Functional methods for hadron spectroscopy

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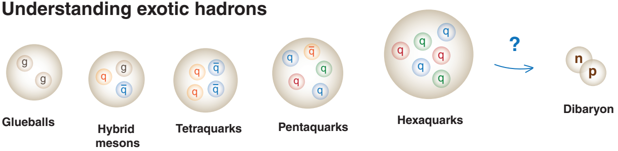
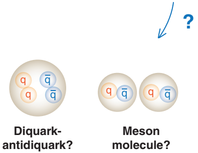
University of Graz

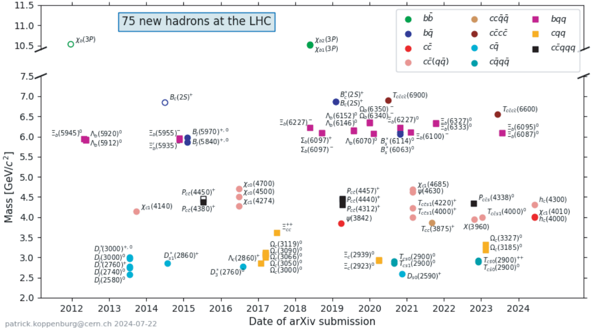
XVIth Quark Confinement and the Hadron Spectrum
Cairns, Australia, August 21, 2024

Motivation

- **Understanding exotic hadrons**







- **New four-quark states:**

  Open-charm $T_{cc}^{+}(3875)$,
  all-charm $T_{c\bar{c}c\bar{c}}(6600)$,
  hidden-charm & open-strange, ...

https://www.nikhef.nl/~pkoppenb/particles.html
https://qwg.ph.nat.tum.de/exoticshub

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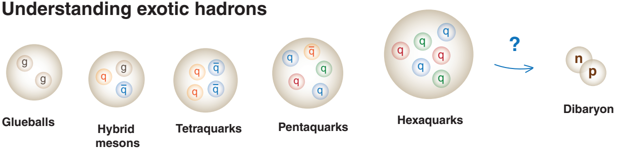
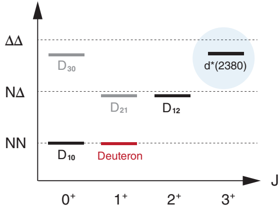
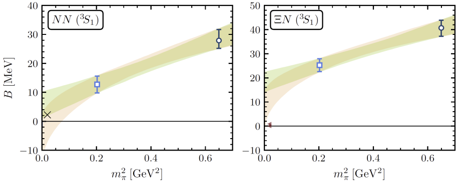


- **Light dibaryons** Dyson, Xuong 1964



Adlarson et al., PRL 106 (2011), PRL 112 (2014), Bashkanov, Brodsky, Clement, PLB 727 (2013), Gal, Garcilazo, PRL 111(2013), . . .

- **Strange dibaryons** NPLQCD, HALQCD, USQCD, PACS-CS, . . .

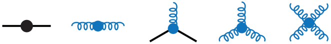


Illa et al., PRD 103 (2021)

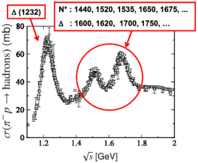
- **Nucleons in nuclei?**
  Short-range correlations, EMC effect?
  CLAS: Duer et al., Nature 560 (2018), Schmidt et al., Nature 578 (2020)

Theory tools

Functional methods
(DSEs & BSEs, FRG, ...)

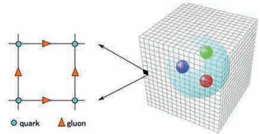


Amplitude analyses

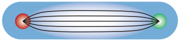


EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

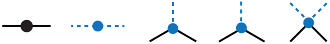
Lattice QCD



Phenomenological models

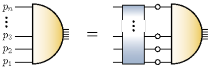


Effective theories
(ChPT, ...)



Functional methods

- Hadronic **bound-state equations** (Bethe-Salpeter & Faddeev eqs)

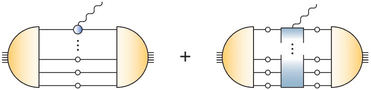


"QFT analogue of Schrödinger eq."

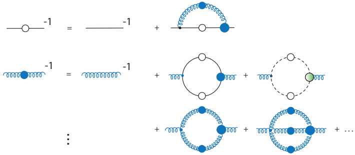
→ hadron masses & "wave functions"

→ **spectroscopy calculations**

- **Structure calculations:** form factors, PDFs, GPDs, TMDs, two-photon processes, ...



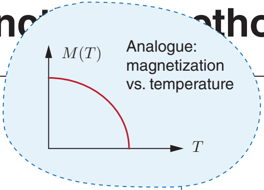
- Ingredients: **QCD's n-point functions,** computed from DSEs (quantum eqs. of motion) or FRG (functional renormalization group)



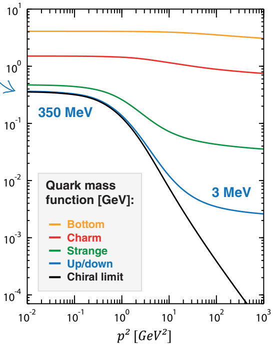
→ Dynamical mass generation, gluon mass gap, confinement, QCD phase diagram, ...

→ see plenary by Fabian Rennecke (Mo)

Func… ethods



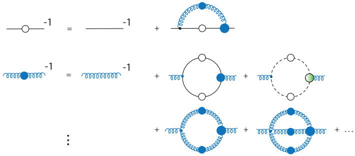
"constituent-quark mass"



current-quark mass

Quark mass

- Ingredients: **QCD's n-point functions,** Satisfy quantum eqs. of motion (DSEs)



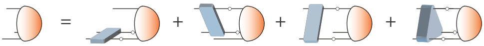
→ Dynamical mass generation, gluon mass gap, confinement, ...

→ see plenary by Craig Roberts (Fri)

Baryons

Three-quark BSE (Faddeev equation) for baryons:

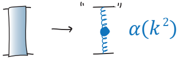
GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)

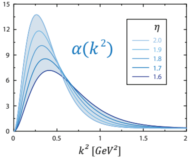


2-body kernel:
fixed in meson sector

3-body kernel:
Leading diagram (3-gluon vertex) vanishes by color trace, higher-order diagrams small (?)
2-quark correlations dominant?

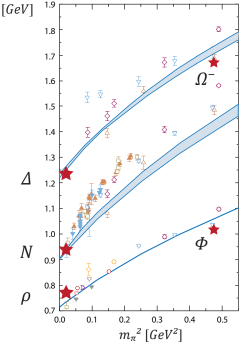
Rainbow-ladder





Scale set by f_π,
shape parameter → bands

Maris, Tandy, PRC 60 (1999)



- Analogous results for many **form factors**

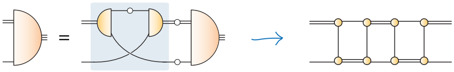
  Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)

- Relativistically, nucleon also has **p waves!**

see also:
Qin, Roberts, Schmidt, PRD 97 (2018)

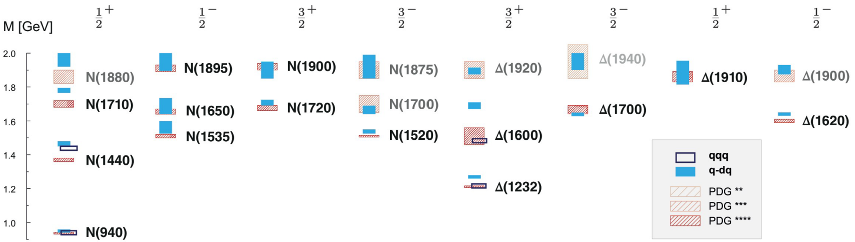
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015)



Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



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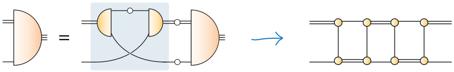
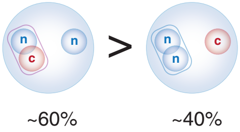


Heavy baryons
Torcato, Arriaga, GE, Peña, FBS 64 (2023)



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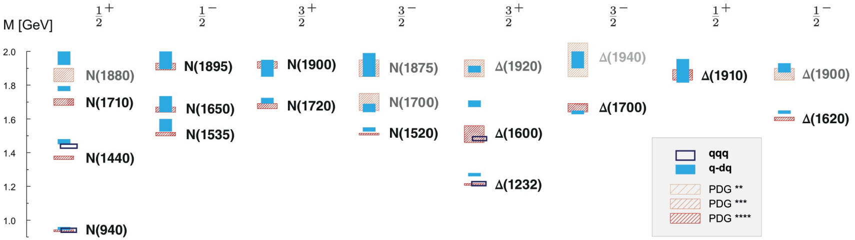
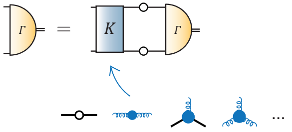


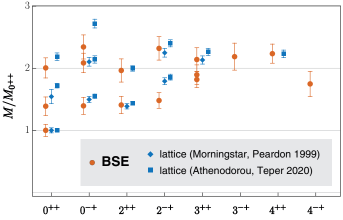
Towards ab-initio

- **Goal:** go towards ab-initio calculations by calculating **higher n-point functions**

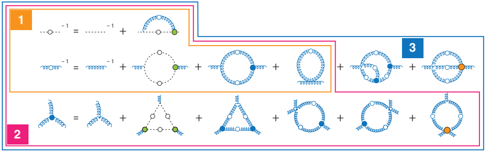


. . . ,
Williams, Fischer, Heupel, PRD 93 (2016),
Cyrol et al., PRD 97 (2018),
Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019),
Aguilar et al., EPJ C 80 (2020),
Huber, PRD 101 (2020),
Qin, Roberts, Chin. Phys. Lett. 38 (2021),
GE, Pawlowski, Silva, PRD 104 (2021),
. . .

- **Glueball spectrum** agrees with lattice QCD
  Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)



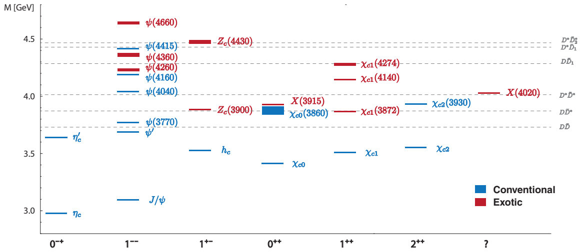
- Coupled **Yang-Mills DSEs**
  Huber, PRD 101 (2020),
  GE, Pawlowski, Silva, PRD 104 (2021)



truncation error:

1 60% 2 10% 3 4%

Exotic mesons



- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Recent additions: all-charm X(6900), open-charm T_{cc}^{+}, ...
- Oldest tetraquark candidates: **light scalar mesons**

Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

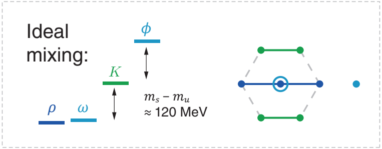
Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

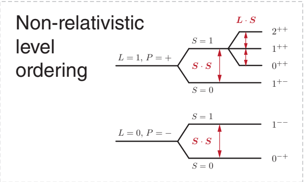
Brambilla, Eidelman, Hanhart et al.,
Phys. Rept. 873 (2020)

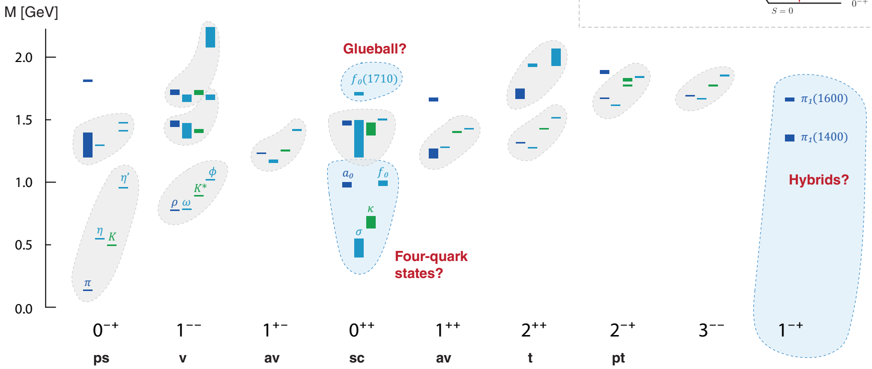
...

Light exotic mesons

Light meson spectrum (PDG 2020)

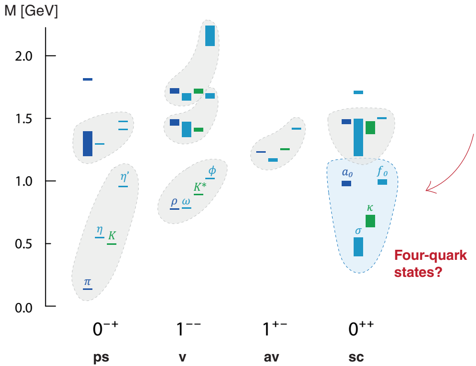




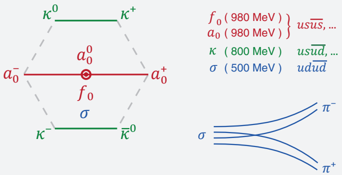


Light exotic mesons

Light meson spectrum
(PDG 2020)



- **Diquark-antidiquark**?
  Explains mass ordering & decay widths
  Jaffe 1977, Close, Tornqvist 2002,
  Maiani, Polosa, Riquer 2004

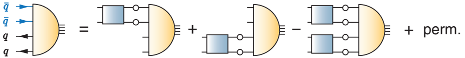


- **Meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-$q\bar{q}$ nature** supported by various approaches
  Pelaez, Phys. Rept. 658 (2016)

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$

$\omega_1 = q \cdot k \qquad \eta_1 = p \cdot P$
$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$
$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$

256 Dirac-Lorentz tensors

2 Color tensors:

$3 \otimes \overline{3}, \quad 6 \otimes \overline{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

$K\,\psi_i = \lambda_i\,\psi_i$

	dim K	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

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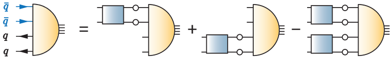

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256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3}, \quad 6\otimes\bar{6}$ or
$1\otimes 1, \quad 8\otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

$f_i\,(\,\mathcal{S}_0\,,$  $)$

Four-quark states

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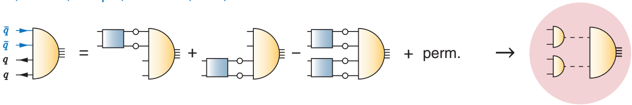
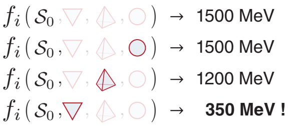
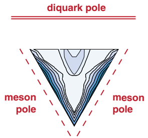
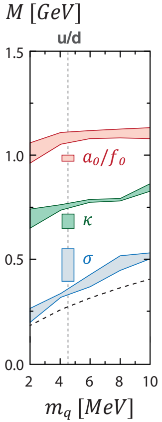


- BSE dynamically generates **meson poles** in BS amplitude:





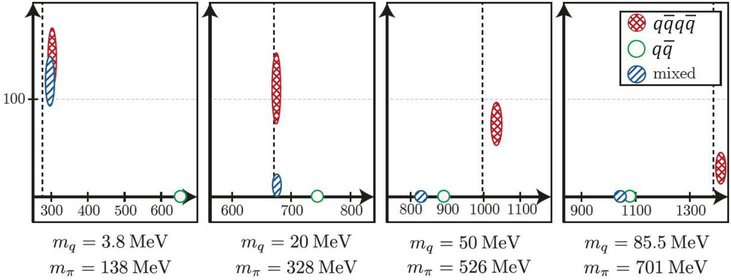


- "Light scalar mesons" look like **meson molecules,**
  diquark-antidiquark components almost negligible.
  Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

Four-quark vs. q$\bar{q}$ dominance

Santowsky, Fischer, PRD 105 (2022)



Light quarks:
$\pi\pi$ dominant

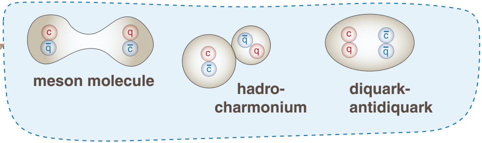
Strange quarks:
$q\bar{q}$ dominant

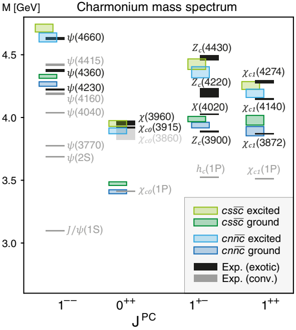
Four-quark states

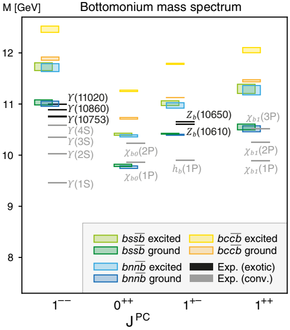
Hidden charm, hidden bottom

Hoffer, GE, Fischer, PRD 109 (2024)

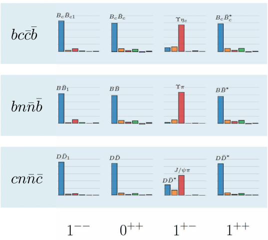
$cq\bar{q}\bar{c}$







Wave-function components:

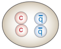


Four-quark states

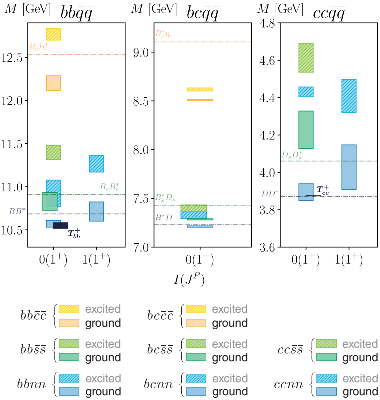
Open-flavor states

Hoffer, GE, Fischer, in preparation

meson molecule



diquark-antidiquark



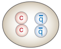
Binding energies (ground states):

Four-quark states

Open-flavor states

Hoffer, GE, Fischer,
in preparation

meson molecule



diquark-antidiquark

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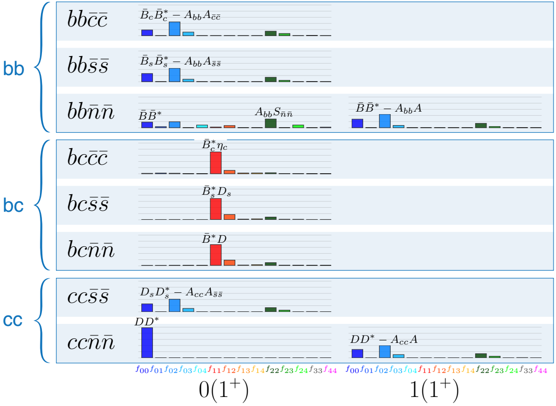


$I(J^P)$	Physical components					
	$\mathbf{1}\otimes\mathbf{1}$		$\bar{\mathbf{3}}\otimes\mathbf{3}$	$\mathbf{8}\otimes\mathbf{8}$		$\bar{\mathbf{6}}\otimes\mathbf{6}$
	f_0	f_1	f_2	f_3	f_4	f_5
$0(1^+)$ $bb\bar{n}\bar{n}$	BB^*	B^*B^*	$A_{bb}S$	BB^*	B^*B^*	$S_{bb}A$
$bc\bar{n}\bar{n}$	BD^*	B^*D	$A_{bc}S$	BD^*	B^*D	$S_{bc}A$
$cc\bar{n}\bar{n}$	DD^*	D^*D^*	$A_{cc}S$	DD^*	D^*D^*	$S_{cc}A$
$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s$	$A_{bc}S_{ss}$
$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)$ $bb\bar{q}\bar{q}$	BB^*	–	$A_{bb}A$	BB^*	–	–
$cc\bar{q}\bar{q}$	DD^*	–	$A_{cc}A$	DD^*	–	–

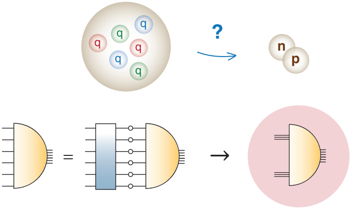
→ T_{bb} is complicated mixture of molecule and diquark-antidiquark

Can **quantify** meson-meson and diquark-antidiquark contributions

→ T_{cc} sits on top of threshold, pure molecule

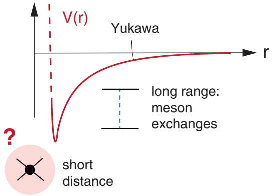
Nucleons in nuclei?

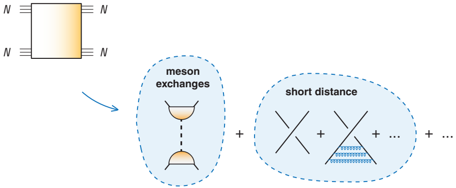
Transition from quarks & gluons to **light nuclei:**



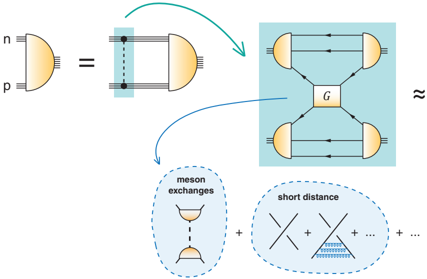
- Relativistic structure of **deuteron?**
- Exotic dibaryons, hypernuclei, short-range correlations, EMC effect ...

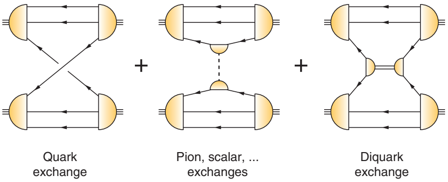
Microscopic origins of **short-range nuclear force?**





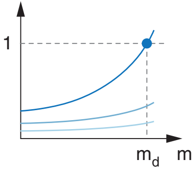
Deuteron



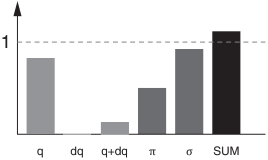


All quantities are renormalized from DSE.
No extra cutoff needed, breakup into quarks automatically induces momentum falloff

Eigenvalue (sketch)



Eigenvalue at $m = m_d$



Diquark exchange is repulsive, almost cancels quark exchange!

preliminary

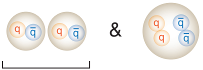
Arriaga, GE, Nunes, Peña, in preparation

OAM composition:

The deuteron is relativistic!

Summary & Outlook

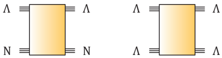
- **Four-quark states:** spectrum & composition



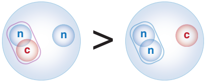
hidden charm

open charm
(unless at threshold)

- **Dibaryons & baryon-baryon interactions:** underway; spectroscopy, form factors, PDFs, scattering amplitudes

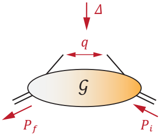


- **Hyperons** and **charmed baryons:** Spectroscopy, form factors, structure

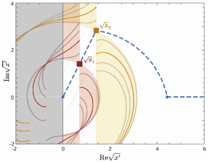


~60% ~40%

- **Hadron structure:** PDFs, GPDs, TMDs



$$\mathcal{G}(z,P,\Delta) = \langle P_f |\, \mathsf{T}\, \Phi(z)\, \mathcal{O}\, \Phi(0)\, | P_i \rangle$$



New method to compute light-front wave functions via contour deformations

Editors' Suggestion:
GE, Ferreira, Stadler, PRD 105 (2022)

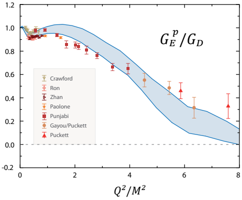
Thank you!

Backup slides

Baryon structure

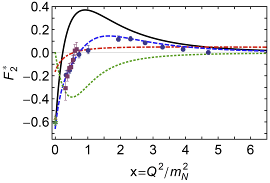
Nucleon electromagnetic FFs

GE, PRD 84 (2011)



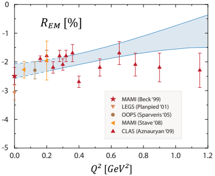
Roper em. transition FFs

Segovia et al., PRL 115 (2015)



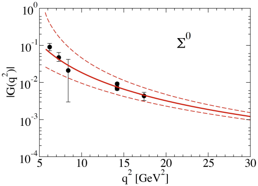
Δ em. transition FFs

GE, Nicmorus, PRD 85 (2012)



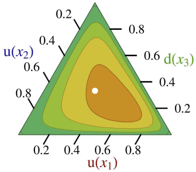
Timelike em. strangeness FFs

Ramalho, Peña, PRD 101 (2020)



Distribution amplitudes

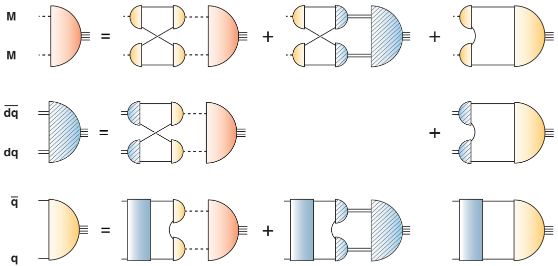
Mezrag, Segovia, Chang, Roberts, PLB 783 (2018)



Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark,**
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)

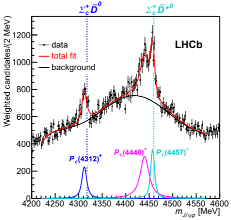


Include **mixing with $q\bar{q}$:**
$\pi\pi$ still dominant

Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)

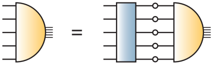
[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Pentaquarks?

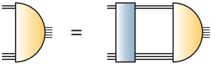


Aaij et al., PRL 112 (2019)

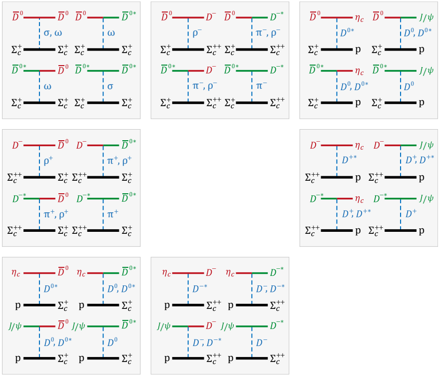
- **5-body equation:** in progress
  GE, Peña, Torres, in preparation



- Meson-baryon equation with hadronic exchanges
  GE, Lourenco, Peña, Stadler, Torres, in preparation

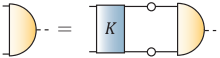


... all couplings calculated dynamically



Diquark correlations

Mesons and diquarks closely related through BSE
Maris, FBS 32 (2002)

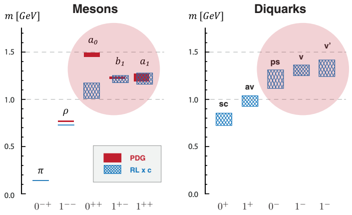


Lowest-lying diquarks are dominant for ground-state octet & decuplet baryons

pseudoscalar mesons	⇔	**scalar diquarks** (~0.8 GeV)
vector mesons	⇔	**axialvector diquarks** (~1 GeV)

Higher-lying diquarks are subleading, but contribute to excited states & remaining channels

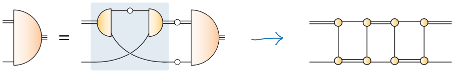
scalar mesons	⇔	**pseudoscalar diquarks** (~1.2 GeV)
axialvector mesons	⇔	**vector diquarks** (~1.3 GeV)



In RL, these are too strongly bound; simulate beyond-RL effects by (one) strength parameter c
Roberts, Chang, Cloet, Roberts, FBS 51 (2011)
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

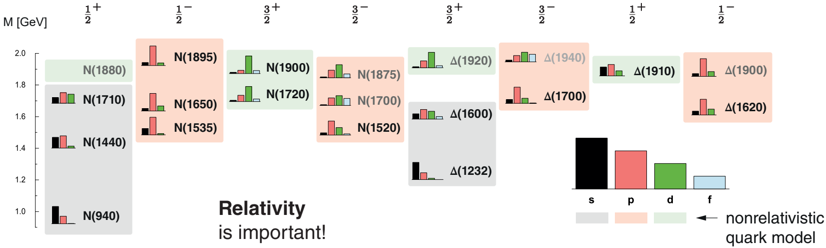
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)



Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), GE, FBS 63 (2022)

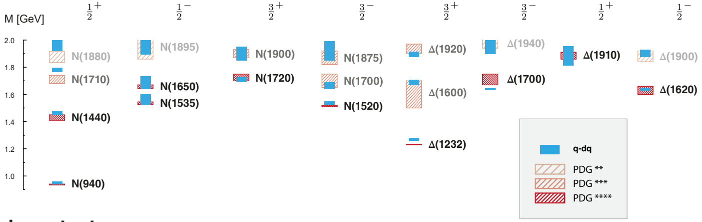


Relativity
is important!

Diquark correlations

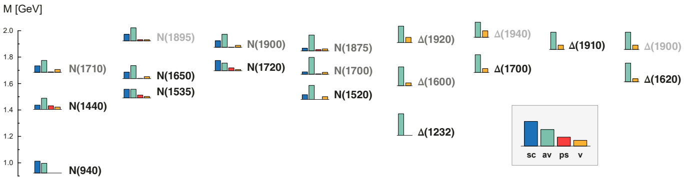
Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquark content:

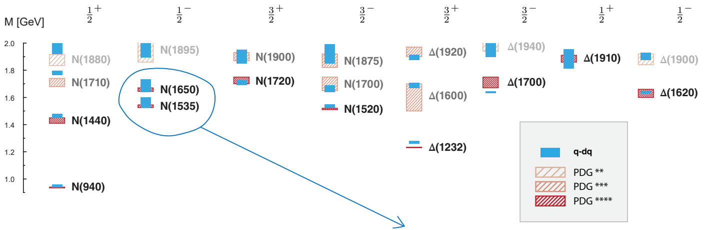
Barabanov et al., PPNP 116 (2021)



Diquark correlations

Light baryon spectrum

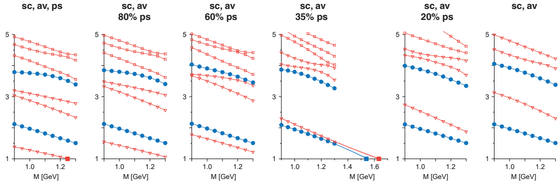
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



RL, all diquarks:
"N(1535)" too low

"Beyond RL":
N(1535), N(1650)

RL, sc+av only:
"N(1650)" too high

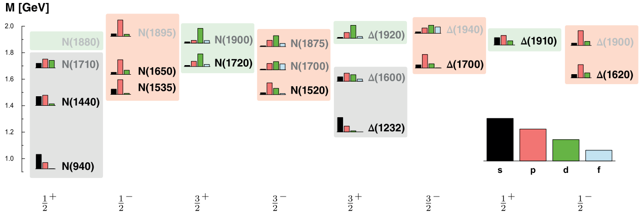


- **Level ordering** determined by diquark dynamics
- Diquarks are not pointlike, also here **rich spectrum!**

Barabanov et al., PPNP 116 (2021)

Relativistic effects

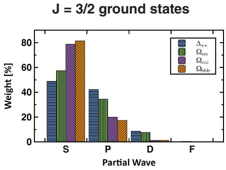
Orbital angular momentum: clear traces of nonrelativistic quark model, but strong relativistic effects (in some cases even dominant)

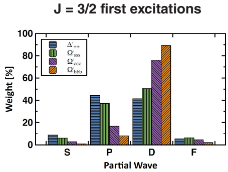


GE, FBS 58 (2017)

Relativistic contributions
even up to bottom baryons!

Qin, Roberts, Schmidt, PRD 97 (2018)





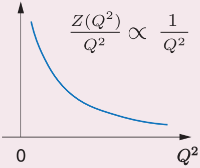
Towards ab-initio

Gluon propagator:

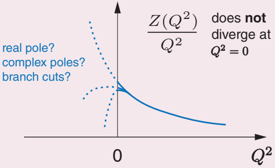
transverse dressing — longitudinal dressing = 1

$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2}\left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) + \xi\,\frac{L(Q^2)}{Q^2}\,\frac{Q^\mu Q^\nu}{Q^2}$$

- **Perturbation theory:** Massless gluon pole



- **Nonperturbative calculations:** Massless pole disappears!



Family of **"decoupling"** solutions, also seen in lattice QCD

Cucchieri, Maas, Mendes, PRD 77 (2008)
Boucaud et al., JHEP 06 (2008)
Bogolubsky et al., PLB 676 (2009)
Fischer, Maas, Pawlowski, Ann. Phys. 324 (2009)
Duarte, Oliveira, Silva, PRD 94 (2016)
Aguilar et al., EPJ C 80 (2020)

Endpoint is **"scaling"** solution, confinement manifest

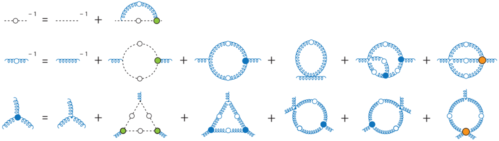
Lerche, Smekal, PRD 65 (2002)
Fischer, Alkofer, PLB 536 (2002)
Alkofer, Fischer, Llanes-Estrada, MPLA 23 (2008)

All solutions show **gluon mass gap**

$$\lim_{r\to\infty}\int\frac{d^3Q}{(2\pi)^3}\,\frac{Z(\boldsymbol{Q}^2)}{\boldsymbol{Q}^2}\,e^{i\,\boldsymbol{x}\cdot\boldsymbol{Q}} \propto e^{-m_{\text{gap}}\,r}$$

Coupled **Yang-Mills DSEs**
GE, Pawlowski, Silva, PRD 104 (2021)

→ Test confinement in hadron observables!

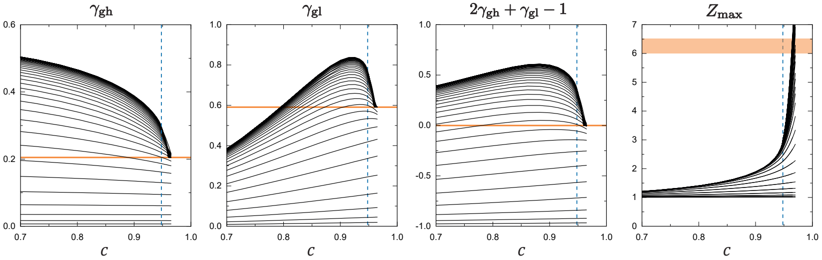


Truncation error

- Set $Z_{3g} \to c\, Z_{3g}$ ... quantifies deviation from STI (without truncation: $c = 1$),
  same effect from “over-renormalizing” 3-gluon vertex
- YM system only converges up to $c_{\mathrm{max}} < 1$
- Anomalous dimensions reproduced for
  - 1 c ~ 0.4
  - 2 c ~ 0.9
  - 3 c ~ 0.96

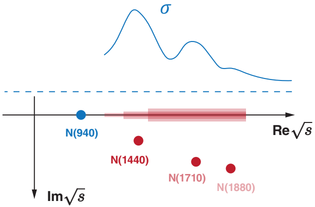
  ⇒ identifies “physical point” for each truncation

  GE, Pawlowski, Silva, PRD 104 (2021)



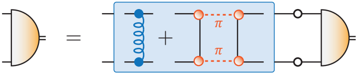
Resonances

- Most hadrons are **resonances** and decay $\Leftrightarrow$ poles in complex momentum plane



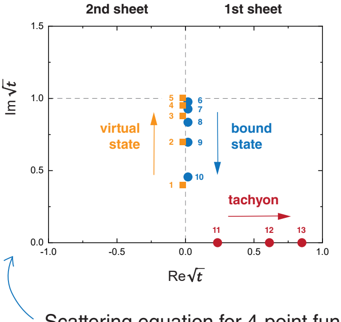
- BSE kernel must include decay channels: ρ meson becomes resonance

Williams, PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019),
Santowsky, GE, Fischer, Wallbott, PRD 102 (2020),
Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

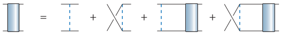


- **Contour deformations** as tool to go beyond thresholds

GE, Duarte, Peña, Stadler, PRD 100 (2019)

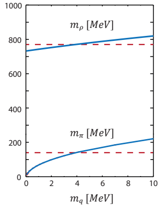


Scattering equation for 4-point function

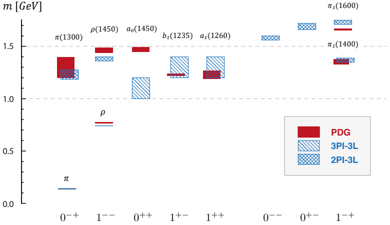


Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

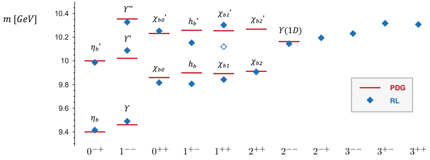


Williams, Fischer, Heupel, PRD 93 (2016)

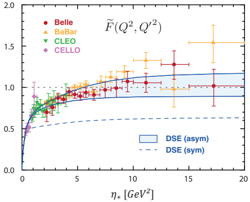
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)

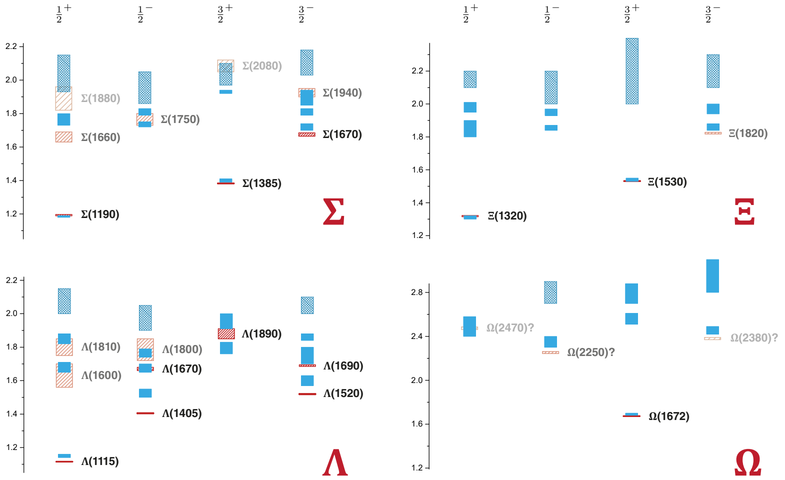


- **Pion transition form factor**



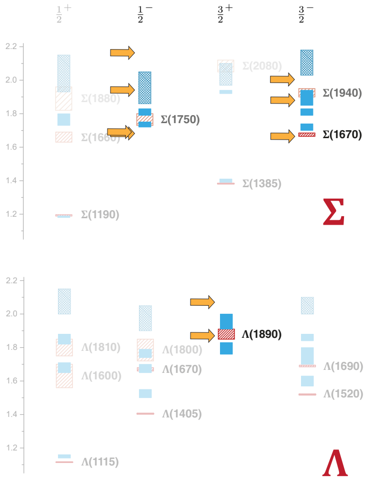
GE, Fischer, Weil, Williams, PLB 774 (2017)

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]

GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017