OSAKA INSTITUTE OF TECHNOLOGY

科研費 KAKENHI JP24K07030

Four-dimensional QCD equation of state at finite chemical potentials

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AM, G. Pihan, B. Schenke, C. Shen, arXiv:2406.11610 [nucl-th]

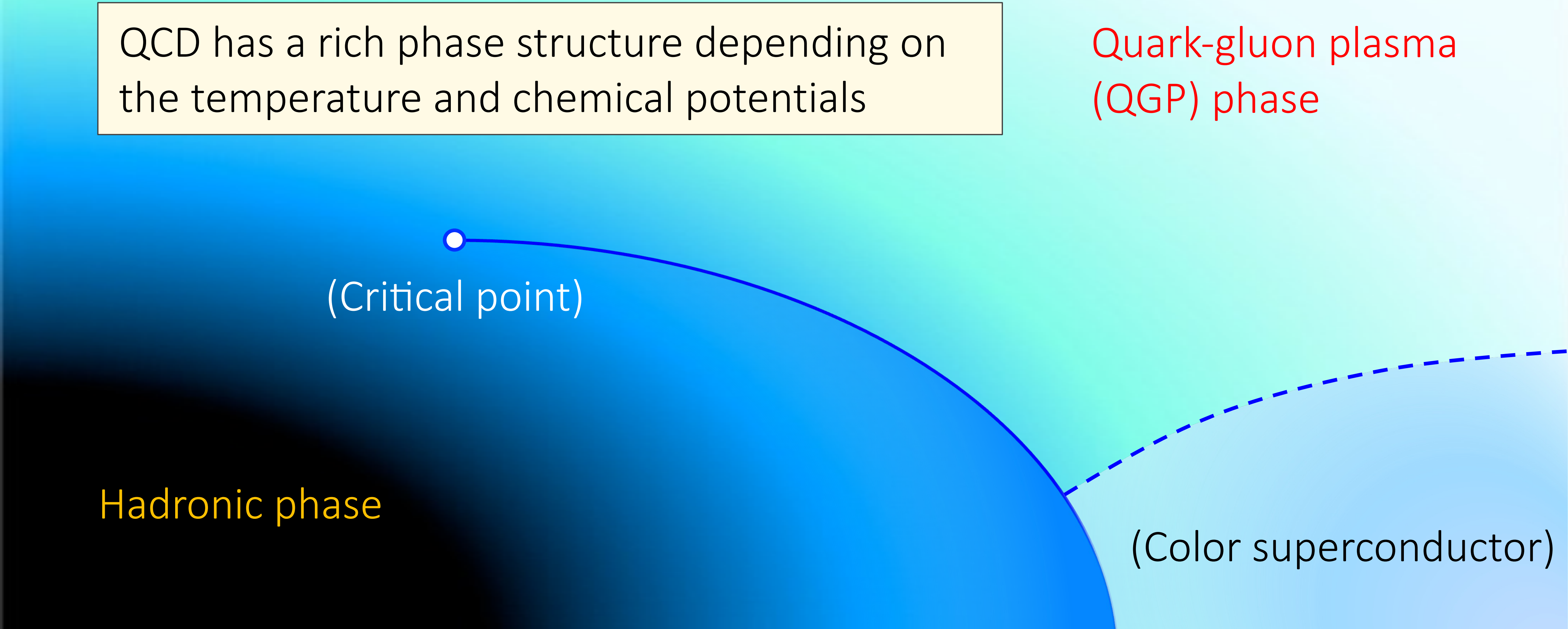
XVIth Quark Confinement and the Hadron Spectrum

21st August 2024, Cairns, Australia

Introduction

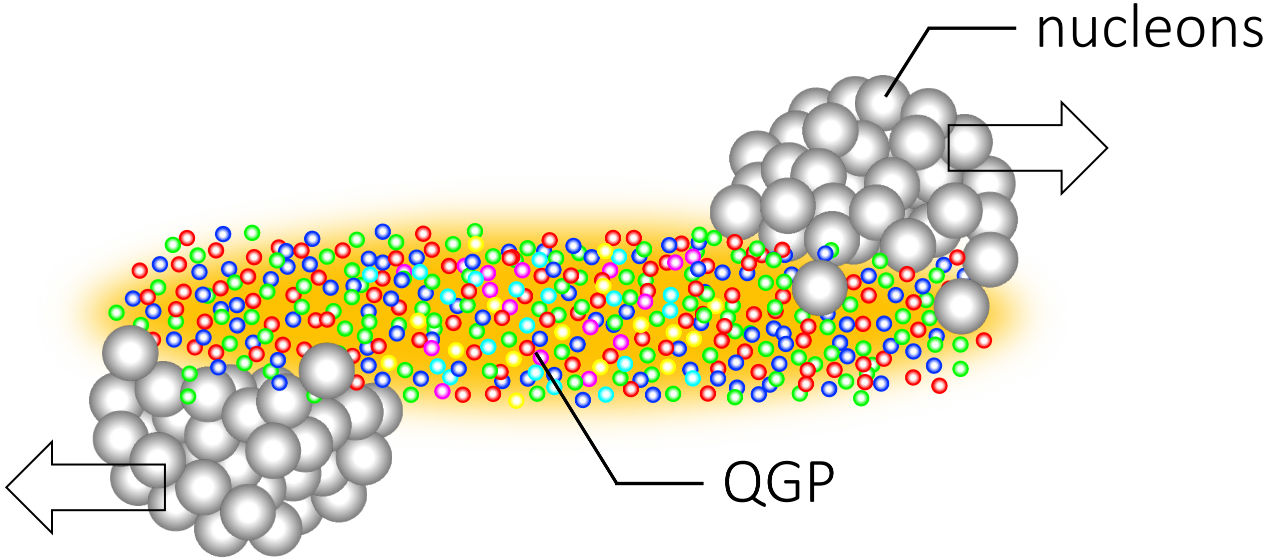
- Exploring the QCD Phase diagram

QCD has a rich phase structure depending on the temperature and chemical potentials



Introduction

- How to make the quark-gluon plasma (QGP)



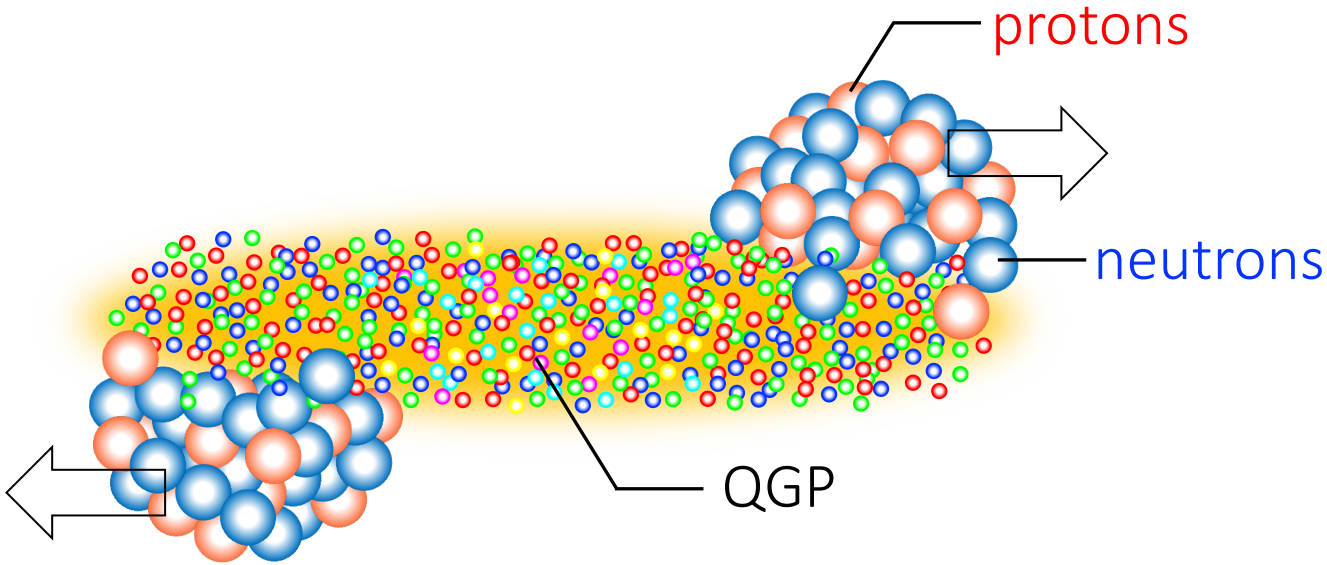
The QGP can be created in nuclear collisions at relativistic energies

BNL Relativistic Heavy Ion Collider (RHIC)

CERN Large Hadron Collider (LHC)

Introduction

- A more precise view of nuclear collisions



Protons and neutrons should be distinguished for precision analyses

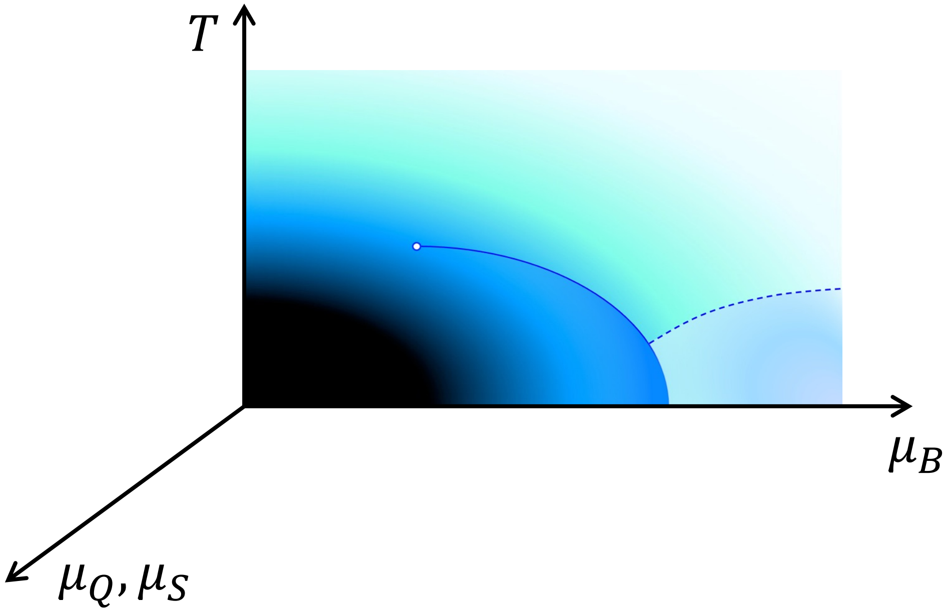
BNL Relativistic Heavy Ion Collider (RHIC)
CERN Large Hadron Collider (LHC)

Nuclear collisions

- Conserved charges

The QGP in nuclear collisions are made of light quarks (u, d, s) ($T \sim 200$ MeV)

Baryon (B) | Electric charge (Q) | Strangeness (S) are conserved

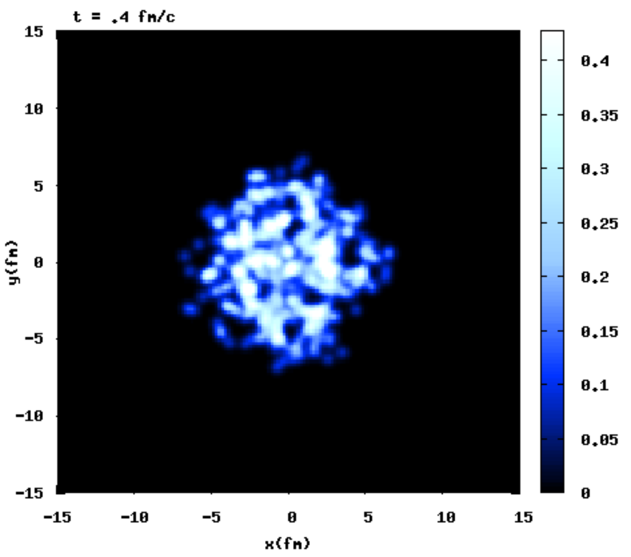


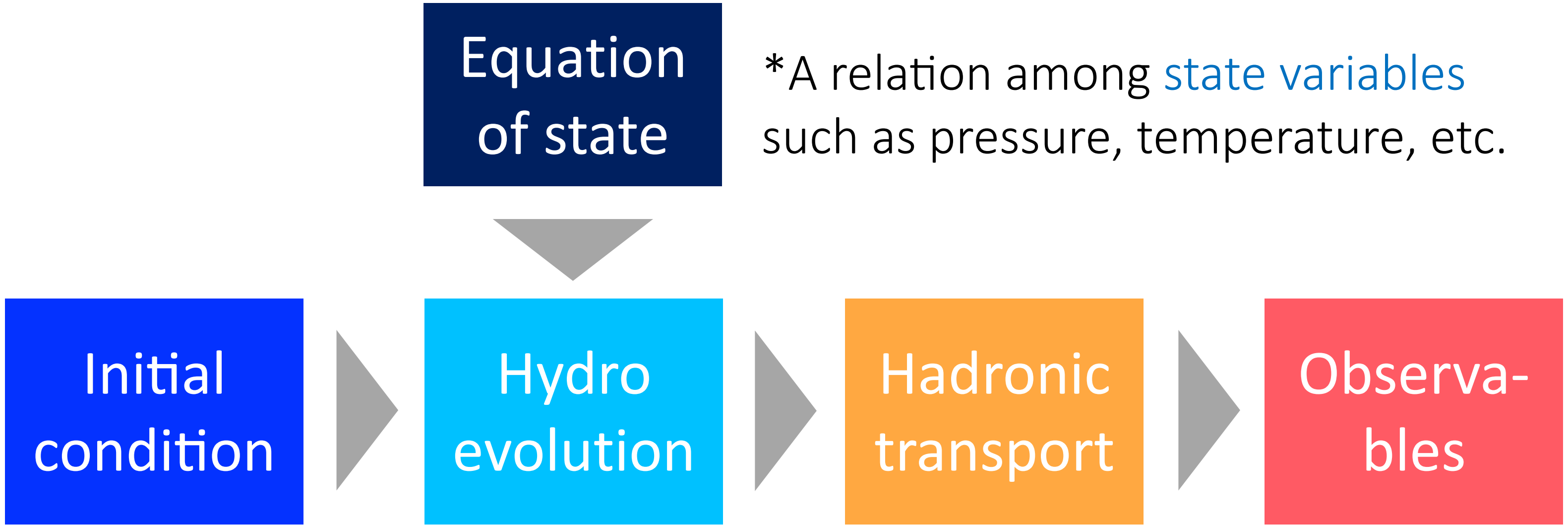
The QCD phase diagram has to be extended to 4 dimensions

- T: Temeperature
- μ_B: Baryon chemical potential
- μ_Q: Charge chemical potential
- μ_S: Strangeness chemical potential

Nuclear collisions

- Relativistic hydrodynamic model



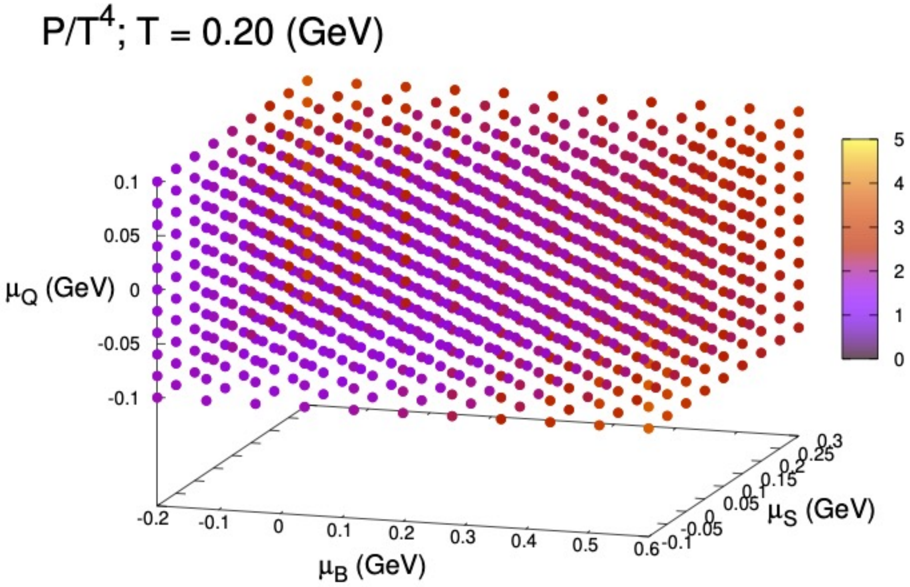


*A relation among state variables such as pressure, temperature, etc.

We construct a 4-dimensional QCD equation of state at finite chemical potentials for nuclear collisions

NEOS-4D

- A Lattice QCD-based equation of state model



- It has B, Q, S charges without constraint, *i.e.*, it is fully 4-dimensional

- Generalization of NEOS BQS, which is tuned to $n_Q = 0.4\, n_B, n_S = 0$ for heavy nuclei (^{197}Au, ^{208}Pb, etc.) Phys. Rev. C **100**, 024907 (2019)

- Protons and neutrons can be distinguished

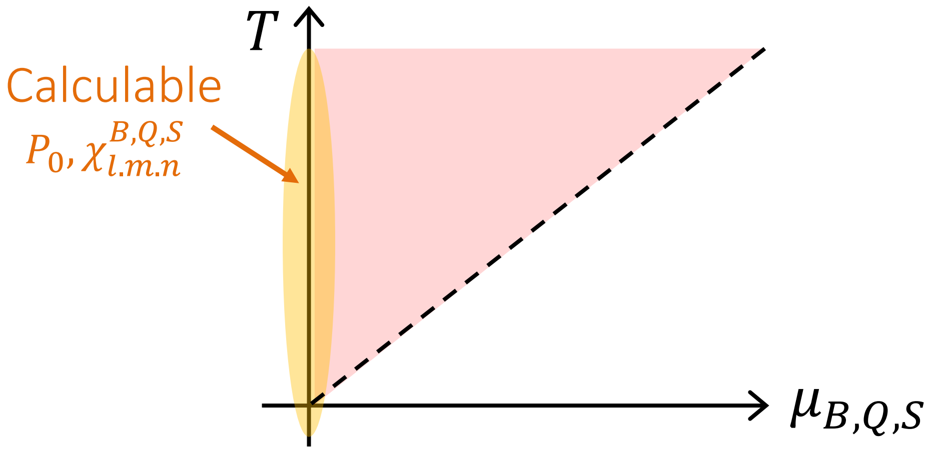
Another approach to the 4D EoS: C. Plumberg et al., arXiv:2405.09648 [nucl-th]

Construction

- QGP phase: Taylor expansion method of Lattice QCD

$$\frac{P_{\rm lat}}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi^{B,Q,S}_{l.m.n}}{l!\,m!\,n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

HotQCD Collaboration, PRD 86, 034509 (2012); PRD 90, 094503 (2014); PRD 92, 074043 (2015); PRD 95, 054504 (2017)



Pro: Ab initio calculation

Con: not reliable when $\frac{\mu}{T}$ is too large

- Susceptibilities up to the 4th order from Lattice QCD
- $\chi^B_6, \chi^{B,Q}_{5,1}, \chi^{B,S}_{5,1}$ parametrized as required by thermodynamic conditions

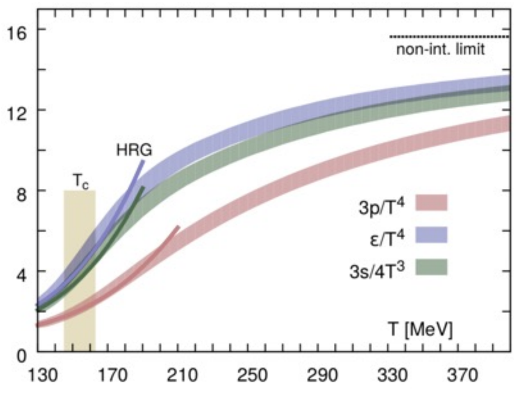
Construction

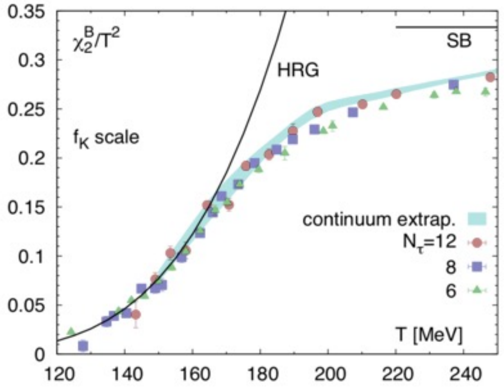
- Hadronic phase: Hadron resonance gas model

$$P_{\mathrm{had}} = \pm T \sum_i \frac{g_i d^3p}{(2\pi)^3} \ln[1 \pm e^{-(E_i-\mu_i)/T}]$$

Particle Data Group: PRD 98, 030001 (2018)

- Hadrons and resonances with u, d, s components with the mass below 2 GeV are used





Pro: Consistent with Lattice QCD

Con: Describes only the hadronic phase

Construction

- The crossover-type EoS is obtained by smoothly connect the two EoS

$$P = \frac{1}{2}\left(1 - \tanh\frac{T - T_c}{\Delta T_c}\right) P_{\mathrm{had}} + \frac{1}{2}\left(1 + \tanh\frac{T - T_c}{\Delta T_c}\right) P_{\mathrm{lat}}$$

Hadron resonance gas model

$$P_{\mathrm{had}} = \pm T \sum_i \frac{g_i d^3p}{(2\pi)^3} \ln[1 \pm e^{-(E_i - \mu_i)/T}]$$

Lattice QCD with Taylor expansion

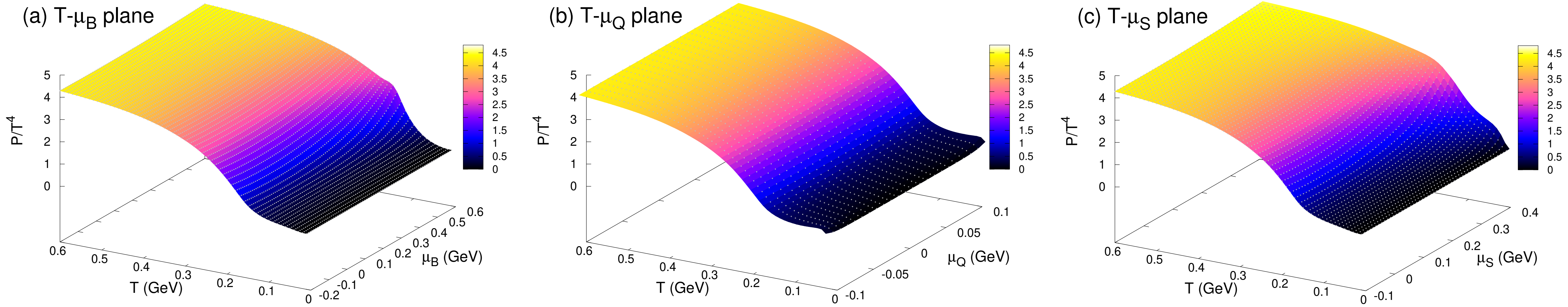
$$\frac{P_{\mathrm{lat}}}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi^{B,Q,S}_{l.m.n}}{l!\,m!\,n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

$T_c(\mu_B) = 0.16 - 0.4(0.139\mu_B^2 + 0.053\mu_B^4)$ GeV, $\Delta T_c = 0.1 T_c(0)$ — J. Cleymans et. al., PRC 73, 034905 (2006)

Numerical results

Results

- Pressure



The dimensionless pressure on 2D slices of the temperature and chemical potentials in the 4D phase space

Phase diagram

- Regions explored in nuclear collisions

s/n_B is constant when entropy and net baryon number are conserved

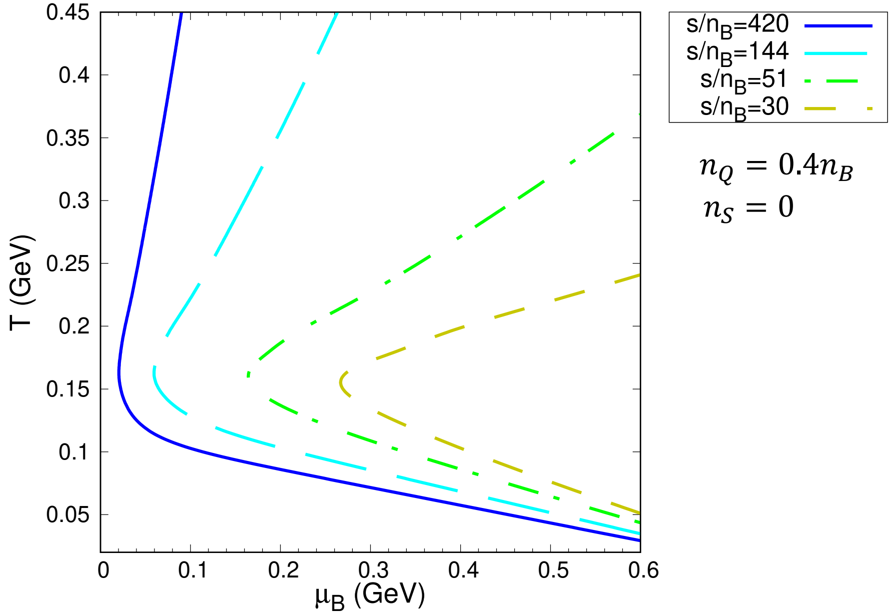
$s/n_B = 420$ → $\sqrt{s_{NN}} = 200$ GeV

$s/n_B = 144$ → $\sqrt{s_{NN}} = 62.4$ GeV

$s/n_B = 51$ → $\sqrt{s_{NN}} = 19.6$ GeV

$s/n_B = 30$ → $\sqrt{s_{NN}} = 14.5$ GeV

J. Gunther et. al., Nucl. Phys. A 967, 720 (2017)



$n_Q = 0.4 n_B$

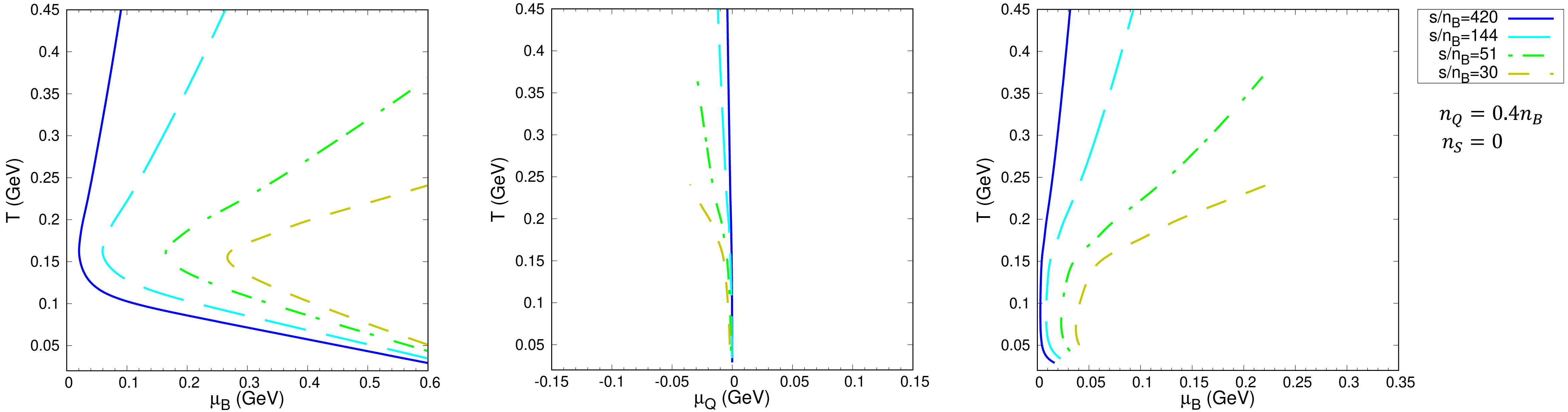
$n_S = 0$

The QGP phase has straight lines because $s/n_B \approx T/\mu_B$

Larger μ_B is required in hadronic phase because protons are heavy

Results

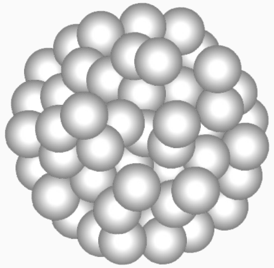
- Trajectories in the phase diagram



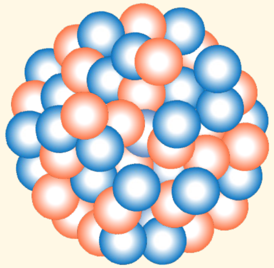
The estimated region explored in nuclear collisions is narrow in μ_Q with the "nucleon" approximation of $n_Q/n_B = 0.4$

Nuclear collisions

- The charge-to-baryon ratio in nuclear collisions



$\frac{n_Q}{n_B} \approx 0.4$ on average for heavy nuclei (^{197}Au, ^{208}Pb) and $n_S = 0$

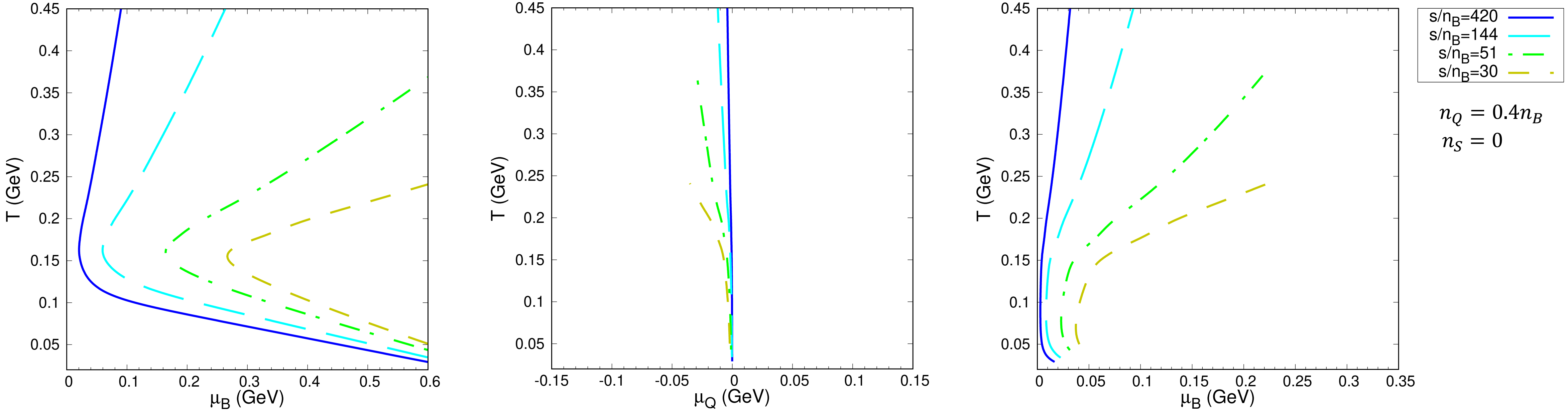


$\frac{n_Q}{n_B} = 1$ in proton-rich regions and $\frac{n_Q}{n_B} = 0$ in neutron-rich regions

⚠ Additional dynamics (e.g. fluctuation, diffusion) can lead to $\frac{n_Q}{n_B} > 1$ or $\frac{n_Q}{n_B} < 0$

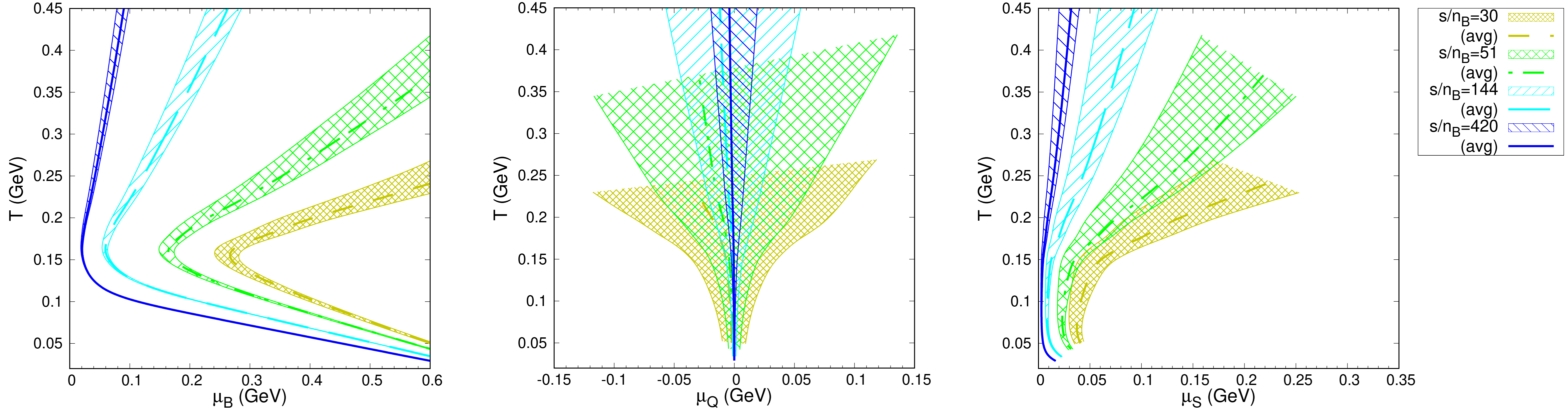
Results

■ Trajectories in the phase diagram



Results

- Trajectories in the phase diagram



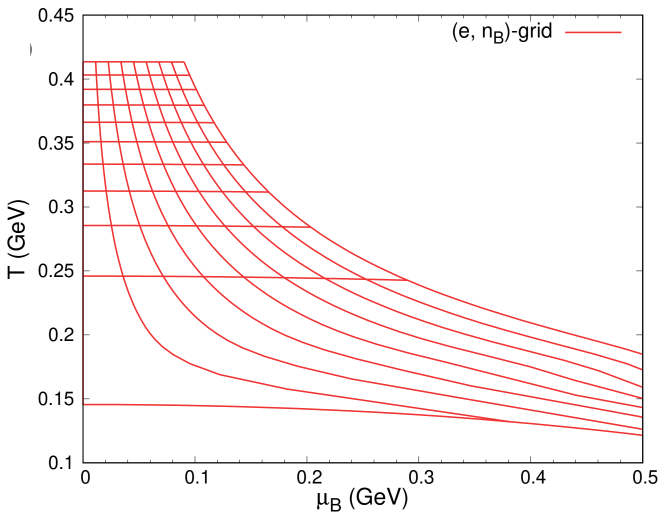
Bands denote the regions between $n_Q/n_B = 1$ and 0; Wide regions of the phase diagram will be explored in colliders

Application to hydrodynamic model

- Hydrodynamic model require $P, T, \mu_B, \mu_Q, \mu_S$ as functions of e, n_B, n_Q, n_S

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_B^\mu = 0, \quad \partial_\mu N_Q^\mu = 0, \quad \partial_\mu N_S^\mu = 0$$

⇨ One often prepares pre-calculated tables of the EoS for efficient numerical simulations



However, a grid with equal spacing in e, n_B, n_Q, n_S results in a warped grid in T, μ_B, μ_Q, μ_S

Covering it leads to a huge redundancy in the 4D case, making hydro simulations difficult

Application

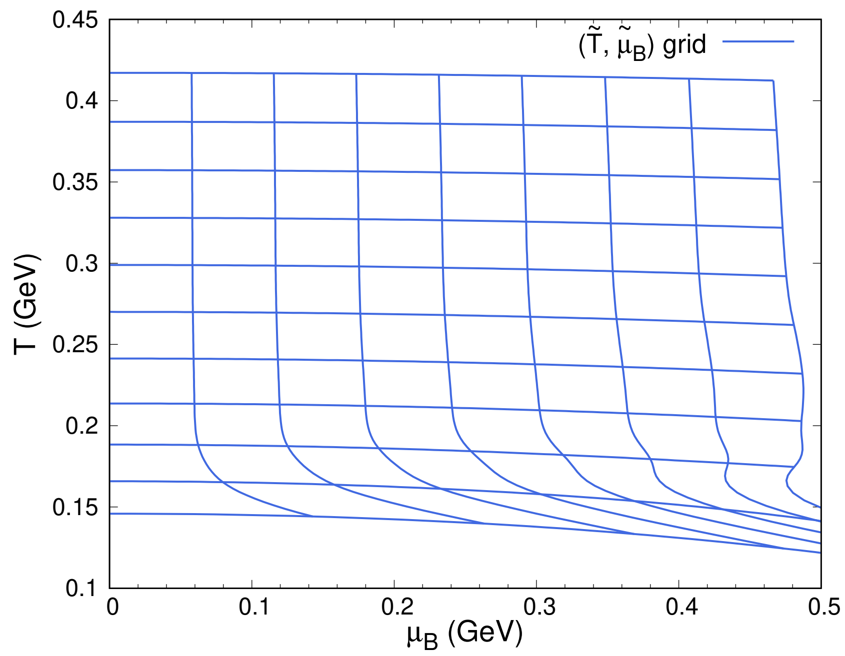
- We introduce $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$, defined as the temperature and chemical potentials of a parton gas with the given e, n_B, n_Q, n_S, for tabulation

$$\tilde{T}(e,n_B,n_Q,n_S) = \left(\frac{12}{19\pi^2}e\right)^{1/4}$$

$$\tilde{\mu}_B(e,n_B,n_Q,n_S) = \frac{5n_B - n_Q + 2n_S}{\tilde{T}^2}$$

$$\tilde{\mu}_Q(e,n_B,n_Q,n_S) = \frac{-n_B + 2n_Q - n_S}{\tilde{T}^2}$$

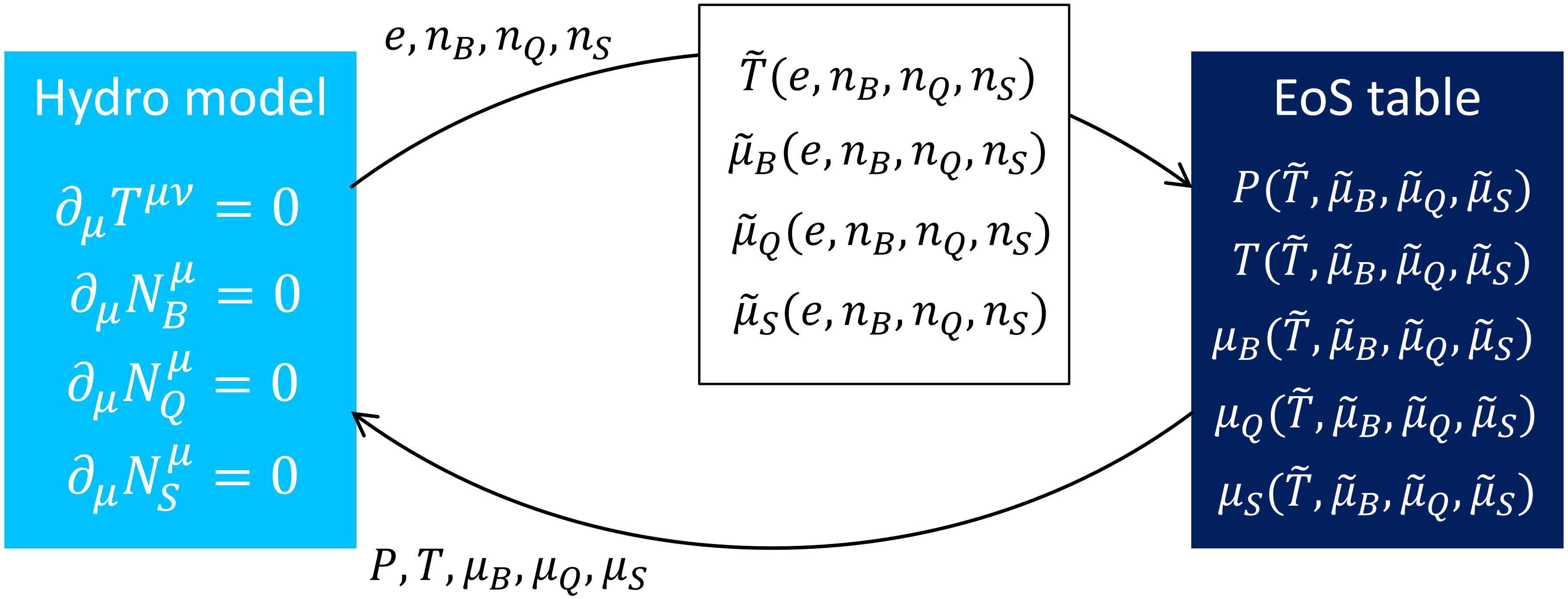
$$\tilde{\mu}_S(e,n_B,n_Q,n_S) = \frac{2n_B - n_Q + 2n_S}{\tilde{T}^2}$$



A grids with equal spacing in $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$ is relatively straight in T, μ_B, μ_Q, μ_S

Application

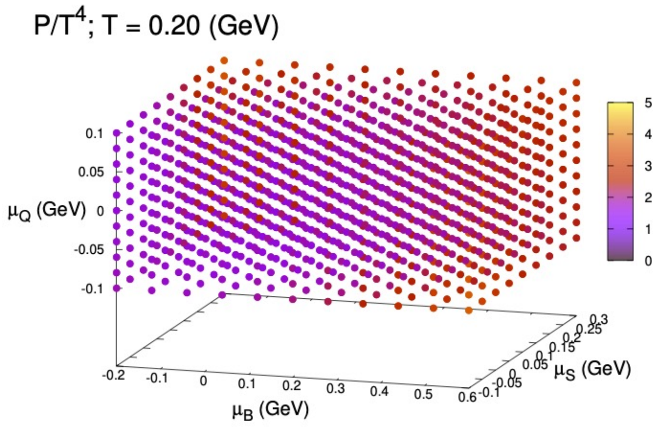
- Schematic of EoS implementation to hydrodynamic model of nuclear collisions



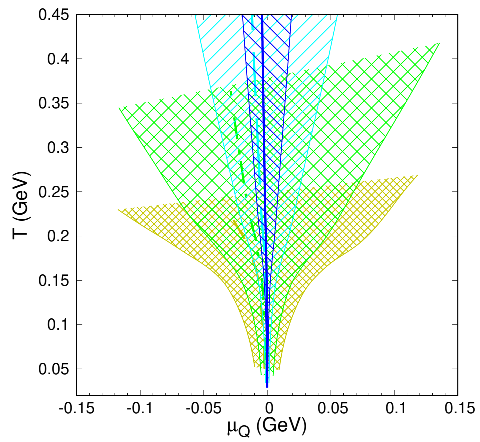
Calculations become efficient; see our recent analyses of isobar collisions for a successful application G. Pihan, AM, B. Schenke, C. Shen, arXiv:2405.19439 [nucl-th]

Summary and outlook

- We have constructed a crossover-type QCD EoS model, NEOS-4D, with net baryon (B), electric charge (Q) and strangeness (S)
  - Lattice QCD results from Taylor expansion method is utilized
  - It is smoothly matched to the hadron resonance gas model at lower temperatures

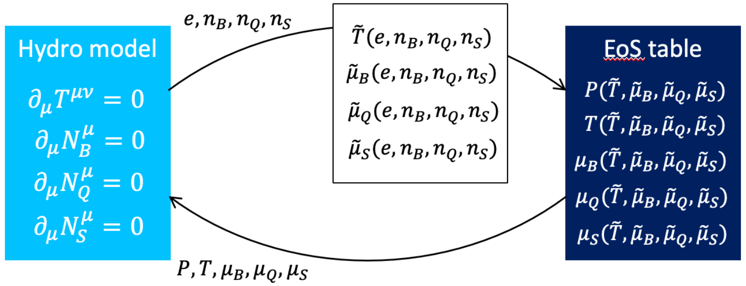


  - One can distinguish protons and neutrons; wide ranges in the T-μ_B-μ_Q-μ_S space are explored



Summary and outlook

- An efficient method of numerical implementation of a 4D EoS to the hydrodynamic model is developed using $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$ variables



Outlook

- Introduction of higher order susceptibilities from Lattice QCD
- Application to the hydrodynamic analyses of nuclear collisions at beam energy scan energies and of different nuclear species
- Estimation of the effects of fluctuations and diffusions

Summary and outlook

- The results of our equation of state model NEOS-4D are publicly available:

https://sites.google.com/view/qcdneos4d/home

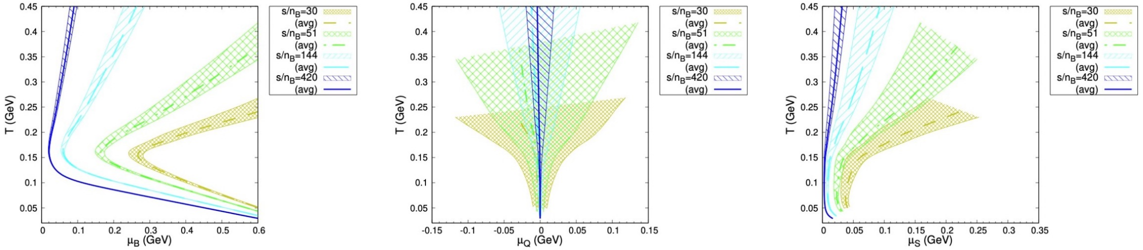
QCD equation of state

QCD equation of state

NEOS-4D

Reference: A. Monnai, G. Pihan, B. Schenke, C. Shen, arXiv:2406.11610 [nucl-th]

NEOS-4D is a four-dimensional extension of the QCD equation of state (EoS) model model NEOS constructed to cover a wide range in the QCD phase diagram with the chemical potentials of net baryon, electric charge, and strangeness that could be explored in relativistic nuclear collisions.



Thank you for listening!