Quantum critical point from competition between the Dirac Kondo effect and chiral symmetry breaking

KH, Daiki Suenaga, Kei Suzuki, Shigehiro Yasui, [2211.16150](2211.16150) (PRB)

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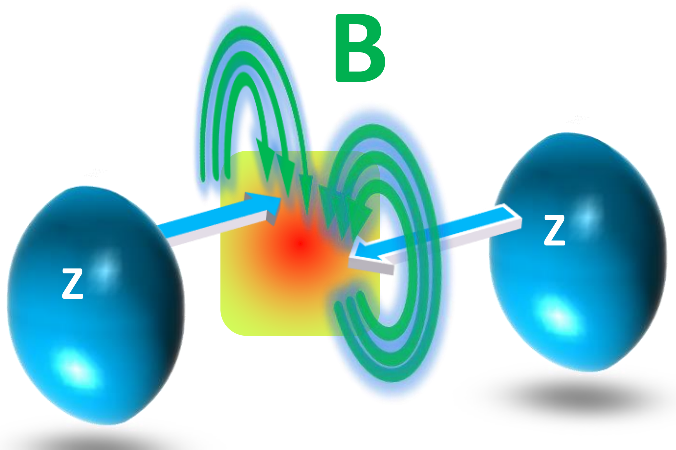
XVI-th Quark Confinement and the Hadron Spectrum
@ Cairns, Aug. 18-22, 2024

Key ingredients



- Magnetic fields

→ Large degeneracy

RHIC, LHC; FAIR, NICA, J-PARC, HIAF
Neutron stars, Lattice QCD simulations

Strong B		High density
- Landau degeneracy	Analogy	- Fermi degeneracy
- Chiral symmetry breaking		- Superconductivity

Competition

- Heavy-quark impurities → The Kondo effect

"Magnetic catalysis" of the chiral symmetry breaking

Chiral condensate increases
in strong (chromo) magnetic fields.

Klevansky, Lemmer (1989)
Tatsumi, Suganuma (1991)
Schramm, Muller, Schramm (1992)

- Moreover, "the chiral symmetry is broken with an infinitesimal attractive interaction, i.e., even in QED."

Gusynin, Miransky, Shovkovy (1995)

Analogy with the BCS theory:
"Superconductivity occurs with an infinitesimal attractive interaction."

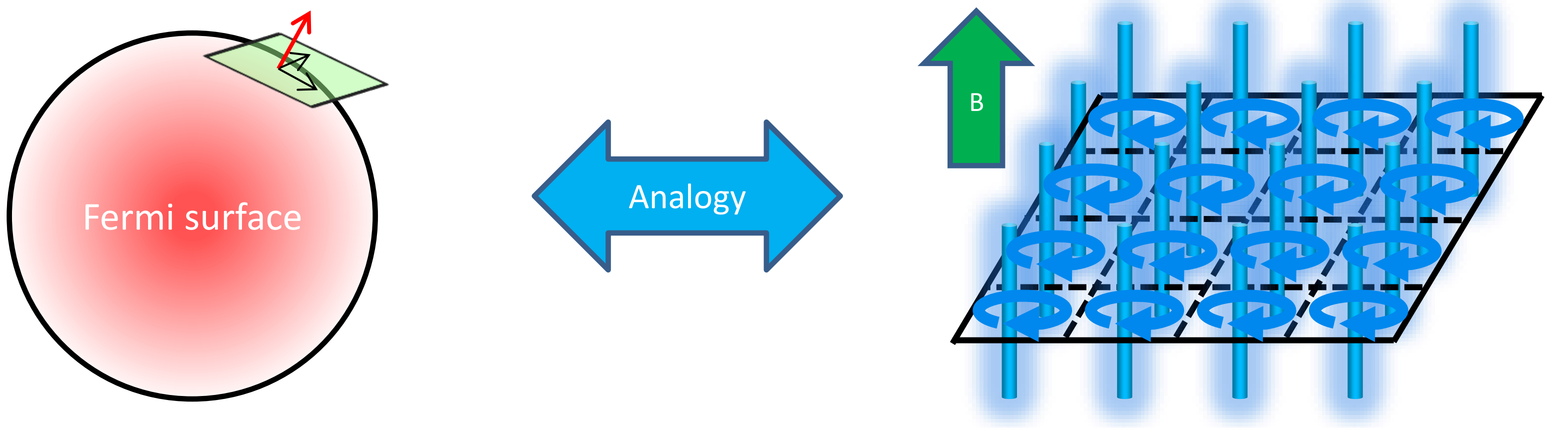
BCS (1957), Shanker, Polchinski (1992)

Dimensional reduction and dynamical chiral symmetry breaking
by a magnetic field in $3 + 1$ dimensions

V.P. Gusynin, V.A. Miransky, I.A. Shovkovy

On the other hand, the dynamics of the fermion pairing in a magnetic field in $3+1$ dimensions is $(1+1)$-dimensional. We recall that, because of the Fermi surface, the dynamics of the electron pairing in BCS theory is also $(1+1)$-dimensional. This analogy is rather deep. In particular, the expression (20) for $m_{\rm dyn}$ can be

Key physics: “Dimensional reduction” near the Fermi surface and in strong B



Degeneracy in 2 dim.

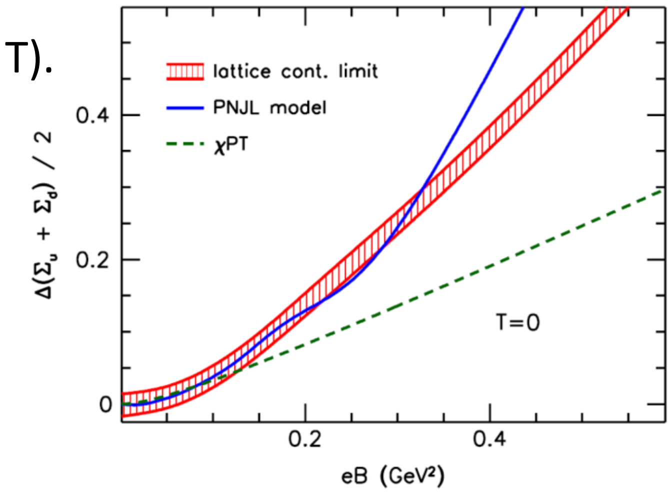
- On the Fermi surface
- In the transverse plane with respect to B

⇒ Kinematics is only determined by the residual (1+1) dim. dynamics.

Bound states inevitably emerge in the effective (1+1) dim. → Cooper pairs (and thus independently of the coupling strength).

QCD phase diagram in strong B from lattice simulations

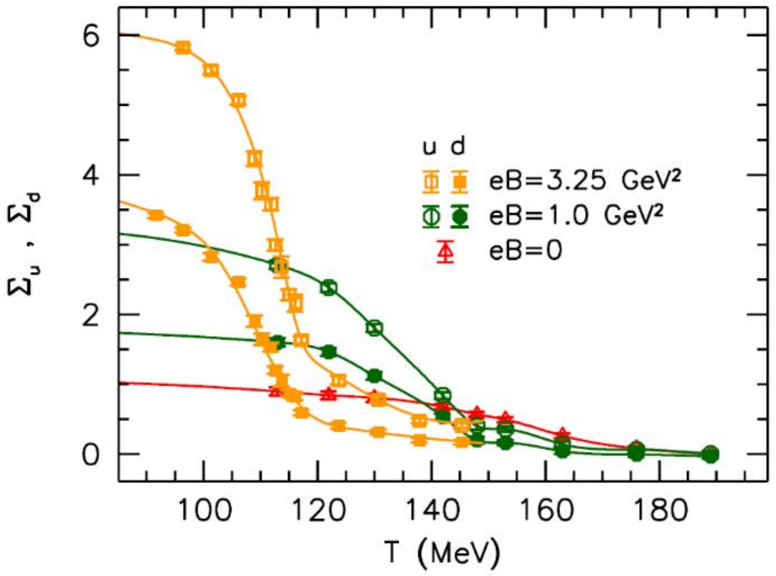
"Magnetic catalysis": Chiral condensate increases (at low T).



Numerous data from lattice collaborations

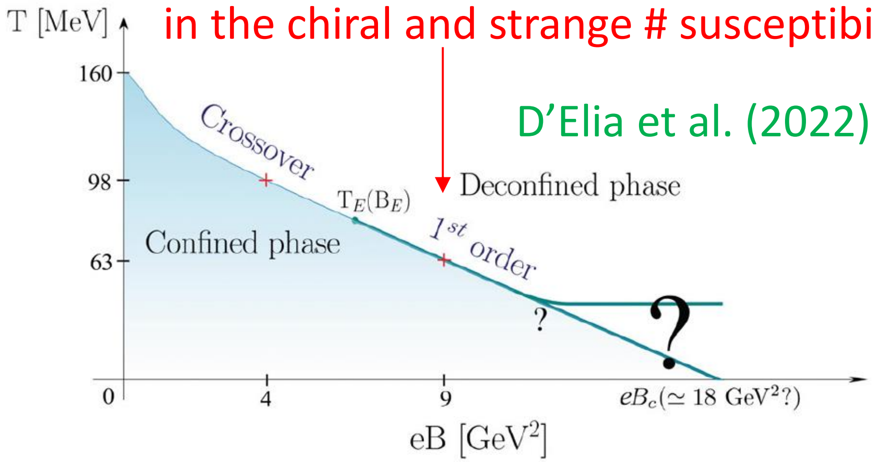
Buividovich, et al. (2010-), D'Elia, et al. (2012-),
Endrodi, et al. (2012-), Ding, et al. (2019-)

"Inverse magnetic catalysis" near the transition T

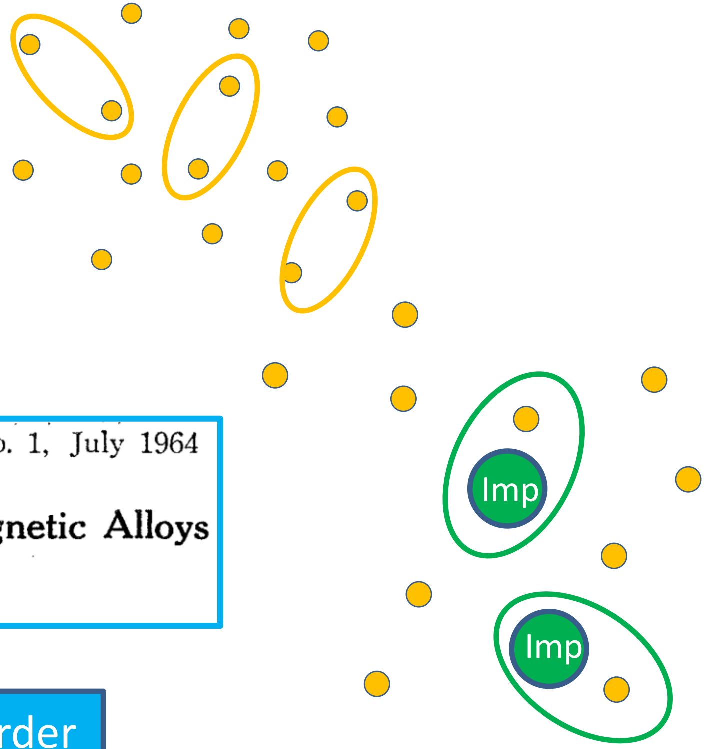


A first-order at 9 GeV^2 was observed
in the chiral and strange # susceptibility.

D'Elia et al. (2022)

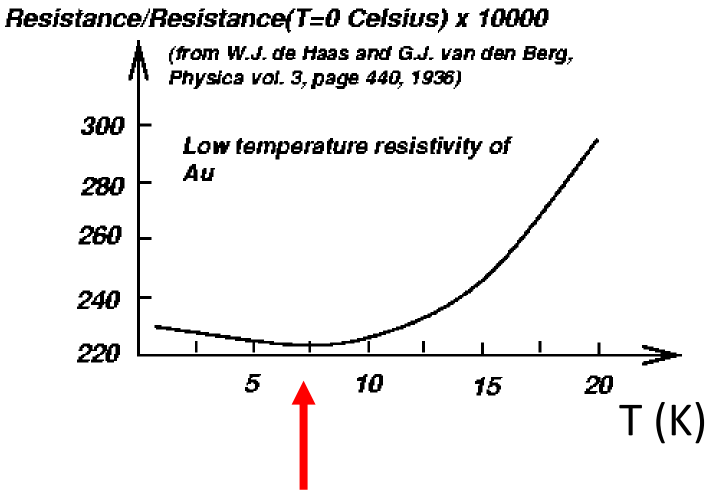


The Kondo effect



Pairing near the Fermi surface

- Cooper pairing: electron-electron scattering
- Kondo condensate or Kondo cloud



Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun Kondo

The key process is the second-order spin-exchange interactions.

- Fermi surface (dim. reduction)
- Quantum effect from loop diagrams
- Non-Abelian (Pauli) matrix

T_K: Kondo temperature (Location of the minimum)

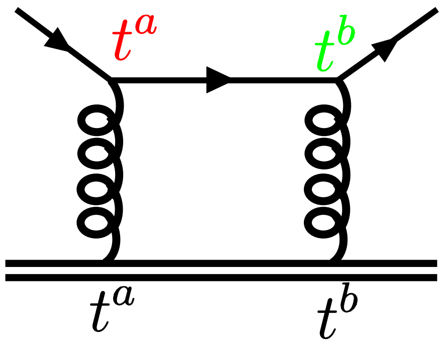
The renormalization group: Critical phenomena and the Kondo problem*†

Kenneth G. Wilson

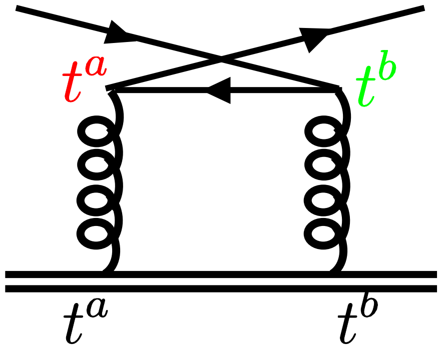
Log enhancement from the NLO scattering amplitudes

Light quark

Heavy quark



$$\bar{u}^k (t^b)^{kj} (t^a)^{ji} u^i$$



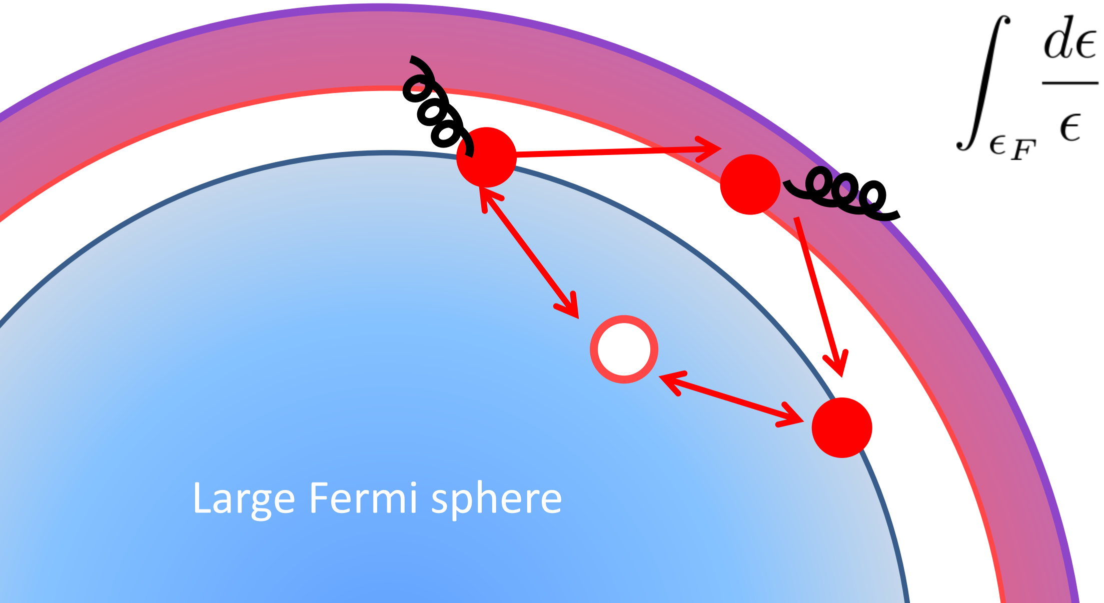
$$\bar{u}^k (t^a)^{kj} (t^b)^{ji} u^i$$

Particle contribution

$$\int_{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim + \log \frac{\Lambda}{\Lambda - d\Lambda}$$

Hole contribution

$$\int^{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim - \log \frac{\Lambda}{\Lambda - d\Lambda}$$



Avoids cancellation with non-Abelian vertices

$$\int d^2\ell_\perp \sim \rho_F = \frac{\mu^2}{2\pi^2}$$

Density of states

Recent developments in the QCD Kondo effect

Finite density

- Renormalization-group analysis, KH, Itakura, Ozaki, Yasui (2015)
- Higher-loop effects on the fixed-point, Kanazawa, Uchino, (2016)
- Phase diagram from mean-field analysis, Yasui, Suzuki, Itakura (2016)
- Mapping to CFT near the fixed-point, Kimura, Ozaki (2017)

Conformal Field Theory

Consequences and more exotic effects:

✓ The Kondo effect in the 2SC phase, KH, Huang, Pisarski (2019)
✓ The Kondo effect induced by the chirality imbalance, Suenaga, Suzuki, Araki, Yasui (2020)
✓ Catalysis of the chiral separation effect, Suenaga, Araki, Suzuki, Yasui (2021)
Cf. Two-color lattice study, Talk by Buividovich in "Hard Problems of Hadron Physics (2021)."
✓ HQ Spin polarization, Suenaga, Araki, Suzuki, Yasui (2022)
….

	Finite density	In magnetic fields
Light fermions	(color) Superconductivity BCS (1957), Shanker, Polchinski (1992)	Magnetic catalysis of chiral condensate Gusynin, Miransky, Shovkovy (1995)
Impurity	QCD Kondo effect KH, Itakura, Ozaki, Yasui (2015)	Magnetically induced Kondo effect Ozaki, Itakura, Kuramoto (2015)

The Kondo effect in magnetic fields

New even in cond. matt. physics

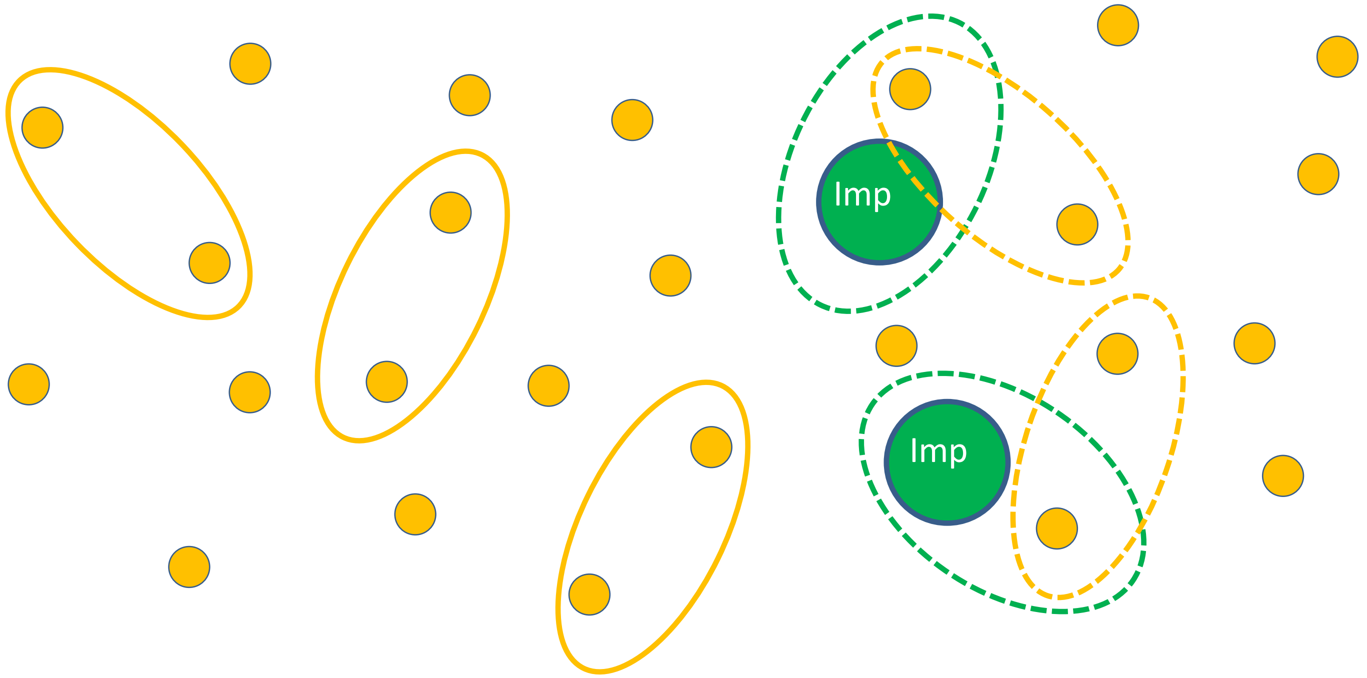
- RG analysis, Ozaki, Itakura, Kuramoto (2015)
- Mapping to CFT, Kimura, Ozaki (2019)

Magnetic catalysis vs Magnetic Kondo effect

- Competition between the two strong statements.

Magnetic catalysis:
Inevitable formation of $<\bar{q}q>$.

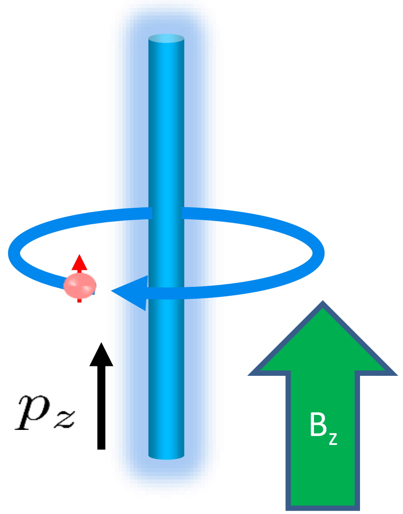
The Kondo effect:
Inevitable formation of $<\bar{Q}q>$.



Cf. Competition at finite density, Suzuki, Yasui, Itakura (2017)

Model

Landau quantization



One light flavor ψ + One heavy flavor Ψ with N colors

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/_{\parallel} - m_l)\psi + \frac{G_{ll}}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right]$$

Light-quark kinetic term in B
(the lowest Landau level, (1+1) dim. along B)

Light-light interactions

$$+\sum_{c=\pm}\left[c\bar{\Psi}_v^c i\partial_0\Psi_v^c + \frac{G_{hl}}{N}\left\{(\bar{\psi}\Psi_v^c)(\bar{\Psi}_v^c\psi) + (\bar{\psi}i\gamma_5\Psi_v^c)(\bar{\Psi}_v^c i\gamma_5\psi)\right\}\right]$$

Heavy quark
+ heavy antiquark

Heavy-light interactions

Heavy-quark kinetic term in the LO heavy-quark expansion (Infinite mass, no spin rotation).

Mean-field analysis

Chiral condensate

$$\langle\bar{\psi}\psi\rangle_{\mathrm{LLL}} \equiv -\frac{N}{G_{ll}}M,$$

Kondo condensate

$$\langle\bar{\psi}\Psi_v^{\pm}\rangle_{\mathrm{LLL}} \equiv \frac{N}{G_{hl}}\Delta$$

Color $\bar{3}$ Kondo condensate is suppressed in the large Nc, Yasui (2017).

- Exact diagonalization of the MF Lagrangian with M and Δ

Four eigenmodes:

$$E_{\pm}(p) \equiv \frac{1}{2}\left(E_p \pm \sqrt{E_p^2 + |2\Delta|^2}\right)$$

$$\tilde{E}_{\pm}(p) \equiv \frac{1}{2}\left(-E_p \pm \sqrt{E_p^2 + |2\Delta|^2}\right)$$

$$E_p \equiv \sqrt{p_z^2 + (m_l + M)^2}.$$

- Thermodynamic potential

$$\tilde{\Omega}(M,\Delta) = \frac{1}{2\tilde{G}_{ll}}M^2 + \frac{1}{2\tilde{G}_{hl}}|2\Delta|^2 - \sum_i \int_{-\Lambda}^{\Lambda} \boxed{\frac{dp_z}{2\pi}} \left[\frac{1}{2}E_i + \frac{1}{\beta}\ln(1 + e^{-\beta E_i})\right]$$

Vacuum

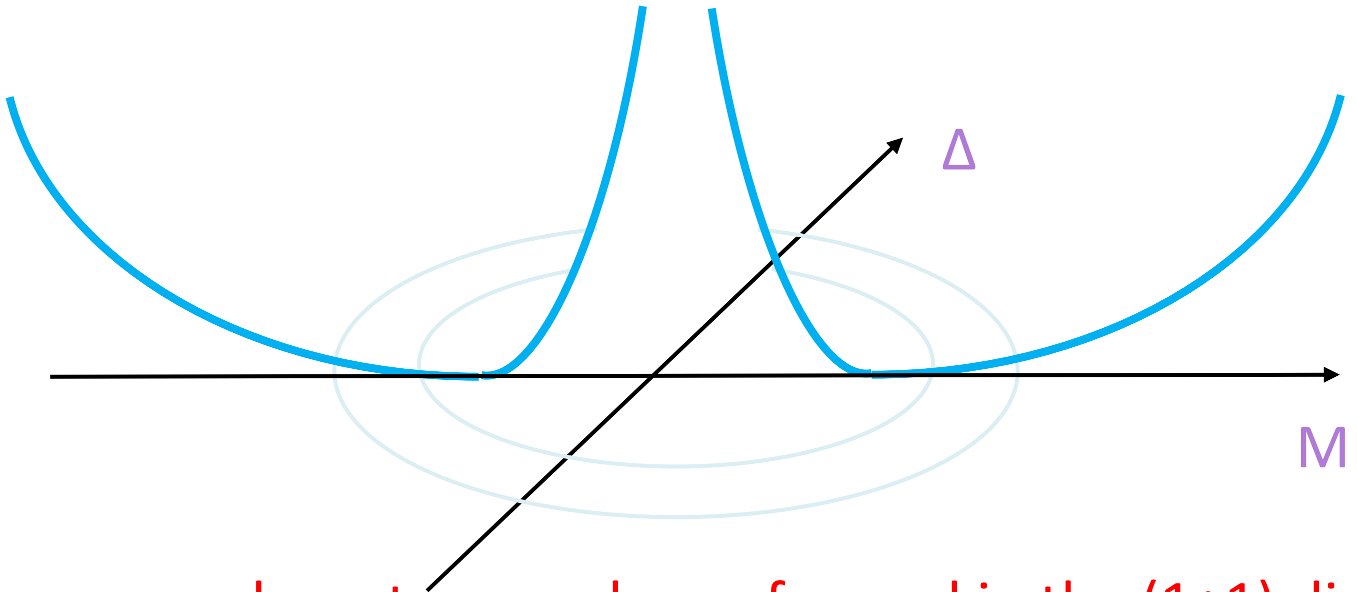
Finite T

Dim. reduction: Momentum integral only in the B direction.

"Log repulsion" at the origin of the potential

Zero T case:

$\sim M^2 \log M, \ \ \Delta^2 \log \Delta$



Nonzero condensates are always favored in the (1+1) dim.

Finite T case:

the same log with the opposite sign

$$\sim \log \frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \log \frac{\Lambda_{\rm UV}}{M} - \log \frac{T}{M}$$

→ Symmetry restoration without the singularity at the origin

Gap equations

- Always has non-trivial solutions at zero T

$$\frac{\partial\Omega}{\partial M} = 0 = \frac{\partial\Omega}{\partial\Delta} \quad \Rightarrow \quad \frac{M}{\tilde{G}_{ll}} + \frac{M+m_l}{2\pi}\ln\Xi = 0, \quad \frac{\Delta}{\tilde{G}_{hl}} + \frac{\Delta}{2\pi}\ln\Xi = 0$$

$$\Xi = \frac{(M+m_l)^2+4\Delta^2}{(\Lambda+\sqrt{(M+m_l)^2+4\Delta^2+\Lambda^2})^2}$$

The magnetic catalysis

Without "heavy-light" interactions
(and in the massless limit):

$$\frac{\pi}{\tilde{G}_{ll}} = \ln\frac{\Lambda+\sqrt{M^2+\Lambda^2}}{M}$$

RHS sweeps from 0 to ∞
due to the log dependence.

The gap eq. always has a nontrivial solution
for any value of G:

$$M = \frac{\Lambda}{\sinh(\pi/\tilde{G}_{ll})} \sim \Lambda e^{-\pi/\tilde{G}_{ll}}$$

The Kondo effect

Without "light-light" interactions,
a nontrivial solution always appears
for Δ in the same manner:

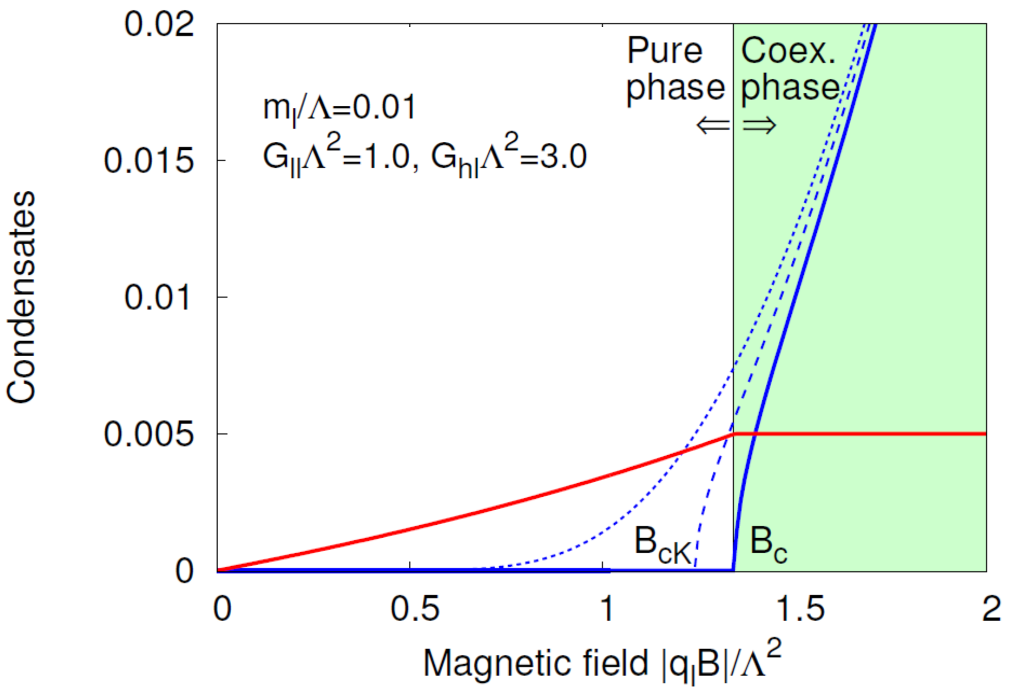
$$\Delta = \frac{\Lambda}{2\sinh(\pi/\tilde{G}_{hl})} \sim \Lambda e^{-\pi/\tilde{G}_{hl}}$$

The same log structure.

Numerical results
(+ analytic results)

Magnetic catalysis vs the Kondo effect at zero T

- Results from numerical search of the global stationary point



- Chiral condensate M
- Kondo condensate Δ

Cf. Δ w/o competition (M=0)

.... Massless limit

- - - - - - - Current q mass

Saturation of the magnetic catalysis

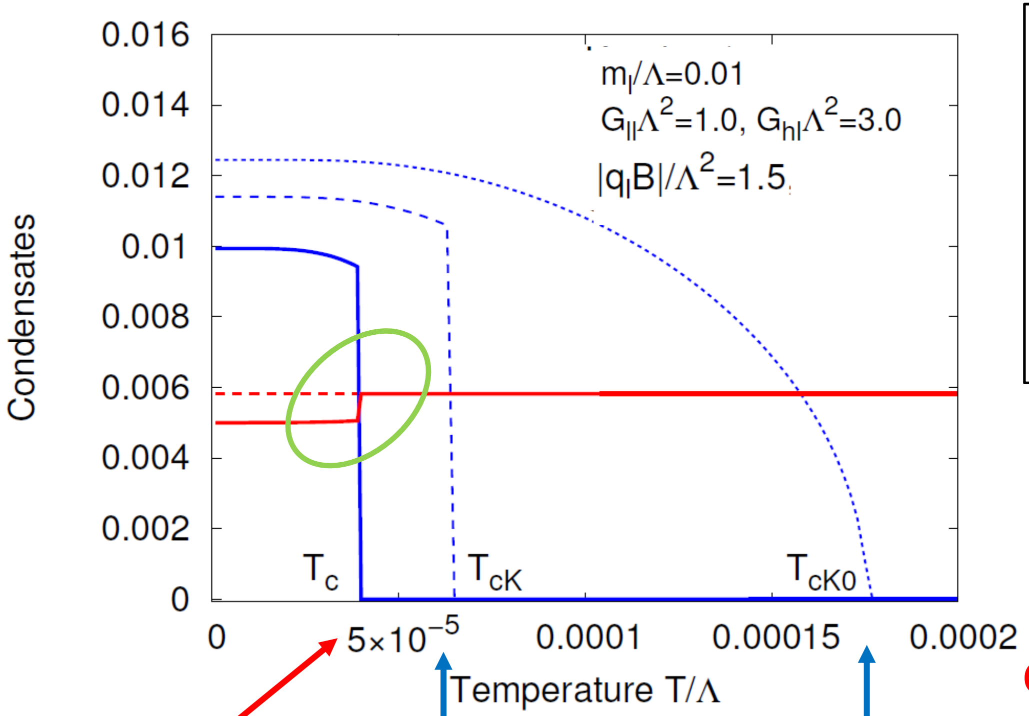
Chiral cond. dominates the Kondo cond.

- Kondo cond. starts growing.
- Chiral cond. saturates at a critical B.

New quantum critical point:
Switch over between the true vacua at zero T

Analytic expression for the critical B $$q_l B_c = \frac{4\pi^3}{G_{hl}\,\mathrm{acrsinh}\left(\Lambda \frac{|G_{hl}-G_{ll}|}{m_l G_{hl}}\right)} \sim \frac{4\pi^3 m_l}{\Lambda |G_{hl}-G_{ll}|}$$

Magnetic catalysis vs Kondo effect at finite T



- Chiral condensate M
- Kondo condensate Δ

Cf. Δ w/o competition (M=0)

Massless limit

Current q mass

Chiral pseudo Tc
(w/o comp., crossover)

Kondo Tc
in competition

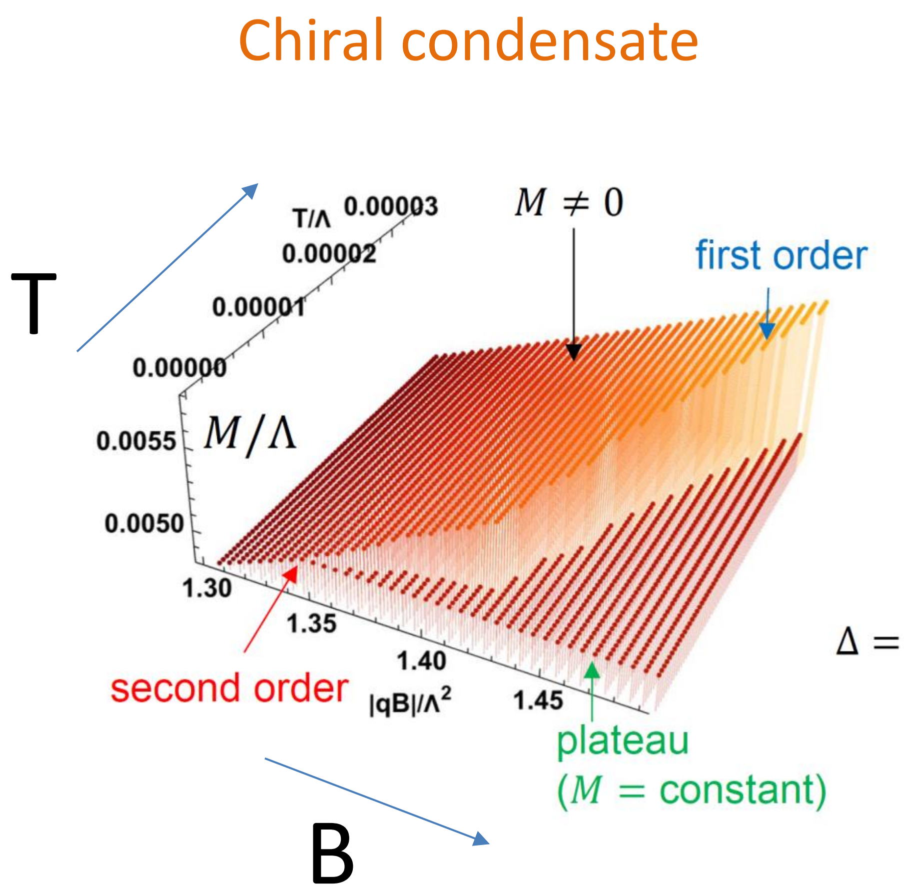
Kondo Tc
(w/o competition)

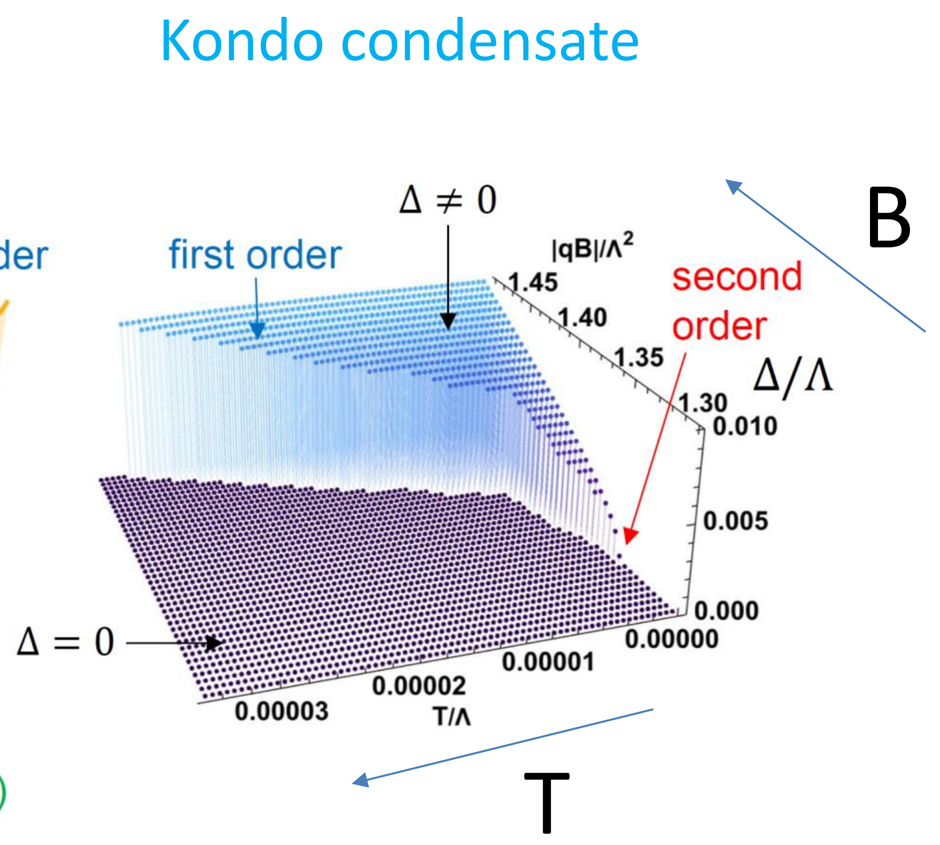
Kondo Tc
(w/o competition, massless limit)

■ The chiral condensate increases as we increase T just after the Kondo cond. melts away.

→ Signature of the end of competition

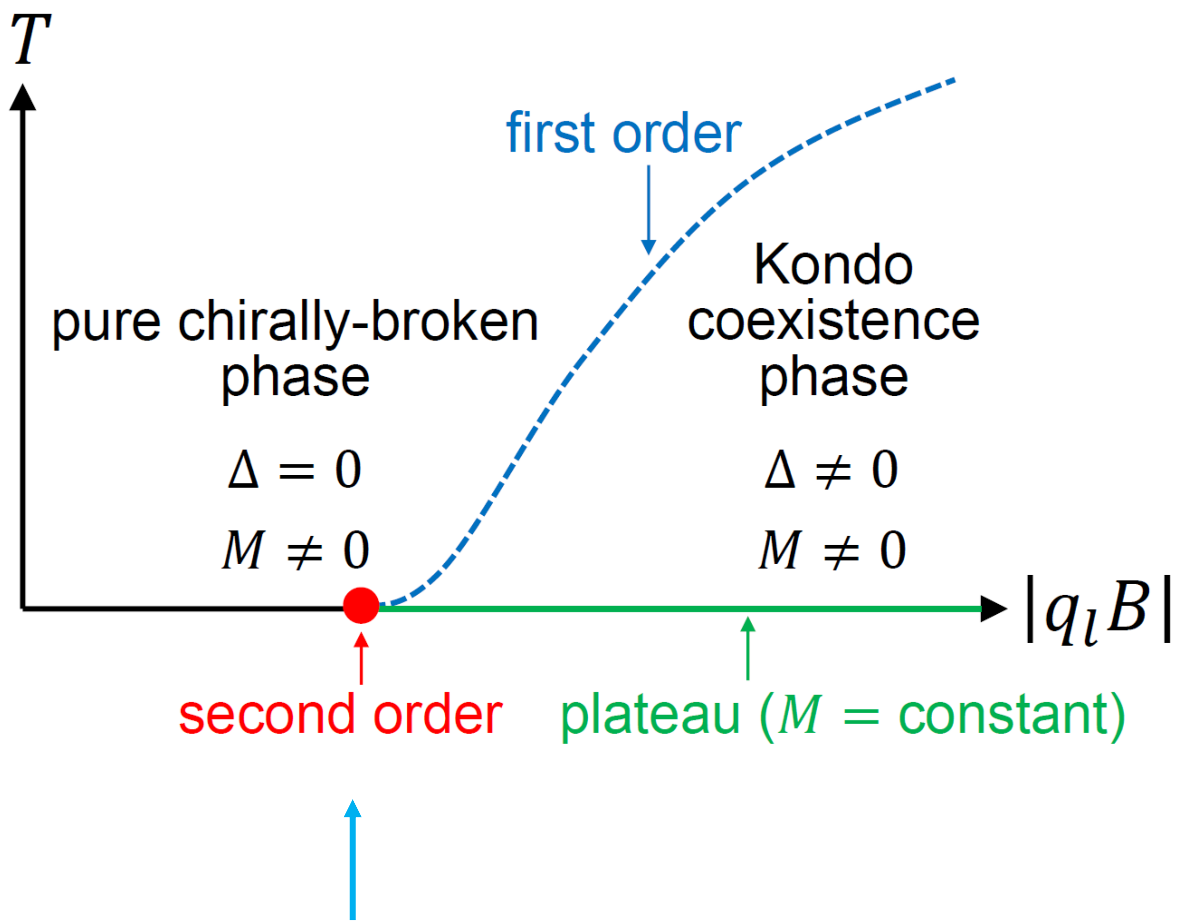
Landscapes at finite T and B
- First-order transition lines





$m_l/\Lambda = 0.01,\ G_{ll}\Lambda^2 = 1.0,\ G_{hl}\Lambda^2 = 3.0$

Phase diagram extracted from the results

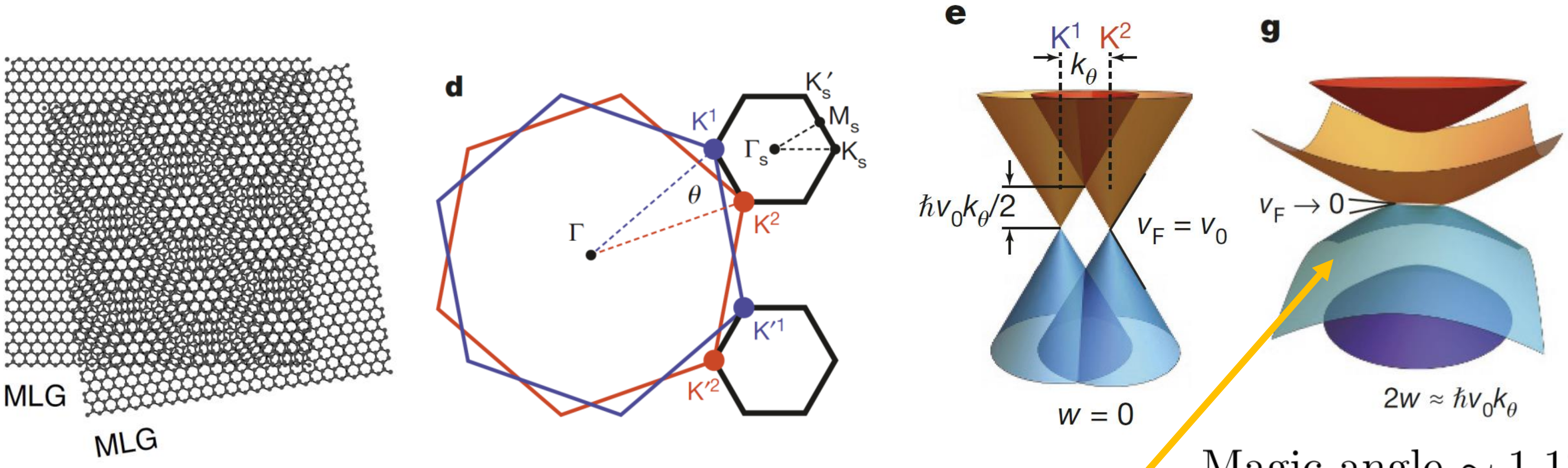


New <u>quantum critical point</u>: Switch over between the true vacua at zero T

Flat bands in the twisted bilayer graphene

Morell, et al. (2010)
Bistritzer and MacDonald (2011)

Moiré patterns



Magic angle $\sim 1.1°$

Flat band

Balents, et al., *Nature Physics* **16** 725–733 (2020)
Cao, et al., *Nature* **556**, 80–84 (2018)

Twisted monolayer–bilayer graphene

Chen et al., *Nature Physics* **17** 374–380 (2021)

Summary

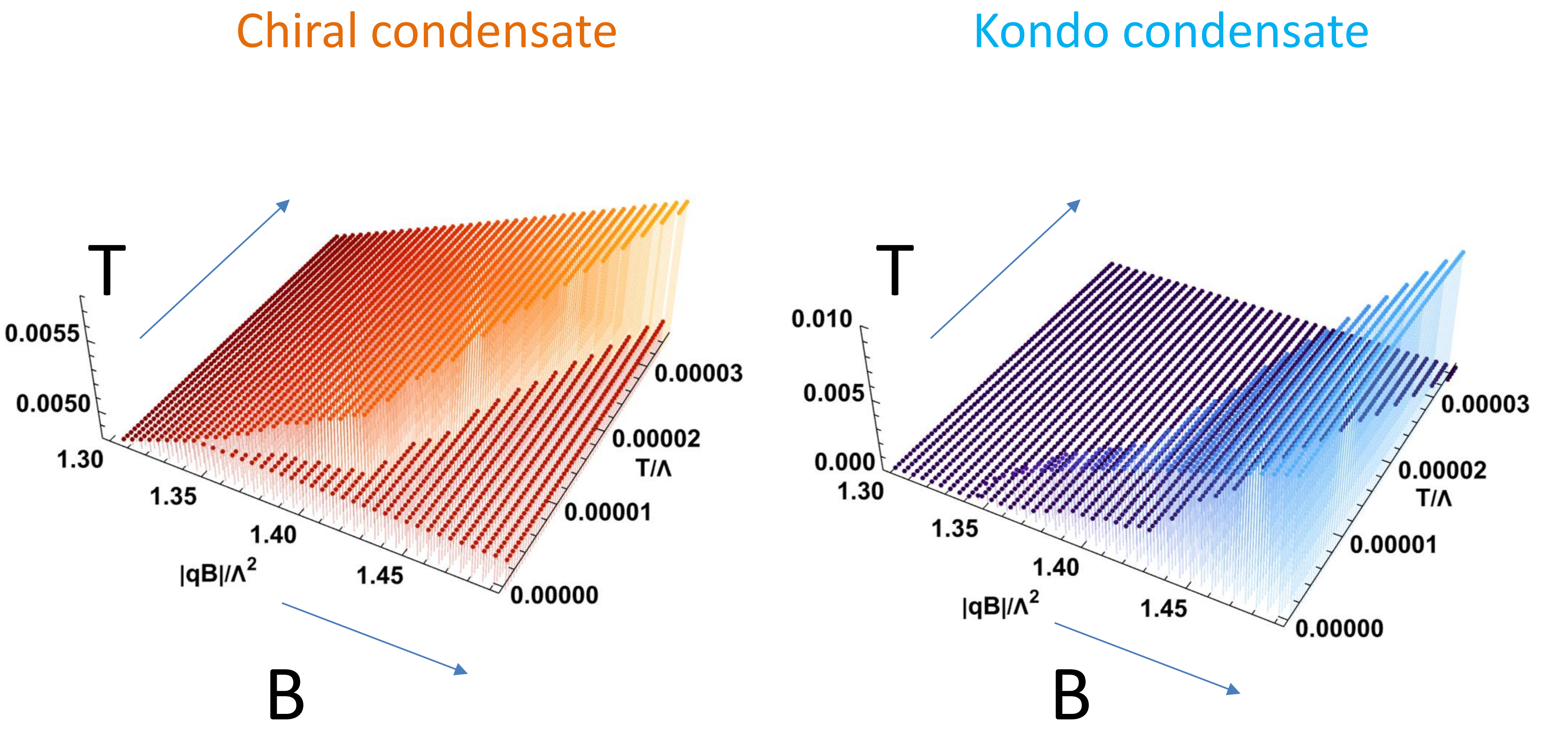
Potential signatures of the competition

- Saturation of the chiral condensate in strong B due to the growth of the Kondo condensate.
- Anomalous increase of the chiral condensate with T just after the end of competition.

Outlook

- Study by lattice simulations (Static thermodynamic quantities)
- Computation of the transport coefficients (Dynamical quantities)
- Applications to the Dirac quasi-particles in cond. matt. systems

Back-up slides


Chiral condensate
Kondo condensate
T
0.0055
0.0050
1.30
1.35
1.40
1.45
|qB|/Λ^2
0.00003
0.00002
T/Λ
0.00001
0.00000
B
T
0.010
0.005
0.000
1.30
1.35
1.40
1.45
|qB|/Λ^2
0.00003
0.00002
T/Λ
0.00001
0.00000
B