Spectral properties of bottomonium at high temperature:

a systematic investigation

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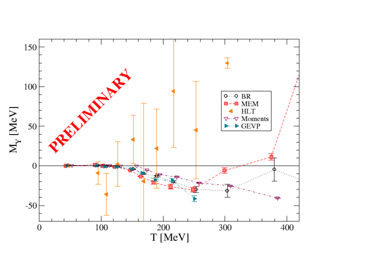
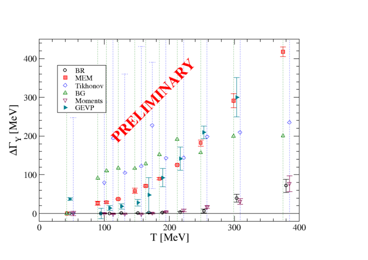
National University of Ireland Maynooth
FASTSUM collaboration

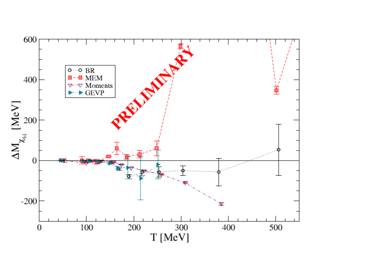
Quark Confinement and the Hadron Spectrum, Cairns, 19–24 August 2024

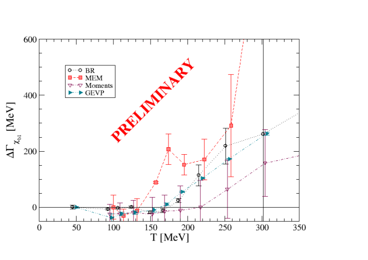
Outline

Summary of results

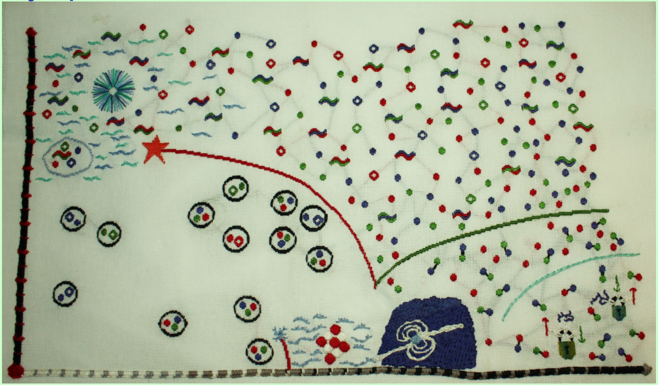








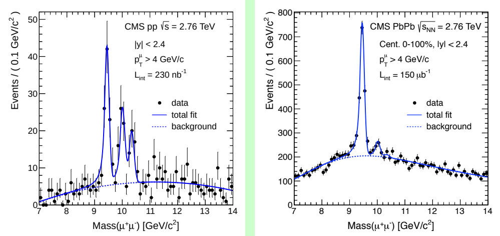
Heavy quarks in the QGP



- Heavy quarks are important probes of medium
- Long history of $c\bar{c}$ studies: experiment, pheno, lattice
- Sequential $b\bar{b}$ suppression observed, numerous studies

Beauty

- Bound states expected to survive up to $T_d^{\Upsilon} \sim 3 - 5\,T_c$
- $\chi_b, \Upsilon(2S)$ melt at $T_d' \lesssim 1.2\,T_c$?
- Sequential suppression observed at CMS, ATLAS, STAR
- Detailed information on mass shifts and widths required for dynamical modelling



Spectral functions

- contain information about the fate of hadrons in the medium
  - stable states $\rho(\omega) \sim \delta(\omega - m)$
  - resonances or thermal width $\rho(\omega) \sim$ lorentzian
  - continuum above threshold

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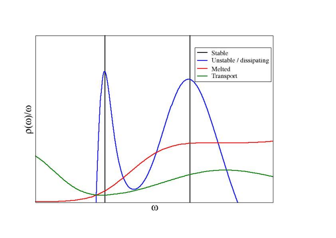

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$$G_\Gamma(\tau, \vec{p}) = \int \rho_\Gamma(\omega, \vec{p}) K(\tau, \omega) d\omega \,,$$

- an ill-posed problem
  - Direct correlator analysis (model driven)
  - Smeared spectral functions
  - Bayesian methods
  - Other methods are available: ML, Cuniberti, Schlessinger, ...

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Here: Focus on mass and width

Dynamical anisotropic lattices

- A large number of points in time direction required to extract spectral information
- For $T = 2T_c$, $\mathcal{O}(10)$ points $\Longrightarrow a_t \sim 0.025$ fm
- Far too expensive with isotropic lattices $a_s = a_t$!
- Fixed-scale approach
  - vary T by varying N_τ (not a)
  - need only 1 $T = 0$ calculation for renormalisation
  - independent handle on temperature

- Introduces 2 additional parameters
- Non-trivial tuning problem
  [PRD **74** 014505 (2006); HadSpec Collab, PRD **79** 034502 (2009)]

Simulation parameters

FASTSUM Gen2L ensemble: $N_f = 2 + 1$ anisotropic clover
[HadSpec, PRD **79** 034502 (2009); FASTSUM, PRD **105** 034504 (2022)]

a_s (fm)	0.112
a_τ (fm)	0.032
ξ	3.45
a_τ^{-1} (GeV)	6.08
m_π (MeV)	239
N_s	32
L_s (fm)	3.6

N_τ	T (MeV)	T/T_c	N_{cfg}
128	47	0.28	1000
64	95	0.57	1000
56	109	0.65	1000
48	127	0.76	1000
40	152	0.91	1000
36	169	1.01	1000
32	190	1.14	1000
28	217	1.30	1000
24	253	1.52	1000
20	304	1.82	1000
16	380	2.28	1000
12	507	3.03	1000
8	760	4.55	1000

NRQCD

Scale separation $M_Q \gg T, M_Q v$

Integrate out hard scales $\longrightarrow$ Effective theory

Expand in orders of heavy quark velocity $\mathbf{v}$; we use $\mathcal{O}(\mathbf{v}^4)$ action

Advantages

- Simple (T-independent) kernel, $G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$
- No zero-modes
- Longer euclidean time range, $\tau_{\text{max}} \approx 1/T$
- High-precision correlators feasible

Disadvantages

- Not renormalisable, requires $Ma_s \gtrsim 1$
- Does not incorporate transport properties
- Energy shift: only energy differences are physical

Time-derivative moments

Basic idea

For a peaked spectral function $\rho(\omega)$ describing a (quasi)particle with mass M and width ω,

$$M = \langle \omega \rangle = \int \omega \rho(\omega) d\omega \,, \quad \Gamma = \mathrm{Var}\omega = \left\langle \omega^2 \right\rangle - \langle \omega \rangle^2$$

In NRQCD this is equivalent to taking time derivatives of the correlator,

$$G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$$
$$G'(\tau) = -\int \omega \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$$
$$G''(\tau) = \int \omega^2 \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$$

Gaussian moments

Assume $\rho(\omega)$ can be approximated by a sum of Gaussians,

$$\rho(\omega; T) = \sum_{i=0}^{\infty} A_i e^{-\frac{(\omega - m_i)^2}{2\Gamma_i^2}},$$

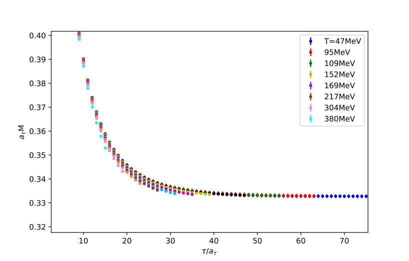
$$\implies \quad G(\tau) = \sum_{i=0}^{\infty} A_i e^{-m_i\tau + \Gamma_i^2\tau^2/2}$$

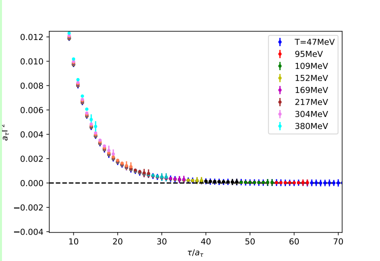
$$= A_0 e^{-m_0\tau + \Gamma_0^2\tau^2/2}\left(1 + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}\right)$$

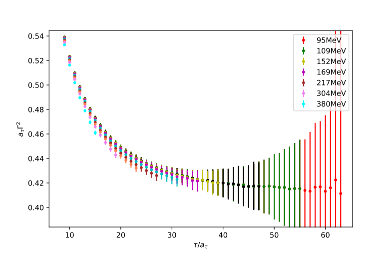
$$\frac{G'(\tau)}{G(\tau)} = \frac{d\log(G(\tau))}{d\tau} = (-m_0 + \Gamma_0^2\tau) + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

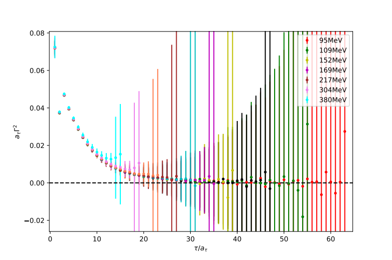
$$\frac{d^2\log(G(\tau))}{d\tau} = \Gamma_0^2 + \sum_{i=1}^{\infty} B_i e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

Moments results



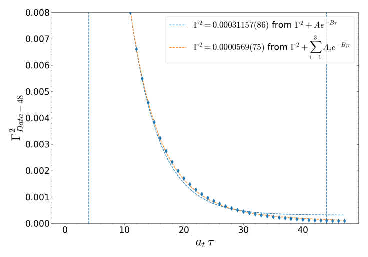


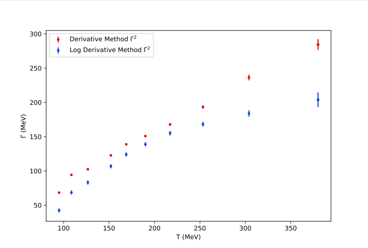




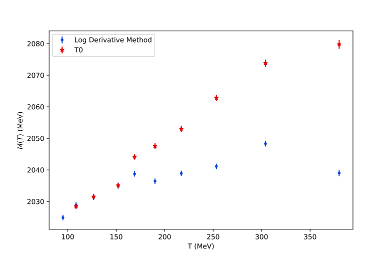
Moments results: systematics

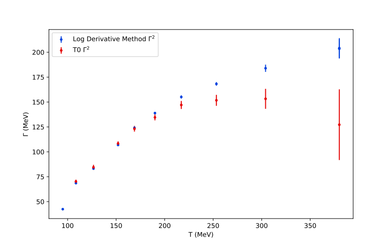
- Effect of higher excited states
- Only single excited state used in analysis
- Derivative $\frac{G''}{G} - \left(\frac{G'}{G}\right)^2$ vs log-derivative $\frac{d^2 \log G}{d\tau}^2$
- Included in systematic error

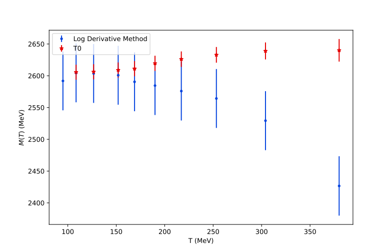


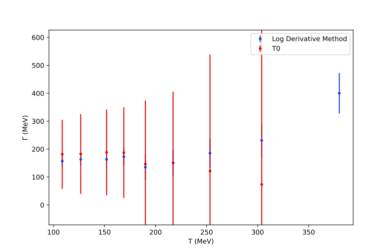


Moments results: disentangling thermal effects









Generalised eigenvalue problem

Correlator matrix

$$G_{ij}(\tau) = \langle\Omega|\mathcal{O}_i\mathcal{O}_j^\dagger|\Omega\rangle = \sum_\alpha \frac{Z_i^\alpha Z_j^{\alpha\,\dagger}}{2E_\alpha} e^{-E_\alpha\tau}$$

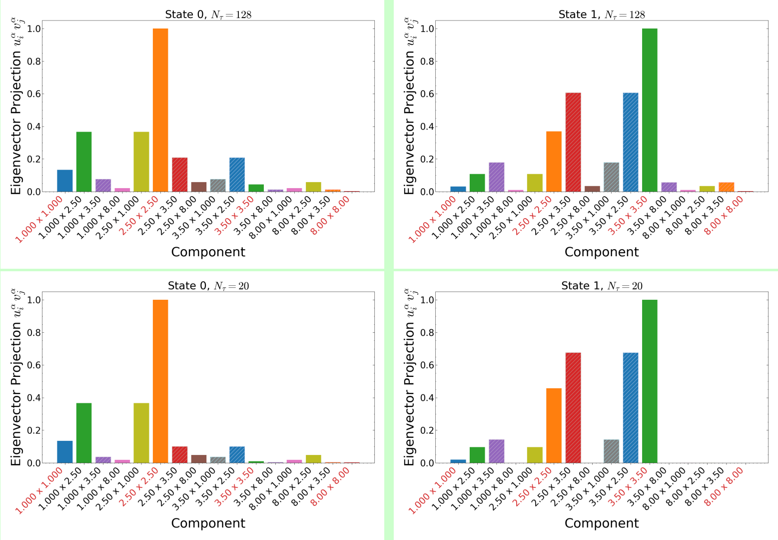
Generalised eigenvalue problem

$$G_{ij}(\tau_0+\delta\tau)u_j^\alpha = e^{-E_\alpha\delta\tau}G_{ij}(\tau_0)u_j^\alpha$$
$$v_i^\alpha G_{ij}(\tau_0+\delta\tau) = e^{-E_\alpha\delta\tau}v_i^\alpha G_{ij}(\tau_0)$$

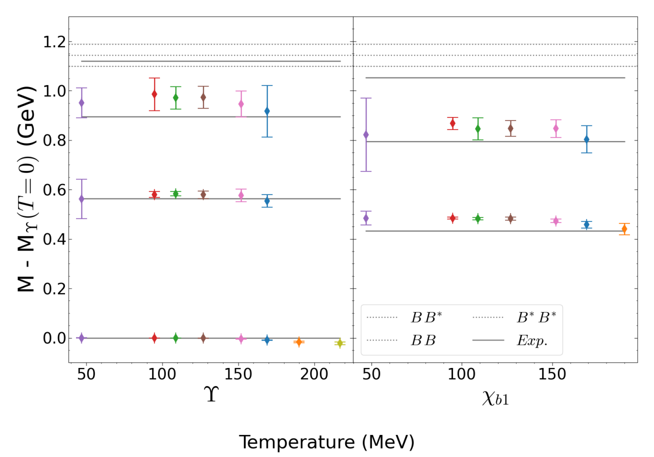
Eigenstate-projected correlator

$$G_\alpha(\tau) = v_i^\alpha G_{ij}(\tau)u_j^\alpha$$

GEVP results: eigenvector contribution

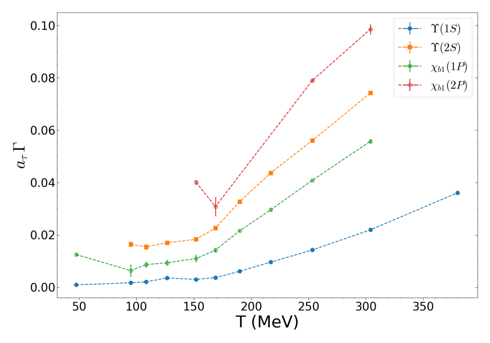


GEVP results: masses



GEVP results: widths

- Apply moments method to projected correlators
- Asymptotic term in fits stable even with noisy data



Linear methods: smeared spectral functions

Introduce smearing function

$$\overline{\Delta}(\omega,\omega_n) = \sum_{\tau=0}^{\tau_{\max}} g_\tau(\omega_n) e^{-\omega\tau} \xrightarrow{N_\tau\to\infty} \delta(\omega-\omega_n)$$

Smeared spectral function is reconstructed as

$$\hat{\rho}(\omega_n) = \sum_{\tau=0}^{\tau_{\max}} g_\tau(\omega_n) e^{\omega\tau} = \int_{\omega_{\min}}^{\infty} d\omega \rho(\omega) g_\tau(\omega_n) e^{-\omega\tau}$$

$$= \int_{\omega_{\min}}^{\infty} d\omega \overline{\Delta}(\omega,\omega_n)\rho(\omega)$$

Determine coefficients g_τ by minimising functional

$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega F[\omega,\omega_n;\overline{\Delta}]$$

A tale of three methods

Coefficients g_τ will become exponentially large and oscillating.

Regularise by minimising functional $W[g_\tau] = A[g_\tau] + \lambda B[g_\tau]$

Tikhonov

$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega(\omega - \omega_n)^2 [\overline{\Delta}(\omega, \omega_n)]^2 \,, \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} I(\tau_1, \tau_2)$$

Backus–Gilbert

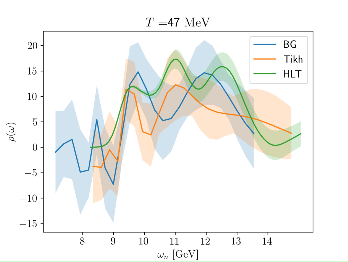
$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega(\omega - \omega_n)^2 [\overline{\Delta}(\omega, \omega_n)]^2 \,, \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} \, \mathrm{Cov}(\tau_1, \tau_2)$$

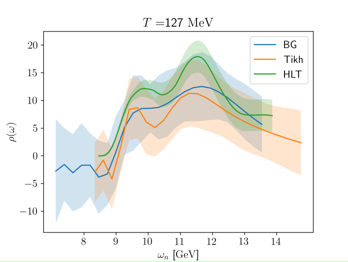
Hansen–Lupo–Tantalo

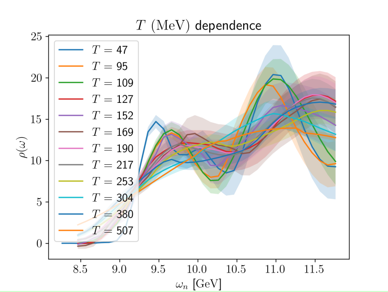
$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega |\overline{\Delta} - \Delta_\sigma|^2 \,, \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} \, \mathrm{Cov}(\tau_1, \tau_2)$$

$$\Delta_\sigma(\omega, \omega_n) = \alpha \exp[-(\omega - \omega_n)^2 / 2\sigma^2]$$

Linear methods results



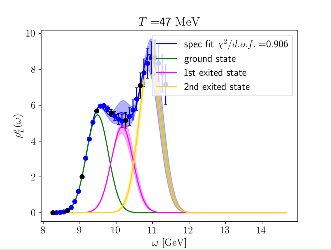




- Consistent results for primary peak positions from three methods
- HLT is better constrained
- Results show peak shift and broadening

HLT fits

- Smearing width σ is input in HLT
  $\longrightarrow$ upper bound on Γ
- Fit spectral function to sum of gaussians, each with width σ
- Number of peaks decreases with increasing T
- σ must be increased



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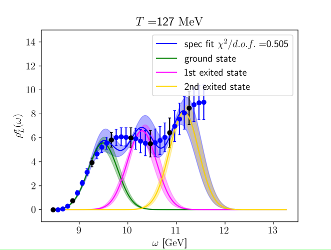

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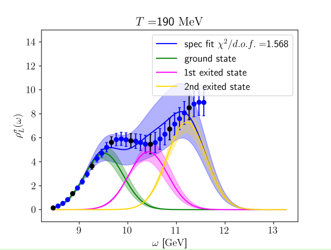

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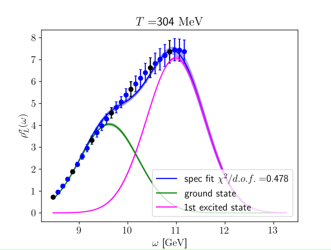

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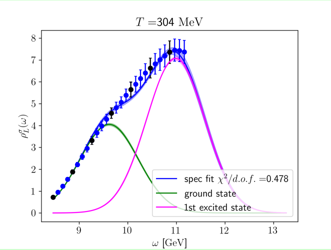
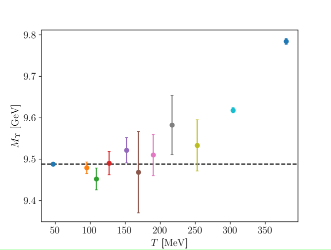




Bayesian spectral function reconstruction

Bayes theorem

$$P(\rho|DI) = \frac{P(D|\rho I)P(\rho|I)}{P(D|I)}$$

Parametrise prior probability by

$$P(\rho|I) \propto e^{\alpha S[\rho]} \quad \Longrightarrow \quad P(\rho|DI) \propto e^{-L[D,\rho]+\alpha S[\rho]}$$

where L is the standard likelihood (χ^2)

Spectral function $\rho(\omega)$ is expressed in terms of default model $m(\omega)$

$$\rho(\omega) = m(\omega)\exp[\sum_{k=1}^{N_b} b_k u_k(\omega)]$$

Bayesian methods

Maximum entropy method (Bryan)

$$S = \int d\omega \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \quad \text{(Shannon–Jaynes entropy)}$$

Singular value decomposition:

$$K(\omega, \tau) \to K(\omega_i, \tau_j) = K_{ij} = U \Xi V^T$$

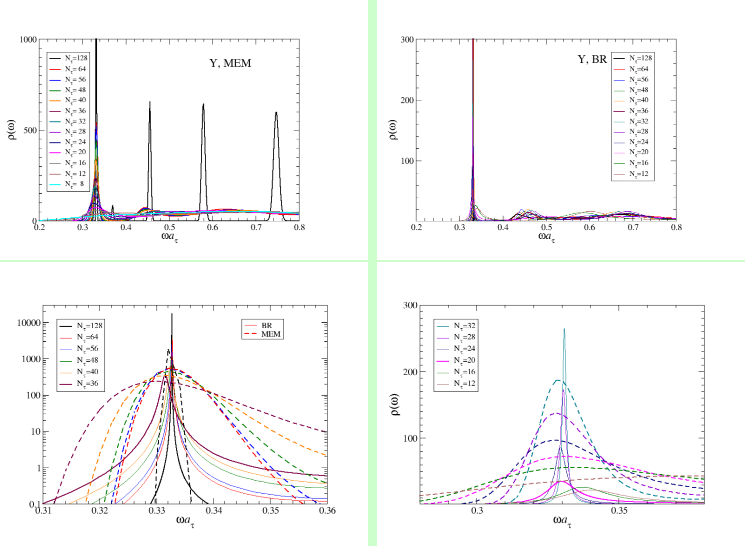
u_k are column vectors of U: $N_b = N_s \leq N_{\text{data}}$

BR method

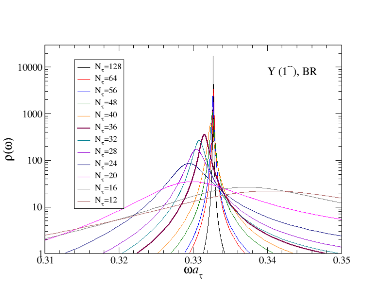
$$S = \int d\omega \Big(1 - \frac{\rho(\omega)}{m(\omega)} + \ln \frac{\rho(\omega)}{m(\omega)} \Big)$$

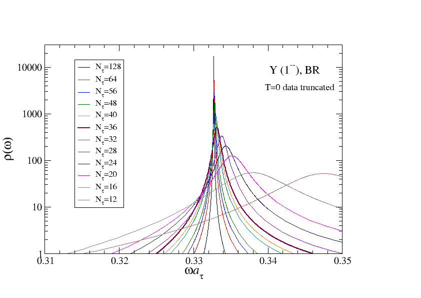
$N_b = N_\omega, u_k(\omega) = \delta_{k\omega}$

MEM vs BR method



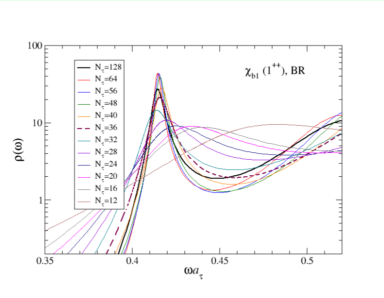
BR results: Υ

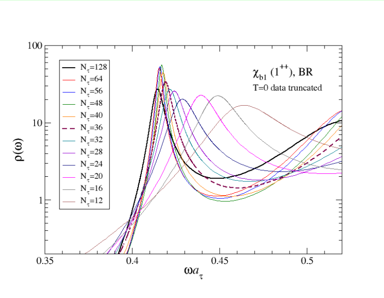




- Clear evidence of negative mass shift
- Finite N_τ artefacts would give positive mass shift
- Broadening similar in magnitude to $T = 0$ truncated data

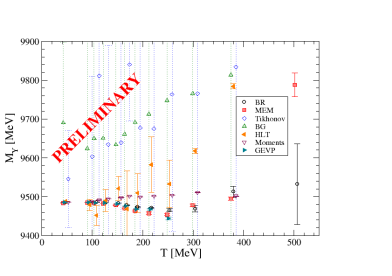
BR results: χ_{b1}

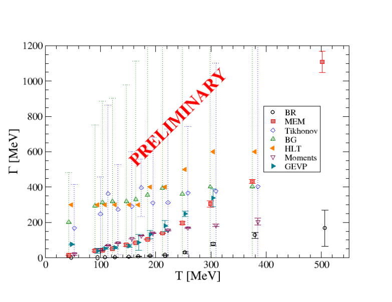


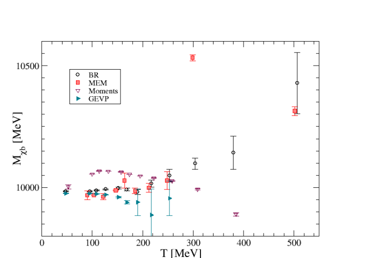


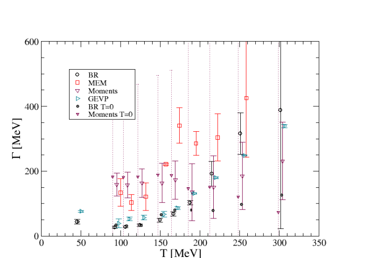
- Clear evidence of thermal broadening
- Finite N_τ artefacts would give positive mass shift — not seen in thermal data

Summary — raw results

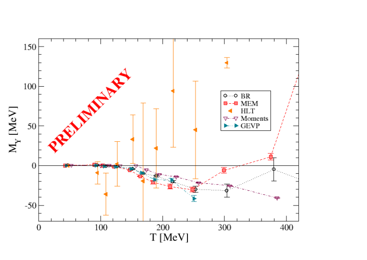


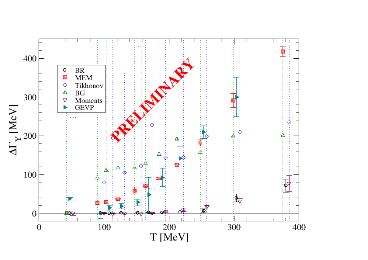


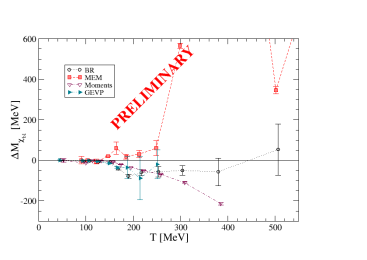


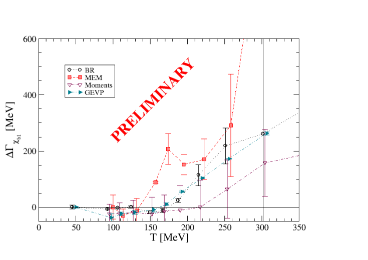


Summary — $T = 0$ subtracted









Summary

- Mass and width of ground state S- and P-wave bottomonium (Υ and χ_{b1}) studied with a range of different methods
- Agreement between correlator and bayesian methods on a negative mass shift of up to 20–40 MeV at $T \sim 250$ MeV.
- Qualitative agreeement for mass and width of χ_{b1}.
- Discrepancy for width of Υ requires further investigation.
- Linear methods have intrinsically larger uncertainties (or better at quantifying them)
- Zero-temperature subtraction is an essential tool

Outlook

- Complete study of systematics, including $T = 0$ subtraction for all methods
- Excited states (talk by Ryan Bignell, thu 1230)
- Repeat with smaller a_τ: Gen3