Complex potential and open system applications in heavy-ions and cold atoms

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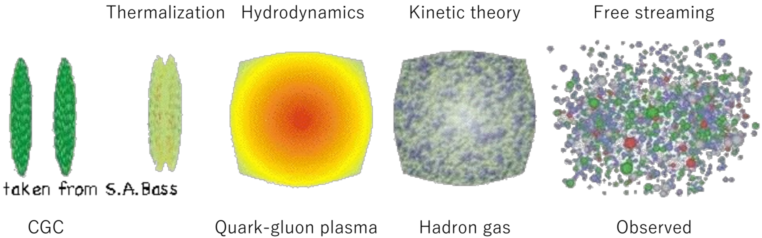
Heavy-ions **[reviewed by Raimond]**

Key achievements

- Formation of Quark-Gluon Plasma (QGP)
- Strongly coupled nature of QGP $\leftarrow$ Nearly perfect fluid $\eta/s \sim 1/4\pi$

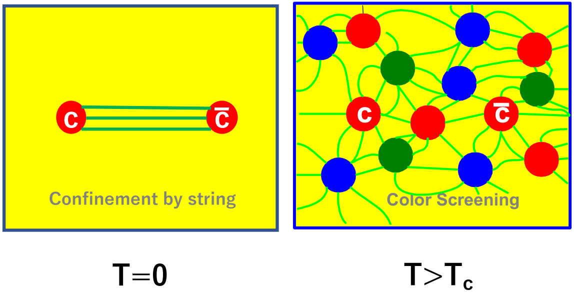


Challenges

- Thermalization in a rapid expansion
- Hydrodynamic collectivity for small systems $N_{\text{particle}} \sim 10^{3-4}$
- Dynamical properties of strongly coupled QGP (why η/s so small? etc)

Quarkonium suppression in 21st century

HQ is a localized probe with color $\rightarrow$ Quarkonium reflects to the color force in medium
Color screening inside QGP = Static effect



- Bound states disappear at high temperatures (J/ψ suppression [Matsui-Satz (86)])
- Experimental data consistent with sequential melting

Question: Dynamical effects, e.g. collisions and gluon absorptions/emissions

In the potential picture, static effect = $V_{\rm Re}$, dynamical effect = $V_{\rm Im}$

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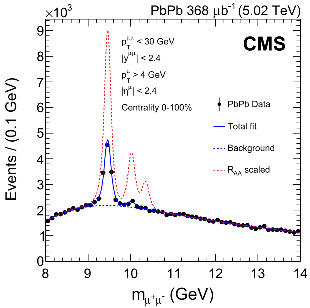

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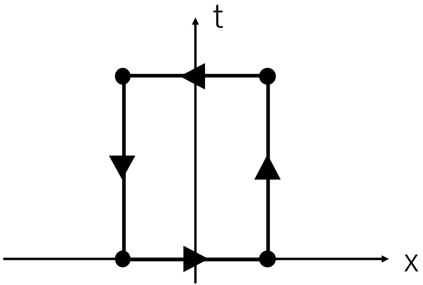
Complex potential: definition

1. Definition using static heavy quark pair ($M = \infty$)

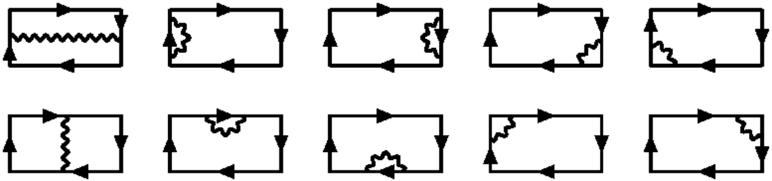
$$\Psi(\boldsymbol{r}, t) = \underbrace{\langle Q_c(\mathbf{0}, t)\, Q(\boldsymbol{r}, t)\, Q^\dagger(\boldsymbol{r}, 0)\, Q_c^\dagger(\mathbf{0}, 0)\rangle_T}_{\text{medium average of } e^{-iV(r;A_{\mathrm{bkg}})t}} \xrightarrow[t\to\infty]{} \underbrace{e^{-iV(\boldsymbol{r})t}\Psi(\boldsymbol{r}, 0)}_{\text{oscillatory damping}}$$

2. Time dependence of real-time thermal Wilson loop at late times



Complex potential in perturbation theory

Leading order (HTL-resummed) perturbation at $r \sim 1/gT$ [Laine et al (07)]



$$V(r) = \underbrace{-\frac{C_F g^2}{4\pi}\left(m_D + \frac{e^{-m_\mathrm{D} r}}{r}\right)}_{\text{mass shift + screening}} \underbrace{-iC_F g^2 T \int \frac{d^3k}{(2\pi)^3} \frac{\pi m_\mathrm{D}^2 (1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}})}{k(k^2 + m_\mathrm{D}^2)^2}}_{\text{Landau damping} \sim \text{collisions}}$$

Next-to-leading order calculation **[see Joan's talk]**

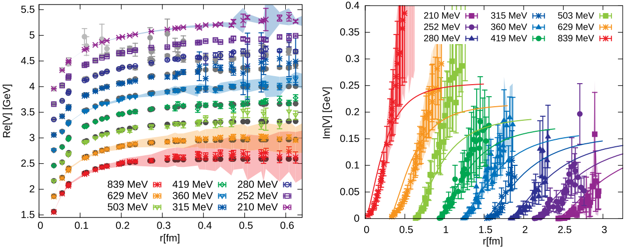
Complex potential on the lattice

Analytic continuation of thermal Wilson loop to imaginary time

$$\underbrace{W(t=-i\tau,r)}_{\text{lattice}}=\int d\omega\, e^{-\omega\tau}\rho(\omega,r),\quad 0\le\tau\le\beta$$

Bayesian reconstruction of $\rho(\omega,r)$ from $W(-i\tau,r)\to V_{\rm Re}(r)+iV_{\rm Im}(r)$

[Jon-Ivar's talk for meson SPF]



[Rothkopf-Burnier (15)]

- $V_{\rm Re}(r)$ screening, $V_{\rm Im}(r)$ increases with r
- Latest result: Johannes's talk, today 2pm-

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Basics of open quantum system [Reviews for quarkonium in QGP: Akamatsu (22), Yao (21), Sharma (21)]

Total System = System $\otimes$ Environment

- System observables are calculated by reduced density matrix $\rho_S(t) = \mathrm{Tr}_E \rho_{\mathrm{tot}}(t)$
- Markovian evolution of the reduced density matrix [Gorini-Kosakowski-Sudarshan (76), Lindblad (76)]

$$\frac{d\rho_S}{dt} = -i[H'_S, \rho_S] + \sum_k L_k \rho_S L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_S - \frac{1}{2} \rho_S L_k^\dagger L_k$$

$$= -iH_{\mathrm{eff}}\rho_S + i\rho_S H_{\mathrm{eff}}^\dagger + \underbrace{\sum_k L_k \rho_S L_k^\dagger}_{\text{quantum jump}}, \qquad \underbrace{H_{\mathrm{eff}} = H'_S - \frac{i}{2}\sum_k L_k^\dagger L_k}_{\text{contains complex potential } V_{\mathrm{Im}}(r)}$$

$$\leftrightarrow \rho_S(t) > 0, \quad \mathrm{Tr}_S \rho_S(t) = 1$$

Remarks on Lindblad equation

- Uncorrelated initial state is assumed $\rho_{\mathrm{tot}}(0) = \rho_S \otimes \rho_E \to$ OK for hard production
- Microscopic derivation assumes system-environment coupling is weak $\to$ not necessarily ...

Quarkonium in weakly coupled QGP [Akamatsu (15)]

Non-relativistic quantum mechanical Hamiltonian from NRQCD

$$H_I = g\left[A_0^a(\boldsymbol{x})(t^a \otimes 1) - A_0^a(\boldsymbol{x}_c)(1 \otimes t^{a*})\right] = \int_{\boldsymbol{k}} \underbrace{\left[e^{i\boldsymbol{k}\cdot\boldsymbol{x}}(t^a \otimes 1) - e^{i\boldsymbol{k}\cdot\boldsymbol{x}_c}(1 \otimes t^{a*})\right]}_{\propto L_{\boldsymbol{k}}^a} \otimes \underbrace{gA_0^a(\boldsymbol{k})}_{\sim \sqrt{\gamma_k}}$$

Lindblad operator in recoilless (quasi-static) limit

- Leading order perturbation at $r \sim 1/gT$

$$L_{\boldsymbol{k}}^a = \sqrt{\gamma_k}\Big[\underbrace{e^{i\boldsymbol{k}\cdot\boldsymbol{x}}(t^a \otimes 1)}_{\text{scattering with } Q} - \underbrace{e^{i\boldsymbol{k}\cdot\boldsymbol{x}_c}(1 \otimes t^{a*})}_{\text{scattering with } Q_c}\Big] + \underbrace{\mathcal{O}(\dot{\boldsymbol{x}}, \dot{\boldsymbol{x}}_c)}_{\text{recoil}\sim\text{dissipation}}$$

- Strength of the coupling γ_k from environment correlator

$$\gamma_k \delta^{ab}\delta(\boldsymbol{k} - \boldsymbol{k}') = g^2 \int_t \langle A_0^a(\boldsymbol{k}, t) A_0^b(-\boldsymbol{k}', 0)\rangle_{T,\mathrm{HTL}}$$

Valid when $g \ll 1$ and $r \sim 1/gT \rightarrow V_{\mathrm{Im}}(r) = -\frac{1}{2}\int_{\boldsymbol{k}} L_{\boldsymbol{k}}^{a\dagger} L_{\boldsymbol{k}}^a$ reproduces the previous result

Quarkonium in the dipole limit [Brambilla et al (17, 18, 20, 22)] **[Ajaharul's talk for QTraj simulation]**

Non-relativistic quantum mechanical Hamiltonian from pNRQCD (after "field redefinitions")

$$H_I = -r_i \Big[\underbrace{\sqrt{\frac{1}{2N_c}}\,(|a\rangle\langle s| + |s\rangle\langle a|)}_{\text{singlet} \leftrightarrow \text{octet}} + \underbrace{\frac{1}{2} d^{abc} |b\rangle\langle c|}_{\text{octet} \leftrightarrow \text{octet}} \Big] \otimes g E_i^a(\boldsymbol{R}) \underbrace{+ (T_A^a)_{bc} |b\rangle\langle c| \otimes g A_0^a(\boldsymbol{R})}_{\text{removed in "field redefinition"}}$$

Lindblad operator in recoilless (quasi-static) limit

- Leading order in dipole size r (non-perturbative in g)

$$L_i^a = \sqrt{\gamma}\, r_i \Big[\sqrt{\frac{1}{2N_c}}\,(|a\rangle\langle s| + |s\rangle\langle a|) + \frac{1}{2} d^{abc} |b\rangle\langle c| \Big] + \underbrace{\mathcal{O}(\dot{r})}_{\text{recoil} \sim \text{dissipation}}$$

$\downarrow$ octet projection $\mathrm{Tr}_{\mathrm{octet}}$

$$L_i^{os} = \sqrt{\frac{\gamma_{os}}{2N_c}}\, r_i |o\rangle\langle s|, \quad L_i^{so} = \sqrt{\frac{\gamma_{so}}{2N_c}}\, r_i |s\rangle\langle o|, \quad L_i^{oo} = \sqrt{\frac{\gamma_{oo}}{4}}\, r_i |o\rangle\langle o|,$$

Valid when quarkonium is small $\rightarrow$ coefficients defined nonperturbatively

Transport coefficients for dipoles: singlet/octet transition rates

Field redefinition = Setting the octet basis at infinite past

$\gamma_{os/so}$ is E-field correlator connected by an adjoint Wilson line [Yao (22)]

[Saga's talk for perturbative calculation, Di-Lun's talk for magnetic correlation]

$$\gamma_{os} = \frac{g^2}{3(N_c^2-1)} \int_t \langle E_i^a(t)\, \underbrace{U_{\mathrm{A}}^{ab}(t,0)}_{\text{adjoint}}\, E_i^b(0)\rangle_T = (N_c^2-1)\gamma_{so}$$

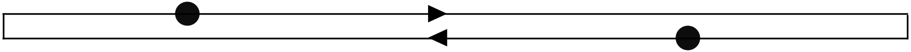
Adjoint Wilson line

- Different from heavy quark momentum diffusion constant

  [CasalderreySolana-Teaney (06), CaronHuot-Moore (08)]

$$\kappa = \frac{g^2}{3N_c} \int_t \langle \mathrm{Tr}\, \underbrace{U_{\mathrm{F}}(-\infty,t)}_{\text{fund.}}\, E_i(t)\, U_{\mathrm{F}}(t,0) E_i(0)\, U_{\mathrm{F}}(0,-\infty)\rangle_T$$

Fundamental Wilson line

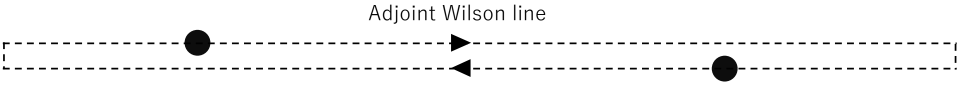


Transport coefficients for dipoles: octet diffusion rate

γ_{oo} is similar to κ, but different from γ_{os}

$$\gamma_{oo} = \frac{g^2}{3(N_c^2-1)} \int_t \langle \mathrm{Tr}\, \underbrace{U_{\mathrm{A}}(-\infty, t)}_{\text{adjoint}} \mathcal{E}_i(t)\, U_{\mathrm{A}}(t,0) \mathcal{E}_i(0)\, U_{\mathrm{A}}(0,-\infty) \rangle_T$$

$$d^{abc} E_i^a =: (\mathcal{E}_i)_{bc}$$

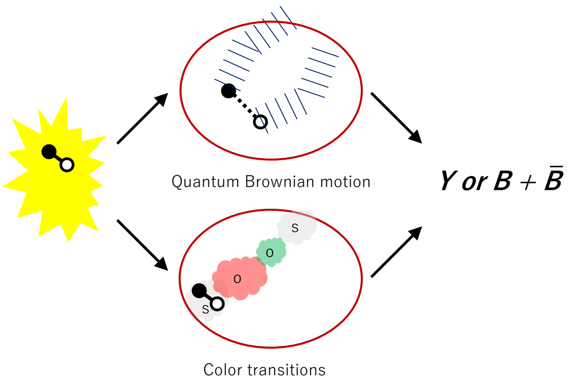


- Recall the interaction Hamiltonian

$$H_I^{oo} = -\frac{1}{2} \boldsymbol{r} \cdot \underbrace{g \boldsymbol{E}^a(\boldsymbol{R}) d^{abc} |b\rangle\langle c|}_{\text{rotation of adjoint color}}, \quad \text{c.f.} \quad H_I^Q = -\boldsymbol{r} \cdot \underbrace{g \boldsymbol{E}^a(\boldsymbol{x}) (t^a)_{ij} |i\rangle\langle j|}_{\text{rotation of fund. color}}$$

Two transport coefficients $\gamma_{so} (\propto \gamma_{os})$ and γ_{oo} in the Lindblad equation

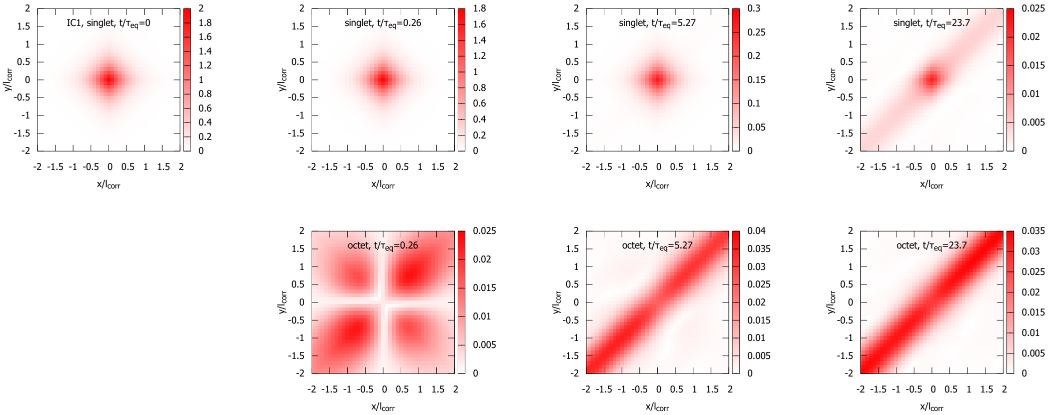
Quantum Brownian motion and color transitions of quarkonium



Reduced density matrix $|\rho_S(x,y,t)|$ in 1D simulation [Miura et al (22)]

Stochastic unravelling: $\rho_S(x,y,t) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \psi_i(x,t)\psi_i^*(y,t)$

Starting from singlet ground state ∼ a bound state jumps in QGP after short formation time



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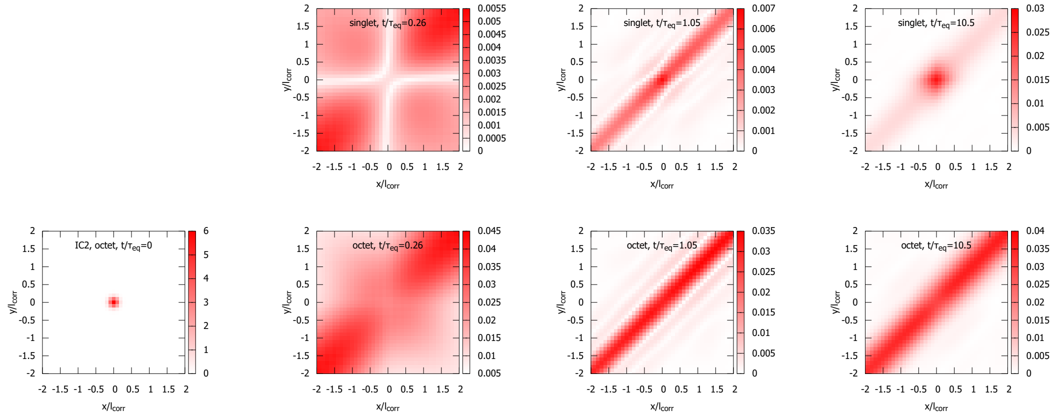
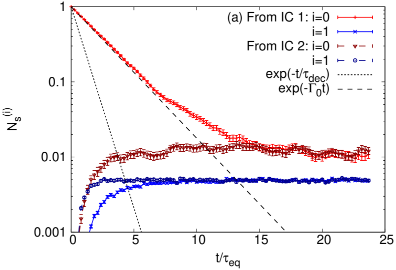
Starting from octet wave packet $\sim$ an octet pair jumps in QGP before forming bound states



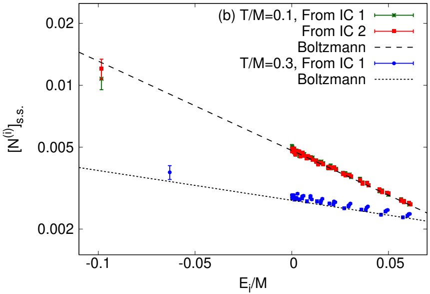
Thermalization of quarkonium, with 1st order recoil [Miura et al (22)]

Evolution of eigenstate occupation



Steady state is independent of initial conditions

Eigenstate occupation in the steady state



Approach the Boltzmann distribution with environment temperatures

Role of recoil in thermalization

1. Interaction Hamiltonian

$$H_I = V_S \otimes V_E$$

2. Lindblad operator with first order recoil

$$L = \sqrt{\gamma}\left(V_S + \frac{i}{4T}\dot{V}_S + \cdots\right) \propto V_S - \frac{1}{4T}[H_S, V_S] + \cdots$$

3. Approximate detailed balance

$$\langle \epsilon_2 | L | \epsilon_1 \rangle \propto \langle \epsilon_2 | V_S | \epsilon_1 \rangle \left(1 - \frac{\epsilon_2 - \epsilon_1}{4T}\right),$$

$$\frac{\Gamma_{1\to 2}}{\Gamma_{2\to 1}} = \frac{|\langle \epsilon_2 | L | \epsilon_1 \rangle|^2}{|\langle \epsilon_1 | L | \epsilon_2 \rangle|^2} = \left(\frac{1 - \frac{\epsilon_2 - \epsilon_1}{4T}}{1 - \frac{\epsilon_1 - \epsilon_2}{4T}}\right)^2 \simeq \exp\left(-\frac{\epsilon_2 - \epsilon_1}{T}\right),$$

$$\because \quad \left(\frac{1 + x/4}{1 - x/4}\right)^2 \simeq 1 + x + \frac{1}{2}x^2 + \underbrace{\frac{3}{16}}_{\simeq 1/6} x^3 + \cdots \simeq e^x$$

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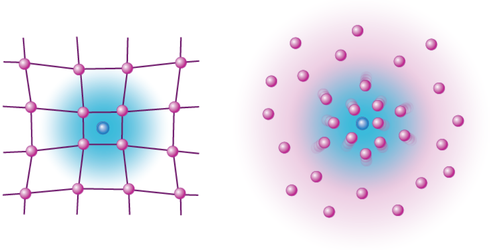
Polarons

- In metals, a conduction electron induces crystal polarization = Polaron [Landau-Pekar (48)]
  - Polaron mass $\gg$ electron mass, e.g. 432 times larger in NaCl
- Polaron broadly means (not necessarily heavy) impurity quasiparticles in cold atomic gas
  - Various mass ratios, e.g. Fermi polaron $^{133}\mathrm{Cs}$ in $^{6}\mathrm{Li}$ gas, Bose polaron $^{40}\mathrm{K}$ in $^{87}\mathrm{Rb}$ gas
  - Tunable coupling $\rightarrow$ attractive polarons, repulsive polarons



Heavy impurities can simulate heavy quark systems in the QGP

Inter-polaron potential in a superfluid

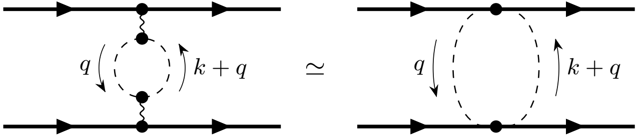
Polarons (Φ) in superfluid phonons (φ)

- Contact interaction nn_Φ, n is conjugate of $U(1)$ phase φ

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{ph}}(\varphi) + \mathcal{L}_{\mathrm{pol}}(\Phi) + g \underbrace{\left[\sqrt{\chi}\partial_t\varphi + \frac{1}{2m}(\boldsymbol{\nabla}\varphi)^2\right]\Phi^\dagger\Phi}_{\text{contact interaction } nn_\Phi},$$

Leading-order potential from two phonon exchange

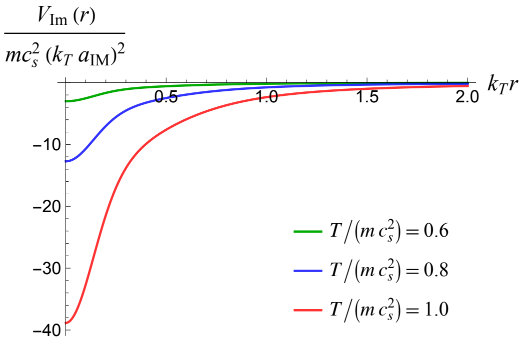
- Real part of the potential $\propto T/r^6$ [Fujii-Hongo-Enss (22)] $\leftarrow$ massless phonons
- Imaginary part of the potential[1] $\propto T/r^2$ [Akamatsu-Endo-Fujii-Hongo (24)] $\leftarrow$ why?

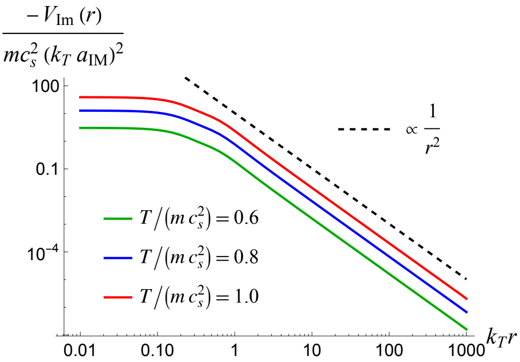


[1] after subtracting $r \to \infty$ value

Universal imaginary potential [Akamatsu-Endo-Fujii-Hongo (24)]

Polarons in a superfluid, $V_{\rm Im}(r) \propto -1/r^2$ at large distance

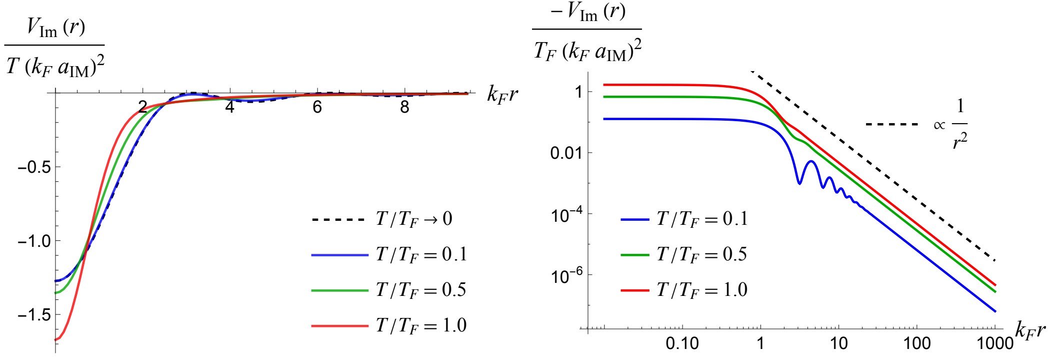




Due to massless nature of phonons?

Universal imaginary potential [Akamatsu-Endo-Fujii-Hongo (24)]

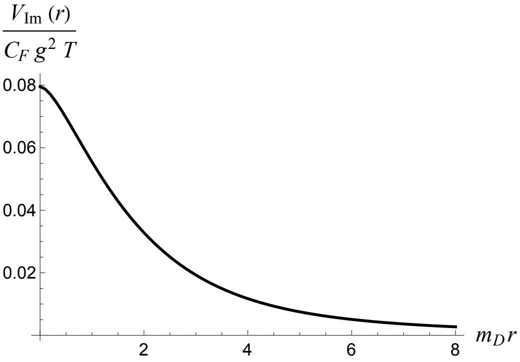
Polarons in a Fermi gas [Sighinolfi et al (22)], $V_{\mathrm{Im}}(r) \propto -1/r^2$ at large distance

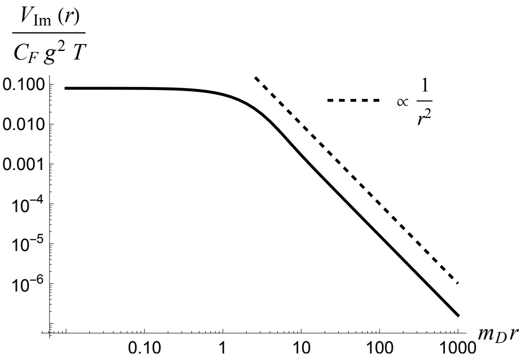


Due to zero-energy excitation of a particle-hole pair?

Universal imaginary potential [Akamatsu-Endo-Fujii-Hongo (24)]

Quarkonium in QGP [Laine+ (07), Beraudo+ (08), Brambilla+ (08)], $V_{\rm Im}(r) \propto +1/r^2$ at large distance

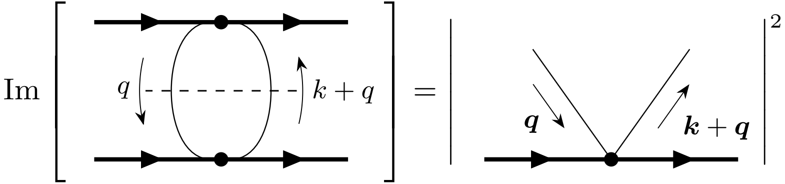




Gluons are massive due to screening → no massless excitations

Physics behind the universal imaginary potential at long distances

Common properties: 2-body collisions



$$\widetilde{V}_{\mathrm{Im}}(\boldsymbol{k}) \propto -\int_{\boldsymbol{q}} |\mathcal{M}_{\boldsymbol{k}+\boldsymbol{q},\boldsymbol{q}}|^2 \underbrace{\delta(E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{q}})}_{\text{instantaneous pot.}} n(E_{\boldsymbol{q}})\big[1 \pm n(E_{\boldsymbol{k}+\boldsymbol{q}})\big],$$

Long distance limit ($k \to 0$)

- Delta function: $\delta(E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{q}}) = \delta(\cos\theta_{\boldsymbol{q}})/v_q k$
- The other parts approach constant $\neq 0$
- In total, $V_{\mathrm{Im}}(\boldsymbol{k}) \propto 1/k \;\to\; V_{\mathrm{Im}}(\boldsymbol{r}) \propto 1/r^2$

Universal imaginary potential $1/r^2$ in the collisional regime

Contents

Summary

- OQS description for quarkonium has been developed
  - Thermalization achieved by including the first recoil effect
  - Application to heavy-ion collisions, different models compared in [Andronic et al (24)]
  - Initial condition in heavy-ion collisions unknown (octet dominant?)
- Challenges:
  - Restricted validity
  - Non-Markovian effects
  - Open many-body systems
- Application to polarons in cold atomic gas
  - Universality $V_{\rm Im}(r) \propto 1/r^2$ in the collisional regime
  - Any interesting questions unique to the cold atomic context?

Can we witness color fields inside QGP by the eyes of octet quarkonium?

Appendix

Field redefinition as a unitary transformation

1. Interaction Hamiltonian in the interaction picture

$$H_I = \cdots + (T_A^a)_{bc}|b\rangle\langle c| \otimes gA_0^a(\boldsymbol{R}, t)$$

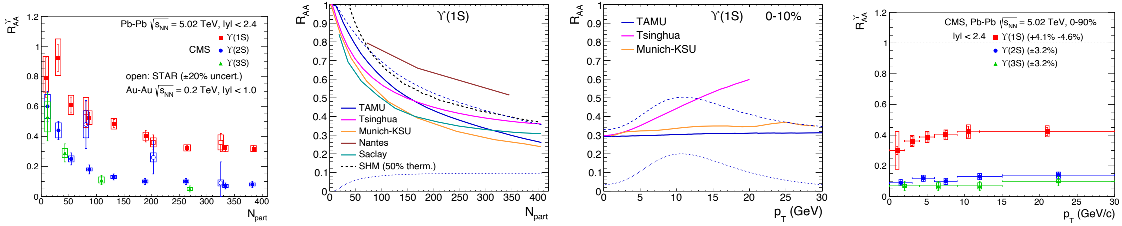
2. Want to eliminate the octet gauge interaction = octet basis set in $t = -\infty$

$$|\Psi'(t)\rangle = U(t)|\Psi(t)\rangle, \quad U(t) = \mathsf{P}\exp\left[-ig\int_t^{-\infty} dt'\, T_A^a \otimes A_0^a(\boldsymbol{R}, t')\right]$$

3. Two-point functions $\langle E(t)E(0)\rangle$ in the $t = -\infty$ basis
4. To get expressions in local octet basis, we need to insert adjoint Wilson lines

Applications in heavy-ion collisions

Phenomenological studies for J/ψ and Υ suppression in heavy-ion collisions [Andronic et al (24)]



- Open system description suitable for Υ (Munich-KSU, Nantes, Saclay)
- For J/ψ, one needs to solve open many-body system of charms