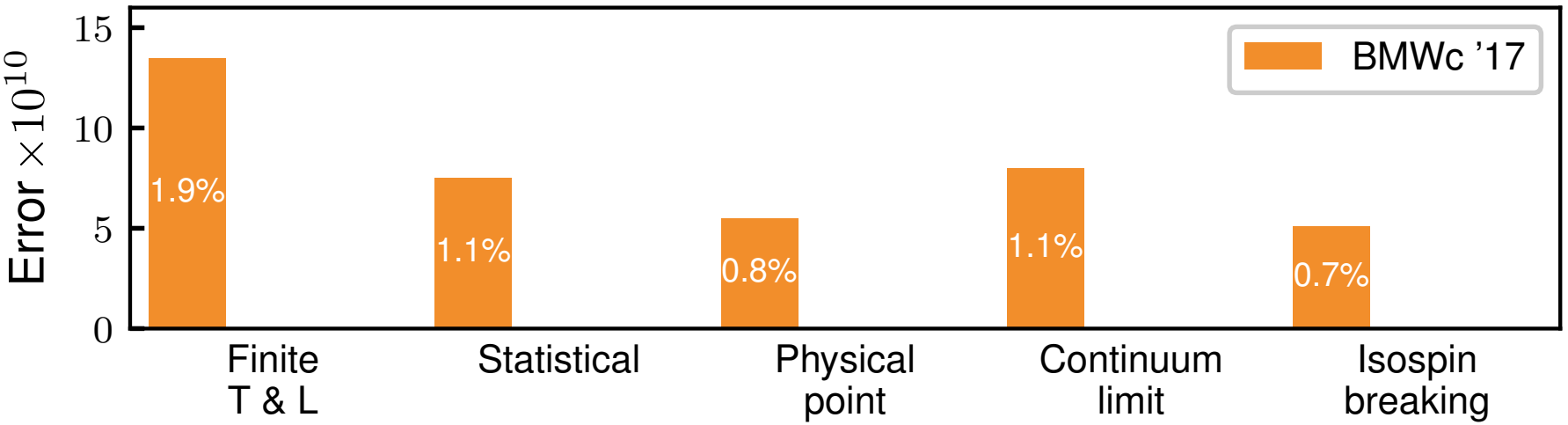


Seven years of progress

- 2020 publication: $3.4\times$ increase in precision over our earlier work [BMWc '17]
- Update this year: further $1.7\times$ increase in precision
- Many improvements needed to attain this precision
- Made possible thanks to the work of many groups around the world

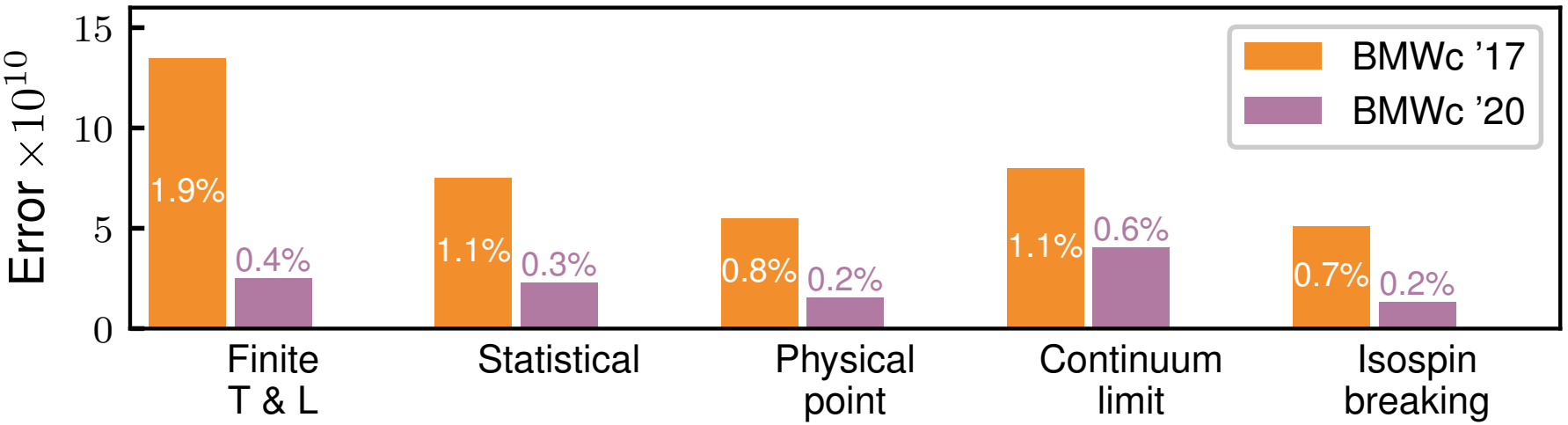


Statistical and systematic errors



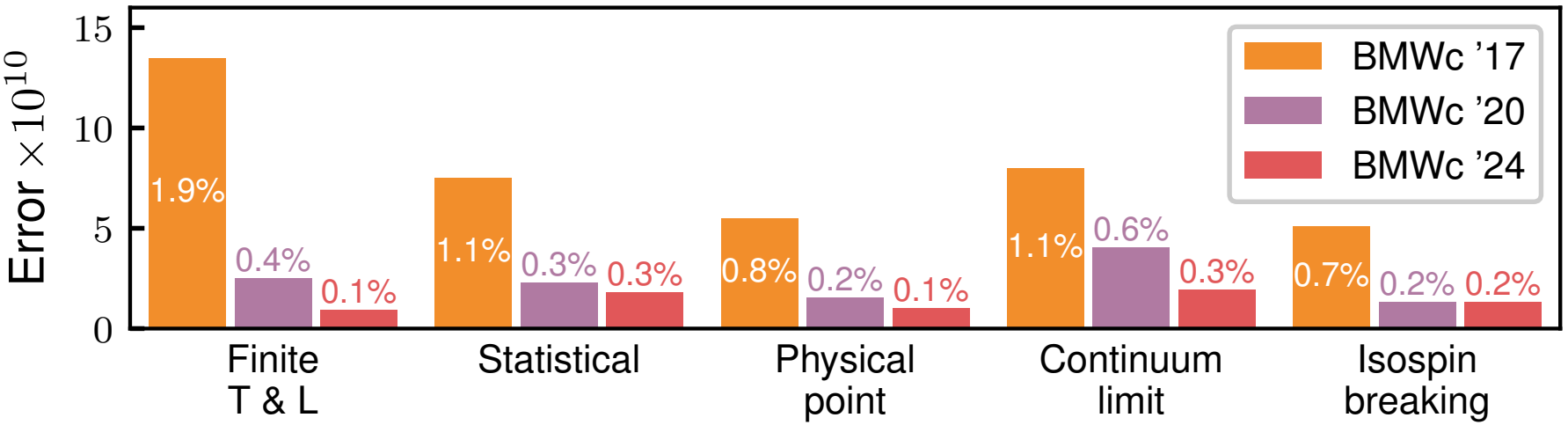
- Five major sources of uncertainty in our first work
- Dominant error from finite-size effects
- Sub-leading uncertainties from
  - Statistical Monte-Carlo sampling of path integral
  - Physical inputs to set the scale and parameters
  - Uncertainty in the continuum limit extrapolation
  - Isospin-breaking effects from $\alpha \neq 0$ and $\delta m \equiv m_d - m_u \neq 0$

Statistical and systematic errors



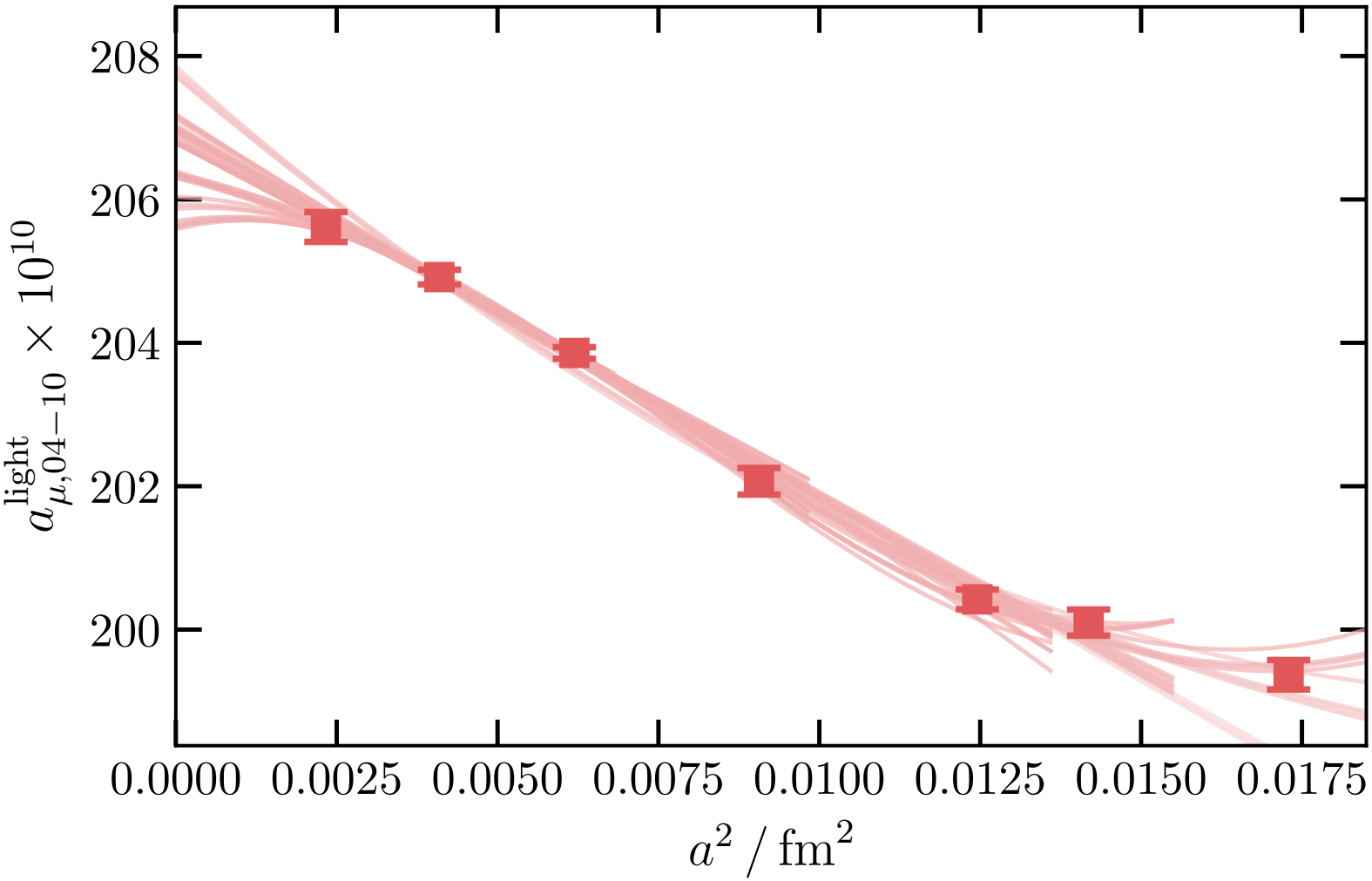
- In 2020 update, finite-size effects addressed using dedicated large-volume simulations ($6\ \mathrm{fm} \to 11\ \mathrm{fm}$)
- Further improvement from algorithmic improvements, extra calculations and updated analysis

Statistical and systematic errors



- Recent update in 2024
- Add a 7th, finer lattice spacing ($a = 0.0483$ fm)
- Break continuum limit into pieces with different systematics
  - Short-distance part with complicated discretisation effects
  - Long-distance part with larger statistical errors
  - Most of the value from intermediate-distance parts with small statistical errors and simple, well-constrained continuum limit

Continuum limit



Combining Data-Driven & Lattice QCD

- Idea: Use data-driven result instead of lattice in tail
- Proposed in RBC '18
- What about problems with data-driven inputs?
- Only take data-driven above 2.8 fm
  - Low-energy region: $> 50\%$ from below ρ peak
  - Here, all experiments agree, and also agree with lattice
- Gives $< 5\%$ of final value, but significant reduction in error
- May be best way to match final experimental precision

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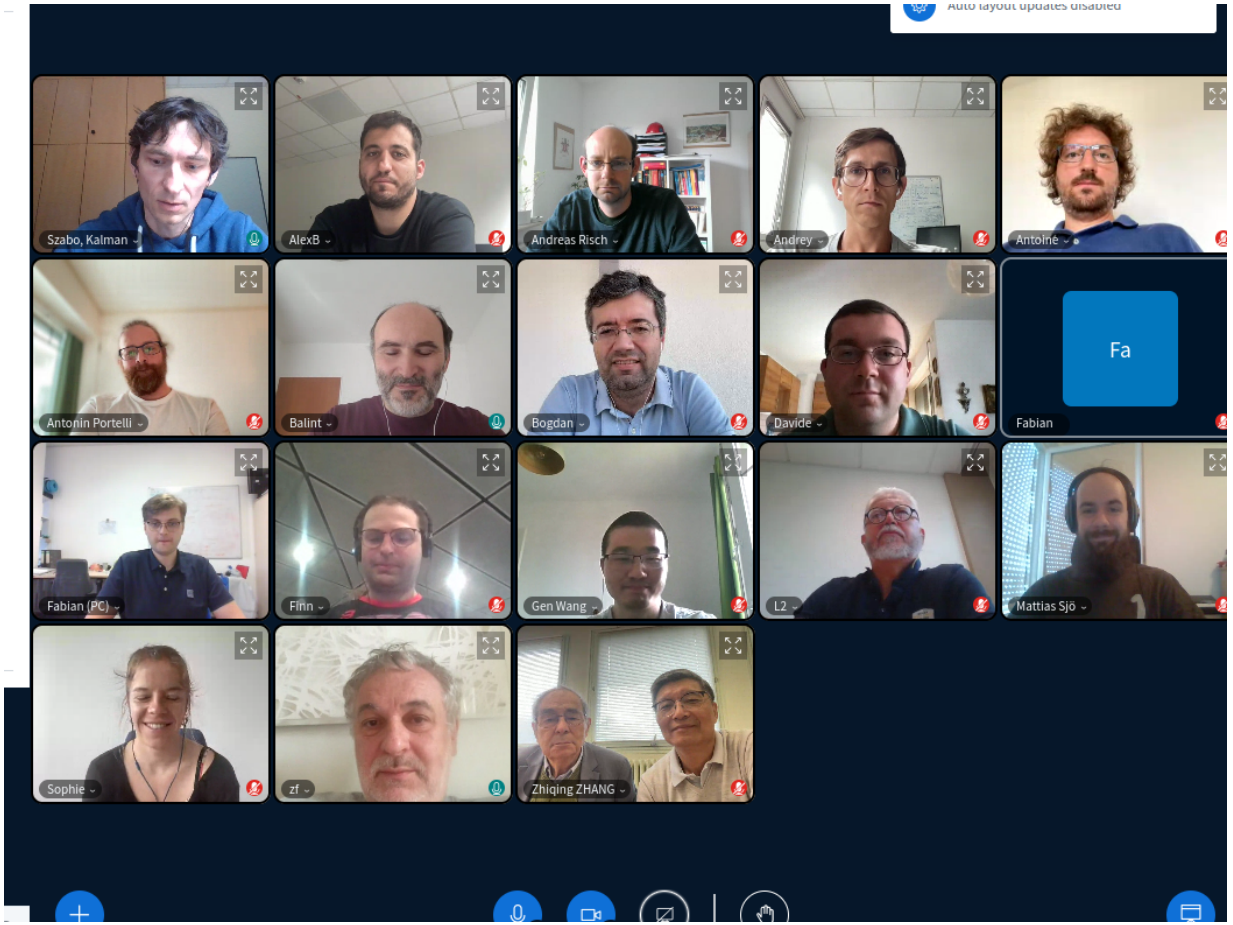
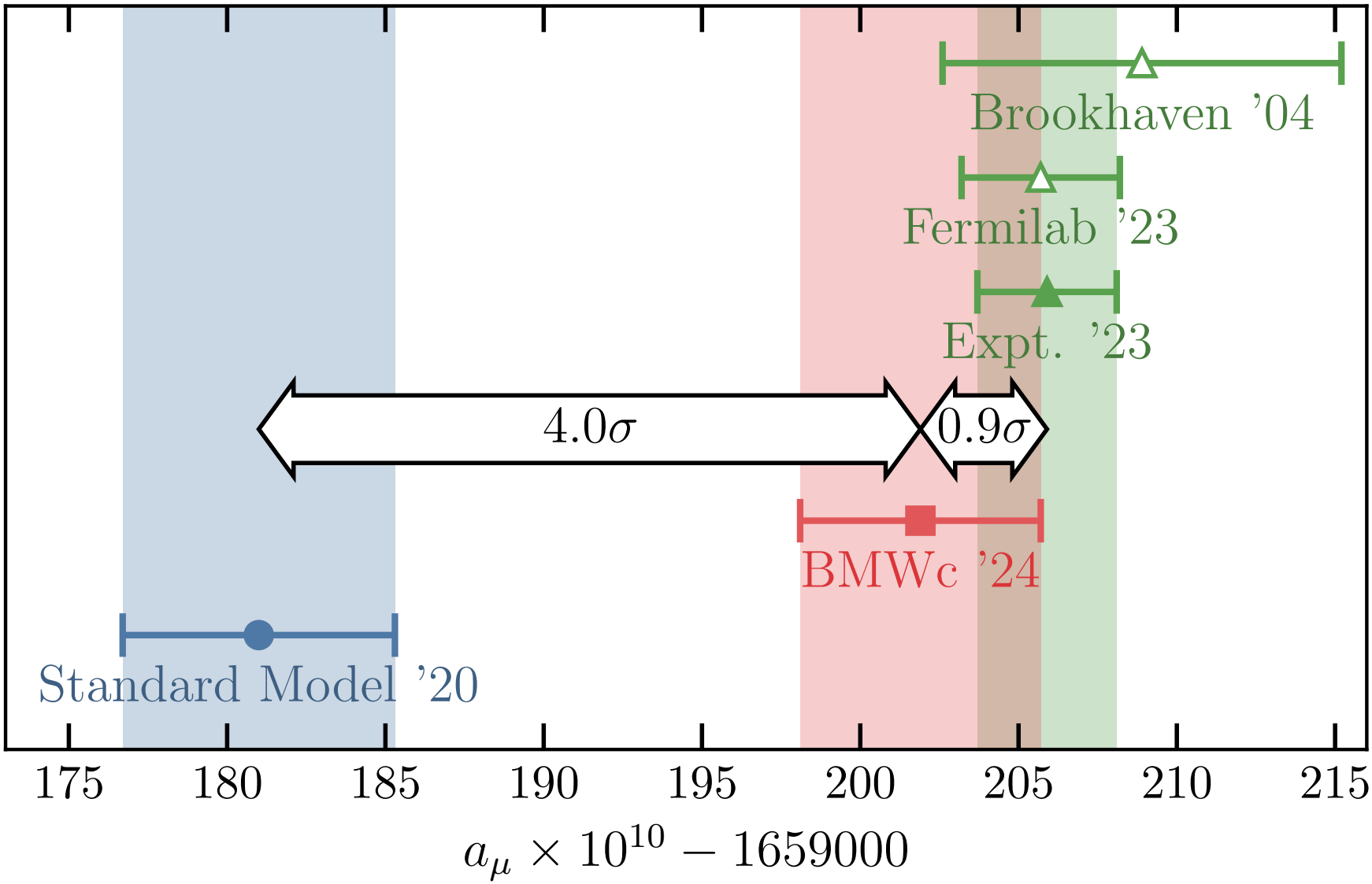
Blinding

- 2024 study was fully blinded
- All a_μ contributions were multiplied by some unknown blinding factor
- Blinding factor was only revealed when analysis was completely finalised and manuscript almost complete

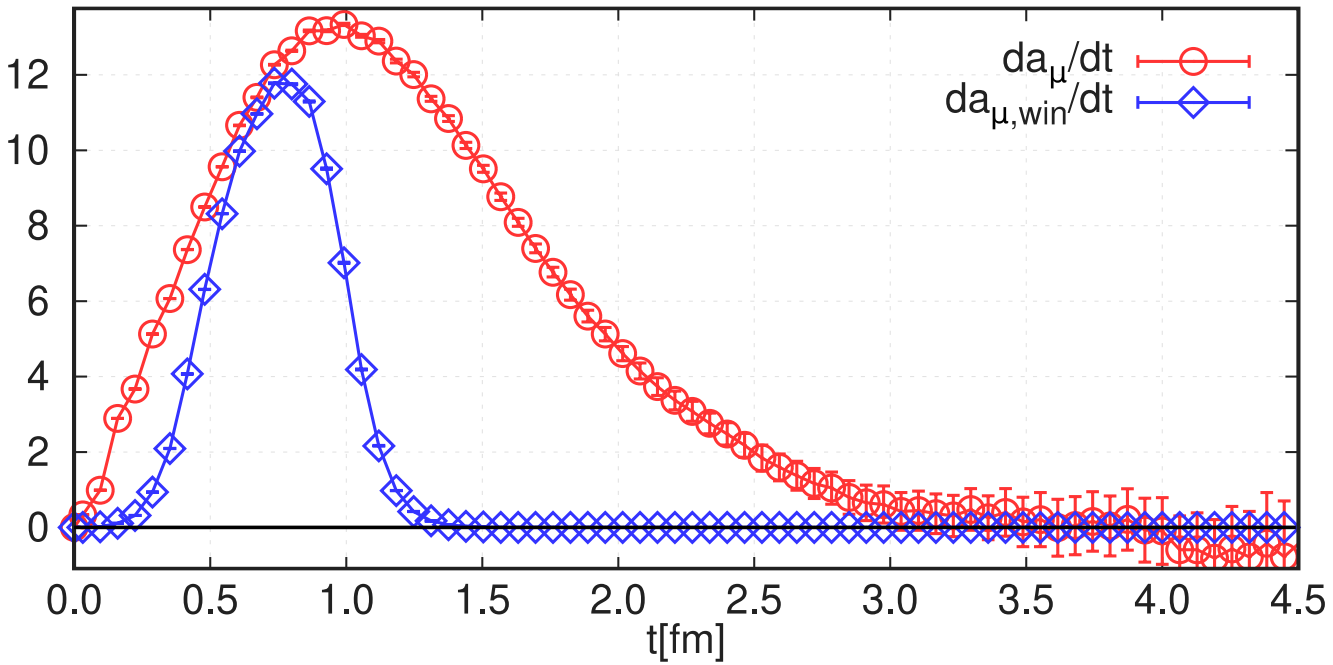


LO-HVP: BMWc ’24



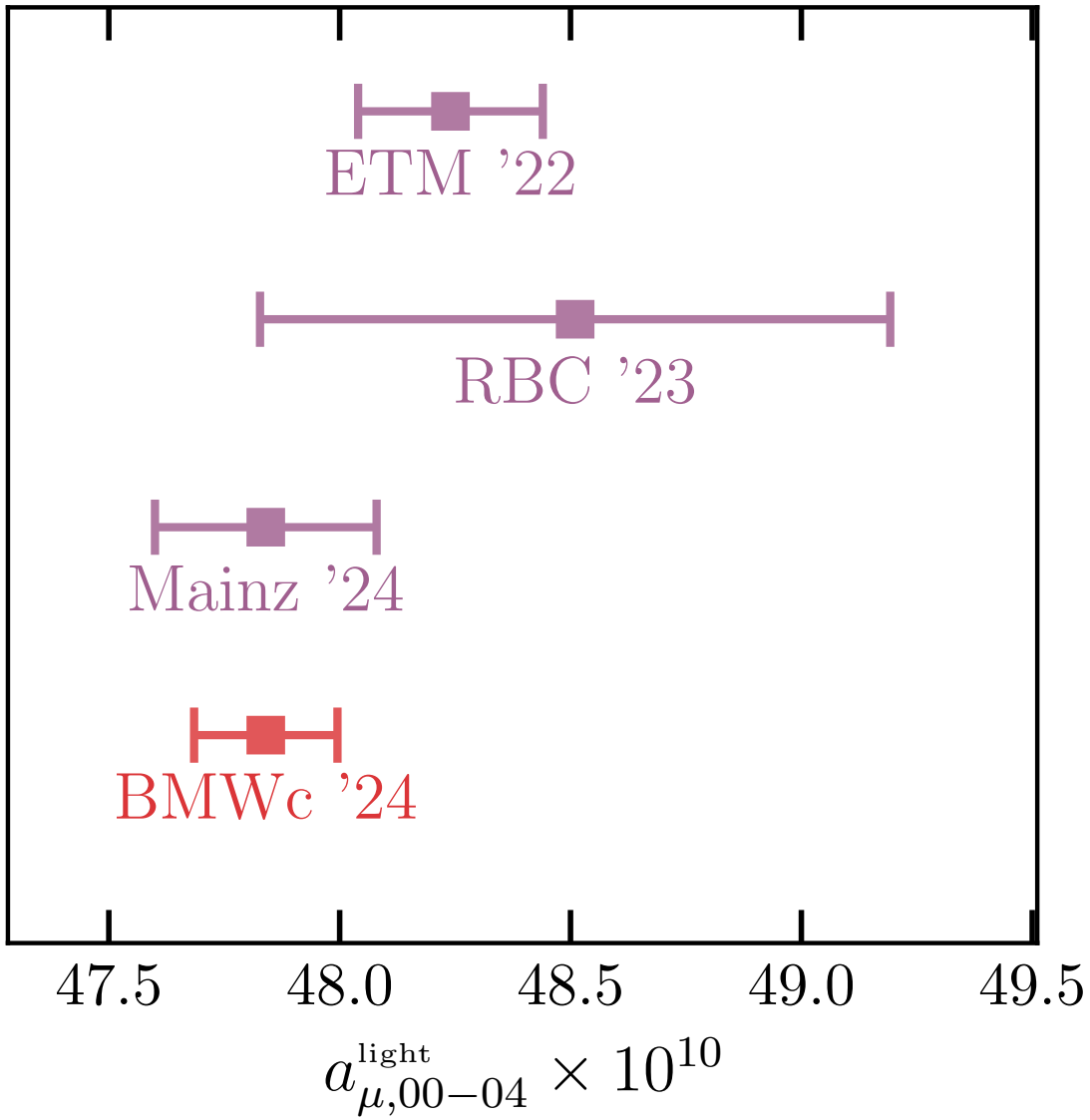
LO-HVP: other lattice groups

- New results from other lattice groups expected in coming months
- New standard: blinded analyses
- RBC/UKQCD have just announced unblinded light connected result, full result coming soon
- ETM, Mainz, FHM making progress but still blinded
- Several partial results have been published



Window Results

- Several groups have published partial calculations
- Good agreement between different lattice groups
- Disagreement with data-driven at intermediate distances



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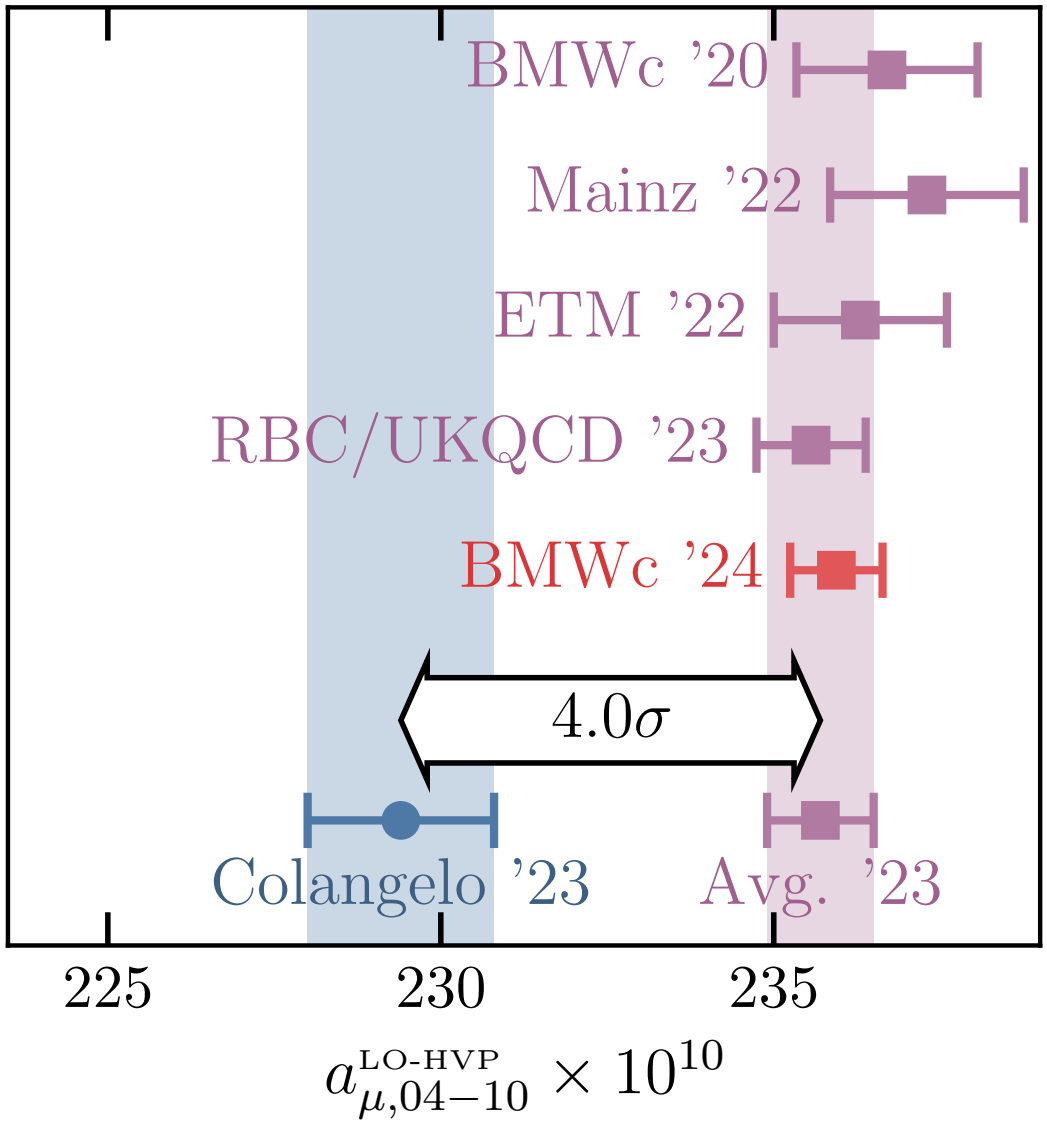

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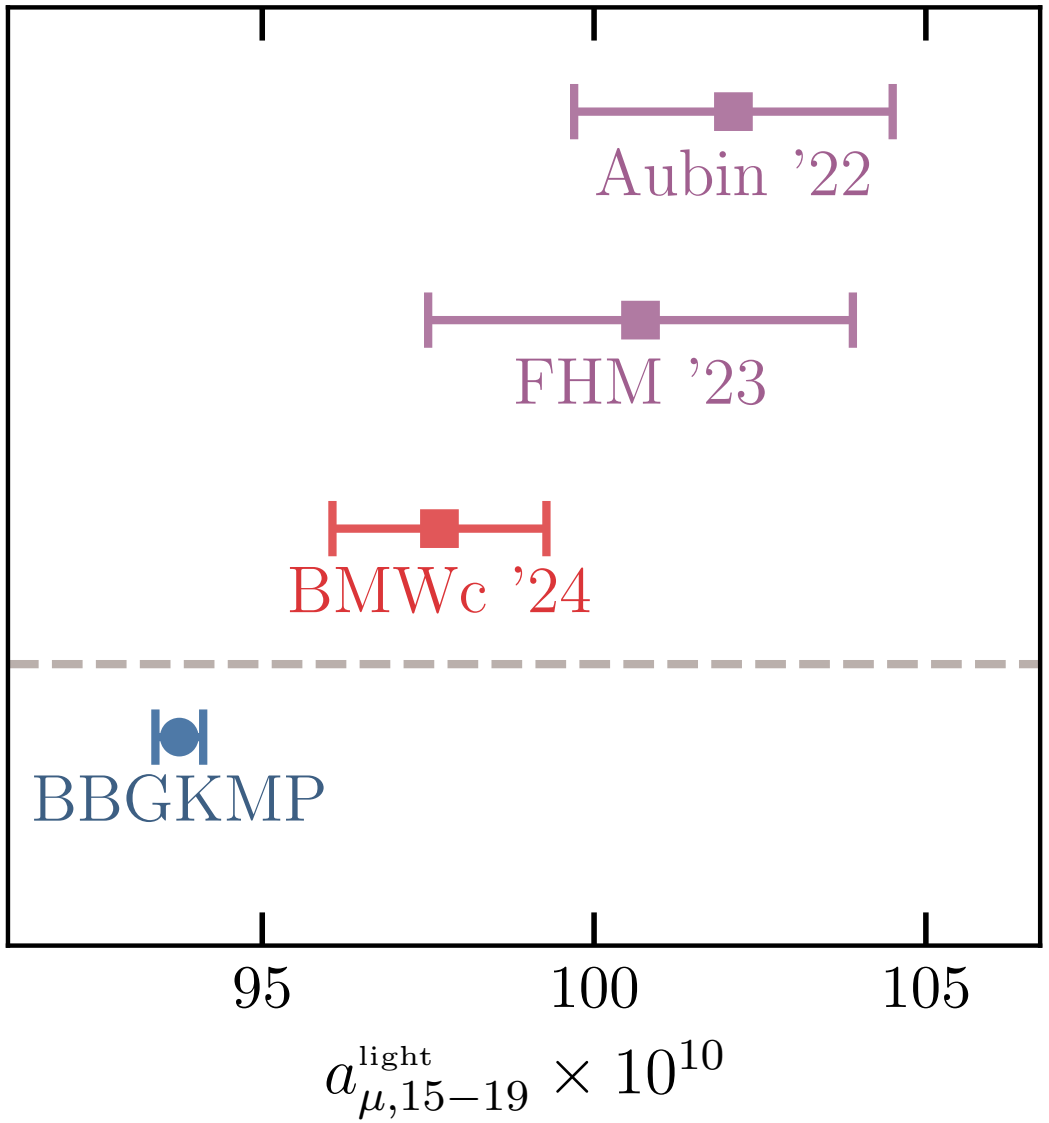


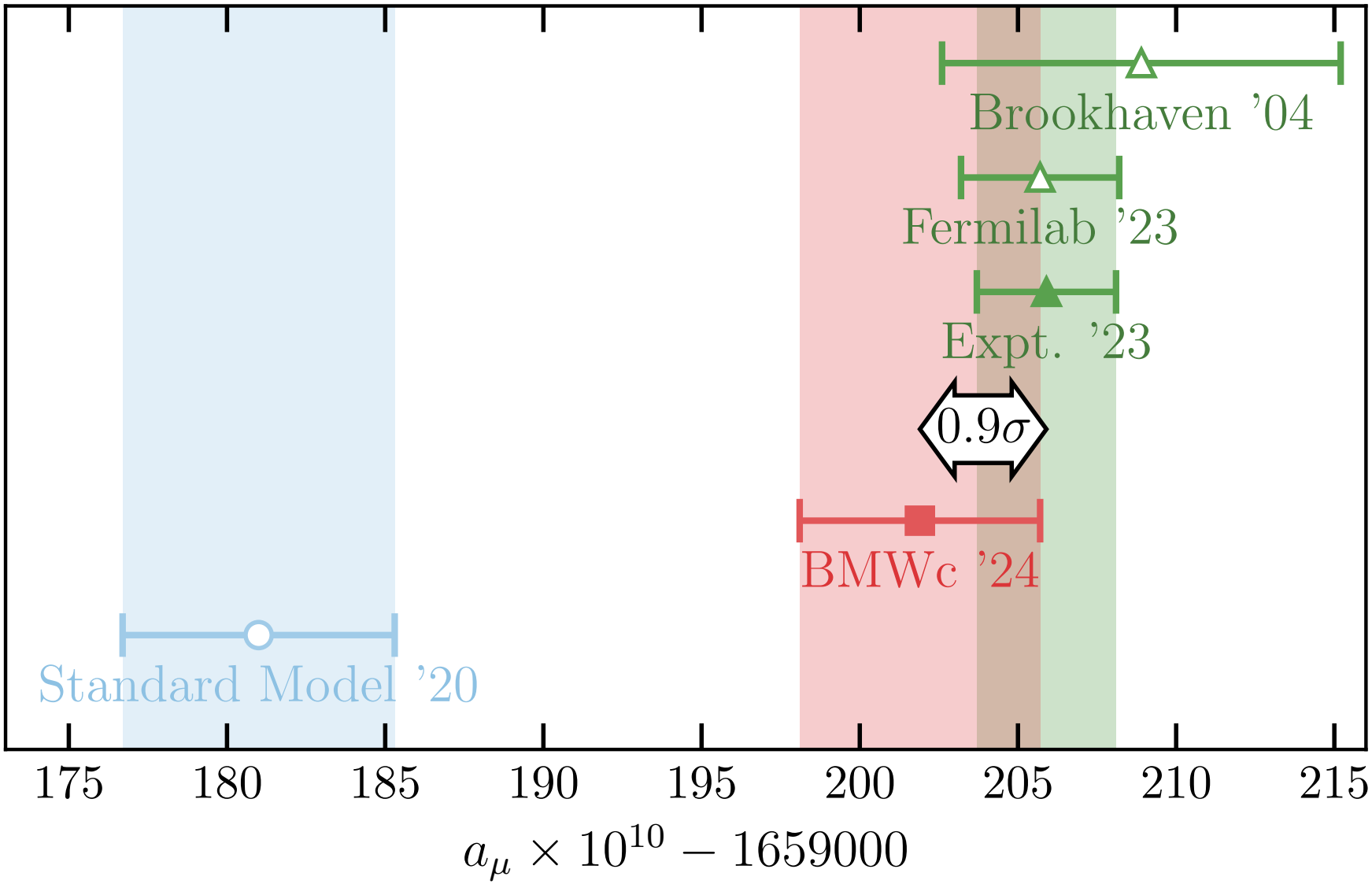
Data-Driven vs Lattice QCD

- Phenomenological estimate shows of IB contributions can't explain discrepancy [Hoferichter et. al. 2307.02532]
- GeV-scale NP might explain tension [2212.03877]

What energies do they disagree at?

- To convert from lattice result to energy spectrum requires inverse Laplace transform: ill-conditioned
- Two possible workarounds:
  - What modifications to the data-driven inputs would give consistency [BMWc/DMZ 2308.04221]
  - Smeared version of data-driven inputs [ETM 2212.12493]
- Both approaches suggest an enhancement near the ρ peak
- Similar to what is seen in CMD-3

Conclusion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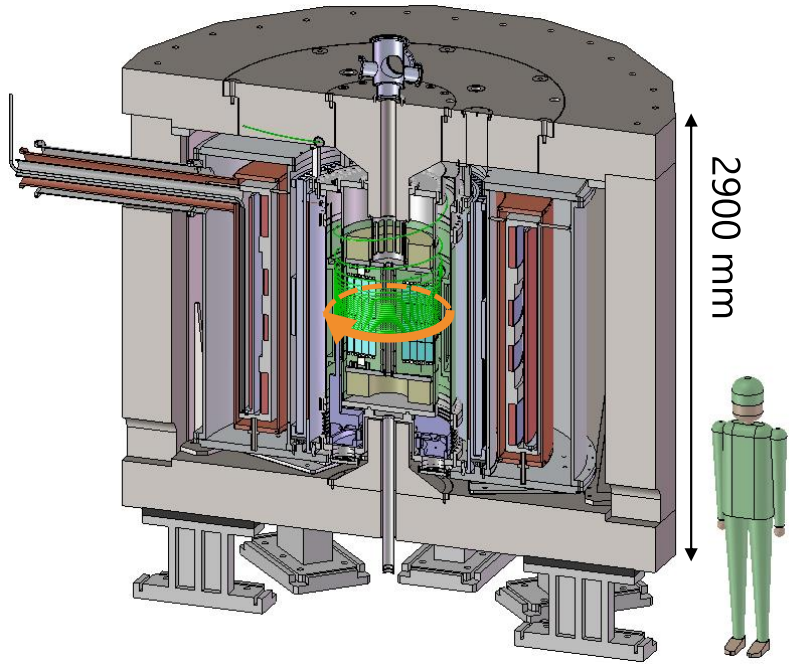
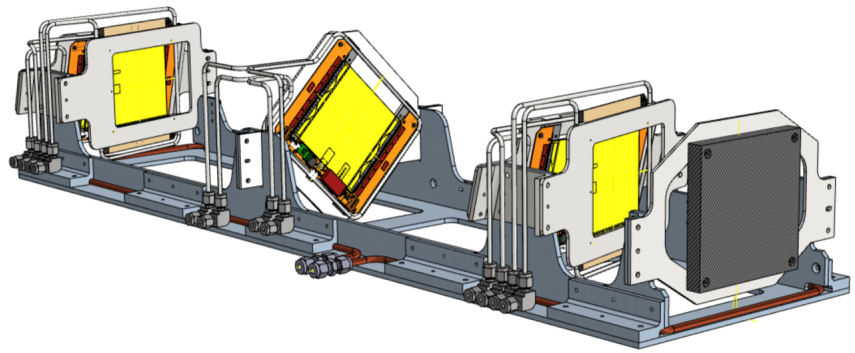
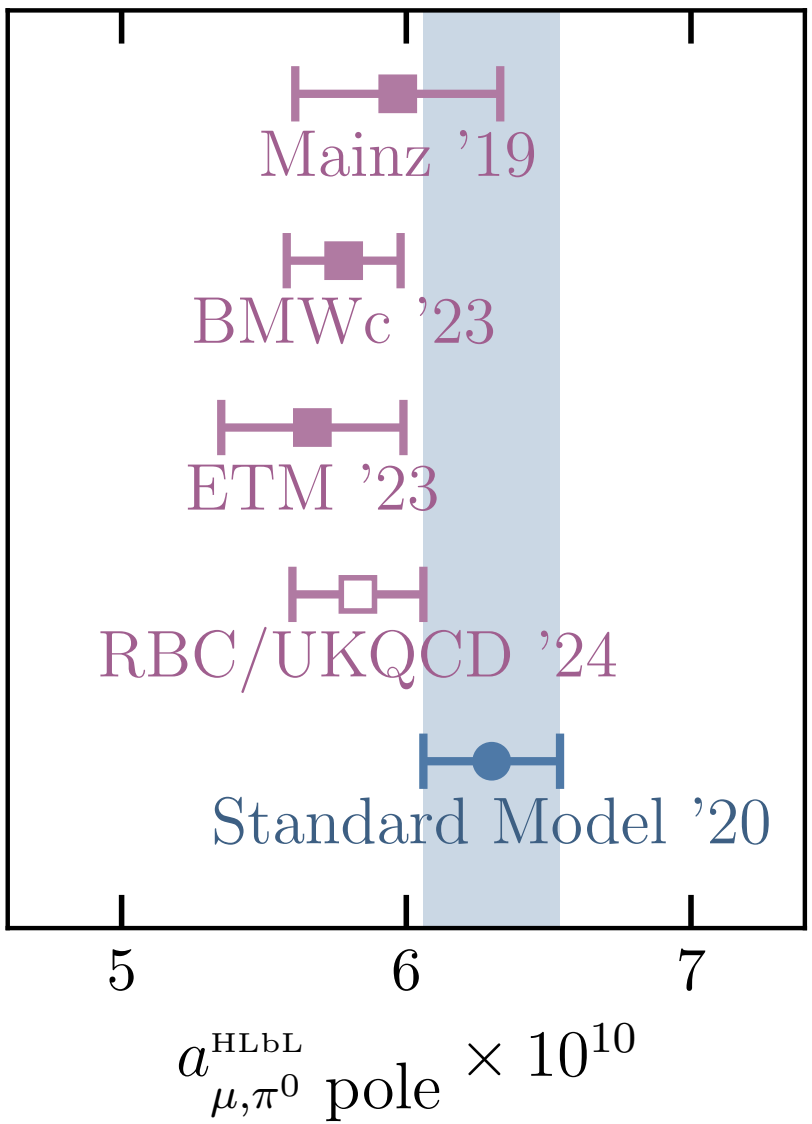
Experimental measurement: Future

- New experiment planned at J-PARC [Just now: K. Aoki]
  - Ultra-cold muon beam from muonium
  - Compact storage ring
  - Very different from BNL/FNAL
  - Expected to start in 2028
- MUonE proposal at CERN
  - Would measure LO-HVP contribution from muon-electron scattering
  - Proof of concept built and tested
  - First phase planned to be online in 2025
  - Full experiment in 2029





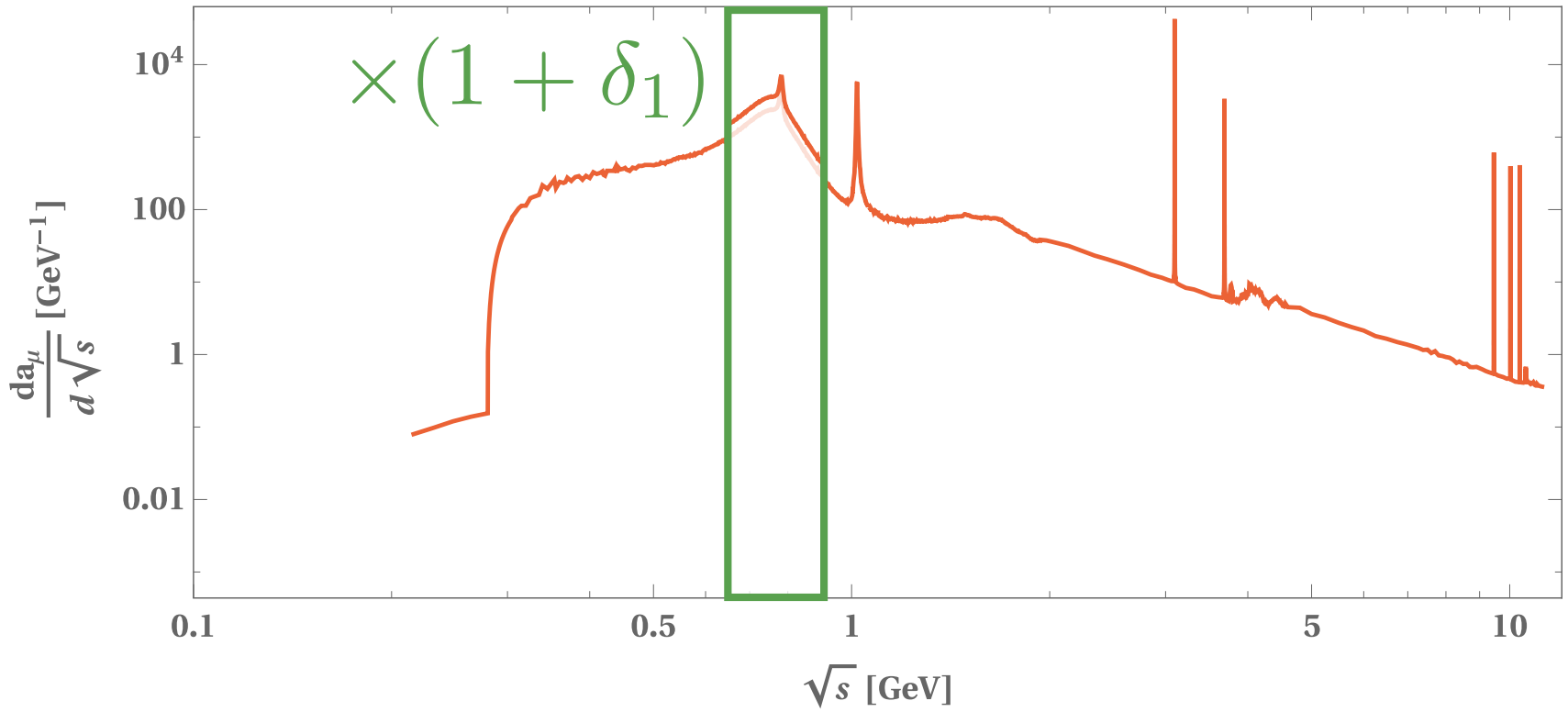
Hadronic Light-by-Light: Pion pole



- Contribution from meson exchange: can use lattice QCD input to dispersive calculations [1903.09471, 2305.04570, 2308.12548]

Data-Driven vs Lattice QCD: Rescaling

- Idea: select an energy range and scale the data inside that region
- Scaling factor of $1 + \delta_1$ for some small δ_1
- Find δ_1 that minimises tensions in $a_\mu^{\mathrm{LO-HVP}}$ and $a_{\mu,\mathrm{win}}^{\mathrm{LO-HVP}}$
- $a_\mu^{\mathrm{LO-HVP}}$ and $a_{\mu,\mathrm{win}}^{\mathrm{LO-HVP}}$ don't constrain high energies
- Consider third observable, related to hadronic running of α



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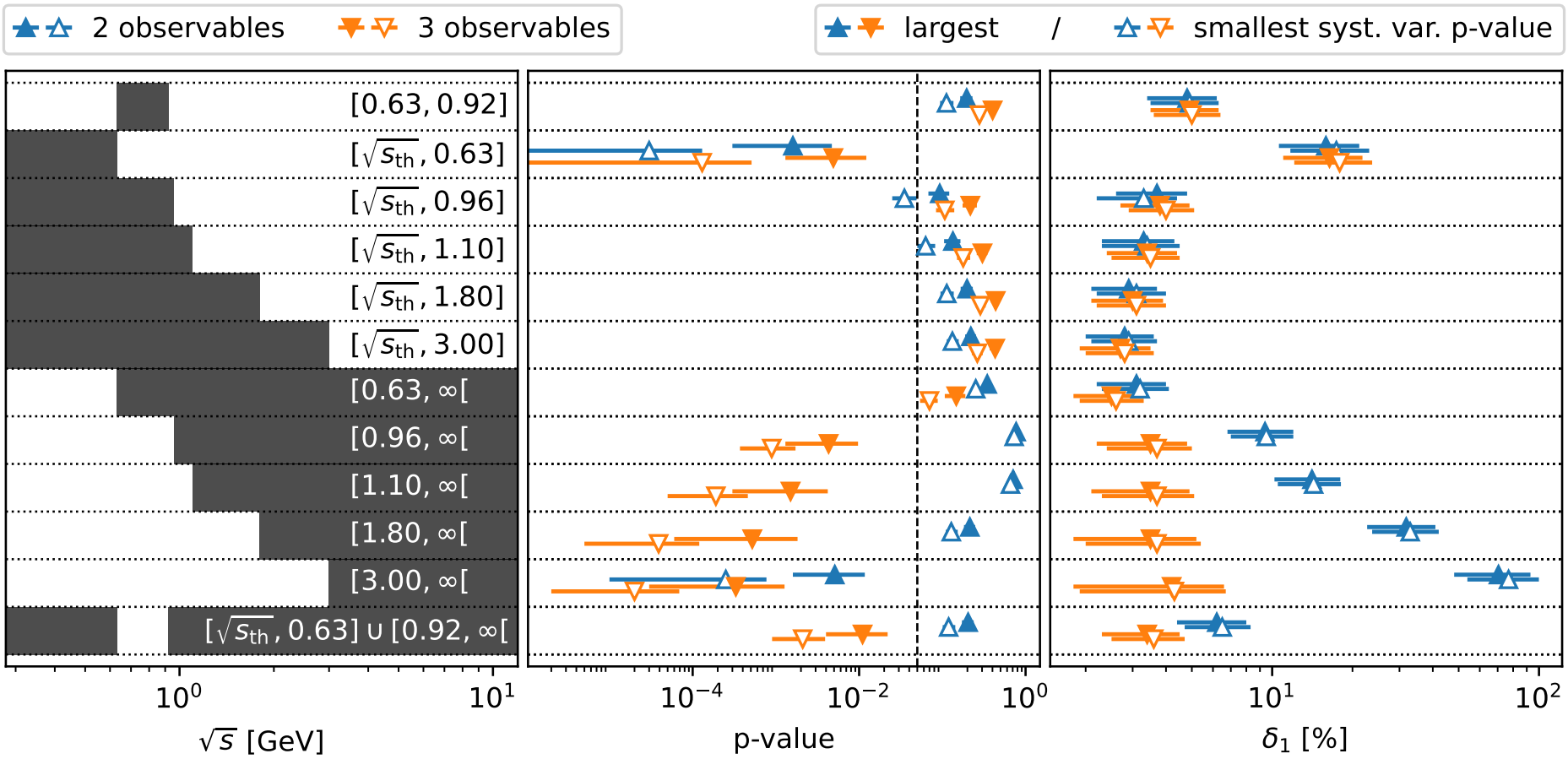
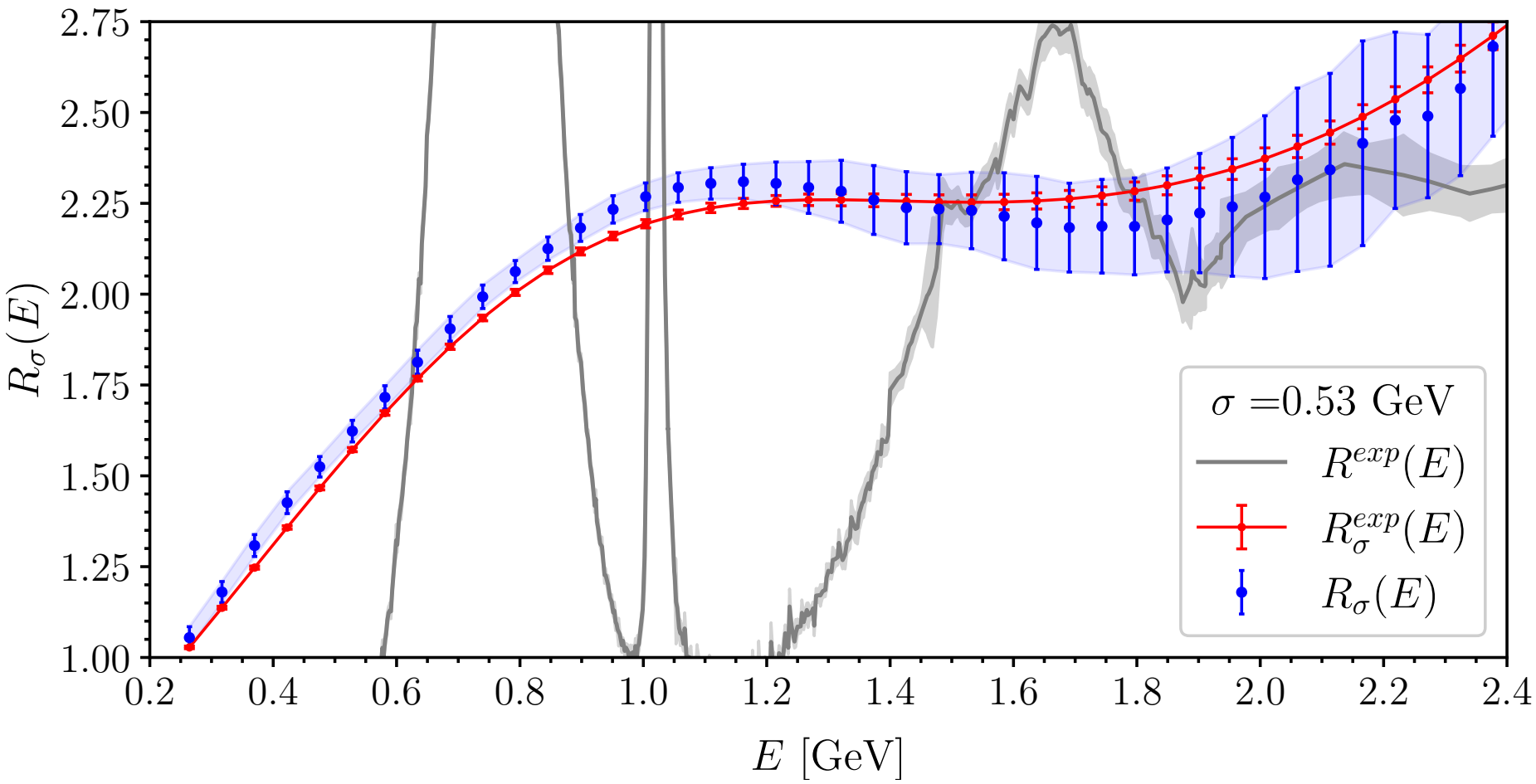


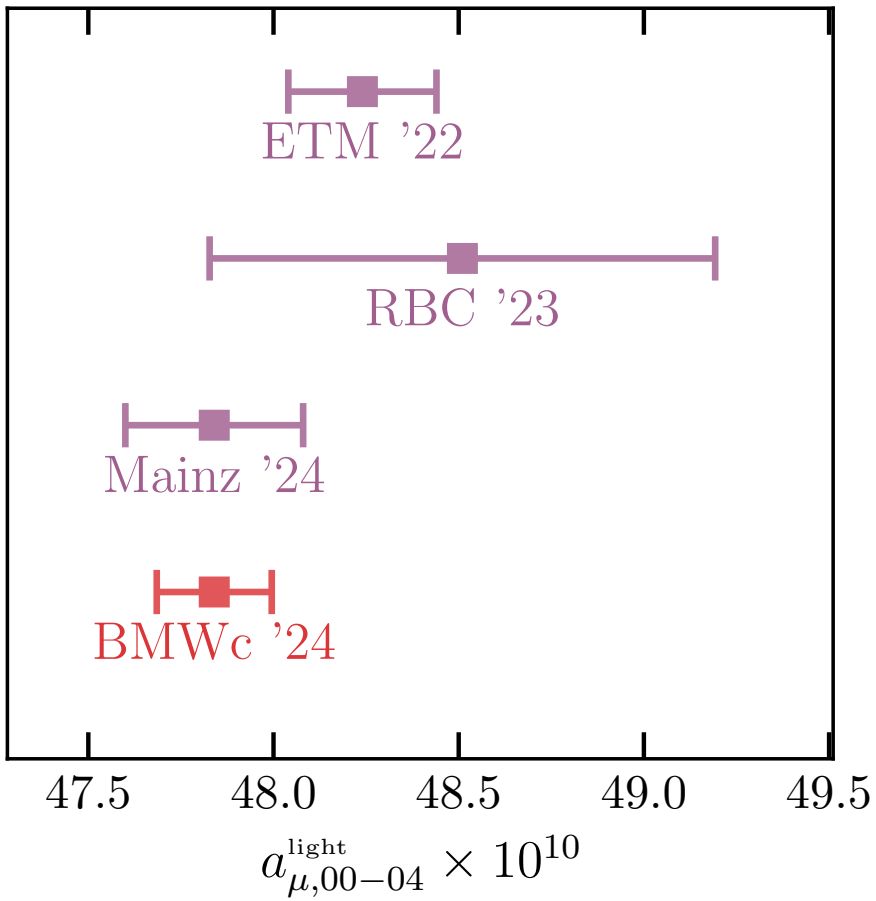
- If rho peak included, have good p-values with δ from 2–5%
- Below the rho peak, have bad p-values
- Above the rho peak, have good p-values for 2 observables and bad p values for 3

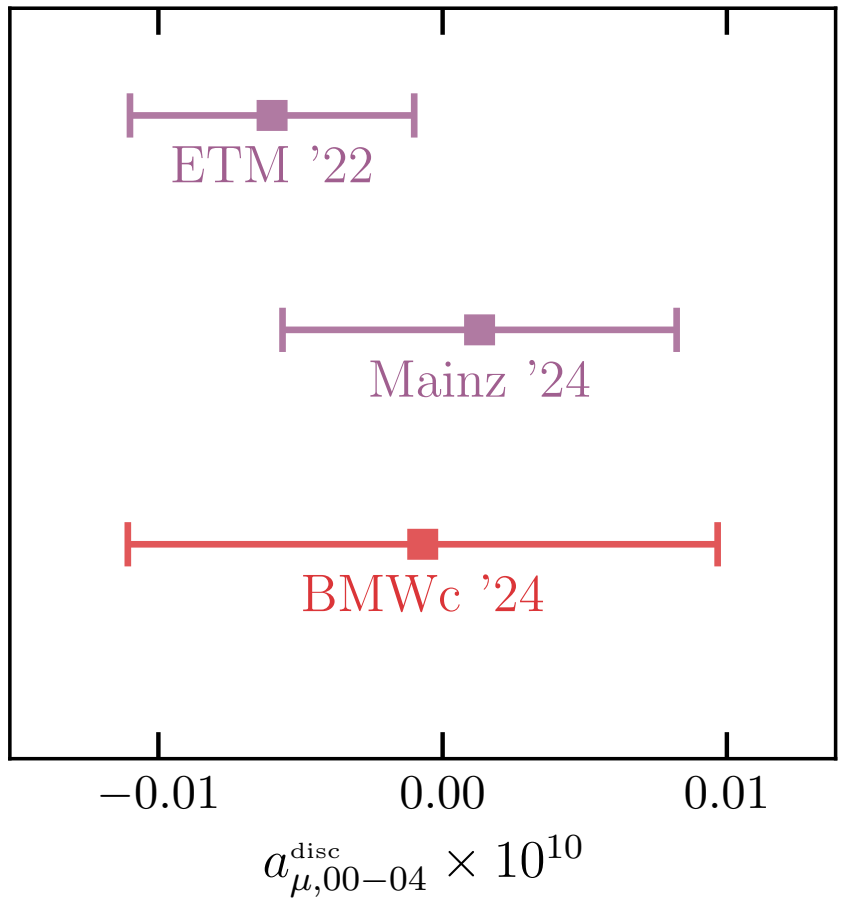
Data-Driven vs Lattice QCD: Smearing



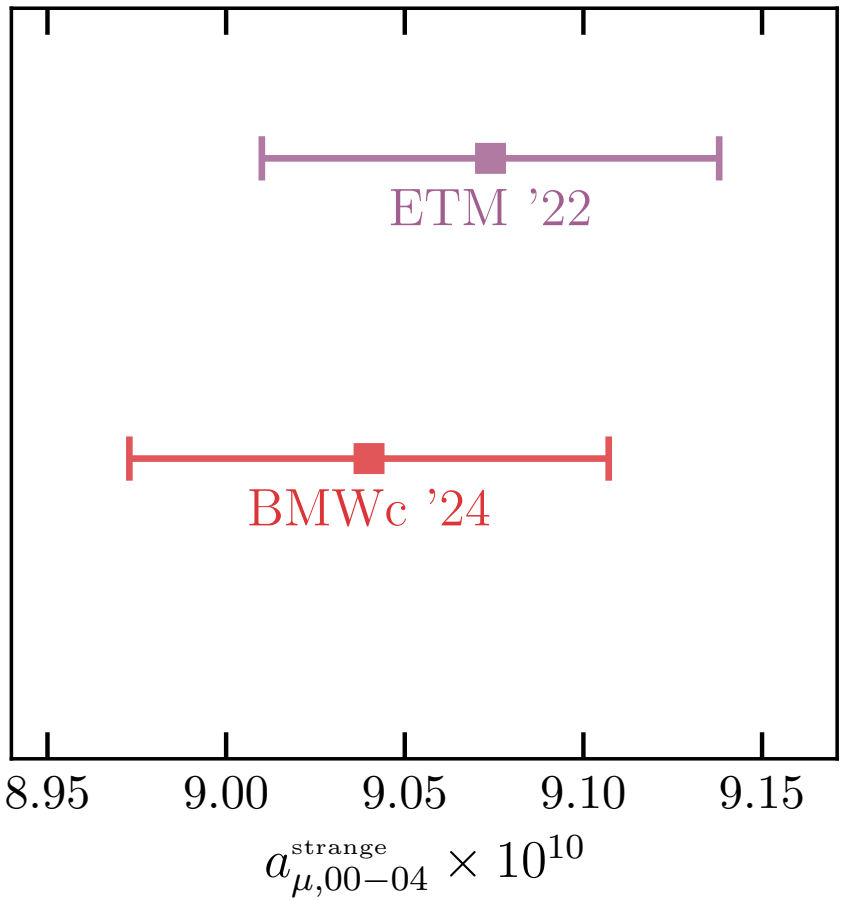
[ETMC 2212.12493]

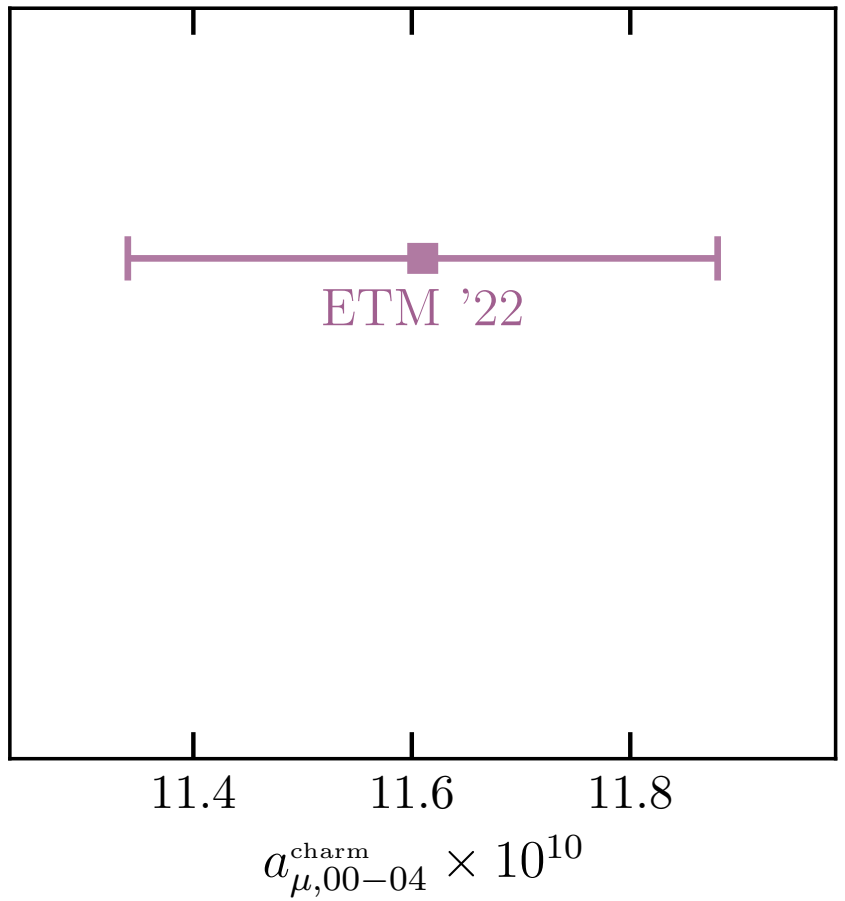
Window Results: 0.0 fm − 0.4 fm



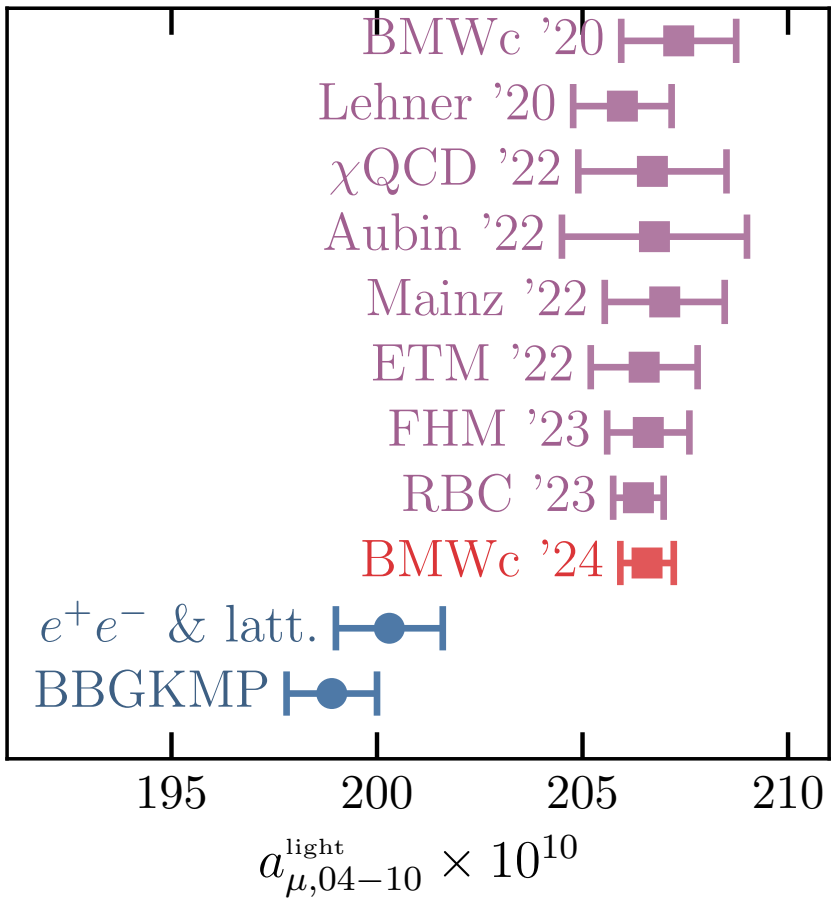


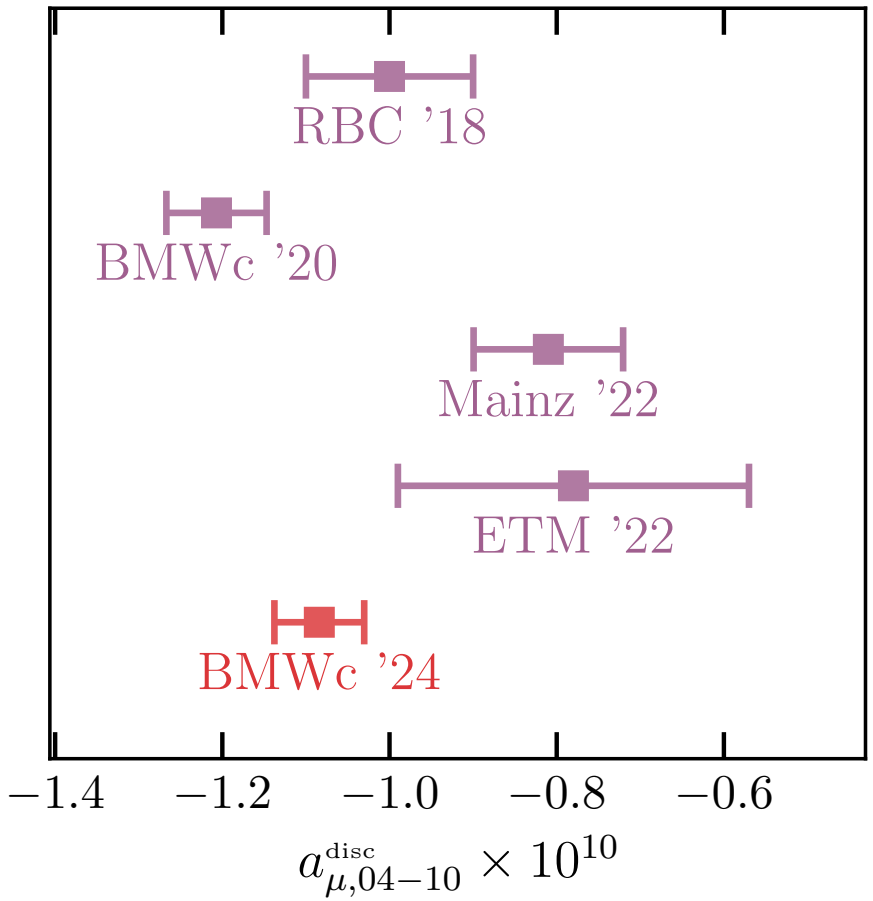
Window Results: 0.0 fm − 0.4 fm





Window Results: 0.4 fm − 1.0 fm





Window Results: 0.4 fm – 1.0 fm