Boundary states and non-Abelian Casimir effect in lattice Yang-Mills theory

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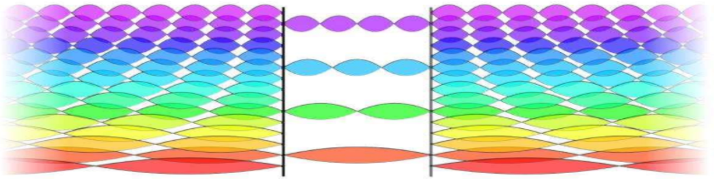
The structure of the talk

The Casimir effect - basics

The emergence of attractive force $F(d)$ between two conducting metallic plates in vacuum.

Predicted in 1948 *H. Casimir, Indag. Math. 10, 1948*.

Measured in 1997 S.K. Lamoreaux, Phys. Rev. Lett. 78, 1997.



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$$E_{\text{tot}} = \sum_k \frac{1}{2}\hbar\omega_k$$

$$\delta E = E(d) - E(\infty) = -\frac{\pi^2 \hbar c}{720 d^3} L^2$$

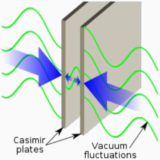
$$\frac{F(d)}{L^2} = -\frac{\pi^2 \hbar c}{240 d^4}$$

Peculiar properties of Casimir vacuum

The problem of spherical geometry

$$\frac{\Delta E}{L^2} = -\frac{\pi^2 \hbar c}{720 d^3}$$

$$\frac{F(d)}{L^2} = -\frac{\pi^2 \hbar c}{240 d^4}$$



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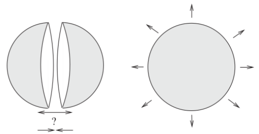
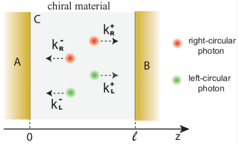


O. Kenneth, I. Klich
Phys.Rev.Lett 97, 16, 2006

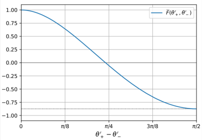
$$\Delta E \simeq +0.09 \frac{\hbar c}{2r}$$

T. Boyer, 1968
Phys. Rev. 174,No.5

Casimir effect in chiral medium



Q. Jiang, F. Wilczek, Phys.Rev.B 99,12,2019



$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{i}{4} g \theta F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\theta'_{\pm} = \arctan(g\theta_{\pm})$$

F. Canfora et. al., JHEP 2022, No. 9

Restructuring of vacuum between plates

Scharnhorst effect

$$\delta c = +\frac{11\pi^2}{90^2} \alpha^2_{\mathrm{e.m.}} \left(\frac{\hbar}{m_e c} \frac{1}{R} \right)^4$$

S. Liberati et al., An. of phys. 298, 1, 2002

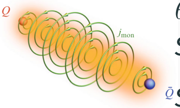
Restoration of chiral symmetry

$$\mathcal{L} = i\bar{\Psi}\partial\!\!\!/\Psi + \frac{g}{2}(\bar{\Psi}\Psi)^2$$

A. Flachi, Phys. Rev. Lett. 110, No. 6, 2013

M. Chernodub et al.,Confinement 2018 proc.; A. Molochkov, Nobel Symp. 167 proc., 2023

Compact QED as model of confinement



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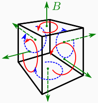
$$\theta_{P_{x,\mu\nu}} = \theta_{x,\mu} + \theta_{x+\hat{\mu},\nu} - \theta_{x+\hat{\nu},\mu} - \theta_{x,\nu}$$

$$S_{\rm nc} = \frac{\beta}{2} \sum_{x\in\Lambda} \sum_{\mu<\nu} \theta_P^2, \ \ \theta_{x,\mu} \in (-\infty, \infty)$$

$$S_{\rm cp} = \beta \sum_{x\in\Lambda} \sum_{\mu<\nu} (1 - \cos\theta_P), \theta_{x,\mu} \in [-\pi, \pi)$$



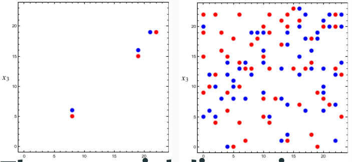
$$\bar{\theta}_P = \theta_P + 2\pi k_P \in [-\pi, \pi)$$

$$j_{x,\mu} = \frac{1}{2\pi} \sum_{\partial C_{x,\mu}} (-1)^P \bar{\theta}_P$$

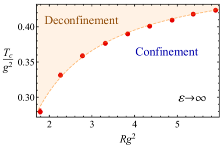
G. 't Hooft, EPS Proc., 1976; S. Mandelstam, Phys.Rep.23, 1976.

A. DeGrand, 1980, Phys.Rev.D 22, N.10

Two spatial dimensions

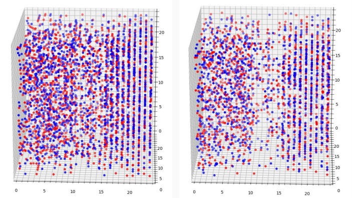


M. Chernodub, V.Goy, A. Molochkov, Phys.Rev.D 95, 2017

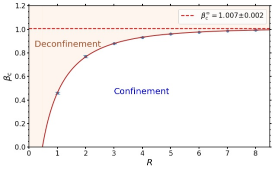


M. Chernodub, V.Goy, A. Molochkov, Phys.Rev.D 96, 2017

Three spatial dimensions

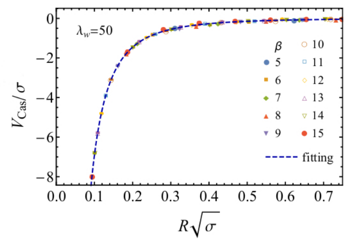


M. Chernodub, V.Goy, A. Molochkov, A. Tanashkin, Phys.Rev.D 105, 2022



New mass scale in SU(2)-gluodynamics in 2 spatial dimensions

Numerical approach



$$V^{\rm fit}_{\rm Cas}(R) = -\frac{3\zeta(3)}{16\pi}\frac{1}{R^2}\frac{1}{(R\sqrt{\sigma})^{\nu}}e^{-M_{\rm Cas}R}$$

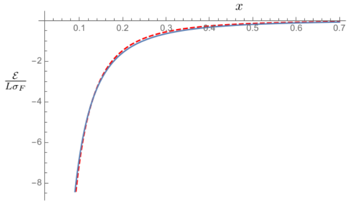
$$M_{\rm Cas} = 1.38(3)\sqrt{\sigma}$$

$$M_{0^{++}} \approx 4.7\sqrt{\sigma}$$

$$\nu = 0.05(2)$$

Chernodub M., Goy V., Molochkov A., Nguyen H.N., Phys.Rev.Lett. 121, No. 19, 2018

Analytic approach



$$\frac{\varepsilon}{L} = -\frac{\dim G}{16\pi R^2}\left[2mR\,{\rm Li}_2\left(e^{-2mR}\right) + {\rm Li}_3\left(e^{-2mR}\right)\right]$$

$$mR = \sqrt{c_A/\pi c_F}\,x, \quad x = R\sqrt{\sigma_F}$$

$$\sigma_F = e^4\frac{c_A c_F}{4\pi}, c_A = N_c, \quad c_F = \frac{N_c^2-1}{2N_c}$$

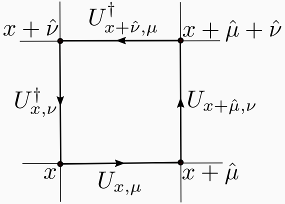
Karabali D., Nair V.P., Phys.Rev.D 98, No. 10, 2018

The action

$$S = \beta \sum_{x\in\Lambda} \sum_{\mu<\nu} (1 - \mathcal{P})$$

$$\mathcal{P} = \frac{1}{N_c} \mathrm{Re}\,\mathrm{tr}\, U_{x,\mu\nu}, \quad \beta = \frac{2N_c}{g^2}$$

$$U_{x,\mu\nu} = U_{x,\mu}\, U_{x+\hat{\mu},\nu}\, U^\dagger_{x+\hat{\nu},\mu}\, U^\dagger_{x,\nu}$$

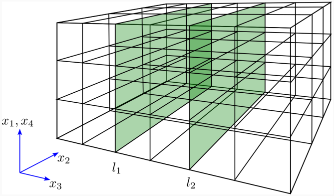


Casimir boundary conditions

$$E^\alpha_\parallel(x)\Big|_{x\in S} = B^\alpha_\perp(x)\Big|_{x\in S} = 0,$$

$$\alpha = 1, \ldots, N_c^2 - 1$$

$$\beta \to \beta_P = \beta[1 + (\varepsilon - 1)\delta_{P,\mathcal{V}}]$$



$$\mathcal{P}_{x,\mu\nu}\Big|_{x_3=l_{1,2}} = 1, (\mu\nu) = \{12, 14, 24\} \Leftrightarrow F_{x,\mu\nu} = 0$$

Glueton – new mass scale in SU(3) gluodynamics in $3+1$ dims

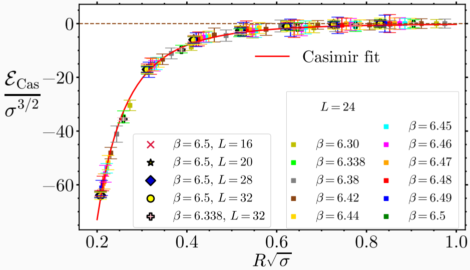
Energy-momentum tensor in Minkowski space

$$T^{\mu\nu} = F^{\mu\alpha}F^{\nu}{}_{\alpha} - \tfrac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

Energy density

$$\mathcal{E} \equiv T^{00} = \frac{1}{2}\left(\boldsymbol{B}^2 + \boldsymbol{E}^2\right) \rightarrow$$

$$T^{44}_{E} = \frac{1}{2}\left(\boldsymbol{B}^2_E - \boldsymbol{E}^2_E\right) .$$



Casimir energy on the lattice:

$$\beta L_s \left(\sum_{i=1}^{3} \langle \mathcal{P}_{i4} \rangle_S - \sum_{i<j=1}^{3} \langle \mathcal{P}_{ij} \rangle_S \right)$$

$$\mathcal{P}_{x,ij} = \tfrac{1}{3}\mathrm{Re\,tr}\, U_{x,ij}$$

$$m_{\mathrm{gt}} = 1.0(1)\sqrt{\sigma} = 0.49(5)\,\mathrm{GeV}$$

$$M_{0^{++}} = 3.41(2)\sqrt{\sigma} = 1.65(3)\,\mathrm{GeV}$$

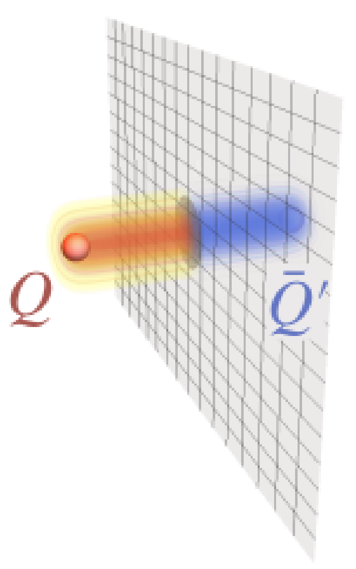
$$\sqrt{\sigma} = 485(6)\ \mathrm{MeV} = [0.407(5)\ \mathrm{fm}]^{-1}$$

A.Athenodorou, M.Teper, JHEP 2020,No.11

$$\mathcal{E}_{\mathrm{Cas}} = -C_0 \frac{2(N_c^2-1)m_{\mathrm{gt}}^2}{8\pi^2 R} \sum_{n=1}^{\infty} \frac{K_2(2n\, m_{\mathrm{gt}} R)}{n^2}$$

D. Karabali, V.P. Nair, Phys. Rev. D 98, 2018
M. Cougo-Pinto et al., Lett. in Math. Phys. 31, No. 4, 1994

Quarkiton - the boundary state of quark and mirror



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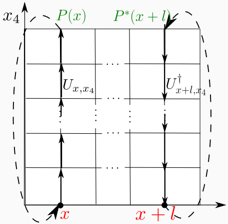
Quarkiton – the boundary state of quark and mirror

Polyakov loop as probe color charge



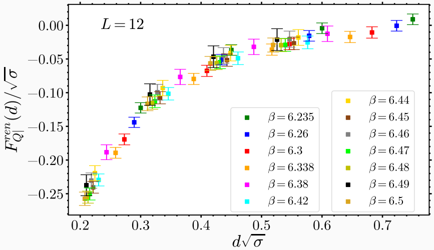
$$O = \mathrm{tr}[P\exp(i\int d^4 z j_\mu(z) A_\mu(z))] \qquad \rightarrow P(x) = \mathrm{tr}\left[\prod_{x_4=0}^{N_t-1} U_{x,x_4}\right]$$
$$j_\mu(z) = (0,0,0,1)\delta(z-x)$$

$$\langle P(x)P^*(x+l)\rangle \propto e^{-aN_t V(al)}(1+\mathcal{O}(e^{-aN_t\Delta E})) = e^{-L_T F_{q\bar{q}}(al)}$$

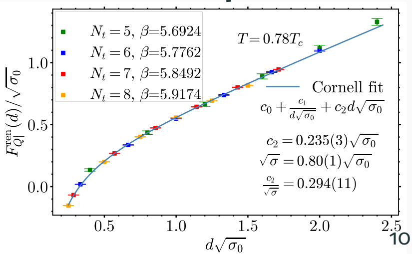
$$\lim_{al\to\infty}\langle P(x)P^*(x+l)\rangle = \langle P(x)\rangle\langle P^*(x+l)\rangle = |\langle P\rangle|^2$$

$$\boxed{\langle\, P\,\rangle_|(d) = \exp\{-aN_t F_{Q|}(d)\}} \qquad T = \frac{1}{aN_t}$$

$F_{Q|}(d)$ – free energy of heavy quark at zero temperature



Free energy of heavy quark at finite temperature



Conclusions

- The new mass scale in the SU(3)-gluodynamics with Casimir boundary conditions is found. It is interpeted as non-pertubative colorless state of gluon and its image in chromometallic mirror. The new particle is called glueton and has mass $m_{\mathrm{gt}} = 1.0(1)\sqrt{\sigma} = 0.49(5)\ \mathrm{GeV}$, that is several times less than mass of ground-state 0^{++} glueball, $M_{0^{++}} = 3.405(21)\sqrt{\sigma} = 1.653(26)\ \mathrm{GeV}$ (Phys. Rev. D 108, 014515, 2023);
- The strong evidence in support of existence of analogues state for quarks (quarkiton) is raised – the Cornell potential excellently decribes the interaction of probe color charge and the mirror;
- The ratio of masses of glueton to 0^{++} glueball is the same as ratio of quarkiton's string tension to the string tension in the absence of the plate at given temperature and equal to 0.294(11)